

# APPLIED CALCULUS

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Third Edition



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## Third Edition

Produced by the Calculus Consortium and initially funded by a National Science Foundation Grant.

Deborah Hughes-Hallett      William G. McCallum  
*University of Arizona*      *University of Arizona*

Andrew M. Gleason      Brad G. Osgood  
*Harvard University*      *Stanford University*

Patti Frazer Lock      Douglas Quinney  
*St. Lawrence University*      *University of Keele*

Daniel E. Flath      Karen Rhea  
*Macalester College*      *University of Michigan*

David O. Lomen      Jeff Tecosky-Feldman  
*University of Arizona*      *Haverford College*

David Lovelock      Thomas W. Tucker  
*University of Arizona*      *Colgate University*

with the assistance of

Otto K. Bretscher  
*Colby College*

Eric Connally  
*Harvard Extension School*

Richard D. Porter  
*Northeastern University*

Sheldon P. Gordon  
*SUNY at Farmingdale*

Andrew Pasquale  
*Chelmsford High School*

Joe B. Thrash  
*University of Southern Mississippi*

Coordinated by  
Elliot J. Marks



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Dedicated to Robin, Kari, Eric, and Dennis,  
and  
Laura, Pearl, Felix, Natt, Isaac, and Matthias.



# PREFACE

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Calculus is one of the greatest achievements of the human intellect. Inspired by problems in astronomy, Newton and Leibniz developed the ideas of calculus 300 years ago. Since then, each century has demonstrated the power of calculus to illuminate questions in mathematics, the physical sciences, engineering, and the social and biological sciences.

Calculus has been so successful because of its extraordinary power to reduce complicated problems to simple rules and procedures. Therein lies the danger in teaching calculus: it is possible to teach the subject as nothing but the rules and procedures—thereby losing sight of both the mathematics and of its practical value. This third edition of *Applied Calculus* continues our effort to refocus the teaching of calculus on concepts as well as procedures.

## A Focused Vision: Conceptual Understanding

Our goal is to provide students with a clear understanding of the ideas of calculus as a solid foundation for subsequent courses. We began work on this book by talking to faculty in business, economics, biology, and a wide range of other fields, as well as to many mathematicians who teach applied calculus. As a result of these discussions we included some new topics and omitted some traditional topics whose inclusion we could not justify. In the process, we also changed the focus of certain topics.

## The Third Edition: Flexibility and the Rule of Four

Since our First Edition struck a new balance between concepts, modeling, and skills, we have helped widen the range of choices for calculus instructors. We have found that the Rule of Four, which emphasizes multiple representations of functions, encourages students to reflect on the meaning of the material.

Students usually learn most when they are active, so we feel the exercises in a text are of central importance. Our problems probe student understanding in areas often taken for granted. The influence of these problems, praised for their creativity and variety, has extended far beyond the users of our textbooks. We have been guided by the following principles.

- Our problems are varied. Some are straightforward and some are challenging. Many problems require students to understand the concepts and cannot be done by following a template in the text.
- The Rule of Four: Where appropriate, topics are presented geometrically, numerically, analytically, and verbally.
- The Key Concept chapters on the derivative and the definite integral (Chapters 2 and 5) can be covered at the outset of the course, right after Chapter 1.
- Because different users often choose very different topics to cover in a one-semester applied calculus course, we have designed this book for either a one-semester course (with much flexibility in choosing topics) or a two-semester course. Sample syllabi are provided in the Instructor's Manual.

## Changes in the Third Edition

The third edition has the same vision as the first two editions. Previous editions were used by a large and diverse group of schools in semester and quarter systems, in large lectures and small classes, in computer labs, small groups, and traditional settings, and with a number of different technologies. In preparing this edition, we solicited comments from a large number of mathematicians who had used the text. We continued to discuss with our colleagues in client disciplines the mathematical needs of their students. We were offered many valuable suggestions, which we have tried to incorporate, while maintaining our original commitment to a focused treatment of a limited number of topics. The changes we have made include:

- Data throughout the book has been updated. More problems have been added throughout; new spreadsheet projects have been added.

- The material in Section 1.4 on taxes has been simplified and easier problems added.
- The Focus on Modeling section on fitting formulas to data now contains an introduction to correlation.
- The subsection on maximizing profit has been moved from Section 2.5 to Section 4.4.
- Section 3.3 on the chain rule has been expanded to provide more practice, especially with functions given graphically.
- Sections 4.3 and 4.4 contain more mid-level problems. The sections on economics in Chapter 4 have been rewritten for clarity.
- In Chapter 9 there is a new project introducing the concept of duality, important in economics.
- Chapter 10 contains a new Focus on Theory section on separation of variables and a new project on the 2003 SARS outbreak.
- There are two new spreadsheet projects: one on whether to buy a new or used car, and one on the outbreak of flu during the First World War.
- A pre-test is included for students whose skills may need a refresher prior to taking the course.

## What Student Background is Expected?

This book is intended for students in business, the social sciences, and the life sciences. We have found the material to be thought-provoking for well-prepared students while still accessible to students with weak algebra backgrounds. Providing numerical and graphical approaches as well as the algebraic gives students several ways of mastering the material. This approach encourages students to persist, thereby lowering failure rates. A pre-test over background material is available in the appendix to the book; an algebra refresher is available at the student book companion site at [www.wiley.com/college/hugheshallett](http://www.wiley.com/college/hugheshallett).

## Technology

We take advantage of computers and graphing calculators to help students learn to think mathematically. For example, using a graphing calculator to zoom in on functions is an excellent way of seeing local linearity. The ability to use technology effectively as a tool is important. Students are expected to use their own judgment to determine where technology is useful.

However, the book does not require any specific software or technology. Instructors have used the materials with graphing calculators, graphing software, and computer algebra systems. Any technology with the ability to graph functions and perform numerical integration will suffice.

## Content

This content represents our vision of how applied calculus can be taught. It is flexible enough to accommodate individual course needs and requirements. Topics can easily be added or deleted, or the order changed.

### Chapter 1: Functions and Change

Chapter 1 introduces the concept of a function and the idea of change, including the distinction between total change and rate of change. All elementary functions are introduced here. Although the functions are probably familiar, the graphical, numerical, verbal, and modeling approach to them is likely to be new. We introduce exponential functions early, since they are fundamental to the understanding of real-world processes.

*Focus on Modeling:* The first section introduces the student to fitting formulas to data and the second section provides further discussion of compound interest and the definition of the number  $e$ .

*Focus on Theory:* This section discusses limits to infinity and end behavior.

### Chapter 2: Rate of Change: The Derivative

Chapter 2 presents the key concept of the derivative according to the Rule of Four. The purpose of this chapter is to give the student a practical understanding of the meaning of the derivative and its interpretation as an

instantaneous rate of change. After finishing this chapter, a student will be able to approximate derivatives numerically by taking difference quotients, visualize derivatives graphically as the slope of the graph, and interpret the meaning of first and second derivatives in various applications. The student will also understand the concept of marginality and recognize the derivative as a function in its own right.

*Focus on Theory:* This section discusses limits and continuity and presents the symbolic definition of the derivative.

### Chapter 3: Short-Cuts to Differentiation

The derivatives of all the functions in Chapter 1 are introduced, as well as the rules for differentiating products, quotients, and composite functions.

*Focus on Theory:* This section uses the definition of the derivative to obtain the differentiation rules.

*Focus on Practice:* This section provides a collection of differentiation problems for skill-building.

### Chapter 4: Using the Derivative

The aim of this chapter is to enable the student to use the derivative in solving problems, including optimization and graphing. It is not necessary to cover all the sections.

### Chapter 5: Accumulated Change: The Definite Integral

Chapter 5 presents the key concept of the definite integral, in the same spirit as Chapter 2.

The purpose of this chapter is to give the student a practical understanding of the definite integral as a limit of Riemann sums, and to bring out the connection between the derivative and the definite integral in the Fundamental Theorem of Calculus. We use the same method as in Chapter 2, introducing the fundamental concept in depth without going into technique. The student will finish the chapter with a good grasp of the definite integral as a limit of Riemann sums, with the ability to approximate a definite integral numerically, and with an understanding of how to interpret the definite integral in various contexts.

Chapter 5 can be covered immediately after Chapter 2 without difficulty.

*Focus on Theory:* This section presents the Second Fundamental Theorem of Calculus and the properties of the definite integral.

### Chapter 6: Using the Definite Integral

This chapter presents applications of the definite integral. It is not meant to be comprehensive and it is not necessary to cover all the sections.

### Chapter 7: Antiderivatives

This chapter covers antiderivatives from a graphical, numerical, and analytical point of view. Integration by substitution is included. The Fundamental Theorem of Calculus is used to evaluate definite integrals and to analyze antiderivatives.

*Focus on Practice:* This section provides a collection of integration problems for skill-building.

### Chapter 8: Probability

This chapter covers probability density functions, cumulative distribution functions, the median, and the mean.

### Chapter 9: Functions of Several Variables

Chapter 9 introduces functions of two variables from several points of view, using contour diagrams, formulas, and tables. It gives students the skills to read contour diagrams and think graphically, to read tables and think numerically, and to apply these skills, along with their algebraic skills, to modeling. The idea of the partial derivative is introduced from graphical, numerical, and symbolic viewpoints. Partial derivatives are then applied to optimization problems, ending with a discussion of constrained optimization using Lagrange multipliers.

*Focus on Theory:* This section uses optimization to derive the formula for the regression line.

### Chapter 10: Mathematical Modeling Using Differential Equations

This chapter introduces differential equations. The emphasis is on modeling, qualitative solutions, and interpretation. This chapter includes applications of systems of differential equations to population models, the spread of disease, and predator-prey interactions.

*Focus on Theory:* This section explains the technique of separation of variables.

### Chapter 11: Geometric Series

This chapter covers geometric series and their applications to business, economics, and the life sciences.

## Supplementary Materials

Supplements for the instructor can be obtained by sending a request on your institutional letterhead to Mathematics Marketing Manager, John Wiley & Sons Inc., 111 River Street, Hoboken, NJ 07030-5774, or by contacting your local Wiley representative. The following supplementary materials are available.

- **Instructor's Manual and Test Bank** containing teaching tips, sample syllabii, calculator programs, overhead transparency masters, and test questions and solutions arranged by section.
- **Instructor's Solution Manual** with complete solutions to all problems.
- **Student's Solution Manual** with complete solutions to half the odd-numbered problems.
- **Student Study Guide** with additional study aids for students that are tied directly to the book.
- **Additional Material for Instructors**, elaborating specially marked points in the text, as well as password protected electronic versions of the instructor ancillaries, can be found on the web at [www.wiley.com/college/hugheshallett](http://www.wiley.com/college/hugheshallett).
- **Additional Material for Students**, at the student book companion site at [www.wiley.com/college/hugheshallett](http://www.wiley.com/college/hugheshallett), includes an algebra refresher and web quizzes.

Getting Started Technology Manual Series:

- **Getting Started with Mathematica**, 2<sup>nd</sup> edn, by C-K. Cheung, G.E. Keough, Robert H. Gross, and Charles Landraitis of Boston College
- **Getting Started with Maple**, 2<sup>nd</sup> edn, by C-K. Cheung, G.E. Keough, both of Boston College, and Michael May of St. Louis University
- **Graphing Calculator Guide for the TI84/83** by Carl Swenson of Seattle University
- **Graphing Calculator Guide for the TI-89** by Carl Swenson of Seattle University

### ConcepTests

ConcepTests, modeled on the pioneering work of Harvard physicist Eric Mazur, are questions designed to promote active learning during class, particularly (but not exclusively) in large lectures. Our evaluation data show students taught with ConcepTests outperformed students taught by traditional lecture methods 73% versus 17% on conceptual questions, and 63% versus 54% on computational problems. A new supplement to *Applied Calculus*, 3<sup>rd</sup> edn, containing ConcepTests by section, is available from your Wiley representative.

### Wiley Faculty Network

A peer-to-peer network of academic faculty dedicated to the effective use of technology in the classroom. This group can help you apply innovative classroom techniques and implement specific software packages. Visit [www.wherefacultyconnect.com](http://www.wherefacultyconnect.com) or ask your Wiley representative for details.

### eGrade Plus

*eGrade Plus* is a powerful online tool that provides a completely integrated suite of teaching and learning resources in one easy-to-use Web site. *eGrade Plus* includes an online version of the text, with fully integrated

electronic versions of all student supplements, including the Student Solutions Manual, the Student Study Guide, and an Algebra and Trigonometry Refresher. Instructors have additional access to electronic versions of the Instructor's Manual, the Instructor's Solutions Manual, additional projects, as well as other valuable resources. *eGrade Plus* also offers an online assessment system with full gradebook capabilities, which contains over 1000 skill-building questions from the Exercise sections in each chapter. Please view our online demo at [www.wiley.com/college/egradeplus](http://www.wiley.com/college/egradeplus). Here you will find additional information about the features and benefits of eGrade Plus, how to request a "test drive" of eGrade Plus, and how to adopt it for class use.

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Daniel E. Flath	Brad G. Osgood	Thomas W. Tucker

## To Students: How to Learn from this Book

- This book may be different from other math textbooks that you have used, so it may be helpful to know about some of the differences in advance. At every stage, this book emphasizes the *meaning* (in practical, graphical or numerical terms) of the symbols you are using. There is much less emphasis on “plug-and-chug” and using formulas, and much more emphasis on the interpretation of these formulas than you may expect. You will often be asked to explain your ideas in words or to explain an answer using graphs.
- The book contains the main ideas of calculus in plain English. Success in using this book will depend on reading, questioning, and thinking hard about the ideas presented. It will be helpful to read the text in detail, not just the worked examples.
- There are few examples in the text that are exactly like the homework problems, so homework problems can’t be done by searching for similar-looking “worked out” examples. Success with the homework will come by grappling with the ideas of calculus.
- For many problems in the book, there is more than one correct approach and more than one correct solution. Sometimes, solving a problem relies on common sense ideas that are not stated in the problem explicitly but which you know from everyday life.
- Some problems in this book assume that you have access to a graphing calculator or computer. There are many situations where you may not be able to find an exact solution to a problem, but you can use a calculator or computer to get a reasonable approximation.
- This book attempts to give equal weight to four methods for describing functions: graphical (a picture), numerical (a table of values), algebraic (a formula), and verbal (words). Sometimes it’s easier to translate a problem given in one form into another. For example, you might replace the graph of a parabola with its equation, or plot a table of values to see its behavior. It is important to be flexible about your approach: if one way of looking at a problem doesn’t work, try another.
- Students using this book have found discussing these problems in small groups helpful. There are a great many problems which are not cut-and-dried; it can help to attack them with the other perspectives your colleagues can provide. If group work is not feasible, see if your instructor can organize a discussion session in which additional problems can be worked on.
- You are probably wondering what you’ll get from the book. The answer is, if you put in a solid effort, you will get a real understanding of one of the crowning achievements of human creativity—calculus—as well as a real sense of the power of mathematics in the age of technology.

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