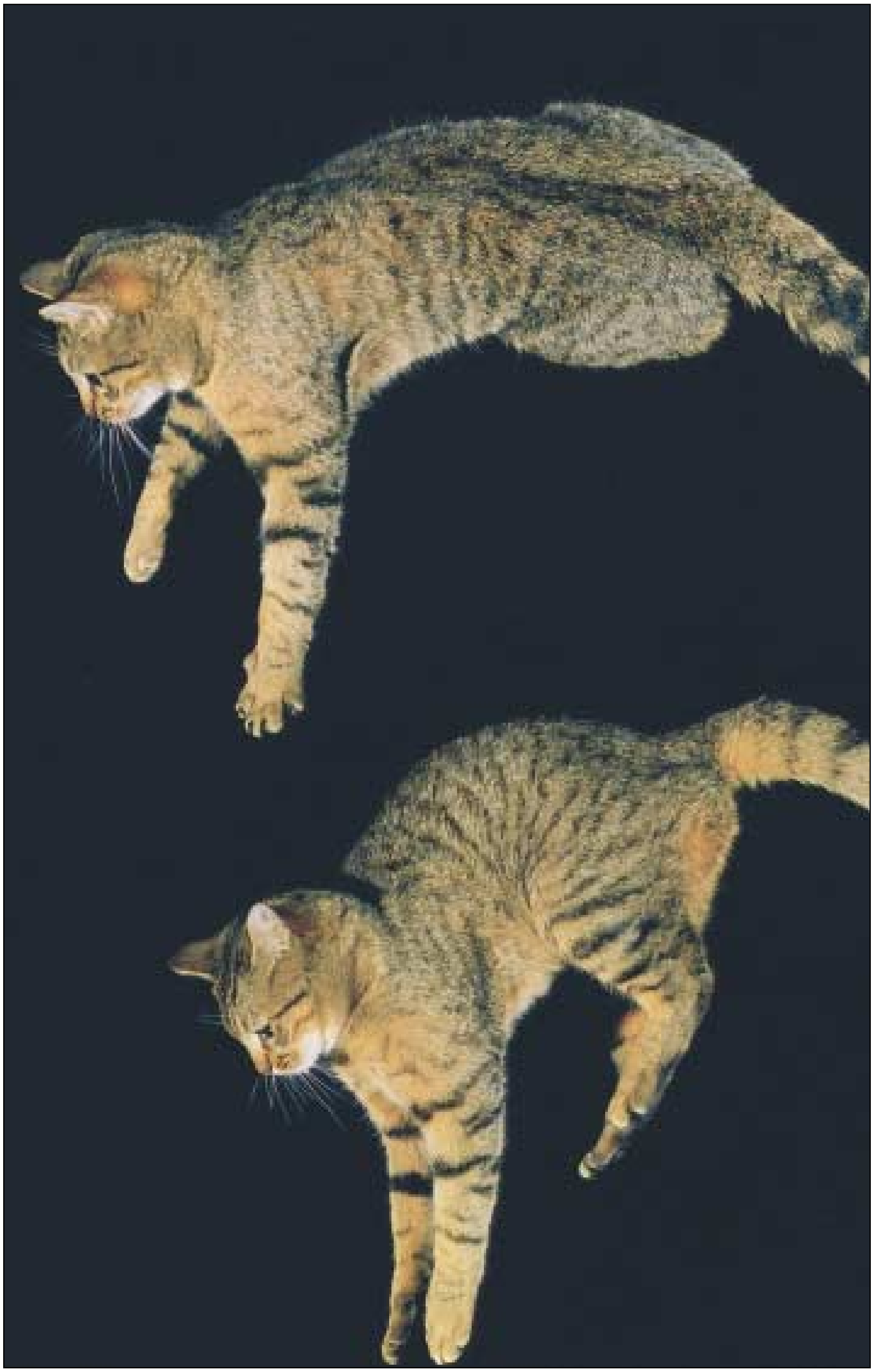
6 Identifying and Using Forces



Cats, who enjoy sleeping on window sills, are often kept in apartment buildings. When a cat accidentally falls out of a window and onto a sidewalk, the extent of injury (such as the number of fractured bones or the certainty of death) *decreases* with height if the fall is more than seven or eight floors. (There is even a record of a cat who fell 32 floors and suffered only slight damage to its thorax and one tooth.)

How can the damage possibly decrease with height?

The answer is in this chapter.

6-1 Combining Everyday Forces

It is common for objects to experience multiple forces that do not act along the same line. We saw examples of this in Section 5-2 in our brief consideration of the motion of a ball falling under the influence of both a gravitational force and air drag forces. In Section 5-7 we discussed the motion involved in the hammer throw and that of a rock rotating on a string. The hammer and the rock experience both a vertical gravitational force and changing centripetal forces that are almost horizontal.

The bola shown in Fig. 6-1 is another example of a system that experiences multiple forces acting in more than one dimension. The bola is a prehistoric weapon devised for capturing relatively large animals. The analysis of the bola's motion as it is whirled about, released, and encounters an animal is very complex. At any given moment the spherical end of a flying bola experiences a gravitational force, the pull of the rope, and an air drag force.

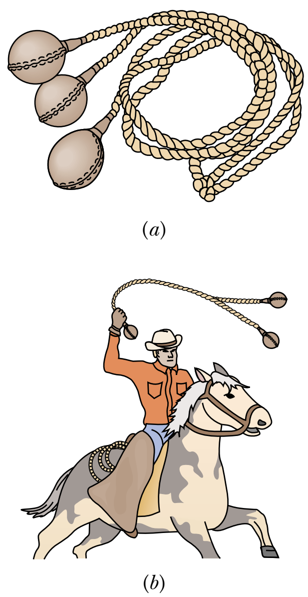


FIGURE 6-1 (*a*) A modern Inuit bola. (*b*) A sketch of a gaucho using a bola.

In this chapter, you will learn more about the characteristics of these everyday forces and how they can be superimposed using vector addition to find net forces. In addition, we will consider how to apply Newton's laws to predict motion and to identify hidden forces. As you will see, the ability to identify forces and use them along with Newton's laws to predict motion is extremely useful for two reasons. First, engineers can use their knowledge of the forces on a system to predict the motion of system components. This ability is vital in the design of a range of devices from bridges to aircraft. Second, the belief physicists have in the validity of Newton's laws of motion leads them to combine acceleration measurements with Newtonian analysis techniques to identify and characterize invisible forces. This approach to the discovery of forces was introduced in Section 3-9.

6-2 Net Force as a Vector Sum

In Chapter 3 we presented experiments that demonstrate that when two or more forces act on an object that moves in a straight line, it is the *net* force that determines how the object's motion will change. For one-dimensional motion the net force turns out to be the vector sum of the forces acting on the object. We call this the **principle of superposition for forces.** If we use the rules of two-dimensional vector addition that we learned about in Chapter 4, can we apply the principle of superposition in cases where the forces do not lie along a single line?

Countless experiments have demonstrated that the principle of superposition also works in two (and three) dimensions. For example, consider the rotating rock discussed in Fig. 5-24. As the rock rotates, it experiences both a gravitational and a string force as shown in Fig. 6-2. We already know that $\vec{F}^{\,grav} = -mg\hat{j}$ where m is the mass of the rock. If we attach a spring scale between the rock and the string, we can measure the string force $\vec{F}^{\,string}$. If the rock is rotating in a circle in a horizontal plane and we measure its centripetal acceleration, we find that it is related to the forces on the rock by

$$\vec{F}^{\,net} = \vec{F}^{\,grav} + \vec{F}^{\,string} = m\vec{a}. \tag{6-1}$$

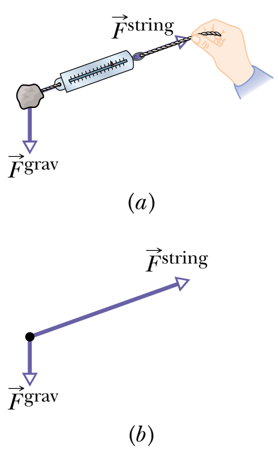


FIGURE 6-2 (*a*) At any particular moment there are two forces on a rock twirled on the end of a string—a gravitational force and a string force. Here the directions of the forces are indicated for the case where the rock rotates in a horizontal plane. (*b*) A free-body diagram showing the tails of the two force vectors at a point that represents the rock on which they act.

Here the net force that leads to the measured acceleration turns out indeed to be the two-dimensional vector sum (or superposition) of the two forces acting on the rock. We can find the vector sum of two or more force vectors by using the graphical method explained in Section 4-3, or we can resolve the vectors into components using the method presented in Section 4-4.

Another way to verify experimentally that the superposition of force vectors in two dimensions is a vector sum is to set up a situation in which the net force in a plane is zero. For example, we can pull on a ring with three force scales in such a way that the ring is stationary. In this case, we know the acceleration of the ring, and hence the net force on the ring is also zero. Every time we do this, we find that the vector sum of three string forces is zero. An example is shown in Fig. 6-3. Here the sum of $F_{A\,x}\hat{\imath}$ and $F_{B\,x}\hat{\imath}$ gives us a vector component that has the same magnitude as $F_{C\,x}\hat{\imath}$ but has the opposite sign so as to cancel it. Thus, the net force is zero as shown in Fig. 6-3*d*. After many such experiments, we become convinced that the net force on an object is the vector sum of the individual forces acting on the object, even if those forces do not act along a single line.

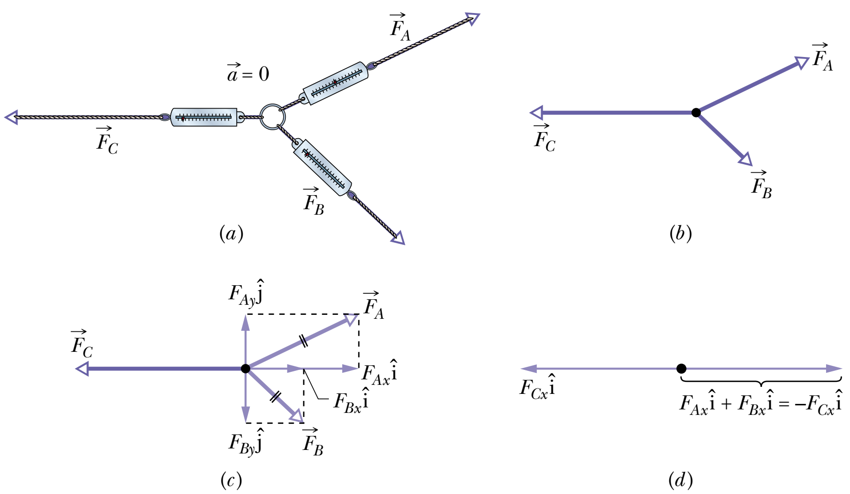


FIGURE 6-3 ■ (*a*) If the ring does not accelerate under the influence of the three forces, we conclude that the net force on it is zero, hence the vector sum of $\vec{F}_A$, $\vec{F}_B$, and $\vec{F}_C$ is zero. (*b*) A free-body diagram showing the tails of the three vectors at a point that represents the center of the ring on which they act. (*c*) Using the component method of resolving the vectors $\vec{F}_A$, $\vec{F}_B$, and $\vec{F}_C$ verifies that their sum is zero. (*d*) The sum of the *x*-components of $F_{A\,x}$ and $F_{B\,x}$ is $-F_{C\,x}$.

Free-Body Diagrams in Two Dimensions

In Chapter 3 we found that it was important to keep track of the magnitudes and directions of the forces acting on an object if we wanted to use Newton's Second Law ($\vec{F}^{\text{ net}} = m\vec{a}$) to determine the object's acceleration. The same is true for cases in which the forces do not lie along a single line. We introduced the idea of using a *free-body diagram* for this purpose in Section 3-7. The procedures for drawing free-body diagrams for two- and three-dimensional forces are similar to those used for one-dimensional forces: (1) Identify the object for which the motion is to be analyzed and represent it as a point. (2) Identify all the forces acting on the object and represent each force vector with an arrow. The tail of each force vector should be on the point. Draw the arrow in the direction of the force. Represent the relative magnitudes of the forces through the relative lengths of the arrows. (3) Label each force vector so that it is clear which force it represents.

Figures 6-2*b* and 6-3*b* are free-body diagrams for the situations depicted in the first part of those figures.

Newton's Second Law in Multiple Dimensions

The preceding example hints at another important point regarding multiple forces acting along different lines; namely, forces (or components of forces) in perpendicular dimensions are independent and separable. That is, Newton's Second Law $\vec{F} = m\vec{a}$ can be written as two (or three) component equations:

$$F_x^{\text{net}} = ma_x, \quad F_y^{\text{net}} = ma_y, \quad \text{and} \quad F_z^{\text{net}} = ma_z.$$

We will focus on two-dimensional examples in this chapter.

This statement regarding the separable nature of forces and components of forces should not be especially surprising. Recall from Chapter 5 that horizontal and vertical motions are independent and separable. That is, an acceleration in one dimension only affects the motion in that dimension. Therefore, we could treat two-dimensional motions as two separate one-dimensional cases. Since net force and acceleration are directly related, the independent and separable nature of acceleration is a direct hint that forces behave this way.

If three forces act on an object, then we can expand $F_x^{net} = ma_x$ to get

$$F_{A\,x} + F_{B\,x} + F_{C\,x} = ma_x,$$

where $F_{A\,x}$ is the x-component of force A, $F_{B\,x}$ is the x-component of force B, and so on. The x-component of the acceleration is a_x. These components are signed scalar quantities. This means that although the components are not vectors, they can still be either positive or negative. So, we need to be careful when we begin substituting in actual values for the components that we include the correct sign.

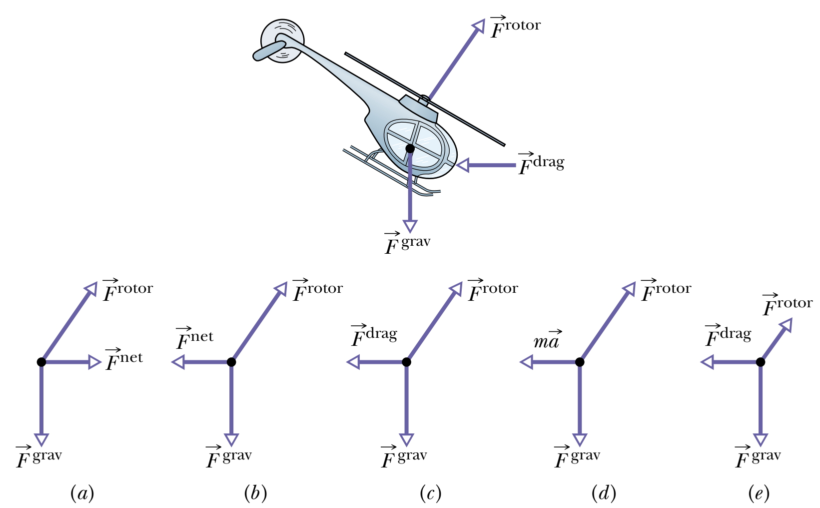
We can use a similar expansion to find ma_y and so on.

A Word about Notation

Recall from earlier chapters that $\vec{F}$ represents a vector. The magnitude (that is, size) of the vector will be represented by $|\vec{F}|$ when we want to stress that the value is always positive. More commonly, the magnitude will simply be represented as F. That is, a vector quantity represented without the arrow over it is the magnitude of the vector, which is always positive. F_x and F_y represent vector components and may be positive or negative depending on what direction $\vec{F}$ points in relation to the chosen coordinate system.

We have already introduced several different forces including gravitational, tension, and friction forces. These are important, everyday forces. In the rest of this chapter, we will add to our list of common forces and discuss those we have already introduced in more detail.

READING EXERCISE 6-1: A helicopter is moving to the right at a constant horizontal velocity due to the force on it caused by its rotor. It also experiences a downward gravitational force and a horizontal drag force as shown in the diagram below. Which of the following diagrams is a correct free-body diagram representing the forces on the helicopter?



TOUCHSTONE EXAMPLE 6-1: Tug-of-War

In a two-dimensional tug-of-war, Alex, Betty, and Charles pull horizontally on an automobile tire at the angles shown in the overhead view of Fig. 6-4*a*. The tire remains stationary in spite of the three pulls. Alex pulls with a force $\vec{F}_A$ of magnitude 220 N, and Charles pulls with force $\vec{F}_C$ of magnitude 170 N. The direction of $\vec{F}_C$ is not given. What is the magnitude of Betty's force $\vec{F}_B$?

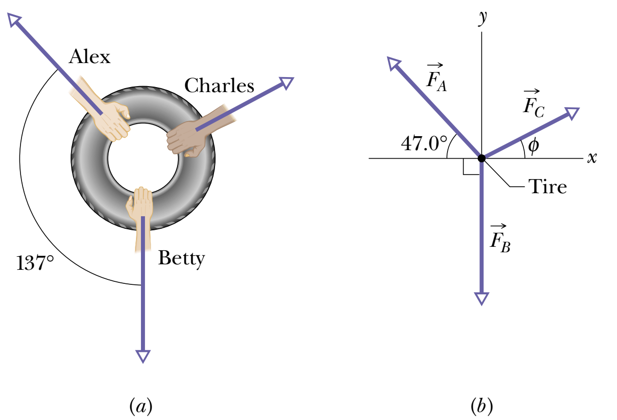


FIGURE 6-4 ■ (*a*) An overhead view of three people pulling on a tire. (*b*) A free-body diagram for the tire.

SOLUTION ■ Because the three forces pulling on the tire do not accelerate the tire, the tire's acceleration is $\vec{a} = 0$ (that is, the forces are in equilibrium). The **Key Idea** here is that we can relate that acceleration to the net force $\vec{F}^{\text{net}}$ on the tire with Newton's Second Law ($\vec{F}^{\text{net}} = m\vec{a}$), which we can write as

$$\vec{F}_A + \vec{F}_B + \vec{F}_C = m(0) = 0,$$

or

$$\vec{F}_B = -\vec{F}_A - \vec{F}_C. \tag{6-2}$$

The free-body diagram for the tire is shown in Fig. 6-4*b*, where we have conveniently centered a coordinate system on the tire and assigned ϕ to the angle between the x axis and $\vec{F}_C$.

We want to solve for the magnitude of $\vec{F}_B$. Although we know both magnitude and direction for $\vec{F}_A$, we know only the magnitude of $\vec{F}_C$ and not its direction. Thus, with unknowns on both sides of Eq. 6-2, we cannot directly solve it on a vector-capable calculator.

Instead we must rewrite Eq. 6-2 in terms of components for either the x or the y axis. If the sum of the forces is zero, it must also be that the sum of the x-components of the forces is zero *and* the sum of the y-components is zero. Since $\vec{F}_B$ is directed along the y axis, we choose that axis and write

$$F_{By} = -F_{Ay} - F_{Cy}.$$

Note that we have dropped the arrows over our symbols and added a subscript "y" here. We did this because we are now dealing with components of the vectors as opposed to the vectors themselves. Evaluating these components with their angles and using the angle 133° (= 180° − 47.0°) for $\vec{F}_A$, we obtain

$$F_B \sin(-90°) = -F_A \sin 133° - F_C \sin\phi,$$

where F_A, F_B, and F_C denote vector magnitudes (not components). Using the given data for the magnitudes, yields

$$-F_B = -(220\text{ N})(\sin 133°) - (170\text{ N})\sin\phi. \tag{6-3}$$

However, we do not know ϕ.

We can find ϕ by rewriting Eq. 6-2 for the x axis as

$$F_{Bx} = -F_{Ax} - F_{Cx}$$

and then as

$$F_B \cos(-90°) = -F_A \cos 133° - F_C \cos\phi,$$

which gives us

$$0 = -(220\text{ N})(\cos 133°) - (170\text{ N})\cos\phi$$

and

$$\phi = \cos^{-1} -\frac{(220\text{ N})(\cos 133°)}{170\text{ N}} = 28.04°.$$

Inserting this into Eq. 6-3, we find

$$F_B = 241\text{ N}. \qquad \text{(Answer)}$$

6-3 Gravitational Force and Weight

Gravitational Force

As we discussed in Section 3-5, gravitational forces result from interactions between masses and can act over long distances. Although gravitational interactions between any two masses are always present, they are only noticeable when at least one of the masses is very large. We have already presented experimental evidence in Section 3-9 that the gravitational pull of the Earth on an object is directly proportional to the object's mass. We use the constant of proportionality, denoted g, to relate the gravitational force to mass from Eq. 3-9:

$$\vec{F}^{\text{grav}} = -mg\hat{\jmath}, \tag{6-4}$$

where the constant g, known as the **local gravitational strength,** is a positive scalar and $\hat{\jmath}$ is a unit vector that points up. The minus sign tells us that the gravitational force points down. Close to the Earth's surface, the value of g is 9.8 N/kg.

Weight

Weight is a commonly used synonym for the magnitude of the gravitational force acting on an object.

> The weight W of a body is a scalar quantity that equals the magnitude $|\vec{F}^{\text{grav}}|$ of the local gravitational force exerted by the Earth or some other massive astronomical object (such as the moon) on the body.

$$W = |\vec{F}^{\text{grav}}| = mg \qquad \text{(weight)} \tag{6-5}$$

To *weigh* a body means to measure its weight. As we mentioned in Section 3-9, we can measure gravitational force and hence weight, using a balance, a spring scale, or an electronic scale. Sometimes scales are marked in mass units. Since the value of g changes as we move away from the Earth, scales are only accurate for measuring mass when the value of g is the same as it is where the scale was calibrated.

Weight must be measured when the body is not accelerating vertically relative to the astronomical object attracting it. For example, you can accurately measure your weight on a scale in your bathroom or on a fast train moving horizontally. However, if you repeat the measurement with the scale in an accelerating elevator, the reading on the scale differs from your weight because of the vertical acceleration. This was first discussed in Section 2-4.

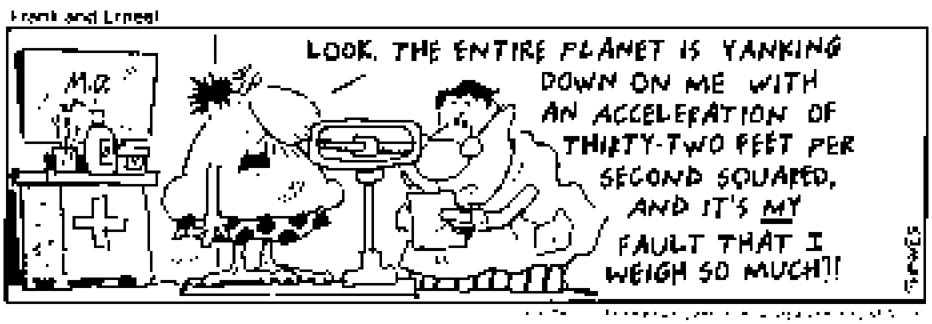
Note that the weight W, which has SI units of newtons, and the local gravitational strength g, which has SI units of newtons per kilogram, are not components of vectors, which can be positive or negative. Instead they are both magnitudes and *are always positive*.

Mass Versus Weight

Unfortunately, everyday speech sometimes leads us to believe that the terms "weight" and "mass" are interchangeable. Although the weight of a body (given by $W = mg$) is proportional to its mass, *weight and mass are not the same thing*. Mass has a standard unit of kilograms whereas weight is the magnitude of a force, with a standard unit of newtons. If you move a body to a location such as the surface of the Moon where the value of the local gravitational strength g is different, the body's mass (how much "stuff" the object is made up of) is *not* different, but its weight is. For example, the weight of a bowling ball with a mass of 7.2 kg is 71 N on Earth. On the Moon, this same bowling ball would have the same mass, but a weight of only 12 N. This is because the local gravitational strength is only about one-sixth of its value on Earth.

READING EXERCISE 6-2: Suppose you are given two different objects, a balance like the one shown in Fig. 3-9 and a spring scale like the one shown in Fig. 3-23. Describe how you could determine whether the two objects have the same mass. What might you do to determine the weight of one of the objects? Is the weight of each object the same as the mass of the object? Is the ratio of the masses the same as the ratio of the weights? ■

READING EXERCISE 6-3: Comment on the accuracy of the statement the patient is making in the *Frank & Ernest* cartoon



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6-4 Contact Forces

As we have mentioned, the gravitational force can act over large distances and exists even if the two interacting objects are not touching. Hence, we sometimes refer to the gravitational force as an "action at a distance" force. In contrast, forces such as tension and frictional forces only exist when there is contact between interacting objects. We call forces of this kind "contact" forces. In order to understand the nature of contact forces between solid objects, it is helpful to learn more about the atomic nature of solids.

An Idealized Model of a Solid

Modern scientists have strong evidence that solids in our everyday world are made of atoms. It is very hard to compress a solid object or pull it apart. The forces between atoms seem to behave like springs. When you push on a spring that is at its natural or equilibrium length, it resists compression by pushing back on you. But when you pull on a spring, it also resists stretching by pulling back on you. This has led physicists to create an idealized model for a solid as an array of atoms held together by forces that behave like very stiff springs, each having an equilibrium length of about 10^{-10} m. A three-dimensional model of a possible array of atoms in a simple solid is shown in Fig. 6-5*a*. This model is explained in more detail in Section 13-5. (As we will see in Chapter 22, the force between atoms in a solid can be understood in terms of the electromagnetic forces between the charged particles in atoms.)

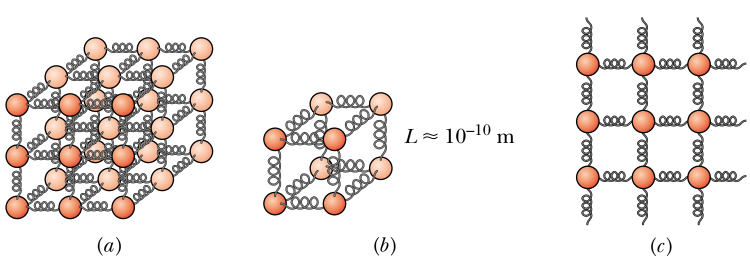


FIGURE 6-5 ■ An idealized model of a solid consisting of atoms separated by tiny springs. (*a*) A model consisting of stiff springs ("atomic bonds") holding balls ("atoms") together. (*b*) Eight atoms at the corner of a cube show the three-dimensional nature of a small hunk of the idealized solid. (*c*) A depiction of a few of the atoms that lie in the plane of the paper.

Using the Model to Understand Contact Forces

How can we use this simplified model to help us understand contact forces? Let's consider what happens when you push on an innerspring mattress (Fig. 6-6). As you push, the springs in the mattress become compressed under your finger and push back

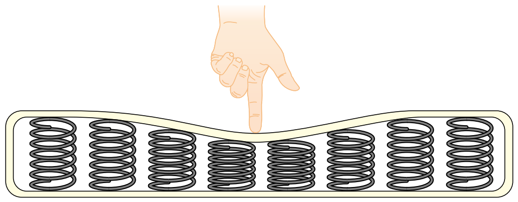


FIGURE 6-6 ■ This physical model of a solid as a matrix of atoms separated by tiny springs behaves rather like an innerspring mattress. Our "solid" is compressed just slightly by the force exerted on it by a finger. According to Newton's Third Law the "solid" then exerts an equal and opposite upward force back on the finger.

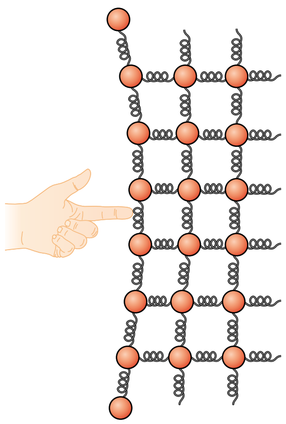


FIGURE 6-7 ■ Compressing an idealized solid wall with a force exerted by a finger. The deformation of the wall is exaggerated. The wall exerts an oppositely directed force with the same magnitude back on the finger.

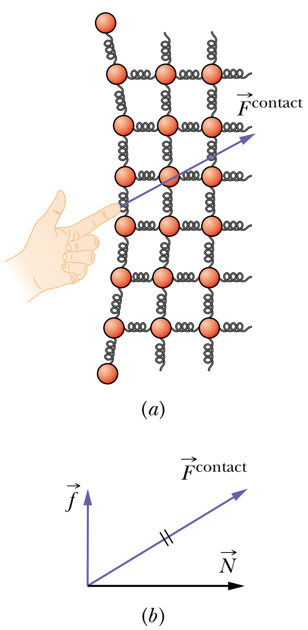


FIGURE 6-8 ■ (*a*) Compressing an idealized solid surface with a contact force that is neither purely perpendicular nor purely parallel to the surface. (*b*) This force exerted on the solid surface can be decomposed into parallel and perpendicular components.

on your finger. According to Newton's Third Law, the force you exert on the mattress springs is equal in magnitude and opposite in direction to the force the mattress springs exert on your finger.

Similarly, if you push on a wall it compresses (it is deformed, bent, or buckled ever so slightly), and it pushes back on you (Fig. 6-7). The compression of the wall is hard to see because its billions and billions of tiny atomic springs are much stiffer than the mattress springs. But the harder you push, the more compressed the wall becomes and the larger the force the wall exerts on your finger. When you push harder, it hurts, because in accord with Newton's Third Law, the surface is also pushing back on your finger with a larger force. You can feel (and see) your finger becoming more and more compressed due to the force exerted on it by the wall. Ouch! Try it!

We call a force exerted perpendicular to a surface a **normal force** and denote it as $\vec{N}$. Note that in this context *normal* is a technical term that derives from a Latin term *norma* meaning "carpenter's square." It is a synonym for perpendicular and does not mean "ordinary."

When one object, such as your finger, exerts a contact force on the surface of another object, such as a wall, the force is not necessarily perpendicular to the object's surface. However, you can decompose the force vector into a parallel component and a perpendicular component as shown in Fig. 6-8*b*. We call the component vector perpendicular to the surface the **normal force.** We call the component vector parallel to the surface the **friction force** and denote it as $\vec{f}$.

In mathematical terms the decomposition of the contact force, $\vec{F}^{\,\text{contact}}$, is given by the sum of the two perpendicular force vectors,

$$\vec{F}^{\,\text{contact}} = \vec{N} + \vec{f}. \qquad (6\text{-}6)$$

The Normal Force

Let's consider a couple of situations in which normal forces are exerted on stationary blocks as shown in Fig. 6-9. The normal force exerted on one object by another object is always directed perpendicular to the surfaces that are in contact and away from the surface of the object exerting the force. We can use our idealized atomic model to explain this. The atoms at the surfaces of the objects that are in close contact interact so as to oppose being pushed closer together. As a result of Newton's Third Law, we can see that:

> When one body exerts a force with a component that is perpendicular to the surface of another body, the other body (even one with a seemingly rigid surface) deforms and pushes back on the first body with an opposing normal force $\vec{N}$ that is also perpendicular to the surfaces that are in contact.

1. A Vertical Wall: A block that is pushed against a wall experiences a normal force from the wall. An example of this is shown in Fig. 6-9*a*. Since the block is not moving,

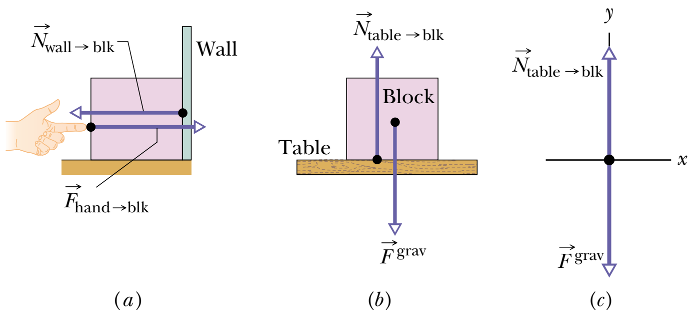


FIGURE 6-9 ■ (*a*) A hand pushes a block into a wall with a force $\vec{F}_{hand \to blk}$. Since the block can't move, it compresses the wall, which pushes back on it with a normal force $\vec{N}_{wall \to blk}$. (*b*) A block resting on a tabletop experiences a normal force $\vec{N}$ perpendicular to the tabletop. (*c*) The corresponding free-body diagram for the block.

the net force on it must be zero. For now we will just consider the horizontal forces on the block given by

$$\vec{F}_{hand \to blk} - \vec{N}_{wall \to blk} = 0 \quad \text{or} \quad \vec{N}_{wall \to blk} = -\vec{F}_{hand \to blk} \qquad \text{(special case 1).} \qquad (6\text{-}7)$$

2. A Horizontal Table: Likewise, any object that rests on a table, shelf, or the ground near the Earth's surface experiences a normal force. Figure 6-9*b* shows an example. A block of mass m lies on a table's horizontal surface. It is not moving in spite of the fact that it has a gravitational force $\vec{F}^{grav}$ on it due to the Earth. In other words, the block should fall but the table is in the way! We must conclude that if the block does not accelerate, the net force on the block must be zero,

$$\vec{F}^{net} = \vec{F}^{grav} + \vec{N}_{table \to blk} = 0 \quad \text{or} \quad \vec{N}_{table \to blk} = -\vec{F}^{grav}_{blk} \qquad \text{(special case 2).}$$

So the table must be pushing up on the block with normal force $\vec{N}_{table \to blk}$ that is equal to $-\vec{F}^{grav}_{blk}$. A free-body diagram for the block is shown in Fig. 6-9*c*. Forces $\vec{F}^{grav}$ and $\vec{N}_{table \to blk}$ are the only two forces on the block, and they are both vertical. We can write Newton's Second Law in terms of components along a positive upward y axis.

The component, F_y^{grav}, of the gravitational force is $-mg$. Whenever there is no vertical acceleration, the magnitude of the normal force of an object resting on a horizontal surface is mg. Since its direction is up, if we use the coordinates shown in Fig. 6-9*c*, then

$$\vec{N}_{table \to blk} = +mg\,\hat{j} \qquad \text{(special case 2).} \qquad (6\text{-}8)$$

Single Normal Force as an Idealization: The normal force exerted by the surface of the table on the block is actually the sum of billions of contact interactions between surface atoms in the table and block. However, the use of a single force vector to summarize external forces that act in the same direction as shown in Fig. 6-9 is a useful simplification. It is conventional to draw a single upward arrow at the point where the middle of the bottom surface of the block touches the table, as shown in Fig. 6-10.

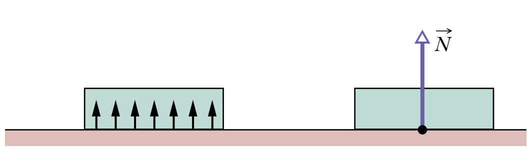


FIGURE 6-10 ■ For simplification, many small force vectors supporting the bottom of the block are replaced by a single large force vector acting through the center of the block.

Normal Force in an Elevator: Suppose a block is placed in an elevator that is accelerating in an upward direction. How would that change the normal force it experiences? In Chapter 2, we discussed how a person riding in such an elevator would feel heavy while accelerating upward and feel light while accelerating downward (see Fig. 2-12). This brings us to the idea of *apparent weight*. A common bathroom scale reading is a measurement of the normal force exerted by the scale on your feet. In normal usage of the scale (in other words, you are standing still on the scale in a space that is not accelerating vertically), the scale will measure your weight. This is because the scale reading (normal force from the scale on your feet) is related to your weight through Newton's $F_y^{net} = ma_y$ relation.

READING EXERCISE 6-4: In Figure 6-9b, is the magnitude of the normal force $\vec{N}$ greater than, less than, or equal to mg if the body and table are in an elevator that is moving upward (a) at constant speed, (b) at increasing speed, and (c) at decreasing speed? ■

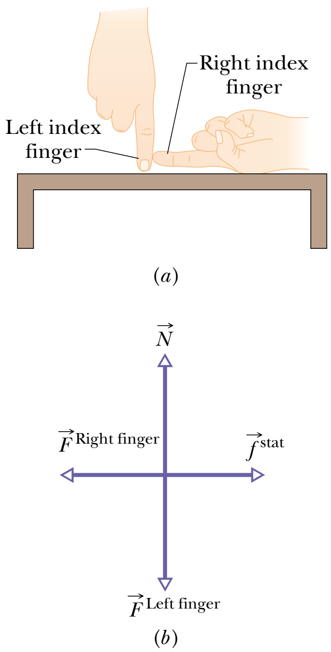


FIGURE 6-11 ■ (*a*) Applying a horizontal force toward the left to an object (such as a fingertip) that is in contact with a surface. (*b*) A free-body diagram showing the forces on the left fingertip. If the object is not accelerating, there must be a friction force on it toward the right that is equal in magnitude and opposite in direction to the force applied on the left fingertip by the right index finger.

The Friction Force Component

Let's consider the friction component of a general contact force. As we discussed earlier, this is the component of the contact force that is parallel to the surface. Suppose the tip of your finger is the object of interest and you would like to study the friction component of the contact force that a fairly smooth table can exert on your fingertip and how it might be related to the normal force the table exerts on it. Tilt your left finger so it is vertical (is at an angle of about 90° with the horizontal). Try the following activities while maintaining the 90° angle with respect to the surface of the table:

Activity 1: Press on the table with your left index finger, first with a small force and then a larger force, and feel the increase in the normal force the table exerts on your fingertip.

Activity 2: Now take the index finger of your right hand and apply enough horizontal force to your left index finger so that it glides along at a constant velocity. (See Figure 6-11.) Is there a horizontal friction force acting? If so, why?

NOTE: In order to make your fingertip slide across the table at constant velocity, you must continually push on it in a horizontal direction. Can your applied force be the only horizontal force on your fingertip? No, because if it were, then your fingertip would accelerate. Thus, if we are not willing to give up on Newton's Second Law, we must assume that there is a second force, directed opposite to the applied force but with the same magnitude, so that the two forces balance out. This idea that a second force exists is represented in both Fig. 6-11 and by the following x-component equations:

$$F_x^{net} = F_x^{r\,finger} + f_x^{stat} = ma_x = 0 \quad \text{so} \quad F_x^{r\,finger} = -f_x^{stat}.$$

Since both forces are purely horizontal, this gives us

$$\vec{F}^{r\,finger} = -\vec{f}^{stat}. \tag{6-9}$$

Activity 3: What happens to the friction force when the normal force on your fingertip increases? Once again, adjust your constant applied force so that your left fingertip is moving at a constant velocity. Next, increase the normal force on your left fingertip just enough so your fingertip stops moving. Then get your left fingertip moving at a constant velocity again by applying more horizontal force with your right finger.

If you do Activity 3 carefully, you should conclude that the friction force on an object opposes the direction of its slipping over the surface and that it is greater when the normal force on the object becomes larger.

Contact friction forces are unavoidable in our daily lives. They are literally everywhere. If we were not able to counteract them, they would stop every moving object and bring to a halt every rotating shaft. On the other hand, if friction were totally absent, we could not walk, travel in a car, or ride a bicycle. In some cases, the effects of friction are very small compared to other forces and can be ignored. In other cases, to simplify a situation, friction is assumed to be negligible even though it may not really be. In either case, if the intention is to ignore the effects of friction, the interface between the object and the surface is called *frictionless.*

Contact friction depends on many factors, and it turns out that friction forces can behave very differently depending on the normal force, the nature of the surfaces that are in contact, and other factors. It is not always obvious when looking at surfaces whether the friction forces will be large or small. Sometimes smooth surfaces have greater friction forces than rough ones.

Understanding the relationship between friction forces and atomic and molecular interactions is a very active field of research in both physics and the engineering sciences. These relationships are not completely understood. Unlike Newton's laws of motion, which scientists believe hold to a high degree of accuracy when applied to everyday objects in our surroundings, some of the characteristics of friction that we describe here are only valid for certain common types of interacting surfaces. *Thus, the friction equations that we present here are sometimes useful approximations, but they do not always apply.*

In the next two subsections, we will explore some common characteristics of *kinetic friction*, in which one surface moves relative to another, and of *static friction*, in which the surfaces in contact are stationary relative to one another.

Kinetic Friction Forces

Imagine that you give a book a quick push and send it sliding across a long horizontal countertop. As you expect, the book slows and then stops. We showed data on this behavior in Section 3-2. What does this observation tell us about the nature of the interaction between the book and the countertop? Based on our definition of velocity and on the data shown in Fig. 3-3, we suspect that the book has a *constant* acceleration. This acceleration is parallel to the surface, and in the direction opposite the book's velocity. Once again, we have no reason to believe that Newton's Second Law is not valid in this situation. Hence, from $F_x^{\text{net}} = ma_x$, we must assume that a contact friction force that is constant acts on the book in the same direction as the acceleration (parallel to the counter surface, in the direction opposite the book's velocity relative to the table) as is shown in Fig. 6-12.

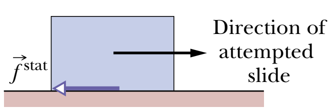


FIGURE 6-12 ■ A friction force $\vec{f}^{\text{kin}}$ opposes the slide of a body over a surface.

In both the example of keeping your fingertip moving at a constant velocity and the example of watching a book with an initial velocity slide to a stop with a constant acceleration, an object is experiencing a **kinetic friction force** $\vec{f}^{\text{kin}}$. The word "kinetic" indicates that the object is moving relative to a surface. The phenomenon of "contact friction" can be explained by assuming that there is an attractive force between the atoms at the surfaces of the two objects. The attraction between two very smooth surfaces such as glass panes is consistent with this assumption and is known as **adhesion.**

What might the kinetic friction force depend on? Imagine sending an object sliding across a countertop as we discussed above. Would the book slow down more or less quickly if we slide the book across a carpeted floor instead of the smooth countertop? Would it slow down more or less quickly if we slide it across ice instead? Does the rate at which an object slows down seem to depend on its velocity? Would the book slow down more or less quickly if it has more mass or an additional applied downward force on it so the normal force between the surfaces is larger?

We can answer some of these questions for several situations by looking at the graph presented in Fig. 3-3. This graph shows the velocity as a function of time for three situations where objects slide to a stop on surfaces. You will likely find it helpful to refer back to that figure now (page [redacted]). We can also draw inferences from the fingertip motions earlier in this section. Here are some observations and conclusions about kinetic friction.

KINETIC FRICTION—SOME OBSERVATIONS AND CONCLUSIONS

OBSERVATION 1 ON THE INFLUENCE OF THE RELATIVE VELOCITY BETWEEN SURFACES: The graphs in Fig. 3-3 tell us that in three situations involving different combinations of objects and surfaces, the objects all slow to a stop with constant acceleration and hence experience a constant kinetic friction force. *Conclusion: Kinetic friction forces appear to be independent of the magnitude of the velocity of the object relative to the surface over which the object is sliding, but act in a direction opposite to the direction of the velocity.*

OBSERVATION 2 ON THE NATURE OF THE SLIDING SURFACES: The graphs in Fig. 3-3 tell us that in three situations involving different combinations of object and surfaces, the rate of the stopping acceleration is different. *Conclusion: Kinetic friction forces appear to depend on the nature of the surfaces that are in contact with one another.*

OBSERVATION 3 ON THE INFLUENCE OF THE NORMAL FORCE: When you completed Activity 3 earlier in this section, you observed that the applied force needed to keep your fingertip moving at a constant velocity increases when the normal force on your fingertip becomes larger. *Conclusion: Kinetic friction forces appear to increase when the normal force on a sliding object increases and thus depend on how hard the objects are being pushed together.*

Is there a mathematical relationship between the magnitude of the kinetic friction force on an object and the magnitude of the normal force the object experiences? A plausible relationship would be that these two force magnitudes are proportional to each other. Let's look at the results of a simple experiment in which we can measure the kinetic friction force as a function of the normal force on a sliding block. In this experiment, we use a spring scale to measure how much horizontal force we need apply to pull a wooden block along at a constant velocity. (See Fig. 6-13.) We can determine the magnitude of the friction force by using the fact that it must be equal to the magnitude of the applied force if the moving block doesn't accelerate (Eq. 6-9). If the table surface is horizontal, and the only other vertical force on the block is the gravitational force, then the normal force is given by $\vec{N} = mg\hat{j}$ (Eq. 6-8). That is, $N_y = mg$. The normal force can be changed by piling more mass on the block. We can then measure the kinetic friction force again.

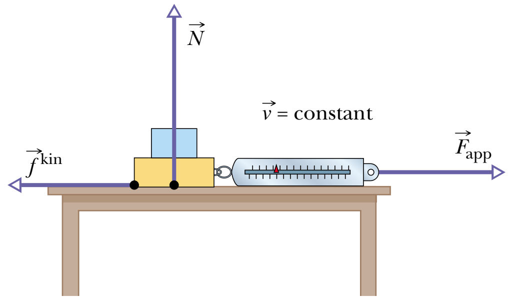


FIGURE 6-13 ■ A block is pulled along at a constant velocity with a horizontal applied force, measured by a spring scale. This force is countered by a kinetic friction force of the same magnitude. Since the tabletop is horizontal, the magnitude of the normal force on the block is equal to the product of its mass m and the gravitational acceleration constant g.

The data shown in Fig. 6-14 reveal that for a Velcro-covered wood block sliding on a Formica table surface, the magnitude of the friction force is proportional to that of the normal force with a constant of proportionality given by $\mu^{kin} = 0.21$. Turning the block on its side to reduce the area in contact does not affect this constant of proportionality.

Results similar to those shown in Fig. 6-14 for many situations reveal that the magnitude of the friction force for dry sliding is usually proportional to the magnitude of the normal forces pressing surfaces together and does not depend on other factors. Thus, for the purposes of the systems we will deal with in this book, the magnitude of the kinetic friction force, $\vec{f}^{kin}$, can be expressed as

$$f^{kin} = \mu^{kin} N, \tag{6-10}$$

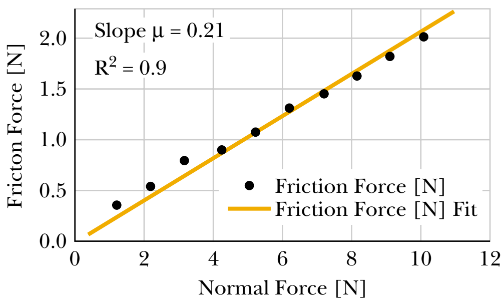


FIGURE 6-14 ■ A graph of data showing that when a block is pulled along at a constant velocity, the magnitude of the kinetic friction force is directly proportional to the normal force exerted by the surface it slides over.

where μ^{kin} is the slope of the linear graph that relates the magnitude of the kinetic friction force f^{kin} and the magnitude of the normal force N. The slope μ^{kin} is called the **coefficient of kinetic friction.**

The coefficient μ^{kin} is a dimensionless scalar that must be determined experimentally. Its value depends on certain properties of both the body and the surface. Hence, the coefficients are usually referred to with the preposition "between," as in "the value of μ^{kin} *between* a book and countertop is 0.04, but the value *between* rock-climbing shoes and rock is as much as 0.9." Based on our observations, we assume that the value of μ^{kin} does not depend on the speed at which the body slides along the surface. Note that $f^{kin} = \mu^{kin}N$ is *not* a vector equation. The direction of $\vec{f}^{\,kin}$ is always parallel to the surface and *opposes* the sliding motion.

Static Friction Forces

Do friction forces continue to act on an object once it stops sliding? The answer to this question is more complicated than simply "yes" or "no." Start out by imagining a large, heavy box sitting on a horizontal, carpeted floor. You push on the box, but the box does not move. Unless we are to believe that Newton's Second Law ($\vec{F}^{\,net} = m\vec{a}$) is not valid in this situation, we must assume that there is some other force acting on the box that is counteracting the application of the push force. That is, there must be a force acting in the opposite direction that is exactly equal in magnitude to the push force. We will call this opposing force a **static friction force.** The word "static" is used to signify that the object is not moving relative to the surface as shown in Fig. 6-15*b-d*.

Now imagine that you push even harder on the box as shown in Fig. 6-15*c* and *d*. The box still does not move. Apparently the friction force can change in magnitude, otherwise it would no longer balance your applied force. In other words, if you push on an object in an attempt to slide it across a surface and the object does not slide, then we know that there is a static friction force. This force acts in the direction opposite the push with the exact same magnitude, regardless of how hard you push. If you stop pushing on the box, that oppositely directed force must disappear. How do we know this? Because if you removed the push force, and the static friction force *did not* disappear as well, then the box would accelerate in the direction of the friction force. We know from everyday observation that this does not happen. So the static friction force appears to be a very strange force that changes magnitude in response to other forces.

This situation is in no way specific to the example of the box on carpet. At the interface between any two solids prior to slipping, the static friction force starts at zero when no applied force is present and increases as the force that tends to produce slipping increases. The static friction force adjusts in magnitude to exactly counteract the applied force (usually a push or pull) at every instant. The static friction force mirrors the applied force. If the applied force is zero, then the static friction force is zero. If the applied force has a horizontal component that is 10 N, the static friction force has a horizontal component that is 10 N. We call forces that behave like the static friction force **passive forces.** Passive forces are forces that change in magnitude in response to other forces.

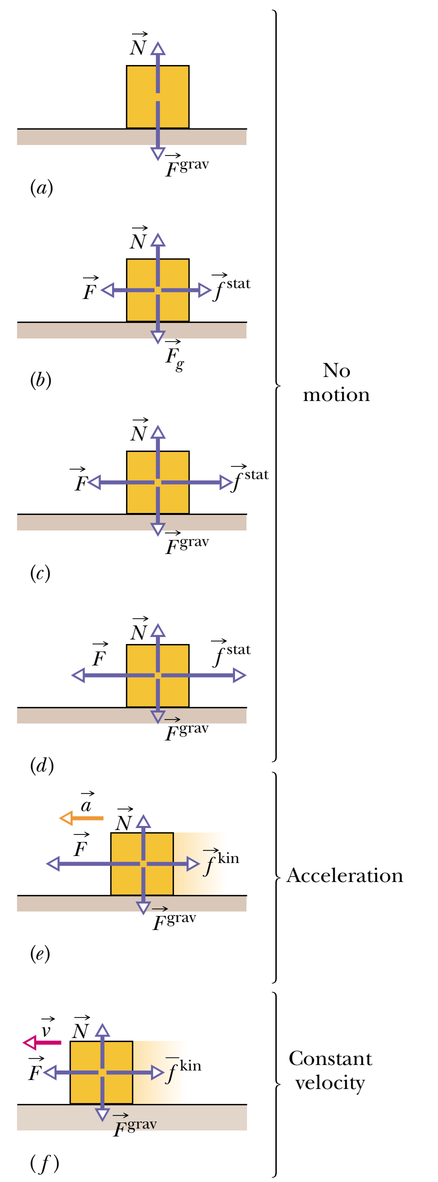


FIGURE 6-15 ■ (*a*) There are no horizontal forces on a stationary block. (*b–d*) An external force $\vec{F}$ applied to the block is balanced by a static friction force $\vec{f}^{\,stat}$. As $\vec{F}$ is increased, $\vec{f}^{\,stat}$ also increases, until $\vec{f}^{\,stat}$ reaches a certain maximum value. (*e*) The block then "breaks away," accelerating suddenly in the direction of $\vec{F}$. (*f*) If the block is now to move with constant velocity, the magnitude $|\vec{F}|$ of the applied force must be reduced from the maximum value it had just before the block broke away.

Now imagine that you push on the box with all your strength. Finally, the box begins to slide. Evidently, there is a maximum magnitude of the static friction force. When you exceed that maximum magnitude, your push force is larger than the opposing static friction force and the box accelerates in the direction of your push. A typical sequence of static frictional force responses to applied forces is shown in Fig. 6-15. This sequence is consistent with the experimental results obtained when an electronic force sensor is used to monitor the force on a block as a function of time. The experimental setup is shown in Fig. 6-16 and a graph of the results is shown in Fig. 6-17.

FIGURE 6-16 ■ The apparatus used for the static friction experiment includes an electronic force sensor attached to a computer data acquisition system (not pictured).

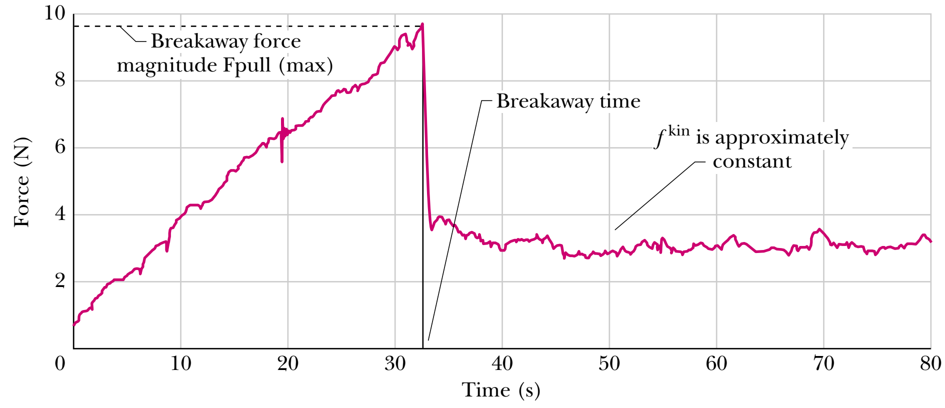


FIGURE 6-17 ■ Graph of the magnitude of the static friction force on a wooden block as a function of time. This force opposes a steadily increasing applied force between 0.0 s and 32 s. At 32 s the block suddenly "breaks away" and starts moving. At about 40 s, it starts moving at a steady velocity as a kinetic friction force with a magnitude that is less than the static force starts acting.

As the pulling force, $\vec{F}^{\text{pull}}$, increases, the block remains at rest. Then, when a "breakaway" force is reached, it moves very suddenly. That is, the magnitude of the friction force, $\vec{f}^{\text{ stat}}$, keeps increasing to oppose the pulling force in accordance with Newton's Second Law until the object "breaks free" and starts to move. hence, we express the magnitude of the static friction vector as

$$f^{\text{stat}} = |\vec{f}^{\text{stat}}| \leq \mu^{\text{stat}} N, \tag{6-11}$$

where μ^{stat} is known as the **coefficient of static friction** and N is the magnitude of the normal force on the body from the surface. Just as for kinetic friction, the coefficient μ^{stat} is dimensionless and determined experimentally. Its value depends on certain properties of both the body and the surface, and so is referred to with the preposition "between."

Usually, the magnitude of the kinetic friction force, which acts when there is motion, is less than the maximum magnitude of the static friction force, which acts when there is no motion. We see this in the data shown in Fig. 6-17. Thus, if you wish the block to move across the surface with a constant speed, you must usually decrease the magnitude of the applied force once the block begins to move, as in Fig. 6-15*f*. Another common behavior for a certain range of applied forces is to see slip-and-stick behavior in which an object breaks away, slides to a stop, breaks away again, and so on. We will not deal with the slip-stick phenomenon in this book.

READING EXERCISE 6-5: Figure 6-17 shows the result of the experiment in which a 295.6 g block with a 500 g mass on it is pulled along a table with a steadily increasing force until it breaks away at $t = 32$ s. (a) What is the coefficient of static friction, μ^{stat}, between the table and the mass? (b) What is the coefficient of kinetic friction μ^{kin}? ■

READING EXERCISE 6-6: A block lies on a floor. (a) What is the magnitude of the friction force exerted on it by the floor if the block is not being pushed? (b) If a horizontal force of 5 N is now applied to the block, but the block does not move, what is the magnitude of the friction force on it? (c) If the maximum value $\vec{f}^{\text{max}}$ of the static friction force on the block is 10 N, will the block move if the magnitude of the horizontally applied force is 8 N? (d) If the magnitude is 12 N? (e) What is the magnitude of the friction force in part (c)? ■

READING EXERCISE 6-7: Discuss and explain the following statement using the terms related to friction forces that are presented above: "If we were not able to counteract them, they would stop every moving object and bring to a halt every rotating shaft. On the other hand, if friction were totally absent, we could not walk or ride a bicycle." ■

Tension

So far we considered contact forces between objects that are not attached and that can be pulled apart fairly easily. Let's consider one more type of contact force—a force that occurs when a long thin object such as a rod or string is attached to other objects at each of its ends. For example, consider a leash with a dog straining at one end and the dog's owner pulling the other end, a handle bolted to a pot that is too massive to move and is being pulled by a cook, or a string with one end attached to a ceiling and the other end attached to a hanging mass. In all three cases, a long narrow object that is stretched is transmitting forces from an object at one of its ends to an object at its other end. We say that a long narrow object that is being pulled taut by opposing forces is under **tension.** In order to use Newton's laws of motion to analyze the forces and motions of the objects that are attached to the ends of strings or rods, we need to understand more about the phenomenon of tension.

What do we observe about tension? Let's consider a stationary rubber band that connects two force probes like that shown in Fig. 6-18. We observe that the forces the rubber band exerts on the force probes at each end have the same magnitude but act in opposite directions.

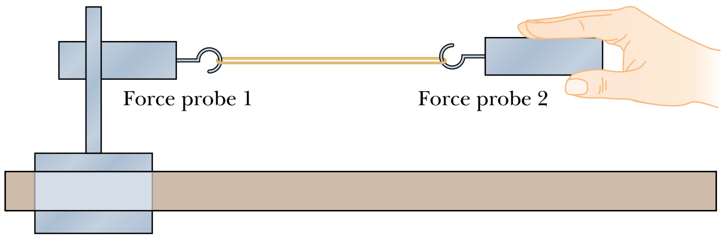


FIGURE 6-18 ■ A rubber band is connecting two force probes. Each probe detects the same magnitude of force, but the force on probe 1 is in the opposite direction of the force on probe 2. This observation is not surprising as it is entirely consistent with Newton's laws.

We also observe that the tension force is present everywhere along the rubber band. Although it is not readily observable, the tension everywhere along the rubber band is in fact equal in magnitude to the applied forces at the ends that caused the rubber band or string to stretch. Thus, when a taut rubber band (that is not accelerating) is attached to an object, it exerts a tension force on the object that is directed along the rubber band and away from the object. This tension expresses itself as a pulling force, but only at the ends of the rubber band. These same observations hold for most long thin connectors including strings, cords, and ropes.

An Atomic Model for Tension Our simple model of solid matter, as consisting of atoms connected by springs, is very helpful in understanding how objects that are under tension can transmit forces. Suppose a very, very thin string, having only one strand of atoms, is connected by small interatomic springs. Figure 6-19*a* shows the natural length of the string. Figure 6-19*b* shows the string when it is extended by equal and opposite forces applied to its ends so that it is not accelerating.

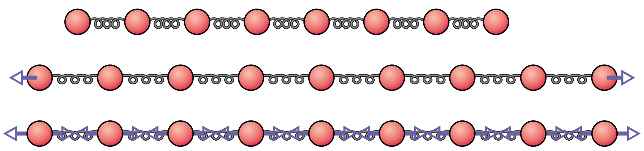


FIGURE 6-19 ■ A string is idealized as a line of atoms with springs representing the mutual interaction forces between them.

In our idealization, we have assumed only one strand of atoms. Obviously, real strings, cords, and ropes have many strands of molecules consisting of complex arrays of atoms. Although many strands will make a string or rope stronger, it will not change the ideas presented in our simple model.

Assuming that our ideal string is not accelerating, each atom must have zero net force on it. The atom on the left end of the string must be experiencing an attractive force from the neighboring atom to its right that is equal in magnitude and opposite in direction to the applied force on the end. However, each stretched spring represents a force of interaction between neighboring atoms that must obey Newton's Third Law. Thus, the leftmost atom must be exerting an attractive force on its neighboring atom that is "equal and opposite" to the force that atom exerts on it. These pairs of mutual interaction forces exist throughout the string, as shown in Fig. 6-19*c*. The magnitude of these interaction forces each atom experiences has been given a special name. It is called the *tension in the string*, which we denote as T. In contrast to the tension force, tension, which we often denote with a T, is a scalar quantity that is always positive with no inherent direction associated with it. Hence, we will often denote a tension force $\vec{T}$ that points (for example) in the positive y direction as $\vec{T} = +T\hat{\text{j}}$ and one that points in a negative direction as $\vec{T} = -T\hat{\text{j}}$.

Next, let's use Newton's laws to examine the effect of tension associated with the motion of a skier being towed by a snowmobile by means of a nylon cord as depicted in Fig. 6-20. We consider two situations—one in which the system is not accelerating and the other in which it is. The snowmobile moves forward when its treads, which are turning, dig into the snow and push against it. However, assume for now that the runners on the skis and those at the front of the snowmobile experience no friction forces.

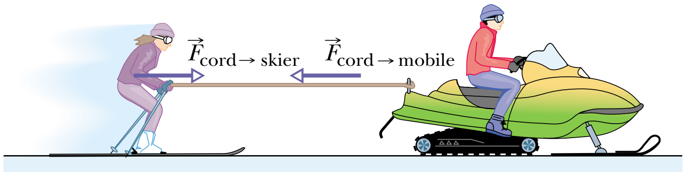


FIGURE 6-20 ■ The cord connecting a skier and a snowmobile exerts oppositely directed forces on the skier and the snowmobile.

Tension for a Nonaccelerating System Remember that if the system is not accelerating, then it moves at a constant velocity. Furthermore, Newton's Third Law tells us that the force between any two objects in the system that are in contact is equal and opposite. For example, at the left end of the cord, the skier feels a pulling force acting along the direction of the cord, which we denote as $\vec{F}_{\text{cord}\rightarrow\text{skier}}$, and the cord experiences an oppositely directed force from the skier, denoted by $\vec{F}_{\text{skier}\rightarrow\text{cord}}$. A similar situation applies to the interaction forces at the right end of the cord, so that

$$\vec{F}_{\text{cord}\rightarrow\text{skier}} = -\vec{F}_{\text{skier}\rightarrow\text{cord}} \quad \text{and} \quad \vec{F}_{\text{cord}\rightarrow\text{mobile}} = -\vec{F}_{\text{mobile}\rightarrow\text{cord}}. \tag{6-12}$$

But we already know from Newton's Second Law for $\vec{a} = 0$ that the net force on the cord must be zero. Since the net force is zero, F_x^{net} (the sum of the x-components of the forces) must be zero. Hence,

$$F^{net}_{cord\,x} = F_{skier \to cord\,x} + F_{mobile \to cord\,x} = 0 \quad \text{or} \quad F_{skier \to cord\,x} = -F_{mobile \to cord\,x}.$$

The forces the skier and snowmobile exert on the cord are purely horizontal. So we have

$$\vec{F}_{skier \to cord} = -\vec{F}_{mobile \to cord}. \tag{6-13}$$

This result agrees with the observation reported in Fig. 6-18. Namely, forces exerted by the ends of a taut cord have the same magnitude. Even more significantly, we can combine Eqs. 6-12 and 6-13 to show that

$$\vec{F}_{skier \to mobile} = -\vec{F}_{mobile \to skier}.$$

When nonaccelerating objects are connected by a string, cord, or rope, they interact in accordance with Newton's Third Law *by means of the connector* as if they are in direct contact.

An Accelerating System Suppose the snowmobile driver pushes in his throttle and increases his velocity at a constant rate. Now the system has an acceleration $\vec{a}$, and the cord connecting the skier to the snowmobile must experience the same acceleration. In terms of the x-components we get

$$F^{net}_{cord\,x} = F_{skier \to cord\,x} + F_{mobile \to cord\,x} = m_{cord}\, a_x. \tag{6-14}$$

Equation 6-14 tells us that if the cord has a nonzero mass, then the force of the snowmobile on the right end of the cord must be greater than the force on the left end to maintain an acceleration. However, in many situations, including this one showing the snowmobile pulling a skier, the mass of the cord is much less than the mass of the entire system.

Taking the direction of motion to be along the positive x axis, we can write the tension forces in terms of the positive scalar T representing the tension,

$$\vec{F}_{skier \to cord} = -T_L\hat{\text{i}} \quad \text{and} \quad \vec{F}_{mobile \to cord} = +T_R\hat{\text{i}}, \tag{6-15}$$

where T_L is the tension on the left side of the cord and T_R is the tension on the right side of the cord. Then we can rewrite Eq. 6-14 in terms of the tension difference and the x-component of acceleration to get $T_R - T_L = m_{cord}\, a_x$. But the snowmobile force, which serves to accelerate the entire system, is given by $F_{mobile \to sys\,x} = m^{tot} a_x$. Solving the last two equations for a_x and rearranging terms gives us the ratio

$$\frac{T_L - T_R}{F_{mobile \to sys\,x}} = \frac{m_{cord}}{m^{tot}}. \tag{6-16}$$

Let's consider the implications of this equation. In most situations, the mass of the cord is much less than the mass of the system. Whenever that is true, the difference in tension at the ends of the cord is much smaller than the force that accelerates the system. For example, assume the skier's mass and the snowmobile's mass together total 200 kg, and the mass of the cord she grips is 1 kg. The ratio of these masses gives us only a 0.5% difference in tension forces at the ends of the cord. For most everyday purposes, this force difference at the ends of an accelerating cord is negligible. In laboratory experiments, masses of between 100 g and 2 kg are typically connected by

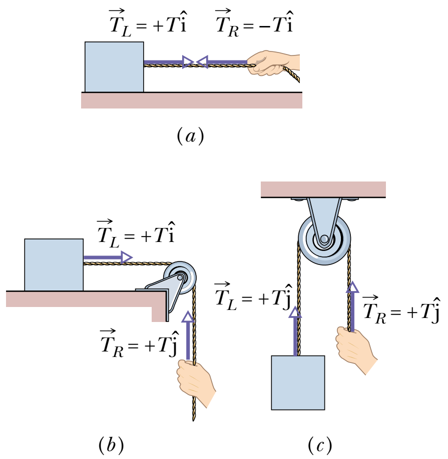


FIGURE 6-21 ■ (*a*) The cord, pulled taut, is under tension. If its mass is negligible, it pulls on the body and the hand with force of magnitude $T = |\vec{T}|$, even if it runs around a massless, frictionless pulley as in (*b*) and (*c*).

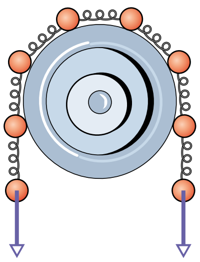


FIGURE 6-22 ■ Tension in a taut string still exists even when it undergoes direction changes.

fishing line capable of sustaining tensions of well over 100 N. A 1 m length of this type of fishing line has a mass of about 0.25 g, so the force differences are usually less than 1%. For cases where the mass of a connecting cord is very small compared to the masses of the objects attached to its ends, we can assume that the tension is *essentially* the same at all points along the cord. When we can legitimately make this simplifying approximation, we say that we have a **massless string.**

Pulleys and Direction Change What happens if a "massless" cord stretched over a "masstess" pulley changes direction as shown in Fig. 6-21*b* and *c*? Is the tension still the same everywhere in the cord? Let's examine our atomic model. If a string is wrapped around a pulley, its direction is different at one end than at the other. However, each tiny segment of the string only changes direction ever so slightly. The direction change is less than it is in Fig. 6-22 where we have only placed eight atoms in the chain. We conclude that the magnitude of the tension forces that are spread throughout the string also do not change significantly when the string bends around other objects. This conclusion is supported by experiments in which spring scales are inserted in various places along a string that bends while it is under tension.

Any solid object that is attached at two ends and pulled can transmit tension forces from one end to another. Some objects are quite elastic, such as rubber bands or weak springs, others are more rigid, such as strings and rods. Small rubber bands, light-duty springs, and strings cannot stand compressive forces. They are so long and narrow that they buckle under compression. Alternatively, rods and heavy springs do not buckle under compression. In some of the analyses that follow, you will be dealing with "massless" strings and springs that buckle under compression forces.

READING EXERCISE 6-8: Consider Figure 6-21*c* and assume that the pulley is massless but the cord is *not.* Is the magnitude of the pull force on the cord exerted by the hand equal to (=), less than (<), or greater than (>) the magnitude of the pull force exerted by the block when the block is moving upward (a) at constant speed, (b) at increasing speed, and (c) at decreasing speed? Explain. ■

TOUCHSTONE EXAMPLE 6-2: Einstein's Elevator

In Fig. 6-23*a*, a passenger of mass $m = 72.2$ kg stands on a platform scale in an elevator cab. We are concerned with the scale readings when the cab is stationary and when it is moving up or down.

(a) Find a general solution for the scale reading, whatever the vertical motion of the cab happens to be.

SOLUTION ■ One **Key Idea** here is that the scale reading is equal to the magnitude of the normal force $\vec{N}$ the scales exert on the passenger. The only other force acting on the passenger is the gravitational force $\vec{F}^{\text{grav}}$, as shown in the free-body diagram of the passenger in Fig. 6-23*b*.

A second **Key Idea** is that we can relate the forces on the passenger to the acceleration $\vec{a}$ of the passenger with Newton's Second Law ($\vec{F}^{\text{net}} = m\vec{a}$). However, recall that we can use this law only in an inertial frame. If the cab accelerates, then it is *not* an inertial frame. So we choose the ground to be our inertial frame and make any measure of the passenger's acceleration relative to it.

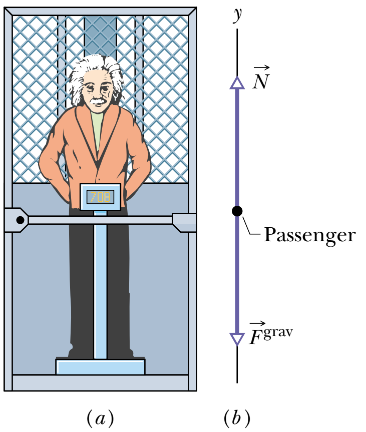


FIGURE 6-23 ■ (*a*) A passenger stands on a platform scale that indicates his weight or apparent weight. (*b*) The free-body diagram for the passenger, showing the normal force $\vec{N}$ on him from the scale and the gravitational force $\vec{F}^{\text{grav}}$.

Because the two forces on the passenger and the passenger's acceleration are all directed vertically, along the *y* axis shown in Fig.

6-23*b*, we can use Newton's Second Law written for *y*-components ($F_y^{\text{net}} = ma_y$) to get

$$N_y + F_y^{\text{grav}} = ma_y$$

or

$$N_y = -F_y^{\text{grav}} + ma_y. \tag{6-17}$$

This tells us that the scale reading, which is equal to N_y (provided $N_y \geq 0$), depends on the vertical acceleration a_y of the cab. Since $\vec{F}^{\text{grav}} = -mg\,\hat{j}$, the *x*-component of the gravitational force, $F_y^{\text{grav}} = -mg$. This gives us

$$N_y = m(g + a_y). \quad \text{(Answer)} \tag{6-18}$$

This tells us that the scale reading is larger than the passenger's static weight, *mg*, when the elevator accelerates upward, since then $a_y > 0$. But if the elevator is accelerating *downward*, then a_y is negative and the scale reads less than the passenger's static weight. This is true, as long as the downward acceleration is smaller than *g*. If the downward acceleration is greater than *g*, $(g + a_y)$ in Eq. 6-18 is a negative value. In that case, $N_y = 0$, since N_y can never be negative. (Why not?)

(b) What does the scale read if the cab is stationary or moving upward at a constant 0.50 m/s?

SOLUTION ■ The **Key Idea** here is that for any constant velocity (zero or otherwise), the acceleration a_y of the passenger is zero. Substituting this and other known values into Eq. 6-18, we find

$$N_y = (72.2\ \text{kg})(9.8\ \text{m/s}^2 + 0) = 708\ \text{N}. \quad \text{(Answer)}$$

This is just the weight of the passenger and is equal to the magnitude F^{grav} of the gravitational force on him.

(c) What does the scale read if the cab accelerates upward at 3.20 m/s² and downward at 3.20 m/s²?

SOLUTION ■ For $a_y = +3.20\ \text{m/s}^2$, Eq. 6-18 gives

$$N_y = (72.2\ \text{kg})(9.8\ \text{m/s}^2 + 3.20\ \text{m/s}^2)$$
$$= 939\ \text{N}, \quad \text{(Answer)}$$

and for $a_y = -3.20\ \text{m/s}^2$, it gives

$$N_y = (72.2\ \text{kg})(9.8\ \text{m/s}^2 - 3.20\ \text{m/s}^2)$$
$$= 477\ \text{N}. \quad \text{(Answer)}$$

So for an upward acceleration (either the cab's upward speed is increasing or its downward speed is decreasing), the scale reading is greater than the passenger's weight. Similarly, for a downward acceleration (either the cab's upward speed is decreasing or its downward speed is increasing), the scale reading is less than the passenger's weight.

(d) During the upward acceleration in part (c), what is the magnitude F^{net} of the net force on the passenger, and what is the magnitude $a_{\text{p, cab}}$ of the passenger's acceleration as measured in the frame of the cab? Does $\vec{F}^{\text{net}} = m\vec{a}_{\text{p, cab}}$?

SOLUTION ■ One **Key Idea** here is that the magnitude F^{grav} of the gravitational force on the passenger does not depend on the motion of the passenger or the cab, so from part (b), F^{grav} is 708 N. From part (c), the magnitude *N* of the normal force on the passenger during the upward acceleration is the 939 N reading on the scale. Thus, the net force on the passenger is

$$F_y^{\text{net}} = N_y + F_y^{\text{grav}} = N - F^{\text{grav}} = 939\ \text{N} - 708\ \text{N} = 231\ \text{N}, \quad \text{(Answer)}$$

during the upward acceleration. However, the acceleration $\vec{a}_{\text{p, cab}}$ of the passenger relative to the frame of the cab is zero. Thus, in the noninertial frame of the accelerating cab, $\vec{F}^{\text{net}}$ is not equal to $m\vec{a}_{\text{p, cab}}$. This is an example of the fact that Newton's Second Law does not hold in noninertial (that is, accelerating) frames of reference.

TOUCHSTONE EXAMPLE 6-3: Pulling a Block

In Fig. 6-24*a*, a hand *H* pulls on a taut horizontal rope *R* (of mass $m = 0.200$ kg) that is attached to a block *B* (of mass $M = 5.00$ kg). The resulting acceleration $\vec{a}$ of the rope and block across the frictionless surface has constant magnitude 0.300 m/s² and is directed to the right. We will call this the positive direction for the *x* axis. Note that this rope is not "massless;" we return to this feature in part (d).

(a) Identify all the third-law force pairs for the horizontal forces in Fig. 6-24*a* and show how the vectors in each pair are related.

SOLUTION ■ The **Key Idea** here is that a third-law force pair arises when two bodies interact; the forces of the pair are equal in magnitude and opposite in direction, and the force on each body is due to the other body. The "exploded view" of Fig. 6-24*b* shows

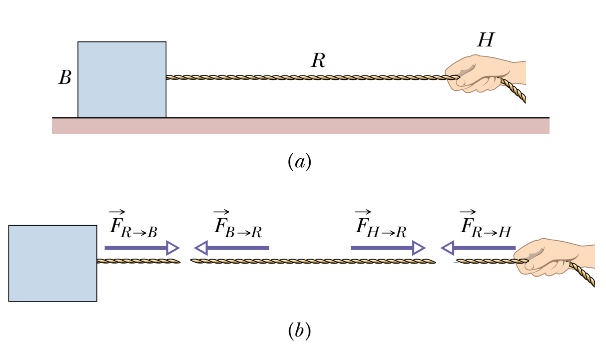


FIGURE 6-24 ■ (*a*) Hand *H* pulls on rope *R*, which is attached to block *B*. (b) An exploded view of block, rope, and hand, with the forces between block and rope and between rope and hand.

that here there are two such force pairs for the horizontal forces. At the hand-rope boundary, we have the force $\vec{F}_{H\to R}$ exerted by the hand on the rope and the force $\vec{F}_{R\to H}$ exerted by the rope on the hand. These forces are a Newton's Third Law force pair and so are equal in magnitude and opposite in direction. They are related by

$$\vec{F}_{H\to R} = -\vec{F}_{R\to H}. \quad \text{(Answer)}$$

Similarly, at the rope–block boundary we have

$$\vec{F}_{R\to B} = -\vec{F}_{B\to R}. \quad \text{(Answer)}$$

(b) What is the magnitude of the force $\vec{F}_{R\to B}$ that the rope exerts on the block?

SOLUTION ■ We know that the block has an acceleration $\vec{a}$ in the positive direction of the x axis. The only force acting on the block along that axis is $\vec{F}_{R\to B}$. The **Key Idea** here is that we can relate force $\vec{F}_{R\to B}$ to acceleration $\vec{a}$ by Newton's Second Law. Because both vectors are along the x axis, we use the x component version of the law ($F_x^{\text{net}} = ma_x$), writing

$$F_{R\to B\,x} = Ma_x.$$

Substituting known values, we find that the magnitude of $\vec{F}_{R\to B}$, which we denote $F_{R\to B}$ and is equal to $F_{R\to B\,x}$, is

$$F_{R\to B} = (5.00\text{ kg})(0.300\text{ m/s}^2) = 1.50\text{ N}. \quad \text{(Answer)}$$

(c) What is the magnitude of the force $\vec{F}_{B\to R}$ that the block exerts on the rope?

SOLUTION ■ From (a), we know that $\vec{F}_{B\to R} = -\vec{F}_{R\to B}$, so $\vec{F}_{B\to R}$ has the magnitude

$$F_{B\to R} = F_{R\to B} = 1.50\text{ N}. \quad \text{(Answer)}$$

(d) What is the magnitude of the force $\vec{F}_{H\to R}$ that the hand exerts on the rope?

SOLUTION ■ A **Key Idea** here is that, with the rope taut, the rope and block form a system on which $\vec{F}_{H\to R}$ acts. The mass of the system is $m + M$. For this system, Newton's Second Law for x-components gives us

$$\begin{aligned} F_{H\to R\,x} &= (m + M)a_x \\ &= (0.200\text{ kg} + 5.00\text{ kg})(0.300\text{ m/s}^2) \\ &= 1.56\text{ N} \quad \text{(Answer)} \quad (6\text{-}19) \end{aligned}$$

Now note that the magnitude of the force $\vec{F}_{H\to R}$ on the rope from the hand (1.56 N) is greater than the magnitude of the force $\vec{F}_{R\to B}$ on the block from the rope [1.50 N, from part (b) above]. The reason is that $\vec{F}_{R\to B}$ must accelerate only the block but $\vec{F}_{H\to R}$ must accelerate both the block and the rope, and the rope's mass m is not negligible. If we let $m \to 0$ in Eq. 6-19, then we find 1.50 N, the same magnitude as at the other end. We often assume that an interconnecting rope is massless so that we can approximate the forces at its two ends as having the same magnitude.

TOUCHSTONE EXAMPLE 6-4: Three Cords

In Fig. 6-25a, a block B of mass $M = 15$ kg hangs by a cord from a knot K of mass m_K, which hangs from a ceiling by means of two other cords. The cords have negligible mass, and the magnitude of the gravitational force on the knot is negligible compared to the gravitational force on the block. What are the tensions in the three cords?

SOLUTION ■ Let's start with the block because it has only one attached cord. The free-body diagram in Fig. 6-25b shows the forces on the block: gravitational force $\vec{F}^{\text{grav}}$ (with a magnitude of Mg) and force $\vec{T}_C$ from the attached cord. A **Key Idea** is that we can relate these forces to the acceleration of the block via Newton's Second Law ($\vec{F}^{\text{net}} = m\vec{a}$). Because the forces are both vertical, we choose the vertical component version of the law, $F_y^{\text{net}} = ma_y$, and write

$$F_y^{\text{net}} = T_{C\,y} + F_y^{\text{grav}} = T_C - Mg = Ma_y.$$

Substituting 0 for the block's acceleration a_y, we find

$$T_{C\,y} - Mg = M(0) = 0.$$

This means that the two forces on the block are equal in magnitude. Substituting for M (= 15.0 kg) and g and solving for $T_{C\,y}$ yields

$$T_{C\,y} = 147\text{ N}. \quad \text{(Answer)}$$

Note: Although $\vec{T}_C$ and $\vec{F}^{\text{grav}}$ are equal in magnitude and opposite in direction, they are *not* a Newton's Third Law force pair. Why?

We next consider the knot in the free-body diagram of Fig. 6-25c, where the negligible gravitational force on the knot is not included. The **Key Idea** here is that we can relate the three other forces acting on the knot to the acceleration of the knot via Newton's Second Law ($\vec{F}^{\text{net}} = m\vec{a}$) by writing

$$\vec{T}_A + \vec{T}_B + \vec{T}_C = m_K\vec{a}_K.$$

Substituting 0 for the knot's acceleration $\vec{a}_K$ yields

$$\vec{T}_A + \vec{T}_B + \vec{T}_C = 0, \quad (6\text{-}20)$$

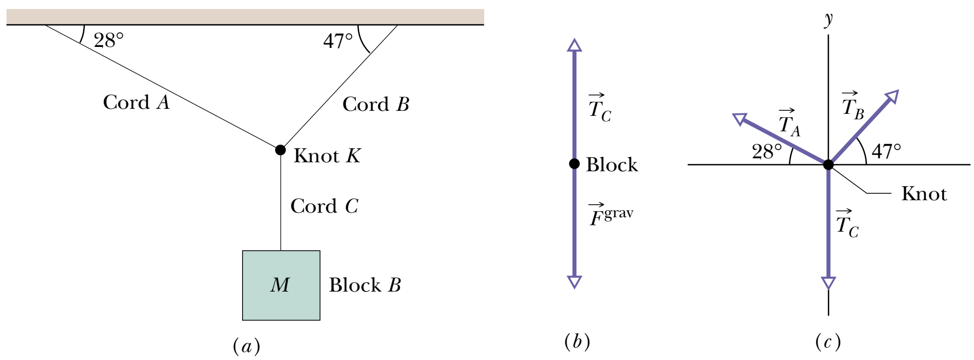


FIGURE 6-25 ■ (*a*) A block of mass *m* hangs from three cords. (*b*) A free-body diagram for the block. (*c*) A free-body diagram for the knot at the intersection of the three cords.

which means that the three forces on the knot are in equilibrium. Although we know both magnitude and angle for $\vec{T}_3$, we know only the angles and not the magnitudes for $\vec{T}_A$ and $\vec{T}_B$. With unknowns in two vectors, we cannot solve Eq. 6-20 for $\vec{T}_A$ or $\vec{T}_B$ directly on a vector-capable calculator. Instead, we rewrite Eq. 6-20 in terms of components along the *x* and *y* axes. For the *x* axis, we have

$$T_{A\,x} + T_{B\,x} + T_{C\,x} = 0,$$

which, using the given data, yields

$$-|\vec{T}_A|\cos 28° + |\vec{T}_B|\cos 47° + 0 = 0, \qquad (6\text{-}21)$$

or alternatively

$$|\vec{T}_A|\cos 152° + |\vec{T}_B|\cos 47° + 0 = 0.$$

Similarly, for the *y* axis we rewrite Eq. 6-20 as

$$T_{A\,y} + T_{B\,y} + T_{C\,y} = 0$$

or

$$|\vec{T}_A|\sin 28° + |\vec{T}_B|\sin 47° - |\vec{T}_C| = 0.$$

Substituting our previous result for T_C then gives us

$$|\vec{T}_A|\sin 28° + |\vec{T}_B|\sin 47° - 147\text{ N} = 0. \qquad (6\text{-}22)$$

We cannot solve Eq. 6-21 or Eq. 6-22 separately because each contains two unknowns, but we can solve them simultaneously because they contain the same two unknowns. Doing so (either by substitution, by adding or subtracting the equations appropriately, or by using the equation-solving capability of a calculator), we discover

$$|\vec{T}_A| = 104\text{ N} \quad \text{and} \quad |\vec{T}_B| = 134\text{ N}. \qquad \text{(Answer)}$$

Thus, the magnitudes of the tensions in the cords are 104 N in cord *A*, 134 N in cord *B*, and 147 N in cord *C*.

6-5 Drag Force and Terminal Speed

If you are riding in a car and put your hand out the window, you feel nothing when the car is first starting up. But as you speed up, the forces on your hand become larger and larger. The force you feel on your hand is called **air drag.** The magnitude of the air drag increases as the velocity of your hand relative to the air increases. Air drag is another common force, but it is only important when an object is moving relatively rapidly.

Air is a fluid. A **fluid** is anything that can flow—generally either a gas or a liquid. When there is a relative velocity between a fluid and a body (either because the body moves through the fluid or because the fluid moves past the body), the body experiences a **drag force** $\vec{D}$ that opposes the relative motion and points in the direction in which the fluid flows relative to the body. Like contact forces, air drag forces are ultimately the result of billions of tiny electromagnetic forces between air molecules and another object.

Here we examine only cases in which air is the fluid, the body is blunt (like your hand or a baseball) rather than slender (like a javelin), and the relative motion is fast enough so that the air becomes turbulent (breaks up into swirls) behind the body. In such cases, experiments reveal that the magnitude $D = |\vec{D}|$ of the drag force is re-

FIGURE 6-26 ■ This skier crouches in an "egg position" to minimize her effective cross-sectional area and thus the air drag acting on her.

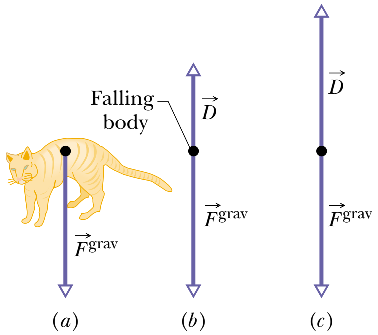


FIGURE 6-27 ■ The forces that act on a body falling through air: (*a*) the body when it has just begun to fall and (*b*) the free-body diagram a little later, after a drag force has developed. (*c*) The drag force has increased until it balances the gravitational force on the body. The body now falls at its constant terminal speed.

lated to the relative speed $v = |\vec{v}|$ by an experimentally determined **drag coefficient** C according to

$$D = \tfrac{1}{2}C\rho A v^2, \qquad (6\text{-}23)$$

where ρ is the air density (mass per volume) and A is the **effective cross-sectional area** of the body (the area of a cross section taken perpendicular to the velocity $\vec{v}$). The drag coefficient C (typical values range from 0.4 to 1.0) is not truly a constant for a given body, because if v varies significantly, the value of C can vary as well. Here, we ignore such complications.

Downhill speed skiers know well that drag depends on the cross-sectional area (A) and speed squared (v^2). To reach high speeds a skier must reduce the drag force as much as possible by, for example, riding the skis in the "egg position" (Fig. 6-26) to minimize cross-sectional area A.

When a blunt body falls from rest through air, the drag force $\vec{D}$ is directed upward; its magnitude gradually increases from zero as the speed of the body increases. This upward force $\vec{D}$ opposes the downward gravitational force, $\vec{F}^{\,grav} = -mg\,\hat{j}$, on the body. We can relate these forces to the body's acceleration by writing Newton's Second Law in terms of vector components for a vertical y axis ($F_y^{\,net} = ma_y$),

$$F_y^{\,net} = (D_y + F_y^{\,grav}) = (+D - mg) = ma_y, \qquad (6\text{-}24)$$

where m is the mass of the body. Experience tells us that D increases as the velocity of the falling object relative to the air increases. As suggested in Fig. 6-27, if the body falls long enough the force magnitudes, D and $F^{\,grav}$, eventually equal each other as shown in Fig. 6-27*c*. According to Eq. 6-24, when this happens $a_y = 0$, and the body's speed no longer increases. The body then falls at a constant speed, called the *terminal speed* v_t. To find the terminal speed, we set $a_y = 0$ in Eq. 6-24 and use that relation for the magnitude of the drag force given by $D = \tfrac{1}{2}C\rho A v^2$ (Eq. 6-23). Then the terminal speed is given by

$$v_t = \sqrt{\frac{2mg}{C\rho A}}. \qquad (6\text{-}25)$$

Table 6-1 gives values of the terminal speed for some common objects.

According to calculations* based on the assumption that $D = \tfrac{1}{2}CA\rho v^2$, a cat must fall about six floors to reach terminal speed. Until it does so, $mg > D_y$ and the cat ac-

TABLE 6-1
Some Terminal Speeds in Air

Object	Terminal Speed (m/s)	95% Distance[a] (m)
Shot (from shot put)	145	2500
Sky diver (typical)	60	430
Baseball	42	210
Tennis ball	31	115
Basketball	20	47
Ping-Pong ball	9	10
Raindrop (radius = 1.5 mm)	7	6
Parachutist (typical)	5	3

[a]This is the distance through which the body must fall from rest to reach 95% of its terminal speed.
Source: Adapted from Peter J. Brancazio, *Sport Science* New York: Simon & Schuster (1984).

celerates downward because of the net downward force. Recall from Chapter 2 that your body is an accelerometer, not a speedometer. Because the cat also senses the acceleration, it is frightened and keeps its feet underneath its body, its head tucked in, and its spine bent upward, making its cross-sectional area (A) small, so its terminal speed v_t becomes relatively large. If the cat maintains this position, it could be injured on landing. However, if the cat shown at the top of the chapter opening photo reaches v_t, its acceleration vanishes so it relaxes, stretching its legs and neck horizontally outward and straightening its spine (it then resembles a flying squirrel). These actions increase its area A and hence the magnitude of the drag force D_y acting on it. The cat begins to slow its descent because now the magnitude of its upward drag force is greater than the downward gravitational force. Eventually, a new, smaller terminal velocity is reached. The decrease in terminal velocity reduces the possibility of serious injury on landing. Just before hitting the ground, the cat pulls its legs back beneath its body to prepare for the landing.

Humans often fall from great heights for fun when sky diving. However, in April 1987, during a jump, sky diver Gregory Robertson noticed that fellow sky diver Debbie Williams had been knocked unconscious in a collision with a third sky diver and was unable to open her parachute. Robertson, who was well above Williams at the time and who had not yet opened his parachute for the 4 km plunge, reoriented his body head-down to minimize his cross-sectional area and maximize his downward speed. Reaching an estimated terminal velocity of 320 km/h, he caught up with Williams and then went into a horizontal "spread eagle" (as shown in Fig. 6-28) to increase his drag force. He could then grab her. He opened her parachute and then, after releasing her, his own, a scant 10 s before impact. Williams received extensive internal injuries due to her lack of control on landing but survived.

FIGURE 6-28 ■ A sky diver in a horizontal "spread eagle" maximizes the air drag.

READING EXERCISE 6-9: Near the ground, is the speed of large raindrops greater than (>), less than (<), or equal to (=) the speed of small raindrops? Assume that all raindrops are spherical and have the same drag coefficient C. **Beware!** More than one factor is involved. ■

6-6 Applying Newton's Laws

Now that you have learned about several types of forces that can act on an object, you have the basic knowledge needed to analyze the accelerations and forces experienced by bodies in an interacting system. However, you will need to use your knowledge in an organized fashion to predict how a system will move or to identify unknown forces based on observations of system motions.

There are several key steps that we suggest you use in performing an analysis. These steps are an extension of those presented in Sections 3-7 and 6-2. The steps are outlined in more detail in Touchstone Example 6-5:

1. Construct a diagram of the system you wish to analyze.
2. Isolate the bodies of interest in the system on your diagram. Identify the types, directions, and approximate magnitudes of the forces acting on each body. Label the forces to indicate the type of force ($\vec{F}^{\text{grav}}$, $\vec{N}$, $\vec{f}$, $\vec{T}$).
3. Construct a free-body diagram representing each body as a point. Place the tails of the labeled force vectors for that body at its point. If possible, show the angles these vectors make with respect to each other as well as the relative magnitudes of the vectors.

*W. O. Whitney and C. J. Mehlhaff, "High-Rise Syndrome in Cats," *The Journal of the American Veterinary Medical Association*, 1987, Vol. 191, pp. 1399–1403.

4. Predict the direction of the acceleration and draw a special acceleration vector in that direction and label it with $\vec{a}$. Then choose a coordinate system so that one axis lies parallel to the direction of the predicted acceleration.
5. Write down Newton's Second Law in vector form for each body in the system. Then decompose the vectors into a pair of one-dimensional equations for each body,

$$\vec{a} = \frac{1}{m}\vec{F}^{\,\text{net}} \Rightarrow \qquad a_x = \frac{1}{m}F_x^{\,\text{net}} \quad \text{and} \quad a_y = \frac{1}{m}F_y^{\,\text{net}}.$$

Remember that we drop the vector notation (arrows) when we write the one-dimensional equations. These equations associate the components of vectors.

6. Solve the set of equations for each dimension (x and y) separately to find the unknown vector components.

In Touchstone Example 6-5 we show how these six steps can be used to find the forces that act on a block of known mass as it slides up an incline.

TOUCHSTONE EXAMPLE 6-5: Sliding Up a Ramp

Figure 6-29 shows a 300 g block on a 30° incline. The block is moving up the incline at a constant velocity because a string that passes over a pulley is attached to a falling mass. If we assume that the mass of the string and pulley are negligible and that there is no friction in the pulley and no friction between the incline and the block. (a) What force is the string exerting on the block during the time that the block is moving up the plane at constant velocity? (b) What is the normal force that the incline is exerting on the block?

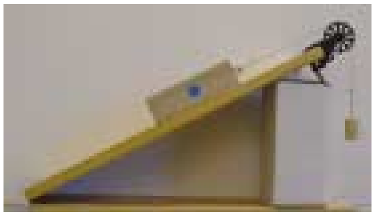


FIGURE 6-29 ■ Photograph of a block on an inclined plane that is moving at a constant velocity.

SOLUTION ■ The **Key Idea** is that because the block is not accelerating, the net force on it must be zero (according to Newton's Second Law). If we follow the steps outlined in Section 6-6 we can identify the forces on the block, choose a coordinate system, and decompose the vectors into components. Since the components along each axis must add up to zero, we can solve our equations for the magnitude and direction of the force of the string on the block.

Step One: Construct a Diagram of the System Figure 6-30*a* shows the essential features of the system of interest needed to answer question (a) including the incline, the block, and the string pulling on the block. The figure is more abstract than the photograph of Fig. 6-29.

Step Two: Isolate the Objects of Interest and Identify the Forces There is only one object of interest in this problem—the block.

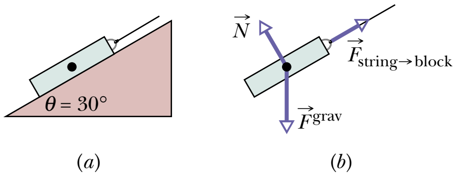


FIGURE 6-30 ■ (*a*) Step one sketch of just those parts of the system of interest for solving the problem. (*b*) Step two sketch of just the block and the forces acting on it with labels.

Thus, we only need to diagram and identify the forces on it. There are three forces acting on the block. First, there is the gravitational force that the Earth exerts on the block that acts vertically downward. Next, there is the normal force that is at right angles (normal to) the surface of the incline. Finally, there is the tension force along the direction of the string that is exerted on the upper end of the block. These forces are shown in Fig. 6-30*b*. *Note*: Although each bit of mass on the block is being pulled downward by the Earth, we can idealize this force and assume it acts at the center of the block. Likewise we assume that the normal force exerted on the block by the inclined plane surface acts like a single force at the middle of the surface of the block that is in contact with the incline. We realize it is the vector sum of billions of smaller normal vectors acting at all points along the surface of contact of the block.

Step Three: Construct a Free-Body Diagram To analyze a system using Newton's Second Law, we draw a free-body diagram for each object in our system. Usually the object experiencing forces is represented by a dot. Then, a vector representing each force that acts on that object is drawn with its tail on the dot. Each vector should be pointing in the direction of the particular force being represented. Also, if the relative magnitudes of the forces are known, the

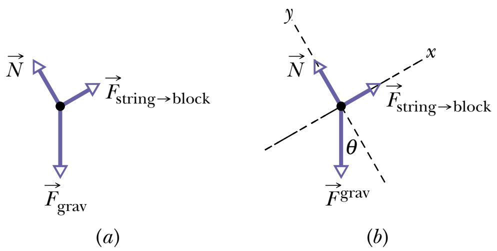


FIGURE 6-31 ■ Steps three and four free-body diagrams for the forces on the block: (*a*) without a coordinate system and (*b*) with a coordinate system.

lengths of the vectors should represent those magnitudes. In this example, we only need a free-body diagram for one object—the block. A clearly labeled arrow showing the predicted direction of the acceleration of the object should also be included. We have no acceleration in this case, so no acceleration vector is included. The free-body diagram for the block is shown in Fig. 6-31*a*.

Step Four: Predict the Direction of the Acceleration and Choose a Coordinate System In choosing the coordinate system for this particular situation, it is useful to break away from our standard practice of having the y axis be a vertical axis and the x axis be horizontal. In general, it is helpful to have one of the axes chosen so it is in the direction of either the acceleration of the object of interest or the forces we are trying to find. One force on the block points up the incline (the string force). Another force is perpendicular to the incline (the normal force). Let's choose "up the incline" as the direction of the positive x axis and a y axis that is perpendicular to the incline (shown in Fig. 6-31*b*). In this coordinate system, only the gravitational force vector will need to be decomposed. Note that using a standard coordinate system would not be incorrect, just less convenient.

We can use some basic geometry to convince ourselves that the gravitational force vector makes an angle of 30° with respect to the negative y axis.

Step Five: Apply Newton's Second Law and Decompose the Force Vectors Recall that the block is moving with constant velocity so the vector sum of the forces acting on it must be zero. Thus we can write

$$\vec{F}^{\,net} = m\vec{a} = 0 \quad \text{so} \quad \vec{F}^{\,grav} + \vec{N} + \vec{F}_{string\to block} = 0.$$

But in order for $\vec{F}^{\,net} = 0$ we must have $F_x^{net} = 0$ and $F_y^{net} = 0$. Therefore,

$$F_x^{grav} + N_x + F_{string\to block\,x} = 0 \quad \text{and}$$
$$F_y^{grav} + N_y + F_{string\to block\,y} = 0.$$

Recall that here F^{grav} denotes the x-component of the gravitational force, N_x denotes the x-component of the normal force, and so on. These components are not vectors and so do not have arrows above them. The only vector that needs decomposition is the gravitational force vector. This decomposition is shown in Fig. 6-32. Since we know by inspection that the gravitational force components are negative, they are expressed with explicit signs as

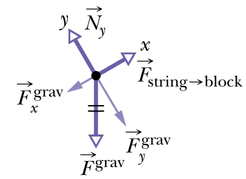


FIGURE 6-32 ■ Decomposition of the gravitational force vector into components along the chosen x and y axes.

$$F_x^{grav} = -|F^{grav}|\sin\theta \quad \text{and} \quad F_y^{grav} = -|F^{grav}|\cos\theta.$$

The angle θ between the downward-pointing force vector and the y axis is 30°. By inspecting the diagram, we see that the normal force vector points along the positive y axis and the tension force vector points along the positive x axis. So, these vectors can be written as

$$\vec{N} = +N_j \quad \text{and} \quad \vec{F}_{string\to block} = +T\hat{\imath},$$

where T is a positive scalar representing the tension in the string and N is a positive scalar representing the magnitude of the normal force. Our expression for $F_x^{net} = 0$ then becomes $+F_{string\to block} - |F^{grav}|\sin\theta = 0$. Our expression for $F_y^{net} = 0$ then becomes $N - |F^{grav}|\cos\theta = 0$. Thus,

$$F_{string\to block} = mg\sin\theta,$$

and $$N = mg\cos\theta.$$

We know how to find the values of the gravitational force components in terms of the mass of the block m, the local gravitational strength constant g, and the angle θ:

$$F_{string\to block} = T = mg\sin\theta = 0.300\text{ kg} \times 9.8\text{ m/s}^2 \times \sin 30° = 1.47\text{ N}$$
$$N = mg\cos\theta = 0.300\text{ kg} \times 9.8\text{ m/s}^2 \times \sin 30° = 2.55\text{ N}.$$

Finally, rounding to 2 significant figures gives us

$$\vec{F}_{string\to block} = +(1.5\text{ N})\hat{\imath} \quad \text{(Answer)}$$
$$\vec{N} = +(2.5\text{ N})\hat{\jmath} \quad \text{(Answer)}$$

A final note: This example shows the basics for a relatively simple analysis. If we had taken friction into account and picked a part of the motion that is accelerated, the problem would have been more complicated. However, the basic steps would be exactly the same. To master the techniques of analysis for more complex situations, you will also need to study the rest of the of touchstone examples in this chapter.

TOUCHSTONE EXAMPLE 6-6: Breaking Loose

Figure 6-33*a* shows a coin of mass m at rest on a book that has been tilted at an angle θ with the horizontal. By experimenting, you find that when θ is increased to 13°, the coin is on the *verge* of sliding down the book, which means that even a slight increase beyond 13° produces sliding. What is the coefficient of static friction μ^{stat} between the coin and the book?

SOLUTION ■ If the book were frictionless, the coin would surely slide down it for any tilt of the book because of the gravitational force on the coin. Thus, one **Key Idea** here is that a frictional force f^{stat} must be holding the coin in place. A second **Key Idea** is that, because the coin is *on the verge* of sliding *down* the book, that force is at its *maximum* magnitude f^{max} and is directed *up* the book. Also, from Eq. 6-11, we know that $f^{max} = \mu^{stat} N$, where N is the magnitude of the normal force $\vec{N}$ on the coin from the book. Thus,

$$f^{max} = \mu^{stat} N,$$

from which

$$\mu^{stat} = \frac{f^{stat}}{N}. \tag{6-26}$$

To evaluate this equation, we need to find the force magnitudes f^{stat} and N. To do that, we use another **Key Idea:** When the coin is on the verge of sliding, it is stationary and thus its acceleration $\vec{a}$ is zero. We can relate this acceleration to the forces on the coin with Newton's Second Law ($\vec{F}^{net} = m\vec{a}$). As shown in the free-body diagram of the coin in Fig. 6-33*b*, these forces are (1) the frictional force $\vec{f}^{stat}$, (2) the normal force $\vec{N}$, and (3) the gravitational force $\vec{F}^{grav}$ on the coin, with magnitude equal to mg. Then, from Newton's Second Law with $\vec{a} = 0$, we have

$$\vec{f}^{stat} + \vec{N} + \vec{F}^{grav} = 0. \tag{6-27}$$

To find f^{stat} and N, we rewrite Eq. 6-27 for components along the x and y axes of the tilted coordinate system in Fig. 6-33*b*. For the x axis and with mg substituted for $|\vec{F}^{grav}|$, we have

$$f_x^{stat} + N_x + F_x^{grav} = f_x^{stat} + 0 - mg \sin\theta = 0,$$

so

$$f_x^{stat} = +mg \sin\theta. \tag{6-28}$$

Similarly, for the y axis we have

$$f_y^{stat} + N_y + F_y^{grav} = 0 + N - mg \cos\theta = 0,$$

so

$$N = +mg \cos\theta. \tag{6-29}$$

Substituting Eqs. 6-28 and 6-29 into Eq. 6-26 produces

$$\mu^{stat} = \frac{mg \sin\theta}{mg \cos\theta} = \tan\theta, \tag{6-30}$$

which here means

$$\mu^{stat} = \tan 13° = 0.23. \quad \text{(Answer)}$$

Actually, you do not need to measure θ to get μ^{stat}. Instead, measure the two lengths shown in Fig. 6-33*a* and then substitute h/d for $\tan\theta$ in Eq. 6-30.

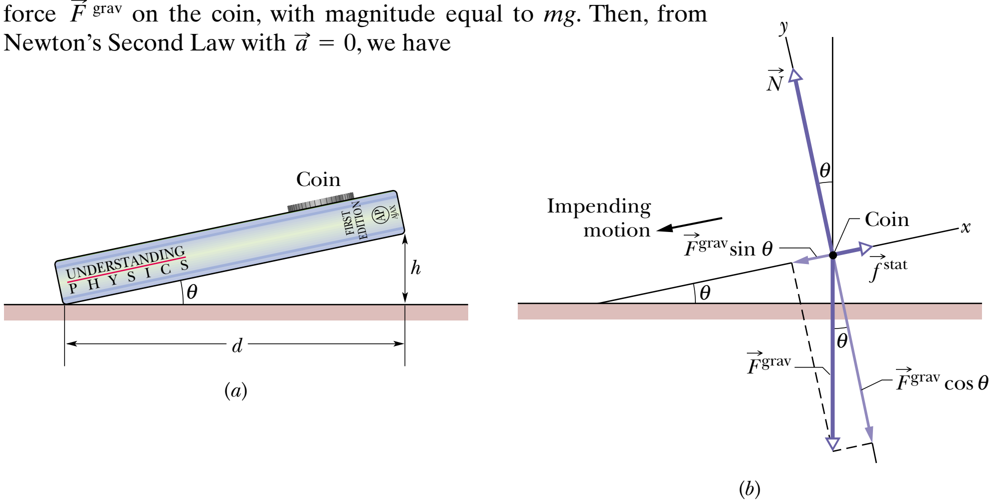


FIGURE 6-33 ■ (*a*) A coin on the verge of sliding down a book. (*b*) A free-body diagram for the coin, showing the three forces (drawn to scale) that act on it. The gravitational force $\vec{F}^{grav}$ is shown resolved into its components along the x and the y axes, whose orientations are chosen to simplify the problem. Component $F_x^{grav} = F^{grav} \sin\theta$ tends to slide the coin down the book. Component $F_y^{grav} = F^{grav} \cos\theta$ presses the coin onto the book.

TOUCHSTONE EXAMPLE 6-7: Accelerated by Friction

A 40 kg slab rests on a frictionless floor. A 10 kg block rests on top of the slab (Fig. 6-34). The coefficient of static friction μ^{stat} between the block and the slab is 0.60, whereas their kinetic friction coefficient μ^{kin} is 0.40. The 10 kg block is pulled by a horizontal force with a magnitude of 100 N. What are the resulting accelerations of (a) the slab and (b) the block?

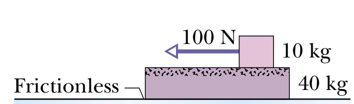


FIGURE 6-34

SOLUTION ■ The first **Key Idea** here is that we should apply Newton's Second Law *separately* to the slab ($m_{\text{slab}} = 40$ kg) and to the block ($m_{\text{slab}} = 10$ kg) to obtain the acceleration of each:

$$\vec{a}_{\text{slab}} = \frac{\vec{F}^{\text{net}}_{\text{slab}}}{m_{\text{slab}}} \quad \text{and} \quad \vec{a}_{\text{block}} = \frac{\vec{F}^{\text{net}}_{\text{block}}}{m_{\text{block}}}.$$

To find the net force on each of these objects, we can draw a free-body diagram for each, as shown in Fig. 6-35. The direction of the frictional force from the slab on the block, $\vec{f}_{\text{slab}\rightarrow\text{block}}$, is determined by considering the direction of the block's impending motion. The direction of the frictional force from the block on the slab, $\vec{f}_{\text{block}\rightarrow\text{slab}}$, is inferred from Newton's Third Law.

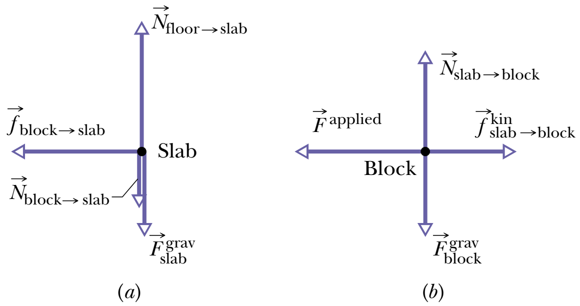


FIGURE 6-35 ■ The free-body diagram showing all the forces acting on (*a*) the 40 kg slab and (*b*) the 10 kg block.

Since we expect the block and the slab to accelerate to the left, let's decide that the positive x axis is pointing to point to the left and the y axis is pointing straight up. Then

$$a_{\text{slab}\,y} = 0 \quad \text{so} \quad F^{\text{net}}_{\text{slab}\,y} = 0, \tag{6-31}$$

and

$$a_{\text{block}\,y} = 0 \quad \text{so} \quad F^{\text{net}}_{\text{block}\,y} = 0. \tag{6-32}$$

To calculate the horizontal accelerations of the block and the slab, we must first determine whether the frictional force of interaction between them is static or kinetic. The **Key Idea** here is that the maximum static frictional force's magnitude is limited to be no larger than $f^{\text{max}} = \mu^{\text{stat}} N$. Applying this to the slab, we find that

$$f^{\text{max}}_{\text{block}\rightarrow\text{slab}} = \mu^{\text{stat}} N_{\text{block}\rightarrow\text{slab}}. \tag{6-33}$$

Newton's Third Law tells us that

$$N_{\text{block}\rightarrow\text{slab}} = N_{\text{slab}\rightarrow\text{block}}, \tag{6-34}$$

and Fig. 6-35*b* and Eq. 6-31 tell us that

$$N_{\text{slab}\rightarrow\text{block}} = F^{\text{grav}}_{\text{block}} = m_{\text{block}}\,g. \tag{6-35}$$

Combining these ideas yields

$$f^{\text{max}}_{\text{block}\rightarrow\text{slab}} = \mu^{\text{stat}} m_{\text{block}}\,g$$
$$= (0.60)(10\text{ kg})(9.8\text{ N/kg}) = 59\text{ N}.$$

If this limit is not exceeded, then static friction would keep the block and the slab locked together, accelerating with a common acceleration

$$a_x = \frac{F_x^{\text{net}}}{(m_{\text{slab}} + m_{\text{block}})}$$
$$= 100\text{ N}/(40\text{ kg} + 10\text{ kg}) = +2.00\text{ m/s}^2.$$

Newton's Second Law (written in terms of x-components) tells us that this acceleration requires $F^{\text{net}}_{\text{slab}\,x} = m_{\text{slab}}\,a_x = (40\text{ kg})(2.00\text{ m/s}^2) = 80.0$ N. But Fig. 6-35*a* shows that the only horizontal force acting on the slab is the frictional force from the block. Since 80 N are required to accelerate the slab but we found the static frictional force is limited to 59 N, we can conclude that the block and the slab *cannot* be locked together by static friction. The block must be sliding to the left on top of the slab. This means that

$$F^{\text{net}}_{\text{block}\,x} = F_x^{\text{app}} + f^{\text{kin}}_{\text{slab}\rightarrow\text{block}}, \tag{6-36}$$

and

$$F^{\text{net}}_{\text{slab}\,x} = f^{\text{kin}}_{\text{block}\rightarrow\text{slab}} = \mu^{\text{kin}} \vec{N}_{\text{block}\rightarrow\text{slab}}. \tag{6-37}$$

Combining Eqs. 6-34, Eq. 6-35, and Eq. 6-37 yields $F^{\text{net}}_{\text{slab}\,x} = \mu^{\text{kin}} m_{\text{block}}\,g$ or

$$\vec{F}^{\text{net}}_{\text{slab}} = (\mu^{\text{kin}} m_{\text{block}}\,g)\,\hat{\text{i}}, \tag{6-38}$$

so

$$\vec{a}_{\text{slab}} = \frac{\vec{F}^{\text{net}}_{\text{slab}}}{m_{\text{slab}}}$$
$$= (0.40)(10\text{ kg})(9.8\text{ m/s}^2)/(40\text{ kg})\hat{\text{i}}$$
$$= (0.98\text{ m/s}^2)\hat{\text{i}} \qquad \text{(Answer)}$$

It's interesting to note that a frictional force causes the slab to speed up, not slow down. The same is true when you start running from rest. To accelerate, you push backwards on the ground with your shoes. The ground, courtesy of Newton's Third Law, pushes forward on you, accelerating you forward. It is actually the static frictional force that the ground exerts on you that accelerates you.

Finally, to calculate the acceleration of the block, we note that the net force on the block in the y direction is zero and that (by Newton's Third Law) $\vec{f}^{\text{kin}}_{\text{slab}\rightarrow\text{block}} = -\vec{f}^{\text{kin}}_{\text{block}\rightarrow\text{slab}}$. Therefore,

$$\vec{F}^{\text{net}}_{\text{block}} = \vec{F}^{\text{app}} + \vec{f}^{\text{kin}}_{\text{slab}\rightarrow\text{block}}$$
$$= \vec{F}^{\text{app}} - \vec{f}^{\text{kin}}_{\text{block}\rightarrow\text{slab}}$$
$$= \vec{F}^{\text{app}} - \mu^{\text{kin}} m_{\text{block}}\,g\,\hat{\text{i}}.$$

So

$$\vec{a}_{\text{block}} = \frac{\vec{F}^{\text{net}}_{\text{block}}}{m_{\text{block}}}$$
$$= \frac{(100\text{ N})\hat{\text{i}} - (0.40)(10\text{ kg})(9.8\text{ m/s}^2)\hat{\text{i}}}{10\text{ kg}}$$
$$= [10\text{ m/s}^2 - (0.4)(9.8\text{ m/s}^2)]\,\hat{\text{i}}$$
$$= (+6.1\text{ m/s}^2)\,\hat{\text{i}}. \qquad \text{(Answer)}$$

TOUCHSTONE EXAMPLE 6-8: Banked Curve

You cannot always count on friction to get your car around a curve, especially if the road is icy or wet. That is why highway curves are banked. Suppose that a car of mass m moves at a constant speed v of 20 m/s around a curve, now banked, whose radius R is 190 m (Fig. 6-36*a*). What bank angle θ makes reliance on friction unnecessary?

SOLUTION ■ A centripetal force must act on the car if the car is to move along the circular path. A **Key Idea** is that the track is banked so as to tilt the normal force $\vec{N}$ on the car toward the center of the circle (Fig. 6-36*b*). Thus, $\vec{N}$ now has a centripetal component N_r, directed inward along a radial axis r. We want to find the value of the bank angle θ such that this centripetal component keeps the car on the circular track without need of friction.

A second key idea is to keep the y axis vertical and the x axis horizontal rather than in the direction of the incline. This enables as to find the radial component of the normal force more easily.

As Fig. 6-36*b* shows (and as you should verify), the angle that $\vec{N}$ makes with the vertical is equal to the bank angle θ of the track. Thus, the radial component N_r is equal to $+N \sin\theta$ where N is the magnitude of the normal force. We can now write Newton's Second Law for components along the r axis ($F_r^{\text{net}} = ma_r$) as

$$+N\sin\theta = m\left(+\frac{v^2}{R}\right). \qquad (6\text{-}39)$$

We cannot solve this equation for the value of θ because it also contains the unknowns N and m.

We next consider the forces and acceleration along the y axis in Fig. 6-36*b*. The vertical component of the normal force is $N_y = N\cos\theta$, the gravitational force $\vec{F}^{\text{grav}}$ on the car is $(-mg)\hat{\jmath}$, and the acceleration of the car along the y axis is zero. Thus, we can write Newton's Second Law for components along the y axis ($F_y^{\text{net}} = ma_y$) as

$$N\cos\theta - mg = m(0),$$

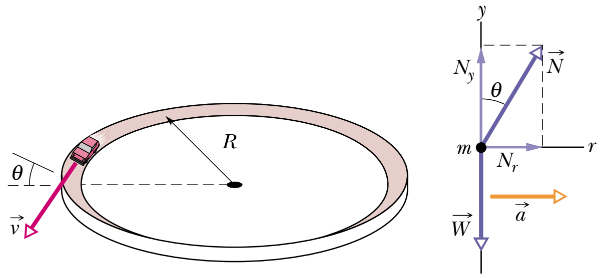
from which

$$N\cos\theta = mg. \qquad (6\text{-}40)$$

This too contains the unknowns N and m, but note that dividing Eq. 6-39 by Eq. 6-40 neatly eliminates both those unknowns. Doing so, and replacing $\sin\theta/\cos\theta$ with $\tan\theta$ and solving for θ, then yield

$$\theta = \tan^{-1}\left(\frac{v^2}{gR}\right)$$

$$= \tan^{-1}\left(\frac{(20\text{ m/s})^2}{(9.8\text{ m/s}^2)(190\text{ m})}\right) = 12°. \qquad \text{(Answer)}$$



(*a*) (*b*)

FIGURE 6-36 ■ (*a*) A car moves around a curved banked road at constant speed. The bank angle is exaggerated for clarity. (*b*) A free-body diagram for the car, assuming that friction between tires and road is zero. The radially inward component of the normal force provides the necessary centripetal force. The resulting acceleration is also radially inward.

6-7 The Fundamental Forces of Nature

According to Newton's Third Law, forces between two objects always act in pairs. In the study of the structure of matter, physicists have used a belief in mutual interactions to study the nature of forces. As we learn more about matter and how it behaves, we explore the nature of forces by observing changes in the motion of objects that interact. Using these observations, scientists have identified only four types of forces.

The most familiar of these forces are the **gravitational force,** of which falling and weight are our most familiar examples, and the **electromagnetic force,** which, at a fundamental level, is the basis of all the other forces we considered in this chapter. The electromagnetic force is the combination of electrical forces and magnetic forces. Electromagnetic forces enable an electrically charged balloon to stick to a wall and a magnet to pick up an iron nail. In fact, aside from the gravitational force, *any* force that we can experience directly as a push or pull is electromagnetic in nature. That is, all such forces, including friction forces, normal forces, contact forces, and tension forces arise from electromagnetic forces exerted by one atom on another. For example, the tension in a taut cord exists only because its atoms attract one another. When pulled apart a bit, while normal forces result from atoms repelling each other when being pushed together.

Only two other fundamental forces are known, and they both act over such short distances that we cannot experience them directly through our senses. They are the **weak force,** which is involved in certain kinds of radioactive decay, and the **strong force,** which binds together the quarks that make up protons and neutrons and is the "glue" that holds together an atomic nucleus.

Physicists have long believed that nature has an underlying simplicity and that the number of fundamental forces can be reduced. Einstein spent most of his working life trying to interpret these forces as different aspects of a single *superforce.* He failed, but in the 1960s and 1970s, other physicists showed that the weak force and the electromagnetic force are different aspects of a single **electroweak force.** The quest for further reduction continues today, at the very forefront of physics. Table 6-2 lists the progress that has been made toward **unification** (as the goal is called) and gives some hints about the future.

TABLE 6-2
The Quest for the Superforce—A Progress Report

Date	Researcher	Achievement
1687	Newton	Showed that the same laws apply to astronomical bodies and to objects on Earth. Unified celestial and terrestrial mechanics.
1820 1830s	Oersted Faraday	Showed, by brilliant experiments, that the then separate sciences of electricity and magnetism are intimately linked.
1873	Maxwell	Unified the sciences of electricity, magnetism, and optics into the single subject of electromagnetism.
1979	Glashow, Salam, Weinberg	Received the Nobel Prize for showing that the weak force and the electromagnetic force could be different aspects of a single *electroweak force.* This combination of forces reduced the number of forces viewed as fundamental forces from four to three.
1984	Rubbia, van der Meer	Received the Nobel Prize for verifying experimentally the predictions of the theory of the electroweak force.
		Work in Progress

Grand unification theories (*GUTs*): Seek to unify the electroweak force and the strong force.
Supersymmetry theories: Seek to unify all forces, including the gravitational force, within a single framework.

Superstring theories: Interpret point-like particles, such as electrons, as being unimaginably tiny, closed loops. Strangely, extra dimensions beyond the familiar four dimensions of space-time appear to be required.

Problems

SEC. 6-2 ■ NET FORCE AS A VECTOR SUM

1. Standard Body If the 1 kg standard body has an acceleration of 2.00 m/s^2 at 20° to the positive direction of the x axis, then what are (a) the x-component and (b) the y-component of the net force on it, and (c) what is the net force in unit-vector notation?

2. Chopping Block Two horizontal forces act on a 2.0 kg chopping block that can slide over a frictionless kitchen counter, which lies in an xy plane. One force is $\vec{F}_A = (3.0\text{ N})\hat{\text{i}} + (4.0\text{ N})\hat{\text{j}}$. Find the acceleration of the chopping block in unit-vector notation when the other force is (a) $\vec{F}_B = (-3.0\text{ N})\hat{\text{i}} + (-4.0\text{ N})\hat{\text{j}}$, (b) $\vec{F}_B = (-3.0\text{ N})\hat{\text{i}} + (4.0\text{ N})\hat{\text{j}}$, and (c) $\vec{F}_B = (3.0\text{ N})\hat{\text{i}} + (-4.0\text{ N})\hat{\text{j}}$.

3. Two Horizontal Forces Only two horizontal forces act on a 3.0 kg body. One force is 9.0 N, acting due east, and the other is 8.0 N, acting 62° north of west. What is the magnitude of the body's acceleration?

4. Two Forces While two forces act on it, a particle is to move at the constant velocity $\vec{v} = (3\text{ m/s})\hat{\text{i}} - (4\text{ m/s})\hat{\text{j}}$. One of the forces is $\vec{F}_A = (2\text{ N})\hat{\text{i}} + (-6\text{ N})\hat{\text{j}}$. What is the other force?

5. Three Forces Three forces act on a particle that moves with unchanging velocity $\vec{v} = (2\text{ m/s})\hat{\text{i}} - (7\text{ m/s})\hat{\text{j}}$. Two of the forces are $\vec{F}_A = (2\text{ N})\hat{\text{i}} + (3\text{ N})\hat{\text{j}}$ and $\vec{F}_B = (-5\text{ N})\hat{\text{i}} + (8\text{ N})\hat{\text{j}}$. What is the third force?

6. Three Astronauts Three astronauts, propelled by jet backpacks, push and guide a 120 kg asteroid toward a processing dock, exerting the forces shown in Fig. 6-37. What is the asteroid's acceleration (a) in unit-vector notation and as (b) a magnitude and (c) a direction?

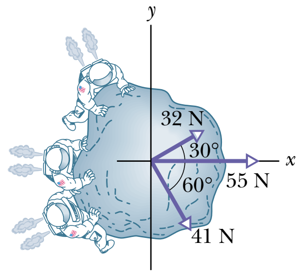


FIGURE 6-37 ■ Problem 6.

7. The Box There are two forces on the 2.0 kg box in the overhead view of Fig. 6-38 but only one is shown. The figure also shows the acceleration of the box. Find the second force (a) in unit-vector notation and as (b) a magnitude and (c) a direction.

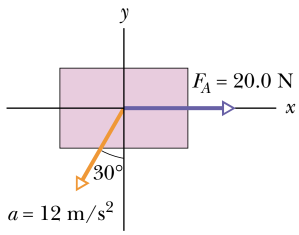


FIGURE 6-38 ■ Problem 7.

8. A Tire Figure 6-39 is an overhead view of a 12 kg tire that is to be pulled by three ropes. One force ($\vec{F}_A$, with magnitude 50 N) is indicated. Orient the other two forces $\vec{F}_B$ and $\vec{F}_C$ so that the magnitude of the resulting acceleration of the tire is least, and find that magnitude if (a) $\vec{F}_B = 30$ N, $\vec{F}_C = 20$ N; (b) $\vec{F}_B = 30$ N, $\vec{F}_C = 10$ N; and (c) $\vec{F}_B = \vec{F}_C = 30$ N.

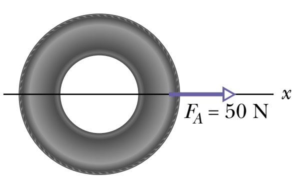


FIGURE 6-39 ■ Problem 8.

SEC. 6-3 ■ GRAVITATIONAL FORCE AND WEIGHT

9. Salami on a Cord (a) An 11.0 kg salami is supported by a cord that runs to a spring scale, which is supported by another cord from the ceiling (Fig. 6-40*a*). What is the reading on the scale, which is marked in weight units? (b) In Fig. 6-40*b* the salami is supported by a cord that runs around a pulley and to a scale. The opposite end of the scale is attached by a cord to a wall. What is the reading on the scale? (c) In Fig. 6-40*c* the wall has been replaced with a second 11.0 kg salami on the left, and the assembly is stationary. What is the reading on the scale now?

10. Spaceship on the Moon A spaceship lifts off vertically from the Moon, where the freefall acceleration is 1.6 m/s². If the spaceship has an upward acceleration of 1.0 m/s² as it lifts off, what is the magnitude of the force of the spaceship on its pilot, who weighs 735 N on Earth?

SEC. ■ 6-4 CONTACT FORCES

11. A Bureau A bedroom bureau with a mass of 45 kg. including drawers and clothing, rests on the floor. (a) If the coefficient of static friction between the bureau and the floor is 0.45, what is the magnitude of the minimum horizontal force that a person must apply to start the bureau moving? (b) If the drawers and clothing, with 17 kg mass, are removed before the bureau is pushed, what is the new minimum magnitude?

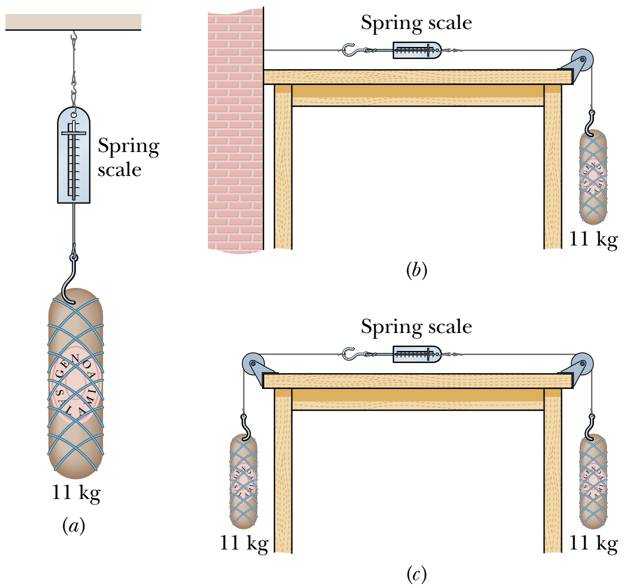


FIGURE 6-40 ■ Problem 9.

12. Scrambled Eggs The coefficient of static friction between Teflon and scrambled eggs is about 0.04. What is the smallest angle from the horizontal that will cause the eggs to slide across the bottom of a Teflon-coated skillet?

13. Baseball Player A baseball player with mass $m = 79$ kg, sliding into second base, is retarded by a frictional force of magnitude 470 N. What is the coefficient of kinetic friction μ^{kin} between the player and the ground?

14. The Mysterious Sliding Stones Along the remote Racetrack Playa in Death Valley. California, stones sometimes gouge out prominent trails in the desert floor, as if they had been migrating (Fig 6-41). For years curiosity mounted about why the stones moved. One explanation was that strong winds during the occasional rainstorms would drag the rough stones over ground softened by rain. When the desert dried out, the trails behind the stones were hard-baked in place. According to measurements, the coefficient of kinetic friction between the stones and the wet playa ground is about 0.80. What horizontal force is needed on a stone of typical mass 20 kg to maintain the stone's motion once a gust has started it moving? (Story continues with Problem 42.)

FIGURE 6-41 ■ Problem 14.

15. A Crate A person pushes horizontally with a force of 220 N on a 55 kg crate to move it across a level floor. The coefficient of kinetic friction is 0.35. (a) What is the magnitude of the frictional force? (b) What is the magnitude of the crate's acceleration?

16. A House on a Hill A house is built on the top of a hill with a nearby 45° slope (Fig. 6-42). An engineering study indicates that the slope angle should be reduced because the top layers of soil along the slope might slip past the lower layers. If the static coefficient of friction between two such layers is 0.5, what is the least angle ϕ through which the present slope should be reduced to prevent slippage?

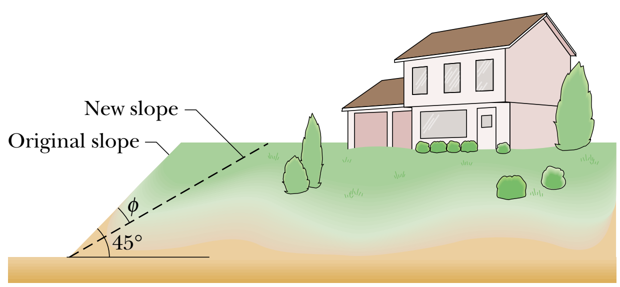


FIGURE 6-42 ■ Problem 16.

17. Hockey Puck A 110 g hockey puck sent sliding over ice is stopped in 15 m by the frictional force on it from the ice. (a) If its initial speed is 6.0 m/s, what is the magnitude of the frictional force? (b) What is the coefficient of friction between the puck and the ice?

18. Rock Climber In Fig. 6-43 a 49 kg rock climber is climbing a "chimney" between two rock slabs. The static coefficient of friction between her shoes and the rock is 1.2; between her back and the rock it is 0.80. She has reduced her push against the rock until her back and her shoes are on the verge of slipping. (a) Draw a free-body diagram of the climber. (b) What is her push against the rock? (c) What fraction of her weight is supported by the frictional force on her shoes?

FIGURE 6-43 ■ Problem 18.

19. Block Against a Wall A 12 N horizontal force $\vec{F}$ pushes a block weighing 5.0 N against a vertical wall (Fig. 6-44). The coefficient of static friction between the wall and the block is 0.60, and the coefficient of kinetic friction is 0.40. Assume that the block is not moving initially. (a) Will the block move? (b) In unit-vector notation, what is the force on the block from the wall?

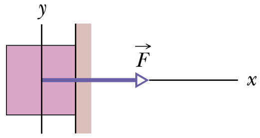


FIGURE 6-44 ■ Problem 19.

20. Block on a Horizontal Surface A 2.5 kg block is initially at rest on a horizontal surface. A 6.0 N horizontal force and a vertical force $\vec{P}$ are applied to the block as shown in Fig. 6-45. The coefficients of friction for the block and surface are $\mu^{\text{stat}} = 0.40$ and $\mu^{\text{kin}} = 0.25$. Determine the magnitude and direction of the frictional force acting on the block if the magnitude of $\vec{P}$ is (a) 8.0 N, (b) 10 N, and (c) 12 N.

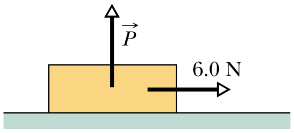


FIGURE 6-45 ■ Problem 20.

21. Pile of Sand A worker wishes to pile a cone of sand onto a circular area in his yard. The radius of the circle is R, and no sand is to spill onto the surrounding area (Fig. 6-46). If μ^{stat} is the static coefficient of friction between each layer of sand along the slope and the sand beneath it (along which is might slip), show that the greatest volume of sand that can be stored in this manner is $\pi\mu^{\text{stat}}R^3/3$. (The volume of a cone is $Ah/3$, where A is the base area and h is the cone's height.)

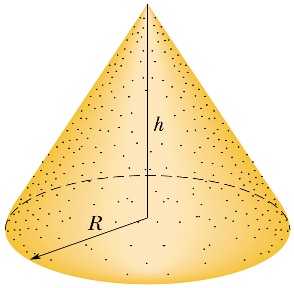


FIGURE 6-46 ■ Problem 21.

22. Worker and Crate A worker pushes horizontally on a 35 kg crate with a force of magnitude 110 N. The coefficient of static friction between the crate and the floor is 0.37. (a) What is the frictional force on the crate from the floor? (b) What is the maximum magnitude $f_{\text{max}}^{\text{stat}}$ of the static frictional force under the circumstances? (c) Does the crate move? (d) Suppose, next, that a second worker pulls directly upward on the crate to help out. What is the least vertical pull that will allow the first worker's 110 N push to move the crate? (e) If, instead, the second worker pulls horizontally to help out, what is the least pull that will get the crate moving?

23. A Crate is Dragged A 68 kg crate is dragged across a floor by pulling on a rope attached to the crate and inclined 15° above the horizontal. (a) If the coefficient of static friction is 0.50, what minimum force magnitude is required from the rope to start the crate moving? (b) If $\mu^{\text{kin}} = 0.35$, what is the magnitude of the initial acceleration of the crate?

FIGURE 6-47 ■ Problem 24.

24. Pig on a Slide A slide-loving pig slides down a certain 35° slide (Fig. 6-47) in twice the time it would take to slide down a frictionless 35° slide. What is the coefficient of kinetic friction between the pig and the slide?

25. Blocks A and B In Fig. 6-48 blocks A and B have weights of 44 N and 22 N, respectively. (a) Determine the minimum weight of block C to keep A from sliding if μ^{stat} between A and the table is 0.20. (b) Block C suddenly is lifted off A. What is the acceleration of block A if μ^{kin} between A and the table is 0.15?

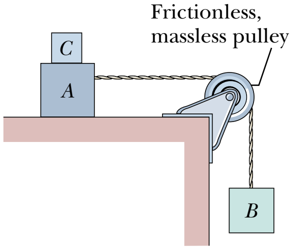


FIGURE 6-48 ■ Problem 25.

26. Block Pushed at an Angle A 3.5 kg block is pushed along a horizontal floor by a force $\vec{F}$ of magnitude 15 N at an angle $\theta = 40°$ with the horizontal (Fig. 6-49). The coefficient of kinetic friction between the block and the floor is 0.25. Calculate

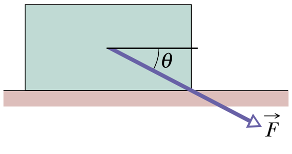


FIGURE 6-49 ■ Problem 26.

the magnitudes of (a) the frictional force on the block from the floor and (b) the acceleration of the block.

27. Mountain Side Figure 6-50 shows the cross section of a road cut into the side of a mountain. The solid line AA' represents a weak bedding plane along which sliding is possible. Block B directly above the highway is separated from uphill rock by a large crack (called a *joint*), so that only friction between the block and the bedding plane prevents sliding. The mass of the block is 1.8×10^7 kg, the *dip angle* θ of the bedding plane is 24°, and the coefficient of static friction between block and plane is 0.63. (a) Show that the block will not slide. (b) Water seeps into the joint and expands upon freezing, exerting on the block a force $\vec{F}$ parallel to AA'. What minimum value of F will trigger a slide?

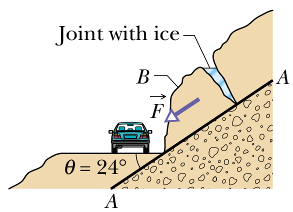


FIGURE 6-50 ■ Problem 27.

28. Penguin Sled A loaded penguin sled weighing 80 N rests on a plane inclined at 20° to the horizontal (Fig. 6-51). Between the sled and the plane, the coefficient of static friction is 0.25, and the coefficient of kinetic friction is 0.15. (a) What is the minimum magnitude of the force $\vec{F}$, parallel to the plane, that will prevent the sled from slipping down the plane? (b) What is the minimum magnitude F that will start the sled moving up the plane? (c) What value of F is required to move the sled up the plane at constant velocity?

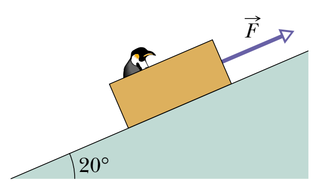


FIGURE 6-51 ■ Problem 28.

29. Block on a Table Block B in Fig. 6-52 weighs 711 N. The coefficient of static friction between block and table is 0.25; assume that the cord between B and the knot is horizontal. Find the maximum weight of block A for which the system will be stationary.

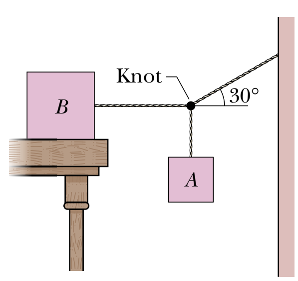


FIGURE 6-52 ■ Problem 29.

30. Force Parallel to a Surface A force $\vec{P}$, parallel to a surface inclined 15° above the horizontal, acts on a 45 N block, as shown in Fig. 6-53. The coefficients of friction for the block and surface are $\mu^{\text{stat}} = 0.50$ and $\mu^{\text{kin}} = 0.34$. If the block is initially at rest, determine the magnitude and direction of the frictional force acting on the block for magnitudes of $\vec{P}$ of (a) 5.0 N, (b) 8.0 N, and (c) 15 N.

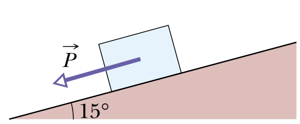


FIGURE 6-53 ■ Problem 30.

31. Body A–Body B Body A in Fig. 6-54 weighs 102 N, and body B weighs 32 N. The coefficients of friction between A and the incline are $\mu^{\text{stat}} = 0.56$ and $\mu^{\text{kin}} = 0.25$. Angle θ is 40°. Find the acceleration of A if (a) A is initially at rest, (b) A is initially moving up the incline, and (c) A is initially moving down the incline.

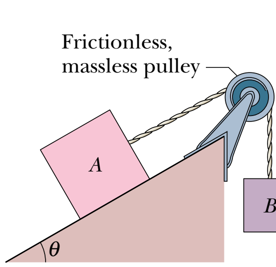


FIGURE 6-54 ■ Problem 31 and 32.

32. Two Blocks and a Pulley In Fig. 6-54, two blocks are connected over a pulley. The mass of block A is 10 kg and the coefficient of kinetic friction between A and the incline is 0.20. Angle θ of the incline is 30°. Block A slides down the incline at constant speed. What is the mass of block B?

33. Two Blocks Massless String Two blocks of weights 3.6 N and 7.2 N are connected by a massless string and slide down a 30° inclined plane. The coefficient of kinetic friction between the lighter block and the plane is 0.10; that between the heavier block and the plane is 0.20. Assuming that the lighter block leads, find (a) the magnitude of the acceleration of the blocks and (b) the tension in the string. (c) Describe the motion if, instead, the heavier block leads.

34. Box of Cheerios In Fig. 6-55, a box of Cheerios® and a box of Wheaties® are accelerated across a horizontal surface by a horizontal force $\vec{F}$ applied to the Cheerios box. The magnitude of the frictional force on the Cheerios box is 2.0 N, and the magnitude of the frictional force on the Wheaties box is 4.0 N. If the magnitude of $\vec{F}$ is 12 N, what is the magnitude of the force on the Wheaties box from the Cheerios box?

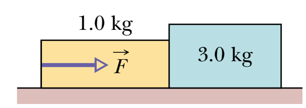


FIGURE 6-55 ■ Problem 34.

35. Blocks Not Attached The two blocks (with $m = 16$ kg and $M = 88$ kg) shown in Fig. 6-56 are not attached. The coefficient of static friction between the blocks is $\mu^{\text{stat}} = 0.38$, but the surface beneath the larger block is frictionless. What is the minimum magnitude of the horizontal force $\vec{F}$ required to keep the smaller block from slipping down the larger block?

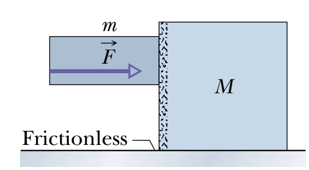


FIGURE 6-56 ■ Problem 35.

36. Aunts and Uncles In Fig. 6-57, a box of ant aunts (total mass $m_A = 1.65$ kg) and a box of ant uncles (total mass $m_B = 3.30$ kg) slide down an inclined plane while attached by a massless rod parallel to the plane. The angle of incline is $\theta = 30°$. The coefficient of kinetic friction between the aunt box and the incline is $\mu_A^{\text{kin}} = 0.226$; that between the uncle box and the incline is $\mu_B^{\text{kin}} = 0.113$. Compute (a) the tension in the rod and (b) the common acceleration of the two boxes. (c) How would the answers to (a) and (b) change if the uncles trailed the aunts?

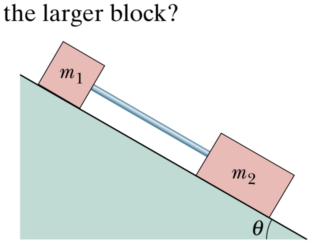


FIGURE 6-57 ■ Problem 36.

37. Block on a Slab A 40 kg slab rests on a frictionless floor. A 10 kg block rests on top of the slab (Fig. 6-58). The coefficient of static friction μ^{stat} between the block and the slab is 0.60, whereas their kinetic friction coefficient μ^{kin} is 0.40. The 10 kg block is pulled by a horizontal force with a magnitude of 100 N. What are the resulting accelerations of (a) the block and (b) the slab?

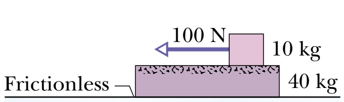


FIGURE 6-58 ■ Problem 37.

38. A Locomotive A locomotive accelerates a 25-car train along a level track. Every car has a mass of 5.0×10^4 kg and is subject to a frictional force $\vec{f}^{\text{stat}} = 250\vec{v}$, where the velocity $\vec{v}$ is in meters per second and the force f is in newtons. At the instant when the speed of the train is 30 km/h, the magnitude of its acceleration is 0.20 m/s^2. (a) What is the tension in the coupling between the first car and the locomotive? (b) If this tension is equal to the maximum force the locomotive can exert on the train, what is the steepest grade up which the locomotive can pull the train at 30 km/h?

39. Crate in a Trough In Fig. 6-59, a crate slides down an inclined right-angled trough. The coefficient of kinetic friction between the crate and the trough is μ^{kin}. What is the acceleration of the crate in terms of μ^{kin}, θ, and g?

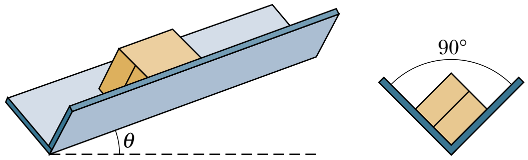


FIGURE 6-59 Problem 39.

40. Box of Sand An initially stationary box of sand is to be pulled across a floor by means of a cable in which the tension should not exceed 1100 N. The coefficient of static friction between the box and the floor is 0.35. (a) What should be the angle between the cable and the horizontal in order to pull the greatest possible amount of sand, and (b) what is the weight of the sand and box in that situation?

41. Boat with Engine Off A 1000 kg boat is traveling at 90 km/h when its engine is shut off. The magnitude of the frictional force $\vec{f}^{\text{kin}}$ between boat and water is proportional to the speed v of the boat; $\vec{f}^{\text{kin}} = 70v$, where v is in meters per second and $\vec{f}^{\text{kin}}$ is in newtons. Find the time required for the boat to slow to 45 km/h.

SEC. 6-5 ■ DRAG FORCE AND TERMINAL SPEED

42. Continuation of Problem 14 First reread the explanation of how the wind might drag desert stones across the playa. Now assume that Eq. 6-23 gives the magnitude of the air drag force on the typical 20 kg stone, which presents a vertical cross-sectional area to the wind of 0.040 m^2 and has a drag coefficient C of 0.80. Take the air density to be 1.21 kg/m^3, and the coefficient of kinetic friction to be 0.80. (a) In kilometers per hour, what wind speed V along the ground is needed to maintain the stone's motion once it has started moving? Because winds along the ground are retarded by the ground, the wind speeds reported for storms are often measured at a height of 10 m. Assume wind speeds are 2.00 times those along the ground, (b) For your answer to (a), what wind speed would be reported for the storm and is that value reasonable for a high-speed wind in a storm?

43. Missile Calculate the drag force on a missile 53 cm in diameter cruising with a speed of 250 m/s at low altitude, where the density of air is 1.2 kg/m^3. Assume $C = 0.75$.

44. Sky Diver The terminal speed of a sky diver is 160 km/h in the spread-eagle position and 310 km/h in the nosedive position. Assuming that the diver's drag coefficient C does not change from one position to the other, find the ratio of the effective cross-sectional area A in the slower position to that in the faster position.

45. Jet Vs. Prop-Driven Transport Calculate the ratio of the drag force on a passenger jet flying with a speed of 1000 km/h at an altitude of 10 km to the drag force on a prop-driven transport flying at half the speed and half the altitude of the jet. At 10 km the density of air is 0.38 kg/m^3, and at 5.0 km it is 0.67 kg/m^3. Assume that the airplanes have the same effective cross-sectional area and the same drag coefficient C.

SEC. 6-6 ■ APPLYING NEWTON'S LAWS

46. Block on an Incline Refer to Fig. 6-29. Let the mass of the block be 8.5 kg and the angle θ be 30°. The block moves at constant velocity. Find (a) the tension in the cord and (b) the normal force acting on the block. (c) If the cord is cut, find the magnitude of the block's acceleration.

47. Electron Moving Horizontally An electron with a speed of 1.2×10^7 m/s moves horizontally into a region where a constant vertical force of 4.5×10^{-16} N acts on it. The mass of the electron is 9.11×10^{-31} kg. Determine the vertical distance the electron is deflected during the time it has moved 30 mm horizontally.

48. Tarzan Tarzan, who weighs 820 N, swings from a cliff at the end of a 20 m vine that hangs from a high tree limb and initially makes an angle of 22° with the vertical. Immediately after Tarzan steps off the cliff, the tension in the vine is 760 N. Choose a coordinate system for which the x axis points horizontally away from the edge of the cliff and the y axis points upward. (a) What is the force of the vine on Tarzan in unit-vector notation? (b) What is the net force acting on Tarzan in unit-vector notation? What are the (c) magnitude and (d) direction of the net force acting on Tarzan? What are the (e) magnitude and (f) direction of Tarzan's acceleration?

49. Skier on a Rope Tow A 50 kg skier is pulled up a frictionless ski slope that makes an angle of 8.0° with the horizontal by holding onto a tow rope that moves parallel to the slope. Determine the magnitude of the force of the rope on the skier at an instant when (a) the rope is moving with a constant speed of 2.0 m/s and (b) the rope is moving with a speed of 2.0 m/s but that speed is increasing at a rate of 0.10 m/s^2.

50. Running Armadillo For sport, a 12 kg armadillo runs onto a large pond of level, frictionless ice with an initial velocity of 5.0 m/s along the positive direction of an x axis. Take its initial position on the ice as being the origin. It slips over the ice while being pushed by a wind with a force of 17 N in the positive direction of the y axis. In unit-vector notation, what are the animal's (a) velocity and (b) position vector when it has slid for 3.0 s?

51. Sphere Suspended from a Cord A sphere of mass 3.0×10^{-4} kg is suspended from a cord. A steady horizontal breeze pushes the sphere so that the cord makes a constant angle of 37° with the vertical. Find (a) the magnitude of that push and (b) the tension in the cord.

52. Skier in the Wind A 40 kg skier comes directly down a frictionless ski slope that is inclined at an angle of 10° with the horizontal while a strong wind blows parallel to the slope. Determine the magnitude and direction of the force of the wind on the skier if (a) the magnitude of the skier's velocity is constant, (b) the magnitude of the skier's velocity is increasing at a rate of 1.0 m/s^2. and (c) the magnitude of the skier's velocity is increasing at a rate of 2.0 m/s^2.

53. Jet Engine A 1400 kg jet engine is fastened to the fuselage of a passenger jet by just three bolts (this is the usual practice). Assume that each bolt supports one-third of the load. (a) Calculate the force on each bolt as the plane waits in line for clearance to take off. (b) During flight, the plane encounters turbulence, which suddenly imparts an upward vertical acceleration of 2.6 m/s^2 to the plane. Calculate the force on each bolt now.

54. Pulling a Crate A worker drags a crate across a factory floor by pulling on a rope tied to the crate (Fig. 6-60). The worker exerts a force of 450 N on the rope, which is inclined at 38° to the horizontal, and the floor exerts a horizontal force of 125 N that opposes the motion. Calculate the magnitude of the acceleration of the crate if (a) its mass is 310 kg and (b) its weight is 310 N.

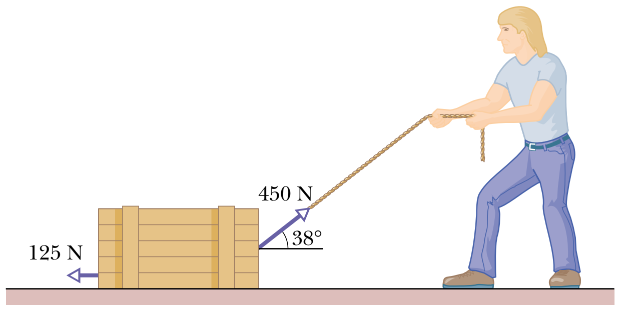


FIGURE 6-60 ■ Problem 54.

55. Motorcycle Rider A motorcycle and 60.0 kg rider accelerate at 3.0 m/s^2 up a ramp inclined 10° above the horizontal. (a) What is the magnitude of the net force acting on the rider? (b) What is the magnitude of the force on the rider from the motorcycle?

56. One on an Incline—One Hanging A block of mass m_A = 3.70 kg on a frictionless inclined plane of angle 30.0° is connected by a cord over a massless, frictionless pulley to a second block of mass m_B = 2.30 kg hanging vertically (Fig. 6-61). What are (a) the magnitude of the acceleration of each block and (b) the direction of the acceleration of the hanging block? (c) What is the tension in the cord?

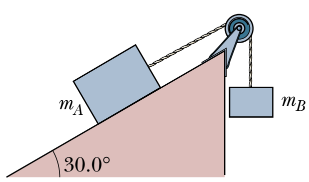


FIGURE 6-61 ■ Problem 56.

57. Pencil Box In Fig. 6-62, a 1.0 kg pencil box on a 30° frictionless incline is connected to a 3.0 kg pen box on a horizontal frictionless surface. The pulley is frictionless and massless. (a) If the magnitude of $\vec{F}$ is 2.3 N, what is the tension in the connecting cord? (b) What is the largest value that the magnitude of $\vec{F}$ may have without the connecting cord becoming slack?

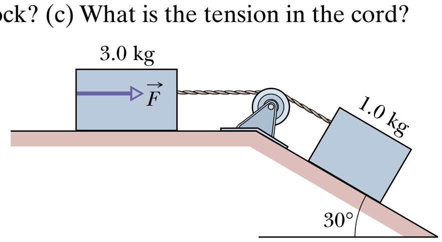
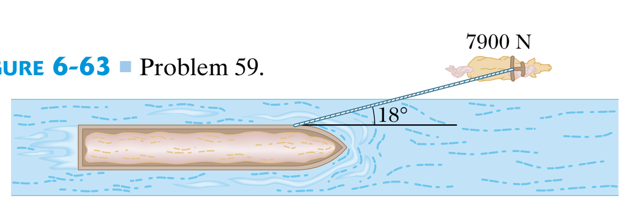


FIGURE 6-62 ■ Problem 57.

58. Projected Up an Incline A block is projected up a frictionless inclined plane with initial speed v_1 = 3.50 m/s. The angle of incline is θ = 32.0°. (a) How far up the plane does it go? (b) How long does it take to get there? (c) What is its speed when it gets back to the bottom?

59. Horse-Drawn Barge In earlier days, horses pulled barges down canals in the manner shown in Fig. 6-63. Suppose the horse pulls on the rope with a force of 7900 N at an angle of 18° to the direction of motion of the barge, which is headed straight along the canal. The mass of the barge is 9500 kg, and its acceleration is 0.12 m/s^2. What are the (a) magnitude and (b) direction of the force on the barge from the water?

FIGURE 6-63 ■ Problem 59.



60. Lifting a Block In Fig. 6-64, a 5.00 kg block is pulled along a horizontal frictionless floor by a cord that exerts a force of magnitude F = 12.0 N at an angle θ = 25.0° above the horizontal. (a) What is the magnitude of the block's acceleration? (b) The force magnitude F is slowly increased. What is its value just before the block is lifted (completely) off the floor? (c) What is the magnitude of the block's acceleration just before it is lifted (completely) off the floor?

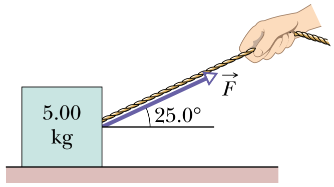


FIGURE 6-64 ■ Problem 60.

61. A Rope Must Sag A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m, as shown in Fig. 6-65. A horizontal force $\vec{F}$ is applied to one end of the rope. (a) Show that the rope *must* sag, even if only by an imperceptible amount. Then, assuming the sag is negligible, find (b) the acceleration of rope and block, (c) the force on the block from the rope, and (d) the tension in the rope at its midpoint.

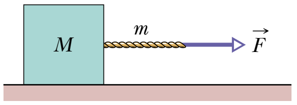


FIGURE 6-65 ■ Problem 61.

62. Crate at Constant Speed In Fig. 6-66, a 100 kg crate is pushed at constant speed up the frictionless 30.0° ramp by a horizontal force $\vec{F}$. What are the magnitudes of (a) $\vec{F}$ and (b) the force on the crate from the ramp?

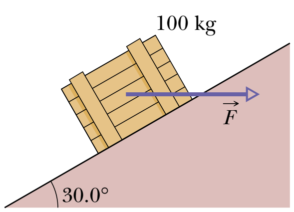


FIGURE 6-66 ■ Problem 62.

63. Alpine Cable Car Figure 6-67 shows a section of an alpine cable-car system. The maximum permissible mass of each car with occupants is 2800 kg. The cars, riding on a support cable, are pulled by a second cable attached to each pylon (support tower); assume the cables are straight. What is the difference in tension between adjacent sections of pull cable if the cars are at the maximum permissible mass and are being accelerated up the 35° incline at 0.81 m/s^2?

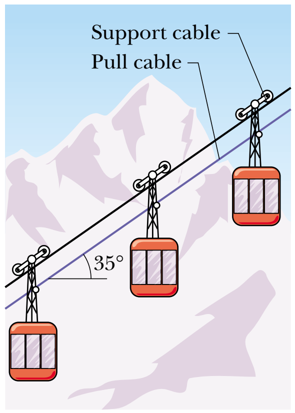


FIGURE 6-67 ■ Problem 63.

64. Bobsled Run During an Olympic bobsled run, the Jamaican team makes a turn of radius 7.6 m at a speed of 96.6 km/h. What is their acceleration in g-units? (1 g-unit = 9.8 m/s^2.

65. Grand Prix Suppose the coefficient of static friction between the road and the tires on a Formula One car is 0.6 during a Grand Prix auto race. What speed will put the car on the verge of sliding as it rounds a level curve of 30.5 m radius?

66. Roller Coaster A roller-coaster car has a mass of 1200 kg when fully loaded with passengers. As the car passes over the top of a circular hill of radius 18 m, its speed is not changing. What are the magnitude and direction of the force of the track on the car at the top of the hill if the car's speed is (a) 11 m/s and (b) 14 m/s?

67. Flat Track What is the smallest radius of an unbanked (flat) track around which a bicyclist can travel if her speed is 29 km/h and the coefficient of static friction between tires and track is 0.32?

68. Amusement Park Ride An amusement park ride consists of a car moving in a vertical circle on the end of a rigid boom of negligible mass. The combined weight of the car and riders is 5.0 kN, and the radius of the circle is 10 m. What are the magnitude and direction of the force of the boom on the car at the top of the circle if the car's speed there is (a) 5.0 m/s and (b) 12 m/s?

69. Puck on a Table A puck of mass *m* slides on a frictionless table while attached to a hanging cylinder of mass *M* by a cord through a hole in the table (Fig. 6-68). What speed keeps the cylinder at rest?

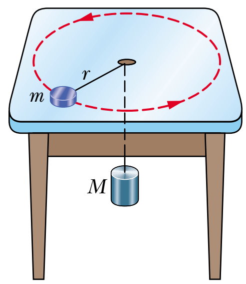


FIGURE 6-68 ■ Problem 69.

70. Bicyclist A bicyclist travels in a circle of radius 25.0 m at a constant speed of 9.00 m/s. The bicycle–rider mass is 85.0 kg. Calculate the magnitudes of (a) the force of friction on the bicycle from the road and (b) the *net* force on the bicycle from the road.

71. Student on Ferris Wheel A student of weight 667 N rides a steadily rotating Ferris wheel (the student sits upright). At the highest point, the magnitude of the normal force $\vec{N}$ on the student from the seat is 556 N. (a) Does the student feel "light" or "heavy" there? (b) What is the magnitude of $\vec{N}$ at the lowest point? (c) What is the magnitude *N* if the wheel's speed is doubled?

72. Old Streetcar An old streetcar rounds a flat corner of radius 9.1 m, at 16 km/h. What angle with the vertical will be made by the loosely hanging hand straps?

73. Flying in a Circle An airplane is flying in a horizontal circle at a speed of 480 km/h. If its wings are tilted 40° to the horizontal, what is the radius of the circle in which the plane is flying? (See Fig. 6-69.) Assume that the required force is provided entirely by an "aerodynamic lift" that is perpendicular to the wing surface.

74. High-Speed Railway A high-speed railway car goes around a flat, horizontal circle of radius 470 m at a constant speed. The magnitudes of the horizontal and vertical components of the force of the car on a 51.0 kg passenger are 210 N and 500 N, respectively. (a) What is the magnitude of the net force (of *all* the forces) on the passenger? (b) What is the speed of the car?

75. Ball Connected to a Rod As shown in Fig. 6-70, a 1.34 kg ball is connected by means of two massless strings to a vertical, rotating rod. The strings are tied to the rod and are taut. The tension in the upper string is 35 N. (a) Draw the free-body diagram for the ball. What are (b) the tension in the lower string, (c) the net force on the ball, and (d) the speed of the ball?

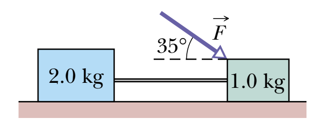


FIGURE 6-70 ■ Problem 75.

76. Pushing the Second Block A 2.0 kg block and a 1.0 kg block are connected by a string and are pushed across a horizontal surface by a force applied to the 1.0 kg block as shown in Fig. 6-71. The coefficient of kinetic friction between the blocks and the horizontal surface is 0.20. If the magnitude of $\vec{F}$ is 20 N, what is the tension in the string that connects the blocks?

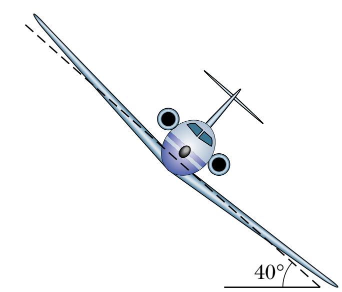


FIGURE 6-71 ■ Problem 76.

Additional Problems

77. Engineering a Highway Curve If a car goes through a curve too fast, the car tends to slide out of the curve, as discussed in Touchstone Example 6-8. For a banked curve with friction, a frictional force acts on a fast car to oppose the tendency to slide out of the curve; the force is directed down the bank (in the direction in which water would drain). Consider a circular curve of radius $R = 200$ m and bank angle θ, where the coefficient of static friction between tires and pavement is μ^{stat}. A car is driven around the curve as shown in Fig. 6-72. (a) Find an expression for the car speed v^{max} that puts the car on the verge of sliding out. (b) On the same graph, plot v^{max} versus angle θ for the range 0° to 50°, first for $\mu^{stat} = 0.60$ (dry pavement) and then for $\mu^{stat} = 0.050$ (wet or icy pavement). In kilometers per hour, evaluate v^{max} for a bank angle of $\theta = 10°$ and for (c) $\mu^{stat} = 0.60$ and (d) $\mu^{stat} = 0.050$. (Now you can see why accidents occur in highway curves when wet or icy conditions are not obvious to drivers, who tend to drive at normal speeds.)

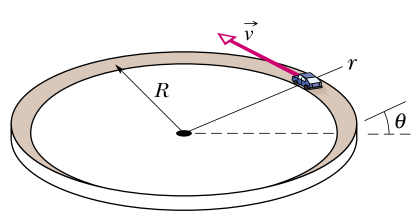


FIGURE 6-72 ■ Problem 77.

78. Change in Conditions In the early afternoon, a car is parked on a street that runs down a steep hill, at an angle of 35.0° relative to the horizontal. Just then the coefficient of static friction between the tires and the street surface is 0.725. Later, after nightfall, a sleet storm hits the area, and the coefficient decreases due to both the ice and a chemical change in the road surface because of the temperature decrease. By what percentage must the coefficient decrease if the car is to be in danger of sliding down the street?

79. Moving People at the Airport While traveling, I passed through Charles de Gaulle Airport in Paris, France. The airport has some interesting devices, including a "people mover"—a moving strip of rubber like a horizontal escalator without steps. It became interesting when


FIGURE 6-73 ■ Problem 79.

the mover entered a plastic tube bent up at an angle to take me to the next terminal. I managed to get a photograph of it (Fig. 6-73). If you were building this people mover for the architect, what material would you choose for the surface of the moving strip? (*Hint*: You want to be sure that people standing on the strip do not tend to slide down it. Figure out what coefficient of friction you need to keep from sliding down and then look up coefficients of friction in tables in reference books to get a material appropriate for the slipperiest shoes.)

80. Expert Witness You testify as an *expert witness* in a case involving an accident in which car A slid into the rear of car B, which was stopped at a red light along a road headed down a hill (Fig. 6-74). You find that the slope of the hill is $\theta = 12.0°$, that the cars were separated by distance $d = 24.0$ m when the driver of car A put the car into a slide (it lacked any automatic anti-brake-lock system), and that the speed of car A at the onset of braking was $v_1 = 18.0$ m/s. With what speed did car A hit car B if the coefficient of kinetic friction was (a) 0.60 (dry road surface) and (b) 0.10 (road surface covered with wet leaves)?

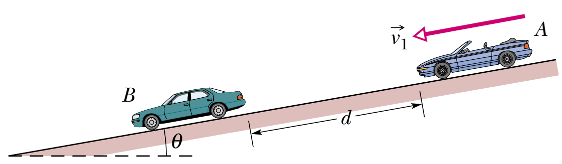


FIGURE 6-74 ■ Problem 80.

81. Luggage Transport Luggage is transported from one location to another in an airport by a conveyor belt. At a certain location, the belt moves down an incline that makes an angle of 2.5° with the horizontal. Assume that with such a slight angle there is no slipping of the luggage. Determine the magnitude and direction of the frictional force by the belt on a box weighing 69 N when the box is on the inclined portion of the belt for the following situations: (a) The belt is stationary. (b) The belt has a speed of 0.65 m/s that is constant. (c) The belt has a speed of 0.65 m/s that is increasing at a rate of 0.20 m/s^2. (d) The belt has a speed of 0.65 m/s that is decreasing at a rate of 0.20 m/s^2. (e) The belt has a speed of 0.65 m/s that is increasing at a rate of 0.57 m/s^2.

82. Bolt on a Rod A bolt is threaded onto one end of a thin horizontal rod, and the rod is then rotated horizontally about its other end. An engineer monitors the motion by flashing a strobe lamp onto the rod and bolt, adjusting the strobe rate until the bolt appears to be in the same eight places during each full rotation of the rod (Fig. 6-75). The strobe rate is 2000 flashes per second; the bolt has mass 30 g and is at radius 3.5 cm. What is the magnitude of the force on the bolt from the rod?

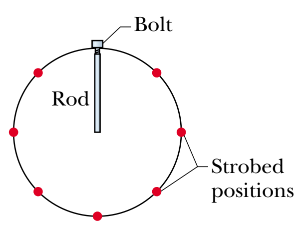


FIGURE 6-75 ■ Problem 82.

83. From the Graph A 4.10 kg block is pushed along a floor by a constant applied force that is horizontal and has a magnitude of 40.0 N. Figure 6-76 gives the block's speed v versus time t as the block moves along an x axis on the floor. What is the coefficient of kinetic friction between the block and the floor?

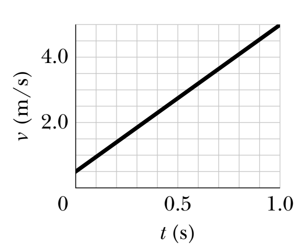


FIGURE 6-76 ■ Problem 83.

84. Tapping a Rolling Ball Figure 6-77 shows a multiple exposure strobe photograph of a ball rolling on a horizontal table. The image marked with a heavy arrow occurs at time $t = 0$ and the ball moves to the right at that instant. Each image of the ball occurs 1/30 s later than the one immediately to its left. Using the coordinate system shown in Fig. 6-77, sketch qualitatively accurate (i.e., we don't care about the values but we do care about the shape) graphs of each of the following variables as a function of time: x coordinate, y coordinate, x-component of velocity, y-component of velocity, x-component of the net force on the ball, and y-component of the net force on the ball. The time at which the "kink" in the path occurs is $t = t_1$. Be sure to note this important time on your graphs.

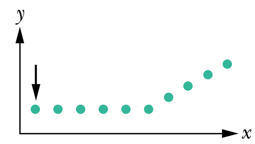


FIGURE 6-77 ■ Problem 84.

85. Ball on a Ramp Figure 6-78 shows a multiple-exposure photograph of a ball rolling up an inclined plane. (The ball is rolling in the dark, the camera lens is held open, and a brief flash occurs every 3/4 sec, four times in total.) The leftmost ball corresponds to an instant just after the ball was released. The rightmost ball is at the highest point the ball reaches.

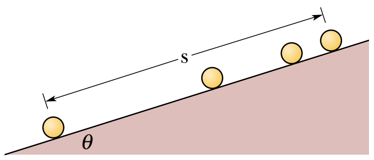


FIGURE 6-78 ■ Problem 85.

(a) Copy this picture on your paper. Draw an arrow at each of the four ball locations to indicate the velocity of the ball at that instant. Make the relative lengths of the arrows indicate the relative magnitudes of the velocities. Explain what is happening ("tell the story" of the picture).
(b) For the instant of time when the ball is at the second position shown from the left, draw a free-body diagram for the ball and indicate all forces acting on it.
(c) If your force diagram doesn't include an arrow pointing up the ramp, explain why the ball keeps rolling up the ramp.
(d) If the mass of the ball is m, what is its acceleration?
(e) If the angle θ is equal to 30°, how long is the distance s?

86. Motion Graphs (a) Suppose you were to push on a bowling ball on a smooth floor at a 45° angle as shown in Fig. 6-79*a* and then leave it alone to roll. Sketch a graph frame like that shown in Fig. 6-79*a*, and then sketch a prediction of the ball's motion both before and after you stop pushing. Note on your graph the point at which you stop pushing and explain the basis for your prediction.

(b) If the initial speed of the ball is 3.5 m/s, what is the magnitude of the x-component of velocity, v_{1x}? Is it positive or negative? What is v_{1y}? Is it positive or negative?
(c) Suppose you and your partner were to tap the ball *very* rapidly. Each set of taps is at right angles to the other as shown in Fig. 6-79*b*. Sketch a graph frame like that shown in Fig. 6-79*b*, and sketch a prediction of the ball's motion on your graph. Explain the basis for your prediction.
(d) Suppose a rocket ship is thrust from a tower at a constant acceleration that has a magnitude of about 9.8 m/s^2 in the x direction and is allowed to fall freely toward Earth in the y direction. Sketch a graph frame like that shown in Fig. 6-79*c*, and sketch a prediction

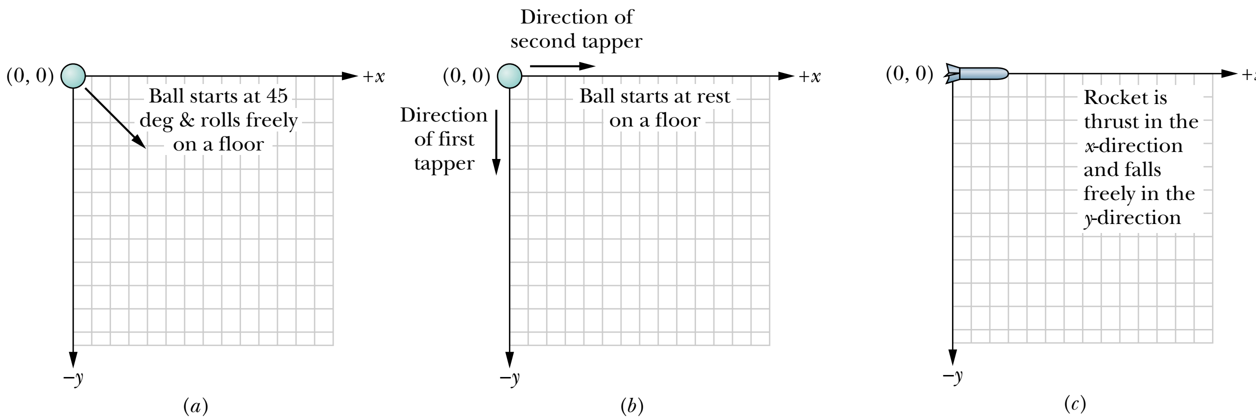


FIGURE 6-79 ■ Problem 86.

of the rocket's motion on your graph. Explain the basis for your prediction.

87. Wanda Lifts Weights Wanda is working out with weights and manages to lift a light rope with a 10 kg mass hanging from it. When she is through lifting the right side of the rope and the left side of the rope each make an angle of $\theta = 15°$ with respect to the horizontal. See Fig. 6-80.

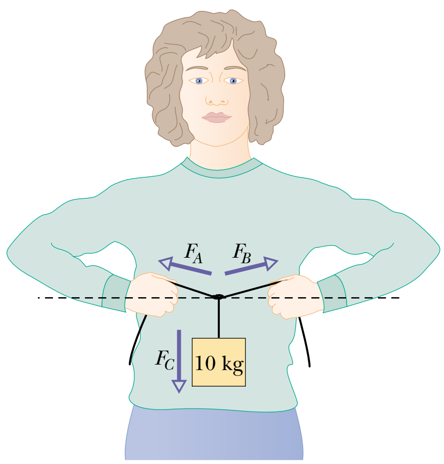


FIGURE 6-80 ■ Problem 87.

(a) Draw a free-body diagram showing the forces on the midpoint of the rope (where it is the lowest).
(b) What are the magnitudes of each of her pulling forces $\vec{F}_A$ and $\vec{F}_B$?
(c) How hard would Wanda have to pull with each hand to raise the 10 kg mass so that the rope becomes perfectly horizontal?

88. Constant Speed on a Race Track The race track shown in Fig. 6-81 has two straight sections connected by semicircular ends. A car is traveling in a clockwise direction around the track at a constant speed. Assume that air resistance is negligible.

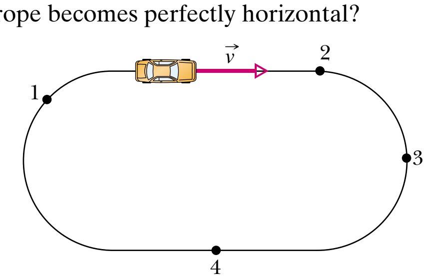


FIGURE 6-81 ■ Problem 88.

Draw three sketches of the race track.
(a) On the first sketch show the velocity vector at each of the numbered points 1–4. Make the relative lengths of the vectors consistent with the relative magnitudes of the velocity at the four points.
(b) On the second sketch show the acceleration vectors at each of the numbered points 1–4. Make the relative lengths of the vectors consistent with the relative magnitudes of the acceleration at the four points. *Hint*: Use the techniques developed in Section 4-3 to draw vectors representing the acceleration or change in velocity.
(c) Horizontal forces are needed to maintain the car's motion around the track. These are provided by road friction and by road forces where the track is banked at the curves. On the third sketch show the vectors representing the required horizontal forces at each of the numbered points 1–4. Make the relative lengths of the vectors consistent with the relative magnitudes of the force at the four points.

Note: This exercise is adapted from A. Arons, *Homework and Test Questions for Introductory Physics Teaching* (New York: Wiley, 1994), Chapter 3.

89. Pulling on the Ceiling Suppose a person exerts a force of 50 N on one end of a rope as shown in Fig. 6-82.

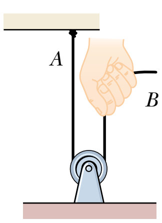


FIGURE 6-82 ■ Problem 89.

(a) What are the magnitude and direction of the force at point A exerted on the rope by the ceiling?
(b) What are the magnitude and direction of the force exerted on the ceiling by the rope? How does the force get transmitted from one end of the rope to the other? What does the stretching of the rope have to do with this?
(c) What are the magnitude and direction of the force the rope exerts on the person's hand at point B?
(d) Draw a diagram with vector arrows indicating the *relative magnitudes* and *directions* of the forces the rope exerts on the ceiling at point A and the force the rope exerts on the person's hand at point B.

90. Thinking About Normal Forces Suppose you push on a flexible piece of stretched fabric with a force of 5.0 N as shown in Fig. 6-83*a*. The fabric assembly is fixed and does not move.

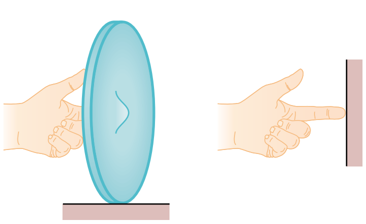


FIGURE 6-83*a* ■ Problem 90.

(a) What are the direction and magnitude of the normal force exerted back on the finger by the sheet? Is this normal force zero? If not, is it larger, smaller, or the same as the normal force would be if the fabric did not stretch?
(b) Discuss the role the stretching of the fabric plays in regard to this normal force.

(c) Suppose you push in the same way on a wall as shown in Fig. 6-83*b*. What are the direction and magnitude of the normal force exerted back on the finger by the wall?
(d) Does the wall stretch noticeably? What causes the wall to be able to exert a force on the finger? How does the wall "know" what force to exert back on the hand?

91. Forces in a Car Suppose you are sitting in a car that is speeding up. Assume the car has rear-wheel drive.

(a) Draw free-body diagrams for your own body, the seat in which you are sitting (apart from the car), the car (apart from the seat), and the road surface where the tires and the road interact.
(b) Describe each force in words; show larger forces with longer arrows.
(c) Identify the third-law pairs of forces.
(d) Explain carefully in your own words the origin of the force imparting acceleration to the car.

92. The Sliding Pizza One day I was coming home late from work and stopped to pick up a pizza for dinner. I put the pizza box on the dashboard of my car and pushed it forward against the windshield and left against the steering wheel to prevent it from falling. (See Fig. 6-84.) Before I started driving, I realized that the box could still slide to the right or back toward the seat. When driving, do I have to worry more about it sliding when I turn left or when I turn right? Do I have to worry more when I speed up or when I slow down? Explain your answer in terms of the physics you have learned.

FIGURE 6-84 ■ Problem 92.

93. The Farmer and the Donkey An old Yiddish joke is told about a farmer in Chelm, a town famous for the lack of wisdom of its inhabitants. One day the farmer was going to the mill to have a bag of wheat ground into flour. He was riding to the mill on his donkey, with the sack of wheat thrown over the donkey's back behind him. On his way, he met a friend. His friend chastised him. "Look at you! You must weigh 200 pounds and that sack of flour must weigh 100. That's a very small donkey! Together, you're too much weight for him to carry!" On his way to the mill the farmer thought about what his friend had said. On his way home, he passed his friend again, confident that this time the friend would be satisfied. The farmer still rode the donkey, but this time he carried the 100 pound bag of flour on his own shoulder!

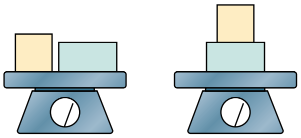


FIGURE 6-85 ■ Problem 93.

Our common sense and intuitions seem to suggest that it doesn't matter how you arrange things; they'll weigh the same. Let's be certain that the Newtonian framework we are developing yields our intuitive result. Analyze the problem by considering the simplified picture shown in Fig. 6-85. Two blocks rest on a scale. One block weighs 10 N, the other 25 N. In case 1 the blocks are arranged on the scale as shown in the figure on the left. In case 2 the blocks are arranged as shown on the right. Each system has come to rest. Analyze the forces on the blocks and on the scale in the two cases by isolating the objects—each block and the scale—and show that according to the principles of Newton's laws, the total force exerted on the scale by both blocks together must be the same in both cases. (*Note*: It's not enough to say: "They have to be the same." That's just restating your intuition. We need to see that *reasoning using only the principles of our Newtonian framework* leads to the same conclusion.)

94. Pulling Two Boxes (a) A worker is trying to pull a pair of heavy crates along the floor with a rope. The rope is attached to the lower crate, which has a mass M. The upper crate has a mass m and the coefficient of static friction between the crate and the floor is μ^{stat}. If the rope is held at an angle θ as shown in Fig. 6-86, what is the magnitude of the maximum force the worker can exert without the box beginning to slide?

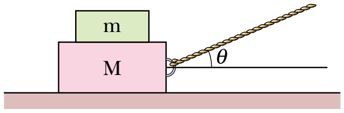


FIGURE 6-86 ■ Problem 94.

(b) The worker knows that the lower crate has a mass of 50 kg and the upper crate has a mass of 10 kg. She finds that if she pulls with a force of 120 N at an angle of 60° she can keep the crates sliding at a constant speed. Can you use this information to find the coefficient of kinetic friction μ^{kin} between the lower crate and the floor? If you can, do it. If you can't, explain why not.
(c) In a different situation, she finds that she can pull a lower crate of mass 30 kg and an upper crate of mass 7.5 kg with a constant velocity of 50 cm/s pulling at an angle of 45°. Can you use this information to find the coefficient of kinetic friction μ^{kin} between the lower crate and the floor? If you can, do it. If you can't, explain why not.

95. Tricking Bill A student, whom we will call Bill, was about to go out on a date when his roommate, Bob, asked him to hold a pail against the ceiling with a broom for a moment. After Bill complied, the roommate mentioned that the pail was filled with water and left. See Fig. 6-87.

(a) Draw a free-body diagram showing all the forces acting on the pail. For each force, be sure you identify the kind of force and the object whose interaction with the pail is responsible for the force.
(b) Suppose Bill wants to slide the pail a few feet to one side so he can get to a chair in the room. Are there any other forces not specified in your answer to part (a) that become relevant?

FIGURE 6-87 ■ Problem 95.

(c) Suppose the pail weighs 1 pound, it has 6 pounds of water in it, the maximum coefficient of static friction $\mu_{\text{broom}}^{\text{stat}}$ between the broom and pail is 0.3, and the maximum coefficient of static friction $\mu_{\text{ceiling}}^{\text{stat}}$ between the pail and the ceiling is 0.5. Can Bill slide the pail? Explain.

96. Friction is Doing *What?* A large block is resting on the table. On top of that block rests another, smaller block, as shown in Fig. 6-88.

You press on the larger block to start it moving. After about 0.25 s, it is moving at a constant speed and the block on the top is not slipping.

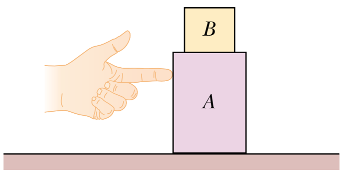


FIGURE 6-88 ■ Problem 96.

(a) Draw a labeled free-body diagram for the two blocks *during the time when they are accelerating*, specifying all the forces acting on the blocks. (Be sure to specify the type of force and the object causing each force.) Wherever you can, compare the magnitudes of forces.
(b) Draw a labeled free-body diagram for the two blocks *during the time when they are moving at a constant speed*, specifying all the forces acting on the blocks. (Be sure to specify the type of force and the object causing each force.) Wherever you can, compare the magnitudes of forces.
(c) Suppose the bottom block has a mass of 0.4 kg and the coefficient of friction between the block and the table is 0.3. The top block has a mass of 0.1 kg and the coefficient of friction between the two blocks is 0.2. What force do you need to exert to keep the blocks moving at a constant speed of 10 cm/s? (You may use $g =$ 10 N/kg and you may treat kinetic and static friction as the same.)

97. Al and George Pushing the Truck George left the lights on in his truck while at a truck stop in Kansas and his battery went dead. Fortunately, his friend Al is there, although Al is driving his Geo Metro. Since the road is very flat, George is able to convince Al to give his truck a long, slow push to get it up to 20 miles/hour. At this speed, George can engage the truck's clutch and the truck's engine should start up. (See Fig. 6-89.)

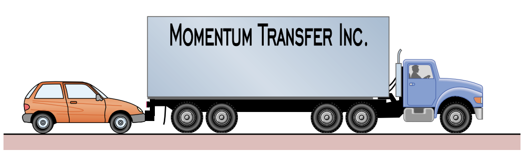


FIGURE 6-89 ■ Problem 97.

(a) Al begins to push the truck. It takes him 5 minutes to get the truck up to a speed of 20 miles/hour. Draw separate free-body diagrams for the Geo and for the truck during the time that Al's Geo is pushing the truck. List all the horizontal forces in order by magnitude from largest to smallest. If any are equal, state that explicitly. Explain your reasoning.
(b) If the truck is accelerating uniformly over the 5 minutes, how far does Al have to push the truck before George can engage the clutch?
(c) Suppose the mass of the truck is 4000 kg, the mass of the car is 800 kg, and the coefficient of static friction between the vehicles and the road is 0.1. At one instant when they are trying to get the truck moving, the car is pushing the truck and exerting a force of 1000 N, but neither vehicle moves. What is the static frictional force between the truck and the road? Explain your reasoning.

98. Pushing a Carriage A young man is pushing a baby carriage at a constant velocity along a level street. A friend comes by to chat and the young man lets go of the carriage. It rolls on for a bit, slows, and comes to a stop. At time $t = 0$ the young man is walking with a constant velocity. At time t_1 he releases the carriage. At time t_2 the carriage comes to rest. Sketch qualitatively accurate (i.e., we don't care about the values but we do care about the shape) graphs of each of the following variables versus time:

(a) position of the carriage, (b) velocity of the carriage, (c) acceleration of the carriage, (d) net force on the carriage, (e) force the man exerts on the carriage, (f) force of friction on the carriage. Be sure to note the important times $t = 0, t_1$, and t_2 on the time axes of your graphs. Take the positive direction to be the direction in which the man was initially walking.

99. A Two-Stage Rocket Students in a school rocketry club have prepared a two-stage rocket. The rocket has two small engines. The first will fire for a time, getting the rocket up partway. Then the first-stage engine drops off, revealing a second engine. After a little time, that engine will fire and take the rocket up even higher.

The rocket starts firing its engines at a time $t = 0$. From that instant, it begins to move upward with a constant acceleration. This continues until time t_1. The rocket drops the first stage and continues upward briefly until time t_2, at which point the second stage begins to fire and the rocket again accelerates upward, this time with a larger (but again constant) acceleration. Sometime during this second period of acceleration, our recording apparatus stops.

Sketch qualitatively accurate (i.e., we don't care about the values but we do care about the shape) graphs of the height of the rocket, y, its velocity, v, its acceleration, a, the force on the rocket that results from the firing of the engine, $\vec{F}$, and the net force on the rocket, $\vec{F}^{\,\text{net}}$. Take the positive direction as upward. Be sure to note times $t = 0, t_1$, and t_2 on the time axes of your graphs.

100. Pushing a Box A worker is pushing a cart along the floor. At first, the worker has to push hard in order to get the cart moving. After a while, it is easier to push. Finally, the worker has to pull back on the cart in order to bring it to a stop before it hits the wall. The force exerted by the worker on the cart is purely horizontal. Take the direction the worker is going as positive.

Figure 6-90 shows graphs of some of the physical variables of the problem. Match the graphs with the variables in the list at the left below. You may use a graph more than once or not at all. *Note*: The time axes are to the same scale, but the ordinates y axes are not.

(a) Friction force
(b) Force exerted by the worker
(c) Net force
(d) Acceleration
(e) Velocity

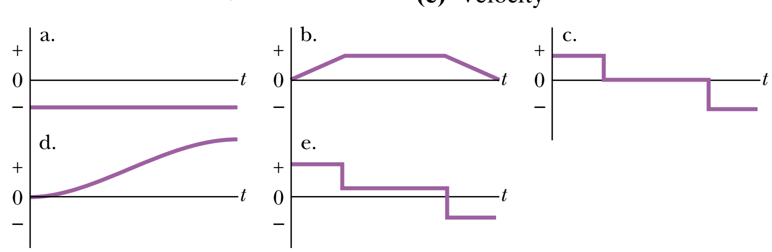


FIGURE 6-90 ■ Problem 100.

101. Comparing a Light and Heavy Object Consider a metal sphere two inches in diameter and a feather. For each quantity in the list below, indicate the relation between the quantity for the sphere and feather. Is it the same, greater, or lesser? Explain in each case why you gave the answer you did.

(a) The gravitational force
(b) The time it will take to fall a given distance in air
(c) The time it will take to fall a given distance in vacuum
(d) The total force on the object when falling in vacuum
(e) The total force on the object when falling in air

102. Hitting the Green A golfer is trying to hit a golf ball onto the green as shown in Fig. 6-91. The green is a horizontal distance s from his tee and it is up on the side of a hill a height h above his tee. When he strikes the ball it leaves the tee at an angle θ to the horizontal. He wants to know with what speed, v_1, the ball must leave the tee in order to reach the height h at the distance s.

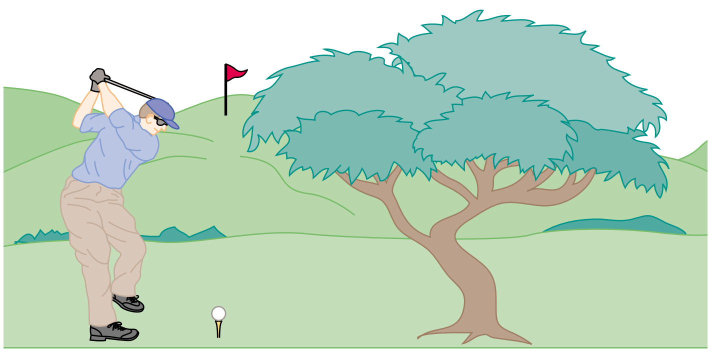


FIGURE 6-91 ■ Problem 102.

(a) Once he has struck the ball, what controls its motion? Write the equations that determine the vector acceleration of the golf ball afler it leaves the tee. Be sure to specify your coordinate system. For this part of the problem you may ignore air resistance.
(b) Solve the equations you have written in (a) to obtain expressions that can be evaluated to give the position of the ball at any time, t.
(c) If the golfer wants his ball to land in the right place, he must hit it so that it leaves the tee with the right speed. Explain how he can calculate it. (Again, you may ignore air resistance.) Find an equation for the initial speed in terms of the problem's givens.
(d) If the ball leaves the tee at an angle of 30°, $s = 100$ m, and $h = 10$ m, find the speed with which the ball leaves the tee.
(e) Now consider the effect of air resistance. Suppose that a good model for the force of air resistance is Newton's drag law,

$$\vec{F} = -b\,|v|\,\vec{v}$$

where $|v|$ stands for the absolute value of the velocity—the speed. Consider three points on the ball's trajectory: halfway up, at its highest point, and halfway down. Discuss the direction of the resistance force at each point. Qualitatively (do not attempt a calculation!), what will the effect of air resistance be on the ball's motion?

103. Air Resistance 1: Dimensional Analysis We know that as an object passes through the air, the air exerts a resistive force on it. Suppose we have a spherical object of radius R and mass m. What might the force plausibly depend on?

- It might depend on the properties of the object. The only ones that seem relevant are m and R.
- It might depend on the object's coordinate and its derivatives: $x, v, a, \ldots$.
- It might depend on the properties of the air, such as the density, ρ.

(a) Explain why it is plausible that the force the air exerts on a sphere depends on R but implausible that it depends on m.
(b) Explain why it is plausible that the force the air exerts depends on the object's speed through it, $|v|$, but not on its position, x, or acceleration, a.
(c) Dimensional analysis is the use of units (e.g., meters, seconds, or newtons) associated with quantities to reason about the relationship between the quantities. Using dimensional analysis, construct a plausible form for the force that air exerts on a spherical body moving through it.

104. Counterweights The use of counterweights to help devices move up and down with a minimum of effort is common in engineering. For example, counterweights are used to help people open and close old-fashioned windows and to move up and down in elevators. Imagine that an engineer working for the Disney Epcot Center is asked to design a ride that allows people to travel up and down a sloped hill to get a view of the entire Epcot Center while other tourists move straight up and down an artificial cliff on the other side of the incline. Our engineer builds a small prototype of his device using a low-friction cart on an inclined track attached to a falling mass. His goal is to see whether he can actually apply Newton's laws to this situation and if it is okay to neglect the effects of friction.

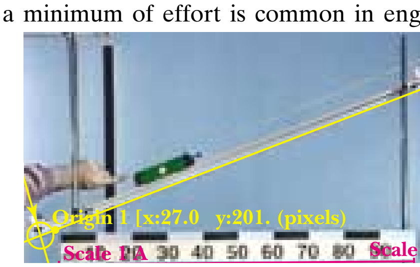


FIGURE 6-92a ■ Problem 104.

In this exercise you will analyze data collected from a digital movie of the situation discussed above and shown in Fig. 6-92a. If you have access to VideoPoint you can view the digital movie yourself. It is entitled PASCO098. Your instructor may provide you with a different but similar movie. The cart in PASCO098 has a mass $m_c = .510$ kg and is accelerated up a ramp that has a 21° incline. A string attached to the cart exerts a force on it. The string transmits a force to the cart because its other end is attached by means of a pulley to a falling mass of $m_f = .184$ kg.

Table 6-3 contains position vs. time measurements for the cart in PASCO098 along an x axis. The x axis is rotated from the horizontal direction so that it lies along the ramp. Using these data you can determine the acceleration, if any, of the cart. (It is best to enter the data into a spreadsheet for analysis.) Finally, you will use Newton's laws along with the information on the angle of the incline and the masses of the cart and the falling mass to determine (theoretically) what the acceleration of the cart is.

TABLE 6-3
Problem 104

Time (sec)	x(m)
0.000	0.002929
0.2050	0.03956
0.4100	0.08465
0.6150	0.1221
0.8200	0.1659
1.025	0.2038
1.230	0.2463
1.435	0.2885
1.640	0.3301
1.845	0.3676
2.050	0.4114
2.255	0.4472
2.460	0.4931
2.665	0.5297
2.870	0.5748
3.075	0.6165
3.280	0.6624

Our goal is to determine whether the theoretically calculated motion and the actual motion (as described by the data in Table 6-3) agree.

(a) Enter the data in Table 6-3 into a spreadsheet program. Determine what kind of motion the cart experiences. Is it a constant velocity? If so what are the magnitude and direction of the velocity? Is the motion a constant acceleration? If so, what are the magnitude and direction of the acceleration? (You may want to use equation-fitting software in answering this question). Cite the evidence that leads you to give the answers you did.
(b) What is the value of the net force on the cart in the x direction (along the incline)?
(c) Sketch a diagram of the cart like that shown in Fig. 6-92*b*. Draw a free-body diagram showing the directions of *all* the forces on the cart including the gravitational force, $\vec{F}^{\,grav}$, the normal force, N, and the string force due to its tension, T.
(d) Consider the situation in which the cart and falling mass move with a constant velocity. Choose a coordinate system in which the positive x axis is directed up along the ramp (rotated from the horizontal). *Assume that there is no friction in the pulley or cart bearings*!

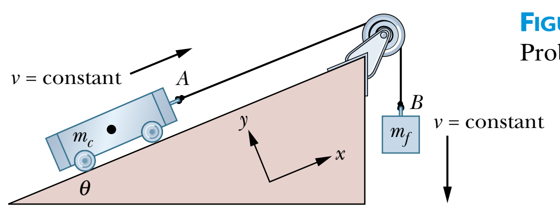


FIGURE 6-92*b* ■ Problem 104.

Show that by taking components of these forces along the x axis the magnitude, F_x^{net}, of the net force on the cart in the x direction, can be calculated using the equation

$$F_x^{\text{net}} = T - m_c g \sin\theta = 0,$$

where the gravitational acceleration constant, g, is $+9.8\ \text{m/s}^2$.
(e) Assume that since the cart and falling mass are connected by the string they have the same magnitude of velocity. Also assume that the tension in the string is the same at all points along the string so that the magnitude of the string force at point A on the cart is the same as the magnitude of the string force at point B on the falling mass. Show that if the net force on the falling mass is zero, then $T - F^{\text{grav}} = 0$, where $F^{\text{grav}} = m_f g$.
(f) Use the equations you derived in parts (d) and (e) to show that if the velocity of the cart and falling mass system are constant, then theoretically $m_f g$ ought to equal $m_c g \sin\theta$.
(g) Use the given values of m_c and m_f (also available on the title screen of the PASCO098 movie) along with the angle of the incline to verify that $m_f g$ and $m_c g \sin\theta$ have the same values to two significant digits. This equality, if it exists, confirms the agreement between theory and experiment.
(h) Also discuss why the answers should only be good to two significant figures.