mathematical task 3.1B Discuss how a 6-year-old would find the answer to the question "What is 7 + 2?" If the 6-year-old were then asked "What is 2 + 7?", how would he find the answer to that question? Is there a difference? Why or why not?

Reflection from Research
When students are taught strategies for thinking and working in mathematics, instead of just basic facts, their computational accuracy, efficiency, and flexibility can be improved (Crespo, Kyriakides, & McGee, 2005).

Thinking Strategies for Learning the Addition Facts The addition table in Figure 3.7 has 100 empty spaces to be filled. The sum of $a+b$ is placed in the intersection of the row labeled a and the column labeled b. For example, since $4+1=5$, a 5 appears in the intersection of the row labeled 4 and the column labeled 1.

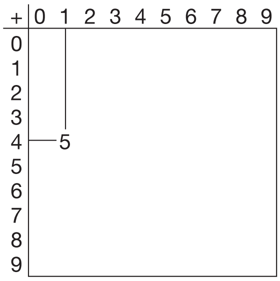

Figure 3.7

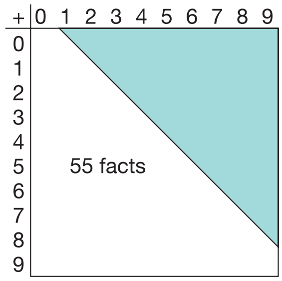

Figure 3.8

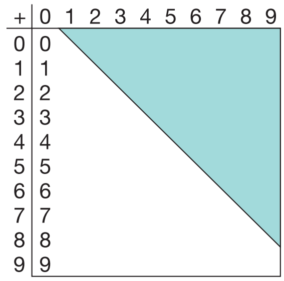

Figure 3.9

1. *Commutativity:* Because of commutativity and the symmetry of the table, a child will automatically know the facts in the shaded region of Figure 3.8 as soon as the child learns the remaining 55 facts. For example, notice that the sum $4+1$ is in the unshaded region, but its corresponding fact $1+4$ is in the shaded region.
2. *Adding zero:* The fact that $a+0=a$ for all whole numbers fills in 10 of the remaining blank spaces in the "zero" column (Figure 3.9)—45 spaces to go.
3. *Counting on by 1 and 2:* Children find sums like $7+1$, $6+2$, $3+1$, and $9+2$ by counting on. For example, to find $9+2$, think 9, then 10, 11. This thinking strategy fills in 17 more spaces in the columns labeled 1 and 2 (Figure 3.10)—28 facts to go.

Common Core – Grade 1
Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

Margin Callouts
Common Core margin notes have been added throughout the text to highlight the correlation between the content of this text and the Common Core standards.

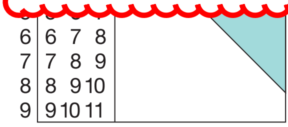

Figure 3.10

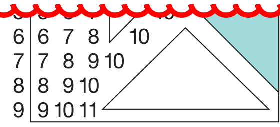

Figure 3.11

Common Core – Kindergarten
Decompose numbers less than or equal to 10 into pairs in more than one way (e.g., $5=2+3$ and $5=4+1$).

4. *Combinations to ten:* Combinations of the ten fingers can be used to find $7+3$, $6+4$, $5+5$, and so on. Notice that now we begin to have some overlap. There are 25 facts left to learn (Figure 3.11).

+	0	1	2	3	4	5	6	7	8	9
0	0									
1	1	2								
2	2	3	4							
3	3	4	5	6						
4	4	5	6		8					
5	5	6	7			10				
6	6	7	8		10		12			
7	7	8	9	10				14		
8	8	9	10						16	
9	9	10	11							18

Figure 3.12

Common Core – Kindergarten

Compose and decompose numbers from 11 to 19 into 10 ones and some further ones such as $18 = 10 + 8$.

5. *Doubles:* $1 + 1 = \mathbf{2}$, $2 + 2 = \mathbf{4}$, $3 + 3 = \mathbf{6}$, and so on. These sums, which appear on the main left-to-right downward diagonal, are easily learned as a consequence of counting by twos: namely, 2, 4, 6, 8, 10, . . . (Figure 3.12). Now there are 19 facts yet to be determined.
6. *Adding ten:* When using base ten pieces as a model, adding 10 amounts to laying down a "long" and saying the new name. For example, $3 + 10$ is 3 units and 1 long, or 13; $7 + 10$ is 17, and so on.
7. *Associativity:* The sum $9 + 5$ can be thought of as $10 + 4$, or 14, because $9 + 5 = 9 + (1 + 4) = (9 + 1) + 4$. Similarly, $8 + 7 = 10 + 5 = 15$, and so on. The rest of the addition table can be filled using associativity (sometimes called *regrouping*) combined with adding 10.
8. *Doubles ±1 and ±2:* This technique overlaps with the others. Many children use it effectively. For example, $7 + 8 = 7 + 7 + 1 = 14 + 1 = 15$, or $8 + 7 = 8 + 8 - 1 = 15$; $5 + 7 = 5 + 5 + 2 = 10 + 2 = 12$, and so on.

By using thinking strategies 6, 7, and 8, the remaining basic addition facts needed to complete the table in Figure 3.12 can be determined.

example 3.2 Use thinking strategies in three different ways to find the sum of $9 + 7$.

SOLUTION

a. $9 + 7 = 9 + (1 + 6) = (9 + 1) + 6 = 10 + 6 = 16$

b. $9 + 7 = (8 + 1) + 7 = 8 + (1 + 7) = 8 + 8 = 16$

c. $9 + 7 = (2 + 7) + 7 = 2 + (7 + 7) = 2 + 14 = 16$

NCTM Standard

All students should develop fluency with basic number combinations for addition and subtraction.

Thus far we have been adding single-digit numbers. However, thinking strategies can be applied to multidigit addition also. Figure 3.13 illustrates how multidigit addition is an extension of single-digit addition. The only difference is that instead of adding units each time, we might be adding longs, flats, and so on. Mentally combine similar pieces, and then exchange as necessary.

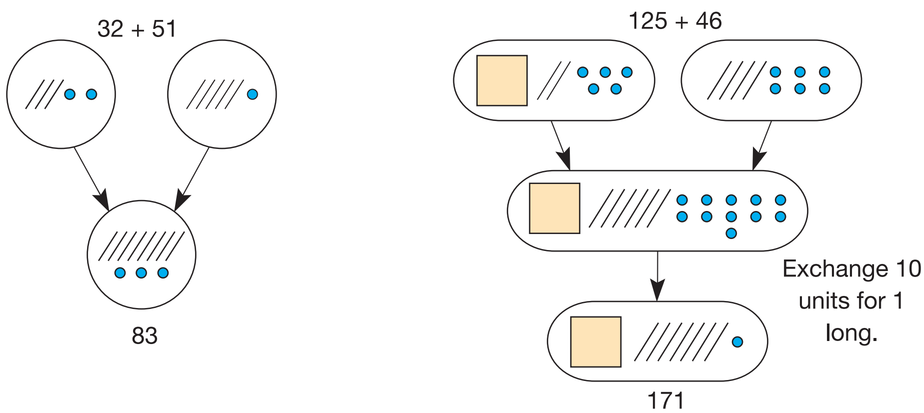

Figure 3.13

The next example illustrates how thinking strategies can be applied to multidigit numbers.