Sample Problems from Solving Statics Problems in MATLAB

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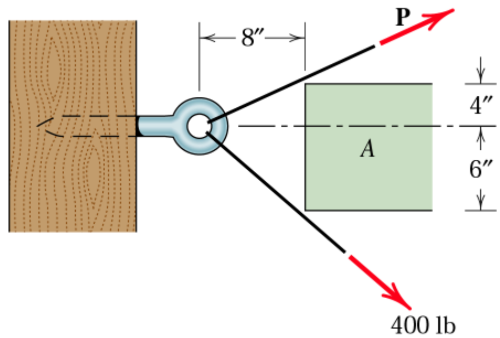
Solving Statics Problems in MATLAB is a supplement to the textbook ***Engineering Mechanics: Statics*** (5th Edition) by J.L. Meriam and L.G. Kraige, Wiley, 2001.

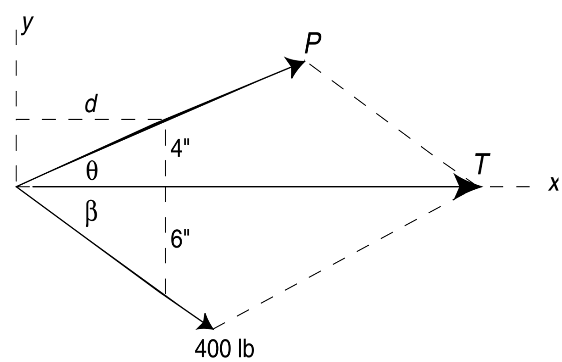
2.1 Problem 2/21 (2D Rectangular Components)

It is desired to remove the spike from the timber by applying force along its horizontal axis. An obstruction *A* prevents direct access, so that two forces, one 400 lb and the other **P** are applied by cables as shown. Here we want to investigate the effects of the distance between the spike and the obstruction on the two forces so replace 8" by *d* in the figure. Determine the magnitude of **P** necessary to insure a resultant **T** directed along the spike. Also find *T*. Plot *P* and *T* as a function of *d* letting *d* range between 2 and 12 inches.



Problem Formulation

The two forces and their resultant are shown on the diagram to the right. The horizontal component of the resultant is *T* while the vertical component is 0. Thus,



$$T = \Sigma F_x = P\cos\theta + 400\cos\beta$$
$$0 = \Sigma F_y = P\sin\theta - 400\sin\beta$$

These two equations can be easily solved for *P* and

$$P = \frac{400\sin\beta}{\sin\theta} \qquad T = 400(\sin\beta\cot\theta + \cos\beta)$$

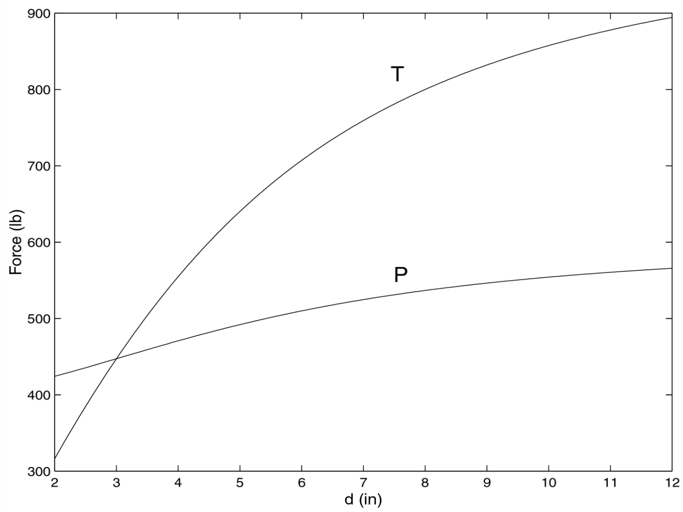
At this point we have *P* and *T* as functions of β and θ. From the figure above we can relate β and θ to *d* as follows.

$$\theta = \tan^{-1}(4/d) \qquad \beta = \tan^{-1}(6/d)$$

One nice thing about using a computer is that it will not be necessary to substitute these results into those above to get *P* and *T* explicitly as functions of *d*. The computer carries out this substitution automatically.

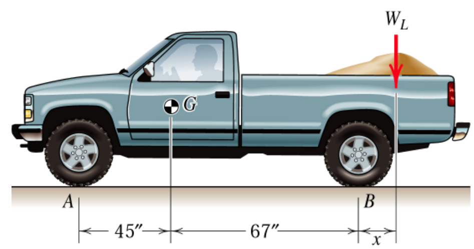
MATLAB Script

```
d=2:0.05:12;
theta=atan(4./d); beta=atan(6./d);
P=400*sin(beta)./sin(theta);
T=400*(sin(beta).*cot(theta)+cos(beta));
plot(d,P,d,T)
xlabel('d (in)'); ylabel('Force (lb)');
```



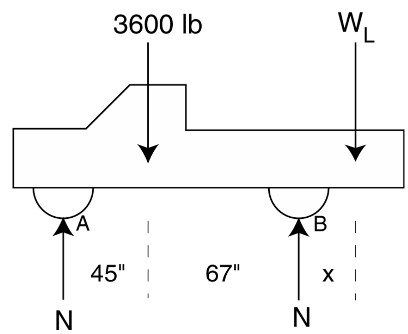
3.1 Problem 3/25 (2D Equilibrium)

The indicated location of the center of gravity of the 3600-lb pickup truck is for the unladen condition. A load W_L whose center of gravity is x inches behind the rear axle is added to the truck. Find the relationship between W_L and x if the normal forces under the front and rear wheels are to be equal. For this case, plot W_L as a function of x for x ranging between 0 and 50 inches.



Problem Formulation

The free-body diagram for the truck is shown to the right. Normally, the two normal forces under the wheels would not be identical, of course. Here we want the relationship between the weight W_L and its location (x) which results in these two forces being equal. This relationship is found from the equilibrium equations.



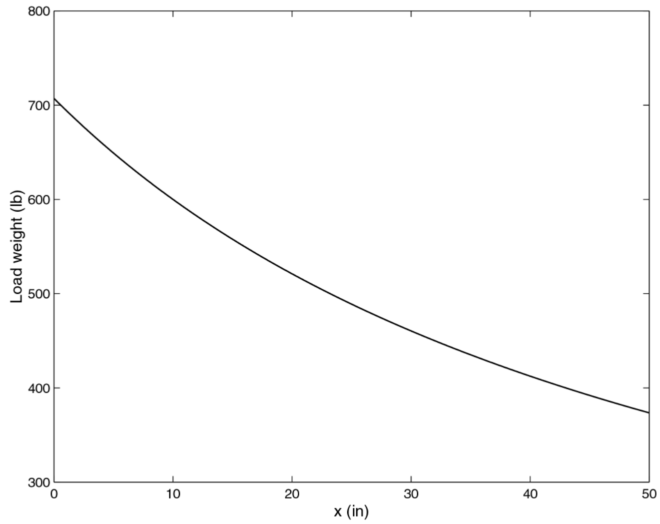
$$\Sigma M_B = 0 = 3600(67) - N(112) - W_L x = 0$$
$$\Sigma F_y = 0 = N + N - 3600 - W_L = 0$$

The second equation can be solved for N and then substituted into the first equation to yield the required relation between W_L and x. This relation can then be solved for W_L with the following result.

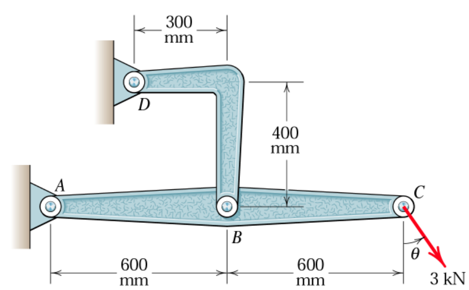
$$W_L = \frac{39600}{56 + x}$$

MATLAB Script

```
x=0:0.05:50;
WL=39600./(56+x);
plot(x,WL)
xlabel('x (in)')
ylabel('Load weight (lb)')
```

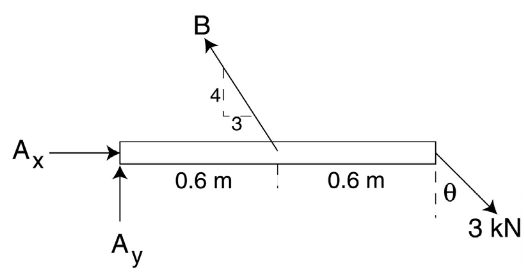


4.5 Problem 4/143 (Frames and Machines)



The structural members support the 3-kN load which may be applied at any angle θ from essentially $-90°$ to $+90°$. The pins at A and B must be designed to support the maximum force transmitted to them. Plot the forces at A and B as a function of θ and determine their maximum values and corresponding angles θ.

Problem Formulation



The free-body diagram is shown to the right. Note that member *BD* is a two-force member, thus the direction of the force **B** is from *B* to *D*. The equilibrium equations are,

$$\circlearrowleft \Sigma M_A = \frac{4}{5}B(0.6) - 3\cos\theta(1.2) = 0$$

$$\Sigma F_x = A_x - \frac{3}{5}B + 3\sin\theta = 0 \qquad \Sigma F_y = A_y + \frac{4}{5}B - 3\cos\theta = 0$$

Solving these equations yields,

$$B = \frac{15}{2}\cos\theta \qquad A_x = \frac{3}{5}B - 3\sin\theta \qquad A_y = 3\cos\theta - \frac{4}{5}B$$

$$A = \sqrt{A_x^2 + A_y^2}$$

Substitution and simplification yields

$$A = 3\sqrt{1 - 3\sin\theta\cos\theta + \frac{9}{4}\cos^2\theta}$$

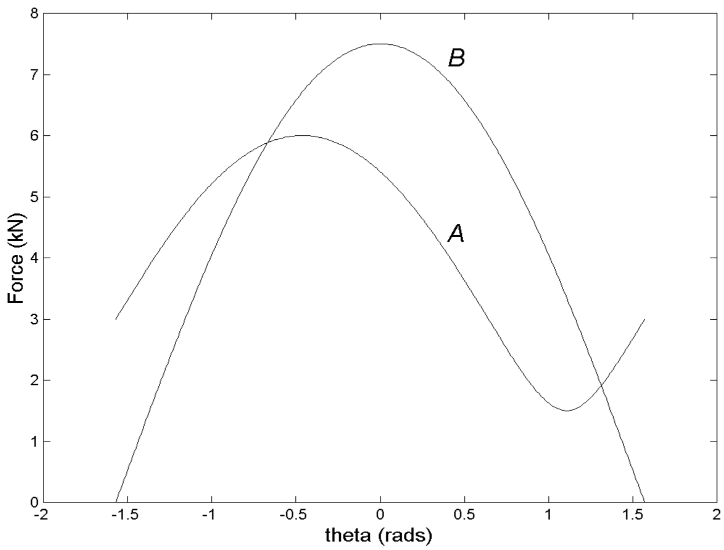
As we will see below, there is really no need to make the substitution above to yield A explicitly as a function of θ. The main reason for making the substitution here is to show that there is no obvious value of θ for which A will be a maximum. The situation is quite different for B since the maximum value for $\cos\theta$ is 1. Thus,

$B_{\max} = 7.5$ kN at $\theta = 0°$

The maximum value of A and the corresponding angle θ will be found in the MATLAB worksheet which follows.

MATLAB Worksheet

```
%%%%%%%%%%
% This script plots A and B versus theta
th=-pi/2:0.01:pi/2;
B=7.5*cos(th);
Ax=0.6*B-3*sin(th);
Ay=3*cos(th)-0.8*B;
A=sqrt(Ax.^2+Ay.^2);
plot(th,B,th,A)
xlabel('theta (rads)')
ylabel('Force (kN)')
% Text is added after generating the plot as described
in Chapter 1.
%%%%%%%%%%
```



As mentioned earlier, the maximum value of *B* is 7.5 kN occurring at $\theta = 0$ degrees. The location for the maximum value of *A* is found by setting the derivative of *A* with respect to θ equal to zero. The resulting θ can then be substituted into *A* to yield the maximum value. We'll use symbolic algebra to obtain the results.

```
EDU» A='3*sqrt(1-3*sin(theta)*cos(theta)+9*cos(theta)^2/4)'
A =
3*sqrt(1-3*sin(theta)*cos(theta)+9*cos(theta)^2/4)
EDU» dA=diff(A,'theta'); % The output is suppressed.
EDU» solve(dA,'theta')
ans =
[    -atan(1/2)]
[ -atan(1/2)+pi]
[       atan(2)]
[    atan(2)-pi]
```

MATLAB has found four solutions. From the graph, the first of these (–atan(1/2)= –0.4636 rads = – 26.6 degrees) corresponds to the maximum over the range plotted.

```
EDU» subs(A,'theta',-atan(1/2))
ans = 6
```

Thus, Amax = 6 kN when theta = - 26.6 degrees.