CHAPTER 7 | *IMPULSE AND MOMENTUM*

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9. ***REASONING*** The impulse applied to the golf ball by the floor can be found from Equation 7.4, the impulse-momentum theorem: $\overline{\mathbf{F}}\,\Delta t = m\mathbf{v}_f - m\mathbf{v}_0$. Only the vertical component of the ball's momentum changes during impact with the floor. In order to use Equation 7.4 directly, we must first find the vertical components of the initial and final velocities. We begin, then, by finding these velocity components.

SOLUTION The figures below show the initial and final velocities of the golf ball.

If we take up as the positive direction, then the vertical components of the initial and final velocities are, respectively, $\mathbf{v}_{0y} = -v_0 \cos 30.0°$ and $\mathbf{v}_{fy} = v_f \cos 30.0°$. Then, from Equation 7.4 the magnitude of the impulse is

$$\overline{F}\Delta t = m(v_{fy} - v_{0y}) = m\left[(v_f \cos 30.0°) - (-v_0 \cos 30.0°)\right]$$

Since $v_0 = v_f = 45$ m/s,

$$\overline{F}\Delta t = 2mv_0 \cos 30.0° = 2(0.047\text{ kg})(45\text{ m/s})(\cos 30.0°) = \boxed{3.7\text{ N}\cdot\text{s}}$$

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17. ***REASONING*** The system consists of the lumberjack and the log. For this system, the sum of the external forces is zero. This is because the weight of the system is balanced by the corresponding normal force (provided by the buoyant force of the water) and the water is assumed to be frictionless. The lumberjack and the log, then, constitute an isolated system, and the principle of conservation of linear momentum holds.

SOLUTION
a. The total linear momentum of the system before the lumberjack begins to move is zero, since all parts of the system are at rest. Momentum conservation requires that the total momentum remains zero during the motion of the lumberjack.

$$\underbrace{m_1 v_{f1} + m_2 v_{f2}}_{\substack{\text{Total momentum}\\ \text{just before the jump}}} = \underbrace{0}_{\text{Initial momentum}}$$

Here the subscripts "1" and "2" refer to the first log and lumberjack, respectively. Let the direction of motion of the lumberjack be the positive direction. Then, solving for v_{1f} gives

$$v_{f1} = -\frac{m_2 v_{f2}}{m_1} = -\frac{(98 \text{ kg})(+3.6 \text{ m/s})}{230 \text{ kg}} = \boxed{-1.5 \text{ m/s}}$$

The minus sign indicates that the first log recoils as the lumberjack jumps off.

b. Now the system is composed of the lumberjack, just before he lands on the second log, and the second log. Gravity acts on the system, but for the short time under consideration while the lumberjack lands, the effects of gravity in changing the linear momentum of the system are negligible. Therefore, to a very good approximation, we can say that the linear momentum of the system is very nearly conserved. In this case, the initial momentum is not zero as it was in part (a); rather the initial momentum of the system is the momentum of the lumberjack just before he lands on the second log. Therefore,

$$\underbrace{m_1 v_{f1} + m_2 v_{f2}}_{\substack{\text{Total momentum} \\ \text{just after lumberjack lands}}} = \underbrace{m_1 v_{01} + m_2 v_{02}}_{\text{Initial momentum}}$$

In this expression, the subscripts "1" and "2" now represent the second log and lumberjack, respectively. Since the second log is initially at rest, $v_{10} = 0$. Furthermore, since the lumberjack and the second log move with a common velocity, $v_{f1} = v_{f2} = v_f$. The statement of momentum conservation then becomes

$$m_1 v_f + m_2 v_f = m_2 v_{02}$$

Solving for v_f, we have

$$v_f = \frac{m_2 v_{02}}{m_1 + m_2} = \frac{(98 \text{ kg})(+3.6 \text{ m/s})}{230 \text{ kg} + 98 \text{ kg}} = \boxed{+1.1 \text{ m/s}}$$

The positive sign indicates that the system moves in the same direction as the original direction of the lumberjack's motion.

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33. ***REASONING*** The system consists of the two balls. The total linear momentum of the two-ball system is conserved because the net external force acting on it is zero. The principle of conservation of linear momentum applies whether or not the collision is elastic.

$$\underbrace{m_1 v_{f1} + m_2 v_{f2}}_{\substack{\text{Total momentum} \\ \text{after collision}}} = \underbrace{m_1 v_{01} + 0}_{\substack{\text{Total momentum} \\ \text{before collision}}}$$

When the collision is elastic, the kinetic energy is also conserved during the collision

$$\underbrace{\tfrac{1}{2} m_1 v_{f1}^2 + \tfrac{1}{2} m_2 v_{f2}^2}_{\substack{\text{Total kinetic energy} \\ \text{after collision}}} = \underbrace{\tfrac{1}{2} m_1 v_{01}^2 + 0}_{\substack{\text{Total kinetic energy} \\ \text{before collision}}}$$

SOLUTION

a. The final velocities for an elastic collision are determined by simultaneously solving the above equations for the final velocities. The procedure is discussed in Example 8 in the text, and leads to Equations 7.8a and 7.8b. According to Equation 7.8:

$$v_{f1} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{01} \qquad \text{and} \qquad v_{f2} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{01}$$

Let the initial direction of motion of the 5.00-kg ball define the positive direction. Substituting the values given in the text, these equations give

$$[5.00\text{-kg ball}] \qquad v_{f1} = \left(\frac{5.00 \text{ kg} - 7.50 \text{ kg}}{5.00 \text{ kg} + 7.50 \text{ kg}} \right)(2.00 \text{ m/s}) = \boxed{-0.400 \text{ m/s}}$$

$$[7.50\text{-kg ball}] \qquad v_{f2} = \left(\frac{2(5.00 \text{ kg})}{5.00 \text{ kg} + 7.50 \text{ kg}} \right)(2.00 \text{ m/s}) = \boxed{+1.60 \text{ m/s}}$$

The signs indicate that, after the collision, the 5.00-kg ball reverses its direction of motion, while the 7.50-kg ball moves in the direction in which the 5.00-kg ball was initially moving.

b. When the collision is completely inelastic, the balls stick together, giving a composite body of mass $m_1 + m_2$ which moves with a velocity v_f. The statement of conservation of linear momentum then becomes

$$\underbrace{(m_1 + m_2)v_f}_{\substack{\text{Total momentum} \\ \text{after collision}}} = \underbrace{m_1 v_{01} + 0}_{\substack{\text{Total momentum} \\ \text{before collision}}}$$

The final velocity of the two balls after the collision is, therefore,

$$v_f = \frac{m_1 v_{01}}{m_1 + m_2} = \frac{(5.00 \text{ kg})(2.00 \text{ m/s})}{5.00 \text{ kg} + 7.50 \text{ kg}} = \boxed{+0.800 \text{ m/s}}$$

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61. ***REASONING*** The cannon and the shell constitute the system. Since no external force hinders the motion of the system after the cannon is unbolted, conservation of linear momentum applies in that case. If we assume that the burning gun powder imparts the same kinetic energy to the system in each case, we have sufficient information to develop a mathematical description of this situation, and solve it for the velocity of the shell fired by the loose cannon.

SOLUTION For the case where the cannon is unbolted, momentum conservation gives

$$\underbrace{m_1 v_{f1} + m_2 v_{f2}}_{\substack{\text{Total momentum} \\ \text{after shell is fired}}} = \underbrace{0}_{\substack{\text{Initial momentum} \\ \text{of system}}} \qquad (1)$$

where the subscripts "1" and "2" refer to the cannon and shell, respectively. In both cases, the burning gun power imparts the same kinetic energy to the system. When the cannon is bolted to the ground, only the shell moves and the kinetic energy imparted to the system is

$$\mathrm{KE} = \tfrac{1}{2} m_{\text{shell}} v_{\text{shell}}^2 = \tfrac{1}{2}(85.0\text{ kg})(551\text{ m/s})^2 = 1.29 \times 10^7\text{ J}$$

The kinetic energy imparted to the system when the cannon is unbolted has the same value and can be written using the same notation as Equation (1):

$$\mathrm{KE} = \tfrac{1}{2} m_1 v_{\text{f1}}^2 + \tfrac{1}{2} m_2 v_{\text{f2}}^2 \tag{2}$$

Solving Equation (1) for v_{f1}, the velocity of the cannon after the shell is fired, and substituting the resulting expression into Equation (2) gives

$$\mathrm{KE} = \frac{m_2^2 v_{\text{f2}}^2}{2\text{m}_1} + \tfrac{1}{2} m_2 v_{\text{f2}}^2 \tag{3}$$

Solving Equation (3) for v_{f2} gives

$$v_{\text{f2}} = \sqrt{\frac{2\,\mathrm{KE}}{m_2\left(\dfrac{m_2}{m_1}+1\right)}} = \sqrt{\frac{2(1.29\times 10^7\text{ J})}{(85\text{ kg})\left[\dfrac{85\text{ kg}}{5.80\times 10^3\text{ kg}}+1\right]}} = \boxed{+547\text{ m/s}}$$