CHAPTER 5

The Theory of Demand

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During the 1990s and early 2000s the tobacco industry became increasingly embroiled in litigation over the damages caused by cigarette smoking. Many states sued tobacco companies to recover health care costs related to smoking. Several tobacco companies agreed to pay billions of dollars to Minnesota, Florida, Mississippi, Texas, New York, and other states. The tobacco companies then faced a difficult question. How would they pay for these legal settlements? Their response was to raise cigarette prices repeatedly.

Why did cigarette producers believe that they could collect more revenues if they raised cigarette prices? And what information would they need to estimate the size of the increase in their revenues from an increase of, say, five cents per pack? As we saw in Chapter 2, firms can predict the effects of a price increase if they know the shape of the market demand curve. An article in the *The Wall Street Journal* summarizes some of the extensive research on the market demand curve for cigarettes.

“...The average price for a pack of cigarettes is about $2. Prices vary by state because of taxes. Analysts say that for every 10 percent price increase, sales volumes drop between 3.5 percent and 4.5 percent. They say that small price increases generally don’t cause most consumers to try to give up smoking, but that they smoke fewer cigarettes each day.”

Based on this information, we would conclude that the price elasticity of demand for cigarettes is approximately \[-0.35\] to \[-0.45\]. Thus the demand for cigarettes is relatively price inelastic. As we learned in Chapter 2, when the demand is relatively inelastic, a small price increase will lead to an increase in sales revenues. In the cigarette market, if the price rises by 10 percent, the sales volumes will fall by about 4 percent. This means that with a 10 percent price increase, the revenues from cigarette sales would increase by about 6 percent. This explains why cigarette producers believed sales revenues would rise if they increased cigarette prices.

We begin with this chapter by using graphical and algebraic approaches to study how a consumer’s demand for a good (such as cigarettes) depends on the prices of all goods and income. We will learn how to use preferences and budget lines to find a consumer’s demand curve. We will then further examine how a change in the price of a good affects the consumer through an income effect and a substitution effect. The income and substitution effects will help us understand three measures used to assess how much better off or worse off a consumer is when a price changes: consumer surplus, compensating variation, and equivalent variation.

After we understand how to find an individual’s demand curve, we ask: Where do market demand curves come from? For many goods the market demand curve is obtained by adding up the demand curves of all of the consumers in the market. However, in some cases a consumer’s demand for a good depends on the number of other people purchasing that good. In that case, we say that there are network externalities that must be taken into account in determining the market demand.

We end this chapter with an application describing how a consumer allocates his time to work and leisure. You might normally expect that a consumer will be willing to work more hours if he is offered a higher wage rate. However, this may not always be true. We will use income and substitution effects to illustrate why a consumer may actually work less when the wage rate rises.

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where demand curves come from? In Chapter 4, we showed how to determine a consumer's optimal basket. Given the consumer's preferences, income, and the prices of all goods, we could ask how much ice cream a consumer will buy each month if the price of a gallon of ice cream is $5. This will be a point on the consumer's demand curve for ice cream. We can find more points on his demand curve by repeating the exercise for different prices of ice cream, asking what his monthly consumption of ice cream will be if the price is $4, $3, or $2 per gallon. Let's see how to do this, using a simplified setting in which our consumer buys only two goods, food and clothing.

THE EFFECTS OF A CHANGE IN PRICE

What happens to the consumer's choice of food when the price of food changes, while the price of clothing and the amount of income remain constant? We have two ways to answer this question, one using the optimal choice diagram in Figure 5.1(a), and the second using the demand curve in Figure 5.1(b). Let's first look at the optimal choice diagram. The graph in Figure 5.1(a) measures the quantity of food consumed (x) on the horizontal axis and the quantity of clothing (y) on the vertical axis. Suppose the consumer's weekly income is $40 and the price of clothing is $4 per unit.

Consider the consumer's choices of food and clothing for three different prices of food. First, suppose the price of food is $4. The budget line that the consumer faces when $P_x = 4$, $P_y = 4$, and $I = 40$ is labeled $BL_1$ in the figure. The slope of her budget line is $-P_x/P_y = -4/4 = -1$. Her optimal basket is $A$, indicating that her weekly consumption of food is 2 units and her optimal consumption of clothing is 8 units.

What happens when the price of food falls to $2. As we learned in Chapter 4, when the price of food falls, the budget line rotates out to $BL_2$. The vertical intercept is the same because income and the price of clothing are unchanged. However, the horizontal intercept of the budget line moves to the right as the price of food falls. The slope of the budget line $BL_2$ is $-P_x/P_y = -2/4 = -1/2$. Her optimal basket is $B$, with a weekly consumption of 10 units of food and 5 units of clothing.

Finally, suppose the price of food falls to $1. The budget line rotates to $BL_3$ in the figure. The slope of the budget line $BL_3$ is $-P_x/P_y = -1/4$. The consumer's optimal basket is $C$, with a weekly consumption of 16 units of food and 6 units of clothing.

One way to describe how changes in the price of food affect the consumer's purchases of both goods is to draw a curve connecting all of the baskets that are optimal as the price of food changes (holding the price of clothing and income constant). This curve is called the price consumption curve.2 Note that the optimal baskets $A$, $B$, and $C$ all lie on the price consumption curve in Figure 5.1(a).

Observe that the consumer is better off as the price of food falls. When the price of food is $4 (and she chooses basket $A$), she reaches the indifference curve $U_1$. When the price of food is $2 (and she chooses basket $B$), her utility rises to $U_2$. If the price of food falls to $1, her utility rises even further to $U_3$.

2In some textbooks the price consumption curve is called the "price expansion path."
5.1 Optimal Choice and Demand

The consumer has a weekly income of $40. The price of clothing $p_y$ is $4$ per unit. (a) When the price of food is $4$, the budget line is $BL_1$. The slope of $BL_1$ is $-2 \left( \frac{p_x}{p_y} \right) = -4/4 = -1$. $BL_2$ and $BL_3$ are the budget lines when the price of food is $2$ and $1$, respectively. The optimal baskets are $A$, $B$, and $C$. The curve connecting the optimal baskets is called the price consumption curve. (b) We can use the optimal choice diagram (a) to draw the demand curve for food. Note that the consumer buys more food as its price falls, so the demand curve for food is downward sloping.
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**Changing Price: Moving Along a Demand Curve**

We can use the optimal choice diagram of Figure 5.1(a) to trace out the demand schedule for food in Figure 5.1(b). In Figure 5.1(b) the *price* of food appears on the vertical axis, and the *quantity* of food on the horizontal axis.

Let’s see how the two graphs are related to each other. When the price of food is $4, the consumer chooses basket *A* in Figure 5.1(a), containing 2 units of food. This tells us that point *A* lies on her demand curve for food in Figure 5.1(b). Similarly, at basket *B* in Figure 5.1(a), the consumer purchases 10 units of food when the price of food is $2. Therefore, point *B* must lie on her demand curve in Figure 5.1(b). Finally, if the price of food falls to $1, the consumer buys 16 units of food, as basket *C* in Figure 5.1(a) indicates. Therefore point *C* must also be on the demand curve. In sum, a decrease in the price of food leads the consumer to move down and to the right along her demand curve for food.

**The Demand Curve Is Also a “Willingness to Pay” Curve**

As you study economics, you will sometimes find it useful to think of a demand curve as a curve that represents a consumer’s “willingness to pay” for a good. To see why this is true, let’s ask how much the consumer would be willing to pay for another unit of food when she is currently at the optimal basket *A* (purchasing 2 units of food) in Figure 5.1(a). Her answer is that she would be willing to pay $4 for another unit of food. Why? At basket *A* her marginal rate of substitution of food for clothing is 1.3 Thus, at basket *A* one more unit of food is worth the same amount to her as one more unit of clothing. Since the price of clothing is $4, the value of an additional unit of food will also be $4. This reasoning helps us to understand why point *A* on the demand curve in Figure 5.1(b) is located at a price of $4. When the consumer is purchasing 2 units of food, the value of another unit of food to her (that is, her “willingness to pay” for another unit of food) is $4.

Note that her $\text{MRS}_{xy}$ falls to 1/2 at basket *B*, and falls even further to 1/4 at basket *C*. The value of additional unit of food is therefore $2 at *B* (when she consumes 10 units of food) and only $1 at basket *C* (when she consumes 16 units of food). In other words, her willingness to pay for an additional unit of food falls as she buys more and more food.

**THE EFFECTS OF A CHANGE IN INCOME**

What happens to the consumer’s choices of food and clothing as *income* changes? Let’s look at the optimal choice diagram in Figure 5.2(a). The graph measures the quantity of food consumed (*x*) on the horizontal axis and the quantity of clothing (*y*) on the vertical axis. Suppose the price of food is $P_x = $2 and the price of clothing is $P_y = $4 per unit. Let’s hold the prices of food and clothing constant. With the prices given, the slope of her budget line is \(-P_x/P_y = -1/2\).

The figure illustrates the consumer’s budget lines and optimal choices of food and clothing for three different levels of income. In Chapter 4 we saw that an increase in income results in an outward, parallel shift of the budget line. Initially,

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3At *A* the indifference curve $U_1$ and the budget line $BL_1$ are tangent to one another, so their slopes are equal. The slope of the budget line is $-P_x/P_y = -1$. Recall that the $\text{MRS}_{xy} = \text{MRS}_{xy}$ at *A* is the negative of the slope of the indifference curve (and the budget line) at that basket. Therefore, $\text{MRS}_{xy} = 1$. 

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5.1 Optimal Choice and Demand

when the consumer’s weekly income is $I_1 = 40, her budget line is $BL_1$. She chooses basket $A$, consuming 10 units of food and 5 units of clothing per week. As her income rises to $I_2 = 68, the budget line shifts out to $BL_2$. She then chooses basket $B$, with a weekly consumption of 18 units of food and 8 units of clothing. If her income increases $I_3 = 92, she faces budget line $BL_3$. Her optimal basket is $C$, with 24 units of food and 11 units of clothing.

One way we can describe how changes in income affect the consumer’s purchases is by drawing a curve that connects all the baskets that are optimal as income changes (keeping prices constant). This curve is called the income consumption curve. Note that the optimal baskets $A, B, and C$ all lie on the income consumption curve in Figure 5.2(a).

5Some textbooks call the income consumption curve the income expansion path.
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Changing Income: Shifting a Demand Curve

We can also use a second way to describe how changes in income affect the consumer’s purchases. In Figure 5.2(a) the consumer purchases more of both goods as her income rises. In other words, an increase in income results in a rightward shift in her demand schedule for each good. In Figure 5.2(b) we can see how a

FIGURE 5.2 The Effects of Changes in Income on Consumption
The consumer buys food at $P_x = $2 per unit and clothing at $P_y = $4 per unit. Both prices are held constant as income varies. (a) The budget lines reflect three different levels of income. The slope of all budget lines is \(-\frac{P_x}{P_y} = -\frac{1}{2}\). $BL_1$ is the budget line when the weekly income is $40. $BL_2$ and $BL_3$ are the budget lines when income is $68 and $92, respectively. As income changes, we can draw a curve connecting the baskets that are optimal (A, B, and C). This curve is called the income consumption curve. (b) The consumer’s demand curve for food shifts out as income rises.
change in income affects her demand curve for food. The price of food appears on the vertical axis, and the quantity of food on the horizontal axis. When the consumer's weekly income is $40 and the price of food is $2, she purchases 10 units of food each week. Thus, point $A'$ must be on her demand curve for food (labeled $D_1$ in Figure 5.2(b)) when her income is $40.

If her income rises to $68, she will purchase 18 units of food. Therefore, her demand for food shifts out to $D_2$. This demand curve must go through point $B'$, because she buys 18 units of food when the price of food is $2. Finally, if her income rises to $92, her demand for food shifts out to $D_3$, which must go through point $C'$ because she buys 24 units of food when the price of food remains at $2.

Using a similar approach, you can also show how the demand curves for clothing shift as income changes. You can do this exercise on your own (see Problem 1 at the end of this chapter).

**Engel Curves**

We have a third way of seeing how a consumer’s choice of a particular good varies with income: We can draw a graph relating the amount of the good consumed to the level of income. We call this graph an Engel curve. We can use the information an income consumption curve contains to construct an Engel curve. In Figure 5.3 we draw the Engel curve relating the amount of food consumed to the consumer’s income. Here the amount of food (x) is on the horizontal axis and the level of income (I) is on the vertical axis. Point $A''$ on the Engel curve shows that the consumer buys 10 units of food when the weekly income is $40. Point $B''$ indicates that she buys 18 units of food when her income is $68. If her weekly income rises to $92, she will buy 24 units of food (point $C''$). Note that we draw the Engel curve holding constant the prices of all goods (the price of food is $2 and the price of clothing is $4). For a different set of prices we would draw a different Engel curve.

![Figure 5.3 Engel Curve](image-url)
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If you look at the income consumption curve in Figure 5.2(a), you will see that the consumer purchases more food when her income rises. When this happens, food is said to be a normal good. A good is normal if a consumer wants to buy more of it when income rises. For a normal good the Engel curve will have a positive slope, as in Figure 5.3.

From Figure 5.2(a) you can also see that clothing is a normal good. Therefore, if you were to draw an Engel curve for clothing, with income on the vertical axis and the amount of clothing on the horizontal axis, the slope of the Engel curve would be positive.

As you might suspect, a consumer may not purchase more of every good as income rises. In fact, a consumer may buy less of some good when income rises.

**FIGURE 5.4 Inferior Good**
Panel (a) shows that, as income rises from $200 to $300, the consumer's weekly consumption of hot dogs increases from 13 (basket A) to 18 (basket B). Hot dogs are a normal good over this range of income. Therefore the Engel curve shown in panel (b) has a positive slope as income rises from $200 to $300. However, as income rises from $300 to $400, the consumer's weekly consumption of hot dogs decreases from 18 to 16 (basket C). Hot dogs are therefore an inferior good over this range of income. The Engel curve thus has a negative slope as income rises from $300 to $400.
Consider a consumer with the preferences for hot dogs and a composite good (“other goods”) depicted in Figure 5.4(a). For low levels of income, this consumer views hot dogs as a normal good. For example, as monthly income rises from $200 to $300, the consumer would change his optimal basket from A to B, buying more hot dogs. However, if income continues to rise, the consumer might prefer to buy fewer hot dogs, and use his increased income instead to buy more of the other goods (such as steak or seafood). The income consumption curve in Figure 5.4(a) illustrates this possibility between baskets B and C. Over this range of the income consumption curve, hot dogs would be an inferior good. A good is inferior if a consumer wants to purchase less of that good when income rises.

Using the information from the income consumption curve in Figure 5.4(a), we can draw the Engel curve for hot dogs in Figure 5.4(b). Note that the Engel curve has a positive slope over the range of incomes for which hot dogs are a normal good, and a negative slope over the range of incomes for which hot dogs are an inferior good.

### LEARNING-BY-DOING EXERCISE 5.1

#### A Normal Good Has a Positive Income Elasticity of Demand

**Problem** In this exercise you will learn that a normal good has a positive income elasticity of demand. A consumer likes to attend rock concerts and consume other goods. Suppose \( x \) measures the number of rock concerts he attends each year, and \( I \) denotes his annual income. Show that the following statement is true: If he views rock concerts as a normal good, then his income elasticity of demand for rock concerts must be positive.

**Solution** In Chapter 2 we learned that the income elasticity of demand is defined as \( E_{x,I} = \frac{(\Delta x/\Delta I)}{(I/x)} \), where all prices are held constant. If rock concerts are a normal good, then \( x \) increases as income \( I \) rises. Therefore \( (\Delta x/\Delta I) > 0 \). (This just tells us that the Engel curve for rock concerts has a positive slope, that is, \( (\Delta I/\Delta x) > 0 \).) Since income \( I \) and the number of rock concerts attended \( x \) are positive, then \( E_{x,I} > 0 \).

**Similar Problem:** Problem 3

This exercise demonstrates a general proposition: If a good is normal, its income elasticity of demand is positive. The converse is also true: Any good whose income elasticity of demand is positive will be a normal good.

Using similar reasoning you can demonstrate that the following statements are also true: Any inferior good has a negative income elasticity of demand. Further, any good with a negative income elasticity of demand will be an inferior good.
During the early nineteenth century, Ireland’s population grew rapidly. Nearly half of the Irish people lived on small farms that produced little income. Many others who were unable to afford their own farms leased land from owners of big estates. But these landlords charged such high rents leased farms also were not profitable.

Because they were poor, many Irish people depended on potatoes as an inexpensive source of nourishment. In *Why Ireland Starved*, noted economic historian Joel Mokyr described the increasing importance of the potato in the Irish diet by the 1840s:

> It is quite unmistakable that the Irish diet was undergoing changes in the first half of the nineteenth century. Eighteenth-century diets, the evergrowing importance of potatoes notwithstanding, seem to have been supplemented by a variety of vegetables, dairy products, and even pork and fish. ... Although glowing reports of the Irish cuisine in the eighteenth century must be deemed unrepresentative since they pertain to the shrinking class of well-to-do farmers, things were clearly worsening in the nineteenth. There was some across-the-board deterioration of diets, due to the reduction of certain supplies, such as dairy products, fish, and vegetables, but the main reason was the relative decline of the number of people who could afford to purchase decent food. The dependency on the potato, while it cut across all classes, was most absolute among the lower two-thirds of the income distribution.

Mokyr’s account suggests that the income expansion path for a typical Irish consumer might have looked like the one in Figure 5.4 (with potatoes on the horizontal axis instead of hot dogs). For people with a low income, potatoes might well have been a normal good. But consumers with higher incomes could afford other types of food, and therefore consumed fewer potatoes.

Given the heavy reliance on potatoes as food and as a source of nourishment, it is not surprising that a crisis occurred between 1845 and 1847, when a plant disease caused the potato crop to fail. During the Irish potato famine, about 750,000 people died of starvation or disease, and hundreds of thousands of others emigrated from Ireland to escape poverty and famine.

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So far in this chapter, we have used a *graphical* approach to show how the amount of a good consumed depends on the levels of prices and income. We have shown how to find the shape of the demand curve when the consumer has a given level of income (as in Figure 5.1), and how the demand curve shifts as the level of income changes (as in Figure 5.2).

We can also describe the demand curve *algebraically*. In other words, given a utility function and a budget constraint, we can show how the amount of a good that is consumed depends on prices and income. The next two exercises show how we can find the equation of the consumer’s demand curve.
LEARNING-BY-DOING EXERCISE 5.2

Finding a Demand Curve (No Corner Points)

A consumer purchases two goods, food and clothing. The utility function is \( U(x, y) = xy \), where \( x \) denotes the amount of food consumed and \( y \) the amount of clothing. The marginal utilities are \( MU_x = y \) and \( MU_y = x \). The price of food is \( P_x \), the price of clothing is \( P_y \), and income is \( I \).

Problem

(a) Show that the equation for the demand curve for food is \( x = I/2P_x \).

Solution  We have already encountered the utility function \( U(x, y) = xy \) in previous chapters. In Learning-By-Doing Exercise 3.1, we learned that the indifference curves for this utility function are bowed in toward the origin and do not intersect the axes. So any optimal basket must be interior, that is, the consumer buys positive amounts of both food and clothing.

How do we determine the optimal choice of food? We know that an interior optimum must satisfy two conditions:

- An optimal basket will be on the budget line. This means that \( P_x x + P_y y = I \).
- Since the optimum is interior, the tangency condition must hold.

From equation (4.4), we know that at a tangency, \( MU_x/MU_y = P_x/P_y \), or with the marginal utilities given,

\[
\frac{y}{x} = \frac{P_x}{P_y},
\]

or more simply \( y = P_x/P_y x \).

So far the solution looks very much like the solution to Learning-By-Doing Exercise 4.2. In that exercise we were interested in finding the optimal consumption of food and clothing given a specific set of prices and income. Now we want to know how much food the consumer buys for any set of prices and income. That is why we are using the exogenous variables \( (P_x, P_y, I) \) instead of actual numbers for the prices and income. So we have two equations with two unknowns (the endogenous variables \( x \) and \( y \)). Let’s substitute \( y = P_x/P_y x \) (coming from the tangency condition) into the budget line \( P_x x + P_y y = I \). This gives us

\[
P_x x + P_y \left( \frac{P_x}{P_y} x \right) = I.
\]

When we solve for the amount of food demanded, \( x \), we find that \( x = I/(2P_x) \). This is the equation of the demand curve for food. Given any numerical values of income and the price of food, we can easily find the quantity of food the consumer will purchase.
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**Problem**

(b) Is food a normal good? Draw the consumer's demand curve for food when the level of income is $I = 120$. Label this demand curve $D_1$. Draw the demand curve when $I = 200$, and label this demand curve $D_2$.

**Solution**

If income is 120, the amount of food demanded will be $x = 120/(2P_x) = 60/P_x$. We can plot points on the demand, as we have done in Figure 5.5. When the price of food is 15, the consumer buys 4 units of food (point $A$ in the graph). At a price of 10, she buys 6 units of food (point $B$ in the graph).

An increase in income shifts the demand schedule to the right. Thus, food is a normal good. At a price of 10 and an income of 200, the consumer buys 10 units of food (point $C$ in the graph).

Note that the consumer always buys some food ($x$ is always positive), no matter how high the price. This is what we expected to see, since there will be no corner-point solution at which she buys no food.

**Similar Problem:** 5.5

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**FIGURE 5.5 Demand Curves for Food at Different Income Levels**

The quantity of food demanded, $x$, depends on the price of food, $P_x$, and on the level of income $I$. The equation representing the demand for food is $x = I/(2P_x)$. When income is 120, the demand curve is $D_1$ in the graph. If the price of food is $15, the consumer buys 4 units of food (point $A$). If the price of food drops to $10, she buys 6 units of food (point $B$). If income rises to $200, the demand curve shifts to the right, to $D_2$. For example, if income is $200 and the price of food is $10, the consumer buys 10 units of food (point $C$).
5.1 Optimal Choice and Demand

Why would we want to go through the exercise of finding the equation of a demand curve, as we have done in this exercise? If we only want to know the optimal amount of food consumed at a specific set of prices and income, we could just use the approach of Learning-By-Doing Exercise 4.2. But, if we want to find out how much food would be purchased for many different prices of food, it would be tedious to repeat that exercise using new numbers. Instead, once we have the equation for the demand curve, we can easily find the quantity of food demanded for any price and income.

**LEARNING-BY-DOING EXERCISE 5.3**

**Finding a Demand Curve (With a Corner-Point Solution)**

A consumer purchases two goods, food and clothing. He has the utility function \( U(x, y) = xy + 10x \), where \( x \) denotes the amount of food consumed and \( y \) the amount clothing. The marginal utilities are \( MU_x = y + 10 \) and \( MU_y = x \). The price of food is \( P_x \), the price of clothing is \( P_y \) and his income is \( I \).

**Problem**

(a) Show that the equation for the consumer’s demand curve for clothing is

\[
y = \begin{cases} \\
\frac{I - 10P_y}{2P_y}, & \text{when } P_y \leq \frac{I}{10} \\
0, & \text{when } P_y > \frac{I}{10}
\end{cases}
\]

**Solution** We have already examined optimal choice with the utility function \( U(x, y) = xy + 10x \) in Learning-By-Doing Exercise 4.3. There we learned that the indifference curves for this utility function are bowed in toward the origin. They also intersect the \( x \)-axis, since the consumer could have a positive level of utility with purchases of food \( (x > 0) \) but no purchases of clothing \( (y = 0) \). So he might not buy any clothing (and choose a corner point) if the price of clothing is too high.

How do we determine the consumer’s optimal choice of clothing? If he is at an interior optimum, we know that his optimal basket will be on the budget line. This means that

\[
P_x x + P_y y = I.
\]

At an interior optimum, the tangency condition must hold. From equation (4.4), we know that at a tangency, \( MU_x/MU_y = P_x/P_y \), or with the marginal utilities given,

\[
\frac{y + 10}{x} = \frac{P_x}{P_y},
\]

or more simply \( P_x x = P_y y + 10P_y \).
So far the solution looks like the solution to Learning-By-Doing Exercise 4.3. Now we want to know how much clothing the consumer buys for any set of prices and income. We therefore use the exogenous variables \((P_x, P_y, \text{ and } I)\) instead of actual numbers for the prices and income. So we have two equations with two unknowns (the endogenous variables \(x\) and \(y\)). Let’s substitute \(P_x x = P_y y + 10P_y\) (coming from the tangency condition) into the budget line \(P_x x + P_y y = I\). This gives us \(2P_y y + 10P_y = I\). When we solve for the amount of clothing demanded, \(y\), we find that \(y = (I - 10P_y)/2P_y\). This is the equation of the consumer’s demand curve for clothing. Note that the consumer will demand a positive amount of clothing when \(I - 10P_y > 0\), or when \(P_y < I/10\).

What happens if \(P_y > I/10\)? Then the consumer will be at a corner point. To see this, let’s compare the marginal utilities per dollar spent on each good. At a corner point at which the consumer buys only food, she will have \(x = I/P_x\) units of food and zero units of clothing (\(y = 0\)). In that case \(MU_x/P_x = (y + 10)/P_x = 10/P_x\). For clothing, \(MU_y/P_y = x/P_y = (I/P_x)y/P_y\). Note that he will buy only food when \((MU_x/P_x) > (MU_y/P_y)\), that is, when \((10/P_x) > (I/P_x)y/P_y\), which simplifies to \(P_y < I/10\). So if \(P_y > I/10\), he will choose a corner point and buy the basket \((x = I/P_x, y = 0)\).

**Problem**

(b) Suppose his income is \(I = 100\). Fill in the following table to show how much clothing he will purchase at each price of clothing (these are points on his demand curve):

<table>
<thead>
<tr>
<th>(P_y)</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Solution** Using the equation for the demand curve we found in (a), the table can be completed as follows:

<table>
<thead>
<tr>
<th>(P_y)</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>20</td>
<td>7.5</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note that the equation for the demand curve helps us easily determine the quantity of clothing that the consumer will demand at any price. That is why we did this exercise. We could have repeated Learning-By-Doing Exercise 4.3 five times (once for each price of clothing) to fill in the table, but that would have required a lot of repetitious work. It is much simpler just to find the equation of the demand curve (as we have done in this exercise), and then use that equation to fill in the table.

**Similar Problem:** Problem 10 is similar.
A decrease in the price of a good affects the consumer in two ways. First, as the price of a good falls, that good becomes cheaper relative to other goods, leading to a substitution effect. For example, if the price of food falls, the consumer may decide to buy more food and less of other goods, because food is now less expensive relative to other goods. Second, as the price goes down, purchasing power increases, since the consumer could buy the same basket of goods with money left over to buy still more goods. This increase in purchasing power affects the consumer much the way it would if income increased, giving rise to an income effect.

These two effects occur at the same time when the price of a good falls. However, we need to distinguish between the two effects to understand better how a price change affects a consumer.

**SUBSTITUTION EFFECT**

Let’s consider what happens when the price of food changes. The substitution effect is the change in the amount of food consumed as the price of food changes, holding constant the level of utility. The substitution effect tells us how much more food the consumer would buy now that food is cheaper relative to clothing.

You can find the substitution effect associated with a price change by following the three steps illustrated in the three optimal choice diagrams in Figure 5.6. Here the consumer buys two goods, food and clothing, both of which have positive marginal utilities. We have drawn the diagrams assuming the price of food decreases.

**Step 1:** Find the initial basket, that is, the basket the consumer chooses at the initial price of food. Figure 5.6(a) illustrates Step 1. Initially, when the price of food is $P_x$, the consumer faces the budget line $BL_1$. When she maximizes utility, she chooses basket $A$ on the indifference curve $U_1$. The initial quantity of food purchased is $x_A$.

**Step 2:** Find the final basket, that is, the basket the consumer chooses at the final price of food. Figure 5.6(b) illustrates Step 2. When the price of food falls to $P_x$, the budget line rotates outward to $BL_2$. The consumer is surely going to be better off as a result of the decrease in the price of food, because the initial basket $A$ lies inside the new budget line $BL_2$. The consumer now chooses basket $C$ and realizes the higher level of utility that the indifference curve $U_2$ represents. The final quantity of food consumed is $x_C$. When the price of food falls, food consumption increases by $(x_C - x_A)$.

**Step 3:** Find the decomposition basket. Figure 5.6(c) illustrates Step 3. We can find the decomposition basket by drawing a budget line parallel to the new budget line $BL_2$ (to reflect the fall in the price of food), but tangent to the initial indifference curve $U_1$ (to keep the initial level of utility unchanged). You can think of this tangent line as the budget line that will help us separate (or decompose) the effect of the price change on the consumption of food $(x_C - x_A)$ into the substitution and income effects. We will therefore call it the decomposition budget line and label it $BL_d$ in Figure 5.6(c). The decomposition budget line $BL_d$ is tangent to the indifference curve $U_1$. At that point of tangency, basket $B$, she purchases $x_B$ units of food.

Since we are holding the level of utility constant, a movement along the initial indifference curve $U_1$ determines the substitution effect. In Figure 5.6(c), the substitution effect is $(x_B - x_A)$. 
As the price of food falls, the substitution effect will lead to an increase in the amount of food purchased. This is true because the consumer has a diminishing marginal rate of substitution of food for clothing, and therefore the indifference curves are bowed in toward the origin. When the price of food falls, the slope of the decomposition budget line is less steep than the original budget line. The decomposition basket $B$ will therefore be to the southeast of the original basket $A$.

The consumer has income $I$, and this income enables her to choose any basket on $BL_1$ when the price of food is $P_{x_1}$, and any basket along $BL_2$ when the price of food is $P_{x_2}$. Note that the decomposition budget line $BL_d$ lies inside the final

![Diagram](image_url)
5.2 Income and Substitution Effects

As the price of food drops from $P_x^1$ to $P_x^2$, the substitution effect leads to an increase in the amount of food consumed from $x_A$ to $x_B$ (so the substitution effect is $x_B - x_A$). Since food is a normal good, the income effect also leads to an increase in food consumption, from $x_B$ to $x_C$ (so the income effect is $x_C - x_B$). When a good is normal, the income and substitution effects reinforce each other. In this case the demand curve for food will be downward sloping. As the price of food decreases from $P_x^1$ to $P_x^2$, the quantity of food will increase from $x_A$ to $x_C$.

**FIGURE 5.6 Income and Substitution Effects Case 1: $x$ is a Normal Good**

As the price of food drops from $P_x$ to $P_x'$, the substitution effect leads to an increase in the amount of food consumed from $x_A$ to $x_B$ (so the substitution effect is $x_B - x_A$). Since food is a normal good, the income effect also leads to an increase in food consumption, from $x_B$ to $x_C$ (so the income effect is $x_C - x_B$). When a good is normal, the income and substitution effects reinforce each other. In this case the demand curve for food will be downward sloping. As the price of food decreases from $P_x$ to $P_x'$, the quantity of food will increase from $x_A$ to $x_C$.

**INCOME EFFECT**

Now let's find the income effect. The income effect is the change in the amount of a good consumed as the consumer's utility changes, holding price constant.
In this example, the movement from $A$ to $B$ does not involve any change in utility because some income is “taken away” as the price of food falls. However, in reality, the consumer does not have to forgo any income when the price of food is lowered. The income effect measures the change in food consumption when the level of income is restored—that is, increased from $I_d$ back to $I$, moving the budget line from $BL_d$ to $BL_2$. In Figure 5.6(c), note what happens when the level of income is increased from $I_d$ to $I$ (and the budget line therefore shifts from $BL_d$ to $BL_2$). The optimal basket would change from $B$ to $C$. The income effect is the corresponding change in the consumption of food, that is, $(x_C - x_B)$.

What is going on here? Initially, the consumer pays a price $P_{x_1}$ and chooses basket $A$. In the final situation the consumer pays a price $P_{x_2}$ and chooses basket $C$. Income does not change. When the price of food falls from $P_{x_1}$ to $P_{x_2}$, the total change on food consumption is $(x_C - x_A)$. In reality the consumer moves directly from basket $A$ to basket $C$ and never actually chooses basket $B$.

However, we have introduced basket $B$ to decompose the total change on food consumption into an income and a substitution effect. The movement from $A$ to $B$ holds utility constant. As the price of food drops, just enough income is “taken away” to make the consumer indifferent between the two baskets. The difference in food consumption as the consumer moves from $A$ to $B$ is the substitution effect.

To find the income effect, the income that is “taken away” in the movement from $A$ to $B$ is then “restored.” The price of food is held constant. As income is restored, the optimal choice changes from $B$ to $C$.

In sum, when the price of food falls from $P_{x_1}$ to $P_{x_2}$, the total change on food consumption is $(x_C - x_A)$. This can be decomposed into the substitution effect $(x_B - x_A)$ and the income effect $(x_C - x_B)$. When we add the substitution effect and the income effect, we get the total change in consumption.

The graphs in Figure 5.6 are drawn for the case (we call it Case 1) in which food is a normal good. As the price of food falls, the income effect leads to an increase in food consumption. As we noted earlier, because the marginal rate of substitution is diminishing, the substitution effect will also lead to increased food consumption. Thus, the income and substitution effects work in the same direction. If the price of food falls, both effects will be positive. The demand curve for food will be downward sloping because the quantity of food purchased will clearly increase when the price of food falls. Similarly, if the price of food were to rise, both effects would be negative. At a higher price of food, the consumer would buy less food.

However, the income and substitution effects do not have to work in the same direction. Consider Case 2, in Figure 5.7. Instead of drawing three more graphs like those in Figure 5.6, we have only drawn the final graph (like Figure 5.6(c)) with the initial, final, and decomposition baskets. Note that basket $C$, the final basket, lies directly above basket $B$, the decomposition basket. As the budget line shifts out from $BL_d$ to $BL_2$, the quantity of food consumed does not change. The income effect is therefore zero because $(x_C - x_B) = 0$. Here a decrease in the price of food leads to a positive substitution effect on food consumption (since $x_B$ is greater than $x_A$), and a zero income effect. The demand curve for food will be downward sloping because more food is purchased at the lower price.
The income and substitution effects might even work in opposite directions. This happens when a good is inferior. Consider Case 3, in Figure 5.8. In this figure we draw the indifference curves so that the income effect is negative. In other words, for this consumer, food is an inferior good. Note that basket $C$, the final basket, lies to the left of basket $B$, the decomposition basket. As the budget line shifts out from $BL_1$ to $BL_2$, the quantity of food consumed decreases. The income effect is therefore negative because $(x_C - x_B) < 0$. Here a decrease in the price of food leads to a positive substitution effect on food consumption (because $x_B$ is greater than $x_A$), and a negative income effect.

Will the demand curve for food be downward sloping for the preferences in Figure 5.8? Yes. The final basket $C$ lies to the right of the initial basket $A$. When the price of food drops from $P_{x_1}$ to $P_{x_2}$, the quantity of food does increase from $x_A$ to $x_C$. The demand curve for food will therefore be downward sloping.

The final case is the rather strange Case 4, in Figure 5.9. Note that we draw the indifference curves for the case in which food is strongly inferior. Basket $C$, the final basket, lies not only to the left of the decomposition basket $B$, but also...
to the left of the initial basket $A$. The income effect is so strongly negative that it more than cancels out the positive substitution effect.

Will the demand curve for food be downward sloping for the preferences in Figure 5.9? No. When the price of food drops from $P_{x_1}$ to $P_{x_2}$, the quantity of food actually decreases from $x_{A}$ to $x_{C}$. The demand curve for food will therefore be upward sloping over the range of prices between $P_{x_1}$ and $P_{x_2}$. Case 4 illustrates the famous case of the Giffen good. A Giffen good is a good that has a demand curve that has a positive slope over part of the curve.

As we have already noted, some goods are inferior for some consumers. As we suggested earlier, your consumption of hot dogs may fall if your income rises, as you decide to eat more steaks and fewer hot dogs. But expenditures on inferior goods typically represent only a small part of a consumer’s income. Income effects for individual goods are usually not large, and the largest income effects are usually associated with goods that are normal rather than inferior, such as food and housing. For an inferior good to have an income effect large enough to

**FIGURE 5.8 Income and Substitution Effects Case 3**

$x$ is an Inferior Good with a Downward-Sloping Demand Curve

As the price of food drops from $P_{x_1}$ to $P_{x_2}$, the substitution effect leads to an increase from $x_A$ to $x_B$ in the amount of food consumed. But food is an inferior good. The income effect on food consumption is negative because $x_C$ is less than $x_B$. When a good is inferior, the income and substitution effects work in opposite directions. Because the substitution effect is larger than the income effect, the demand curve for food will still be downward sloping. As the price of food decreases from $P_{x_1}$ to $P_{x_2}$, the quantity of food will increase from $x_A$ to $x_C$. 
5.2 Income and Substitution Effects

offset the substitution effect, the income elasticity of demand would have to be negative and the expenditures on the good would need to represent a large part of the consumer's budget. Thus, while the Giffen good is intriguing as a theoretical possibility, it is not of much practical concern.

**EXAMPLE 5.3 How Do Rats Respond to Changes in Prices?**

In Chapter 2 we cited studies showing that people have negatively sloped demand curves for goods and services, and that many goods are adequate substitutes for one another. In the early 1980s several economists conducted experiments designed to ask how rats would respond to changes in relative prices. In one famous experiment, white rats were offered root beer and collins mix in different containers. To extract a unit of the beverage, a rat had to “pay a price” by pushing a lever a certain number of times. The researchers allowed the rat a specified number of pushes per day. This was the rat’s income.
CHAPTER 5 The Theory of Demand

Each rat was then able to choose its initial basket of the beverages. Then the experimenters altered the relative prices of the beverages by changing the number of times the rat needed to push the lever to extract a unit of each beverage. The rat’s income was adjusted so that it would allow a rat to consume its initial basket. The researchers found that the rats altered their consumption patterns to choose more of the beverage with the lower relative price. The choices the rats made indicated that they were willing to substitute one beverage for the other when the relative prices of the beverages changed.

In another experiment, rats were offered a similar set of choices between food and water. When relative prices were changed, the rats were willing to engage in some limited substitution toward the good with the lower relative price. But the cross-price elasticities of demand were much lower in this experiment because food and water are not good substitutes for one another.

In a third study, researchers designed an experiment to see if they could confirm the existence of a Giffen good for rats. When the rats were offered a choice between quinine water and root beer, researchers discovered that quinine water was an inferior good. They reduced the rats’ incomes to low levels, and set prices so that the rats spent most of their budget on quinine water. This was the right environment for the potential discovery of a Giffen good. Theory predicts that we are most likely to observe a Giffen good when an inferior good (quinine water) also comprises a large part of a consumer’s expenditures. When researchers lowered the price of quinine water, they found that the rats did in fact extract less quinine water, using their increased wealth to choose more root beer. The researchers concluded that for rats quinine water was a Giffen good.7


While researchers have not yet confirmed the existence of a Giffen good for human beings, some economists have suggested that the Irish potato famine (see Example 5.2) came close to creating the right environment. However, as Joel Mokyr observed, “For people with a very low income, potatoes might have well been a normal good. But consumers with higher levels of income could afford other types of food, and therefore consumed fewer potatoes.” Thus, while expenditures on potatoes did constitute a large part of consumer expenditures, they may not have been inferior at low incomes. This may explain why researchers have not shown the potato to have been a Giffen good at that time.

LEARNING-BY-DOING EXERCISE 5.4

Numerical Example of Income and Substitution Effects

In Learning-By-Doing Exercises 4.2 and 5.2, we met a consumer who purchases two goods, food and clothing. She has the utility function \( U(x, y) = xy \), where \( x \) denotes the amount of food consumed and \( y \) the amount clothing. Her marginal utilities are \( MU_x = y \) and \( MU_y = x \). Now suppose that she has an in-
come of $72 per week and that the price of clothing is $1 per unit. Suppose that the price of food is initially $9 per unit, and that the price subsequently falls to $4 per unit.

Problem Find the numerical values of the income and substitution effects on food consumption, and graph the results.

Solution To find the income and substitution effects, we follow the three steps we identified earlier in this section.

Step 1: Find the initial consumption basket $A$ when the price of food is $9.

To find the amount of food and clothing consumed, we know that two conditions must satisfied at an optimum. First, an optimal basket will be on the budget line. This means that $P_x x + P_y y = I$, or with the given information

$$9x + y = 72.$$ 

Second, since the optimum is interior, the tangency condition must hold. From equation (4.4), we know that at a tangency, $MU_x/MU_y = P_x/P_y$, or with the given information,

$$\frac{y}{x} = \frac{9}{1},$$

or more simply $y = 9x$.

So we have two equations with two unknowns: $9x + y = 72$ (coming from the budget line) and $y = 9x$ (coming from the tangency condition). Together these imply that $x = 4$, and $y = 36$. So the consumer’s optimal basket involves the purchase of 4 units of food and 36 units of clothing each month, as basket $A$ in Figure 5.10 indicates.

Now, suppose you had already worked out the equation for the demand curve for food from Learning-By-Doing Example 5.2. There we found that the demand curve for food $x = I/2P_x$. When the income is $I = 72$ and the price of a unit of food is $9$, the equation for demand tells us that the consumer will buy 4 units of food. This food costs her $P_x x = ($9 per unit of food)(4 units of food) = $36$. She spends the rest of her income ($72 total income - $36 spent on food) on clothing. So she spends $36 on clothing, and each unit of clothing costs her $1. She therefore buys 36 units of clothing.

We summarize the information about the initial basket $A$ in Table 5.1. The initial level of utility is $U_1 = xy = (4)(36) = 144$. So the indifference curve passing through the initial basket $A$ has a value of 144. The slopes of the indifference curve and budget line at basket $A$ are $-9$, and she will need to spend $72 to purchase basket $A$ when the price of a unit of food is $9$, and the price of a unit of clothing is $1$.

Step 2: Find the final consumption basket $C$ when the price of food is $4$.

We repeat part (a) of this exercise, now with the price of a unit of food of $4$. So we have two equations with two unknowns:

$$4x + y = 72$$ (coming from the budget line)

$$y = 4x$$ (coming from the tangency condition)
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FIGURE 5.10 Income and Substitution Effects
As the price of food drops from $9 to $4, the substitution effect leads to an increase in food consumption from 4 (at the initial basket A) to 6 (at the decomposition basket B). The substitution effect is therefore 2. We measure the income effect by the change in food consumption as the consumer moves from the decomposition basket B (where 6 units of food are purchased) to the final basket C (where 9 units of food are bought). The income effect is therefore 3.

TABLE 5.1
Optimal Baskets for Learning-By-Doing Exercise 5.4

<table>
<thead>
<tr>
<th>Basket</th>
<th>x</th>
<th>y</th>
<th>U = xy</th>
<th>( \frac{MU_x}{MU_y} = \frac{p_x}{p_y} )</th>
<th>Expenditure ( P_x x + P_y y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>36</td>
<td>144</td>
<td>( \frac{9}{1} = \frac{9}{1} )</td>
<td>(9)(4) + (1)(36) = 72</td>
</tr>
<tr>
<td>B</td>
<td>6</td>
<td>24</td>
<td>144</td>
<td>( \frac{4}{1} = \frac{4}{1} )</td>
<td>(4)(6) + (1)(24) = 48</td>
</tr>
<tr>
<td>C</td>
<td>9</td>
<td>36</td>
<td>324</td>
<td>( \frac{4}{1} = \frac{4}{1} )</td>
<td>(4)(9) + (1)(36) = 72</td>
</tr>
</tbody>
</table>
5.2 Income and Substitution Effects

Solving the two equations together yields the following:

\[ 4x + 4x = 72 \]
\[ 8x = 72 \]

Then \( x = 9 \), and \( y = 4x \), or 36.

So the consumer’s optimal basket involves the purchase of 9 units of food and 36 units of clothing each month, as basket \( C \) in Figure 5.10 indicates.

The last row of Table 5.1 summarizes the information about the initial basket \( C \). The final level of utility is \( U_2 = xy = (9)(36) = 324 \). So the indifference curve passing through the initial basket \( C \) has a value of 324. The slopes of the indifference curve and budget line at basket \( C \) are \(-4\), and the consumer will need to spend $72 to purchase basket \( C \) when the price of a unit of food is $4, and the price of a unit of clothing is $1.

**Step 3:** Find the decomposition basket \( B \). The decomposition basket must satisfy two conditions. First, it must lie on the original indifference curve. Therefore, the amounts of food and clothing must yield a level of utility equal to \( U_1 \), which is 144. At basket \( B \) the amounts of food and clothing must therefore satisfy \( xy = 144 \). Second, at basket \( B \) the indifference curve and decomposition budget line must be tangent to one another. The final price of food determines the slope of the decomposition budget line. The tangency is will occur when \( \frac{MU_x}{MU_y} = \frac{P_x}{P_y} \), or when \( y/x = 4/1 \). The tangency condition requires that \( y = 4x \).

We now have all of the information we need to find the decomposition basket. We know that the purchases of food and clothing at \( B \) must allow the consumer to reach the initial level of utility \( (xy = 144) \) and that the tangency condition with the new price of food will be satisfied \(( y = 4x \)\). These two equations tell us that the decomposition basket \( B \) will contain 6 units of food and 24 units of clothing \(( x = 6 \) and \( y = 24 \)\).

We summarize the information about the decomposition basket in Table 5.1. At basket \( B \) the level of utility is 144, and the slopes of the indifference curve and decomposition budget lines are both \(-4\). The last column indicates that if the consumer were to buy basket \( B \) when the price of food is $4 and the price of clothing is 1, she would need to spend only $48. Because the decomposition budget line is tangent to the initial indifference curve \( U_1 \), basket \( B \) represents the choice she would make if she wants to minimize her total expenditure when (1) she faces the new price of food of $4, and (2) she wants to remain on the indifference curve \( U_1 \).

Baskets \( A \) and \( B \) both lie on the same indifference curve. Therefore, the consumer would be equally happy with either of the following two situations: (1) having an income of $72 and paying $9 per unit of food (and buying basket \( A \)), or (2) having an income of $48 and paying $4 per unit of food (and buying basket \( B \)).

Now we can measure the income and substitution effects. The substitution effect is the increase in food purchased as the consumer slides around the initial indifference curve, moving from basket \( A \) (at which she purchases 4 units of food) to basket \( B \) (at which she purchases 6 units of food). The substitution effect is therefore +2 units of food.
The income effect is the increase in food purchased as she moves from basket $B$ (at which she purchases 6 units of food) to basket $C$ (at which she purchases 9 units of food). The income effect is therefore +3 units of food.

Figure 5.10 graphs the income and substitution effects. In this exercise food is a normal good. As expected, the income and substitution effects have the same sign. The consumer's demand curve for food is downward sloping because the quantity of food she purchases increases when the price of food falls.

**Similar Problem:** No similar problem.
5.2 Income and Substitution Effects

$y$, other goods

$\text{Income effect} = x_C - x_B$

$\text{Substitution effect} = x_B - x_A$

FIGURE 5.11 Income and Substitution Effects

At the initial price of housing the budget line is $BL_1$, and the consumer buys basket $A$ and receives a level of utility $U_1$. When the price of housing doubles, the budget line becomes $BL_2$, and the consumer purchases basket $C$, reaching the indifference curve $U_2$. The decomposition budget line $BL_d$ is parallel to $BL_2$, and the decomposition basket $B$ is located where $BL_d$ is tangent to the initial indifference curve $U_1$. The substitution effect is $(x_B - x_A)$, because housing consumption increases from $x_A$ (at the initial basket $A$) to $x_B$ (at the decomposition basket $B$). The income effect $(x_C - x_B)$ is measured by the change in housing consumption as the consumer moves from the decomposition basket $B$ to the final basket $C$.

LEARNING-BY-DOING EXERCISE 5.6

Income and Substitution Effects with a Quasilinear Utility Function

A college student who loves chocolate has a budget of $10 per day, and out of that income she purchases chocolate and other goods. $x$ measures the number of ounces of chocolate she purchases. $y$ measures the number of units of the composite good that she buys. The price of the composite good is 1.
The utility function $U(x, y) = 2\sqrt{x} + y$ represents the student’s preferences. You may recall from Chapter 3 that this is a quasi-linear utility function of two goods because it is a linear function of one of its arguments (the composite good, $y$), but not a linear function of the amount of chocolate she buys. For this utility function, $MU_x = 1/\sqrt{x}$ and $MU_y = 1$.

**Problem**

(a) Suppose the price of chocolate is initially $0.50 per ounce. How many ounces of chocolate and how many units of the composite good are in the student’s optimal consumption basket?

**Solutions** At an interior optimum, $MU_x/MU_y = P_x/P_y$, which tells us that $1/\sqrt{x} = P_x$. The student’s demand for $x$ is therefore $x = 1/(P_x)^2$. When the price of $x$ is $0.50 per ounce, she buys $1/(0.5)^2 = 4$ ounces of chocolate per day.

We can find the number of units of the composite good from the budget line, $P_xx + P_yy = I$. With the information given, the budget line is $(0.5)(4) + (1)y = 10$, so the student buys $y = 8$ units of the composite good.

**Problem**

(b) Suppose the price of chocolate drops to $0.20 per ounce. How many ounces of chocolate and how many units of the composite good are in the optimal consumption basket?

**Solution** We use the demand curve for chocolate from part (a) to find her demand for chocolate when the price falls to $0.20 per ounce. She buys $1/(0.2)^2 = 25$ ounces of chocolate at the lower price. Her budget constraint is now $(0.2)(25) + (1)y = 10$, so she buys $5$ units of the composite good.

**Problem**

(c) What are the substitution and income effects that result from the decline in the price of chocolate? Illustrate these effects on a clearly labeled graph.

**Solution** In the first two parts of this problem we already found all we need to know about the initial basket $A$ and the final basket $C$. Figure 5.12 shows these baskets. Table 5.2 summarizes the information about these baskets.

To find the income and substitution effects, we need to find the decomposition basket $B$. We know two things about the tangency at basket $B$:

1. The utility at $B$ must be the same as at the initial basket $A$. Therefore we know that $2\sqrt{x} + y = 12$.
2. The decomposition budget line must have the same slope as the final budget line at the point of tangency. Therefore $MU_x/MU_y = P_x/P_y$, or $1/\sqrt{x}/1 = 0.20/1$. These two conditions tell us that $x = 25$ and $y = 2$ at the decomposition basket $B$. Figure 5.12 also plots this basket.
In the graph we can measure the size of the substitution effect by the change in chocolate purchased as the consumer moves from the initial basket $A$ (where she consumes 4 ounces of chocolate) to the decomposition basket $B$ (where she consumes 25 ounces of chocolate). The substitution effect on chocolate is therefore is +21 ounces. The income effect is zero because she consumes the same amount of chocolate at $B$ and $C$.

**Similar Problem:** Problem 6

**FIGURE 5.12 Income and Substitution Effects**

At the initial price of chocolate ($P_x = $0.50 per ounce) the budget line is $BL_1$. The consumer buys basket $A$, containing 4 ounces of chocolate, and receives a level of utility $U_1 = 12$. When the price of chocolate falls to $0.50 per ounce, the budget line becomes $BL_2$. The consumer purchases basket $C$, containing 25 ounces of chocolate, and has the utility $U_2 = 15$. The decomposition budget line $BL_d$ is parallel to $BL_2$, and the decomposition basket $B$ is located where $BL_d$ is tangent to the initial indifference curve $U_1$. The substitution effect is +21 ounces because chocolate consumption increases from 4 ounces (at the initial basket $A$) to 25 ounces (at the decomposition basket $B$). The income effect is measured by the change in chocolate bought as the consumer moves from the decomposition basket $B$ (where 25 ounces of chocolate are purchased) to the final basket $C$ (where she still buys 25 ounces of chocolate). The income effect is therefore zero.
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You can see this more formally as follows: If the utility function is quasilinear, then

\[ U(x, y) = H11005 f(x) \]

where \( k \) is a constant. For this utility function

\[ MU_y = H11005 k. \]

Let \( MU_x = \) the marginal utility of \( x \). At an interior optimum,

\[ MU_x / MU_y = H11005 P_x / P_y. \]

Which tells us that

\[ MU_x / k = H11005 P_x / P_y. \]

If income changes (and prices are constant), then \( MU_x \) must remain constant. This can happen only if \( x \) remains constant because \( MU_x \) would change if \( x \) were to change. Therefore, as long as the optimal basket is interior as income varies, the optimal choice of \( x \) will remain constant. This means that \( \Delta x / \Delta I = 0 \), and also that the income elasticity is zero.

Learning-By-Doing Exercise 5.6 illustrates one of the properties of a quasilinear utility function with a constant marginal utility of \( y \) and indifference curves that are bowed in toward the origin. When prices are constant, at an interior optimum the consumer will purchase the same amount of \( x \) as income varies. In other words, the income consumption curve will be a vertical line in the graph. This means that the income effect associated with a price change on \( x \) will be zero, as in Case 2, in Figure 5.7.

A consumer will buy a good when the purchase makes him better off. Consumer surplus is the difference between the maximum amount a consumer is willing to pay for a good and what must actually be paid if the good is purchased in the marketplace. It measures how much better off the consumer will be when he purchases the good.

Let’s begin with an example. Suppose you are considering buying an automobile and that you will either buy one automobile or no automobile at all. You are willing to pay up to $15,000 for it. But you can buy that automobile for $12,000 in the marketplace. Because your willingness to pay exceeds the amount you will actually have to pay for it, you will buy it. When you do, you will walk away from the marketplace with a consumer surplus of $3,000 from that purchase. Your consumer surplus is your net economic benefit from making the purchase, that is, the maximum amount you would be willing to pay ($15,000) less the amount you actually have to pay ($12,000).

Of course, for many types of commodities you might want to consume more than one unit. You will have a demand curve for such a commodity, and, as we have already pointed out, it represents your willingness to pay for the good. For example, suppose you like to play tennis, and that you must rent the tennis court for an hour each time you play. Your demand curve for court time appears in

<table>
<thead>
<tr>
<th>Basket</th>
<th>( x )</th>
<th>( y )</th>
<th>( U )</th>
<th>( M_{U_x} / M_{U_y} )</th>
<th>( P_x / P_y )</th>
<th>Expenditure</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>1/\sqrt{4} = 0.50</td>
<td>1</td>
<td>(0.50)(4) + (1)(8) = 10</td>
</tr>
<tr>
<td>B</td>
<td>25</td>
<td>2</td>
<td>12</td>
<td>1/\sqrt{25} = 0.20</td>
<td>1</td>
<td>(0.20)(25) + (1)(2) = 7</td>
</tr>
<tr>
<td>C</td>
<td>25</td>
<td>5</td>
<td>15</td>
<td>1/\sqrt{25} = 0.20</td>
<td>1</td>
<td>(0.20)(25) + (1)(5) = 10</td>
</tr>
</tbody>
</table>

You can see this more formally as follows: If the utility function is quasilinear, then \( U(x, y) = f(x) + k y \), where \( k \) is a constant. For this utility function \( M_{U_x} = k \). Let \( M_{U_x} \) be the marginal utility of \( x \). At an interior optimum, \( M_{U_x} / M_{U_y} = P_x / P_y \), which tells us that \( M_{U_x} / k = P_x / P_y \). If income changes (and prices are constant), then \( M_{U_x} \) must remain constant. This can happen only if \( x \) remains constant because \( M_{U_x} \) would change if \( x \) were to change. Therefore, as long as the optimal basket is interior as income varies, the optimal choice of \( x \) will remain constant. This means that \( \Delta x / \Delta I = 0 \), and also that the income elasticity is zero.
Figure 5.13. It shows that you would be willing to pay up to $25 for the first hour of court time each month. For the second hour in the month, you are willing to pay $23, and for the third hour $21. Your demand schedule is downward sloping because you have a diminishing marginal utility for playing tennis.

Suppose you must pay $10 per hour to rent the court. At that price your demand curve indicates that you will play tennis for 8 hours during the month. You are willing to pay $11 for the eighth hour, but only $9 for the ninth hour, and even less for additional hours. You are therefore not willing to play more than 8 hours when court time costs $10 per hour.

How much consumer surplus do you get from playing tennis each month? To find out, you add the surpluses from each of the units you consume. Your consumer surplus from the first hour is $15, that is, the $25 you are willing to pay minus the $10 you actually must pay. The consumer surplus from the second hour is $13. The consumer surplus from using the court for the 8 hours during the month is then $64 (the sum of the consumer surpluses for each of the 8 hours, or $15 + $13 + $11 + $9 + $7 + $5 + $3 + $1).

As the example illustrates, the consumer surplus is the area below the demand schedule and above the price that the consumer must pay for the good. We represented the demand schedule here as a series of “steps” to help us illustrate the consumer surplus from each unit purchased. Of course, more generally, a demand function may be a smooth curve, often represented as an algebraic equation. The concept of consumer surplus is the same for a smooth demand curve.

As we shall show, the area under a demand curve exactly measures net benefits for a consumer only if the consumer experiences no income effect over the range of price change. This may often be a reasonable assumption. However, if the assumption is not satisfied, then the area under the demand curve will not measure the consumer’s net benefits exactly. For the moment, let’s not worry about this complication. We assume that there is no income effect.
LEARNING-BY-DOING EXERCISE 5.7

Consumer Surplus

Suppose the following equation represents a consumer’s monthly demand schedule for milk: \( Q = 40 - 4P \), where \( Q \) is the number of gallons of milk purchased when the price is \( P \) dollars per gallon. Note that the income effect here is zero because the level of income does not appear in the demand function.

Problem

(a) What is the consumer surplus per month if the price of milk is $3 per gallon?

Solution

Figure 5.14 shows the demand curve for milk. When the price is $3, the consumer will buy 28 gallons of milk. The consumer surplus is the area under the demand curve and above the price of $3. Since the demand curve given in the exercise is a straight line, the area representing the consumer surplus is triangle \( G \). The area of \( G \) is \( \frac{1}{2}(10 - 3)(28) = 98 \).

Problem

(b) What is the increase in consumer surplus if the price falls to $2 per gallon?

\[ \text{Area of triangle } G = \frac{1}{2}(10 - 3)(28) = 98 \]

\[ \text{Area of triangle } H = \frac{1}{2}(2)(28) = 28 \]

\[ \text{Area of triangle } I = \frac{1}{2}(2)(2) = 2 \]

\[ \text{Total increase in consumer surplus} = 98 + 28 + 2 = 128 \]
5.3 Consumer Surplus

If we have a demand curve, we can calculate the consumer surplus for any price we might choose. Let’s do this using the demand curve for milk in Learning-By-Doing Exercise 5.7. If the price is $P$, the consumer will buy \((40 - 4P)\) gallons of milk. The consumer surplus will be area \(F\) in Figure 5.15. The height of the consumer surplus triangle is \((10 - P)\), and the base of the triangle (the quantity demanded) is \((40 - 4P)\). The consumer surplus is therefore

\[
CS(P) = \frac{1}{2}(10 - P)(40 - 4P),
\]

which simplifies to

\[
CS(P) = 200 - 40P + 2P^2.
\]

We can verify that this formula gives us the consumer surplus when the price of milk is $3 per gallon. Then

\[
CS(3) = 200 - 40(3) + 2(3)^2 = 98,
\]

which is the same answer we calculated in part (a). Similarly, when the price is $2, \(CS(2) = 200 - 40(2) + 2(2)^2 = 128\), which is the same value of consumer surplus we determined in (b).

Note, too, that the consumer surplus will be zero if the price is $10 because none of the demand curve lies above that price. Our formula also confirms this because

\[
CS(10) = 200 - 40(10) + 2(10)^2 = 0.
\]

**Solution** If the price drops from $3 to $2, the consumer will buy 32 gallons of milk. Consumer surplus will *increase* by the areas \(H\) and \(I\). The increase will therefore be $28 + $2 = $30. The total consumer surplus will now be $128, that is, the sum of areas \(G\), \(H\), and \(I\).

**Similar Problem:**

If we have a demand curve, we can calculate the consumer surplus for any price we might choose. Let’s do this using the demand curve for milk in Learning-By-Doing Exercise 5.7. If the price is \(P\), the consumer will buy \((40 - 4P)\) gallons of milk. The consumer surplus will be area \(F\) in Figure 5.15. The height of the consumer surplus triangle is \((10 - P)\), and the base of the triangle (the quantity demanded) is \((40 - 4P)\). The consumer surplus is therefore

\[
CS(P) = \frac{1}{2}(10 - P)(40 - 4P),
\]

which simplifies to

\[
CS(P) = 200 - 40P + 2P^2.
\]

We can verify that this formula gives us the consumer surplus when the price of milk is $3 per gallon. Then

\[
CS(3) = 200 - 40(3) + 2(3)^2 = 98,
\]

which is the same answer we calculated in part (a). Similarly, when the price is $2, \(CS(2) = 200 - 40(2) + 2(2)^2 = 128\), which is the same value of consumer surplus we determined in (b).

Note, too, that the consumer surplus will be zero if the price is $10 because none of the demand curve lies above that price. Our formula also confirms this because

\[
CS(10) = 200 - 40(10) + 2(10)^2 = 0.
\]
UNDERSTANDING CONSUMER SURPLUS FROM THE OPTIMAL CHOICE DIAGRAM

We have now learned how to understand consumer surplus as the area under a demand curve and above the price paid. But we also need to see how to measure the net benefits to a consumer using the diagram of optimal choice.

Compensating Variation and Equivalent Variation

How can we estimate the monetary value that a consumer would assign to a change in the price of a good, for example, if the price of a good falls from \( P_1 \) to \( P_2 \)? Here, we study two ways to answer this question. First, one could ask: How much money would the consumer be willing to give up after the price reduction to make her just as well off as she was before the price change? We call the change in income necessary to restore the consumer to the initial level of utility the compensating variation. It is the change in income that would just compensate her for the price change.\(^9\)

In an optimal choice diagram, such as Figure 5.16, the compensating variation is the difference between the consumer’s income (the amount of income necessary to buy \( A \) at the old price or \( C \) at the new price) and the amount she would have to spend to purchase the decomposition basket \( B \) at the new price. Recall that basket \( B \) is determined by finding where a line parallel to the final budget line is tangent to the initial indifference curve.

We could also measure the monetary effect of a price change in a different way. We could ask how much money we would have to give the consumer before a price reduction to keep her as well off as she would be after the price change. We call the change in income necessary to hold the consumer at the final level of utility as price changes the equivalent variation.\(^10\) To see the equivalent variation on the optimal choice diagram, we need to introduce another basket (call it \( E \)) on the final indifference curve. To determine basket \( E \), we need to find where a line parallel to the initial budget line is tangent to the final indifference curve. In an optimal consumption diagram, the equivalent variation is the difference between the consumer’s income (again, the amount of income necessary to buy \( A \) at the old price or \( C \) at the new price) and the expenditure needed to purchase basket \( E \) at the old price.

In graphical terms, the compensating and equivalent variations are simply two different ways of measuring the distance between the initial and final indifference curves (see Figure 5.16(a)). If the price of clothing (the good the \( y \) axis measures) is 1, the compensating variation will be the length of the segment \( KL \). If the price of \( y \) is $1, then the segment \( OK \) measures the consumer’s income. The segment \( OL \) measures the expenditures she would need to buy basket \( B \) at the new price of food. The difference (the segment \( KL \)) is the compensating variation. Because baskets \( B \) and \( A \) are on the same indifference curve, the consumer would accept a reduction in income of \( KL \) if she can buy food a lower price.

\(^9\)If the price of the good were to rise from \( P_1 \) to \( P_2 \), we would have to answer the following questions to find out the size of the compensating variation: How much money (i.e., additional income) would we have to give the consumer after the price increase to make her just as well off as she was before the price change?\(^9\)

\(^10\)Suppose the price were to increase (instead of decrease). The equivalent variation would tell us how much money the consumer would be willing to give up before the price increase to keep her as well off as she would be after the price change.
5.3 Consumer Surplus

FIGURE 5.16 Compensating and Equivalent Variations
In (a), there are positive income effects (C lies to the right of B, and E lies to the right of A). In this case the equivalent variation (the length of the segment JK) is larger than the compensating variation (KL). The price of y is 1. In panel (b), the utility function is quasi-linear. Therefore, the indifference curves are parallel as y increases, and there is no income effect (C lies directly above B, and E lies directly above A). The compensating variation (JK) and equivalent variations (KL) are equal. The price of y is 1.
The equivalent variation will be the length of the segment $KJ$. As before, the segment $OK$ measures the consumer’s income. The segment $OJ$ measures the expenditures she would need to buy basket $E$ at the old price of food. The difference (the segment $KJ$) is the equivalent variation. Because baskets $E$ and $C$ are on the same indifference curve, she would require an extra income of $JK$ if she is asked to buy food at the initial higher price instead of at the lower final price.

In general the sizes of the compensating variation (the segment $KL$) and the equivalent variation (the segment $KJ$) will not be the same. That is why one must be careful when trying to measure the monetary value that a consumer associates with a price change.

However, if the utility function is quasilinear, the equivalent and compensating variations will always be the same. For a quasilinear utility function, the vertical distance between any two indifference curves will be the same, regardless of the amount of food consumed. With a quasilinear utility function (illustrated in Figure 5.16(b)), there is no income effect on $x$ as the price of $x$ changes. Basket $C$ will always lie directly above (or below) basket $B$. Basket $E$ will always lie directly above (or below) basket $A$. The vertical distance between the indifference curves will always be the same, whether measured as the length of the segment $AE$ or the length of the segment $BC$. The length of $BC$ is the same as the length of $KL$, the compensating variation. The length of $AE$ is the same as the length of $KJ$, the equivalent variation. That is why the equivalent and compensating variations will be identical when the utility function is quasi-linear.

We now work through an exercise that illustrate the following important point: If there is no income effect, the compensating variation and equivalent variation will give us the same measure of the monetary value that a consumer would assign to the reduction in price of the good. Further, the area under the demand curve (the consumer surplus) will be the same as the compensating variation and equivalent variation. Therefore, with no income effect the area under the demand curve exactly measures both the compensating and equivalent variations.

### LEARNING-BY-DOING EXERCISE 5.8

**Compensating and Equivalent Variations with No Income Effect**

In Learning-By-Doing Exercise 5.6, a student consumed chocolate and “other goods” with the quasi-linear utility function $U(x, y) = 2\sqrt{x} + y$. She had an income of $10 per day, and the price of the composite good $y$ was $1 per unit.

---

11Suppose the utility function $U(x, y)$ is quasi-linear, so that $U(x, y) = f(x) + ky$, where $k$ is some positive constant. Since $U$ always increases by $k$ units whenever $y$ increases by one unit, we know that $MU_y = k$. Therefore, the marginal utility of $y$ is constant. For any given level of $x$, $\Delta U = k\Delta y$. So the vertical distance between indifference curves will be $y_2 - y_1 = (U_2 - U_1)/k$. Note that this vertical distance between indifference curves is the same for all values of $x$. That is why the indifference curves are parallel as we increase $y$. 


5.3 Consumer Surplus

**Problem**

(a) What is the compensating variation of the reduction in the price of chocolate?

**Solution** Consider the optimal choice diagram in Figure 5.17. The compensating variation answers the following question: How much money would the consumer be willing to give up after the price reduction to make her just as well off as she was before the price change? The compensating variation is the difference between her income ($10) and what she would have to spend to

![Figure 5.17: Compensating and Equivalent Variations with No Income Effect](image-url)

FIGURE 5.17 Compensating and Equivalent Variations with No Income Effect

Here the price of chocolate drops from $0.50 to $0.20 per ounce. With an income of $10, the consumer could purchase \( A \) (on the indifference curve \( U_1 \)) at the initial price. After the price reduction she can buy an equally satisfactory basket \( B \) with only $7. Therefore, her compensating variation is $3, which is how much money she would give up after the price reduction to remain just as well off as she was before the price change. The same consumer could buy basket \( C \) at the final price of chocolate ($0.20 per ounce) with an income of $10. She could then reach indifference curve \( U_2 \). If the price of chocolate were $0.50, the least costly basket on the indifference curve \( U_2 \) is basket \( E \), which would cost her $13. Therefore, her equivalent variation is $3 ($13 − $10), which is how much extra income she would need before the price reduction to be as well off as she would be at the lower income ($10) after the price change.
CHAPTER 5  The Theory of Demand

As we have already noted, if there is an income effect, the compensating variation and equivalent variation will give us the different measures of the monetary value that a consumer would assign to the reduction in price of the good. These measures will generally be different from the area under the demand curve. If the income effect is small, the equivalent and compensating variations may be close to one another, and then the area under the demand curve will be a good approximation (although not an exact measure) of the compensating and equivalent variations.
5.3 Consumer Surplus

When the price of chocolate falls from $0.50 per ounce to $0.20 per ounce, the consumer increases consumption from 4 ounces to 25 ounces per day. Her consumer surplus increases by the shaded area, that is, by $3 per day.

**LEARNING-BY-DOING EXERCISE 5.9**

**Compensating and Equivalent Variations with an Income Effect**

In Learning-By-Doing Exercise 5.4 the consumer had the utility function $U(x, y) = xy$. She had an income of $72 per day, and the price of clothing (measured by $y$) was $1 per unit. Suppose the price of food falls from $9 to $4 per unit.

**Problem**

(a) What is the compensating variation of the reduction in the price of food?

**Solution** Consider the optimal choice diagram in Figure 5.19. The compensating variation answers the following question: How much money would the consumer be willing to give up after the price reduction to make her just as well off as she was before the price change? The compensating variation is the difference between her income ($72) and what she would have to spend to purchase the decomposition basket $B$ at the new price of food of $4. How much money would she need to spend to purchase basket $B$ at the new price? The
CHAPTER 5  The Theory of Demand

The answer is \( P_x x + P_y y = 4(6) + 1(24) = 48 \). The consumer would be willing to have her income reduced from $72 to $48 (a change of $24) if the price of food falls from $9 to $4. Therefore, the compensating variation associated with the price reduction is $24.

Problem

(b) What is the equivalent variation of the reduction in the price of food?

Solution

Now we need to determine the equivalent variation. How can we find the basket \( E \) in Figure 5.19? We know two things. First, basket \( E \) lies on the final indifference curve. Therefore, we know that \( U = x y = 324 \). Second, the tangency condition tells us that at \( E \), the slope of the indifference curve

![FIGURE 5.19 Compensating and Equivalent Variation with an Income Effect](image)
Now let’s see what happens if we measure consumer surplus using the area under the demand curve for chocolate. In Learning-By-Doing Exercise 5.4, we showed that her demand for chocolate is \( x = \frac{100}{2} I / P_x \). Figure 5.20 shows her demand curve when her income is $72. As the price of chocolate falls from $9 to $4 per unit, her consumption rises from 4 units to 9 units. If we use the shaded area in Figure 5.20 to measure the increase in consumer surplus, we would conclude that her consumer surplus increases by about $29.20. Note that the size of the shaded area ($29.20) is different from both the compensating variation ($24) and the equivalent variation ($36).

This exercise illustrates that the area under the demand curve will not measure exactly either the compensating variation or the equivalent variation when the income effect is not zero. Generally speaking, the compensating variation and equivalent variation will become closer to each other (and to the area under the demand curve) when the income effect is small.

**Similar Problem:** No similar problem.

---

**FIGURE 5.20 Approximate Consumer Surplus**

When the price of food falls from $9 per unit to $4 per unit, the consumer increases her food consumption from 4 units to 9 units. If we measure the shaded area to estimate her increase in consumer surplus, we would calculate an increase in consumer surplus of $29.20. However, this is greater than the compensating variation ($24) and less than the equivalent variation ($36). The size of the area under the demand curve differs from both the compensating variation and the equivalent variation because the income effect is not small.
CHAPTER 5 The Theory of Demand

5.4 MARKET DEMAND

Businesses often have information about the demands for their products in different market “segments.” For example, a seller of computer software may know that there are academic users of software and business customers. If the seller wants to know the market demand curve for its software, it can add up the demands for the two segments to find the market demand. Similarly, a rental car agency may be aware that there are two types of customers in its market, business travelers and vacation travelers. If the agency knows the demand curves for the two segments, it can add their demands to determine the market demand for rental cars.

Where do market demand curves come from? We illustrate an important principle in this section: The market demand curve is the horizontal sum of the demands of the individual consumers. This principal holds whether two consumers, three consumers, or a million consumers are in the market.

Let’s work through an example how to derive a market demand from individual consumer demands. To keep it simple, suppose only two consumers are in the market for orange juice. The first is “health conscious” and likes orange juice because of its nutritional value and its taste. The second column of Table 5.3 tells us how many liters of orange juice he would buy each month at the prices listed in the first column. The second user (a “casual consumer” of orange juice) also likes its taste, but is less concerned about its nutritional value. The third column of Table 5.3 tells us how many liters of orange juice she would buy each month at the prices listed in the first column.

To find the total amount consumed in the market at any price, we simply add the quantities that each consumer would purchase at that price. For example, if the market price is $5 per liter, neither consumer will buy orange juice. If the price is between $3 and $5, only the health-conscious consumer will buy it. Thus, if the price in the market is $4 per liter, he will buy 3 liters, and the market demand will also be 3 liters. If the price in the market is $3 per liter, the market demand will be 6 liters.

Finally, if the market price is below $3, both consumers will purchase orange juice. If the price is $2 per liter, the market demand will be 11 liters. At a price of $1 the market demand will be 16 liters.

In Figure 5.21 we show both the demand curve for each consumer and the market demand. The thick line is the demand curve is for the market for orange juice.

### TABLE 5.3
Market Demand for Orange Juice

<table>
<thead>
<tr>
<th>Price ($/liter)</th>
<th>Health Conscious (Liters/Month)</th>
<th>Casual (Liters/Month)</th>
<th>Market Demand (Liters/Month)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td>4</td>
<td>16</td>
</tr>
</tbody>
</table>
Finally, we can describe the three demand curves algebraically. Let $Q_h$ be the quantity demanded by the health conscious consumer, $Q_c$ be the quantity demanded by the casual consumer, and $Q_m$ be the quantity demanded in the whole market (which contains only the two consumers in this exercise). See if you can write down the three demand functions $Q_h(P)$, $Q_c(P)$ and $Q_m(P)$.

As you can see in Figure 5.21, the demand curve for the health conscious consumer is a straight line; he buys orange juice only when the price is below $5 per liter. The equation of the demand curve for this segment is

$$Q_h(P) = \begin{cases} 15 - 3P, & \text{when } P \leq 5 \\ 0, & \text{when } P > 5 \end{cases}$$

The demand curve for the casual consumer is also a straight line; she buys orange juice only when the price is below $3 per liter. The equation of the demand curve for this segment is

$$Q_c(P) = \begin{cases} 6 - 2P, & \text{when } P \leq 3 \\ 0, & \text{when } P > 3 \end{cases}$$

As the graph shows, the market demand curve is \textit{kinked}, with connecting straight lines. When the price is higher than $5, neither consumer buys orange juice. When the price is between $3 and $5, only the health conscious consumer
buys orange juice. Therefore, over this range of prices, the market demand curve is the same as the demand curve for the health conscious consumer. Finally, when the price is below $3, both consumers buy orange juice. So the market demand $Q_m(P)$ is just the sum of the segment demands $Q_h(P) + Q_c(P) = (15 - 3P) + (6 - 2P) = 21 - 5P$. Therefore, the market demand $Q_m(P)$ is

$$Q_m(P) = \begin{cases} 
21 - 5P, & \text{when } P \leq 3 \\
15 - 3P, & \text{when } 3 \leq P \leq 5 \\
0, & \text{when } P > 5 
\end{cases}$$

You can verify that the descriptions of the demand curves in the table, in the graph and with algebra are all consistent with one another.

The exercise demonstrates that you must be careful when you add segment demands to get a market demand curve. First, since the construction of a market demand curve involves adding quantities, you must write the demand curves in the normal form (with $Q$ expressed as a function of $P$), rather than using the inverse form of the demand (with $P$ written as a function of $Q$).

Second, you must pay attention to the range of prices for which the underlying segment demands are positive when you add the segment demands algebraically. If you simply added the equations for the segment demands to get the market demand $Q_m = Q_h(P) + Q_c(P)$, this expression would not be valid for a price above $3$. For example, if the price is $4$, the expression $Q_m = 21 - 5P$ would tell you that the quantity demanded in the market would be 1 liter. Yet this is incorrect, because according to Table 5.3, the correct quantity demanded in the market and that price is 3 liters. See if you can figure out why this approach leads to an error. (If you give up, look at the footnote.)

$^{12}$The error arises because we derived the market demand equation $Q_m = 21 - 5P$ by adding $Q_h(P) = 15 - 3P$ and $Q_c(P) = 6 - 2P$. According to these segment demand equations, when $P = 4$, $Q_h(P) = 3$ and $Q_c(P) = -2$. Sure enough the sum is 1. But you are assuming that the casual consumer demands a quantity of orange juice "2 liters of orange juice when the price is $4", and this is economic nonsense! The expression for the demand of the casual consumer $Q_c(P) = 6 - 2P$ is not valid at a price of $4$. At this price, $Q_c(P) = 0$, not $-2$. 

5.5 NETWORK EXTERNALITIES

Thus far we have been assuming that one person’s demand for a good is independent of everyone else’s demand. For example, the amount of chocolate a consumer wants to purchase depends on the consumer’s income, the price of chocolate, and possibly other prices, but not on anyone else’s demand for chocolate. This assumption enables us to find the market demand curve for a good by adding up the demand curves of all of the consumers in the market.

Yet, for some goods, a consumer’s demand does depend on how many other people purchase the good. In that case, there are network externalities. If one consumer’s demand for a good increases with the number of other consumers who buy the good, the externality is positive. If the amount a consumer demands increases when fewer other consumers have the good, the externality is negative. Many goods or services have network externalities.
EXAMPLE 5.4

Network Externalities in Physical Networks

We can easily understand why products like telephones or a fax machines have positive network externalities. A telephone is useless unless at least one other telephone exists to communicate with. For most people, a telephone becomes more useful as the number of other people with telephones increases.

Imagine a telephone network with only two subscribers. Each person on the network would be able to call only one other person. Thus, it would be possible to make two calls on that network. If we add a third subscriber to the network, each person can call two other people. It will now be possible to make a total of six calls because each of the three subscribers can call the other two subscribers. More generally, in a telephone network with \( N \) subscribers, the addition of another subscriber allows for \( 2N \) additional calls to be made. If consumers value being able to call (and be called by) more people, then the value of telephone service to an individual rises as the number of the subscribers increases.

Many settings beyond telephone and fax networks have positive network externalities. Positive network externalities are perhaps nowhere more pronounced than in the case of the Internet. In the 1990s the number of sites on the Internet and the number of individuals and firms with access to it grew rapidly. For most people, the value of access to the Internet increases as the number of sites and other people with Internet access grows. These strong positive network externalities have surely helped Internet traffic snowball. Some estimates suggest that the amount of traffic on the Internet doubles every 120 days.

Although we can often find network externalities in physical networks (as in Example 5.3), we may also see them in other settings (sometimes called virtual networks because there is no physical connection among consumers). For example, a piece of computer software (such as Microsoft Word) has some value to prepare written documents even if that software had only one user. However, the product becomes more valuable to any one user when it has many other users. A virtual network of users makes it possible to exchange and process documents with the software.

A virtual network may also be present if a good or service requires two complementary components to have value. For example, a computer operating system, such as Microsoft Windows 98, only has value if software applications exist that can run on the operating system. The operating system is more valuable when many applications can be used with the operating system. A software application also has a higher value if it runs on a widely accepted operating system.

Finally, positive network externalities can occur if a good or service is a fad. We often see fads for goods and services that affect lifestyles, such as fashions of clothing, children’s toys, or beer. Advertisers and marketers often try to highlight the popularity of a product as part of its image.

Figure 5.22 illustrates the effects of a positive network externality. The graph shows a set of market demand curves for connections to the Internet. For this
The bandwagon effect is a positive network externality that refers to the increase in the quantity of a good demanded as more consumers buy it. What happens to the demand for access to the Internet if the monthly charge for access falls from $20 to $10? Without network externalities, the quantity demanded would increase from 30 to 38 million subscribers because of the pure price effect. But this increase in subscribers leads even more people to want access because they can reach more people with services such as e-mail. The positive externality (a bandwagon effect) adds another 22 million subscribers to the Internet.

For example, let’s assume that a connection to the Internet refers to a subscription to a provider of access to the Internet, such as the connections America Online or Microsoft Network provides. The curve $D_{30}$ represents the demand if consumers believe that 30 million subscribers have access to the internet. The curve $D_{60}$ represents the demand if consumers believe that 60 million subscribers have access. Suppose initially that access costs $20 per month, and that there are 30 million subscribers (point A in the graph).

What happens if the monthly price of access drops to $10? If there were no positive network externality, the quantity demanded will simply change as we move along $D_{30}$. The quantity of subscriptions will grow to 38 million lines (point B in the graph). However, there is a positive network externality; as more people use e-mail and other Internet features, more people will want to sign up. Therefore, at the lower price, the number of consumers wanting access will be even greater than a movement along $D_{30}$ to point B would indicate. The total number of subscriptions actually demanded at a price of $10 per month will grow to 60 million (point C in the graph). The total effect of the price decrease is an increase of 30 million subscribers. The total effect is the pure price effect of 8 million new subscribers (moving from point A to point B) plus a bandwagon effect of 22 million new subscribers (moving from point B to point C). A bandwagon effect refers to the increased quantity demanded as more consumers are connected to the internet. Thus, a demand curve observed with positive network externalities (such as the heavy demand curve in Figure 5.22) is more elastic than a demand curve with no network externalities (such as $D_{30}$).
For some the quantity demanded may decrease when more people have the good. Then there is a negative network externality. Rare items, such as Stradivarius violins, Babe Ruth baseball cards, and expensive automobiles are examples of such goods. These goods enjoy a snob effect. The snob effect is a negative network externality that refers to the decrease in the quantity of a good that is demanded as more consumers buy it. A snob effect may arise because consumers value being one of the few to own a particular type of good. We might also see the snob effect if the value of a good or service diminishes because congestion increases when more people purchase that good or service.

Figure 5.23 shows the effects of a snob effect. The graph illustrates a set of market demand curves for memberships to a health and fitness club. The curve $D_{1000}$ represents the demand if consumers believe the club has 1,000 members. Similarly, the curve $D_{1300}$ shows the demand if consumers believe it has 1,300 members. Suppose initially a membership costs $1,200 per year, and that the club has 1,000 members (point A in the graph).

What happens if the membership price decreases to $900? If consumers believe that the number of members will stay at 1,000, 1,800 would actually want to join the club (point B in the graph). However, the fitness club will become more congested as more members join, and would shift the demand curve inward. The total number of memberships actually demanded at a price of $900 per month will grow only to 1,300 (point C in the graph). The total effect of the price decrease is an increase of 300 members. The total effect is the pure price effect of...
800 new members (moving from point $A$ to point $B$) plus a snob effect of $-500$ members (moving from point $B$ to point $C$). A demand curve observed with negative network externalities (such as the thick demand curve in Figure 5.23) is less elastic than a demand curve without network externalities (such as $D_{1000}$).

As we have already seen, the model of optimal consumer choice has many everyday applications. Let’s examine a consumer’s choice of how much to work.

Let’s divide the day into two parts, the hours when an individual works and the hours where he pursues leisure. Why does the consumer work at all? Because he works, he earns an income, and he uses the income to pay for the activities he enjoys in his leisure time. The term leisure includes all nonwork activities, such as eating, sleeping, recreation, and entertainment. We assume that the consumer likes leisure activities.

Let’s suppose the consumer chooses to work $L$ hours per day. Since a day has 24 hours, the time available for leisure will be the time that remains after work, that is, $(24 - L)$ hours.

The consumer is paid an hourly wage rate $w$. Thus, his total daily income will be $wL$. He uses the income to purchase units of a composite good measured by $y$. The price of each unit of the composite good is $1$.

The consumer’s utility $U$ depends on the amount of leisure time and the number of units of the composite good he can buy. We can represent the consumer’s decision on the optimal choice diagram in Figure 5.24. On the horizontal axis, we plot the number of hours of leisure each day, which must be no greater than 24 hours. On the vertical axis we represent the number of units of the composite good that he may purchase from his income. Since the price of the composite good is $1$, the vertical axis also measures the consumer’s income.

To find an optimal choice of leisure and other goods, we need a set of indifference curves and a budget constraint. The figure shows a set indifference curves for which the marginal utility of leisure and the composite good are both positive. Thus $U_5 > U_4 > U_3 > U_2 > U_1$. The indifference curves are bowed in toward the origin, so there is also a diminishing marginal rate of substitution.

The consumer’s budget line for this problem will tell us all the combinations of the composite good $y$ and hours of leisure $(24 - L)$ that the consumer can choose. If the consumer does no work, he will have 24 hours of leisure, but no income to spend on the composite good. This corresponds to point $A$ on the budget line in the graph.

The location of the rest of the budget line depends on the wage rate $w$. Suppose the wage rate is $5$ per hour. This means that for every unit of leisure the consumer gives up to work, he can buy 5 units of the composite good. The budget line will have a slope of $-5$. If the consumer works 24 hours per day, his income would be $120$, and could buy 120 units of the composite good. This is $B$ on the budget line. The consumer’s optimal choice will then be at basket $E$. The diagram tells us that when the wage rate is $5$, the consumer will work 8 hours.

For any wage rate, the slope of the budget line is $-w$. In the figure budget lines are drawn for five different values of the wage rate ($5$, $10$, $15$, $20$, and $25$). The graph shows the optimal choice for each wage rate. As the wage rate rises from $5$ to $15$, the number of hours of leisure falls. However, as the wage rate continues to rise, the consumer begins to increase his choice of leisure time.
5.6 The Choice of Labor and Leisure

THE BACKWARD-BENDING SUPPLY OF LABOR

Since a day has only 24 hours, the consumer’s choice about the amount of leisure time is also a choice about the amount of labor he will supply. The optimal choice diagram in Figure 5.24 contains enough information to enable us to construct a curve showing how much labor the consumer will supply at any wage rate. In other words, we can draw the consumer’s supply of labor curve. Figure 5.25 shows this curve.

The points $E'$, $F'$, $G'$, $H'$, and $I'$ in Figure 5.25 correspond respectively to points $E$, $F$, $G$, $H$, and $I$ in Figure 5.24. For example, when the wage rate is $5, the consumer will work 8 hours. The supply of labor rises for wage rates up to $15. When the wage rate reaches $15, the consumer works 11 hours.

However, look what happens when wage rates exceed $15. While we normally think that a higher price (remember, the wage rate is the price of labor) stimulates supply for most goods and services, here a higher wage rate decreases
the quantity of labor the consumer supplies. The supply of labor curve is *backward bending* for wage rates above $15. For example, if the wage rate rises from $15 to $25, the consumer decreases work time from 11 hours to 9 hours.

Why might there be a backward bending region on the supply of labor curve? To understand this think about the income and substitution effects associated with a change in the wage rate.

Look at the optimal choice diagram in Figure 5.24. Instead of having a fixed income, our consumer has a fixed amount of time in the day, 24 hours. That is why the horizontal intercept of the budget line stays at 24 hours, regardless of the wage rate. An hour of work always “costs” the consumer an hour of leisure, no matter what the wage rate is.

However, an increase in the wage rate makes a unit of the composite good look less expensive to the consumer. If the wage rate doubles, the consumer needs to work only half as long to buy as much of the composite good as before. That is why the vertical intercept of the budget line moves up as the wage rate rises. The increase in the wage rate therefore leads to an upward rotation of the budget line, as Figure 5.24 shows.

Two kinds of effects are associated with a wage increase. First, the amount of work required to buy a unit of the composite good falls. This effect alone would induce the consumer to substitute more of the composite good for leisure. The substitution effect leads to less leisure, and therefore more labor.

Second, the consumer feels as though he has more income, again because it takes less work to buy a unit of the composite good. So an income effect will be associated with the wage increase. Since leisure is a normal good for most people, the income effect on the amount of leisure will be positive. This means that the income effect on the amount of labor will be negative.

Now let’s examine the income and substitution effects of a wage increase from $15 to $25. In Figure 5.26, we have drawn the initial budget line (with the wage
rate of $15) and shown the optimal consumption of leisure (13 hours) at point G. The number of hours worked is therefore 11 hours.

Next we draw the final budget line (with the wage rate of $25). The optimal consumption of leisure is 15 hours at point I. Therefore, the consumer works 9 hours.

Finally, we draw the decomposition budget line. This line will be tangent to the initial indifference curve (U₃) and parallel to the final budget line. At the decomposition basket (point J) the number of hours of leisure is 12 hours, and the number of hours worked is therefore also 12 hours.

The substitution effect on leisure is thus $1$ hour (the change in leisure as we move from G to J). The income effect on leisure is $+3$ hours (the change in leisure as we move from J to I). Since the income effect outweighs the substitution effect, the net effect of the change in the wage rate on the amount of leisure is $+2$ hours. Put another way, the net effect of the increase in the wage rate on the amount of labor is $-2$ hours.

**FIGURE 5.26** Optimal Choice of Labor and Leisure

This figure helps us understand why the supply of labor curve in Figure 5.25 is backward bending for wage rates above $15. Suppose the wage rate rises from $15 to $25. The substitution effect reduces the amount of leisure (and increases the amount of work) by 1 hour. But the income effect increases the amount of leisure (and decreases the amount of work) by 3 hours. Thus, although the substitution effect would induce the consumer to work more at higher wages, he will actually work less because the income effect outweighs the substitution effect.
EXAMPLE 14.4

The Supply of Nursing Services

Medical groups and hospitals have long had difficulty attracting enough workers. They have often increased pay to stimulate the supply of medical workers. Yet raising wage rates may not always increase the amount of labor supplied.

In 1991 the *The Wall Street Journal* described some of these difficulties in an article titled “Medical Groups Use Pay Boosts, Other Means to Find More Workers.” According to the article, the American Hospital Association concludes, “Pay rises may have worsened the nursing shortage in Massachusetts by enabling nurses to work fewer hours.”

Why might this have happened? We can use Figure 5.26, which we have already seen, to depict an optimal choice diagram for an individual nurse who is deciding how much to work. A higher wage may induce a consumer to pursue more leisure, and thus fewer hours worked. In other words, many nurses may be on the backward-bending region of their supply curve for labor.

Since wage increases alone do not always attract more workers, employers have resorted to other strategies. For example, the article in *The Wall Street Journal* states that the M.D. Anderson Cancer Center at the University of Texas gave employees a $500 bonus if they referred new applicants who took “hard-to-fill” jobs. The Texas Heart Institute in Houston recruited nurses partly by showcasing prospects for promotion. The University of Pittsburgh Medical Center started an “adopt-a-high-school” program to encourage students to enter the health care sector, and reimbursed employees’ tuition fees when they enrolled in programs to increase their skills.

In sum, we will see the backward bending region of the labor supply curve when the income effect associated with a wage increase outweighs the substitution effect. We will observe the upward-sloping region of the labor supply curve when the substitution effect outweighs the income effect.

5.7

CONSUMER PRICE INDEX

The Consumer Price Index (CPI) is one of the most important sources of information about trends in consumer prices and inflation in the United States. It is often viewed as a measure of the change in the cost of living and is used extensively for economic analysis in both the private and public sector. For example, in contracts among individuals and firms, the prices at which goods are exchanged are often adjusted over time to reflect changes in the CPI. In negotiations between labor unions and employers, adjustments in wage rates often reflect past or expected future changes in the CPI.

The CPI also has an important impact on the budget of the federal government. On the expenditure side, the government uses the CPI to adjust payments to Social Security recipients, to retired government workers, and for many entitlement programs such as food stamps and school lunches. As the CPI rises, the government’s payments increase. And, changes in the CPI also affect how much money the gov-
government collects through taxes. For example, individual income tax brackets are adjusted for inflation using the CPI. As the CPI increases, tax revenues decrease.

Measuring the CPI is not easy. Let’s construct a simple example to see what factors might be desirable in designing a CPI. Suppose we have only one consumer, who buys only two goods, food and clothing. In the year 1, a unit of food cost \( P_{F1} = $3 \), and a unit of clothing cost \( P_{C1} = $8 \). The consumer had an income of $480, and faced the budget line \( BL_1 \). He purchased basket \( A \), containing 80 units of food and 30 units of clothing. Figure 5.27 depicts the optimal basket \( A \), located on indifference curve \( U_1 \).

Now suppose that in year 2, the prices of food and clothing increase to \( P_{F2} = $6 \) and \( P_{C2} = $9 \). How much income will the consumer need in year 2 to be as well off as in year 1, that is, to reach the indifference curve \( U_1 \)? The new budget line he requires \( (BL_2) \) will be tangent to \( U_1 \) and have a slope reflecting the new prices, \( -\frac{P_{F2}}{P_{C2}} = -2/3 \). At the new prices, the least costly combination of food and clothing on the indifference curve is at basket \( B \), with 60 units of food and 40 units of clothing. The total expenditure necessary to buy basket \( B \) at the new prices is \( P_{F2}(60) + P_{C2}(40) = $720 \).

In principle, the CPI should measure the percentage increase in expenditures that would be necessary for the consumer to remain as well off in year 2 as he was in year 1. In the example, the expenditures increased from $480 in year one to

**FIGURE 5.27** Substitution Bias in the Consumer Price Index

In year 1 the consumer has an income of $450, the price of food is $3, and the price of clothing is $8. The consumer chooses basket \( A \). In year 2 the price of food rises to $6, and the price of clothing rises to $9. The consumer could maintain his initial level of utility at the new prices by purchasing basket \( B \), costing $720. An ideal cost of living index would be 1.5 \( (\frac{$720}{480}) \), telling us that the cost of living has increased by 50%. By contrast, the Laspeyers index assumes the consumer does not substitute clothing for food as relative prices change. If the consumer continues to buy basket \( A \) at the new prices, he would need an income of $750. The Laspeyers index \( (\frac{$750}{480} = 1.56) \) suggests that the consumer’s cost of living has increased by about 56%, which overstates the actual increase in the cost of living.
$720 in year 2. The “ideal” CPI would be the ratio of the new expenses to the old expenses, that is $720/$480 = 1.5. In other words, at the higher prices, it would take 50% more expenditure in year 2 to make the consumer as well off as he was in year 1. In this sense the “cost of living” in year 2 is 50% greater in year 2 than it was in year 1. In calculating this ideal CPI, we would need to recognize that the consumer would substitute more clothing for food when the price of food rises relative to the price of clothing, moving from the initial basket \( A \) to basket \( B \).

Note that to determine the ideal CPI, the government would need to collect data on the old prices and the new prices and on changes in the composition of the basket (how much food and clothing are consumed). But, considering the huge number of goods and services in the economy, this is an enormous amount of data to collect. It is difficult enough to collect data on the way so many prices change over time, and even more difficult to collect information on the changes in the baskets that consumers actually purchase.

In practice, to simplify the measurement of the CPI, the government has historically calculated the change in expenditures necessary to buy a fixed basket as prices change, where the fixed basket is the amount of food and clothing purchased in year 1. In our example, the fixed basket is \( A \). The income necessary to buy basket \( A \) at the new prices is \( P_{F2}F_1 + P_{C2}C_1 = (\$6)(80) + (\$9)(30) = \$750 \). If he were given \$750 with the new prices, he would face the budget line \( BL_3 \). If we were to calculate a CPI using the fixed basket \( A \), the ratio of the new expenses to the old expenses, that is, \( \$750/\$480 = 1.5625 \). This index tells us that the consumer’s expenditures would need to increase by 56.25% to buy the fixed basket (that is, the basket purchased in year 1) at the new prices.

As the example shows, the index based on the fixed basket overcompensates the consumer for the higher prices. Economists refer to the overstatement of the increase in the cost of living as the “substitution bias.” By assuming that the consumer’s basket is fixed at the initial levels of consumption, the index ignores the possible substitution that consumers will make toward goods that are relatively less expensive in a later year. In fact, if the consumer were given an income of \$750 instead of \$720 in year 2, he could choose a basket such as \( C \) on \( BL_3 \) and make himself better off than he was at \( A \).

**Example 5.6**

The Substitution Bias in the Consumer Price Index

While economists have long argued that the Consumer Price Index overstates changes in the cost of living, the bias in the CPI took center stage in the 1990s when Congress tried to balance the budget. In 1995 Alan Greenspan, the Chairman of the Federal Reserve, brought this controversy to the fore when he told Con-
progress that the official CPI may be overstating the increase in the true cost of living by perhaps 0.5 to 1.5 percent. The Senate Finance Committee appointed a panel to study the magnitude of the bias. The panel concluded that the CPI overstates the cost of living by about 1.1 percentage points.

While estimates of the impact of the substitution bias are necessarily imprecise, they are potentially very important. Greenspan estimated that if the annual level of inflation adjustments to indexed programs and taxes were reduced by 1 percentage point, the annual level of the deficit would be lowered by as much as $55 billion after five years. The Office of Management and Budget estimated that in fiscal year 1996, a one percent increase in the index led to an increase in government expenditures of about $5.7 billion, as well as a decrease in tax revenues of about $2.5 billion.

The government has long been aware of the need to periodically update the “fixed basket” used in the CPI calculation. In fact the basket has been revised approximately every ten years. At the dawn of the new millennium the government continues to investigate ways to improve how it calculates the Consumer Price Index.


CHAPTER SUMMARY

• The income effect for a good is the change in the amount of that good that a consumer would buy as her purchasing power changes, holding price constant. If the good is normal, the income effect will reinforce (move in the same direction) as the substitution effect. If the good is inferior, the income effect will move in the direction opposite from the substitution effect.

• If the good is so strongly inferior that the income effect outweighs the substitution effect, the demand curve would have an upward slope over some prices. Such a good is called a Giffen good. Although the Giffen good is of theoretical interest, it is not of much practical importance.

• Consumer surplus is the difference between what a consumer is willing to pay for a good and what he must pay for it. Without income effects, consumer surplus provides a monetary measure of how much better off the consumer will be when he purchases a good. On a graph the consumer surplus will be the area under an ordinary demand curve and above the price of the good. Changes in consumer surplus can also measure how much better off or worse off a consumer is if the price changes. LBD 5.7

• The compensating variation measures how much money the consumer would be willing to give up after a reduction in the price of a good to make her just as well off as she was before the price change. The equivalent variation measures how much money we would have to give the consumer before a price reduction to keep her as well off as she would be after the price change. If there is an income effect, the compensating variation and equivalent variation will differ, and these measures will also be different from the area under the ordinary demand curve. If the income effect is small, the equivalent and compensating variations may be close to one another, and the area under an ordinary demand curve will be a good approximation (although not an exact measure) of consumer surplus. LBD 5.9

• Without an income effect, the compensating variation and equivalent variation will give us the same measure of the monetary value that a consumer would assign to a change in the price of the good. The area under an
ordinary demand curve will be the same as the compensating variation and equivalent variation. **LBD 5.8**

- The market demand curve for a good is the horizontal sum of the demands of all of the individual consumers in the market (assuming there are no network externalities).

- The bandwagon effect is a positive network externality. With a bandwagon effect, the quantity of a good that is demanded increases as more consumers buy it. The snob effect is a negative network externality. With a snob effect the quantity of a good that is demanded decreases as more consumers buy it.

### REVIEW QUESTIONS

1. What is a price consumption curve for a good?
2. How does a price consumption curve differ from an income consumption curve?
3. What can you say about the income elasticity of demand of a normal good? Of an inferior good?
4. If indifference curves are bowed in toward the origin and the price of a good drops, can the substitution effect ever lead to less consumption of the good?
5. Suppose a consumer purchases only three goods, food, clothing and shelter. Could all three goods be normal? Could all three goods be inferior? Explain.
6. Does economic theory require that a demand curve always be downward sloping? If not, under what circumstances might the demand have an upward slope over some region of prices?
7. What is consumer surplus?
8. Two different ways of measuring the monetary value that a consumer would assign to the change in price of the good are (1) the compensating variation and (2) the equivalent variation. What is the difference between the two measures, and when would these measures be equal?
9. Consider the following four statements. Which might be an example of a positive network externality? Which might be an example of a negative network externality?
   (i) People eat hot dogs because they like the taste, and hot dogs are filling.
   (ii) As soon as Zack discovered that everybody else was eating hot dogs, he stopped buying them.
   (iii) Sally wouldn’t think of buying hot dogs until she realized that all her friends were eating them.
   (iv) When personal income grew by 10 percent, hot dog sales fell.
10. Why might an individual supply less labor (demand more leisure) as the wage rate rises?

### PROBLEMS

1. Figure 5.2(a) shows a consumer’s optimal choices of food and clothing for three values of weekly income: \( I_1 = $40 \), \( I_2 = $68 \) and \( I_3 = $92 \). Figure 5.2(b) illustrates how the consumer’s demand curves for food shift as income changes. Draw three demand curves for clothing (one for each level of income) to illustrate how changes in income affect the consumer’s purchases of clothing.

2. Use the income consumption curve in Figure 5.2(a) to draw the Engel curve for clothing, assuming the price of food is \( P_f \) and the price of clothing is \( P_c \).

3. Show that the following statements are true:
   a) An inferior good has a negative income elasticity of demand.
   b) A good whose income elasticity of demand is negative will be an inferior good.

4. If the demand for a product is perfectly price inelastic, what does the corresponding price consumption curve look like? Draw a graph to show the price consumption curve.

5. Suzy purchases two goods, food and clothing. She has the utility function \( U(x, y) = xy \), where \( x \) denotes the amount of food consumed and \( y \) the amount clothing. The marginal utilities for this utility function are \( MU_x = y \) and \( MU_y = x \).
   a) Show that the equation for her demand curve for clothing is \( y = I/2P \).
b) Is clothing a normal good? Draw her demand curve for clothing when the level of income is $I = 200$. Label this demand curve $D_1$. Draw the demand curve when $I = 300$, and label this demand curve $D_2$.

c) What can be said about the cross-price elasticity of demand of food with respect to the price of clothing?

6. Rick purchases two goods, food and clothing. He has a diminishing marginal rate of substitution of food for clothing. Let $x$ denote the amount of food consumed, and $y$ the amount clothing. Suppose the price of food increases from $P_x$, to $P_x'$. On a clearly labeled graph, illustrate the income and substitution effects of the price change on the consumption of food. Do so for each of the following cases:

   a) Case 1: Food is a normal good.
   b) Case 2: The income elasticity of demand for food is zero.
   c) Case 3: Food is an inferior good, but not a Giffen good.
   d) Case 4: Food is a Giffen good.

7. Some texts define a “luxury good” as a good for which the income elasticity of demand is greater than 1. Suppose that a consumer purchases only two goods. Can both goods be luxury goods? Explain.

8. Scott consumes only two goods, steak and ale. When the price of steak falls, he buys more steak and more ale. On an optimal choice diagram (with budget lines and indifference curves), illustrate this pattern of consumption.

9. Dave consumes only two goods, coffee and doughnuts. When the price of coffee falls, he buys more coffee and more doughnuts.

   a) On an optimal choice diagram (with budget lines and indifference curves), illustrate this pattern of consumption.
   b) Is this purchasing behavior consistent with a quasi-linear utility function? Explain.

10. Suppose that a consumer’s utility function is $U(x, y) = xy + 10y$. The price of $x$ is $P_x$, and the price of $y$ is $P_y$, with both prices positive. The consumer has income $I$. (This problem shows that an optimal consumption choice need not be interior, and may be at a corner point.)

    a) Assume first that we are at an interior optimum. Show that the demand schedule for $x$ can be written as $x = I/2P_x - 5$.
    b) Suppose now that $I = 100$. Since $x$ must never be negative, what is the maximum value of $P_x$ for which this consumer would ever purchase any $x$?
    c) Suppose $P_x = 20$ and $P_y = 20$. On a graph illustrating the optimal consumption bundle of $x$ and $y$, show that since $P_x$ exceeds the value you calculated in part b, this corresponds to a corner point at which the consumer purchases only $y$. (In fact, the consumer would purchase $y = I/P_y = 5$ units of $y$ and no units of $x$.)
    d) Compare the marginal rate of substitution of $x$ for $y$ with the ratio $(P_x/P_y)$ at the optimum in part c. Does this verify that the consumer would reduce utility if she purchased a positive amount of $x$?
    e) Assuming income remains at 100, draw the demand schedule for $x$ for all values of $P_x$. Does its location depend on the $P_y$?

11. Suppose rental cars have two market segments, business travelers and vacation travelers. The demand curve for rental cars by business travelers is $Q_b = 35 - 0.25P$, where $Q_b$ is the quantity demanded by business travelers (in thousands of cars) when the rental price is $P$ dollars per day. No business customers will rent cars if the price exceeds $140 per day.

    The demand curve for rental cars by vacation is, $Q_v = 120 - 1.5P$, where $Q_v$ is the quantity demanded by vacation travelers (in thousands of cars) when the rental price is $P$ dollars per day. No vacation customers will rent cars if the price exceeds $80 per day.

    a) Fill in the table to find the quantities demanded in the market at each price in the table.

<table>
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<th>Price ($/day)</th>
<th>Business (000 cars/day)</th>
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b) Graph the demand curves for each segment, and draw the market demand curve for rental cars.

c) Describe the market demand curve algebraically. In other words, show how the quantity demanded in the market $Q_m$ depends on $P$. Make sure that your algebraic equation for the market demand is consistent with your answers to parts a and b.

d) If the price of a rental car is $60, what is the consumer surplus in each market segment?

12. One million consumers like to rent movie videos in Pulmonia. Each has an identical demand curve for movies. The price of a rental is $P$. At a given price, will
the market demand be more elastic or less elastic than the demand curve for any individual. (Assume there are no network externalities.)

13. Joe’s income consumption curve for tea is a vertical line on an optimal choice diagram with tea on the horizontal axis and other goods on the horizontal axis.
   a) Show that Joe’s demand curve for tea must be downward sloping.
   b) When the price of tea drops from $9 to $8 per pound, the change in Joe’s consumer surplus (i.e., the change in the area under the demand curve) is $30 per month. Would you expect the compensating variation and the equivalent variation resulting for the price decrease to be near $30? Explain.

14. Consider the optimal choice of labor and leisure discussed in the text. Suppose the consumer can work the first 8 hours of the day at a wage rate of $10 per hour, but receives an overtime wage rate of $20 for additional time worked.
   a) On an optimal choice diagram, draw the budget constraint. [Hint: It is not a straight line.]
   b) Draw a set of indifference curves that would make it optimal for him to work four hours of overtime each day.