CHAPTER 6

Accounting and the Time Value of Money
LECTURE OUTLINE

This chapter can be covered in two to three class sessions. Most students have had previous exposure to single sum problems and ordinary annuities, but annuities due and deferred annuities will be new material for most students. The first class session can be used for lecture on the chapter concepts, and for discussing Illustration 6-5.

Illustration 6-5 can be distributed to students as a self-contained 6-page handout. It uses 10 sample problems to demonstrate a 5-step solution method that can be used to solve any of the problems discussed in the chapter.

Some students with prior background in math or finance courses may prefer to use exponential formulas rather than interest tables to find interest factors. Other students with sophisticated calculators may prefer to "let the calculator do the work." Remind students that whether they use interest tables, exponential formulas, or internal calculator routines, they cannot solve problems correctly unless they can correctly identify the type of problem, the number of periods, and the interest rate involved. Students often have no difficulty with problems that are worded. "At 6%, what is the present value of an annuity due of 20 payments of $10,000 each?" but they may not know how to proceed if the same problem is worded: "What amount must be deposited now in an account paying 12% if it is desired to make 20 semiannual withdrawals of $10,000 each beginning today?" Emphasize to students the importance of properly setting up the problem.

The second and third class sessions can be used for determining solutions to more complex problems, including deferred annuities, bond valuation and other accounting applications. Some of the journal entries for the accounting applications can be discussed briefly.

The following lecture outline is appropriate for this chapter.

A. Introduction.

1. Discuss the importance of the time value of money.

2. Describe accounting applications of time value concepts: bonds, pensions, leases, long-term notes.

3. Describe personal applications of time value concepts: purchasing a home, planning for retirement, evaluating alternative investments.
B. Nature of Interest.

1. **Interest** is payment for the use of money. It is the excess cash received or repaid over and above the **principal** (amount lent or borrowed).

2. Interest rates are generally stated on an **annual** basis unless indicated otherwise.

3. Choosing an appropriate interest rate:
   
   a. is not always obvious.
   
   b. three components of interest:

      (1) pure rate of interest (2%–4%).
      
      (2) credit risk rate of interest (0%–5%).
      
      (3) expected inflation rate of interest (0%–?%).

C. Simple Interest.

Illustration 6-1 can be used to distinguish between simple interest and compound interest.

1. **Simple interest** is computed on the amount of the principal only.

2. Simple interest = $p \times i \times n$ where

   $p = \text{principal}$
   
   $i = \text{rate of interest for a single period}$
   
   $n = \text{number of periods}$
D. Compound Interest.

1. **Compound interest** is computed on the principal and on any interest earned that has not been paid or withdrawn.

2. Discuss the power of time and compounding. (e.g., note 6 on text page 282 indicates that at 6% compounded annually, $24 grows to $13 billion in 345 years. At 6% simple interest, $24 would grow to only $520 in 345 years.) $520 = $24 + ($24 x .06 x 345).

3. The term **period** should be used instead of **years**.

   a. Interest may be compounded more than once a year:

<table>
<thead>
<tr>
<th>If interest is compounded</th>
<th>Number of compounding periods per year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annually</td>
<td>1</td>
</tr>
<tr>
<td>Semiannually</td>
<td>2</td>
</tr>
<tr>
<td>Quarterly</td>
<td>4</td>
</tr>
<tr>
<td>Monthly</td>
<td>12</td>
</tr>
</tbody>
</table>

   b. Adjustment when interest is compounded more than once a year.

   (1) Compute the compounding period **interest rate**: divide the annual interest rate by the number of compounding periods per year.

   (2) Compute the total number of compounding **periods**: multiply the number of years by the number of compounding periods per year.
E. Terminology Used in Compound Interest Problems.

1. Describe the four **fundamental variables** in compound interest problems:

   ![TEACHING TIP]
   
   **Illustration 6-2** depicts a time diagram that identifies the four variables that are fundamental to all compound interest problems. **Illustration 6-3** can be used to show students how the four fundamental variables relate to the time value of money concepts.

   a. Rate of interest.

   b. Number of time periods.

   c. Future amount.

   d. Present value.

2. Describe the difference between **single sum** and **annuity** problems.

   a. Single sum problems involve a single amount of money that either exists now or will in the future.

   b. Annuity problems involve a series of equal periodic payments or receipts called **rents**.

      (1) In an **ordinary annuity** the rents occur at the **end** of each period. The first rent will occur one period from now.

      (2) In an **annuity due** the rents occur at the **beginning** of each period. The first rent will occur now.

      (3) In a **deferred annuity** the rents occur in the **future**. In an ordinary annuity of n rents deferred for y periods the first rent will occur \((y + 1)\) periods from now. In an annuity due of n rents deferred for y periods the first rent will occur \(y\) periods from now.
F. Use of Compound Interest Tables.

1. The tables contain interest factors that simplify the computation of compound interest. Example: If $1,000 is deposited today at 9% compound interest, the balance in 3 years can be determined:

   a. By repetitive calculation—

   First year: $1,000 + ($1,000 x .09) = $1,090.
   Second year: $1,090 + ($1,090 x .09) = $1,188.
   Third year: $1,188 + ($1,188 x .09) = $1,295 (rounded).

   b. By use of exponential formulas—

   $1,000 x (1.09)^3 = $1,295 (rounded).

   c. By use of internal calculator routines or computer programs.

   d. By obtaining the 1.29503 interest factor from Table 6-1 for 3 periods at 9% and performing the appropriate computation—

   $1,000 x 1.29503 = $1,295.03.

2. Describe the five interest tables provided in the text:

   a. Table 6-1: Future Amount of 1.

   b. Table 6-2: Present Value of 1.

   c. Table 6-3: Future Amount of an Ordinary Annuity of 1.

   d. Table 6-4: Present Value of an Ordinary Annuity of 1.

   e. Table 6-5: Present Value of an Annuity Due of 1.
G. Steps in Solving Compound Interest Problems.

**TEACHING TIP**

Use *Illustration 6-4* to discuss the 5-step solution method that can be used to solve any compound interest problem.

1. Emphasize the importance of performing Steps 1 and 2 correctly. Whether students use interest tables, exponential formulas, or calculator routines, they cannot solve problems correctly unless they can correctly identify the type of problem, the number of periods, and the interest rate involved.

H. Specific Problems—Single Sum Problems.

**TEACHING TIP**

Problems 1, 2 and 3 in *Illustration 6-5* demonstrate single sum problem situations.

1. Formula for future amount:

\[
FV = PV(FVF_{n,i})
\]

2. Formula for present value:

\[
PV = FV(PVF_{n,i})
\]

3. Point out that the present value is always a smaller quantity than the future amount.

4. The process of finding the future amount is called **accumulation**. The process of finding the present value is called **discounting**.
5. The factors in Table 6-2 are the reciprocal of corresponding factors in Table 6-1. Therefore all single sum problems can be solved by using either Table 6-1 or 6-2. For example, if the future amount is known and the present value is to be solved for, the present value can be found:

   a. by multiplying the known future amount by the appropriate factor from Table 6-2, or

   b. by dividing the known future amount by the appropriate factor from Table 6-2.

I. Specific Problems—Ordinary Annuities.

   **TEACHING TIP**

   Problems 4, 5 and 8 in Illustration 6-5 demonstrate ordinary annuity problem situations.

1. Formula for future value of an ordinary annuity

   \[
   \text{Future value of ordinary annuity} = \text{periodic rent} \times \text{future value of ordinary annuity of factor for } n \text{ periods at } i\% \\
   FVOA = R(FVF - OA_{n,i})
   \]

2. Formula for present value of an ordinary annuity:

   \[
   \text{Present value of ordinary annuity} = \text{periodic rent} \times \text{present value of ordinary annuity factor for } n \text{ periods at } i\% \\
   PVOA = R(PVF - OA_{n,i})
   \]

3. Point out that the present value of an ordinary annuity is always smaller than the future amount of a similar annuity.

4. The factors in Tables 6-3 and 6-4 are **not** reciprocals of each other.
5. In annuity problems, the rents, interest payments, and number of periods must all be stated on the same basis. For example, if interest is compounded semiannually, then \( n = \) the number of semiannual rents paid or received, \( i = \) the annual interest rate divided by 2, and \( R = \) the amount of rent paid or received every 6 months.

6. Some confusion may arise in annuity problems because of two different meanings of the word "period."

   a. **For the purpose of looking up interest factors**, \( n \) equals the number of "periods" and is **always** equal to the number of rents.

   b. In the phrase "when computing the future amount of an ordinary annuity the number of compounding periods is one less than the number of rents," the term "periods" refers to "compounding periods" or "interest-bearing periods." This refers to the number of times interest is earned on the principal and any accumulated interest. This usage of the term "period" is useful for distinguishing between ordinary annuities and annuities due. This usage is intended to explain why the adjustment of factors from Table 6-3 is done the way it is when the problem involves the future amount of an annuity due.

J. Specific Problems—Annuities Due.

   **TEACHING TIP**

   Problems 6 and 7 in Illustration 6-5 demonstrate annuity due problem situations.

1. Formula for future value of annuity due:

   \[
   \text{Future value of annuity due} = \text{periodic rent} \times \text{future value of ordinary annuity factor for } n \ \text{periods at } i \% \times (1 + i)
   \]

   \[
   FVAD = R \times (FVF - OA_{n,i}) \times (1 + i)
   \]

   a. An interest table is not provided for the future amount of an annuity due.
b. Example: At 9%, what is the future amount of an annuity due of 7 payments of $3,000 each?

\[ \text{Future Amount} = \text{Periodic Rent} \times \text{Future Value of Annuity Due Factor} \times (1 + i) \]

\[ \text{Future Amount} = 3,000 \times 9.20044 \times 1.09 = 30,085.44 \]

2. Formula for present value of annuity due:

\[ \text{Present Value of Annuity Due} = \text{Periodic Rent} \times (PVF - AD_{n,i}) \]

3. Point out that:

a. The present value of an annuity due is always smaller than the future amount of a similar annuity due.

b. The future amount (present value) of an annuity due is always larger than the future amount (present value) of a similar ordinary annuity with the same interest rate and number of rents.

K. Specific Problems—Deferred Annuities.

**TEACHING TIP**

Problems 9 and 10 in Illustration 6-5 demonstrate deferred annuity problem situations.

1. A deferred annuity does not begin to produce rents until two or more periods have expired.

2. A deferred annuity problem can occur in either an ordinary annuity situation or an annuity due situation.

   a. In order to keep the presentation straightforward, only the ordinary annuity situation has been illustrated in the text and in Illustration 6-5.
b. The differences between the two situations are as follows:

<table>
<thead>
<tr>
<th></th>
<th>Ordinary Annuity of n Rents Deferred for y Periods</th>
<th>Annuity Due of n Rents Deferred for y Periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>First rent occurs</td>
<td>(y + 1) periods from now</td>
<td>y periods from now</td>
</tr>
<tr>
<td>Last rent occurs</td>
<td>(y + n) periods from now</td>
<td>(y + n – 1) periods from now</td>
</tr>
<tr>
<td>Future amount is measured as</td>
<td>immediately after the last rent</td>
<td>one period after the last rent</td>
</tr>
</tbody>
</table>

c. If a deferred annuity involves solving for a **present value**, the distinction between an ordinary annuity and an annuity due has no practical significance. (i.e., see Problem 10 in Illustration 6-5. This can be set up either as the present value of an **ordinary annuity** of 4 rents deferred 3 periods, as was done in Illustration 6-5, or it can be set up as the present value of an **annuity due** of 4 rents deferred 4 periods. If the latter is done, different combinations of factors will be used, but the same answer will be obtained.)

d. However, if a deferred annuity involves solving for a **future amount**, the distinction between an ordinary annuity and an annuity due is important. The following formula is required:

\[
FVAD - d = R \times (FVF - OA_{n,i}) \times (1 + i) \text{ where}
\]

\[
FVAD - d = \text{future amount of an annuity due of } n \text{ rents deferred for } y \text{ periods}
\]

\[
R = \text{periodic rents}
\]

\[
(FVF - OA_{n,i}) = \text{factor from Table 6-3 for } n \text{ periods at } i \%
\]

\[
(1 + i) = 1 \text{ plus the interest rate}
\]

To illustrate, suppose Problem 9 in Illustration 6-5 had been worded as follows:

As of the beginning of his first year in college, a student plans to deposit $1,000 in an 8% account at the end of his third, fourth, and fifth years in school. What will be the balance in the account one year after the last deposit?
The time diagram for this revised problem is:

```
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>1,000</th>
<th></th>
<th>1,000</th>
<th></th>
<th>1,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>
```

The revised problem involves solving for the future amount of an annuity due of 3 rents deferred for 3 periods. The solution is

\[
FVAD - d = R \times (FVF - OA_{3.8\%}) (1.08) = 1,000 \times 3.24640 \times 1.08 \\
= 3,506.11.
\]

To sum up: In solving for the future amount of a deferred annuity —

1. If the future amount is to be determined immediately after the last rent, the problem may be thought of as an ordinary annuity. The computation described in the text and in Illustration 6-5 is adequate.

2. If the future amount is to be determined one period after the last rent, the problem should be thought of as an annuity due. An adjustment must be made to accumulate interest for one more period.

L. Specific Problems—Bond Valuation Problems.

1. Discuss the distinction between the stated rate of interest and the market or effective yield rate of interest:

   a. The **stated rate** of interest is used to determine the periodic amount of cash interest paid.

   b. The **market or effective yield rate** of interest is used to value the bonds. This is the rate which is looked up in the present value tables.
2. The example in the text demonstrates valuation of bonds which pay interest annually.

**TEACHING TIP**

Illustration 6-6 provides an example of a bond valuation problem in which bond interest is paid semiannually.