CHAPTER 3

Resistive Circuits

Preview The resistor, with resistance *R*, is an element commonly used in most electric circuits. In this chapter we consider the analysis of circuits consisting of resistors and sources.

In addition to Ohm's law, we need two laws for relating (1) current flow at connected terminals and (2) the sum of voltages around a closed circuit path. These two laws were developed by Gustav Kirchhoff in 1847.

Using Kirchhoff's laws and Ohm's law, we are able to complete the analysis of resistive circuits and determine the currents and voltages at desired points in the circuit. This analysis may be accomplished for circuits with both independent and dependent sources.

3.1 Design Challenge

Adjustable Voltage Source

A circuit is required to provide an adjustable voltage. The specifications for this circuit are:

1. It should be possible to adjust the voltage to any value between −5 V and +5 V. It should not be possible to accidentally obtain a voltage outside this range.
2. The load current will be negligible.
3. The circuit should use as little power as possible.

The available components are:

1. Potentiometers: resistance values of 10 kΩ, 20 kΩ, and 50 kΩ are in stock.
2. A large assortment of standard 2 percent resistors having values between 10 Ω and 1 MΩ (see Appendix E).
3. Two power supplies (voltage sources): one 12-V supply and one −12-V supply; each rated for a maximum current of 100 mA (milliamps).

Describe the Situation and the Assumptions

Figure 3.1-1 shows the situation. The voltage v is the adjustable voltage. The circuit that uses the output of the circuit being designed is frequently called the "load." In this case, the load current is negligible, so $i = 0$.

State the Goal

A circuit providing the adjustable voltage

$$-5 \text{ V} \leq v \leq +5 \text{ V}$$

must be designed using the available components.

Generate a Plan

Make the following observations.

1. The adjustability of a potentiometer can be used to obtain an adjustable voltage v.
2. Both power supplies must be used so that the adjustable voltage can have both positive and negative values.
3. The terminals of the potentiometer cannot be connected directly to the power supplies because the voltage v is not allowed to be as large as 12 V or −12 V.

These observations suggest the circuit shown in Figure 3.1-2*a*. The circuit in Figure 3.1-2*b* is obtained by using the simplest model for each component in Figure 3.1-2*a*.

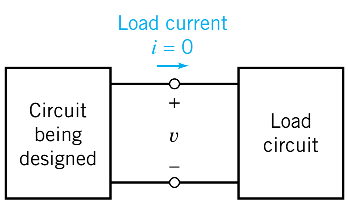

FIGURE 3.1-1 The circuit being designed provides an adjustable voltage, v, to the load circuit.

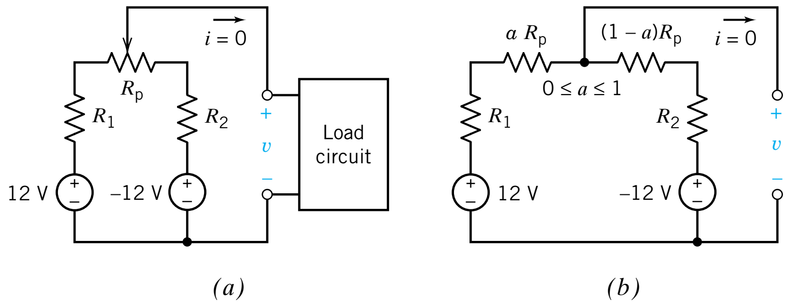

FIGURE 3.1-2
(*a*) A proposed circuit for producing the variable voltage *v*, and (*b*) the equivalent circuit after the potentiometer is modeled.

To complete the design, values need to be specified for R_1, R_2, and R_p. Then, several results need to be checked and adjustments made, if necessary.

1. Can the voltage *v* be adjusted to any value in the range −5 V to 5 V?
2. Are the voltage source currents less than 100 mA? This condition must be satisfied if the power supplies are to be modeled as ideal voltage sources.
3. Is it possible to reduce the power absorbed by R_1, R_2, and R_p?

Act on the Plan

Taking action on this plan requires analyzing the circuit in Figure 3.1-2*b*. Analysis of this type of circuit is discussed in this chapter. We will return to this problem at the end of this chapter.

3.2 Electric Circuit Applications

Overseas communications have always been of great importance to nations. One of the most brilliant chapters of the history of electrical technology was the development of underwater electric cable circuits. Underwater electric cables were used to carry electric telegraph communications. In late 1852 England and Ireland were connected by cable, and a year later there was a cable between Scotland and Ireland. In June 1853 a cable was strung between England and Holland, a distance of 115 miles.

It was Cyrus Field and Samuel Morse who saw the potential for a submarine cable across the Atlantic. By 1857 Field had organized a firm to complete the transatlantic telegraph cable and issued a contract for the production of 2500 miles of cable. The cable laying began in June 1858. After several false starts, a cable was laid across the Atlantic by August 5, 1858. However, this cable failed after only a month of operation.

Another series of cable-laying projects commenced, and by September 1865 a successful Atlantic cable was in place. This cable stretched over 3000 miles, from England to eastern Canada. There followed a flurry of cable laying. Approximately 150,000 km (90,000 mi) were in use by 1870, linking all continents and all major islands. An example of modern undersea cable is shown in Figure 3.2-1.

One of the greatest uses of electricity in the late 1800s was for electric railways. In 1884 the Sprague Electric Railway was incorporated. Sprague built an electric railway for Richmond, Virginia, in 1888. By 1902 the horse-drawn street trolley was obsolete, and there were

FIGURE 3.2-1
Examples of undersea cable. Courtesy of Bell Laboratories.

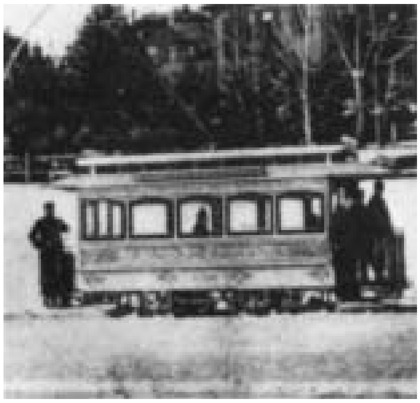

FIGURE 3.2-2
View of the Sprague Electric Railway car on the Brookline branch of the Boston system about 1900. This electric railway branch operates as an electric trolley railroad today with modern electric cars. Courtesy of General Electric Company.

22,576 miles of electric railway track in the United States. A 1900 electric railway is shown in Figure 3.2-2.

3.3 Kirchhoff's Laws

An electric circuit consists of circuit elements that are connected together. The places where the elements are connected to each other are called nodes. Figure 3.3-1*a* shows an electric circuit that consists of six elements connected together at four nodes. It is common practice to draw electric circuits using straight lines and to position the elements horizontally or vertically as shown in Figure 3.3-1*b*.

The circuit is shown again in Figure 3.3-1*c,* this time emphasizing the nodes. Notice that redrawing the circuit using straight lines and horizontal and vertical elements has changed the way that the nodes are represented. In Figure 3.3-1*a,* nodes are represented as points. In Figures 3.3-1*b,c,* nodes are represented using both points and straight-line segments.

The same circuit can be drawn in several different ways. One drawing of a circuit might look much different from another drawing of the same circuit. How can we tell when two circuit drawings represent the same circuit? Informally, we say that two circuit drawings represent the same circuit if corresponding elements are connected to corresponding nodes. More formally, we say that circuit drawings A and B represent the same circuit when the following three conditions are met.

1. There is a one-to-one correspondence between the nodes of drawing A and the nodes of drawing B. (A one-to-one correspondence is a matching. In this one-to-one correspondence, each node in drawing A is matched to exactly one node of drawing B, and vice versa. The position of the nodes is not important.)
2. There is a one-to-one correspondence between the elements of drawing A and the elements of drawing B.
3. Corresponding elements are connected to corresponding nodes.

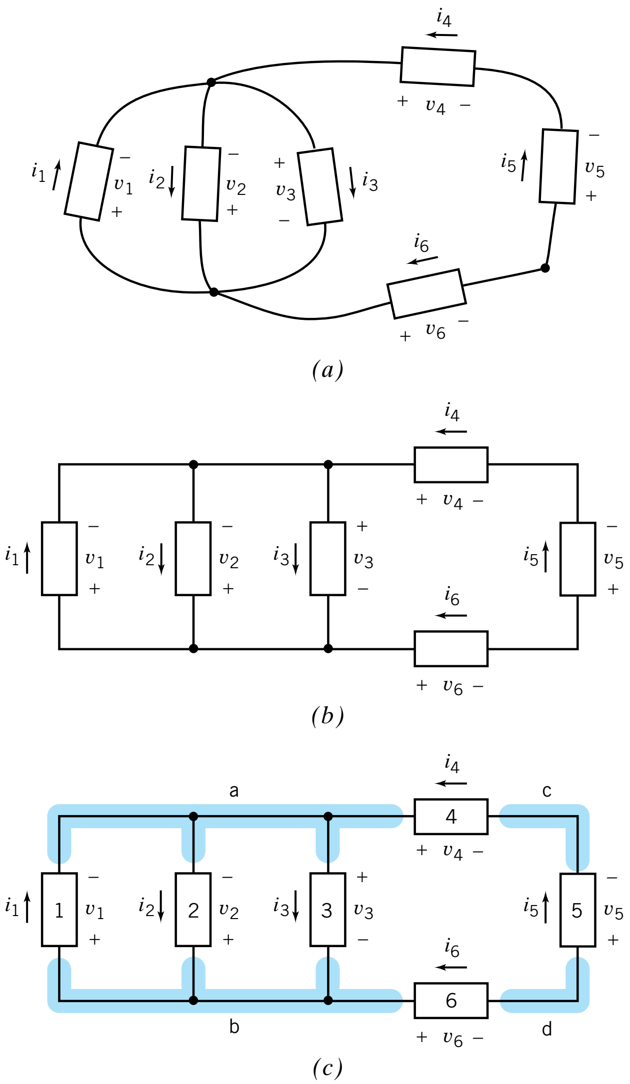

FIGURE 3.3-1
(*a*) An electric circuit. (*b*) The same circuit, redrawn using straight lines and horizontal and vertical elements. (*c*) The circuit after labeling the nodes and elements.

Example 3.3-1

Figure 3.3-2 shows four circuit drawings. Which of these drawings, if any, represent the same circuit as the circuit drawing in Figure 3.3-1*c*?

Solution

The circuit drawing shown in Figure 3.3-2*a* has five nodes, labeled r, s, t, u and v. The circuit drawing in Figure 3.3-1*c* has four nodes. Since the two drawings have different numbers of nodes, there cannot be a one-to-one correspondence between the nodes of the two drawings. Hence theses drawings represent different circuits.

The circuit drawing shown in Figure 3.3-2*b* has four nodes and six elements, the same numbers of nodes and elements as the circuit drawing in Figure 3.3-1*c*. The nodes in Figure 3.3-2*b* have been labeled in the same way as the corresponding nodes in Figure 3.3-1*c*. For example, node c in Figure 3.3-2*b* corresponds to node c in Figure 3.3-1*c*. The elements in Figure 3.3-2*b* have been labeled in the same way as the corresponding elements in Figure 3.3-1*c*. For example, element 5 in Figure 3.3-2*b* corresponds to element 5 in Figure 3.3-1*c*. Corresponding elements are indeed connected to corresponding nodes. For example, element 2 is connected to nodes a and b, both in Figure 3.3-2*b* and in Figure 3.3-1*c*. Consequently, Figure 3.3-2*b* and Figure 3.3-1*c* represent the same circuit.

The circuit drawing shown in Figure 3.3-2*c* has four nodes and six elements, the same numbers of nodes and elements as the circuit drawing in Figure 3.3-1*c*. The nodes and elements in Figure 3.3-2*c* have been labeled in the same

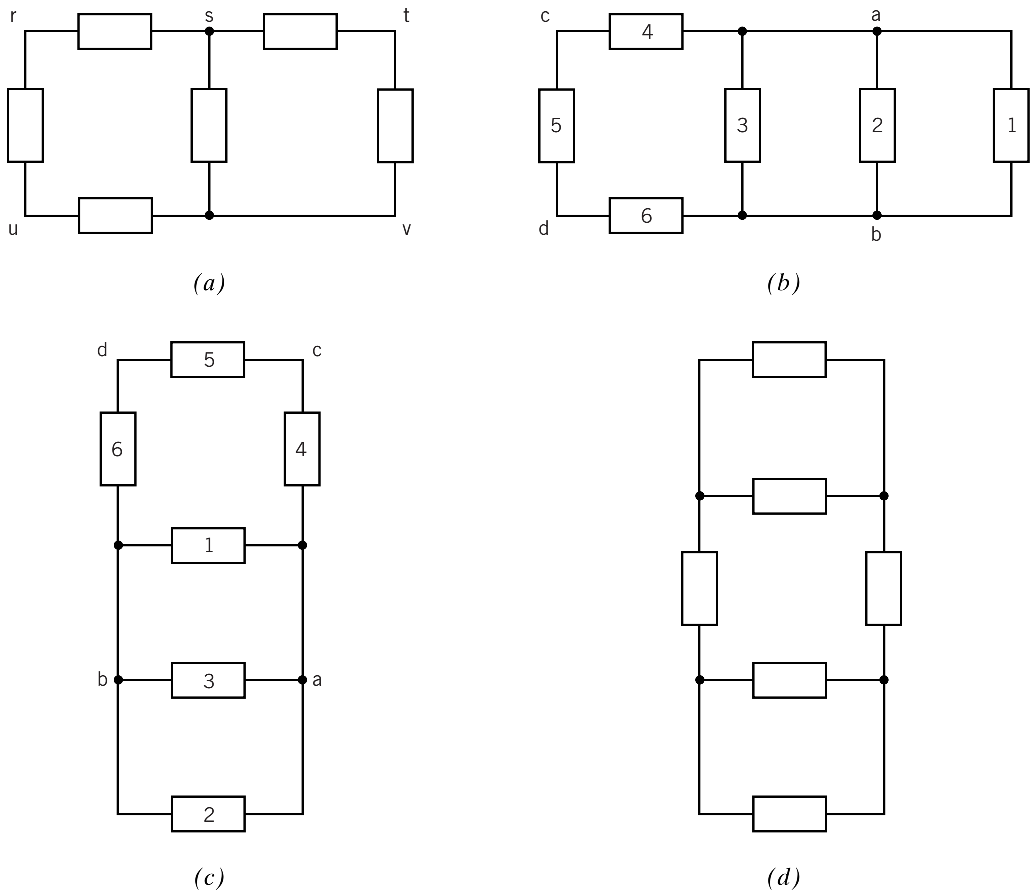

FIGURE 3.3-2
Four circuit drawings.

way as the corresponding nodes and elements in Figure 3.3-1*c*. Corresponding elements are indeed connected to corresponding nodes. Therefore Figure 3.3-2*c* and Figure 3.3-1*c* represent the same circuit.

The circuit drawing shown in Figure 3.3-2*d* has four nodes and six elements, the same numbers of nodes and elements as the circuit drawing in Figure 3.3-1*c*. However, the nodes and elements of Figure 3.3-2*d* cannot be labeled so that corresponding elements of Figure 3.3-1*c* are connected to corresponding nodes. (For example, in Figure 3.3-1*c* three elements are connected between the same pair of nodes, a and b. That does not happen in Figure 3.3-2*d*.) Consequently, Figure 3.3-2*d* and Figure 3.3-1*c* represent different circuits.

In 1847, Gustav Robert Kirchhoff, a professor at the University of Berlin, formulated two important laws that provide the foundation for analysis of electric circuits. These laws are referred to as *Kirchhoff's current law* (KCL) and *Kirchhoff's voltage law* (KVL) in his honor. Kirchhoff's laws are a consequence of conservation of charge and conservation of energy. Gustav Robert Kirchhoff is pictured in Figure 3.3-3.

Kirchhoff's current law states that the algebraic sum of the currents entering any node is identically zero for all instants of time.

Kirchhoff's current law (KCL): The algebraic sum of the currents into a node at any instant is zero.

The phrase *algebraic sum* indicates that we must take reference directions into account as we add up the currents of elements connected to a particular node. One way to take reference directions into account is to use a plus sign when the current is directed away from the node and a minus sign when the current is directed toward the node. For example, consider the circuit shown in Figure 3.3-1*c*. Four elements of this circuit—elements 1, 2, 3, and 4—are connected to node a. By Kirchhoff's current law, the algebraic sum of the element currents i_1, i_2, i_3, and i_4 must be zero. Currents i_2 and i_3 are directed away from node a, so we will use a plus sign for i_2 and i_3. In contrast, currents i_1 and i_4 are directed toward node a, so we will use a minus sign for i_1 and i_4. The KCL equation for node a of Figure 3.3-1*c* is

$$-i_1 + i_2 + i_3 - i_4 = 0 \tag{3.3-1}$$

An alternate way of obtaining the algebraic sum of the currents into a node is to set the sum of all the currents directed away from node equal to the sum of all the currents directed toward that node. Using this technique, the KCL equation for node a of Figure 3.3-1*c* is

$$i_2 + i_3 = i_1 + i_4 \tag{3.3-2}$$

Clearly, Eqs 3.3-1 and 3.3-2 are equivalent.

Similarly, the Kirchhoff's current law equation for node b of Figure 3.3-1*c* is

$$i_1 = i_2 + i_3 + i_6$$

Before we can state Kirchhoff's voltage law, we need the definition of a loop. A *loop* is a closed path through a circuit that does not encounter any intermediate node more than once. For example, starting at node a in Figure 3.3-1*c*, we can move through element 4 to node c, then through element 5 to node d, through element 6 to node b, and finally through element 3 back to node a. We have a closed path, and we did not encounter any of the intermediate nodes—b, c, or d—more than once. Consequently, elements 3, 4, 5, and 6 comprise a loop. Similarly, elements 1, 4, 5, and 6 comprise a loop of the circuit shown in Figure 3.3-1*c*. Elements 1 and 3 comprise yet another loop of this circuit. The circuit has three other loops: elements 1 and 2, elements 2 and 3, and elements 2, 4, 5, and 6.

We are now ready to state Kirchhoff's voltage law.

Kirchhoff's voltage law (KVL): The algebraic sum of the voltages around any loop in a circuit is identically zero for all time.

The phrase *algebraic sum* indicates that we must take polarity into account as we add up the voltages of elements that comprise a loop. One way to take polarity into account is to move around the loop in the clockwise direction while observing the polarities of the element voltages. We write the voltage with a plus sign when we encounter the + of the voltage polarity before the –. In contrast, we write the voltage with a minus sign when we encounter the – of the voltage polarity before the +. For example, consider the circuit shown in Figure 3.3-1*c*. Elements 3, 4, 5, and 6 comprise a loop of the circuit. By Kirchhoff's voltage law, the algebraic sum of the element voltages v_3, v_4, v_5, and v_6 must be zero. As we move around the loop in the clockwise direction, we encounter the + of v_4 before the –, the – of v_5 before the +, the – of v_6 before the +, and the – of v_3 before the +. Consequently, we use a minus sign for v_3, v_5, and v_6 and a plus sign for v_4. The KCL equation for this loop of Figure 3.3-1*c* is

$$v_4 - v_5 - v_6 - v_3 = 0$$

Similarly, the Kirchhoff's voltage law equation for the loop consisting of elements 1, 4, 5, and 6 is

$$v_4 - v_5 - v_6 + v_1 = 0$$

The Kirchhoff's voltage law equation for the loop consisting of elements 1 and 2 is

$$-v_2 + v_1 = 0$$

FIGURE 3.3-3 Gustav Robert Kirchhoff (1824–1887). Kirchhoff stated two laws in 1847 regarding the current and voltage in an electrical circuit. Courtesy of the Smithsonian Institution.

Example 3.3-2

Consider the circuit shown in Figure 3.3-4*a*. Determine the power supplied by element *C* and the power received by element *D*.

Solution

Figure 3.3-4*a* provides a value for the current in element *C* but not for the voltage, *v*, across element *C*. The voltage and current of element *C* given in Figure 3.3-4*a* adhere to the passive convention, so the product of this voltage and current is the power *received* by element *C*. Similarly, Figure 3.3-4*a* provides a value for the voltage across element *D* but not for the current, *i*, in element *D*. The voltage and current of element *D* given in Figure 3.3-4*a* do not adhere to the passive convention, so the product of this voltage and current is the power *supplied* by element *D*.

We need to determine the voltage, *v*, across element *C* and the current, *i*, in element *D*. We will use Kirchhoff's laws to determine values of *v* and *i*. First, we identify and label the nodes of the circuit as shown in Figure 3.3-4*b*.

Apply Kirchhoff's voltage law (KVL) to the loop consisting of elements *C*, *D*, and *B* to get

$$-v - (-4) - 6 = 0 \qquad \Rightarrow \qquad v = -2 \text{ V}$$

The value of the current in element *C* in Figure 3.3-4*b* is 7 A. The voltage and current of element *C* given in Figure 3.3-4*b* adhere to the passive convention, so

$$p_C = v(7) = (-2)(7) = -14 \text{ W}$$

is the power *received* by element *C*. Therefore element *C* *supplies* 14 W.

Next, apply Kirchhoff's current law (KCL) at node b to get

$$7 + (-10) + i = 0 \qquad \Rightarrow \qquad i = 3 \text{ A}$$

The value of the voltage across element *D* in Figure 3.3-4*b* is −4 V. The voltage and current of element *D* given in Figure 3.3-4*b* do not adhere to the passive convention, so the power *supplied* by element *F* is given by

$$p_D = (-4)\, i = (-4)(3) = -12 \text{ W}$$

Therefore, element *D* *receives* 12 W.

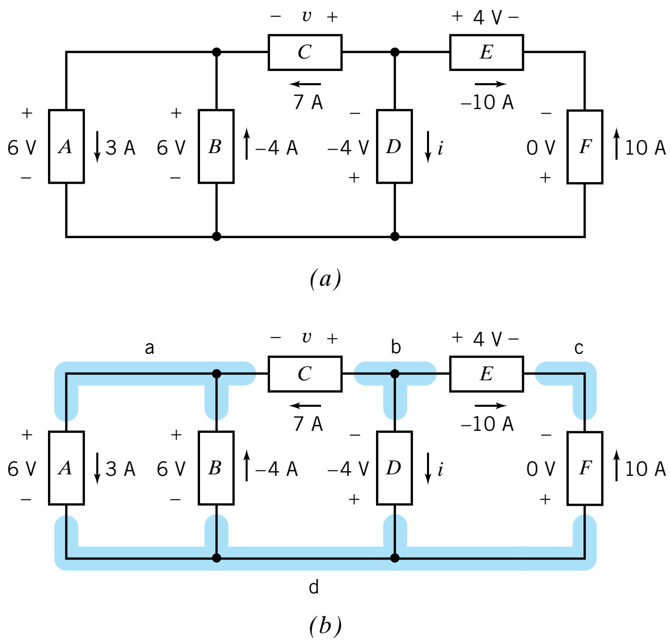

FIGURE 3.3-4
(*a*) The circuit considered in Example 3.3-2 and (*b*) the circuit redrawn to emphasize the nodes.

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Example 3.3-3

Consider the circuit shown in Figure 3.3-5. Notice that the passive convention was used to assign reference directions to the resistor voltages and currents. This anticipates using Ohm's law. Find each current and each voltage when $R_1 = 8\ \Omega$, $v_2 = -10$ V, $i_3 = 2$ A, and $R_3 = 1\ \Omega$. Also, determine the resistance R_2.

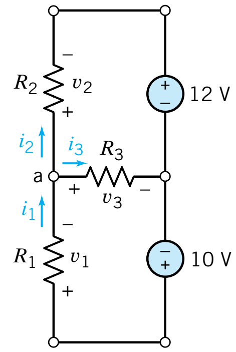

FIGURE 3.3-5 Circuit with two constant-voltage sources.

Solution

The sum of the currents entering node a is

$$i_1 - i_2 - i_3 = 0$$

Using Ohm's law for R_3, we find that

$$v_3 = R_3 i_3 = 1(2) = 2\ \text{V}$$

Kirchhoff's voltage law for the bottom loop incorporating v_1, v_3, and the 10-V source is

$$-10 + v_1 + v_3 = 0$$

Therefore,

$$v_1 = 10 - v_3 = 8\ \text{V}$$

Ohm's law for the resistor R_1 is

$$v_1 = R_1 i_1$$

or

$$i_1 = v_1/R_1 = 8/8 = 1\ \text{A}$$

Since we have now found $i_1 = 1$ A and $i_3 = 2$ A as originally stated, then

$$i_2 = i_1 - i_3 = 1 - 2 = -1\ \text{A}$$

We can now find the resistance R_2 from

$$v_2 = R_2 i_2$$

or

$$R_2 = v_2/i_2 = -10/-1 = 10\ \Omega$$

Example 3.3-4

Determine the value of the current, in amps, measured by the ammeter in Figure 3.3-6*a*.

Solution

An ideal ammeter is equivalent to a short circuit. The current measured by the ammeter is the current in the short circuit. Figure 3.3-6*b* shows the circuit after replacing the ammeter by the equivalent short circuit.

The circuit has been redrawn in Figure 3.3-7 to label the nodes of the circuit. This circuit consists of a voltage source, a dependent current source, two resistors, and two short circuits. One of the short circuits is the controlling element of the CCCS and the other short circuit is a model of the ammeter.

Applying KCL twice, once at node **d** and again at node **a** shows that the current in the voltage source and the current in the 4-Ω resistor are both equal to i_a. These currents are labeled in Figure 3.3-7. Applying KCL again, at node c, shows that the current in the 2 Ω resistor is equal to i_m. This current is labeled in Figure 3.3-7.

Next, Ohm's law tells us that the voltage across the 4-Ω resistor is equal to 4 i_a and that the voltage across the 2-Ω resistor is equal to 2 i_m. Both of these voltages are labeled in Figure 3.3-7.

Applying KCL at node **b** gives

$$-i_a - 3i_a - i_m = 0$$

Applying KVL to closed path **a - b - c - e - d - a** gives

$$0 = -4i_a + 2i_m - 12 = -4\left(-\frac{1}{4} i_m\right) + 2i_m - 12 = 3i_m - 12$$

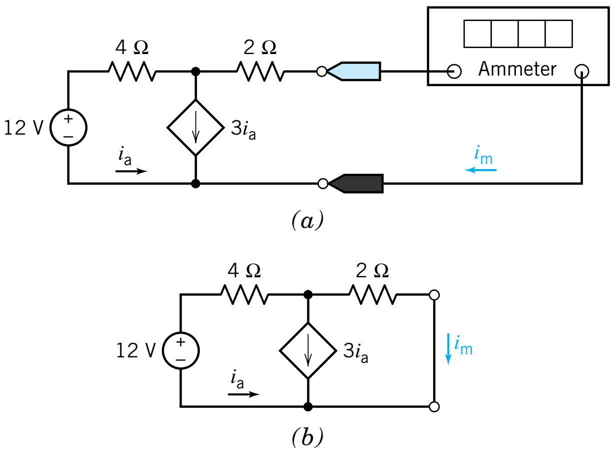

FIGURE 3.3-6
(*a*) A circuit with dependent source and an ammeter. (*b*) The equivalent circuit after replacing the ammeter by a short circuit.

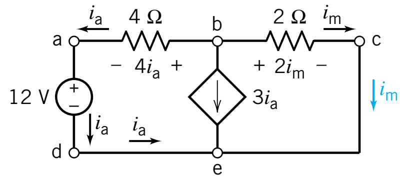

FIGURE 3.3-7
The circuit of Figure 3.3-6 after labeling the nodes and some element currents and voltages.

Finally, solving this equation gives

$$i_m = 4 \text{ A}$$

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Example 3.3-5

Determine the value of the voltage, in volts, measured by the voltmeter in Figure 3.3-8*a*.

Solution

An ideal voltmeter is equivalent to an open circuit. The voltage measured by the voltmeter is the voltage across the open circuit. Figure 3.3-8*b* shows the circuit after replacing the voltmeter by the equivalent open circuit.

The circuit has been redrawn in Figure 3.3-9 to label the nodes of the circuit. This circuit consists of a voltage source, a dependent voltage source, two resistors, a short circuit, and an open circuit. The short circuit is the controlling element of the CCVS and the open circuit is a model of the voltmeter.

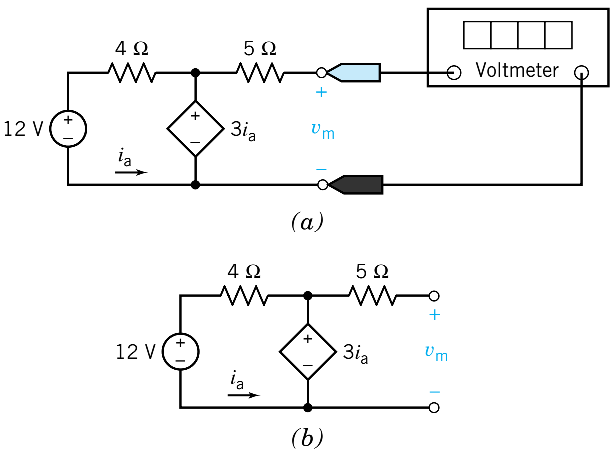

FIGURE 3.3-8
(*a*) A circuit with dependent source and a voltmeter. (*b*) The equivalent circuit after replacing the voltmeter by a open circuit.

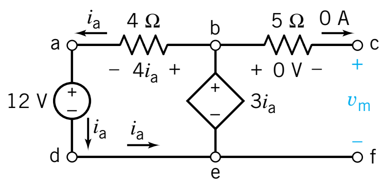

FIGURE 3.3-9
The circuit of Figure 3.3-8*b* after labeling the nodes and some element currents and voltages.

Applying KCL twice, once at node **d** and again at node **a,** shows that the current in the voltage source and the current in the 4-Ω resistor are both equal to i_a. These currents are labeled in Figure 3.3-9. Applying KCL again, at node c, shows that the current in the 5-Ω resistor is equal to the current in the open circuit, that is, zero. This current is labeled in Figure 3.3-9. Ohm's law tells us that the voltage across the 5-Ω resistor is also equal to zero. Next, applying KVL to the closed path **b - c - f - e - b** gives $v_m = 3\,i_a$.

Applying KVL to the closed path **a - b - e - d - a** gives

$$-4\,i_a + 3\,i_a - 12 = 0$$

so

$$i_a = -12 \text{ A}$$

Finally

$$v_m = 3\,i_a = 3(-12) = -36 \text{ V}$$

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Exercise 3.3-1 Determine the values of i_3, i_4, i_6, v_2, v_4, and v_6 in Figure E 3.3-1.
Answer: $i_3 = -3$ A, $i_4 = 3$ A, $i_6 = 4$ A, $v_2 = -3$ V, $v_4 = -6$ V, $v_6 = 6$ V

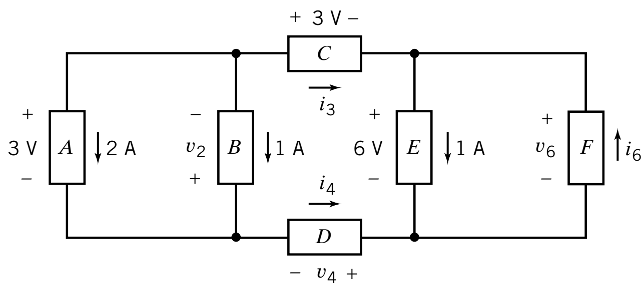

FIGURE E 3.3-1

Exercise 3.3-2 Determine the current i in Figure E 3.3-2.
Answer: $i = 4$ A

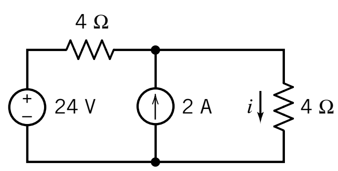

FIGURE E 3.3-2

Exercise 3.3-3 Determine the value of the current i_m in Figure E 3.3-3*a*.
Hint: Apply KVL to the closed path **a - b - d - c - a** in Figure E 3.3-3*b* to determine v_a. Then apply KCL at node b to find i_m
Answer: $i_m = 9$ A.

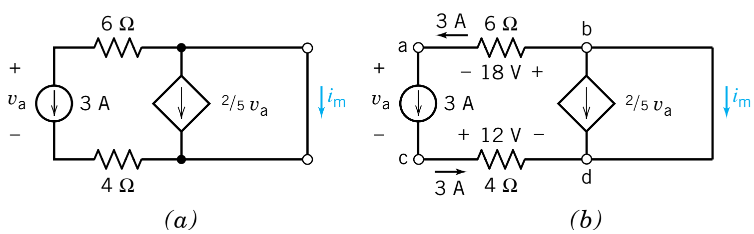

FIGURE E 3.3-3 (*a*) A circuit containing a VCCS. (*b*) The circuit after labeling the nodes and some element currents and voltages.

Exercise 3.3-4 Determine the value of the voltage v_m in Figure E 3.3-4*a*.
Hint: Apply KVL to the closed path **a - b - d - c - a** in Figure E 3.3-4*b* to determine v_a.
Answer: $v_m = 24$ V

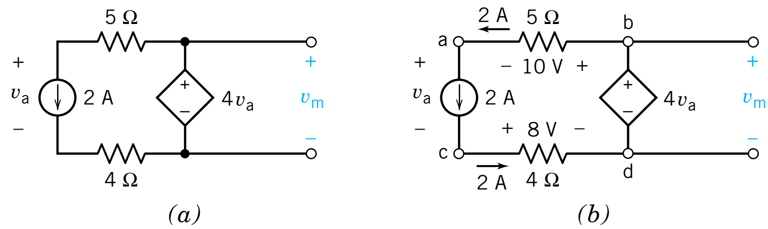

FIGURE E 3.3-4
(*a*) A circuit containing a VCVS. (*b*) The circuit after labeling the nodes and some element currents and voltages.

3.4 A Single-Loop Circuit—The Voltage Divider

Let us consider a single-loop circuit, as shown in Figure 3.4-1. In anticipation of using Ohm's law, the passive convention has been used to assign reference directions to resistor voltages and currents. Using KCL at each node, we obtain

$$\text{a:} \quad i_s - i_1 = 0 \tag{3.4-1}$$
$$\text{b:} \quad i_1 - i_2 = 0 \tag{3.4-2}$$
$$\text{c:} \quad i_2 - i_3 = 0 \tag{3.4-3}$$
$$\text{d:} \quad i_3 - i_s = 0 \tag{3.4-4}$$

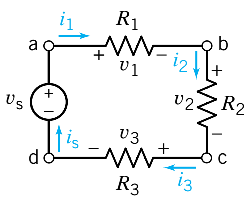

FIGURE 3.4-1
Single-loop circuit with a voltage source v_s.

We have four equations, but any one of the four can be derived from the other three equations. In any circuit with n nodes, $n - 1$ independent current equations can be derived from Kirchhoff's current law.

Of course, we also note that

$$i_s = i_1 = i_2 = i_3$$

so that the current i_1 can be said to be the loop current and flows continuously around the loop from a to b to c to d and back to a.

The resistors in Figure 3.4-1 are connected in *series*. Notice, for example, that resistors R_1 and R_2 are both connected to node b and that no other circuit elements are connected to node b. Consequently, $i_1 = i_2$, so both resistors have the same current. A similar argument shows that resistors R_2 and R_3 are also connected in series. Noticing that R_2 is connected in series with both R_1 and R_3, we say that all three resistors are connected in series. The order of series resistors is not important. For example, the voltages and currents of the three resistors in Figure 3.4-1 will not change if we interchange the positions R_2 and R_3.

The defining characteristic of series elements is that they have the same current. To identify a pair of series elements, we look for two elements connected to single a node that has no other elements connected to it.

In order to determine i_1, we use KVL around the loop to obtain

$$-v_s + v_1 + v_2 + v_3 = 0 \tag{3.4-5}$$

where v_1 is the voltage across the resistor R_1. Using Ohm's law for each resistor, Eq. 3.4-5 can be written as

$$-v_s + i_1R_1 + i_1R_2 + i_1R_3 = 0$$

Solving for i_1, we have

$$i_1 = \frac{v_s}{R_1 + R_2 + R_3}$$

Thus, the voltage across the nth resistor R_n is v_n and can be obtained as

$$v_n = i_1 R_n = \frac{v_s R_n}{R_1 + R_2 + R_3} \tag{3.4-6}$$

For example, the voltage across resistor R_2 is

$$v_2 = \frac{R_2}{R_1 + R_2 + R_3} v_s$$

Thus, the voltage appearing across one of a series connection of resistors connected in series with a voltage source will be the ratio of its resistance to the total resistance times the source voltage. This circuit demonstrates the principle of *voltage division,* and the circuit is called a *voltage divider.*

In general, we may represent the voltage divider principle by the equation

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v_s \tag{3.4-7}$$

where the voltage is across the nth resistor of N resistors connected in series.

Example 3.4-1

Let us consider the circuit shown in Figure 3.4-2 and determine the resistance R_2 required so that the voltage across R_2 will be one-fourth of the source voltage when $R_1 = 9\ \Omega$. Determine the current flowing when $v_s = 12$ V.

Solution

The voltage across resistor R_2 will be

$$v_2 = \frac{R_2}{R_1 + R_2} v_s$$

Since we desire $v_2/v_s = 1/4$, we have

$$\frac{R_2}{R_1 + R_2} = \frac{1}{4}$$

or

$$R_1 = 3R_2$$

Since $R_1 = 9\ \Omega$, we require that $R_2 = 3\ \Omega$. Using KVL around the loop, we have

$$-v_s + v_1 + v_2 = 0$$

or

$$v_s = iR_1 + iR_2$$

Therefore,

$$i = \frac{v_s}{R_1 + R_2} \tag{3.4-8}$$

or

$$i = \frac{12}{12} = 1\text{ A}$$

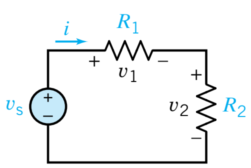

FIGURE 3.4-2
Voltage divider circuit with $R_1 = 9\ \Omega$.

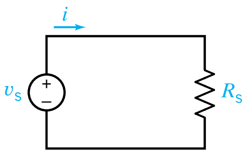

FIGURE 3.4-3
Equivalent circuit for a series connection of resistors.

Let us consider the simple circuit of the voltage source connected to a resistance R_s as shown in Figure 3.4-3. For this circuit

$$i = \frac{v_s}{R_s} \tag{3.4-9}$$

Comparing Eqs. 3.4-8 and 3.4-9, we see that the currents are identical when

$$R_s = R_1 + R_2$$

The resistance R_s is said to be an *equivalent resistance* of the series connection of resistors R_1 and R_2. In general, the equivalent resistance of a series of N resistors is

$$R_s = R_1 + R_2 + \cdots + R_N \tag{3.4-10}$$

In this specific case

$$R_s = R_1 + R_2 = 9 + 3 = 12\ \Omega$$

Each resistor in Figure 3.4-2 absorbs power, so the power absorbed by R_1 is

$$p_1 = \frac{v_1^2}{R_1}$$

and the power absorbed by the second resistor is

$$p_2 = \frac{v_2^2}{R_2}$$

The total power absorbed by the two resistors is

$$p = p_1 + p_2 = \frac{v_1^2}{R_1} + \frac{v_2^2}{R_2} \tag{3.4-11}$$

However, according to the voltage divider principle,

$$v_n = \frac{R_n}{R_1 + R_2} v_s$$

Then we may rewrite Eq. 3.4-11 as

$$p = \frac{R_1}{(R_1 + R_2)^2} v_s^2 + \frac{R_2}{(R_1 + R_2)^2} v_s^2$$

Since $R_1 + R_2 = R_s$, the equivalent series resistance, we have

$$p = \frac{R_1 + R_2}{R_s^2} v_s^2 = \frac{v_s^2}{R_s}$$

Thus, the total power absorbed by the two series resistors is equal to the power absorbed by the equivalent resistance R_s. The power absorbed by the two resistors is equal to that supplied by the source v_s. We show this by noting that the current is

$$i = \frac{v_s}{R_s}$$

and therefore the power supplied is

$$p = v_s i = \frac{v_s^2}{R_s}$$

Example 3.4-2

For the circuit of Figure 3.4-4*a*, find the current measured by the ammeter. Then show that the power absorbed by the two resistors is equal to that supplied by the source.

Solution

Figure 3.4-4*b* shows the circuit after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter, i_m. Applying KVL gives

$$15 + 5i_m + 10i_m = 0$$

The current measured by the ammeter is

$$i_m = -\frac{15}{5+10} = -1\ \text{A}$$

(Why is i_m negative? Why can't we just divide the source voltage by the equivalent resistance? Recall that when we use Ohm's law, the voltage and current must adhere to the passive convention. In this case, the current calculated by dividing the source voltage by the equivalent resistance does not have the same reference direction as i_m, and so we need a minus sign.)

The total power absorbed by the two resistors is

$$p_R = 5i_m^2 + 10i_m^2 = 15(1^2) = 15\ \text{W}$$

The power supplied by the source is

$$p_s = -v_s\, i_m = -15(-1) = 15\ \text{W}$$

Thus, the power supplied by the source is equal to that absorbed by the series connection of resistors.

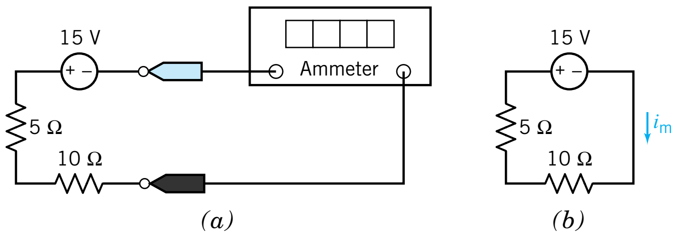

FIGURE 3.4-4 (*a*) A circuit containing series resistors. (*b*) The circuit after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter, i_m.

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Exercise 3.4-1 For the circuit of Figure E 3.4-1, find the voltage v_3 and the current i and show that the power delivered to the three resistors is equal to that supplied by the source.

Answer: $v_3 = 3$ V, $i = 1$ A

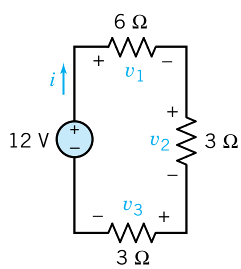


FIGURE E 3.4-1 Circuit with three series resistors (for Exercise 3.4-1).

Exercise 3.4-2 Consider the voltage divider shown in Figure E 3.4-2 when $R_1 = 6\ \Omega$. It is desired that the output power absorbed by $R_1 = 6\ \Omega$ be 6 W. Find the voltage v_o and the required source v_s.

Answer: $v_s = 14$ V, $v_o = 6$ V

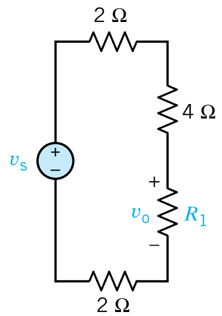
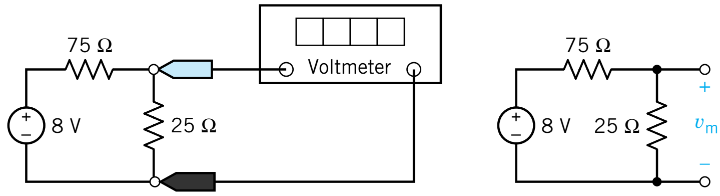


FIGURE E 3.4-2
Voltage divider for Exercise 3.4-2.

Exercise 3.4-3 Determine the voltage measured by the voltmeter in the circuit shown in Figure E 3.4-3*a*.

Hint: Figure E 3.4-3*b* shows the circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m.

Answer: $v_m = 2$ V



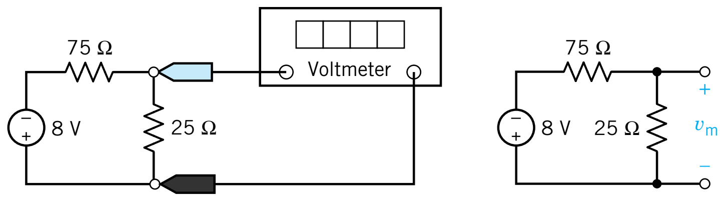
(*a*) (*b*)

FIGURE E 3.4-3
(*a*) A voltage divider. (*b*) The voltage divider after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m.

Exercise 3.4-4 Determine the voltage measured by the voltmeter in the circuit shown in Figure E 3.4-4*a*.

Hint: Figure E 3.4-4*b* shows the circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m.

Answer: $v_m = -2$ V



(*a*) (*b*)

FIGURE E 3.4-4
(*a*) A voltage divider. (*b*) The voltage divider after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m.

3.5 Parallel Resistors and Current Division

Circuit elements, such as resistors, are connected in *parallel* when the voltage across each element is identical. The resistors in Figure 3.5-1 are connected in *parallel.* Notice, for example, that resistors R_1 and R_2 are each connected to both node a and node b. Consequently, $v_1 = v_2$, so both resistors have the same voltage. A similar argument shows that resistors R_2 and R_3 are also connected in parallel. Noticing that R_2 is connected in parallel with both R_1 and R_3, we say that all three resistors are connected in parallel. The order of parallel resistors is not important. For example, the voltages and currents of the three resistors in Figure 3.5-1 will not change if we interchange the positions R_2 and R_3.

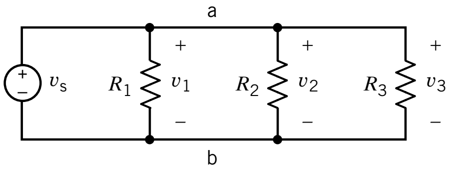


FIGURE 3.5-1
A circuit with parallel resistors.

The defining characteristic of parallel elements is that they have the same voltage. To identify a pair of parallel elements, we look for two elements connected between the same pair of nodes.

Consider the circuit with two resistors and a current source shown in Figure 3.5-2. Note that both resistors are connected to terminals a and b and that the voltage v appears across each parallel element. In anticipation of using Ohm's law, the passive convention is used to assign reference directions to the resistor voltages and currents. We may write KCL at node a (or at node b) to obtain

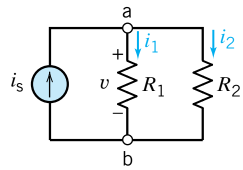


FIGURE 3.5-2
Parallel circuit with a current source.

$$i_s - i_1 - i_2 = 0$$

or

$$i_s = i_1 + i_2$$

However, from Ohm's law

$$i_1 = \frac{v}{R_1} \quad \text{and} \quad i_2 = \frac{v}{R_2}$$

Then

$$i_s = \frac{v}{R_1} + \frac{v}{R_2} \tag{3.5-1}$$

Recall that we defined conductance G as the inverse of resistance R. We may therefore rewrite Eq. 3.5-1 as

$$i_s = G_1 v + G_2 v = (G_1 + G_2)v \tag{3.5-2}$$

Thus, the equivalent circuit for this parallel circuit is a conductance G_p, as shown in Figure 3.5-3, where

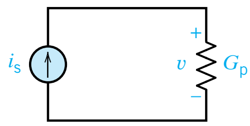


FIGURE 3.5-3
Equivalent circuit for a parallel circuit.

$$G_p = G_1 + G_2$$

The equivalent resistance for the two-resistor circuit is found from

$$G_p = \frac{1}{R_1} + \frac{1}{R_2}$$

Since $G_p = 1/R_p$, we have

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

or

$$R_p = \frac{R_1 R_2}{R_1 + R_2} \tag{3.5-3}$$

Note that the total conductance, G_p, increases as additional parallel elements are added and that the total resistance, R_p, declines as each resistor is added.

The circuit shown in Figure 3.5-2 is called a *current divider* circuit since it divides the source current. Note that

$$i_1 = G_1 v \tag{3.5-4}$$

Also, since $i_s = (G_1 + G_2)v$, we solve for v, obtaining

$$v = \frac{i_s}{G_1 + G_2} \tag{3.5-5}$$

Substituting v from Eq. 3.5-5 into Eq. 3.5-4, we obtain

$$i_1 = \frac{G_1 i_s}{G_1 + G_2} \tag{3.5-6}$$

Similarly,

$$i_2 = \frac{G_2 i_s}{G_1 + G_2}$$

Note that we may use $G_2 = 1/R_2$ and $G_1 = 1/R_1$ to obtain the current i_2 in terms of two resistances as follows:

$$i_2 = \frac{R_1 i_s}{R_1 + R_2}$$

The current of the source divides between conductances G_1 and G_2 in proportion to their conductance values.

Let us consider the more general case of current division with a set of N parallel conductors as shown in Figure 3.5-4. The KCL gives

$$i_s = i_1 + i_2 + i_3 + \cdots + i_N \tag{3.5-7}$$

for which

$$i_n = G_n v \tag{3.5-8}$$

for $n = 1, \cdots, N$. We may write Eq. 3.5-7 as

$$i_s = (G_1 + G_2 + G_3 + \cdots + G_N)v \tag{3.5-9}$$

Therefore

$$i_s = v\sum_{n=1}^{N} G_n \tag{3.5-10}$$

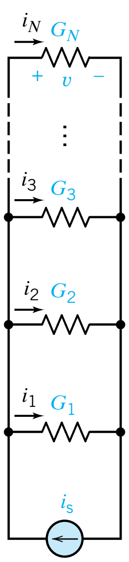


FIGURE 3.5-4 Set of N parallel conductances with a current source i_s.

Since $i_n = G_n v$, we may obtain v from Eq. 3.5-10 and substitute it in Eq. 3.5-8, obtaining

$$i_n = \frac{G_n i_s}{\sum_{n=1}^{N} G_n} \tag{3.5-11}$$

Recall that the equivalent circuit, Figure 3.5-3, has an equivalent conductance G_p such that

$$G_p = \sum_{n=1}^{N} G_n \tag{3.5-12}$$

Therefore

$$i_n = \frac{G_n i_s}{G_p} \tag{3.5-13}$$

which is the basic equation for the current divider with N conductances. Of course, Eq. 3.5-12 can be rewritten as

$$\frac{1}{R_p} = \sum_{n=1}^{N} \frac{1}{R_n} \tag{3.5-14}$$

Example 3.5-1

For the circuit in Figure 3.5-5 find (a) the current in each branch, (b) the equivalent circuit, and (c) the voltage v. The resistors are

$$R_1 = \frac{1}{2}\,\Omega, \quad R_2 = \frac{1}{4}\,\Omega, \quad R_3 = \frac{1}{8}\,\Omega$$

Solution

The current divider follows the equation

$$i_n = \frac{G_n i_s}{G_p}$$

so it is wise to find the equivalent circuit, as shown in Figure 3.5-6, with its equivalent conductance G_p. We have

$$G_p = \sum_{n=1}^{N} G_n = G_1 + G_2 + G_3 = 2 + 4 + 8 = 14\ \text{S}$$

Recall that the units for conductance are siemens (S). Then

$$i_1 = \frac{G_1 i_s}{G_p} = \frac{2}{14}(28) = 4\ \text{A}$$

Similarly,

$$i_2 = \frac{G_2 i_s}{G_p} = \frac{4(28)}{14} = 8\ \text{A}$$

and

$$i_3 = \frac{G_3 i_s}{G_p} = 16\ \text{A}$$

Since $i_n = G_n v$, we have

$$v = \frac{i_1}{G_1} = \frac{4}{2} = 2\ \text{V}$$

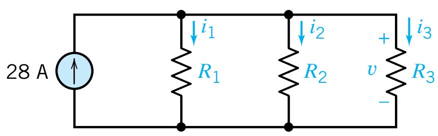


FIGURE 3.5-5
Parallel circuit for Example 3.5-1.

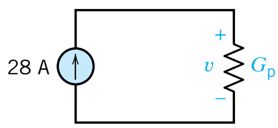


FIGURE 3.5-6
Equivalent circuit for the parallel circuit of Figure 3.5-5.

Example 3.5-2

For the circuit of Figure 3.5-7a, find the voltage measured by the voltmeter. Then show that the power absorbed by the two resistors is equal to that supplied by the source.

Solution

Figure 3.5-7b shows the circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m. The two resistors are connected in parallel and can be replaced with a single equivalent resistor. The resistance of this equivalent resistor is calculated as

$$\frac{40 \cdot 10}{40 + 10} = 8\,\Omega$$

Figure 3.5-7*c* shows the circuit after the parallel resistors have been replaced by the equivalent resistor. The current in the equivalent resistor is 250 mA, directed upward. This current and the voltage v_m do not adhere to the passive convention. The current in the equivalent resistance can also be expressed as −250 mA, directed downward. This current and the voltage v_m do adhere to the passive convention. Ohm's law gives

$$v_m = 8(-0.25) = -2\text{ V}$$

The voltage v_m in Figure 3.5-7*b* is equal to the voltage v_m in Figure 3.5-7*c*. This is a consequence of the equivalence of the 8-Ω resistor to the parallel combination of the 40-Ω and 10-Ω resistors. Looking at Figure 3.5-7*b,* the power absorbed by the resistors is

$$p_R = \frac{v_m^2}{40} + \frac{v_m^2}{10} = \frac{2^2}{40} + \frac{2^2}{10} = 0.1 + 0.4 = 0.5\text{ W}$$

The power supplied by the current source is

$$p_s = 2(0.25) = 0.5\text{ W}$$

Thus, the power absorbed by the two resistors is equal to that supplied by the source.

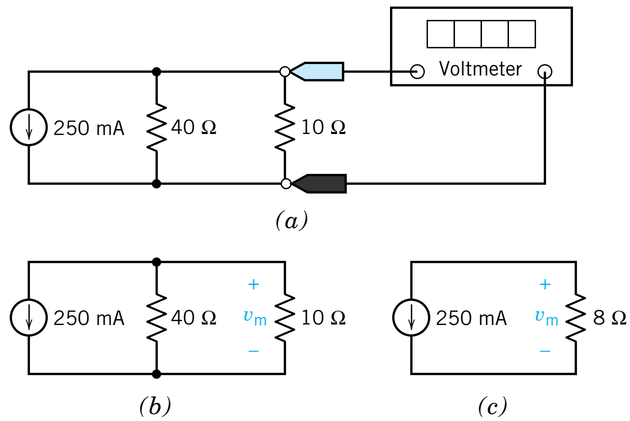


FIGURE 3.5-7
(*a*) A circuit containing parallel resistors. (*b*) The circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m. (*c*) The circuit after the parallel resistors have been replaced by an equivalent resistance.

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Exercise 3.5-1 A resistor network consisting of parallel resistors is shown in a package used for printed circuit board electronics in Figure E 3.5-1*a.* This package is only 2 cm × 0.7 cm, and each resistor is 1 kΩ. The circuit is connected to use four resistors as shown in Figure E 3.5-1*b.* Find the equivalent circuit for this network. Determine the current in each resistor when i_s = 1 mA.

Answer: R_p = 250 Ω

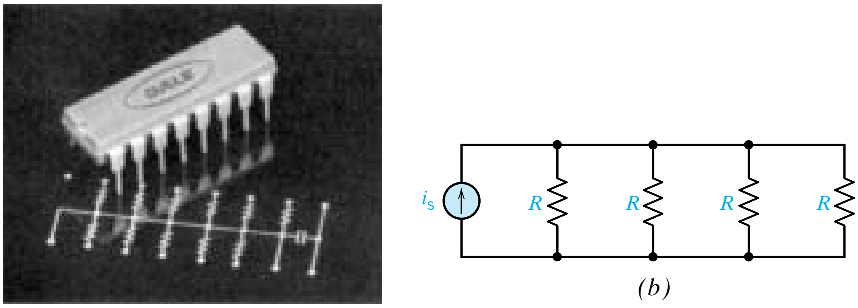


FIGURE E 3.5-1
(*a*) A parallel resistor network. Courtesy of Dale Electronics. (*b*) The connected circuit uses four resistors where $R = 1\ \text{k}\Omega$.

Exercise 3.5-2 Determine the current measured by the ammeter in the circuit shown in Figure E 3.5-2*a*.

Hint: Figure E 3.5-2*b* shows the circuit after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter, i_m.

Answer: $i_m = -1$ A

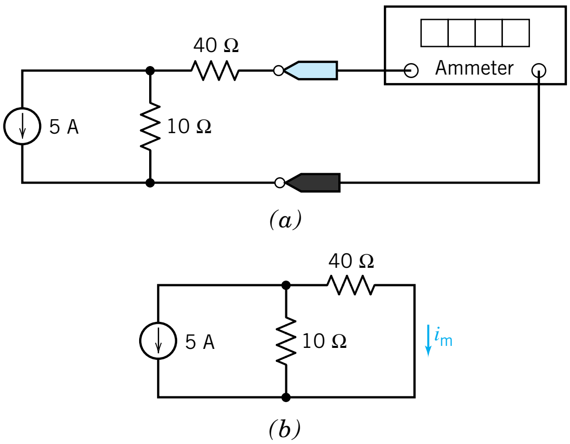


FIGURE E 3.5-2
(*a*) A current divider. (*b*) The current divider after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter, i_m.

3.6 Series Voltage Sources and Parallel Current Sources

Voltage sources connected in series are equivalent to a single voltage source. The voltage of the equivalent voltage source is equal to the sum of voltages of the series voltage sources.

Consider the circuit shown in Figure 3.6-1*a*. Notice that the currents of both voltage sources are equal. Accordingly, define the current, i_s, to be

$$i_s = i_a = i_b \tag{3.6-1}$$

Next, define the voltage, v_s, to be

$$v_s = v_a + v_b \tag{3.6-2}$$

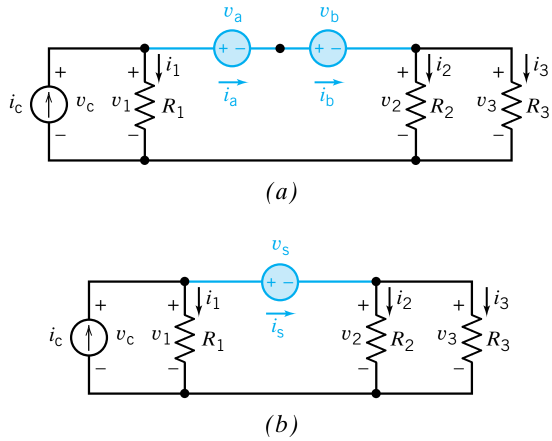


FIGURE 3.6-1
(*a*) A circuit containing voltage sources connected in series and (*b*) an equivalent circuit.

Using KCL, KVL, and Ohm's law, we can represent the circuit in Figure 3.6-1*a* by the equations

$$i_c = \frac{v_1}{R_1} + i_s \tag{3.6-3}$$

$$i_s = \frac{v_2}{R_2} + i_3 \tag{3.6-4}$$

$$v_c = v_1 \tag{3.6-5}$$

$$v_1 = v_s + v_2 \tag{3.6-6}$$

$$v_2 = i_3 R_3 \tag{3.6-7}$$

where $i_s = i_a = i_b$ and $v_s = v_a + v_b$. These same equations result from applying KCL, KVL, and Ohm's law to the circuit in Figure 3.6-1*b*. If $i_s = i_a = i_b$ and $v_s = v_a + v_b$, then the circuits shown in Figures 3.6-1*a* and 3.6-1*b* are equivalent because they are both represented by the same equations.

For example, suppose that $i_c = 4$ A, $R_1 = 2\ \Omega$, $R_2 = 6\ \Omega$, $R_3 = 3\ \Omega$, $v_a = 1$ V, and $v_b = 3$ V. The equations describing the circuit in Figure 3.6-1*a* become

$$4 = \frac{v_1}{2} + i_s \tag{3.6-8}$$

$$i_s = \frac{v_2}{6} + i_3 \tag{3.6-9}$$

$$v_c = v_1 \tag{3.6-10}$$

$$v_1 = 4 + v_2 \tag{3.6-11}$$

$$v_2 = 3 i_3 \tag{3.6-12}$$

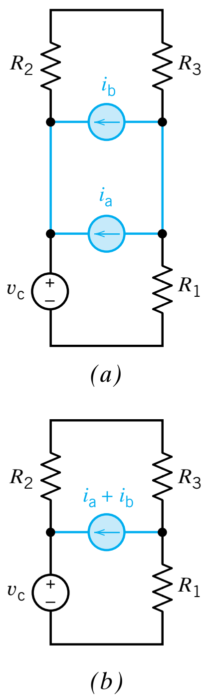
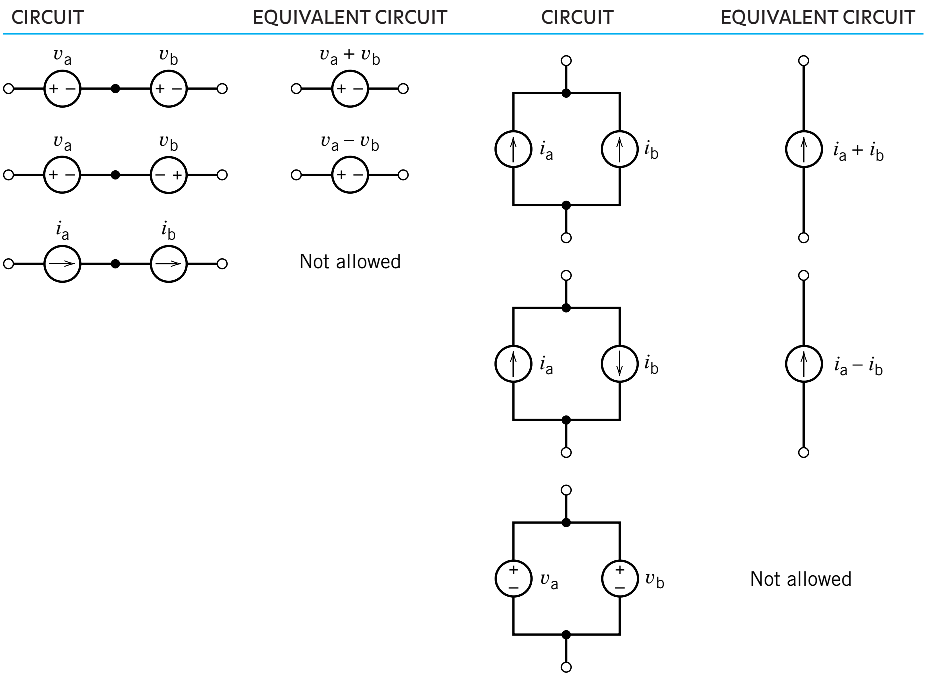


FIGURE 3.6-2
(*a*) A circuit containing parallel current sources and (*b*) an equivalent circuit.

The solution to this set of equations is $v_1 = 6$ V, $i_s = 1$ A, $i_3 = 0.66$ A, $v_2 = 2$ V, and $v_c = 6$ V. Eqs. 3.6-8 to 3.6-12 also describe the circuit in Figure 3.6-1*b*. Thus, $v_1 = 6$ V, $i_s = 1$ A, $i_3 = 0.66$ A, $v_2 = 2$ V, and $v_c = 6$ V in both circuits. Replacing series voltage sources by a single, equivalent voltage source does not change the voltage or current of other elements of the circuit.

Figure 3.6-2*a* shows a circuit containing parallel current sources. The circuit in Figure 3.6-2*b* is obtained by replacing these parallel current sources by a single, equivalent current source. The current of the equivalent current source is equal to the sum of the currents of the parallel current sources.

Table 3.6-1 Parallel and Series Voltage and Current Sources



We are not allowed to connect independent current sources in series. Series elements have the same current. This restriction prevents series current sources from being independent. Similarly, we are not allowed to connect independent voltage sources in parallel.

Table 3.6-1 summarizes the parallel and series connections of current and voltage sources.

3.7 Circuit Analysis

In this section we consider the analysis of a circuit by replacing a set of resistors with an equivalent resistance, thus reducing the network to a form easily analyzed.

Consider the circuit shown in Figure 3.7-1. Note that it includes a set of resistors that is in series and another set of resistors that is in parallel. It is desired to find the output voltage v_o, so we wish to reduce the circuit to the equivalent circuit shown in Figure 3.7-2.

Two circuits are **equivalent** if they exhibit identical characteristics at the same two terminals.

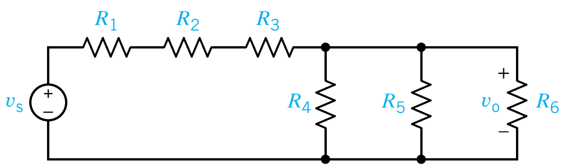


FIGURE 3.7-1
Circuit with a set of series resistors and a set of parallel resistors.

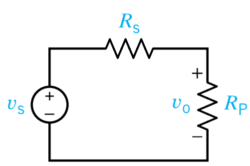


FIGURE 3.7-2
Equivalent circuit for the circuit of Figure 3.7-1.

We note that the equivalent series resistance is

$$R_s = R_1 + R_2 + R_3$$

and the equivalent parallel resistance is

$$R_p = \frac{1}{G_p}$$

where $$G_p = G_4 + G_5 + G_6$$

Then, using the voltage divider principle, with Figure 3.7-2, we have

$$v_o = \frac{R_p}{R_s + R_p} v_s$$

If a circuit has several combinations of sets of parallel resistors and sets of series resistors, one works through several steps, reducing the network to its simplest form.

Example 3.7-1

Consider the circuit shown in Figure 3.7-3. Find the current i_1 when

$$R_4 = 2\ \Omega \quad \text{and} \quad R_2 = R_3 = 8\ \Omega$$

Solution

Since the objective is to find i_1, we will attempt to reduce the circuit so that the 3-Ω resistor is in parallel with one resistor and the current source i_s. Then we can use the current divider principle to obtain i_1. Since R_2 and R_3 are in parallel, we find an equivalent resistance as

$$R_{p1} = \frac{R_2 R_3}{R_2 + R_3} = 4\ \Omega$$

Then adding R_{p1} to R_4, we have a series equivalent resistor

$$R_s = R_4 + R_{p1} = 2 + 4 = 6\ \Omega$$

Now the R_s resistor is in parallel with three resistors as shown in Figure 3.7-3*b*. However, we wish to obtain the equivalent circuit as shown in Figure 3.7-4, so that we can find i_1. Therefore, we combine the 9-Ω resistor, the 18-Ω resistor, and R_s shown to the right of terminals a–b in Figure 3.7-3*b* into one parallel equivalent conductance G_{p2}. Thus, we find

$$G_{p2} = \frac{1}{9} + \frac{1}{18} + \frac{1}{R_s} = \frac{1}{9} + \frac{1}{18} + \frac{1}{6} = \frac{1}{3}\ \text{S}$$

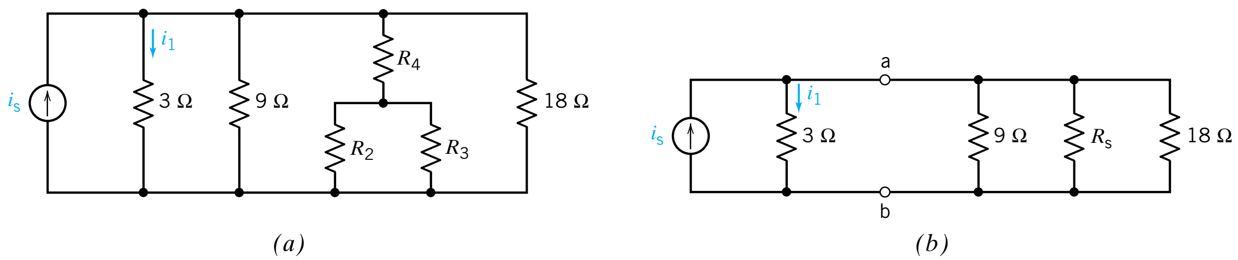


FIGURE 3.7-3
(*a*) Circuit for Example 3.7-1. (*b*) Partially reduced circuit for Example 3.7-1.

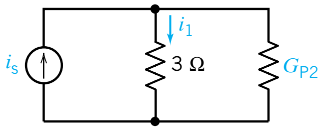


FIGURE 3.7-4
Equivalent circuit for Figure 3.7-3.

Then, using the current divider principle,

$$i_1 = \frac{G_1 i_s}{G_p}$$

where

$$G_p = G_1 + G_{p2} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

Therefore,

$$i_1 = \frac{1/3}{2/3} i_s = \frac{1}{2} i_s$$

Example 3.7-2

The circuit in Figure 3.7-5*a* contains an ohmmeter. An ohmmeter is an instrument that measures resistance in ohms. The ohmmeter will measure the equivalent resistance of the resistor circuit connected to its terminals. Determine the resistance measured by the ohmmeter in Figure 3.7-5*a*.

Solution

Working from left to right, the 30-Ω resistor is parallel to the 60-Ω resistor. The equivalent resistance is

$$\frac{60 \cdot 30}{60 + 30} = 20\,\Omega$$

In Figure 3.7-5*b* the parallel combination of the 30-Ω and 60-Ω resistors has been replaced with the equivalent 20-Ω resistor. Now the two 20-Ω resistors are in series. The equivalent resistance is

$$20 + 20 = 40\ \Omega$$

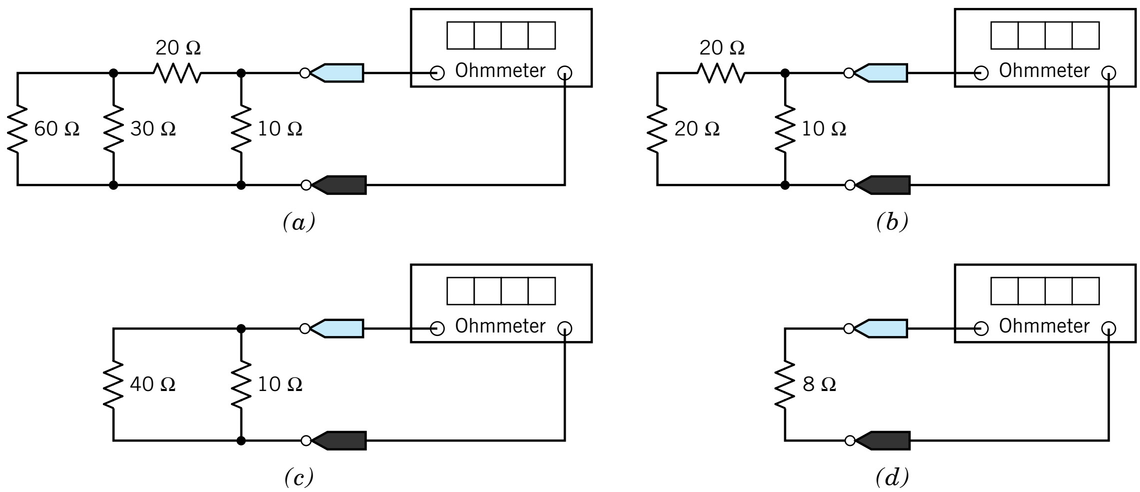


FIGURE 3.7-5

In Figure 3.7-5*c* the series combination of the two 20-Ω resistors has been replaced with the equivalent 40-Ω resistor. Now the 40-Ω resistor is parallel to the 10-Ω resistor. The equivalent resistance is

$$\frac{40 \cdot 10}{40 + 10} = 8\,\Omega$$

In Figure 3.7-5*d* the parallel combination of the 40-Ω and 10-Ω resistors has been replaced with the equivalent 8-Ω resistor. Thus, the ohmmeter measures a resistance equal to 8 Ω.

Try It Yourself! More Problems and Worked Examples Are in the **Electric Circuit Study Applets**

In general, we may find the equivalent resistance (or conductance) for a portion of a circuit consisting only of resistors and then replace that portion of the circuit with the equivalent resistance. For example, consider the circuit shown in Figure 3.7-6. We use $R_{\text{eq x-y}}$ to denote the equivalent resistance seen looking into terminals x–y. We note that the equivalent resistance to the right of terminals c–d is

$$R_{\text{eq c-d}} = \frac{15(6+4)}{15+(6+4)} = \frac{150}{25} = 6\,\Omega$$

Then the equivalent resistance of the circuit to the right of terminals a–b is

$$R_{\text{eq a-b}} = 4 + R_{\text{eq c-d}} = 4 + 6 = 10\,\Omega$$

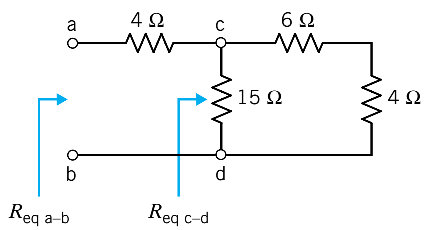


FIGURE 3.7-6
The equivalent resistance looking into terminals c–d is denoted as $R_{\text{eq c-d}}$.

Exercise 3.7-1 Determine the resistance measured by the ohmmeter in Figure E 3.7-1.

Answer: $\dfrac{(30+30)\cdot 30}{(30+30)+30} + 30 = 50\,\Omega$

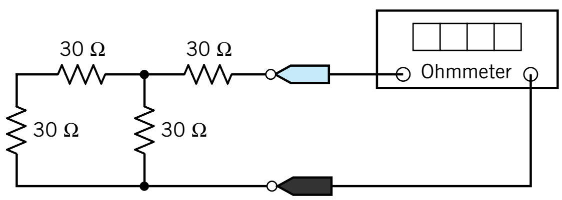


FIGURE E 3.7-1

Exercise 3.7-2 Determine the resistance measured by the ohmmeter in Figure E 3.7-2.

Answer: $12 + \dfrac{40 \cdot 10}{40 + 10} + 4 = 24\,\Omega$

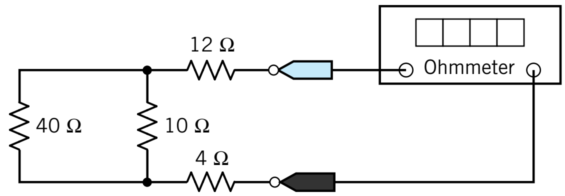


FIGURE E 3.7-2

Exercise 3.7-3 Determine the resistance measured by the ohmmeter in Figure E 3.7-3.

Answer: $\dfrac{(60 + 60 + 60) \cdot 60}{(60 + 60 + 60) + 60} = 45\,\Omega$

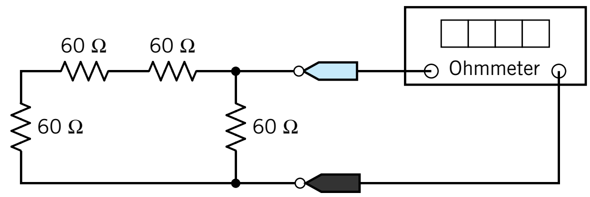


FIGURE E 3.7-3

3.8 Analyzing Resistive Circuits Using MATLAB

The computer program MATLAB is a tool for making mathematical calculations. In this section MATLAB is used to solve the equations encountered when analyzing a resistive circuit. Consider the resistive circuit shown in Figure 3.8-1*a*. The goal is to determine the value of the input voltage, V_s, required to cause the current I to be 1 A.

(Subscripts can't be used in the MATLAB input file. Thus V_s and R_p in Figure 3.8-1 become Vs and Rp in the MATLAB input file. We have been using lowercase letters to represent element voltages and currents, but in MATLAB examples we will use capital

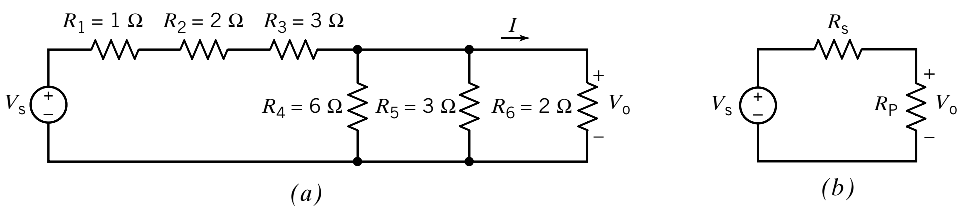


FIGURE 3.8-1
(*a*) A resistive circuit and (*b*) an equivalent circuit.

letters to represent currents and voltages to improve the readability of the MATLAB input file. For example, the input and output voltage are denoted as V_s and V_o rather than v_s and v_o.)

Resistors R_1, R_2, and R_3 are connected in series and can be replaced by an equivalent resistor, R_s, given by

$$R_s = R_1 + R_2 + R_3 \tag{3.8-1}$$

Resistors R_4, R_5, and R_6 are connected in parallel and can be replaced by an equivalent resistor, R_p, given by

$$R_p = \frac{1}{\dfrac{1}{R_4} + \dfrac{1}{R_5} + \dfrac{1}{R_6}} \tag{3.8-2}$$

Figure 3.8-1*b* shows the circuit after R_1, R_2, and R_3 are replaced by R_s and R_4, R_5, and R_6 are replaced by R_P. Applying voltage division to the circuit in Figure 3.8-1*b* gives

$$V_o = \frac{R_p}{R_s + R_p} V_s \tag{3.8-3}$$

where V_o is the voltage across R_p in Figure 3.8-1*b* and is also the voltage across the parallel resistors in Figure 3.8-1*a*. Ohm's law indicates that the current in R_6 is given by

$$I = \frac{V_o}{R_6} \tag{3.8-4}$$

Figure 3.8-2 shows a plot of the output current I versus the input voltage V_s. This plot shows that I will be 1 A when $V_s = 14$ V. Figure 3.8-3 shows the MATLAB input file that was used to obtain Figure 3.8-2. The MATLAB program first causes V_s to vary over a range of voltages. Next, MATLAB calculates the value of I corresponding to each value of V_s using Eqs. 3.8-1 through 3.8-4. Finally, MATLAB plots the current I versus the voltage V_s.

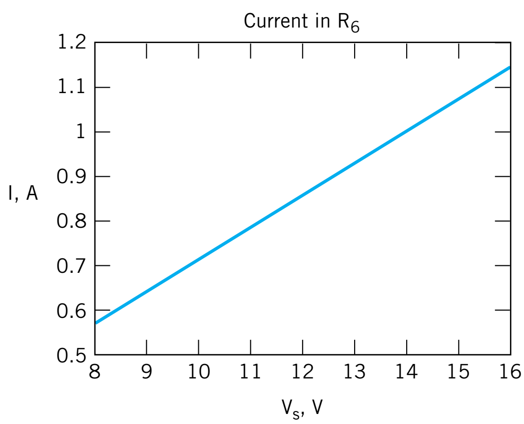


FIGURE 3.8-2
Plot of I versus V_s for the circuit shown in Figure 3.8-1.

```
% Analyzing Resistive Circuits Using MATLAB - ch3ex.m
%----------------------------------------------------------------
% Vary the input voltage from 8 to 16 volts in 0.1 volt steps.
%----------------------------------------------------------------
Vs = 8:0.1:16;
%----------------------------------------------------------------
%               Enter values of the resistances.
%----------------------------------------------------------------
R1 = 1; R2 = 2; R3 = 3;      % series resistors, ohms
R4 = 6; R5 = 3; R6 = 2;      % parallel resistors, ohms
%----------------------------------------------------------------
%    Find the current, I, corresponding to each value of Vs.
%----------------------------------------------------------------
Rs = R1 + R2 + R3;                      % Equation 3.8-1
Rp = 1 / (1/R4 +1/R5 +1/R6);            % Equation 3.8-2

for k = 1:length(Vs)
    Vo(k) = Vs(k) * Rp / (Rp + Rs);     % Equation 3.8-3
    I(k)   = Vo(k) / R6;                % Equation 3.8-4
end
%----------------------------------------------------------------
%                        Plot I versus Vs
%----------------------------------------------------------------
plot(Vs, I)
grid
xlabel('Vs, V'), ylabel('I,A')
title('Current in R6')
```

FIGURE 3.8-3
MATLAB input file used to obtain the plot of I versus V_s shown in Figure 3.8-2.

3.9 Verification Example

The circuit shown in Figure 3.9-1*a* was analyzed by writing and solving a set of simultaneous equations:

$$12 = v_2 + 4i_3, \quad i_4 = \frac{v_2}{5} + i_3$$
$$v_5 = 4i_3, \quad \frac{v_5}{2} = i_4 + 5i_4$$

A computer and the program Mathcad (Mathcad User's Guide, 1991) was used to solve the equations as shown in Figure 3.9-1*b*. It was determined that

$$v_2 = -60 \text{ V}, \quad i_3 = 18 \text{ A}, \quad i_4 = 6 \text{ A}, \quad v_5 = 72 \text{ V}$$

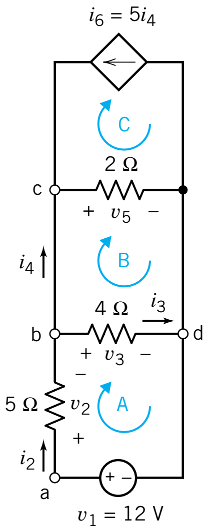
Are these currents and voltages correct?

The current i_2 can be calculated from v_2, i_3, i_4, and v_5 in a couple of different ways. First, Ohm's law gives

$$i_2 = \frac{v_2}{5} = \frac{-60}{5} = -12 \text{ A}$$

Next, applying KCL at node b gives

$$i_2 = i_3 + i_4 = 18 + 6 = 24 \text{ A}$$



(a)

v2 := 0   i3 := 0   i4 := 0   v5 := 0

Given

$12 \approx v2 + 4 \cdot i3$ Apply KVL to loop A.

$i4 \approx \dfrac{v2}{5} + i3$ Apply KCL at node b.

$v5 \approx 4 \cdot i3$ Apply KVL to loop B.

$\dfrac{v5}{2} \approx i4 + 5 \cdot i4$ Apply KCL at node c.

$$\text{Find } (v2, i3, i4, v5) = \begin{bmatrix} -60 \\ 18 \\ 6 \\ 72 \end{bmatrix}$$

(b)

FIGURE 3.9-1
(*a*) An example circuit and (*b*) computer analysis using Mathcad.

Clearly, i_2 cannot be both −12 and 24 A, so the values calculated for v_2, i_3, i_4, and v_5 cannot be correct. Checking the equations used to calculate v_2, i_3, i_4, and v_5, we find a sign error in the KCL equation corresponding to node b. This equation should be

$$i_4 = \frac{v_2}{5} - i_3$$

After making this correction, v_2, i_3, i_4, and v_5 are calculated to be

$$v_2 = 7.5 \text{ V}, \quad i_3 = 1.125 \text{ A}, \quad i_4 = 0.375 \text{ A}, \quad v_5 = 4.5 \text{ V}$$

Now
$$i_2 = \frac{v_2}{5} = \frac{7.5}{5} = 1.5$$

and
$$i_2 = i_3 + i_4 = 1.125 + 0.375 = 1.5$$

This checks as we expected.

As an additional check, consider v_3. First, Ohm's law gives

$$v_3 = 4i_3 = 4(1.125) = 4.5$$

Next, applying KVL to the loop consisting of the voltage source and the 4-Ω and 5-Ω resistors gives

$$v_3 = 12 - v_2 = 12 - 7.5 = 4.5$$

Finally, applying KVL to the loop consisting of the 2-Ω and 4-Ω resistors gives

$$v_3 = v_5 = 4.5$$

The results of these calculations agree with each other, indicating that

$$v_2 = 7.5 \text{ V}, \quad i_3 = 1.125 \text{ A}, \quad i_4 = 0.375 \text{ A}, \quad v_5 = 4.5 \text{ V}$$

are the correct values.

3.10 Design Challenge Solution

Adjustable Voltage Source

A circuit is required to provide an adjustable voltage. The specifications for this circuit are:

1. It should be possible to adjust the voltage to any value between −5 V and +5 V. It should not be possible to accidentally obtain a voltage outside this range.
2. The load current will be negligible.
3. The circuit should use as little power as possible.

The available components are:

1. Potentiometers: resistance values of 10 kΩ, 20 kΩ, and 50 kΩ are in stock
2. A large assortment of standard 2 percent resistors having values between 10 Ω and 1 MΩ (see Appendix E)
3. Two power supplies (voltage sources): one 12-V supply and one −12-V supply, both rated at 100 mA (maximum)

Describe the Situation and the Assumptions

Figure 3.10-1 shows the situation. The voltage v is the adjustable voltage. The circuit that uses the output of the circuit being designed is frequently called the "load." In this case, the load current is negligible, so $i = 0$.

State the Goal

A circuit providing the adjustable voltage

$$-5\text{ V} \le v \le +5\text{ V}$$

must be designed using the available components.

Generate a Plan

Make the following observations.

1. The adjustability of a potentiometer can be used to obtain an adjustable voltage v.
2. Both power supplies must be used so that the adjustable voltage can have both positive and negative values.
3. The terminals of the potentiometer cannot be connected directly to the power supplies because the voltage v is not allowed to be as large as 12 V or −12 V.

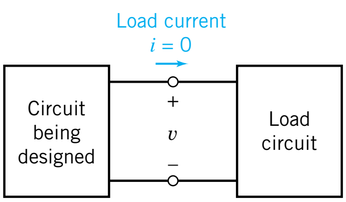


FIGURE 3.10-1 The circuit being designed provides an adjustable voltage, v, to the load circuit.

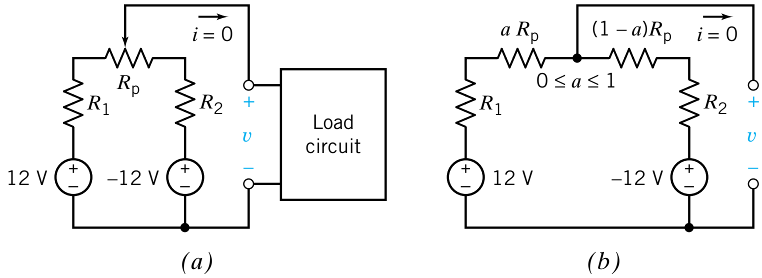


FIGURE 3.10-2
(*a*) A proposed circuit for producing the variable voltage, v, and (*b*) the equivalent circuit after the potentiometer is modeled.

These observations suggest the circuit shown in Figure 3.10-2*a*. The circuit in Figure 3.10-2*b* is obtained by using the simplest model for each component in Figure 3.10-2*a*.

To complete the design, values need to be specified for R_1, R_2, and R_p. Then, several results need to be checked and adjustments made, if necessary.

1. Can the voltage v be adjusted to any value in the range −5 V to +5 V?
2. Are the voltage source currents less than 100 mA? This condition must be satisfied if the power supplies are to be modeled as ideal voltage sources.
3. Is it possible to reduce the power absorbed by R_1, R_2, and R_p?

Act on the Plan

It seems likely the R_1 and R_2 will have the same value, so let $R_1 = R_2 = R$. Then it is convenient to redraw Figure 3.10-2*b* as shown in Figure 3.10-3.

Applying KVL to the outside loop yields

$$-12 + Ri_a + aR_p\,i_a + (1-a)\,R_p\,i_a + Ri_a - 12 = 0$$

so

$$i_a = \frac{24}{2R + R_p}$$

Next, applying KVL to the left loop gives

$$v = 12 - (R + aR_p)i_a$$

Substituting for i_a gives

$$v = 12 - \frac{24(R + aR_p)}{2R + R_p}$$

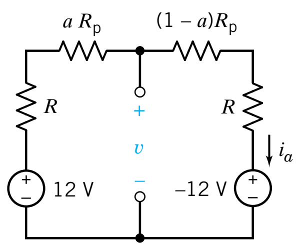


FIGURE 3.10-3
The circuit after setting $R_1 = R_2 = R$.

When $a = 0$, v must be 5 V, so

$$5 = 12 - \frac{24R}{2R + R_p}$$

Solving for R gives

$$R = 0.7R_p$$

Suppose the potentiometer resistance is selected to be $R_p = 20\ \text{k}\Omega$, the middle of the three available values. Then

$$R = 14\ \text{k}\Omega$$

Verify the Proposed Solution

As a check, notice that when $a = 1$

$$v = 12 - \left(\frac{14\ \text{k} + 20\ \text{k}}{28\ \text{k} + 20\ \text{k}}\right)24 = -5$$

as required. The specification that

$$-5\ \text{V} \le v \le 5\ \text{V}$$

has been satisfied. The power absorbed by the three resistances is

$$p = i_a^2(2R + R_p) = \frac{24^2}{2R + R_p}$$

so

$$p = 12\ \text{mW}$$

Notice that this power can be reduced by choosing R_p to be as large as possible, 50 kΩ in this case. Changing R_p to 50 kΩ requires a new value of R:

$$R = 0.7 \times R_p = 35\ \text{k}\Omega$$

Since

$$-5\ \text{V} = 12 - \left(\frac{35\ \text{k} + 50\ \text{k}}{70\ \text{k} + 50\ \text{k}}\right)24 \le v \le 12 - \left(\frac{35\ \text{k}}{70\ \text{k} + 50\ \text{k}}\right)24 = 5\ \text{V}$$

the specification that

$$-5\ \text{V} \le v \le 5\ \text{V}$$

has been satisfied. The power absorbed by the three resistances is now

$$p = \frac{24^2}{50\ \text{k} + 70\ \text{k}} = 5\ \text{mW}$$

Finally, the power supply current is

$$i_a = \frac{24}{50\ \text{k} + 70\ \text{k}} = 0.2\ \text{mA}$$

which is well below that 100 mA that the voltage sources are able to supply. The design is complete.

3.11 Summary

- Gustav Robert Kirchhoff formulated the laws that enable us to study a circuit. Kirchhoff's current law (KCL) states that the algebraic sum of all the currents entering a node is zero. Kirchhoff's voltage law (KVL) states that the algebraic sum of all the voltage around a closed path (loop) is zero.
- Two special circuits of interest are the series circuit and the parallel circuit. Table 3.11-1 summarizes the results from this chapter regarding series and parallel elements.

  ⇒ The first row of the table shows series resistors connected to a circuit. These series resistors can be replaced by an equivalent resistor. Doing so does not disturb the circuit connected to the series resistors. The element voltages and currents in this circuit don't change. In particular, the terminal voltage and current (labeled v and i, respectively, in Table 3.11-1) do not change when the series resistors are replaced by the equivalent resistor. Indeed, this is what is meant by "equivalent."

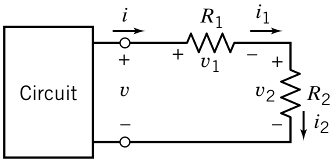
  ⇒ Replacing parallel resistors by an equivalent resistor does not change any voltage or current in the circuit connected to the parallel resistors.

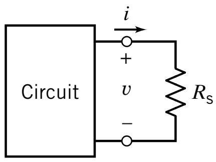
  ⇒ Replacing series voltage sources by an equivalent voltage source does not disturb the circuit connected to the series voltage sources.

  ⇒ Replacing parallel current sources by an equivalent current source does not disturb the circuit connected to the parallel current sources.
- A circuit consisting of series resistors is sometimes called a voltage divider because the voltage across the series resistors divides between the individual resistors. Similarly, a circuit consisting of parallel resistors is called a current divider because the current through the parallel resistors divides between the individual resistors. Table 3.11-1 provides the equations describing voltage division for series resistors and current division for parallel resistors.

Table 3.11-1 Equivalent Circuits for Series and Parallel Elements

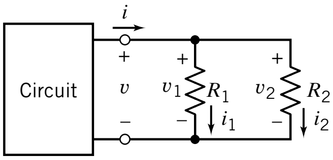
Series resistors

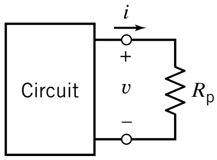




$i = i_1 = i_2$, $v_1 = \dfrac{R_1}{R_1 + R_2}\, v$, and $v_2 = \dfrac{R_2}{R_1 + R_2}\, v$ $R_s = R_1 + R_2$ and $v = R_s i$

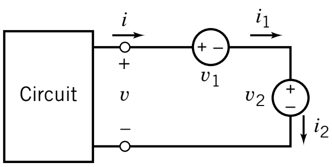
Parallel resistors

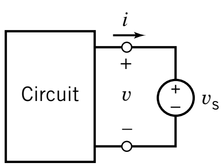




$v = v_1 = v_2$, $i_1 = \dfrac{R_2}{R_1 + R_2}\, i$, and $i_2 = \dfrac{R_1}{R_1 + R_2}\, i$ $R_p = \dfrac{R_1 R_2}{R_1 + R_2}$ and $v = R_p i$

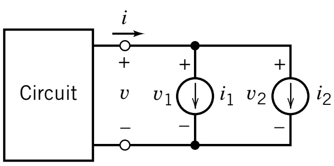
Series voltage sources

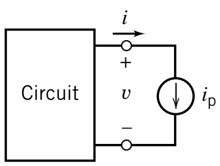




$i = i_1 = i_2$ and $v = v_1 + v_2$ $v_s = v_1 + v_2$

Parallel current sources





$v = v_1 = v_2$ and $i = i_1 + i_2$ $i_p = i_1 + i_2$

Problems

Section 3.3 Kirchhoff's Laws

P 3.3-1 Consider the circuit shown in Figure P 3.3-1. Determine the values of the power supplied by branch *B* and the power supplied by branch *F*.

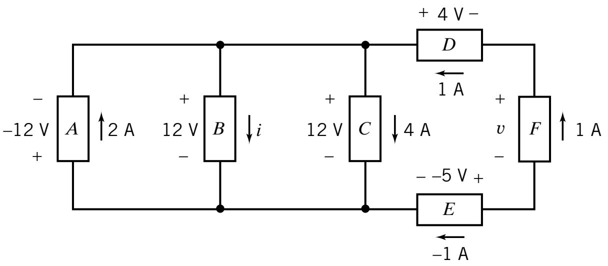


FIGURE P 3.3-1

P 3.3-2 Determine the values of i_2, i_4, v_2, v_3, and v_6 in Figure P 3.3-2.

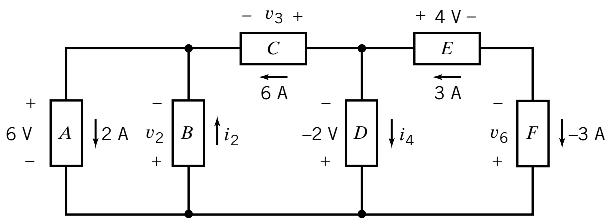


FIGURE P 3.3-2

P 3.3-3 Consider the circuit shown in Figure P 3.3-3.

(a) Suppose that $R_1 = 6\ \Omega$ and $R_2 = 3\ \Omega$. Find the current i and the voltage v.

(b) Suppose, instead, that $i = 1.5$ A and $v = 2$ V. Determine the resistances R_1 and R_2.

(c) Suppose, instead, that the voltage source supplies 24 W of power and that the current source supplies 9 W of power. Determine the current *i*, the voltage *v*, and the resistances R_1 and R_2.

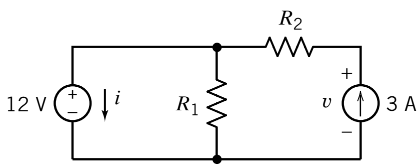


FIGURE P 3.3-3

P 3.3-4 Determine the power absorbed by each of the resistors in the circuit shown in Figure P 3.3-4.

Answer: The 4-Ω resistor absorbs 100 W, the 6-Ω resistor absorbs 24 W, and the 8-Ω resistor absorbs 72 W.

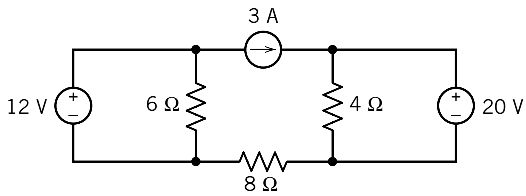


FIGURE P 3.3-4

P 3.3-5 Determine the power absorbed by each of the resistors in the circuit shown in Figure P 3.3-5.

Answer: The 4-Ω resistor absorbs 16 W, the 6-Ω resistor absorbs 24 W, and the 8-Ω resistor absorbs 8 W.

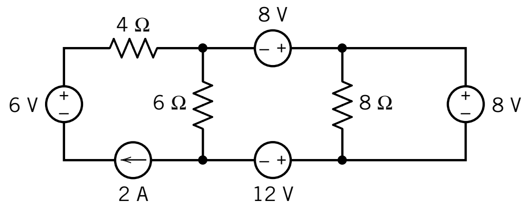


FIGURE P 3.3-5

P 3.3-6 Determine the power supplied by each current source in the circuit of Figure P 3.3-6.

Answer: The 2-mA current source supplies 6 mW and the 1-mA current source supplies −7 mW.

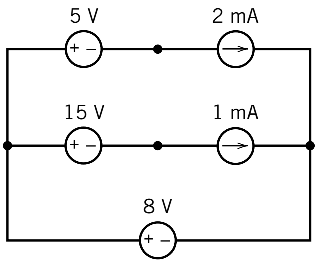


FIGURE P 3.3-6

P 3.3-7 Determine the power supplied by each voltage source in the circuit of Figure P 3.3-7.

Answer: The 2-V voltage source supplies 2 mW and the 3-V voltage source supplies −6 mW.

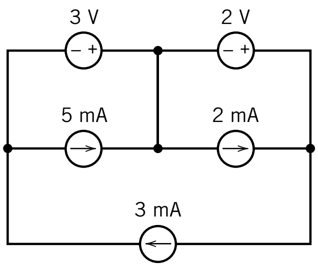


FIGURE P 3.3-7

P 3.3-8 What is the value of the resistance R in Figure P 3.3-8?
Hint: Assume an ideal ammeter. An ideal ammeter is equivalent to a short circuit.
Answer: $R = 4\ \Omega$

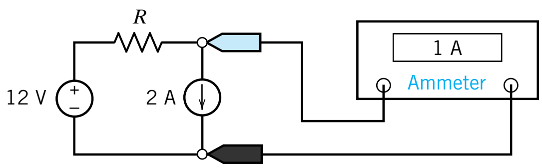


FIGURE P 3.3-8

P 3.3-9 The voltmeter in Figure P 3.3-9 measures the value of the voltage across the current source to be 56 V. What is the value of the resistance R?
Hint: Assume an ideal voltmeter. An ideal voltmeter is equivalent to an open circuit.
Answer: $R = 10\ \Omega$

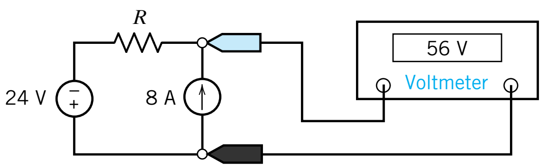


FIGURE P 3.3-9

P 3.3-10 Determine the values of the resistances R_1 and R_2 in Figure P 3.3-10.

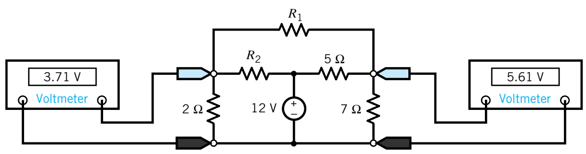


FIGURE P 3.3-10

Section 3.4 A Single-Loop Circuit—The Voltage Divider

P 3.4-1 Use voltage division to determine the voltages v_1, v_2, v_3, and v_4 in the circuit shown in Figure P 3.4-1.

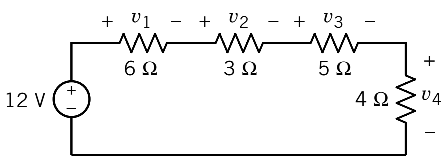


FIGURE P 3.4-1

P 3.4-2 Consider the circuits shown in Figure P 3.4-2.

(a) Determine the value of the resistance R in Figure P 3.4-2b that makes the circuit in Figure P 3.4-2b equivalent to the circuit in Figure P 3.4-2a.

(b) Determine the current i in Figure P 3.4-2b. Because the circuits are equivalent, the current i in Figure P 3.4-2a is equal to the current i in Figure P 3.4-2b.

(c) Determine the power supplied by the voltage source.

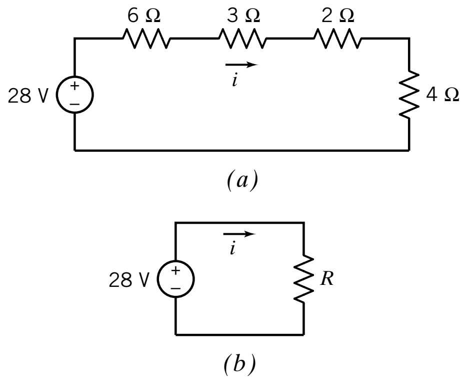


FIGURE P 3.4-2

P 3.4-3 The ideal voltmeter in the circuit shown in Figure P 3.4-3 measures the voltage v.

(a) Suppose $R_2 = 100\ \Omega$. Determine the value of R_1.

(b) Suppose, instead, $R_1 = 100\ \Omega$. Determine the value of R_2.

(c) Suppose, instead, that the voltage source supplies 1.2 W of power. Determine the values of both R_1 and R_2.

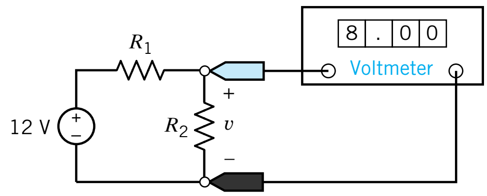


FIGURE P 3.4-3

P 3.4-4 Determine the voltage v in the circuit shown in Figure P 3.4-4.

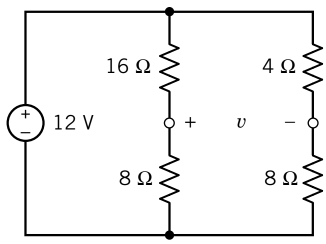


FIGURE P 3.4-4

P 3.4-5 The model of a cable and load resistor connected to a source is shown in Figure P 3.4-5. Determine the appropriate cable resistance, R, so that the output voltage, v_o, remains between 9 V and 13 V when the source voltage, v_s, varies between 20 V and 28 V. The cable resistance can only assume integer values in the range $20 < R < 100\ \Omega$.

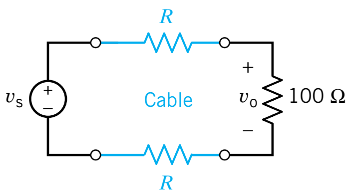


FIGURE P 3.4-5
Circuit with a cable.

P 3.4-6 The input to the circuit shown in Figure P 3.4-6 is the voltage of the voltage source, v_a. The output of this circuit is the voltage measured by the voltmeter, v_b. This circuit produces an output that is proportional to the input, that is

$$v_b = k\, v_a$$

where k is the constant of proportionality.

(a) Determine the value of the output, v_b, when $R = 240\ \Omega$ and $v_a = 18$ V.

(b) Determine the value of the power supplied by the voltage source when $R = 240\ \Omega$ and $v_a = 18$ V.

(c) Determine the value of the resistance, R, required to cause the output to be $v_b = 2$ V when the input is $v_a = 18$ V.

(d) Determine the value of the resistance, R, required to cause $v_b = 0.2\, v_a$ (that is, the value of the constant of proportionality is $k = \frac{2}{10}$).

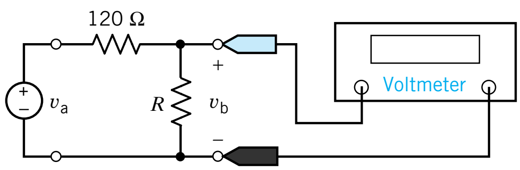


FIGURE P 3.4-6

Section 3.5 Parallel Resistors and Current Division

P 3.5-1 Use current division to determine the currents i_1, i_2, i_3, and i_4 in the circuit shown in Figure P 3.5-1.

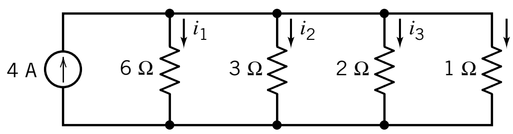


FIGURE P 3.5-1

P 3.5-2 Consider the circuits shown in Figure P 3.5-2.

(a) Determine the value of the resistance R in Figure P 3.5-2*b* that makes the circuit in Figure P 3.4-2*b* equivalent to the circuit in Figure P 3.5-2*a*.

(b) Determine the voltage v in Figure P 3.5-2*b*. Because the circuits are equivalent, the voltage v in Figure P 3.5-2*a* is equal to the voltage v in Figure P 3.5-2*b*.

(c) Determine the power supplied by the current source.

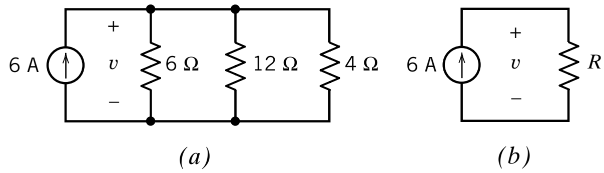


FIGURE P 3.5-2

P 3.5-3 The ideal voltmeter in the circuit shown in Figure P 3.5-3 measures the voltage v.

(a) Suppose $R_2 = 12\ \Omega$. Determine the value of R_1 and of the current i.

(b) Suppose, instead, $R_1 = 12\ \Omega$. Determine the value of R_2 and of the current i.

(c) Instead, choose R_1 and R_2 to minimize the power absorbed by any one resistor.

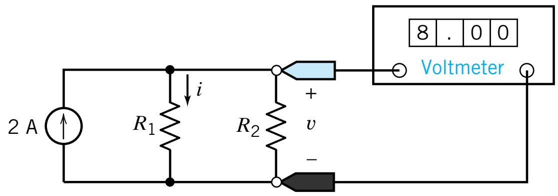


FIGURE P 3.5-3

P 3.5-4 Determine the current i in the circuit shown in Figure P 3.5-4.

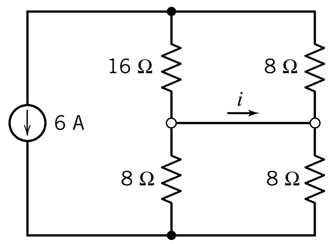


FIGURE P 3.5-4

P 3.5-5 Consider the circuit shown in Figure P 3.5-5 when $4\ \Omega \le R_1 \le 6\ \Omega$ and $R_2 = 10\ \Omega$. Select the source i_s so that v_o remains between 9 V and 13 V.

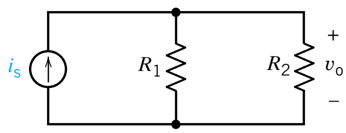


FIGURE P 3.5-5

P 3.5-6. The input to the circuit shown in Figure P 3.5-6 is the current of the current source, i_a. The output of this circuit is the current measured by the ammeter, i_b. This circuit produces an output that is proportional to the input, that is

$$i_b = k\, i_a$$

where k is the constant of proportionality.

(a) Determine the value of the output, i_b, when $R = 24\ \Omega$ and $i_a = 1.8$ A.
(b) Determine the value of the resistance, R, required to cause the output to be $i_b = 1.6$ A when the input is $i_a = 2$ A.
(c) Determine the value of the resistance, R, required to cause $i_b = 0.4\ i_a$ (that is, the value of the constant of proportionality is $k = \frac{4}{10}$).

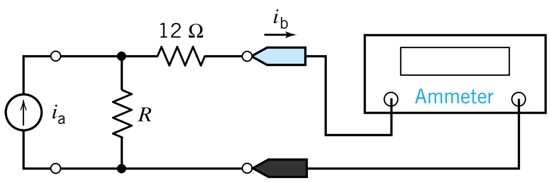


FIGURE P 3.5-6

Section 3.7 Circuit Analysis

P 3.7-1 The circuit shown in Figure P 3.7-1a has been divided into two parts. In Figure P 3.7-1b, the right-hand part has been replaced with an equivalent circuit. The left-hand part of the circuit has not been changed.

(a) Determine the value of the resistance R in Figure P 3.7-1b that makes the circuit in Figure P 3.7-1b equivalent to the circuit in Figure P 3.7-1a.
(b) Find the current i and the voltage v shown in Figure P 3.7-1b. Because of the equivalence, the current i and the voltage v shown in Figure P 3.7-1a are equal to the current i and the voltage v shown in Figure P 3.7-1b.
(c) Find the current i_2 shown in Figure P 3.7-1a using current division.

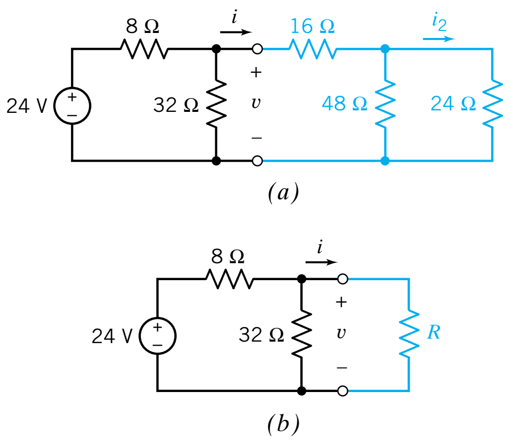


FIGURE P 3.7-1

P 3.7-2 The circuit shown in Figure P 3.7-2a has been divided into three parts. In Figure P 3.7-2b, the rightmost part has been replaced with an equivalent circuit. The rest of the circuit has not been changed. The circuit is simplified further in Figure 3.7-2c. Now the middle and rightmost parts have been replaced by a single equivalent resistance. The leftmost part of the circuit is still unchanged.

(a) Determine the value of the resistance R_1 in Figure P 3.7-2b that makes the circuit in Figure P 3.7-2b equivalent to the circuit in Figure P 3.7-2a.
(b) Determine the value of the resistance R_2 in Figure P 3.7-2c that makes the circuit in Figure P 3.7-2c equivalent to the circuit in Figure P 3.7-2b.
(c) Find the current i_1 and the voltage v_1 shown in Figure P 3.7-2c. Because of the equivalence, the current i_1 and the voltage v_1 shown in Figure P 3.7-2b are equal to the current i_1 and the voltage v_1 shown in Figure P 3.7-2c.
Hint: $24 = 6(i_1 - 2) + i_1 R_2$
(d) Find the current i_2 and the voltage v_2 shown in Figure P 3.7-2b. Because of the equivalence, the current i_2 and the voltage v_2 shown in Figure P 3.7-2a are equal to the current i_2 and the voltage v_2 shown in Figure P 3.7-2b.
Hint: Use current division to calculate i_2 from i_1.
(e) Determine the power absorbed by the 3-Ω resistance shown at the right of Figure P 3.7-2a.

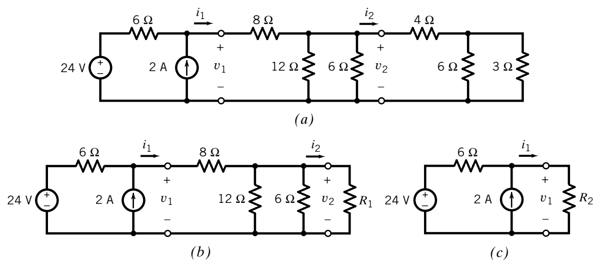


FIGURE P 3.7-2

P 3.7-3 Find i using appropriate circuit reductions and the current divider principle for the circuit of Figure P 3.7-3.

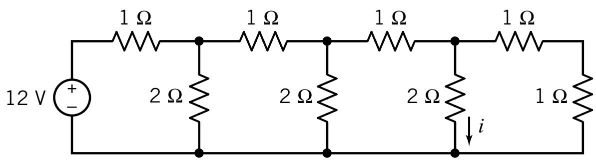


FIGURE P 3.7-3

P 3.7-4 (a) Determine values of R_1 and R_2 in Figure P 3.7-4b that make the circuit in Figure P 3.7-4b equivalent to the circuit in Figure P 3.7-4a.
(b) Analyze the circuit in Figure P 3.7-4b to determine the values of the currents i_a and i_b
(c) Because the circuits are equivalent, the currents i_a and i_b shown in Figure P 3.7-4b are equal to the currents i_a and i_b shown in Figure P 3.7-4a. Use this fact to determine values of the voltage v_1 and current i_2 shown in Figure P 3.7-4a.

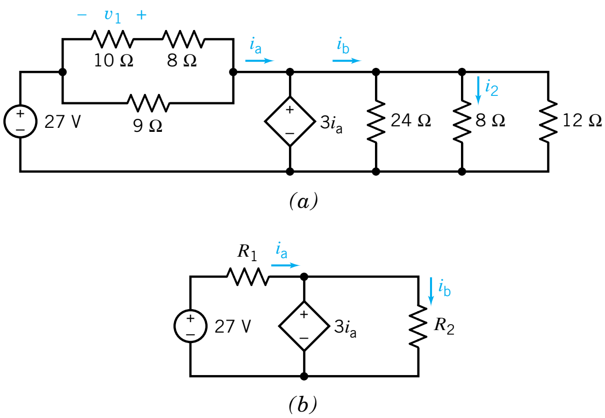


FIGURE P 3.7-4

P 3.7-5 The voltmeter in the circuit shown in Figure P 3.7-5 shows that the voltage across the 30-Ω resistor is 6 volts. Determine the value of the resistance R_1.

Hint: Use the voltage division twice.

Answer: $R_1 = 40\ \Omega$

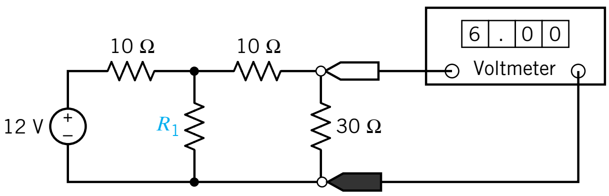


FIGURE P 3.7-5

P 3.7-6 Determine the voltages v_a and v_c and the currents i_b and i_d for the circuit shown in Figure P 3.7-6.

Answer: $v_a = -2$ V, $v_c = 6$ V, $i_b = -16$ mA, and $i_d = 2$ mA

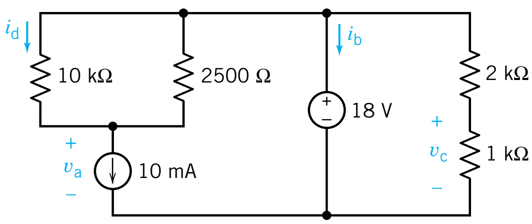


FIGURE P 3.7-6

P 3.7-7 Determine the value of the resistance R in Figure 3.7-7.

Answer: $R = 28$ kΩ

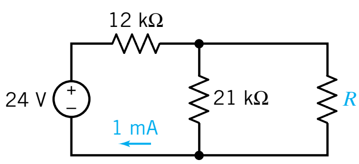


FIGURE P 3.7-7

P 3.7-8 Most of us are familiar with the effects of a mild electric shock. The effects of a severe shock can be devastating and often fatal. Shock results when current is passed through the body. A person can be modeled as a network of resistances. Consider the model circuit shown in Figure P 3.7-8. Determine the voltage developed across the heart and the current flowing through the heart of the person when he or she firmly grasps one end of a voltage source whose other end is connected to the floor. The heart is represented by R_h. The floor has resistance to current flow equal to R_f, and the person is standing barefoot on the floor. This type of accident might occur at a swimming pool or boat dock. The upper-body resistance R_u and lower-body resistance R_L vary from person to person.

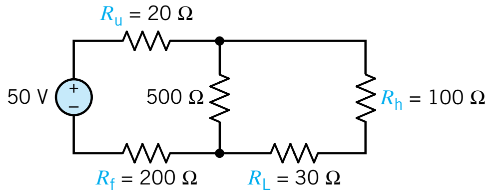


FIGURE P 3.7-8
The resistance of the heart. $R_h = 100\ \Omega$

P 3.7-9 Determine the value of the current i in Figure 3.7-9.

Answer: $i = 0.5$ mA

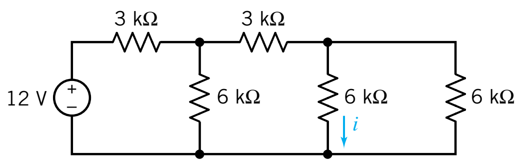


FIGURE P 3.7-9

P 3.7-10 Determine the values of i_a, i_b, and v_c in Figure P 3.7-10.

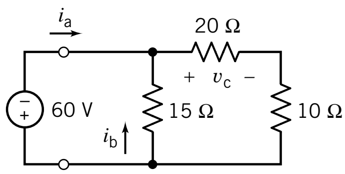


FIGURE P 3.7-10

P 3.7-11 Find i and $R_{eq\ a-b}$ if $v_{ab} = 40$ V in the circuit of Figure P 3.7-11.

Answer: $R_{eq\ a-b} = 8\ \Omega$, $i = 5/6$ A

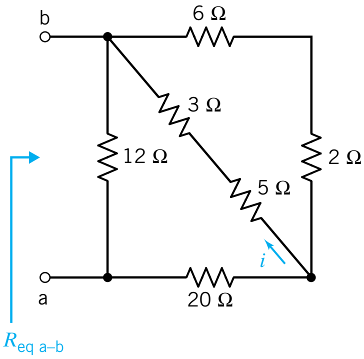


FIGURE P 3.7-11

P 3.7-12 The ohmmeter in Figure P 3.7-12 measures the equivalent resistance, R_{eq}, of the resistor circuit. The value of the equivalent resistance, R_{eq}, depends on the value of the resistance R.

(a) Determine the value of the equivalent resistance, R_{eq}, when $R = 18\ \Omega$.

(b) Determine the value of the resistance R required to cause the equivalent resistance to be $R_{eq} = 18\ \Omega$.

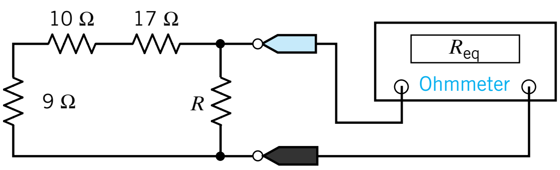


FIGURE P 3.7-12

P 3.7-13 The source $v_s = 240$ volts is connected to three equal resistors as shown in Figure P 3.7-13. Determine R when the voltage source delivers 1920 W to the resistors.

Answer: $R = 45\ \Omega$

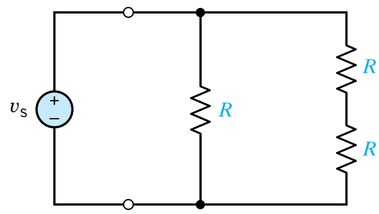


FIGURE P 3.7-13

P 3.7-14 Find the R_{eq} at terminals a–b in Figure P 3.7-14. Also determine i, i_1, and i_2.

Answer: $R_{eq} = 8\ \Omega$, $i = 5$ A, $i_1 = 5/3$ A, $i_2 = 5/2$ A

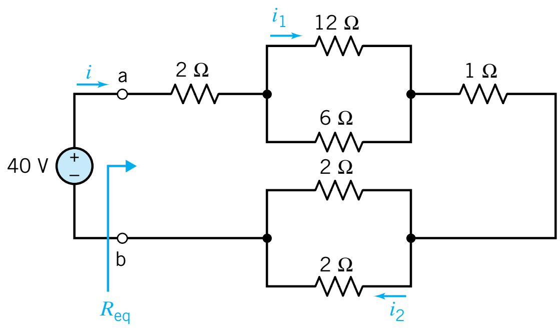


FIGURE P 3.7-14

Verification Problems

VP 3-1 A computer analysis program, used for the circuit of Figure VP 3.1, provides the following branch currents and voltages: i_1 A = −0.833, i_2 A = −0.333, i_3 A = −1.167, and $v = -2.0$ V. Are these answers correct?

Hint: Verify that KCL is satisfied at the center node and that KVL is satisfied around the outside loop consisting of the two 6-Ω resistors and the voltage source.

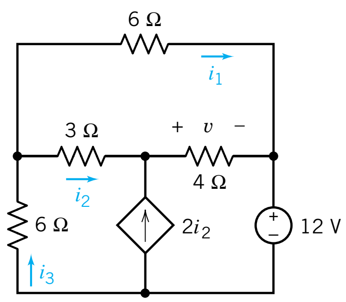


FIGURE VP 3.1

VP 3-2 The circuit of Figure VP 3.2 was assigned as a homework problem. The answer in the back of the textbook says the current, i, is 1.25 A. Verify this answer using current division.

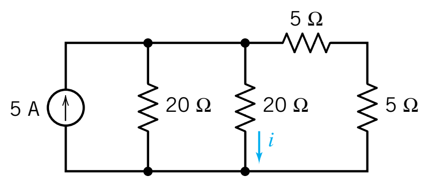


FIGURE VP 3.2

VP 3-3 The circuit of Figure VP 3.3 was built in the lab and v_o was measured to be 6.25 V. Verify this measurement using the voltage divider principle.

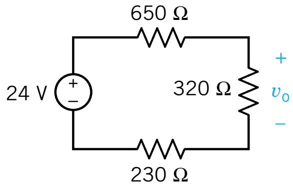


FIGURE VP 3.3

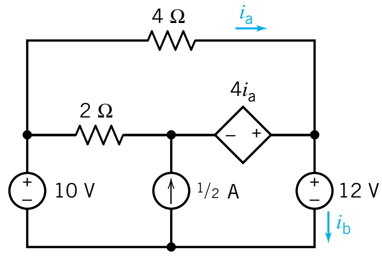


FIGURE VP 3.5

VP 3-4 The circuit of Figure VP 3.4 represents an auto's electrical system. A report states that $i_H = 9$ A, $i_B = -9$ A, and $i_A = 19.1$ A. Verify that this result is correct.
Hint: Verify that KCL is satisfied at each node and that KVL is satisfied around each loop.

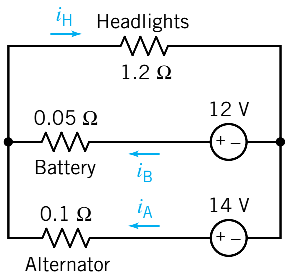


FIGURE VP 3.4
Electric circuit model of an automobile's electrical system.

VP 3-5 Computer analysis of the circuit in Figure VP 3.5 shows that $i_a = -0.5$ mA and $i_b = -2$ mA. Was the computer analysis done correctly?
Hint: Verify that the KVL equations for all three meshes are satisfied when $i_a = -0.5$ mA and $i_b = -2$ mA.

VP 3-6 Computer analysis of the circuit in Figure VP 3.6 shows that $i_a = 0.5$ mA and $i_b = 4.5$ mA. Was the computer analysis done correctly?
Hint: First, verify that the KCL equations for all five nodes are satisfied when $i_a = 0.5$ mA and $i_b = 4.5$ mA. Next, verify that the KVL equation for the lower left mesh (a-e-d-a) is satisfied. (The KVL equations for the other meshes aren't useful because each involves an unknown voltage.)

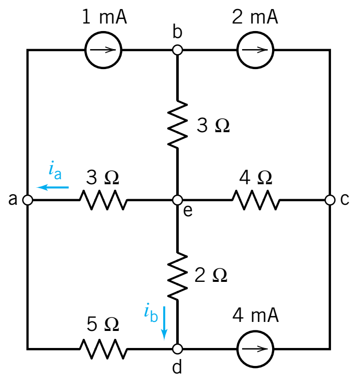


FIGURE VP 3.6

Design Problems

DP 3-1 The circuit shown in Figure DP 3.1 uses a potentiometer to produce a variable voltage. The voltage v_m varies as a knob connected to the wiper of the potentiometer is turned. Specify the resistances R_1 and R_2 so that the following three requirements are satisfied:

1. The voltage v_m varies from 8 V to 12 V as the wiper moves from one end of the potentiometer to the other end of the potentiometer.
2. The voltage source supplies less than 0.5 W of power.
3. Each of R_1, R_2, and R_P dissipate less than 0.25 W.

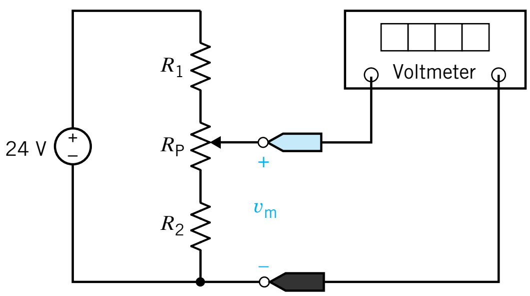


FIGURE DP 3.1

DP 3-2 The resistance R_L in Figure DP 3.2 is the equivalent resistance of a pressure transducer. This resistance is specified to be 200 Ω ± 5 percent. That is, 190 Ω ≤ R_L ≤ 210 Ω. The voltage source is a 12 V ± 1 percent source capable of supplying 5 W. Design this circuit, using 5 percent, 1/8-watt resistors for R_1 and R_2, so that the voltage across R_L is

$$v_o = 4 \text{ V} \pm 10\%$$

(A 5 percent, 1/8-watt 100-Ω resistor has a resistance between 95 and 105 Ω and can safely dissipate 1/8-W continuously.)