On September 26, 1993, Dave Munday, a diesel mechanic by trade, went over the Canadian edge of Niagara Falls for the second time, freely falling 48 m to the water (and rocks) below. On this attempt, he rode in a steel chamber with an airhole. Munday, keen on surviving this plunge that had killed other stuntmen, had done considerable research on the physics and engineering aspects of the plunge.

If he fell straight down, how could we predict the speed at which he would hit the water?

The answer is in this chapter.
2-1 Motion

The world, and everything in it, moves. Even a seemingly stationary thing, such as a roadway, moves because the Earth is moving. Not only is the Earth rotating and orbiting the Sun, but the Sun is also moving through space. The motion of objects can take many different forms. For example, a moving object’s path might be a straight line, a curve, a circle, or something more complicated. The entity in motion might be something simple, like a ball, or something complex, like a human being or galaxy.

In physics, when we want to understand a phenomenon such as motion, we begin by exploring relatively simple motions. For this reason, in the study of motion we start with **kinematics**, which focuses on describing motion, rather than on **dynamics**, which deals with the causes of motion. Further, we begin our study of kinematics by developing the concepts required to measure motion and mathematical tools needed to describe them in one dimension (or in 1D). Only then do we extend our study to include a consideration of the causes of motion and motions in two and three dimensions. Further simplifications are helpful. Thus, in this chapter, our description of the motion of objects is restricted in two ways:

1. **The motion of the object is along a straight line.** The motion may be purely vertical (that of a falling stone), purely horizontal (that of a car on a level highway), or slanted (that of an airplane rising at an angle from a runway), but it must be a straight line.

2. **The object is effectively a particle** because its size and shape are not important to its motion. By “particle” we mean either: (a) a point-like object with dimensions that are small compared to the distance over which it moves (such as the size of the Earth relative to its orbit around the Sun), (b) an extended object in which all its parts move together (such as a falling basketball that is not spinning), or (c) that we are only interested in the path of a special point associated with the object (such as the belt buckle on a walking person).

We will start by introducing very precise definitions of words commonly used to describe motion like speed, velocity, and acceleration. These definitions may conflict with the way these terms are used in everyday speech. However, by using precise definitions rather than our casual definitions, we will be able to describe and predict the characteristics of common motions in graphical and mathematical terms. These mathematical descriptions of phenomena form the basic vocabulary of physics and engineering.

Although our treatment may seem ridiculously formal, we need to provide a foundation for the analysis of more complex and interesting motions.

**READING EXERCISE 2-1:** Which of the following motions are along a straight line: (a) a string of carts traveling up and down along a roller coaster, (b) a cannonball shot straight up, (c) a car traveling along a straight city street, (d) a ball rolling along a straight ramp tilted at a 45° angle.

**READING EXERCISE 2-2:** In reality there are no point particles. Rank the following everyday items from most particle-like to least particle-like: (a) a 2-m-tall long jumper relative to a 25 m distance covered in a jump, (b) a piece of lead shot from a shotgun shell relative to its range of 5 m, (c) the Earth of diameter $13 \times 10^6$ m relative to the approximate diameter of its orbit about the Sun of $3 \times 10^{11}$ m.
Defining a Coordinate System

In order to study motion along a straight line, we must be able to specify the location of an object and how it changes over time. A convenient way to locate a point of interest on an object is to define a coordinate system. Houses in Costa Rican towns are commonly located with addresses such as “200 meters east of the Post Office.” In order to locate a house, a distance scale must be agreed upon (meters are used in the example), a reference point or origin must be specified (in this case the Post Office), and a direction (in this case east). Thus, in locating an object that can move along a straight line, it is convenient to specify its position by choosing a one-dimensional coordinate system. The system consists of a point of reference known as the origin (or zero point), a line that passes through the chosen origin called a coordinate axis, one direction along the coordinate axis, chosen as positive and the other direction as negative, and the units we use to measure a quantity. We have labeled the coordinate axis as the \( x \) axis, in Fig. 2-1, and placed an origin on it. The direction of increasing numbers (coordinates) is called the positive direction, which is toward the right in Fig. 2-1. The opposite direction is the negative direction.

Figure 2-1 is drawn in the traditional fashion, with negative coordinates to the left of the origin and positive coordinates to the right. It is also traditional in physics to use meters as the standard scale for distance. However, we have freedom to choose other units and to decide which side of the origin is labeled with negative coordinates and which is labeled with positive coordinates. Furthermore, we can choose to define an \( x \) axis that is vertical rather than horizontal, or inclined at some angle. In short, we are free to make choices about how we define our coordinate system.

Good choices make describing a situation much easier. For example, in our consideration of motion along a straight line, we would want to align the axis of our one-dimensional coordinate system along the line of motion. In Chapters 5 and 6, when we consider motions in two dimensions, we will be using more complex coordinate systems with a set of mutually perpendicular coordinate axes. Choosing a coordinate system that is appropriate to the physical situation being described can simplify your mathematical description of the situation. To describe a particle moving in a circle, you would probably choose a two-dimensional coordinate system in the plane of the circle with the origin placed at its center.

Defining Position as a Vector Quantity

The reason for choosing our standard one-dimensional coordinate axis and orienting it along the direction of motion is to be able to define the position of an object relative to our chosen origin, and then be able to keep track of how its position changes as the object moves. It turns out that the position of an object relative to a coordinate system can be described by a mathematical entity known as a **vector**. This is because, in order to find the position of an object, we must specify both how far and in which direction the object is from the origin of a coordinate system.

A **vector** is a mathematical entity that has both a magnitude and a direction. Vectors can be added, subtracted, multiplied, and transformed according to well-defined mathematical rules.

There are other physical quantities that also behave like vectors such as velocity, acceleration, force, momentum, and electric and magnetic fields.

However, not all physical quantities that have signs associated with them are vectors. For example, temperatures do not need to be described in terms of a coordinate
A SCALAR is defined as a mathematical quantity whose value does not depend on the orientation of a coordinate system and has no direction associated with it.

In general, a one-dimensional vector can be represented by an arrow. The length of the arrow, which is inherently positive, represents the magnitude of the vector and the direction in which the arrow points represents the direction associated with the vector.

We begin this study of motion by introducing you to the properties of one-dimensional position and displacement vectors and some of the formal methods for representing and manipulating them. These formal methods for working with vectors will prove to be very useful later when working with two- and three-dimensional vectors.

A one-dimensional position vector is defined by the location of the origin of a chosen one-dimensional coordinate system and of the object of interest. The magnitude of the position vector is a scalar that denotes the distance between the object and the origin. For example, an object that has a position vector of magnitude 5 m could be located at the point +5 m or −5 m from the origin.

On a conventional x axis, the direction of the position vector is positive when the object is located to the right of the origin and negative when the object is located to the left of the origin. For example, in the system shown in Fig. 2-1, if a particle is located at a distance of 3 m to the left of the origin, its position vector has a magnitude of 3 m and a direction that is negative. One of many ways to represent a position vector is to draw an arrow from the origin to the object’s location, as shown in Fig. 2-2, for an object that is 1.5 m to the left of the origin. Since the length of a vector arrow represents the magnitude of the vector, its length should be proportional to the distance from the origin to the object of interest. In addition, the direction of the arrow should be from the origin to the object.

Instead of using an arrow, a position vector can be represented mathematically. In order to develop a useful mathematical representation we need to define a unit vector associated with our x axis.

A UNIT VECTOR FOR A COORDINATE AXIS is a dimensionless vector that points in the direction along a coordinate axis that is chosen to be positive.

It is customary to represent a unit vector that points along the positive x axis with the symbol \( \hat{i} \) (i-hat), although some texts use the symbol \( \hat{x} \) (x-hat) instead. When considering three-dimensional vectors, the unit vectors pointing along the designated positive y axis and z axis are denoted by \( \hat{j} \) and \( \hat{k} \), respectively.

These vectors are called “unit vectors” because they have a dimensionless value of one. However, you should not confuse the use of word “unit” with a physical unit. Unit vectors should be shown on coordinate axes as small pointers with no physical units, such as meters, associated with them. This is shown in Fig. 2-3 for the x axis unit vector. Since the scale used in the coordinate system has units, it is essential that the units always be associated with the number describing the location of an object along an axis. Figure 2-3 also shows how the unit vector is used to create a position vector corresponding to an object located at position −1.5 meters on our x axis. To do this we stretch or multiply the unit vector by the magnitude of the position vector, which
is 1.5 m. Note that we are using the coordinate axis to describe a position in meters relative to an origin, so it is essential to include the units with the number. This multiplication of the dimensionless unit vector by 1.5 m creates a 1.5-m-long vector that points in the same direction as the unit vector. It is denoted by $(1.5 \text{ m})\hat{i}$. However, the vector we want to create points in the negative direction, so the vector pointing in the positive direction must be inverted using a minus sign. The position vector we have created is denoted as $\mathbf{x}$. It can be divided into two parts—a vector component and a unit vector,

$$\mathbf{x} = (-1.5 \text{ m})\hat{i}.$$ 

In this example, the $x$-component of the position vector, denoted as $x$, is $-1.5 \text{ m}$.

Here the quantity 1.5 m with no minus sign in front of it is known as the magnitude of this position vector. In general, the magnitude is denoted as $|\mathbf{x}|$. Thus, the one-dimensional position vector for the situation shown in Fig. 2-3 is denoted mathematically using the following symbols:

$$\mathbf{x} = x\hat{i} = (-1.5 \text{ m})\hat{i}.$$ 

The $x$-component of a position vector, denoted $x$, can be positive or negative depending on which side of the origin the particle is. Thus, in one dimension in terms of absolute values, the vector component $x$ is either $+|x|$ or $-|x|$, depending on the object’s location.

In general, a component of a vector along an axis, such as $x$ in this case, is not a scalar since our $x$-component will change sign if we choose to reverse the orientation of our chosen coordinate system. In contrast, the magnitude of a position vector is always positive, and it only tells us how far away the object is from the origin, so the magnitude of a vector is always a scalar quantity. The sign of the component ($+$ or $-$) tells us in which direction the vector is pointing. The sign will be negative if the object is to the left of the origin and positive if it is to the right of the origin.

### Defining Displacement as a Vector Quantity

The study of motion is primarily about how an object’s location changes over time under the influence of forces. In physics the concept of change has an exact mathematical definition.

**Change** is defined as the difference between the state of a physical system (typically called the final state) and its state at an earlier time (typically called the initial state).

This definition of change is used to define displacement.

**Displacement** is defined as the change of an object’s position that occurs during a period of time.
Since position can be represented as a vector quantity, displacement is the difference between two vectors, and thus, is also a vector. So, in the case of motion along a line, an object moving from an “initial” position $\vec{x}_1$ to another “final” position $\vec{x}_2$ at a later time is said to undergo a displacement $\Delta \vec{r}$, given by the difference of two position vectors

$$\Delta \vec{r} = \vec{x}_2 - \vec{x}_1 = \Delta \vec{x}$$

(displacement vector), (2-1)

where the symbol $\Delta$ is used to represent a change in a quantity, and the symbol “=” signifies that the displacement $\Delta \vec{r}$ is given by $\vec{x}_2 - \vec{x}_1$ because we have chosen to define it that way.

As you will see when we begin to work with vectors in two and three dimensions, it is convenient to consider subtraction as the addition of one vector to another that has been inverted by multiplying the vector component by $-1$. We can use this idea of defining subtraction as the addition of an inverted vector to find displacements. Let’s consider three situations:

(a) A particle moves along a line from $\vec{x}_1 = (5 \text{ m})\hat{i}$ to $\vec{x}_2 = (12 \text{ m})\hat{i}$. Since $\Delta \vec{r} = \vec{x}_2 - \vec{x}_1 = \vec{x}_2 + (-\vec{x}_1)$,

$$\Delta \vec{r} = (12 \text{ m})\hat{i} - (5 \text{ m})\hat{i} = (12 \text{ m})\hat{i} + (-5 \text{ m})\hat{i} = (7 \text{ m})\hat{i}.$$  

The positive result indicates that the motion is in the positive direction (toward the right in Fig. 2-4a).

(b) A particle moves from $\vec{x}_1 = (12 \text{ m})\hat{i}$ to $\vec{x}_2 = (5 \text{ m})\hat{i}$. Since $\Delta \vec{r} = \vec{x}_2 - \vec{x}_1 = \vec{x}_2 + (-\vec{x}_1)$,

$$\Delta \vec{r} = (5 \text{ m})\hat{i} - (12 \text{ m})\hat{i} = (5 \text{ m})\hat{i} + (-12 \text{ m})\hat{i} = (-7 \text{ m})\hat{i}.$$  

The negative result indicates that the displacement of the particle is in the negative direction (toward the left in Fig. 2-4b).

(c) A particle starts at 5 m, moves to 2 m, and then returns to 5 m. The displacement for the full trip is given by $\Delta \vec{r} = \vec{x}_2 - \vec{x}_1 = \vec{x}_2 + (-\vec{x}_1)$, where $\vec{x}_1 = (5 \text{ m})\hat{i}$ and $\vec{x}_2 = (5 \text{ m})\hat{i}$:

$$\Delta \vec{r} = (5 \text{ m})\hat{i} + (-5 \text{ m})\hat{i} = (0 \text{ m})\hat{i}$$

and the particle’s position hasn’t changed, as in Fig. 2-4c. Since displacement involves only the original and final positions, the actual number of meters traced out by the particle while moving back and forth is immaterial.

If we ignore the sign of a particle’s displacement (and thus its direction), we are left with the magnitude of the displacement. This is the distance between the original and final positions and is always positive. It is important to remember that displacement (or any other vector) has not been completely described until we state its direction.

We use the notation $\Delta \vec{r}$ for displacement because when we have motion in more than one dimension, the notation for the position vector is $\vec{r}$. For a one-dimensional motion along a straight line, we can also represent the displacement as $\Delta \vec{r}$. The magnitude of displacement is represented by surrounding the displacement vector symbol with absolute value signs:

$$|\text{magnitude of displacement}| = |\Delta \vec{r}| \quad \text{or} \quad |\Delta \vec{x}|$$

**Reading Exercise 2-3:** Can a particle that moves from one position with a negative value, to another position with a negative value, undergo a positive displacement?
Three pairs of initial and final positions along an x axis represent the location of objects at two successive times: (pair 1) \(-3\, \text{m},\ +5\, \text{m}\); (pair 2) \(-3\, \text{m},\ -7\, \text{m}\); (pair 3) \(7\, \text{m},\ -3\, \text{m}\).

(a) Which pairs give a negative displacement?

**SOLUTION** The Key Idea here is that the displacement is negative when the final position lies to the left of the initial position. As shown in Fig. 2-5, this happens when the final position is more negative than the initial position. Looking at pair 1, we see that the final position, \(+5\, \text{m}\), is positive while the initial position, \(-3\, \text{m}\), is negative. This means that the displacement is from left (more negative) to right (more positive) and so the displacement is positive for pair 1.

(b) Calculate the value of the displacement in each case using vector notation.

**SOLUTION** The Key Idea here is to use Eq. 2-2 to calculate the displacement for each pair of positions. It tells us the difference between the final position and the initial position, in that order,

\[ \Delta \vec{x} = \vec{x}_2 - \vec{x}_1 \quad \text{(displacement)}. \]  

For pair 1 the final position is \(\vec{x}_2 = (+5\, \text{m})\hat{i}\) and the initial position is \(\vec{x}_1 = (-3\, \text{m})\hat{i}\), so the displacement between these two positions is just

\[ \Delta \vec{x} = (+5\, \text{m})\hat{i} - (-3\, \text{m})\hat{i} = (+5\, \text{m})\hat{i} + (3\, \text{m})\hat{i} = (+8\, \text{m})\hat{i}. \]

(Answer)

For pair 2 the same argument yields

\[ \Delta \vec{x} = (-7\, \text{m})\hat{i} - (-3\, \text{m})\hat{i} = (-7\, \text{m})\hat{i} + (3\, \text{m})\hat{i} = (-4\, \text{m})\hat{i}. \]

(Answer)

Finally, the displacement for pair 3 is

\[ \Delta \vec{x} = (-3\, \text{m})\hat{i} - (+7\, \text{m})\hat{i} = (-3\, \text{m})\hat{i} + (-7\, \text{m})\hat{i} = (-10\, \text{m})\hat{i}. \]

(Answer)

(c) What is the magnitude of each position vector?

**SOLUTION** Of the six position vectors given, one of them—namely \(\vec{x}_1 = (-3\, \text{m})\hat{i}\)—appears in all three pairs. The remaining three positions are \(\vec{x}_2 = (+5\, \text{m})\hat{i}\), \(\vec{x}_3 = (-7\, \text{m})\hat{i}\), and \(\vec{x}_4 = (+7\, \text{m})\hat{i}\). The Key Idea here is that the magnitude of a position vector just tells us how far the point lies from the origin without regard to whether it lies to the left or to the right of the origin. Thus the magnitude of our first position vector is \(3\, \text{m}\) (Answer) since the position specified by \(\vec{x} = (-3\, \text{m})\hat{i}\) is \(3\, \text{m}\) to the left of the origin. It’s not \(-3\, \text{m}\), because magnitudes only specify distance from the origin, not direction.

For the same reason, the magnitude of the second position vector is just \(5\, \text{m}\) (Answer) while the magnitude of the third and the fourth are each \(7\, \text{m}\). (Answer) The fact that the third point lies \(7\, \text{m}\) to the left of the origin while the fourth lies \(7\, \text{m}\) to the right doesn’t matter here.

(d) What is the value of the x-component of each of these position vectors?

**SOLUTION** To answer this question you need to remember what is meant by the component of a vector. The key equation relating a vector in one dimension to its component along its direction is \(\vec{x} = x\, \hat{i}\), where \(\vec{x}\) (with the arrow over it) is the vector itself and \(x\) (with no arrow over it) is the component of the vector in the direction specified by the unit vector \(\hat{i}\). So the component of \(\vec{x} = (-3\, \text{m})\hat{i}\) is \(-3\, \text{m}\), while that of \(\vec{x} = (+5\, \text{m})\hat{i}\) is just \(+5\, \text{m}\), and \(\vec{x} = (-7\, \text{m})\hat{i}\) has as its component along the \(\hat{i}\) direction \(-7\, \text{m}\) while for \(\vec{x} = (+7\, \text{m})\hat{i}\) it’s just \(+7\, \text{m}\). In other words, the component of a vector in the direction of \(\hat{i}\) is just the signed number (with its units) that multiplies \(\hat{i}\). (Answer)

### 2-3 Velocity and Speed

Suppose a student stands still or speeds up and slows down along a straight line. How can we describe accurately and efficiently where she is and how fast she is moving? We will explore several ways to do this.
Representing Motion in Diagrams and Graphs

Motion Diagrams: Now that you have learned about position and displacement, it is quite easy to describe the motion of an object using pictures or sketches to chart how position changes over time. Such a representation is called a motion diagram. For example, Fig. 2-6 shows a student whom we treat as if she were concentrated into a particle located at the back of her belt. She is standing still at a position \( \vec{x} = (-2.00 \text{ m})\hat{i} \) from a point on a sidewalk that we choose as our origin. Figure 2-7 shows a more complex diagram describing the student in motion. Suppose we see that just as we start timing her progress with a stopwatch (so \( t = 0.0 \text{ s} \)), the back of her belt is 2.47 m to the left of our origin. The \( x \)-component of her position is then \( x = -2.47 \text{ m} \). The student then moves toward the origin, almost reaches the origin at \( t = 1.5 \text{ s} \), and then continues moving to the right so that her \( x \)-component of position has increasingly positive values. It is important to recognize that just as we chose an origin and direction for our coordinate axis, we also chose an origin in time. If we had chosen to start our timing 12 seconds earlier, then the new motion diagram would show the back of her belt as being at \( x = -2.47 \text{ m} \) at \( t = 12 \text{ s} \).

Graphs: Another way to describe how the position of an object changes as time passes is with a graph. In such a graph, the \( x \)-component of the object’s position, \( x \), can be plotted as a function of time, \( t \). This position–time graph has alternate names such as a graph of \( x \) as a function of \( t \), \( x(t) \), or \( x \) vs. \( t \). For example, Fig. 2-8 shows a graph of the student standing still with the back of her belt located at a horizontal position of \(-2.00 \text{ m}\) from a spot on the sidewalk that is chosen as the origin.

The graph of steady motion shown in Fig. 2-8 is not more informative than the picture or a comment that the student is standing still for 3 seconds at a certain location. But it’s another story when we consider the graph of a motion. Figure 2-9 is a graph of a student’s \( x \)-component of position as a function of time. It represents the same information depicted in the motion diagram in Fig. 2-7. Data on the student’s motion are first recorded at \( t = 0.0 \text{ s} \) when the \( x \)-component of her position is \( x = -2.47 \text{ m} \). The student then moves toward \( x = 0.00 \text{ m} \), passes through that point at about \( t = 1.5 \text{ s}\), and then moves on to increasingly larger positive values of \( x \) while slowing down.
Although the graph of the student’s motion in Fig. 2-9 seems abstract and quite unlike a motion diagram, it is richer in information. For example, the graph allows us to estimate the motion of the student at times between those for which position measurements were made. Equally important, we can use the graph to tell us how fast the student moves at various times, and we deal with this aspect of motion graphs next.

What can motion diagrams and \( x \) vs. \( t \) graphs tell us about how fast and in what direction something moves along a line? It is clear from an examination of the motion diagram at the bottom of Fig. 2-9 that the student covers the most distance and so appears to be moving most rapidly between the two times \( t_1 = 1.0 \) s and \( t_2 = 1.5 \) s. But this time interval is also where the slope (or steepness) of the graph has the greatest magnitude. Recall from mathematics that the average slope of a curve between two points is defined as the ratio of the change in the variable plotted on the vertical axis (in this case the \( x \)-component of her position) to the change in the variable plotted on the horizontal axis (in this case the time). Hence, on position vs. time graphs (such as those shown in Fig. 2-8 and Fig. 2-9),

\[
\text{average slope} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \quad \text{(definition of average slope).} \tag{2-3}
\]

Since time moves forward, \( t_2 > t_1 \), so \( \Delta t \) always has a positive value. Thus, a slope will be positive whenever \( x_2 > x_1 \), so \( \Delta x \) is positive. In this case a straight line connecting the two points on the graph slants upward toward the right when the student is moving along the positive \( x \)-direction. On the other hand, if the student were to move “backwards” in the direction along the \( x \) axis we chose to call negative, then \( x_2 < x_1 \). In this case, the slope between the two times would be negative and the line connecting the points would slant downward to the right.

**Average Velocity**

For motion along a straight line, the steepness of the slope in an \( x \) vs. \( t \) graph over a time interval from \( t_1 \) to \( t_2 \) tells us “how fast” a particle moves. The direction of motion is indicated by the sign of the slope (positive or negative). Thus, this slope or ratio \( \Delta x/\Delta t \) is a special quantity that tells us how fast and in what direction something moves. We haven’t given the ratio \( \Delta x/\Delta t \) a name yet. We do this to emphasize the fact...
that the ideas associated with figuring out how fast and in what direction something moves are more important than the names we assign to them. However, it is inconvenient not to have a name. The common name for this ratio is \textit{average velocity}, which is defined as the ratio of displacement vector $\Delta \vec{x}$ for the motion of interest to the time interval $\Delta t$ in which it occurs. This vector can be expressed in equation form as

$$\langle \vec{v} \rangle = \frac{\Delta \vec{x}}{\Delta t} = \frac{\Delta x}{\Delta t} \hat{i} = \frac{x_2 - x_1}{t_2 - t_1} \hat{i} \quad \text{(definition of 1D average velocity),} \quad (2-4)$$

where $x_2$ and $x_1$ are components of the position vectors at the final and initial times. Here we use angle brackets $\langle \rangle$ to denote the average of a quantity. Also, we use the special symbol “$\equiv$” for equality to emphasize that the term on the left is equal to the term on the right by definition. The time change is a positive scalar quantity because we never need to specify its direction explicitly. In defining $\langle \vec{v} \rangle$ we are basically multiplying the displacement vector, $\Delta \vec{x}$, by the scalar $(1/\Delta t)$. This action gives us a new vector that points in the same direction as the displacement vector.

![Figure 2-10](image)

\textbf{Figure 2-10} Calculation of the slope of the line that connects the points on the curve at $t_1 = 1.0$ s and $t_2 = 1.5$ s. The $x$-component of the average velocity is given by this slope.

Figure 2-10 shows how to find the average velocity for the student motion represented by the graph shown in Fig. 2-9 between the times $t_1 = 1.0$ s and $t_2 = 1.5$ s. The average velocity during that time interval is

$$\langle \vec{v} \rangle = \frac{\Delta x}{\Delta t} \hat{i} = \frac{x_2 - x_1}{t_2 - t_1} \hat{i} = \frac{-0.22 \text{ m} - (-1.24 \text{ m})}{1.5 \text{ s} - 1.0 \text{ s}} \hat{i} = (2.04 \text{ m/s}) \hat{i}.$$  

The $x$-component of the average velocity along the line of motion, $\langle v_x \rangle = 2.04$ m/s, is simply the slope of the straight line that connects the point on the curve at the beginning of our chosen interval and the point on the curve at the end of the interval. Since our student is speeding up and slowing down, the values of $\langle \vec{v} \rangle$ and $\langle v_x \rangle$ will in general be different when calculated using other time intervals.

\textbf{Average Speed}

Sometimes we don’t care about the direction of an object’s motion but simply want to keep track of the distance covered. For instance, we might want to know the total distance a student walks (number of steps times distance covered in each step). Our student could be pacing back and forth wearing out her shoes without having a vector displacement. Similarly, average speed, $\langle s \rangle$, is a different way of describing “how fast” an object moves. Whereas the average velocity involves the particle’s displacement $\Delta \vec{x}$, which is a vector quantity, the average speed involves the total distance covered (for example, the product of the length of a step and the number of steps the student took), which is independent of direction. So \textbf{average speed} is defined as
Since neither the total distance traveled nor the time interval over which the travel occurred has an associated direction, average speed does not include direction information. Both the total distance and the time period are always positive, so average speed is always positive too. Thus, an object that moves back and forth along a line can have no vector displacement, so it has zero velocity but a rather high average speed. At other times, while the object is moving in only one direction, the average speed is the same as the magnitude of the average velocity \( \frac{\Delta x}{\Delta t} \). However, as you can demonstrate in Reading Exercise 2-4, when an object doubles back on its path, the average speed is not simply the magnitude of the average velocity \( |\vec{v}| \).

**Instantaneous Velocity and Speed**

You have now seen two ways to describe how fast something moves: average velocity and average speed, both of which are measured over a time interval \( \Delta t \). Clearly, however, something might speed up and slow down during that time interval. For example, in Fig. 2-9 we see that the student is moving more slowly at \( t = 0.0 \) s than she is at \( t = 1.5 \) s, so her velocity seems to be changing during the time interval between 0.0 s and 1.5 s. The average slope of the line seems to be increasing during this time interval. Can we refine our definition of velocity in such a way that we can determine the student’s true velocity at any one “instant” in time? We envision something like the almost instantaneous speedometer readings we get as a car speeds up and slows down.

Defining an instant and instantaneous velocity is not a trivial task. As we noted in Chapter 1, the time interval of 1 second is defined by counting oscillations of radiation absorbed by a cesium atom. In general, even our everyday clocks work by counting oscillations in an electronic crystal, pendulum, and so on. We associate “instants in time” with positions on the hands of a clock, and “time intervals” with changes in the position of the hands.

For the purpose of finding a velocity at an instant, we can attempt to make the time interval we use in our calculation so small that it has almost zero duration. Of course the displacement we calculate also becomes very small. So **instantaneous velocity** along a line—like average velocity—is still defined in terms of the ratio of \( \Delta x/\Delta t \). But we have this ratio passing to a limit where \( \Delta t \) gets closer and closer to zero. Using standard calculus notation for this limit gives us the following definition:

\[
\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt} \tag{2-6}
\]

In the language of calculus, the **instantaneous velocity** is the rate at which a particle’s position vector, \( \vec{x} \), is changing with time at a given instant.

In passing to the limit the ratio \( \Delta \vec{x}/\Delta t \) is not necessarily small, since both the numerator and denominator are getting small together. The first part of this expression,

\[
\vec{v} = \nu_1 \hat{i} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t},
\]

tells us that we can find the (instantaneous) velocity of an object by taking the slope of a graph of the position component vs. time at the point associated with that
moment in time. If the graph is a curve rather than a straight line, the *slope at a point* is actually the tangent to the line at that point. Alternatively, the second part of the expression, shown in Eq. 2-6,

\[ \vec{v} = \frac{d\vec{x}}{dt} \]

indicates that, if we can approximate the relationship between \( \vec{x} \) and \( t \) as a continuous mathematical function such as \( \vec{x} = (3.0\text{ m/s}^2)t^2 \), we can also find the object’s instantaneous velocity by taking a derivative with respect to time of the object’s position \( \vec{x} \). When \( \vec{x} \) varies continuously as time marches on, we often denote \( \vec{x} \) as a position function \( \vec{x}(t) \) to remind us that it varies with time.

**Instantaneous speed**, which is typically called simply *speed*, is just the magnitude of the instantaneous velocity vector, \( |\vec{v}| \). Speed is a scalar quantity consisting of the velocity value that has been stripped of any indication of the direction the object is moving, either in words or via an algebraic sign. A velocity of \( (+5\text{ m/s})\hat{i} \) and one of \( (-5\text{ m/s})\hat{i} \) both have an associated speed of 5 m/s.

### READING EXERCISE 2-4:
Suppose that you drive 10 mi due east to a store. You suddenly realize that you forgot your money. You turn around and drive the 10 mi due west back to your home and then return to the store. The total trip took 30 min. (a) What is your average velocity for the entire trip? (Set up a coordinate system and express your result in vector notation.) (b) What was your average speed for the entire trip? (c) Discuss why you obtained different values for average velocity and average speed.

### READING EXERCISE 2-5:
Suppose that you are driving and look down at your speedometer. What does the speedometer tell you—average speed, instantaneous speed, average velocity, instantaneous velocity—or something else? Explain.

### READING EXERCISE 2-6:
The following equations give the position component, \( x(t) \), along the \( x \) axis of a particle’s motion in four situations (in each equation, \( x \) is in meters, \( t \) is in seconds, and \( t > 0 \)): (1) \( x = (3\text{ m/s})t - (2\text{ m}) \); (2) \( x = (-4\text{ m/s}^2)t^2 - (2\text{ m}) \); (3) \( x = (-4\text{ m/s}^2)t^2 \); and (4) \( x = -2\text{ m} \).

(a) In which situations is the velocity \( \vec{v} \) of the particle constant? (b) In which is the vector \( \vec{v} \) pointing in the negative \( x \) direction?

### READING EXERCISE 2-7:
In Touchstone Example 2-2, suppose that right after refueling the truck you drive back to \( x_1 \) at 35 km/h. What is the average velocity for your entire trip?

## TOUCHSTONE EXAMPLE 2-2: Out of Gas

You drive a beat-up pickup truck along a straight road for 8.4 km at 70 km/h, at which point the truck runs out of gasoline and stops. Over the next 30 min, you walk another 2.0 km farther along the road to a gasoline station.

(a) What is your overall displacement from the beginning of your drive to your arrival at the station?

**SOLUTION** Assume, for convenience, that you move in the positive direction along an \( x \) axis, from a first position of \( x_1 = 0 \) to a second position of \( x_2 \) at the station. That second position must be at \( x_2 = 8.4\text{ km} + 2.0\text{ km} = 10.4\text{ km} \). Then the **Key Idea** here is that your displacement \( \Delta x \) along the \( x \) axis is the second position minus the first position. From Eq. 2-2, we have

\[ \Delta x = x_2 - x_1 = 10.4\text{ km} - 0 = 10.4\text{ km} \quad \text{(Answer)} \]

Thus, your overall displacement is 10.4 km in the positive direction of the \( x \) axis.

(b) What is the time interval \( \Delta t \) from the beginning of your drive to your arrival at the station?
**SOLUTION** We already know the time interval \( \Delta t_{\text{dr}} (= 0.50 \text{ h}) \) for the walk, but we lack the time interval \( \Delta t_{\text{dr}} \) for the drive. However, we know that for the drive the displacement \( \Delta x_{\text{dr}} \) is 8.4 km and the average velocity \( \langle v_{\text{dr}} \rangle \) is 70 km/h. A **Key Idea** to use here comes from Eq. 2-4: This average velocity is the ratio of the displacement for the drive to the time interval for the drive,

\[
\langle v_{\text{dr}} \rangle = \frac{\Delta x_{\text{dr}}}{\Delta t_{\text{dr}}}.
\]

Rearranging and substituting data then give us

\[
\Delta t_{\text{dr}} = \frac{\Delta x_{\text{dr}}}{\langle v_{\text{dr}} \rangle} = \frac{8.4 \text{ km}}{70 \text{ km/h}} = 0.12 \text{ h}.
\]

Therefore,

\[
\Delta t = \Delta t_{\text{dr}} + \Delta t_{\text{wlk}}
\]

\[
= 0.12 \text{ h} + 0.50 \text{ h} = 0.62 \text{ h}.
\]

(c) What is your average velocity \( \langle v \rangle \) from the beginning of your drive to your arrival at the station? Find it both numerically and graphically.

**SOLUTION** The **Key Idea** here again comes from Eq. 2-4: \( \langle v \rangle \) for the entire trip is the ratio of the displacement of 10.4 km for the entire trip to the time interval of 0.62 h for the entire trip. With Eq. 2-4, we find it is

\[
\langle v \rangle = \frac{\Delta x}{\Delta t} = \frac{10.4 \text{ km}}{0.62 \text{ h}} \quad \text{(Answer)}
\]

\[
= 16.8 \text{ km/h} \approx 17 \text{ km/h}.
\]

To find \( \langle v \rangle \) graphically, first we graph \( x(t) \) as shown in Fig. 2-11, where the beginning and arrival points on the graph are the origin and the point labeled “Station.” The **Key Idea** here is that your average velocity in the \( x \) direction is the slope of the straight line connecting those points; that is, it is the ratio of the rise \( (\Delta x = 10.4 \text{ km}) \) to the run \( (\Delta t = 0.62 \text{ h}) \), which gives us \( \langle v \rangle = 16.8 \text{ km/h} \).

(d) Suppose that to pump the gasoline, pay for it, and walk back to the truck takes you another 45 min. What is your average speed from the beginning of your drive to your return to the truck with the gasoline?

**SOLUTION** The **Key Idea** here is that your average speed is the ratio of the total distance you move to the total time interval you take to make that move. The total distance is 8.4 km + 2.0 km + 2.0 km = 12.4 km. The total time interval is 0.12 h + 0.50 h + 0.75 h = 1.37 h. Thus, Eq. 2-5 gives us

\[
\langle s \rangle = \frac{12.4 \text{ km}}{1.37 \text{ h}} = 9.1 \text{ km/h}. \quad \text{(Answer)}
\]

---

**2-4 Describing Velocity Change**

The student shown in Fig. 2-9 is clearly speeding up and slowing down as she walks. We know that the slope of her position vs. time graph over small time intervals keeps changing. Now that we have defined velocity, it is meaningful to develop a mathematical description of how fast velocity changes. We see two approaches to describing velocity change. We could determine velocity change over an interval of displacement magnitude, \( |\Delta x| \), and use \( \Delta \vec{v}/|\Delta x| \) as our measure. Alternatively, we could use the ratio of velocity change to the interval of time, \( \Delta t \), over which the change occurs or \( (\Delta \vec{v}/\Delta t) \). This is analogous to our definition of velocity.

Both of our proposals are possible ways of describing velocity change—neither is right or wrong. In the fourth century B.C.E., Aristotle believed that the ratio of velocity change to distance change was probably constant for any falling objects. Almost 2000 years later, the Italian scientist Galileo did experiments with ramps to slow down the motion of rolling objects. Instead he found that it was the second ratio, \( \Delta \vec{v}/\Delta t \), that was constant.
Our modern definition of acceleration is based on Galileo’s idea that $\Delta \vec{v}/\Delta t$ is the most useful concept in the description of velocity changes in falling objects.

Whenever a particle’s velocity changes, we define it as having an **acceleration**. The **average acceleration**, $\vec{a}$, over an interval $\Delta t$ is defined as

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t} \quad \text{(definition of 1D average acceleration).} \quad (2-7)$$

When the particle moves along a line (that is, an $x$ axis in one-dimensional motion),

$$\vec{a} = \frac{(v_{2x} - v_{1x})}{(t_2 - t_1)} \hat{i}.$$

It is important to note that an object is accelerated even if all that changes is only the **direction** of its velocity and not its speed. Directional changes are important as well.

### Instantaneous Acceleration

If we want to determine how velocity changes during an instant of time, we need to define **instantaneous acceleration** (or simply **acceleration**) in a way that is similar to the way we defined instantaneous velocity:

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \quad \text{(definition of 1D instantaneous acceleration).} \quad (2-8)$$

In the language of calculus, the **ACCELERATION** of a particle at any instant is the rate at which its velocity is changing at that instant.

Using this definition, we can determine the acceleration by taking a time derivative of the velocity, $\vec{v}$. Furthermore, since velocity of an object moving along a line is the derivative of the position, $\vec{x}$, with respect to time, we can write

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt} \left( \frac{d\vec{x}}{dt} \right) = \frac{d^2 \vec{x}}{dt^2} \quad \text{(1D instantaneous acceleration).} \quad (2-9)$$

Equation 2-9 tells us that the instantaneous acceleration of a particle at any instant is equal to the second derivative of its position, $\vec{x}$, with respect to time. Note that if the object is moving along an $x$ axis, then its acceleration can be expressed in terms of the $x$-component of its acceleration and the unit vector $\hat{i}$ along the $x$ axis as

$$\vec{a} = a_x \hat{i} = \frac{dv_x}{dt} \hat{i} \quad \text{so} \quad a_x = \frac{dv_x}{dt}.$$

Figure 2-12c shows a plot of the $x$-component of acceleration of an elevator cab. Compare the graph of the $x$-component of acceleration as a function of time ($a_x$ vs. $t$) with the graph of the $x$-component of velocity as a function of time ($v_x$ vs. $t$) in part b. Each point on the $a_x$ vs. $t$ graph is the derivative (slope or tangent) of the corresponding point on the $v_x$ vs. $t$ graph. When $v_x$ is constant (at either 0 or 4 m/s), its time derivative is zero and hence so is the acceleration. When the cab first begins to move, the $v_x$ vs. $t$ graph has a positive derivative (the slope is positive), which means that $a_x$ is positive. When the cab slows to a stop, the derivative or slope of the $v_x$ vs. $t$ graph is negative; that is, $a_x$ is negative. Next compare the slopes of the $v_x$ vs. $t$ graphs during the
two acceleration periods. The slope associated with the cab’s stopping is steeper, because the cab stops in half the time it took to get up to speed. The steeper slope means that the magnitude of the stopping acceleration is larger than that of the acceleration as the car is speeding up, as indicated in Fig. 2-12c.

Acceleration has both a magnitude and a direction and so it is a vector quantity. The algebraic sign of its component $a_x$ represents the direction of velocity change along the chosen $v_x$ axis. When acceleration and velocity are in the same direction (have the same sign) the object will speed up. If acceleration and velocity are in opposite directions (and have opposite signs) the object will slow down.

It is important to realize that speeding up is not always associated with an acceleration that is positive. Likewise, slowing down is not always associated with an acceleration that is negative. The relative directions of an object’s velocity and acceleration determine whether the object will speed up or slow down.

Since acceleration is defined as any change in velocity over time, whenever an object moving in a straight line has an acceleration it is either speeding up, slowing down, or turning around. Beware! In listening to common everyday language, you will probably hear the word acceleration used only to describe speeding up and the word deceleration to mean slowing down. It’s best in studying physics to use the more formal definition of acceleration as a vector quantity that describes both the magnitude...
and direction of any type of velocity change. In short, an object is accelerating when it is slowing down as well as when it is speeding up. We suggest avoiding the use of the term deceleration while trying to learn the formal language of physics.

The fundamental unit of acceleration must be a velocity (displacement/time) divided by a time, which turns out to be displacement divided by time squared. Displacement is measured in meters and time in seconds in the SI system described in Chapter 1. Thus, the “official” unit of acceleration is m/s². You may encounter other units. For example, large accelerations are often expressed in terms of “g” units where g is directly related to the magnitude of the acceleration of a falling object near the Earth’s surface. A g unit is given by

\[ 1 \text{ g} = 9.8 \text{ m/s}^2. \] (2-10)

On a roller coaster, you have brief accelerations up to 3g, which, in standard SI units, is \((3)(9.8 \text{ m/s}^2)\) or about 29 m/s². A more extreme example is shown in the photographs of Fig. 2-13, which were taken while a rocket sled was rapidly accelerated along a track and then rapidly braked to a stop.

**Figure 2-13** Colonel J.P. Stapp in a rocket sled as it is brought up to high speed (acceleration out of the page) and then very rapidly braked (acceleration into the page).

**READING EXERCISE 2-8:** A cat moves along an x axis. What is the sign of its acceleration if it is moving (a) in the positive direction with increasing speed, (b) in the positive direction with decreasing speed, (c) in the negative direction with increasing speed, and (d) in the negative direction with decreasing speed?

**TOUCHSTONE EXAMPLE 2-3: Position and Motion**

A particle’s position on the x axis of Fig. 2-1 is given by

\[ x = 4 \text{ m} - (27 \text{ m/s}) t + (1 \text{ m/s}^3) t^2, \]

with x in meters and t in seconds.

(a) Find the particle’s velocity function \( v_x(t) \) and acceleration function \( a_x(t) \).

**SOLUTION** One Key Idea is that to get the velocity function \( v_x(t) \), we differentiate the position function \( x(t) \) with respect to time. Here we find

\[ v_x = -(27 \text{ m/s}) + 3 \cdot (1 \text{ m/s}^3) t^2 = -(27 \text{ m/s}) + (3 \text{ m/s}^3) t^2 \] (Answer)

with \( v_x \) in meters per second.

Another Key Idea is that to get the acceleration function \( a_x(t) \), we differentiate the velocity function \( v_x(t) \) with respect to time. This gives us
when you apply its brakes steadily to bring it to a smooth stop; an airplane when after being startled. 

Taking off or when completing a smooth landing; or a dolphin that speeds up suddenly 

Some examples of motions that yield similar graphs to those shown in Fig. 2-14 in-

pose you measure the times and corresponding positions for an object that you sus-

negligible. 

direction at the same rate 

of the ratio 

accelerated objects. 

Because constant accelerations are common, it is useful to derive a special set of kinematic equations to describe the motion of any object that is moving along a line with a constant acceleration. We can use the definitions of acceleration and velocity and an assumption about average velocity to derive the kinematic equations. These equations allow us to use known values of the vector components describing positions, velocities, and accelerations, along with time intervals to predict the motions of constantly accelerated objects. 

Derivation of the Kinematic Equations 

Because constant accelerations are common, it is useful to derive a special set of kinematic equations to describe the motion of any object that is moving along a line with a constant acceleration. We can use the definitions of acceleration and velocity and an assumption about average velocity to derive the kinematic equations. These equations allow us to use known values of the vector components describing positions, velocities, and accelerations, along with time intervals to predict the motions of constantly accelerated objects. 

SOLUTION The key idea is to examine the expressions for x(t), v(t), and a(t). 

At t = 0, the particle is at x(0) = +4 m and is moving with a velocity of v(0) = −27 m/s—that is, in the negative direction of the x axis. Its acceleration is a(0) = 0, because just then the particle’s velocity is not changing. 

For 0 < t < 3 s, the particle still has a negative velocity, so it continues to move in the negative direction. However, its acceleration is no longer 0 but is increasing and positive. Because the signs of the velocity and the acceleration are opposite, the particle must be slowing. 

Indeed, we already know that it turns around at t = 3 s. Just then the particle is as far to the left of the origin in Fig. 2-1 as it will ever get. Substituting t = 3 s into the expression for x(t), we find that the particle’s position just then is x = −50 m. Its acceleration is still positive. 

For t > 3 s, the particle moves to the right on the axis. Its acceleration remains positive and grows progressively larger in magnitude. The velocity is now positive, and it too grows progressively larger in magnitude.
Let’s start the derivation by noting that when the acceleration is constant, the average and instantaneous accelerations are equal. As usual we place our \( x \) axis along the line of the motion. We can now use vector notation to write

\[
\vec{a} = a_x \hat{i} = \langle \vec{a} \rangle,
\]

so that

\[
\langle \vec{a} \rangle = \frac{(v_{2x} - v_{1x})}{t_2 - t_1} \hat{i},
\]

where \( a_x \) is the component of acceleration along the line of motion of the object. We can use the definition of average acceleration (Eq. 2-7) to express the acceleration component \( a_x \) in terms of the object’s velocity components along the line of motion, where \( v_{2x} \) and \( v_{1x} \) are the object’s velocity components along the line of motion,

\[
a_x = \frac{(v_{2x} - v_{1x})}{t_2 - t_1}.
\]

This expression allows us to derive the kinematic equations in terms of the vector components needed to construct the actual one-dimensional velocity and acceleration vectors. The subscripts 1 and 2 in most of the equations in this chapter, including Eq. 2-12, refer to initial and final times, positions, and velocities.

If we solve Eq. 2-12 for \( v_{2x} \), then the \( x \)-component of velocity at time \( t_2 \) is

\[
v_{2x} = v_{1x} + a_x(t_2 - t_1) = v_{1x} + a_x \Delta t \quad \text{(primary kinematic [\( a_x = \text{constant} \)] equation)},
\]

or

\[
\Delta v_x = a \Delta t.
\]

This equation is the first of two primary equations that we will derive for use in analyzing motions involving constant acceleration. Before we move on, we should think carefully about what the expression \( t_2 - t_1 \) represents in this equation: It represents the time interval in which we are tracking the motion.

In a manner similar to what we have done above, we can rewrite Eq. 2-4, the expression for the average velocity along the \( x \) axis,

\[
\langle v \rangle = \langle v_x \rangle \hat{i} = \frac{\Delta x}{\Delta t} \hat{i} = \frac{(x_2 - x_1)}{t_2 - t_1} \hat{i},
\]

Hence, the \( x \)-component of the average velocity is given by

\[
\langle v_x \rangle = \frac{(x_2 - x_1)}{(t_2 - t_1)}.
\]

Solving for \( x_2 \) gives

\[
x_2 = x_1 + \langle v_x \rangle(t_2 - t_1).
\]

In this equation \( v_x \) is the \( x \)-component of the position of the particle at \( t = t_1 \) and \( \langle v_x \rangle \) is the component along the \( x \) axis of average velocity between \( t = t_1 \) and a later time \( t = t_2 \). Note that unless the velocity is constant, the average velocity component along the \( x \) axis, \( \langle v_x \rangle \), is not equal to the instantaneous velocity component, \( v_x \).

However, we do have a plausible alternative for expressing the average velocity component in the special case when the acceleration is constant. Figure 2-15 depicts the fact that velocity increases in a linear fashion over time for a constant acceleration. It seems reasonable to assume that the component along the \( x \) axis of the \textit{average} velocity over any time interval is the average of the components for the in-
stantaneous velocity at the beginning of the interval, \(v_{1x}\), and the instantaneous velocity component at the end of the interval, \(v_{2x}\). So we expect that when a velocity increases linearly, the average velocity component over a given time interval will be

\[
\langle v_x \rangle = \frac{v_{1x} + v_{2x}}{2}.
\]  

(2-15)

Using Eq. 2-13, we can substitute \(v_{1x} + a_x(t_2 - t_1)\) for \(v_{2x}\) to get

\[
\langle v_x \rangle = \frac{1}{2} \left[ v_{1x} + v_{1x} + a_x(t_2 - t_1) \right] = v_{1x} + \frac{1}{2} a_x(t_2 - t_1).
\]  

(2-16)

Finally, substituting this equation into Eq. 2-14 yields

\[
x_2 - x_1 = v_{1x}(t_2 - t_1) + \frac{1}{2} a_x(t_2 - t_1)^2 \quad \text{(primary kinematic \([a_x = \text{constant}]\) equation), (2-17)}
\]

or

\[
\Delta x = v_{1x} \Delta t + \frac{1}{2} a_x \Delta t^2
\]

This is our second primary equation describing motion with constant acceleration. Figures 2-14a and 2-16 show plots of Eq. 2-17.

These two equations are very useful in the calculation of unknown quantities that can be used to characterize constantly accelerated motion. There are five or six quantities contained in our primary equations (Eqs. 2-13 and 2-17). The simplest kinematic calculations involve situations in which all but one of the quantities is known in one of the primary equations. In more complex situations, both equations are needed. Typically for a complex situation, we need to calculate more than one unknown. To do this, we find the first unknown using one of the primary equations and use the result in the other equations to find the second unknown. This method is illustrated in the next section and in Touchstone Examples 2-4 and 2-6.

The primary equations above, \(v_{2x} = v_{1x} + a_x(t_2 - t_1) = v_{1x} + a_x \Delta t\) (Eq. 2-13), and \(x_2 - x_1 = v_{1x}(t_2 - t_1) + \frac{1}{2} a_x(t_2 - t_1)^2\) (Eq. 2-17), are derived directly from the definitions of velocity and acceleration, with the condition that the acceleration is constant. These two equations can be combined in three ways to yield three additional equations. For example, solving for \(v_{1x}\) in \(v_{2x} = v_{1x} + a_x(t_2 - t_1)\) and substituting the result into \(x_2 - x_1 = v_{1x}(t_2 - t_1) + \frac{1}{2} a_x(t_2 - t_1)^2\) gives us

\[
v_{2x}^2 = v_{1x}^2 + 2a_x(x_2 - x_1).
\]

We recommend that you learn the two primary equations and use them to derive other equations as needed. Then you will not need to remember so much. Table 2-1 lists our two primary equations. Note that a really nice alternative to using the two
equations in Table 2-1 is to use the first of the equations (Eq. 2-13) along with the expression for the average velocity component in Eq. 2-15, 

\[ \langle v_x \rangle = \frac{\Delta x}{\Delta t} = \frac{v_{1x} + v_{2x}}{2} \]  

(an alternative "primary" equation).

to derive all the other needed equations. The derivations of the kinematic equations that we present here are not rigorous mathematical proofs but rather what we call plausibility arguments. However, we know from the application of the kinematic equations to constantly accelerated motions that they do adequately describe these motions.

### Analyzing the Niagara Falls Plunge

At the beginning of this chapter we asked questions about the motion of the steel chamber holding Dave Munday as he plunged into the water after falling 48 m from the top of Niagara Falls. How long did the fall take? That is, \( \Delta t \)? How fast was the chamber moving when it hit the water? (What is \( v \)?) As you will learn in Chapter 3, if no significant air drag is present, objects near the surface of the Earth fall at a constant acceleration of magnitude \( |a_x| = 9.8 \text{ m/s}^2 \). Thus, the kinematic equations can be used to calculate the time of fall and the impact speed.

Let’s start by defining our coordinate system. We will take the \( x \) axis to be a vertical or up–down axis that is aligned with the downward path of the steel chamber. We place the origin at the bottom of the falls and define up to be positive as shown in Fig. 2-17. (Later when considering motions in two and three dimensions, we will often denote vertical axes as \( y \) axes and horizontal axes as \( x \) axes, but these changes in symbols will not affect the results of calculations.)

We know that the value of the vertical displacement is given by

\[ x_2 - x_1 = (0 \text{ m}) - (+48 \text{ m}) = -48 \text{ m} \]

and that the velocity is getting larger in magnitude in the downward (negative direction). Since the velocity is downward and the object is speeding up, the vertical acceleration is also downward (in the negative direction). Its component along the axis of motion is given by \( a_x = -9.8 \text{ m/s}^2 \). Finally, we assume that Dave Munday’s capsule dropped from rest, so \( v_{1x} = 0 \text{ m/s} \). Thus we can find the time of fall \( (\Delta t = t_2 - t_1) \) using Eq. 2-17. Solving this equation for the time elapsed during the fall \( (t_2 - t_1) \) when the initial velocity \( v_{1x} \) is zero gives

\[ \Delta t = t_2 - t_1 = \sqrt{\frac{2(x_2 - x_1)}{a_x}} = \sqrt{\frac{2(-48 \text{ m})}{-9.8 \text{ m/s}^2}} = 3.13 \text{ s} = 3.1 \text{ s}. \]

This is a fast trip indeed!
Next we can use the time interval of the fall in the other primary kinematic equation, Eq. 2-13, to find the velocity at impact. This gives a component of impact velocity at the end of the fall of

\[ v_{2x} = v_{1x} + a_x (t_2 - t_1) = 0 \text{ m/s} + (-9.8 \text{ m/s}^2)(3.13 \text{ s}) = -31 \text{ m/s}. \]

The minus sign indicates that the impact velocity component is negative and is, therefore, in the downward direction. In vector notation, the velocity \( \vec{v} \) is thus \( \vec{v} = (-31 \text{ m/s}) \hat{j} \). Note that this is a speed of about 69 mi/hr. Since the time interval was put into the calculation of velocity of impact as an intermediate value, we retained an extra significant figure to use in the next calculation.

**READING EXERCISE 2-9:** The following equations give the \( x \)-component of position \( x(t) \) of a particle in meters (denoted m) as a function of time in seconds for four situations:

1. \( x = (3 \text{ m/s})t - 4 \text{ m} \)
2. \( x = (-5 \text{ m/s}^2)t^3 + (4 \text{ m/s})t + 6 \text{ m} \)
3. \( x = (2 \text{ m/s}^2)t^2 - (4 \text{ m/s})t \)
4. \( x = (5 \text{ m/s}^2)t^3 - 3 \text{ m} \)

To which of these situations do the equations of Table 2-1 apply? Explain.

---

**TOUCHSTONE EXAMPLE 2-4: Slowing Down**

Spotting a police car, you brake your Porsche from a speed of 100 km/h to a speed of 80.0 km/h during a displacement of 88.0 m, at a constant acceleration.

(a) What is that acceleration?

**SOLUTION** Assume that the motion is along the positive direction of an \( x \) axis. For simplicity, let us take the beginning of the braking to be at time \( t_1 = 0 \), at position \( x_1 \). The **Key Idea** here is that, with the acceleration constant, we can relate the car’s acceleration to its velocity and displacement via the basic constant acceleration equations (Eqs. 2-13 and 2-17). The initial velocity is \( v_{1x} = 100 \text{ km/h} = 27.78 \text{ m/s} \), the displacement is \( x_2 - x_1 = 88.0 \) m, and the velocity at the end of that displacement is \( v_{2x} = 80.0 \text{ km/h} = 22.22 \text{ m/s} \). However, we do not know the acceleration \( a_x \) and time \( t_2 \), which appear in both basic equations, so we must solve those equations simultaneously.

To eliminate the unknown \( t_2 \), we use Eq. 2-13 to write

\[ t_2 - t_1 = \frac{v_{2x} - v_{1x}}{a_x}, \quad (2-18) \]

and then we substitute this expression into Eq. 2-17 to write

\[ x_2 - x_1 = v_{1x} \left( \frac{v_{2x} - v_{1x}}{a_x} \right) + \frac{1}{2} \left( \frac{v_{2x} - v_{1x}}{a_x} \right)^2. \]

Solving for \( a_x \) and substituting known data then yields

\[ a_x = \frac{v_{2x}^2 - v_{1x}^2}{2(x_2 - x_1)} = \frac{(22.22 \text{ m/s})^2 - (27.78 \text{ m/s})^2}{2(88.0 \text{ m})} = -1.58 \text{ m/s}^2. \]

(b) How much time is required for the given decrease in speed?

**SOLUTION** Now that we know \( a_x \), we can use Eq. 2-18 to solve for \( t_2 \):

\[ t_2 - t_1 = \frac{v_{2x} - v_{1x}}{a_x} = \frac{22.22 \text{ m/s} - 27.78 \text{ m/s}^2}{-1.58 \text{ m/s}^2} = 3.52 \text{ s}. \]  

(Answer)

If you are initially speeding and trying to slow to the speed limit, there is plenty of time for the police officer to measure your excess speed.

You can use one of the alternate equations for motion with a constant acceleration, Eq. 2-15, to check this result. The **Key Idea** here is that the distance traveled is just the product of the average velocity and the elapsed time, when the acceleration is constant. The Porsche traveled 88.0 m while it slowed from 100 km/h down to 80 km/h. Thus its average velocity while it covered the 88.0 m was

\[ \langle v_i \rangle = \frac{(100 \text{ km/h} + 80 \text{ km/h})}{2} = 90 \text{ km/h} \times \left( \frac{1000 \text{ m}}{1 \text{ km}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) = 25.0 \text{ m/s}, \]

so the time it took to slow down was just

\[ t_2 - t_1 = \frac{x_2 - x_1}{\langle v_i \rangle} = \frac{88.0 \text{ m}}{25.0 \text{ m/s}} = 3.52 \text{ s}, \]

(Answer)

which still isn’t enough time to avoid that speeding ticket!
Suppose that you gave a box sitting on a carpeted floor a push and then recorded its position three times per second as it slid to a stop. The table gives the results of such a measurement. Let's analyze the position vs. time data for the box sliding on the carpet and use curve fitting and calculus to obtain the velocity measurements. We will use Excel spreadsheet software to perform our analysis, but other computer- or calculator-based fitting or modeling software can be used.

(a) Draw a graph of the $x$ vs. $t$ data and discuss whether the relationship appears to be linear or not.

**SOLUTION** The Key Idea here is that the relationship between two variables is linear if the graph of the data points lie more or less along a straight line. There are many ways to graph the data for examination: by hand, with a graphing calculator, with a spreadsheet graphing routine, or with other graphing software such as Data Studio (available from PASCO scientific) or Graphical Analysis (available from Vernier Software and Technology). The graph in Fig. 2-18 that shows a curve and so the relationship between position, $x$, as a function of time is not linear.

![Graph of position vs. time for a box sliding on carpet](image1)

**Figure 2-18** Solution to Touchstone Example 2-5(a). A graph of position versus time for a box sliding across a carpet.

(b) Draw a motion diagram of the box as it comes to rest on the carpet.

**SOLUTION** The Key Idea here is to use the data to sketch the position along a line at equal time intervals. In Fig. 2-19, the black circles represent the location of the rear of the box at intervals of 1/30 of a second.

![Motion diagram for a box sliding across a carpet](image2)

**Figure 2-19** Solution to Touchstone Example 2-5(b). A motion diagram for a box sliding across a carpet.

(c) Is the acceleration constant? If so, what is its component along the $x$ axis?

**SOLUTION** The Key Idea here is to explore whether or not the relationship between position and time of the box as it slides to a stop can be described with a quadratic (parabolic) function of time as described in Eq. 2-17. This can be done by entering the data that are given into a spreadsheet or graphing calculator and either doing a quadratic model or a fit to the data. The outcome of a quadratic model is shown in Fig. 2-20. The $x$-model column contains the results of calculating $x$ using the equation $x = v_1 t + \frac{1}{2} a t^2$ for each of the times in the first column using the initial position, velocity and acceleration data shown in the boxes. The line shows the model data. If the kinematic equation fits the data, then we can conclude that the acceleration component is a constant given by $a_x = -6.6 \text{ m/s}^2$. Thus the acceleration is in the negative $y$ direction.

![Graph of position vs. time for a box sliding on carpet](image3)

**Figure 2-20** Solution to Touchstone Example (c). Data and a graph of position as function of time for a box sliding over carpet. Actual data is compared to a model of what is expected from Eq. 2-17 (assumed constant acceleration). The value of acceleration which produced the best match between the model and actual data is $-6.6 \text{ m/s}^2$. 

<table>
<thead>
<tr>
<th>$a$</th>
<th>$-6.6 \text{ m/s}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_1$</td>
<td>1.6 (m/s)</td>
</tr>
<tr>
<td>$x_1$</td>
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</table>
Problems

In several of the problems that follow you are asked to graph position, velocity, and acceleration versus time. Usually a sketch will suffice, appropriately labeled and with straight and curved portions apparent. If you have a computer or graphing calculator, you might use it to produce the graph.

SEC. 2-3 ■ VELOCITY AND SPEED

1. Fastball If a baseball pitcher throws a fastball at a horizontal speed of 160 km/h, how long does the ball take to reach home plate 18.4 m away?

2. Fastest Bicycle A world speed record for bicycles was set in 1992 by Chris Huber riding Cheetah, a high-tech bicycle built by three mechanical engineering graduates. The record (average) speed was 110.6 km/h through a measured length of 200.0 m on a desert road. At the end of the run, Huber commented, “Cogito ergo zoom!” (I think, therefore I go fast!) What was Huber’s elapsed time through the 200.0 m?

3. Auto Trip An automobile travels on a straight road for 40 km at 30 km/h. It then continues in the same direction for another 40 km at 60 km/h. (a) What is the average velocity of the car during this 80 km trip? (Assume that it moves in the positive x direction.) (b) What is the average speed? (c) Graph x vs. t and indicate how the average velocity is found on the graph.

4. Radar Avoidance A top-gun pilot, practicing radar avoidance maneuvers, is manually flying horizontally at 1300 km/h, just 35 m above the level ground. Suddenly, the plane encounters terrain that slopes gently upward at 4.3°, an amount difficult to detect visually (Fig. 2-22). How much time does the pilot have to make a correction to avoid flying into the ground?

TOUCHSTONE EXAMPLE 2-6: Distance Covered

Figure 2-21b shows a graph of a person riding on a low-friction cart being pulled along with a bungee cord as shown in Fig. 2-21a. Use information from the two graphs and the kinematic equations to determine approximately how far the student moved in the time interval between 1.1 s and 2.0 s.

SOLUTION ■ The Key Idea is that the initial velocity can be determined from the velocity vs. time graph on the left and the acceleration during the time interval can be determined from the acceleration vs. time graph on the right (or by finding the slope of the velocity vs. time graph on the left during the time interval). Note that the velocity at $t_1 = 1.1$ s is given by $v_{1x} = 0.4$ m/s. The acceleration during the time interval of interest is given by $a_x = 0.4$ m/s$^2$. Since the acceleration is constant over the time interval of interest, we can use the data in Eq. 2-17 to get

$$x_2 - x_1 = v_{1x}(t_2 - t_1) + \frac{1}{2} a_x (t_2 - t_1)^2$$

$$= (0.4 \text{ m/s})(2.0 \text{ s} - 1.1 \text{ s}) + \frac{1}{2}(0.4 \text{ m/s}^2)(2.0 \text{ s} - 1.1 \text{ s})^2$$

$$\approx 0.5 \text{ m.}$$  
(Answer)

Half a meter is not very far!

FIGURE 2-21 ■ (a) A person riding on a low-friction cart is pulled by another person who exerts a constant force along a straight line by keeping the length of a bungee cord constant. (b) These graphs show velocity and acceleration components vs. time for a rider on a cart. For the first 0.5 s (region A) the cart is at rest. Between 0.5 s and 1.1 s (region B) the cord is beginning to stretch. Between 1.1 s and 2.0 s (region C) a constant force is acting and the acceleration is also constant.
5. On Interstate You drive on Interstate 10 from San Antonio to Houston, half the time at 55 km/h and the other half at 90 km/h. On the way back you travel half the distance at 55 km/h and the other half at 90 km/h. What is your average speed (a) from San Antonio to Houston, (b) from Houston back to San Antonio, and (c) for the entire trip? (d) What is your average velocity for the entire trip? (e) Sketch x vs. t for (a), assuming the motion is all in the positive x direction. Indicate how the average velocity can be found on the sketch.

6. Walk Then Run Compute your average velocity in the following two cases: (a) You walk 73.2 m at a speed of 1.22 m/s and then run 73.2 m at a speed of 3.05 m/s along a straight track. (b) You walk for 1.00 min at a speed of 1.22 m/s and then run for 1.00 min at 3.05 m/s along a straight track. (c) Graph x vs. t for both cases and indicate how the average velocity is found on the graph.

7. Position and Time The position of an object moving along an x axis is given by \( x = (3 \text{ m/s})t - (4 \text{ m/s}^2)t^2 + (1 \text{ m/s}^3)t^3 \), where \( x \) is in meters and \( t \) in seconds. (a) What is the position of the object at \( t = 1, 2, 3, \) and 4 s? (b) What is the object’s displacement between \( t_0 = 0 \) and \( t_f = 4 \text{ s} \)? (c) What is its average velocity for the time interval from \( t_1 = 2 \text{ s} \) to \( t_2 = 4 \text{ s} \)? (d) Graph x vs. t for \( 0 \leq t \leq 4 \text{ s} \) and indicate how the answer for (c) can be found on the graph.

8. Two Trains and a Bird Two trains, each having a speed of 30 km/h, are headed at each other on the same straight track. A bird that can fly 60 km/h flies off the front of one train when they are 60 km apart and heads directly for the other train. On reaching the other train it flies directly back to the first train, and so forth. (We have no idea why a bird would behave in this way.) What is the total distance the bird travels?

9. Two Winners On two different tracks, the winners of the 1 kilometer race ran their races in 2 min, 27.95 s and 2 min, 28.15 s. In order to conclude that the runner with the shorter time was indeed faster, how much longer can the other track be in actual length?

10. Scampering Armadillo The graph in Fig. 2-23 is for an armadillo that scampers left (negative direction of x) and right along an x axis. (a) When, if ever, is the animal to the left of the origin on the axis? When, if ever, is its velocity (b) negative, (c) positive, or (d) zero?

11. Position and Time (a) If a particle’s position is given by \( x = 4m - (12 \text{ m/s})t + (3 \text{ m/s}^2)t^2 \) (where \( t \) is in seconds and \( x \) is in meters), what is its velocity at \( t_1 = 1 \text{ s} \)? (b) Is it moving in the positive or negative direction of \( x \) just then? (c) What is its speed just then? (d) Is the speed larger or smaller at later times? (Try answering the next two questions without further calculation.) (e) Is there ever an instant when the velocity is zero? (f) Is there a time after \( t_3 = 3 \text{ s} \) when the particle is moving in the negative direction of \( x \)?

12. Particle Position and Time The position of a particle moving along the x axis is given in centimeters by \( x = 9.75m + (1.5 \text{ m/s}^2)t^3 \) where \( t \) is in seconds. Calculate (a) the average velocity during the time interval \( t = 2.00 \text{ s} \) to \( t = 3.00 \text{ s} \); (b) the instantaneous velocity at \( t = 2.00 \text{ s} \); (c) the instantaneous velocity at \( t = 3.00 \text{ s} \); (d) the instantaneous velocity at \( t = 2.50 \text{ s} \); and (e) the instantaneous velocity when the particle is midway between its positions at \( t = 2.00 \text{ s} \) and \( t = 3.00 \text{ s} \). (f) Graph x vs. t and indicate your answers graphically.

13. Velocity–Time Graph How far does the runner whose velocity–time graph is shown in Fig. 2-24 travel in 16 s?

14. Various Motions Sketch a graph that is a possible description of position as a function of time for a particle that moves along the x axis and, at \( t = 1 \text{ s} \), has (a) zero velocity and positive acceleration; (b) zero velocity and negative acceleration; (c) negative velocity and positive acceleration; (d) negative velocity and negative acceleration. (e) For which of these situations is the speed of the particle increasing at \( t = 1 \text{ s} \)?

15. Two Similar Expressions What do the quantities (a) \( (dx/dt)^2 \) and (b) \( d^2x/dt^2 \) represent? (c) What are their SI units?

16. Frightened Ostrich A frightened ostrich moves in a straight line with velocity described by the velocity–time graph of Fig. 2-25. Sketch acceleration vs. time.

17. Speed Then and Now A particle had a speed of 18 m/s at a certain time, and 2.4 s later its speed was 30 m/s in the opposite direction. What were the magnitude and direction of the average acceleration of the particle during this 2.4 s interval?

18. Stand Then Walk From \( t_0 = 0 \) to \( t_5 = 5.00 \text{ min} \), a man stands still, and from \( t_5 = 5.00 \text{ min} \) to \( t_{10} = 10.0 \text{ min} \), he walks briskly in a straight line at a constant speed of 2.20 m/s. What are (a) his average velocity \( \langle v \rangle \) and (b) his average acceleration \( \langle a \rangle \) in the time interval 2.00 min to 8.00 min? What are (c) \( \langle v \rangle \) and (d) \( \langle a \rangle \) in the time interval 3.00 min to 9.00 min? (e) Sketch \( x \) vs. \( t \) and \( v \) vs. \( t \), and indicate how the answers to (a) through (d) can be obtained from the graphs.

19. Particle Position and Time The position of a particle moving along the x axis depends on the time according to the equation \( x = ct^2 - bt^3 \), where \( x \) is in meters and \( t \) in seconds. (a) What units must \( c \) and \( b \) have? Let their numerical values be 3.0 and 2.0, respectively. (b) At what time does the particle reach its maximum positive x position? From \( t_0 = 0 \text{ s} \) to \( t_4 = 4.0 \text{ s} \), (c) what distance does the particle move and (d) what is its displacement? At \( t = 1.0\), 2.0, 3.0, and 4.0 s, what are (e) its velocities and (f) its accelerations?

20. Driver and Rider An automobile driver on a straight road increases the speed at a constant rate from 25 km/h to 55 km/h in 0.50 min. A bicycle rider on a straight road speeds up at a constant rate from rest to 30 km/h in 0.50 min. Calculate their accelerations.

21. Stopping a Muon A muon (an elementary particle) moving in a straight line enters a region with a speed of \( 5.00 \times 10^8 \text{ m/s} \) and...
then is slowed at the rate of $1.25 \times 10^{14}$ m/s$^2$. (a) How far does the muon take to stop? (b) Graph $x$ vs. $t$ and $v$ vs. $t$ for the muon.

22. Rattlesnake Striking The head of a rattlesnake can accelerate at 50 m/s$^2$ in striking a victim. If a car could do as well, how long would it take to reach a speed of 100 km/h from rest?

23. Accelerating an Electron An electron has a constant acceleration of +3.2 m/s$^2$. At a certain instant its velocity is +9.6 m/s. What is its velocity (a) 2.5 s earlier and (b) 2.5 s later?

24. Speeding Bullet The speed of a bullet is measured to be 640 m/s as the bullet emerges from a barrel of length 1.20 m. Assuming constant acceleration, find the time that the barrel spends in the barrel after it is fired.

25. Comfortable Acceleration Suppose a rocket ship in deep space moves with constant acceleration equal to 9.8 m/s$^2$, which gives the illusion of normal gravity during the flight. (a) If it starts from rest, how long will it take to acquire a speed one-tenth that of light, which travels at $3.0 \times 10^8$ m/s? (b) How far will it travel in so doing?

26. Taking Off A jumbo jet must reach a speed of 360 km/h on the runway for takeoff. What is the least constant acceleration needed for takeoff from a 1.80 km runway?

27. Even Faster Electrons An electron with initial velocity $v_1 = 1.50 \times 10^5$ m/s enters a region 1.0 cm long where it is electrically accelerated (Fig. 2-26). It emerges with velocity $v_2 = 5.70 \times 10^5$ m/s. What is its acceleration, assumed constant? (Such a process occurs in conventional television sets.)

28. Stopping Col. Stapp A world’s land speed record was set by Colonel John P. Stapp when in March 1954 he rode a rocket-propelled sled that moved along a track at 1020 km/h. He and the sled were brought to a stop in 1.4 s. (See Fig. 2-13) In g units, what acceleration did he experience while stopping?

29. Speed Trap The brakes on your automobile are capable of slowing down your car at a rate of 5.2 m/s$^2$. (a) If you are going 137 km/h and suddenly see a state trooper, what is the minimum time in which you can get your car under the 90 km/h speed limit? The answer reveals the futility of braking to keep your high speed from being detected with a radar or laser gun.) (b) Graph $x$ vs. $t$ and $v$ vs. $t$ for such a deceleration.

30. Judging Acceleration Figure 2-27 depicts the motion of a particle moving along an $x$ axis with a constant acceleration. What are the magnitude and direction of the particle’s acceleration?

31. Hitting a Wall A car traveling 56.0 km/h is 24.0 m from a barrier when the driver slams on the brakes. The car hits the barrier 2.00 s later. (a) What is the car’s constant acceleration before impact? (b) How fast is the car traveling at impact?

32. Red and Green Trains A red train traveling at 72 km/h and a green train traveling at 144 km/h are headed toward one another along a straight, level track. When they are 950 m apart, each engineer sees the other’s train and applies the brakes. The brakes slow each train at the rate of 1.0 m/s$^2$. Is there a collision? If so, what is the speed of each train at impact? If not, what is the separation between the trains when they stop?

33. Between Two Points A car moving with constant acceleration covered the distance between two points 60.0 m apart in 6.00 s. Its speed as it passes the second point was 15.0 m/s. (a) What was the speed at the first point? (b) What was the acceleration? (c) At what time distance from the first point was the car at rest? (d) Graph $x$ vs. $t$ and $v$ vs. $t$ for the car from rest ($t_0 = 0$).

34. Chasing a Truck At the instant the traffic light turns green, an automobile starts with a constant acceleration $a$ of 2.2 m/s$^2$. At the same instant a truck, traveling with a constant speed of 9.5 m/s, overtakes and passes the automobile. (a) How far beyond the traffic signal will the automobile overtake the truck? (b) How fast will the car be traveling at that instant?

35. Reaction Time To stop a car, first you require a certain reaction time to begin braking; then the car slows under the constant braking. Suppose that the total distance moved by your car during these two phases is 56.7 m when its initial speed is 80.5 km/h, and 24.4 in when its initial speed is 48.3 km/h. What are (a) your reaction time and (b) the magnitude of the deceleration?

36. Avoiding a Collision When a high-speed passenger train traveling at 161 km/h rounds a bend, the engineer is shocked to see that a locomotive has improperly entered onto the track from a siding and is a distance $D = 676$ m ahead (Fig. 2-28). The locomotive is moving at 29.0 km/h. The engineer of the high-speed train immediately applies the brakes. (a) What must be the magnitude of the resulting constant acceleration if a collision is to be just avoided? (b) Assume that the engineer is at $x = 0$ when, at $t = 0$, he first spots the locomotive. Sketch the $x(t)$ curves representing the locomotive and, high-speed train for the situations in which a collision is just avoided and is not quite avoided.

37. Going Up An elevator cab in the New York Marquis Marriott has a total run of 190 m. Its maximum speed is 305 m/min. Its acceleration (both speeding up and slowing) has a magnitude of 1.22 m/s$^2$. (a) How far does the cab move while accelerating to full speed from rest? (b) How long does it take to make the nonstop 190 m run, starting and ending at rest?

38. Shuffleboard Disk A shuffleboard disk is accelerated at a constant rate from rest to a speed of 6.0 m/s over a 1.8 m distance by a player using a cue. At this point the disk loses contact with the cue and slows at a constant rate of 2.5 m/s$^2$ until it stops. (a) How much
time elapses from when the disk begins to accelerate until it stops? (b) What total distance does the disk travel?

39. Electric Vehicle An electric vehicle starts from rest and accelerates at a rate of 2.0 m/s² in a straight line until it reaches a speed of 20 m/s. The vehicle then slows at a constant rate of 1.0 m/s² until it stops. (a) How much time elapses from start to stop? (b) How far does the vehicle travel from start to stop?

40. Red Car—Green Car In Fig. 2-29 a red car and a green car, identical except for the color, move toward each other in adjacent lanes and parallel to an x axis. At time \( t = 0 \), the red car is at \( x = 0 \) and the green car is at \( x = 220 \) m. If the red car has a constant velocity of 20 km/h, the cars pass each other at \( x = 44.5 \) m, and if it has a constant velocity of 40 km/h, they pass each other at \( x = 76.6 \) m. What are (a) the initial velocity and (b) the acceleration of the green car?

Additional Problems

42. Kids in the Back! An unrestrained child is playing on the front seat of a car that is traveling in a residential neighborhood at 35 km/h. (How many mi/h is this? Is this car going too fast?) A small dog runs across the road and the driver applies the brakes, stopping the car quickly and missing the dog. Estimate the speed with which the child strikes the dashboard, presuming that the car stops before the child does so. Compare this speed with that of the world-record 100 m dash, which is run in about 10 s.

43. The Passat GLX Test results (Car & Driver, February 1993, p. 48) on a Volkswagen Passat GLX show that when the brakes are fully applied it has an average braking acceleration of magnitude 8.9 m/s². If a preoccupied driver who is moving at a speed of 42 mph looks up suddenly and sees a stop light 30 m in front of him, will he have sufficient time to stop? The weight of the Volkswagen is 3 152 lb.

44. Velocity and Pace When we drive a car we usually describe our motion in terms of speed or velocity. A speed limit, such as 60 mi/h, is a speed. When runners or joggers describe their motion, they often do so in terms of a pace—how long it takes to go a given distance. A 4-min mile (or better, “4 minutes/mile”) is an example of a pace.

(a) Express the speed 60 mi/h as a pace in min/mi.
(b) I walk on my treadmill at a pace of 17 min/mi. What is my speed in mi/h?
(c) If I travel at a speed, \( v \), given in mi/h, what is my pace, \( p \), given in min/mi? (Write an equation that would permit easy conversion.)

45. Spirit of America The 9000 lb Spirit of America (designed to be the world’s fastest car) accelerated from rest to a final velocity of 756 mph in a time of 45 s. What would the acceleration have been in meters per second? What distance would the driver, Craig Breedlove, have covered?

46. Driving to New York You and a friend decide to drive to New York from College Park, Maryland (near Washington, D.C.) on Saturday over the Thanksgiving break to go to a concert with some friends who live there. You figure you have to reach the vicinity of the city at 5 p.m. in order to meet your friends in time for dinner before the concert. It’s about 220 mi from the entrance to Route 95 to the vicinity of New York City. You would like to get on the highway about noon and stop for a bite to eat along the way. What does your average velocity have to be? If you keep an approximately constant speed (not a realistic assumption!), what should your speedometer read while you are driving?

47. NASA Internship You are working as a student intern for the National Aeronautics and Space Administration (NASA) and your supervisor wants you to perform an indirect calculation of the upward velocity of the space shuttle relative to the Earth’s surface just 5.5 s after it is launched when it has an altitude of 100 m. In order to obtain data, one of the engineers has wired a streamlined flare to the side of the shuttle that is gently released by remote control after 5.5 s. If the flare hits the ground 8.5 s after it is released, what is the upward velocity of the flare (and hence of the shuttle) at the time of its release? (Neglect any effects of air resistance on the flare.)

Note: Although the flare idea is fictional, the data on a typical shuttle altitude and velocity at 5.5 s are straight from NASA!

48. Cell Phone Fight You are arguing over a cell phone while trailing an unmarked police car by 25 m; both your car and the police car are traveling at 110 km/h. Your argument diverts your attention and begins braking. (b) If you too brake at 5.0 m/s², what is your speed when you hit the police car?

49. Reaction Distance When a driver brings a car to a stop by braking as hard as possible, the stopping distance can be regarded as the sum of a “reaction distance,” which is initial speed multiplied by the driver’s reaction time, and a “braking distance,” which is the distance traveled during braking. The following table gives typical values. (a) What reaction time is the driver assumed to have? (b) What is the car’s stopping distance if the initial speed is 25 m/s?

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<tr>
<th>Initial Speed (m/s)</th>
<th>Reaction Distance (m)</th>
<th>Braking Distance (m)</th>
<th>Stopping Distance (m)</th>
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</tbody>
</table>
50. Tailgating In this problem we analyze the phenomenon of “tailgating” in a car on a highway at high speeds. This means traveling too close behind the car ahead of you. Tailgating leads to multiple car crashes when one of the cars in a line suddenly slows down. The question we want to answer is: “How close is too close?”

To answer this question, let’s suppose you are driving on the highway at a speed of 100 km/h (a bit more than 60 mi/h). The driver ahead of you suddenly puts on his brakes. We need to calculate a number of things: how long it takes you to respond; how far you travel in that time, and how far the other car travels in that time.

(a) First let’s estimate how long it takes you to respond. Two times are involved: how long it takes from the time you notice something happening till you start to move to the brake, and how long it takes to move your foot to the brake. You will need a ruler to do this. Take the ruler and have a friend hold it from the one end hanging straight down. Place your thumb and forefinger opposite the bottom of the ruler. As your friend releases the ruler suddenly, try to catch it with your thumb and forefinger. Measure how far it falls before you catch it. Do this three times and take the average distance. Assuming the ruler is falling freely without air resistance (not a bad assumption), calculate how much time it takes you to catch it, \( t_1 \). Now estimate the time, \( t_2 \), it takes you to move your foot to the brake pedal. Your reaction time is \( t_1 + t_2 \).

(b) If you brake hard and fast, you can bring a typical car to rest from 100 km/h (about 60 mi/h) in 5 seconds.

1. Calculate your acceleration, \(-a_0\) assuming that it is constant.
2. Suppose the driver ahead of you begins to brake with an acceleration \(-a_0\). How far will he travel before he comes to a stop? (Hint: How much time will it take him to stop? What will be his average velocity over this time interval?)
3. Now we can put these results together into a fairly realistic situation. You are driving on the highway at 100 km/hr and there is a driver in front of you going at the same speed.

1. You see him start to slow immediately (an unreasonable but simplifying assumption). If you are also traveling 100 km/h, how far (in meters) do you travel before you begin to brake? If you can also produce the acceleration \(-a_0\) when you brake, what will be the total distance you travel before you come to a stop?
2. If you don’t notice the driver ahead of you beginning to brake for 1 s, how much additional distance will you travel?
3. Discuss, on the basis of these calculations, what you think is a safe distance to stay behind a car at 60 mi/h. Express your distance in “car lengths” (about 15 ft). Would you include a safety factor beyond what you have calculated here? How much?

51. Testing the Motion Detector A motion detector that may be used in physics laboratories is shown in Fig. 2-30. It measures the distance to the nearest object by using a speaker and a microphone. The speaker clicks 30 times a second. The microphone detects the sound bouncing back from the nearest object in front of it. The computer calculates the time delay between making the sound and receiving the echo. It knows the speed of sound (about 343 m/s at room temperature), and from that it can calculate the distance to the object from the time delay.

(a) If the nearest object in front of the detector is too far away, the echo will not get back before a second click is emitted. Once that happens, the computer has no way of knowing that the echo isn’t an echo from the second click and that the detector isn’t giving correct results any more. How far away does the object have to be before that happens?

(b) The speed of sound changes a little bit with temperature. Let’s try to get an idea of how important this is. At room temperature (72 °F) the speed of sound is about 343 m/s. At 62 °F it is about 1% smaller. Suppose we are measuring an object that is really 1.5 meters away at 72 °F. What is the time delay \( \Delta t \) that the computer detects before the echo returns? Now suppose the temperature is 62 °F. If the computer detects a time delay of \( \Delta t \) but (because it doesn’t know the temperature) calculates the distance using the speed of sound appropriate for 72 °F, how far away does the computer report the object to be?

52. Hitting a Bowling Ball A bowling ball sits on a hard floor at a point that we take to be the origin. The ball is hit some number of times by a hammer. The ball moves along a line back and forth across the floor as a result of the hits. (See Fig. 2-31.) The region to the right of the origin is taken to be positive, but during its motion the ball is at times on both sides of the origin. After the ball has been moving for a while, a motion detector like the one discussed in Problem 51 is started and takes the following graph of the ball’s velocity.

![Figure 2-30](image.png)  Problem 51.

![Figure 2-31](image.png)  Problem 52.

Answer the following questions with the symbols L (left), R (right), N (neither), or C (can’t say which). Each question refers only to the time interval displayed by the computer.

(a) At which side of the origin is the ball for the time marked A?
(b) At the time marked B, in which direction is the ball moving?
(c) Between the times A and C, what is the direction of the ball’s displacement?
(d) The ball receives a hit at the time marked D. In what direction is the ball moving after that hit?

53. Waking the Balrog In *The Fellowship of the Ring*, the hobbit Peregrine Took (Pippin for short) drops a rock into a well while the travelers are in the caves of Moria. This wakes a balrog (a bad thing) and causes all kinds of trouble. Pippin hears the rock hit the floor at a 7.5 s after he drops it.

(a) Ignoring the time it takes the sound to get back up, how deep is the well?
(b) It is quite cool in the caves of Moria, and the speed of sound in air changes with temperature. Take the speed of sound to be 340 m/s (it is pretty cool in that part of Moria). Was it OK to ignore the time it takes sound to get back up? Discuss and support your answer with a calculation.

54. Two Balls, Passing in the Night* Figure 2-32 represents the position vs. clock reading of the motion of two balls, A and B,

moving on parallel tracks. Carefully sketch the figure on your homework paper and answer the following questions:

(a) Along the $t$ axis, mark with the symbol $t_A$ any instant or instants at which one ball is passing the other.
(b) Which ball is moving faster at clock reading $t_B$?
(c) Mark with the symbol $c$ any instant or instants at which the balls have the same velocity.
(d) Over the period of time shown in the diagram, which of the following is true of ball $B$? Explain your answer.
   1. It is speeding up all the time.
   2. It is slowing down all the time.
   3. It is speeding up part of the time and slowing down part of the time.

55. Graph for a Cart on a Tilted Airtrack—With Spring The graph in Fig. 2-33 below shows the velocity graph of a cart moving on an air track. The track has a spring at one end and has its other end raised. The cart is started sliding up the track by pressing it against the spring and releasing it. The clock is started just as the cart leaves the spring. Take the direction the cart is moving in initially to be the positive $x$ direction and take the bottom of the spring to be the origin.

Letters point to six points on the velocity curve. For the physical situations described below, identify which of the letters corresponds to the situation described. You may use each letter more than once, more than one letter may be used for each answer, or none may be appropriate. If none is appropriate, use the letter N.

(a) This point occurs when the cart is at its highest point on the track.
(b) At this point, the cart is instantaneously not moving.
(c) This is a point when the cart is in contact with the spring.
(d) At this point, the cart is moving down the track toward the origin.
(e) At this point, the cart has acceleration of zero.

56. Rolling Up and Down A ball is launched up a ramp by a spring as shown in Fig. 2-34. At the time when the clock starts, the ball is near the bottom of the ramp and is rolling up the ramp as shown. It goes to the top and then rolls back down. For the graphs shown in Fig. 2-34, the horizontal axis represents the time. The vertical axis is unspecified.

For each of the following quantities, select the letter of the graph that could provide a correct graph of the quantity for the ball in the situation shown (if the vertical axis were assigned the proper units). Use the $x$ and $y$ coordinates shown in the picture. If none of the graphs could work, write N.

(a) The $x$-component of the ball’s position __________
(b) The $y$-component of the ball’s velocity __________
(c) The $x$-component of the ball’s acceleration __________
(d) The $y$-component of the normal force the ramp exerts on the ball __________
(e) The $x$-component of the ball’s velocity __________
(f) The $x$-component of the force of gravity acting on the ball __________

57. Model Rocket A model rocket, propelled by burning fuel, takes off vertically. Plot qualitatively (numbers not required) graphs of $y$, $v$, and $a$ versus $t$ for the rocket’s flight. Indicate when the fuel is exhausted, when the rocket reaches maximum height, and when it returns to the ground.

58. Rock Climber At time $t = 0$, a rock climber accidentally allows a piton to fall freely from a high point on the rock wall to the valley below him. Then, after a short delay, his climbing partner, who is 10 m higher on the wall, throws a piton downward. The positions $y$ of the pitons versus $t$ during the fall are given in Fig. 2-35. With what speed was the second piton thrown?

59. Two Trains As two trains move along a track, their conductors suddenly notice that they are headed toward each other. Figure 2-36 gives their velocities $v$ as functions of time $t$ as the conductors slow the trains.
The slowing processes begin when the trains are 200 m apart. What is their separation when both trains have stopped?

60. Runaway Balloon As a runaway scientific balloon ascends at 19.6 m/s, one of its instrument packages breaks free of a harness and free-falls. Figure 2-37 gives the vertical velocity of the package versus time, from before it breaks free to when it reaches the ground. (a) What maximum height above the break-free point does it rise? (b) How high was the break-free point above the ground?

61. Position Function Two A particle moves along the x axis with position function \( x(t) \) as shown in Fig. 2-38. Make rough sketches of the particle’s velocity versus time and its acceleration versus time for this motion.

62. Velocity Curve Figure 2-39 gives the velocity \( v(t) \) versus time \( t \) (s) for a particle moving along an x axis. The area between the time axis and the plotted curve is given for the two portions of the graph. At \( t = t_A \) (at one of the crossing points in the plotted figure), the particle’s position is \( x = 14 \) m. What is its position at (a) \( t = 0 \) and (b) \( t = t_B \)?

63. The Motion Detector Rag This assignment is based on the Physics Pholk Song CD distributed by Pasco scientific. These songs are also available through the Dickinson College Web site at http://physics.dickinson.edu.

(a) Refer to the motion described in the first verse of the Motion Detector Rag; namely, you are moving for the same amount of time that you are standing. Sketch a position vs. time graph for this motion. Also, describe the shape of the graph in words.

(b) Refer to the motion described in the second verse of the Motion Detector Rag. In this verse, you are making a “steep down-slope,” then a “gentle up-slope,” and last a flat line. You spend the same amount of time engaged in each of these actions. Sketch a position vs. time graph of this motion. Also, describe what you are doing in words. That is, are you standing still, moving away from the origin (or motion detector), moving toward the origin (or motion detector)? Which motion is the most rapid, and so on?

(c) Refer to the motion described in the third verse of the Motion Detector Rag. You start from rest and move away from the motion detector at an acceleration of +1.0 m/s^2 for 5 seconds. Sketch the acceleration vs. time graph to this motion. Sketch the corresponding velocity vs. time graph. Sketch the shape of the corresponding position vs. time graph.

64. Hockey Puck At time \( t = 0 \), a hockey puck is sent sliding over a frozen lake, directly into a strong wind. Figure 2-40 gives the velocity \( v \) of the puck vs. time, as the puck moves along a single axis. At \( t = 14 \) s, what is its position relative to its position at \( t = 0 \)?

65. Describing One-Dimensional Velocity Changes In each of the following situations you will be asked to refer to the mathematical definitions and the concepts associated with the number line. Note that being more positive is the same as being less negative, and so on.

(a) Suppose an object undergoes a change in velocity from +1 m/s to +4 m/s. Is its velocity becoming more positive or less positive? What is meant by more positive? Less positive? Is the acceleration positive or negative?

(b) Suppose an object undergoes a change in velocity from −4 m/s to −1 m/s. Is its velocity becoming more positive or less positive? What is meant by more positive? Less positive? Is the acceleration positive or negative?

(c) Suppose an object is turning around so that it undergoes a change in velocity from −2 m/s to +2 m/s. Is its velocity becoming more positive or less positive than it was before? What is meant by more positive? Less positive? Is it undergoing an acceleration while it is turning around? Is the acceleration positive or negative?

(d) Another object is turning around so that it undergoes a change in velocity from +1 m/s to −1 m/s. Is its velocity becoming more positive or less positive than it was before? What is meant by more positive? Less positive? Is it undergoing an acceleration while it is turning around? Is the acceleration positive or negative?

66. Bowling Ball Graph A bowling ball was set into motion on a fairly smooth level surface, and data were collected for the total distance covered by the ball at each of four times. These data are shown below.

<table>
<thead>
<tr>
<th>Average</th>
<th>Time (s)</th>
<th>Distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>0.92</td>
<td>2.0</td>
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</tr>
<tr>
<td>1.85</td>
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</tr>
<tr>
<td>2.87</td>
<td>6.0</td>
<td></td>
</tr>
</tbody>
</table>

(a) Plot the data points on a graph.

(b) Use a ruler to draw a straight line that passes as close as possible to the data points you have graphed.

(c) Using methods you were taught in algebra, calculate the value of the slope, \( m \), and find the value of the intercept, \( b \), of the line you have sketched through the data.

67. Modeling Bowling Ball Motion A bowling ball is set into motion on a smooth level surface, and data were collected for the total distance covered by the ball at each of four times. These data are shown in the table in Problem 66. Your job is to learn to use a spreadsheet program — for example, Microsoft Excel — to create a mathematical model of the bowling ball motion data shown. You are to find what you think is the best value for the slope, \( m \), and the y-intercept, \( b \). Practicing with a tutorial worksheet entitled MODTUT.XLS will help you to learn about the process of
modeling for a linear relationship. Ask your instructor where to find this tutorial worksheet.

After using the tutorial, you can create a model for the bowling ball data given above. To do this:

(a) Open a new worksheet and enter a title for your bowling ball graph.
(b) Set the y-label to Distance (m) and the x-label to Time (s).
(c) Refer to the data table above. Enter the measured times for the bowling ball in the Time (s) column (formerly x-label).
(d) Set the y-exp column to D-data (m) and enter the measured distances for the bowling ball (probably something like 0.00 m, 2.00 m, 4.00 m, and 6.00 m.).
(e) Place the symbol m (for slope) in the cell B1. Place the symbol b (for y-intercept) in cell B2.
(f) Set the y-theory column to D-model (m) and then put the appropriate equation for a straight line of the form Distance = $m \times$ Time + b in cells C7 through C12. Be sure to refer to cells C1 for slope and C2 for y-intercept as absolutes; that is, use $C$1 and $C$2 when referring to them.
(g) Use the spreadsheet graphing feature to create a graph of the data in the D-exp and D-theory columns as a function of the data in the Time column.
(h) Change the values in cells Cl and C2 until your theoretical line matches as closely as possible your red experimental data points in the graph window.
(i) Discuss the meaning of the slope of a graph of distance vs. time. What does it tell you about the motion of the bowling ball?

68. A Strange Motion  After doing a number of the exercises with carts and fans on ramps, it is easy to draw the conclusion that everything that moves is moving at either a constant velocity or a constant acceleration. Let's examine the horizontal motion of a triangular frame with a pendulum at its center that has been given a push. It undergoes an unusual motion. You should determine whether or not it is moving at either a constant velocity or constant acceleration. (Note: You may want to look at the motion of the triangular frame by viewing the digital movie entitled PASCO070. This movie is included on the VideoPoint compact disk. If you are not using VideoPoint, your instructor may make the movie available to you some other way.)

The images in Fig. 2-41 are taken from the 7th, 16th, and 25th frames of that movie.

Data for the position of the center of the horizontal bar of the triangle were taken every tenth of a second during its first second of motion. The origin was placed at the zero centimeter mark of a fixed meter stick. These data follow.

(a) Examine the position vs. time graph of the data shown above. Does the triangle appear to have a constant velocity throughout the first second? A constant acceleration? Why or why not?
(b) Discuss the nature of the motion based on the shape of the graph. At approximately what time, if any, is the triangle changing direction? At approximately what time does it have the greatest negative velocity? The greatest positive velocity? Explain the reasons for your answers.
(c) Use the data table and the definition of average velocity to calculate the average velocity of the triangle at each of the times between 0.100 s and 0.900 s. In this case you should use the position just before the indicated time and the position just after the indicated time in your calculation. For example, to calculate the average velocity at $t_2 = 0.100$ s, use $x_3 = 44.5$ cm and $x_1 = 52.1$ cm along with the differences of the times at $t_3$ and $t_1$. Hint: Use only times and positions in the gray boxes to get a velocity in a gray box and use only times and positions in the white boxes to get a velocity in a white box.
(d) Since people usually refer to velocity as distance divided by time, maybe we can calculate the average velocities as simply $x_3/t_1$, $x_2/t_2$, $x_3/t_3$, and so on. This would be easier. Is this an equivalent method for

<table>
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<th>Pr#</th>
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<th>x(cm)</th>
<th>$&lt;\Delta&gt;$ (cm/s)</th>
</tr>
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<tbody>
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<td>39.1</td>
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<td></td>
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<tr>
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<td></td>
</tr>
<tr>
<td>19</td>
<td>7</td>
<td>0.600</td>
<td>36.9</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>8</td>
<td>0.700</td>
<td>43.0</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>9</td>
<td>0.800</td>
<td>49.2</td>
<td></td>
</tr>
<tr>
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<td>10</td>
<td>0.900</td>
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<td>1.000</td>
<td>54.4</td>
<td>no entry</td>
</tr>
</tbody>
</table>

**Figure 2-41** = Problem 68.
finding the velocities at the different times? Try using this method of calculation if you are not sure. Give reasons for your answer.

(e) Often, when an oddly shaped but smooth graph is obtained from data it is possible to fit a polynomial to it. For example, a fourth-order polynomial that fits the data is

\[ x = (-376 \text{ cm/s}^3)t^4 + (719 \text{ cm/s}^2)t^3 - (347 \text{ cm/s})t^2 + (5.63 \text{ cm/s})t + 52.1 \text{ cm} \]

Using this polynomial approximation, find the instantaneous velocity at \( t = 0.700 \text{ s} \). Comment on how your answer compares to the average velocity you calculated at 0.700 s. Are the two values close? Is that what you expect?

69. Cedar Point At the Cedar Point Amusement Park in Ohio, a cage containing people is moving at a high initial velocity as the result of a previous free fall. It changes direction on a curved track and then coasts in a horizontal direction until the brakes are applied. This situation is depicted in a digital movie entitled DSON002. (Note: This movie is included on the VideoPoint compact disk. If you are not using VideoPoint, your instructor may make the movie available to you some other way.)

(a) Use video analysis software to gather data for the horizontal positions of the tail of the cage in meters as a function of time. Don’t forget to use the scale on the title screen of the movie so your results are in meters rather than pixels. Summarize this data in a table or in a printout attached to your homework.

(b) Transfer your data to a spreadsheet and do a parabolic model to show that within 5% or better \( x = (-7.5 \text{ m/s}^2)t^2 + (22.5 \text{ m/s})t + 2.38 \text{ m} \). Please attach a printout of this model and graph with your name on it to your submission as “proof of completion.”(Note: Your judgments about the location of the cage tail may lead to slightly different results.)

(c) Use the equation you found along with its interpretation as embodied in the first kinematic equation to determine the horizontal acceleration, \( a \), of the cage as it slows down. What is its initial horizontal velocity, \( v_i \), at time \( t = 0 \text{ s} \)? What is the initial position, \( x_i \), of the cage?

(d) The movie ends before the cage comes to a complete stop. Use your knowledge of \( a, v_i, \) and \( x_i \) along with kinematic equations to determine the horizontal position of the cage when it comes to a complete stop so that the final velocity of the cage is given by \( v_f = 0.00 \text{ m/s} \).

70. Three Digital Movies Three digital movies depicting the motions of four single objects have been selected for you to examine using a video-analysis program. They are as follows:

PASCO004: A cart moves on an upper track while another moves on a track just below.

PASCO153: A metal ball attached to a string swings gently.

HRSY003: A boat with people moves in a water trough at Hershey Amusement Park.

Please examine the horizontal motion of each object carefully by viewing the digital movies. In other words, just examine the motion in the \( x \) direction (and ignore any slight motions in the \( y \) direction). You may use VideoPoint, VideoGraph, or World-in-Motion digital analysis software and a spreadsheet to analyze the motion in more detail if needed. Based on what you have learned so far, there is more than one analysis method that can be used to answer the questions that follow. Note: Since we are interested only in the nature of these motions (not exact values) you do not need to scale any of the movies. Working in pixel units is fine.

(a) Which of these four objects (upper cart, lower cart, metal ball, or boat), if any, move at a constant horizontal velocity? Cite the evidence for your conclusions.

(b) Which of these four objects, if any, move at a constant horizontal acceleration? Cite the evidence for your conclusions.

(c) Which of these four objects, if any, move at neither a constant horizontal velocity nor acceleration? Cite the evidence for your conclusions.

(d) The kinematic equations are very useful for describing motions. Which of the four motions, if any, cannot be described using the kinematic equations? Explain the reasons for your answer.

71. Speeding Up or Slowing Down Figure 2-42 shows the velocity vs. time graph for an object constrained to move in one dimension. The positive direction is to the right.

(a) At what times, or during what time periods, is the object speeding up?

(b) At what times, or during what time periods, is the object slowing down?

(c) At what times, or during what time periods, does the object have a constant velocity?

(d) At what times, or during what time periods, is the object at rest?

If there is no time or time period for which a given condition exists, state that explicitly.

72. Right or Left Figure 2-42 shows the velocity vs. time graph for an object constrained to move along a line. The positive direction is to the right.

(a) At what times, or during what time periods, is the object speeding up and moving to the right?

(b) At what times, or during what time periods, is the object slowing down and moving to the right?

(c) At what times, or during what time periods, does the object have a constant velocity to the right?

(d) At what times, or during what time periods, is the object speeding up and moving to the left?

(e) At what times, or during what time periods, is the object slowing down and moving to the left?

(f) At what times, or during what time periods, does the object have a constant velocity to the left?

If there is no time or time period for which a given condition exists, state that explicitly.

73. Constant Acceleration Figure 2-42 shows the velocity vs. time graph for an object constrained to move along a line. The positive direction is to the right.

(a) At what times, or during what time periods, is the object’s acceleration zero?
(b) At what times, or during what time periods, is the object’s acceleration constant?
(c) At what times, or during what time periods, is the object’s acceleration changing?

If there is no time or time period for which a given condition exists, state that explicitly.

74. Acceleration to the Right or Left Figure 2-42 shows the velocity vs. time graph for an object constrained to move along a line. The positive direction is to the right.

(a) At what times, or during what time periods, is the object’s acceleration increasing and directed to the right?
(b) At what times, or during what time periods, is the object’s acceleration decreasing and directed to the right?
(c) At what times, or during what time periods, does the object have a constant acceleration to the right?
(d) At what times, or during what time periods, is the object’s acceleration increasing and directed to the left?
(e) At what times, or during what time periods, is the object’s acceleration decreasing and directed to the left?
(d) At what times, or during what time periods, does the object have a constant acceleration to the left?

If there is no time or time period for which a given condition exists, state that explicitly.