A NEWSLETTER FOR THE CALCULUS CONSORTIUM BASED AT HARVARD UNIVERSITY

# FOCUS ON CALCULUS

## Comments on Calculus Reform

*Peter D. Lax, New York University*

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SPRING 1995 ISSUE No. 8



There is, at long last, a consensus that the teaching of calculus in American colleges has been asleep for fifty years. Involving a group of reformers in the creation of the AP Calculus Exam is the strongest indication yet that the sleeper is beginning to stir.

During this long slumber, a tremendous amount of detritus has accumulated in the standard calculus course. Vestigial remains and rococo excrescencies obscure its main ideas and applications, and swell the typical calculus text to a length of 1,000 pages or more. The first and most urgent task is to ruthlessly remove all the weeds. The second task, more ambitious and far more challenging, is to create meaningful applications.

To aid in the task of removing ineffective material from the calculus course, and to add to the excellent guidelines provided by the Calculus Consortium based at Harvard, I offer my own list of topics that we, as reformers, should address.

- Let's forget about the word "asymptote." No working mathematician thinks in such terms; we say that a function tends to a constant as $x \to \infty$ tends to ∞ as $x$ tends to $a$.
- Instead of talking about functions continuous at a point, we should discuss uniform continuity on an interval. It is a far easier concept to grasp, and more useful. We should also acquaint students with uniform convergence.
- Most calculus books call a function differentiable if it has a derivative at every point of an interval. This is a bizarre class; no self-respecting analyst would touch it with a 10-foot pole. The natural class is $C^1$ it could be defined as functions for which the difference quotient $D_h f$ tends uniformly to a limit. Most calculus books insist on proving that a differentiable function is continuous. We should keep away from proving things no reasonable person would doubt.
- Let's admit that related rates is the same thing as the chain rule. Calling the same thing by two different names is confusing.
- No differentials, please.
- No L'Hopital's Rule. We are not in the business of teaching rules, but understanding. The thing to be understood is the behavior of a single function near a zero; the behavior of the quotient of two such functions can then be deduced.
- The important trig functions are sine and cosine, for they describe vibrations. The tangent function is secondary. Cotangent, secant and cosecant are merely names for the reciprocals.
- Techniques of integrations should be presented briefly, as ways of transforming one integral into another form, not as a quest for the Holy Grail, the antiderivative.

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## A *Mathematica* Laboratory Component for the CCH Approach

*John W. Hagood, Northern Arizona University*

Originating in two experimental sections five years ago, a computer lab using M*atheniatica* now accompanies all sections of first and second semester calculus at Northern Arizona University. The lab exercises are intended to improve visual perception and understanding of concepts, encourage exploration, and permit sophisticated computations and applications while making them less tedious.

Several years of experimenting with the structure of the labs led to our current organization. Each four-credit section meets four days per week with at least half of one period used to introduce the lab. Students then spend from one to two hours completing the lab during open lab hours in the presence of an experienced lab aide. This organization permits the course in

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**FOCUS ON CALCULUS**

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Readers are encouraged to share their experiences in changing the way calculus is taught by contributing articles for publication in future issues. Please address your correspondence to Editor, FOCUS ON CALCULUS, and mail it to Debra Riegert at John Wiley & Sons, Inc.

The text materials being written by the Calculus Consortium based at Harvard University (CCH) are being published by John Wiley & Sons, Inc. The Preliminary Edition of *Calculus* by Hughes-Hallett, Gleason, et al. is now available for test site use. For information about this text, contact your local Wiley representative, or call the CCH Hotline at (212) 850-6700, ext. 5727.



# FROM THE PUBLISHER

It's spring (in some places more than others) and time to register for the fourth annual summer Conference on the Teaching of Calculus. As you read about it below, you'll notice that it has a new name: The Conference on the Teaching of Mathematics. This year's conference will reflect the impact of reform calculus on the broader mathematics curriculum and include presentations and workshops on the courses that precede and follow calculus. The program promises to be even more information-filled than before, so please use the coupon on the facing page to get your registration in now.

The Conference's shift in focus is mirrored by Wiley's publishing program, too. We are now developing "reform" texts in many of these same areas and will have news of them to share in the months to come. Two of them, *Applied Calculus* and *Multivariable Calculus,* are highlighted on pages 7 and 8.

Also in this issue are detailed looks at the use of projects and computer technology with the CCH text, for which we heartily thank the contributors. If you have information to share, or need more information from us, please contact us at the address at left, or via e-mail at math@jwiley.com. ▲

# Fourth Conference on the Teaching of Mathematics and TICAP Conference

The Calculus Consortium based at Harvard University, in conjunction with the National Science Foundation (NSF), and John Wiley and Sons, Inc., announces the Fourth Conference on the Teaching of Mathematics on June 23-24, 1995, in San Jose, California. Two and four-year college, university, and secondary school faculty are welcome. Hosted by Stanford University, and chaired by Brad Osgood, the conference offers something of interest for everyone involved in the way mathematics is taught. This year's conference will broaden its focus from calculus to include other courses in undergraduate mathematics. A variety of sessions are planned, including differential equations, precalculus, college algebra, linear algebra, interdisciplinary programs, vector calculus, writing in the curriculum, and student reactions to calculus reform.

Keynote speaker Peter Lax, of New York University, will lead off an exciting group of speakers and panelists that includes Frank Avenoso, Nassau Community College; Simon Bernua, University of Texas at El Paso; John Dossey, Illinois State University; Judy Clark and Linda Kime, University of Massachusetts, Boston; Ken Millett, UC Santa Barbara; Lawrence Moore and David Smith, Duke University; Keith Stroyan, University of Iowa; Carl Swenson, Seattle University; Jerry UhI, University of Illinois; and Paul Zorn, St. Olaf's College.

A conference for Advanced Placement calculus teachers entitled "AP's Next Generation" will follow on June 25, 199. This day-long meeting is supported partially by the project Technology Intensive Calculus for AP (TICAP).

Contributed papers of about 15 minutes, an important part of the upcoming conference, are invited in the following categories: Changing Pedagogy, Using Technology, Assessing Students Knowledge, Evaluating New Courses, Special Topics, Topics in Transition: Before Calculus, Calculus, and Beyond Calculus.

If you are interested in submitting a paper, please send the title of the paper, the category from the above list, and a 25-word abstract to Karen or Joe Thrash, Mathematics, University of Southern Mississippi, 730 East Beach Boulevard, Long Beach, MS 39560. You may also e-mail your submissions to calculus@bull.cc.usm.edu, but please be sure to include your U.S. mailing address. Deadline for submissions is April22, 1994. For more information, please contact Kim Lemmonds at John Wiley and Sons, Inc., 605 Third Avenue, New York, NY 10158-0012, telephone (212) 850-6711, ore-mail at math@jwiley.com. To register, please use the coupon on page 3. ▲

# Comments on Reform

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- Go easy on volumes and surface areas of figures of revolution.
- Convergent sequences should be discussed at the very beginning. There is no need to make infinite series a special topic, except for Taylor series.
- Get rid of the special curves, such as the folium of Descartes, the cardoid, etc. We are not teaching taxonomy.

Most of what is presented in old texts as an application of calculus utterly lack an element of surprise. The reform texts do much better, but even here there is lots of room for improvement. To this end, I offer two additional suggestions: first, numerical methods themselves are splendid applications of calculus and second, most technical aspects of calculus should be finished in the first semester with the second semester devoted to in-depth discussions of substantial applications. Although the old guard may resist, I believe that those who understand the issues of calculus reform, especially those who use calculus in their mathematics, have an obligation in the next few years to try out the new approaches.

*This article is adapted from an address given by Peter B. Lax at a meeting about the Advanced Placement exam attended by representatives of calculus is reform projects, tile College Board, and the Educational Testing Service.* ▲

# *Mathematica* Laboratory

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structor to interact with students at the start of each lab. We've found this approach to be more effective than staffing the lab with a separate instructor because students are more likely to value the lab if the primary instructor is present to relate the activities to the course.

We began experimenting with computer exercises in 1990 by installing *Derive* on 20 computers in open university labs. Although this experiment involved only a few sections of calculus, using campus-wide computers was helpful in demonstrating the need for a separate mathematics lab. Through a series of external (NSF-ILI) and internal grants, we were able to acquire 20 Macintosh computers and *Mathematica* for our own lab. To fund regular replacements on a three to four year cycle, each calculus section carries a $30 lab fee. A member of the faculty serves as lab coordinator to oversee maintenance, seek funds, acquire and install equipment and software, appoint and supervise lab aides, and handle scheduling.

### Exercises for the Lab

Many computer lab manuals for calculus are now available from textbook publishers, including three separate manuals for the CCH approach published by Wiley for use with *Matheniatica, Maple,* or *Derive.* In 1990, written lab manuals were scarce, so four faculty members collaborated to produce exercises and projects for our labs. These generally parallel the CCH material, covering some topics in depth and reinforcing other topics that are covered more thoroughly in class. Topics that are covered almost exclusively in the lab include power functions, polynomials and rational functions, continuity, differentiability, development of the derivatives of trigonometric and exponential functions, families of functions, Newton's method, Euler's method, intervals of convergence of Taylor series, and geometric series. Since the lab has only about half as many computers as students in each section, group work is encouraged. Even when some computers are vacant, we've found that most students tend to work in groups.

The following examples of exercises illustrate the nature of our lab component.

- Investigate scale, dominance and end behavior, first by graphing $f(x) = x^4 - 15x^2 + 15x + 7$ on intervals of lengths 0.02, 2, 10, and 80; then by graphing and investigating table data for the same function $f(x)$ and $g(x)=e^{0.2x}$ simultaneously or as a ratio as $x \to \infty$
- Investigate local linearity and approximate the derivative of a function at a point by zooming in on the graph.
- Analyze a family of functions such as $f(x) = bxe^{-bx}$ to determine the effect of

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# *Mathematica* Laboratory

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the parameter or to fit a member of the family to given data.

- Develop the derivatives of sine, cosine, and exponential functions by using the graph of an approximate derivative such as

$$fp(x) = \frac{f(x+0.001) - f(x)}{0.001},$$

- investigating the limit of the difference quotient via tables, and finally using the formal limit feature of *Mathematica.*
- Study minima and maxima and the shapes of curves using instructive but complicated examples.
- Investigate genuine applications involving messy computations, such as finding the extremes of the apparent velocity and acceleration of a rocket rising with constant vertical acceleration in order to determine specifications for tracking devices.
- Compare approximate integration techniques.
- Discover limitations of the computer by integrating across a singularity.
- Investigate the nature of solutions of differential equations using slope fields. We use the package *MacMath* for this because of ease of use, but *Mathematica* can be used.
- Investigate intervals of convergence of Taylor series from the graphs of several Taylor polynomials.
- Demonstrate the divergence of the harmonic series by successively adding terms from one power of three to the next. This causes the partial sum to increase by about one each time.
- Investigate very basic objects and concepts in three dimensions.
- Analyze functions of two variables using cross sections and contour plots.

When we next revise the lab exercises, we intend to highlight the learning objectives of each lab to help the students focus on what they should be gaining from their efforts and, where possible, to increase the exploratory nature of the labs.

## Student Reaction

Student response to our labs is varied. Some students can't get enough of the labs while others question their value. The latter group includes those who struggle with the *Mathematica* syntax throughout the term or those who believe the labs are busy work. Students surveyed last spring generally found the labs to be very helpful in developing visual perception and of some value for doing meaningful problems that would not otherwise be possible, increasing understanding of concepts, providing practice in the use of calculus, and reinforcing ideas presented in class. Lab exercises most appreciated by the students tend to be those with a major visual or graphical emphasis such as three-dimensional graphs, contour plots, slope field analysis of differential equations, the zoom approach to analyzing the derivative, and the analysis of the nature of graphs using first and second derivatives.

The computer lab component has become an important part of our calculus sequence. It has taken and continues to take some time to write exercises and to set up and manage the facility, but we believe the effect on student learning is well worth the effort. ▲

# Students Save with *Mathematica*

John Wiley & Sons, Inc. and Wolfram Research, Inc. are pleased to announce an exclusive offer for calculus instructors and students. Selected Wiley calculus texts are now available together with the Student Version of Wolfram's numeric, symbolic, and graphic computational software, *Mathematica,* at one phenomenal package price.

*Mathematica* is a complete computational environment for learning calculus and mathematics. The Student Version of *Mathematica* has all the capabilities of the professional version of *Mathematica,* although it runs a bit slower. Over 1,000 built-in functions help students find numeric and symbolic solutions to mathematics problems quickly and accurately. Students using *Mathematica* gain a solid understanding of mathematical concepts because they can graph and visualize equations in two and three dimensions. *Mathematica's* user-friendly interface lets students combine live computations, graphics, and text in a single electronic document which can be printed out or submitted electronically to an instructor via the school network.
With this special value, your students can save approximately $50 and have the convenience of using *Mathematica* on their own personal computer. *Mathematica* student versions are available for both Macintosh and Microsoft Windows (no numeric coprocessor is required) and complete student documentation is included in the package, including a student user manual and *Mathematica: The Student Book* by Stephen Wolfram. Several Wiley calculus texts are available shrink-wrapped with the Student Version of *Mathematica.*

The following texts are available with *Mathematica* for a package list price of $129.95:

***Calculus,*** by the Calculus Consortium based at Harvard University: Deborah Hughes-Hallett, Andrew Gleason, et al. (order #12335-8)

***Calculus with Analytic Geometry, SE*** (#12329-3) and *Calculus with Analytic Geometry*, ***Brief SE*** (#12332-3), by Howard Anton, Drexel University

***Salas & Hille's Calculus: One Variable, SE*** **(#12338-2)** and ***Calculus: One and Several Variables, 7E*** (#12339-0), revised by Garret Etgen, University of Houston

The following texts are available with *Matheniatica* for a package list price of $99.95:

***Multivariable Calculus, Draft Version*** (#12333-1) and ***Multivariable Calculus, Preliminary Edition*** (#12334-X, available August, 1995) by the Calculus Consortium based at Harvard University: William G. McCallum, Deborah HughesHallett, Andrew Gleason, et al.

To order any of these book and software packages, ask your bookstore to use the package order codes listed above. For more information on this exclusive offer, please contact your local Wiley representative or send a message to us by e-mail at math@jwiley.com. ▲

# Student Group Projects at Willamette University

*Richard Iltis, Willainette University*

Willamette University is a private liberal arts college of 1600 undergraduate students located in Salem, Oregon. Each year, the six faculty in our mathematics department teach about 12 sections of Calculus I, Calculus II, and a one-semester calculus course. There are 20 to 30 students in a typical class.

Several years ago we began some individual, piecemeal attempts to improve our calculus courses, but our efforts did not coalesce until we adopted the CCH text in the summer of 1992. In our attempts to emphasize concepts rather than skills and to focus the class on the students rather than the teacher, we had already tried using collaborative learning, integrating technology, and encouraging writing. It was the CCH text, along with student group projects, that made all the pieces fit together.

After spending the summer of 1992 attending workshops, reading, and having long discussions, we were still exploring and learning about calculus reform when classes began in the fall. We were very excited about the possibilities: the experience reminded me of a time in graduate school when I finally understood epsilon-delta arguments (after seeing them for the third time). I couldn't wait to show my students the true light of calculus reform!

The first semester, the four of us teaching Calculus I were primed to give common exams and projects: we assigned five projects and gave three mid-semester exams. The students were overwhelmed and told us that, although they learned a lot, there was too much work. Now most of us assign three or four projects and give two mid-semester exams. We learned that it is important for students to have sufficient time to work on their projects and that a good effort requires a meaningful reward. If we want students to take projects as seriously as exams, then projects must count as much as exams toward a final grade. The advice of the many mathematics teachers who have successfully implemented projects, especially that of the faculty at Ithaca College and New Mexico State University, has helped to guide us in their use.

## Why We Assign Projects

Projects are integral to making students active learners: they teach students that all problems are not well formulated with tidy answers. We strive for open-ended problems, but find it difficult to formulate good problems that are both open-ended and approachable. Our projects emphasize concepts and often integrate ideas from different calculus topics. Because these problems are more complex than most students could master on their own, the strength of cooperative effort is emphasized. We feel that cooperative learning is important because it is likely that the students will have to work in groups as well as by themselves after they leave school.

## How We Assign Projects

To emphasize that projects are going to be an important part of the course, we often assign the first project on the first day of class. Along with a syllabus, students receive instructions for working together on projects. It is also helpful to have some class exercises to help students socialize since students do not automatically know how to work together and need some guidance.

We form groups of two, three, or four students in different ways: randomly, by student preference, or by combining students with different talents and experiences. I have learned that random assignments often lead to new student friendships, and when more students in a class know each other, class discussions are livelier. Some change in groups is good, but I do not change group members for each project.

Projects have a firm due date about two weeks after being assigned, and I require a preliminary report to ensure that the groups get started and are on the right track. Groups are encouraged to consult with me whenever they wish.

For the first project, each group meets with me in my office to discuss their progress, plus any questions or ambiguities, a week before the due date. Each group submits one report, which must be typed with a word processor. Formulas and graphs may be drawn by hand, but many students use computers instead. Reports must be well-written in proper English, and be understandable to a calculus student unfamiliar with the project. We emphasize that a good report consists of more than figuring out what needs to be

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# Software Resources on the Internet

Over 40 different mathematical software programs are available from the University of Arizona free of charge. The Arizona Software spans material of interest to students and instructors who are involved with high school and college mathematics. The complete collection includes MS-DOS programs that occupy about 11.5MB of hard disk space. The programs require minimal hardware (CGA graphics, 640K RAM, no mouse) and an easy-to-use menu system maybe downloaded with the software to make it easy for students to access.

The Arizona Software is available on the network fileserver math.arizona.edu. Before connecting, it is recommended that you request complete instructions via e-mail from software@math.arizona.edu. Novices will find the instructions very helpful.
Arizona also hosts four calculus reform newsgroups: math.cch (CCH discussions), math.az.ftp-listing (software available via FTP from math.arizona.edu), math.teaching.styles (discussion of teaching methods), and math.teaching.technology (discussion of technology-based methods). For more information, send e-mail to newsgroups@math.arizona.edu. ▲

# Student Group Projects

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calculated and completing the calculations—a good report will also discuss the relationships and concepts involved.

Since it may be difficult for all group members to participate in writing the report, I also have some individual assignments that require written responses. To ensure that each group member pays attention and understands the project as a whole, my colleague Steve Prothero warns students that an in-class oral report will be required from one randomly chosen member of each group—the reports are given later that day. Again, it is important that projects count for a significant portion of the course grade.

## More Project Tips

- We believe that it is important to explain repeatedly to students the purpose of projects and how they connect with the course.
- Students like project assignments that are couched in a story, whether it is fact or fiction. A story engages the imagination and encourages students to be creative.
- Since we want the projects to emphasize the rule of three, we state that a good analysis will view the problem from multiple perspectives.
- We assume that technology will be used in the projects and we require TI-81 or TI-85 graphing calculators in our calculus classes.
- In order to encourage students to work cooperatively, I try to avoid describing the project with a list of questions that could easily be divided among the group members.

## Project Sources

Some sources of project ideas are: chapter review problems in *Calculus* by HughesHallett, et al.; Peter D.Taylor's *Calculus* (especially Chapter 1); *Calculus* by Hubert, Maceli, Robinson, Schwartz, and Seltzer; *Sttident Research Projects in Calculus* by Cohen, Gaughan, Knoebel, Kurtz, and Pengelley; and MAA *Notes* on calculus. Another possibility is to use an application that will not be covered in class such as marginal analysis in economics or density functions in probability. The problems do not need to be complicated. For example, we have successfully asked, "At what depth would a Japanese fishing float sit?" This is a problem from a standard calculus text; it requires finding the volume of a segment of a sphere and solving a cubic equation.

## Project Examples

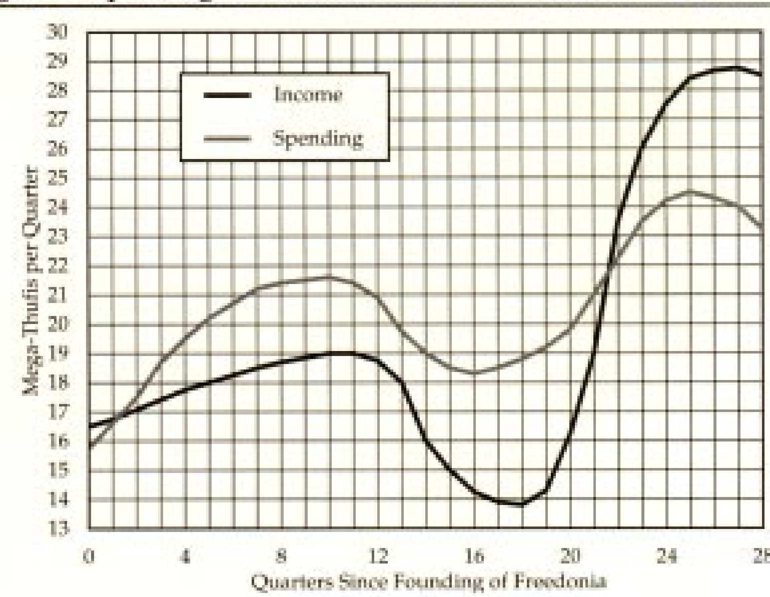
The following project, devised by my colleague Mark Janeba, emphasizes relationships between derivatives and integrals. There are many variations on this project; e.g., calorie intake and metabolism rates for a person during a 24-hour period. *Since its founding, the nation of Freedonia has kept quarterly records of both government income and government spending. When the government spends more than its income in a quarter, we call the difference a deficit; this difference is made up from borrowing, or from savings if any, and the total amount borrowed at any given point is called the national debt. When income exceeds spending for a quarter, we call the difference a surplus (a negative deficit). If a surplus occurs when there is an outstanding debt, the surplus is used to reduce the total debt (at least in Freedonia). Since the monetary unit of Freedonia is the Thufi, quarterly surpluses and deficits have units of Thufis per quarter. Figure 1 shows the income and spending data for Freedonia in the first 28 quarters since its founding. The overall assignment is to discuss the fiscal soundness of the government of Freedonia over this period. Include specifics about when debt and deficit are most, are rising, and rising fastest. In later years, a former Freedonian president will reminisce about his administration, by saying that "while it was not possible to reduce the deficit, we reduced the rate it was growing." At what possible quarters might lie have been elected?*

Another colleague, Frank Zizza, asked students to find a project of their own with the following instructions: *Choose a physical problem. For example, population growth, Newton's law of cooling, bank accounts, or (see any of over* 100 *books in the library on the subject of differential equitations). Derive a differential equation that models the physical problem. This is the most difficult part and you should be very thorough. Solve the differential equation.*

## Student Reactions

Students like projects because it gives them a chance to work together. As instructors, we like projects because students get involved in doing significant mathematics and show that they are willing to reach beyond the classroom and text. I have been surprised by how often students begin with a table of data and use computer software to make calculations and graphs, or use a computer or their calculators to fit curves to the data—all without instruction. ▲

**Figure 1 Spending and Income for the Government of Freedonia**



# Applied Calculus for Business, Life, and Social Sciences

*Patti Frazer Lock, St. Lawrence University*

The Preliminary Edition of a new CCH text, *Applied Calculus,* is scheduled to be available for review in November 1995, and for adoption in Spring 1996 courses. This new calculus text is designed for courses offered to business, life, and social science students. In an applied calculus course, building an understanding of the concepts of calculus is of paramount importance. This text will bring the CCH conceptual approach to students who need to be able to apply calculus in a wide variety of practical situations.

*Applied Calculus* remains true to the spirit of the CCH *Calculus* text. The emphasis is on developing a conceptual understanding of the ideas in calculus, and on using the "rule of three"—ideas are presented from graphical, numerical, and analytical viewpoints wherever possible.

Indeed, it is apparent that graphical and numerical understanding is of even greater value for this audience, while the analytical and symbolic manipulation aspects of the course are of lesser importance.

Examples and problems reflecting applications in business, life, and social sciences are used throughout the text. In addition, because these students often have somewhat weaker mathematical backgrounds, easier examples and problems have been included throughout the book. Students in business, life, and social sciences are far more likely to encounter functions in graphical or numerical form than as a symbolic formula. Therefore, examples and problems requiring difficult algebraic manipulations have been virtually eliminated from the text and have been replaced with more conceptual problems and more problems involving graphs and tables of data.

The topics covered in the text reflect the needs of this audience. Multivariable functions are essential to these students, and the last two chapters of the book enable students to develop a thorough understanding of these functions and their derivatives. On the other hand, the material on the integral has been dramatically reduced.

The conceptual understanding of the definite integral as area and as total change is covered in depth, but all definite integrals are evaluated numerically rather than symbolically to reflect what is actually done in practice.

The traditional applied calculus course is packed with so many topics that there is very little time for developing in-depth understanding. The CCH *Applied Calculus* text has been streamlined as much as possible to ensure that the emphasis is on understanding the important concepts rather than on covering a large amount of dense material. The approach to this book has been the same as the approach taken with the original CCH *Calculus* text: talk to the user departments and then begin with a clean slate and include only material that is needed for an understanding of the concepts of calculus or by the user departments. We have tried to allow time in the syllabus for instructors to slow down and focus on student understanding.

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## Tentative Table of Contents for *Applied Calculus*

## Applied Calculus

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*Applied Calculus* is designed for a one-year course, and a tentative table of contents is included on page 7. Input on this project is welcome; in particular, we would love to hear from anyone with interesting and relevant examples and problems that would be appropriate for this book. Comments and suggestions may be sent to Patti Frazer Lock, Department of Mathematics, St. Lawrence University, Canton, NY 13617, e-mail: plock@vm.stlawu.edu. For more information ***about Applied Calcu***lus, please contact your Wiley representative, use the CCH Hotline at (212) 850-6700, ext. 5727, or send an e-mail message to math@jwiley.com. ▲

# Upcoming Faculty Workshops

If your institution will be using the CCH materials for the first time in the 1995-1996 academic year, you will want to attend a special two-day workshop preceding this year's conference in San Jose. On June 21 and 22, the workshop will focus specifically on the Calculus Consortium's approach and teaching materials, and offer valuable hands-on sessions and lively discussion of implementation issues. There will also be a one-day workshop on June 22 for instructors using the multi-variable material for the first time in the 1995-1996 academic year.

Workshop participants are encouraged to stay in San Jose and attend the conference on June 23 and 24. The $55 conference registration fee will be waived for all workshop participants. (There is no fee for the faculty development workshop.)

For more information and to receive an application form, please send a description of your department's plans for using the CCH materials in the coming year and a list of those people planning to attend the workshop to Herman 0. Sudholz, Calculus Consortium, Science Center #325, One Oxford Street, Cambridge, MA 02138; telephone (617) 496-5421. Workshop space is limited so don't delay! ▲

# The Challenge of CCH Multivariable Calculus

*William G. McCallum, University of Arizona*

One of the biggest challenges in developing the CCH book on multivariable calculus has been dealing with the wonderfully complex circle of ideas leading up to the divergence theorem and Stokes' theorem. The ideas are tricky enough that even experienced research mathematicians can go astray (believe me). On the other hand, they have such a pleasing geometric flavor that they can be given a good treatment at the undergraduate level—if only one can nudge them into the right arrangement.

Take, for example, the notion of divergence. Traditionally one learns the formula for this in Cartesian coordinates, then the statement of the divergence theorem, and that's about it. Those who go on to physics and engineering gain a deeper appreciation of the physical meaning of the divergence from the study of electromagnetism or fluid flow. Those who go on to higher studies in mathematics learn to understand the divergence as a special case of the exterior derivative. Unfortunately, ten or fifteen years down the road, the latter are teaching the next generation of the former, and the only common ground is, once again, the formula. And so the cycle repeats itself. In the CCH book we have decided to make a thorough presentation of divergence, initially as the outflow per unit volume, then in the guise that is more natural for the study of fluid flow, namely as the rate of change of a moving volume element. We hope students will end up with a better appreciation of what the divergence means.

For a thorough treatment of the buildup from vector fields, through line integrals and flux integrals, to the divergence theorem and Stokes' theorem, one needs more time than is usually allowed. In the current rewrite for the preliminary edition, we have tried to structure this material so that it is clear what can be missed in a quick treatment, and so that there are a number of different stopping points. For example, the chapter on line integrals now has a treatment of Green's theorem; some may wish to stop right there. Others may wish to dip lightly into this chapter, and into the subsequent chapter on flux integrals, in order to save room for the grand finale chapter on Stokes' theorem.

Either way, the student will have a more geometric understanding of the notion of vector fields, line integrals, and flux integrals, which should be useful in future courses. The general approach we used in *Calculus* (giving greater prominence to geometric and numerical understanding) has turned out to be a good guiding principle in the multivariable book as well. ▲

**FOCUS ON CALCULUS**

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