17. **House Mortgage** A couple wish to purchase a house for $200,000 with a down payment of $40,000. They can amortize the balance either at 8% for 20 years or at 9% for 25 years. Which monthly payment is greater? For which loan is the total interest paid greater? After 10 years, which loan provides the greater equity?

18. **House Mortgage** A couple have decided to purchase a $250,000 house using a down payment of $20,000. They can amortize the balance at 8% for 25 years. (a) What is their monthly payment? (b) What is the total interest paid? (c) What is their equity after 5 years? (d) What is the equity after 20 years?

19. **Planning Retirement** John is 45 years old and wants to retire at 65. He wishes to make monthly deposits in an account paying 9% compounded monthly so when he retires he can withdraw $300 a month for 30 years. How much should John deposit each month?

20. **Cost of a Lottery** The grand prize in an Illinois lottery is $6,000,000, which will be paid out in 20 equal annual payments of $300,000 each. Assume the first payment of $300,000 is made, leaving the state with the obligation to pay out $5,700,000 in 19 equal yearly payments of $300,000 each. How much does the state need to deposit in an account paying 6% compounded annually to achieve this?

21. **College Expenses** Dan works during the summer to help with expenses at school the following year. He is able to save $100 each week for 12 weeks, and he invests it at 6% compounded weekly.
   (a) How much does he have after 12 weeks?
   (b) When school starts, Dan will begin to withdraw equal amounts from this account each week. What is the most Dan can withdraw each week for 34 weeks?

22. **Analyzing a Town House Purchase** Mike and Yola have just purchased a town house for $200,000. They obtain financing with the following terms: a 20% down payment and the balance to be amortized over 30 years at 9%.
   (a) What is their down payment?
   (b) What is the loan amount?
   (c) How much is their monthly payment on the loan?
   (d) How much total interest do they pay over the life of the loan?
   (e) If they pay an additional $100 each month toward the loan, when will the loan be paid?
   (f) With the $100 additional monthly payment, how much total interest is paid over the life of the loan?

23. **House Mortgage** Mr. Smith obtained a 25-year mortgage on a house. The monthly payments are $2247.57 (principal and interest) and are based on a 7% interest rate. How much did Mr. Smith borrow? How much interest will be paid?

25. **Car Payments** A car costs $12,000. You put 20% down and amortize the rest with equal monthly payments over a 3-year period at 15% to be compounded monthly. What will the monthly payment be?

26. **Cost of Furniture** Jay pays $320 per month for 36 months for furniture, making no down payment. If the interest charged is 0.5% per month on the unpaid balance, what was the original cost of the furniture? How much interest did he pay?

27. **Paying for Restaurant Equipment** A restaurant owner buys equipment costing $20,000. If the owner pays 10% down and amortizes the rest with equal monthly payments over 4 years at 12% compounded monthly, what will be the monthly payments? How much interest is paid?

28. **Refinancing a Mortgage** A house that was bought 8 years ago for $150,000 is now worth $300,000. Originally, the house was financed by paying 20% down with the rest financed through a 25-year mortgage at 10.5% interest. The owner (after making 96 equal monthly payments) is in need of cash, and would like to refinance the house. The finance company is willing to loan 80% of the new value of the house amortized over 25 years with the same interest rate. How much cash will the owner receive after paying the balance of the original loan?

29. **Comparing Mortgages** A home buyer is purchasing a $140,000 house. The down payment will be 20% of the price of the house, and the remainder will be financed by a 30-year mortgage at a rate of 9.8% interest compounded monthly. What will the monthly payment be? Compare the monthly payments and the total amounts of interest paid if a 15-year mortgage is chosen instead of a 30-year mortgage.

30. **Mortgage Payments** Mr. and Mrs. Hoch are interested in building a house that will cost $180,000. They intend to use the $60,000 equity in their present house as a down payment on the new one and will finance the rest with a 25-year mortgage at an interest rate of 10.2% compounded monthly. How large will their monthly payment be on the new house?

Problems 31 and 32 require logarithms.

31. **IRA Withdrawals** How long will it take to exhaust an IRA of $100,000 if you withdraw $2000 every month? Assume a rate of interest of 5% compounded monthly.

32. **IRA Withdrawals** How long will it take to exhaust an IRA of $200,000 if you withdraw $3000 every month? Assume a rate of interest of 4% compounded monthly.

33. **Refinancing a Home Loan** Suppose that a couple decide to refinance a 30-year home loan, after making monthly payments for 5 years. The original loan amount was $312,000 with an annual interest rate of 6.825%, compounded
Annuities and Amortization Using Recursive Sequences

PREPARING FOR THIS SECTION

Before getting started, review the following:

> Recursive Sequences (Appendix A, Section A.2, pp. xx–xx)

OBJECTIVES

1. Use sequences and a graphing utility to solve annuity problems
2. Use sequences and a graphing utility to solve amortization problems

Annuities

In Section 5.2 we developed the compound interest formula, which gives the future value when a fixed amount of money is deposited in an account that pays interest compounded periodically. Often, though, money is invested in equal amounts at periodic intervals. An annuity is a sequence of equal periodic deposits. The periodic deposits may be made annually, quarterly, monthly, or daily.

When deposits are made at the same time that the interest is credited, the annuity is called ordinary. We will only deal with ordinary annuities here. The amount of an annuity is the sum of all deposits made plus all interest paid.

Suppose that the initial amount deposited in an annuity is $M$, the periodic deposit is $P$, and the per annum rate of interest is $r\%$ (expressed as a decimal) compounded $N$ times per year*. The periodic deposit is made at the same time that the interest is credited.

34. Refinancing a Home Loan Suppose that a couple decide to refinance a 30-year home loan, after making monthly payments for 3 years. The original loan amount was $285,000 with an annual interest rate of 6.75%, compounded monthly. This couple accept an offer from a California bank to refinance their loan with no closing costs. The new loan will be a 15-year loan but will have a lower interest rate, 5.5%, compounded monthly. With this refinancing, by how much will the monthly payments be reduced? Over the full term of the new loan, how much total interest will be saved?


35. Prepaying a Home Loan After making minimum payments for 4 years on a 30-year home loan, a couple decide to pay an additional $150 per month toward the principal. The original loan amount was $235,000 with an annual interest rate of 6.125%, compounded monthly. By how many months (rounded to the nearest tenth) will the term of this loan be reduced with this additional monthly payment? Over the life of this loan, how much interest will be saved?


36. Adjustable Rate Mortgages A couple decide to accept an adjustable rate, 30-year mortgage, in which the interest rate is fixed for the first 5 years and then may be adjusted annually beginning with year 6. Suppose that this couple borrow $305,000 with the annual interest rate, compounded monthly, for the first 5 years set at 5.5%. Suppose that the interest rate is increased to 6.75% for the remaining term of the loan. Calculate the monthly payment during the first 5 years and after the first 5 years. Find the amount of interest that would be paid over the term of this loan.


*We use $N$ to represent the number of times interest is compounded per annum instead of $n$, since $n$ is the traditional symbol used with sequences to denote the term of the sequence.
so \( N \) deposits are made per year. The amount \( A_n \) of the annuity after \( n \) deposits will equal the amount of the annuity after \( n - 1 \) deposits, \( A_{n-1} \), plus the interest earned on this amount plus the periodic deposit \( P \). That is,

\[
A_n = A_{n-1} + \frac{r}{N} A_{n-1} + P = \left( 1 + \frac{r}{N} \right) A_{n-1} + P
\]

We have established the following result:

**Annuity Formula**

If \( A_0 = M \) represents the initial amount deposited in an annuity that earns \( r\% \) per annum compounded \( N \) times per year, and if \( P \) is the periodic deposit made at each payment period, then the amount \( A_n \) of the annuity after \( n \) deposits is given by the recursive sequence

\[
A_0 = M, \quad A_n = \left( 1 + \frac{r}{N} \right) A_{n-1} + P, \quad n \geq 1
\]

Formula (1) may be explained as follows: the money in the account initially, \( A_0 \), is \( M \); the money in the account after \( n - 1 \) payments, \( A_{n-1} \), earns interest \( \frac{r}{N} \) during the \( n \)th period; so when the periodic payment of \( P \) dollars is added, the amount after \( n \) payments, \( A_n \), is obtained.

**EXAMPLE 1 Saving for Spring Break**

A trip to Cancun during spring break will cost $450 and full payment is due March 2. To have the money, a student, on September 1, deposits $100 in a savings account that pays 4% per annum compounded monthly. On the first of each month, the student deposits $50 in this account.

(a) Find a recursive sequence that explains how much is in the account after \( n \) months.

(b) Use the TABLE feature to list the amounts of the annuity for the first 6 months.

(c) After the deposit on March 1 is made, is there enough in the account to pay for the Cancun trip?

(d) If the student deposits $60 each month, will there be enough for the trip?

**SOLUTION**

(a) The initial amount deposited in the account is \( A_0 = $100 \). The monthly deposit is \( P = $50 \), and the per annum rate of interest is \( r = 0.04 \) compounded \( N = 12 \) times per year. The amount \( A_n \) in the account after \( n \) monthly deposits is given by the recursive sequence

\[
A_0 = 100, \quad A_n = \left( 1 + \frac{r}{N} \right) A_{n-1} + P = \left( 1 + \frac{0.04}{12} \right) A_{n-1} + 50
\]

(b) In SEQuence mode on a TI-83, enter the sequence \{\( A_n \)\} and create Table 8. On September 1 (\( n = 0 \)), there is $100 in the account. After the first payment on
October 1, the value of the account is $150.33. After the second payment on November 1, the value of the account is $200.83. After the third payment on December 1, the value of the account is $251.50, and so on.

(c) On March 1 ($n = 6$), there is only $404.53, not enough to pay for the trip to Cancun.

(d) If the periodic deposit, $P$, is $60, then on March 1, there is $465.03 in the account, enough for the trip. See Table 9.

\begin{table}[h]
\begin{tabular}{|c|c|}
\hline
$n$ & $A_n$ \\
\hline
$0$ & $100$ \\
$1$ & $50.33$ \\
$2$ & $20.83$ \\
$3$ & $5.15$ \\
$4$ & $1.65$ \\
$5$ & $0.42$ \\
$6$ & $0.11$ \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\begin{tabular}{|c|c|}
\hline
$n$ & $A_n$ \\
\hline
$0$ & $100$ \\
$1$ & $100.33$ \\
$2$ & $100.87$ \\
$3$ & $101.98$ \\
$4$ & $103.64$ \\
$5$ & $105.86$ \\
$6$ & $108.03$ \\
\hline
\end{tabular}
\end{table}

NOW WORK PROBLEM 5.

Amortization

Recursive sequences can also be used to compute information about loans. When equal periodic payments are made to pay off a loan, the loan is said to be amortized.

Amortization Formula

If $B$ is borrowed at an interest rate of $r\%$ (expressed as a decimal) per annum compounded monthly, the balance $A_n$ due after $n$ monthly payments of $P$ is given by the recursive sequence

$$A_0 = B, \quad A_n = \left(1 + \frac{r}{12}\right)A_{n-1} - P, \quad n \geq 1$$

Formula (2) may be explained as follows: The initial loan balance is $B$. The balance due $A_n$ after $n$ payments will equal the balance due previously, $A_{n-1}$, plus the interest charged on that amount reduced by the periodic payment $P$.

EXAMPLE 2  Mortgage Payments

John and Wanda borrowed $180,000 at 7\% per annum compounded monthly for 30 years to purchase a home. Their monthly payment is determined to be $1197.54.

(a) Find a recursive formula that represents their balance after each payment of $1197.54 has been made.

(b) Determine their balance after the first payment is made.

(c) When will their balance be below $170,000? 

(a) We use Formula (2) with $A_0 = 180,000$, $r = 0.07$, and $P = 1197.54$. Then

$$A_0 = 180,000 \quad A_n = \left(1 + \frac{0.07}{12}\right)A_{n-1} - 1197.54$$
In SEQuence mode on a TI-83, enter the sequence \( \{A_n\} \) and create Table 10. After the first payment is made, the balance is \( A_1 = 179,852 \).

Scroll down until the balance is below $170,000. See Table 11. After the 58th payment is made \( (n = 58) \), the balance is below $170,000.

**TABLE 10**

<table>
<thead>
<tr>
<th>( n )</th>
<th>( U(\chi) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>180000</td>
</tr>
<tr>
<td>1</td>
<td>179852</td>
</tr>
<tr>
<td>2</td>
<td>179704</td>
</tr>
<tr>
<td>3</td>
<td>179556</td>
</tr>
<tr>
<td>4</td>
<td>179408</td>
</tr>
</tbody>
</table>

**TABLE 11**

<table>
<thead>
<tr>
<th>( n )</th>
<th>( U(\chi) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>52</td>
<td>170000</td>
</tr>
<tr>
<td>53</td>
<td>170000</td>
</tr>
<tr>
<td>54</td>
<td>170000</td>
</tr>
<tr>
<td>55</td>
<td>170000</td>
</tr>
<tr>
<td>56</td>
<td>170000</td>
</tr>
<tr>
<td>57</td>
<td>170000</td>
</tr>
<tr>
<td>58</td>
<td>170000</td>
</tr>
</tbody>
</table>

**EXERCISE 5.5** Answers to Odd-Numbered Problems Begin on Page AN-00.

1. **Credit Card** Debt John has a balance of $3000 on his credit card that charges 1% interest per month on any unpaid balance. John can afford to pay $100 toward the balance each month. His balance each month after making a $100 payment is given by the recursively defined sequence

   \[
   B_0 = 3000, \quad B_n = 1.01 B_{n-1} - 100
   \]

   (a) Determine John’s balance after making the first payment. That is, determine \( B_1 \).

   (b) Using a graphing utility, determine when John’s balance will be below $2000. How many payments of $100 have been made?

   (c) Using a graphing utility, determine when John will pay off the balance. What is the total of all the payments?

   (d) What was John’s interest expense?

2. **Car Loans** Phil bought a car by taking out a loan for $18,500 at 0.5% interest per month. Phil’s normal monthly payment is $434.47 per month, but he decides that he can afford to pay $100 extra toward the balance each month. His balance each month is given by the recursively defined sequence

   \[
   B_0 = 18,500, \quad B_n = 1.005 B_{n-1} - 534.47
   \]

   (a) Determine Phil’s balance after making the first payment. That is, determine \( B_1 \).

   (b) Using a graphing utility, determine when Phil’s balance will be below $10,000. How many payments of $534.47 have been made?

   (c) Using a graphing utility, determine when Phil will pay off the balance. What is the total of all the payments?

   (d) What was Phil’s interest expense?

3. **Trout Population** A pond currently has 2000 trout in it. A fish hatchery decides to add an additional 20 trout each month. In addition, it is known that the trout population is growing 3% per month. The size of the population after \( n \) months is given by the recursively defined sequence

   \[
   p_0 = 2000, \quad p_n = 1.03 p_{n-1} + 20
   \]

   (a) How many trout are in the pond at the end of the second month? That is, what is \( p_2 \)?

   (b) Using a graphing utility, determine how long it will be before the trout population reaches 5000.

4. **Environmental Control** The Environmental Protection Agency (EPA) determines that Maple Lake has 250 tons of pollutants as a result of industrial waste and that 10% of the pollutants present is neutralized by solar oxidation every year. The EPA imposes new pollution control laws that result in 15 tons of new pollutants entering the lake each year. The amount of pollutants in the lake at the end of each year is given by the recursively defined sequence

   \[
   p_0 = 250, \quad p_n = 0.9 p_{n-1} + 15
   \]

   (a) Determine the amount of pollutants in the lake at the end of the second year. That is, determine \( p_2 \).

   (b) Using a graphing utility, provide pollutant amounts for the next 20 years.

   (c) What is the equilibrium level of pollution in Maple Lake? That is, what is \( \lim_{n \to \infty} p_n \)?

5. **Roth IRA** On January 1, 1999, Bob decides to place $500 at the end of each quarter into a Roth Individual Retirement Account.

   (a) Find a recursive formula that represents Bob’s balance at the end of each quarter if the rate of return is assumed to be 8% per annum compounded quarterly.

   (b) How long will it be before the value of the account exceeds $100,000?

   (c) What will be the value of the account in 25 years when Bob retires?
6. **Education IRA** On January 1, 1999, John's parents decide to place $45 at the end of each month into an Education IRA.

(a) Find a recursive formula that represents the balance at the end of each month if the rate of return is assumed to be 6% per annum compounded monthly.

(b) How long will it be before the value of the account exceeds $4000?

(c) What will be the value of the account in 16 years when John goes to college?

7. **Home Loan** Bill and Laura borrowed $150,000 at 6% per annum compounded monthly for 30 years to purchase a home. Their monthly payment is determined to be $899.33.

(a) Find a recursive formula for their balance after each monthly payment has been made.

(b) Determine Bill and Laura’s balance after the first payment.

(c) Using a graphing utility, create a table showing Bill and Laura’s balance after each monthly payment.

(d) Using a graphing utility, determine when Bill and Laura’s balance will be below $140,000.

(e) Using a graphing utility, determine when Bill and Laura will pay off the balance.

(f) Determine Bill and Laura’s interest expense when the loan is paid.

(g) Suppose that Bill and Laura decide to pay an additional $100 each month on their loan. Answer parts (a) to (f) under this scenario.

(h) Is it worthwhile for Bill and Laura to pay the additional $100? Explain.

8. **Home Loan** Jodi and Jeff borrowed $120,000 at 6.5% per annum compounded monthly for 30 years to purchase a home. Their monthly payment is determined to be $758.48.

(a) Find a recursive formula for their balance after each monthly payment has been made.

(b) Determine Jodi and Jeff’s balance after the first payment.

(c) Using a graphing utility, create a table showing Jodi and Jeff’s balance after each monthly payment.

(d) Using a graphing utility, determine when Jodi and Jeff’s balance will be below $100,000.

(e) Using a graphing utility, determine when Jodi and Jeff will pay off the balance.

(f) Determine Jodi and Jeff’s interest expense when the loan is paid.

(g) Suppose that Jodi and Jeff decide to pay an additional $100 each month on their loan. Answer parts (a) to (f) under this scenario.

(h) Is it worthwhile for Jodi and Jeff to pay the additional $100? Explain.

---

5.6 **Applications: Leasing; Capital Expenditure; Bonds**

**OBJECTIVE** 1 Solve applied problems

In this section we solve various applied problems.

**Leasing**

**EXAMPLE 1 Lease or Purchase**

A corporation may obtain a particular machine either by leasing it for 4 years (the useful life) at an annual rent of $1000 or by purchasing the machine for $3000.

(a) Which alternative is preferable if the corporation can invest money at 10% per annum?

(b) What if it can invest at 14% per annum?

**SOLUTION**

(a) Suppose the corporation may invest money at 10% per annum. The present value of an annuity of $1000 for 4 years at 10% equals $3169.87, which exceeds the purchase price. Therefore, purchase is preferable.

(b) Suppose the corporation may invest at 14% per annum. The present value of an annuity of $1000 for 4 years at 14% equals $2913.71, which is less than the purchase price. Leasing is preferable.
Capital Expenditure

EXAMPLE 2 Selecting Equipment

A corporation is faced with a choice between two machines, both of which are designed to improve operations by saving on labor costs. Machine A costs $8000 and will generate an annual labor savings of $2000. Machine B costs $6000 and will save $1800 annually. Machine A has a useful life of 7 years while machine B has a useful life of only 5 years. Assuming that the time value of money (the investment opportunity rate) of the corporation is 10% per annum, which machine is preferable? (Assume annual compounding and that the savings is realized at the end of each year.)

SOLUTION

Machine A costs $8000 and has a life of 7 years. Since an annuity of $1 for 7 years at 10% interest has a present value of $4.87, the cost of machine A may be considered the present value of an annuity:

\[
\frac{8000}{4.87} = 1642.71
\]

The $1642.71 may be termed the equivalent annual cost of machine A. Similarly, the equivalent annual cost of machine B may be calculated by reference to the present value of an annuity of 5 years, namely,

\[
\frac{6000}{3.79} = 1583.11
\]

The net annual savings of each machine is given in Table 12:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor savings</td>
<td>$2000.00</td>
<td>$1800.00</td>
</tr>
<tr>
<td>Equivalent annual cost</td>
<td>1642.71</td>
<td>1583.11</td>
</tr>
<tr>
<td>Net savings</td>
<td>$ 357.29</td>
<td>$ 216.89</td>
</tr>
</tbody>
</table>

Machine A is preferable.

NOW WORK PROBLEM 3.

Bonds

We begin our third application with some definitions of terms concerning corporate bonds.

Face Amount (Face Value or Par Value)

The face amount or denomination of a bond (normally $1000) is the amount paid to the bondholder at maturity. It is also the amount usually paid by the bondholder when the bond is originally issued.
Nominal interest is normally quoted as an annual percentage of the face amount. Nominal interest payments are conventionally made semiannually, so semiannual periods are used for compound interest calculations. For example, if a bond has a face amount of $1000 and a coupon rate of 8%, then every 6 months the owner of the bond would receive \((1000)(0.08)(\frac{1}{2}) = 40\).

But, because of market conditions, such as the current prime rate of interest, or the discount rate set by the Federal Reserve Board, or changes in the credit rating of the company issuing the bond, the price of a bond will fluctuate. When the bond price is higher than the face amount, it is trading at a *premium*; when it is lower, it is trading at a *discount*. For example, a bond with a face amount of $1000 and a coupon rate of 8% may trade in the marketplace at a price of $1100, which means the *true yield* is less than 8%.

To obtain the *true interest rate* of a bond, we view the bond as a combination of an annuity of semiannual interest payments plus a single future amount payable at maturity. The price of a bond is therefore the sum of the present value of the annuity of semiannual interest payments plus the present value of the single future payment at maturity. This present value is calculated by discounting at the true interest rate and assuming semiannual discounting periods.

**EXAMPLE 3 Pricing Bonds**

A bond has a face amount of $1000 and matures in 10 years. The nominal interest rate is 8.5%. What is the price of the bond to yield a true interest rate of 8%?

**SOLUTION**

**STEP 1** Calculate the amount of each semiannual interest payment:

\[(1000)(\frac{1}{2})(0.085) = 42.50\]

**STEP 2** Calculate the present value of the annuity of semiannual payments:

- Amount of each payment from Step 1: \$42.50
- Number of payments (2 \(\times\) 10 years): 20
- True interest rate per period (half of stated true rate): 4%
- Factor from formula for \(V\):
  \[\frac{1 - (1 + .04)^{-20}}{.04} = 13.5903\]
- Present value of interest payments \((42.50)(13.5903)\): \$577.59

**STEP 3** Calculate the present value of the amount payable at maturity:

- Amount payable at maturity: \$1000
- Number of semiannual compounding periods before maturity: 20
- True interest rate per period: 4%
- Factor from formula for \((1 + i)^{-n}\):
  \[(1 + .04)^{-20} = 0.45639\]
- Present value of maturity value \((1000)(0.45639)\): \$456.39
STEP 4 Determine the price of the bond:

- Present value of interest payments $577.59
- Present value of maturity payment 456.39
- Price of bond (Add) $1033.98

NOW WORK PROBLEM 7.

Use Excel to Solve Example 3.

SOLUTION

STEP 1 Calculate the amount of each semiannual interest payment:

\[
\frac{(1000)(.5)(0.085)}{1000} = 42.50
\]

STEP 2 Calculate the present value of the annuity of semiannual payments. Use the Excel function \(\text{PV}(\text{rate}, \text{nper}, \text{pmt}, \text{fv}, \text{type})\).

- \text{rate} is the interest rate per period, 4%.
- \text{nper} is the total number of payments, 20.
- \text{pmt} is the payment made each period, $42.50.
- \text{fv} is the future value of the annuity or loan, 0.
- \text{type} is set to 0 for an ordinary annuity.

The result is given in the Excel spreadsheet below.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>STEP 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Annual interest rate</strong></td>
<td><strong>Period interest rate</strong></td>
<td><strong>Number of periods</strong></td>
<td><strong>Future Value</strong></td>
<td><strong>Payments</strong></td>
</tr>
<tr>
<td>8.00%</td>
<td>4.000%</td>
<td>20</td>
<td>$0.00</td>
<td>$42.50</td>
</tr>
<tr>
<td>Present Value of Interest Payments</td>
<td>($577.59)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

STEP 3 Calculate the present value of the amount payable at maturity. Use the Excel function \(\text{PV}(\text{rate}, \text{nper}, \text{pmt}, \text{fv}, \text{type})\).

- \text{rate} is the interest rate per period, 4%.
- \text{nper} is the total number of payments, 20.
- \text{pmt} is the payment made each period, $0.00.
- \text{fv} is the future value of the annuity or loan, 1000.
- \text{type} is set to 0 for an ordinary annuity.
STEP 4  Determine the price of the bond. Add the two present values. The entire problem’s Excel spreadsheet is given below.

![Excel Spreadsheet Screenshot]

NOW WORK PROBLEM 7 USING EXCEL.

EXERCISE 5.6  Answers to Odd-Numbered Problems Begin on Page AN-00.

1. Leasing Problem  A corporation may obtain a machine either by leasing it for 5 years (the useful life) at an annual rent of $2000 or by purchasing the machine for $8100. If the corporation can borrow money at 10% per annum, which alternative is preferable?

2. If the corporation in Problem 1 can borrow money at 14% per annum, which alternative is preferable?

3. Capital Expenditure Analysis  Machine A costs $10,000 and has a useful life of 8 years, and machine B costs $8000 and has a useful life of 6 years. Suppose machine A generates an annual labor savings of $2000 while machine B generates an annual labor savings of $1800. Assuming the time value of money (investment opportunity rate) is 10% per annum, which machine is preferable?
4. In Problem 3, if the time value of money is 14% per annum, which machine is preferable?

5. **Corporate Bonds** A bond has a face amount of $1000 and matures in 15 years. The nominal interest rate is 9%. What is the price of the bond that will yield an effective interest rate of 8%?

6. For the bond in Problem 5 what is the price of the bond to yield an effective interest rate of 10%?

7. **Treasury Notes** Determine the selling price, per $1000 maturity value, of a 10-year treasury note with a nominal interest rate of 4.000% and a true interest rate of 4.095%.

   *Source:* U.S. Treasury. (This is an actual U.S. Treasury note auctioned in November 2002.)

8. **Treasury Notes** Determine the selling price, per $1000 maturity value, of a 5-year treasury note with a nominal interest rate of 3.000% and a true interest rate of 3.030%.

   *Source:* U.S. Treasury. (This is an actual U.S. Treasury note auctioned in August 2002.)

9. **Treasury Notes** Determine the true interest rate of a 10-year treasury note with a nominal interest rate of 4.375% if the selling price, per $1000 maturity value, is $998.80.

   *Source:* U.S. Treasury. (This is an actual U.S. Treasury note auctioned in August 2002.)

10. **Treasury Notes** Determine the true interest rate of a 5-year treasury note with a nominal interest rate of 3.250% if the selling price, per $1000 maturity value, is $995.52.

    *Source:* U.S. Treasury. (This is an actual U.S. Treasury note auctioned in August 2002.)

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### Chapter 5 Review

**OBJECTIVES**

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IMPORTANT FORMULAS

Simple Interest Formula \[ I = Prt \]
Discounted Loans \[ R = L - Lrt \]
Compound Interest Formula \[ A_n = R(1 + i)^n \]
Amount of an Annuity \[ A = P \frac{(1 + i)^n - 1}{i} \]

Present Value of an Annuity \[ V = P \frac{1 - (1 + i)^{-n}}{i} \]
Amortization \[ P = V \frac{i}{1 - (1 + i)^{-n}} \]

TRUE–FALSE ITEMS Answers are on page AN-00.

T F 1. Simple interest is interest computed on the principal for the entire period it is borrowed.
T F 2. The amount \( A \) accrued on a principal \( P \) after five payment periods at \( i \) interest per payment period is \( A = P(1 + i)^5 \).
T F 3. The effective rate of interest for 10% compounded daily is about 9.58%.
T F 4. The present value of an annuity is the sum of all the present values of the payment less any interest.

FILL IN THE BLANKS Answers are on page AN-00.

1. For a discounted loan, the amount the borrower receives is called the __________.
2. In the formula \( P = A(1 + i)^{-n} \), \( P \) is called the __________ __________ of the amount \( A \) due at the end of \( n \) interest periods at \( i \) interest per payment period.
3. The amount of an __________ is the sum of all deposits made plus all interest accumulated.
4. A loan with a fixed rate of interest is __________ if both principal and interest are paid by a sequence of equal payments over equal periods of time.

REVIEW EXERCISES

In Problems 1–9, calculate the indicated quantity.

1. 3% of 500
2. 20% of 1200
3. 140% of 250
4. What percent of 200 is 40?
5. What percent of 350 is 75?
6. What percent of 50 is 125?
7. 12 is 15% of what number?
8. 25 is 6% of what number?
9. 11 is 0.5% of what number?

10. **Sales Tax** The sales tax in New York City is 8.5%. If the total charge for a DVD player (including tax) is $162.75, what does the DVD player itself cost?
11. **Sales Tax** Indiana has a sales tax of 6%. Dan purchased gifts for his family worth $330.00. How much sales tax will Dan have to pay on his purchase?
12. Find the interest \( I \) charged and amount \( A \) due, if $400 is borrowed for 9 months at 12% simple interest.
13. Dan borrows $500 at 9% per annum simple interest for 1 year and 2 months. What is the interest charged, and what is the amount due?
14. **Interest Due** Jim borrows $14,000 for a period of 4 years at 6% simple interest. Determine the interest due on the loan.
15. **Loan Amount** Warren needs $15,000 for a new machine for his auto repair shop. He obtains a 2-year discounted loan at 12% interest. How much must he repay to settle his debt?
16. **Treasury Bills** How much should a bank bid for a 15-month, $5,000 treasury bill in order to earn 2.5% simple interest?
17. Find the amount of an investment of $100 after 2 years and 3 months at 10% compounded monthly.
18. Mike places $200 in a savings account that pays 4% per annum compounded monthly. How much is in his account after 9 months?
19. **Choosing a Car Loan** A car dealer offers Mike the choice of two loans:

(a) $3000 for 3 years at 12% per annum simple interest
(b) $3000 for 3 years at 10% per annum compounded monthly
(c) Which loan costs Mike the least?

20. A mutual fund pays 9% per annum compounded monthly. How much should I invest now so that 2 years from now I will have $100 in the account?

21. **Saving for a Bicycle** Katy wants to buy a bicycle that costs $75 and will purchase it in 6 months. How much should she put in her savings account for this if she can get 10% per annum compounded monthly?

22. **Doubling Money** Marcia has $220,000 saved for her retirement. How long will it take for the investment to double in value if it earns 6% compounded semiannually?

23. **Doubling Money** What annual rate of interest will allow an investment to double in 12 years?

24. **Effective Rate of Interest** A bank advertises that it pays 3 1/2% interest compounded monthly. What is the effective rate of interest?

25. **Effective Rate of Interest** What interest rate compounded quarterly has an effective interest rate of 6%?

26. **Saving for a Car** Mike decides he needs $500 1 year from now to buy a used car. If he can invest at 8% compounded monthly, how much should he save each month to buy the car?

27. **Saving for a House** Mr. and Mrs. Corey are newlyweds and want to purchase a home, but they need a down payment of $40,000. If they want to buy their home in 2 years, how much should they save each month in their savings account that pays 3% per annum compounded monthly?

28. **True Cost of a Car** Mike has just purchased a used car and will make equal payments of $50 per month for 18 months at 12% per annum charged monthly. How much did the car actually cost? Assume no down payment.

29. **House Mortgage** Mr. and Mrs. Ostedt have just purchased an $400,000 home and made a 25% down payment. The balance can be amortized at 10% for 25 years.

(a) What are the monthly payments?
(b) How much interest will be paid?
(c) What is their equity after 5 years?

30. **Inheritance Payouts** An inheritance of $25,000 is to be paid in equal amounts over a 5-year period at the end of each year. If the $25,000 can be invested at 10% per annum, what is the annual payment?

31. **House Mortgage** A mortgage of $125,000 is to be amortized at 9% per annum for 25 years. What are the monthly payments? What is the equity after 10 years?

32. **Paying Off Construction Bonds** A state has $8,000,000 worth of construction bonds that are due in 25 years. What annual sinking fund deposit is needed if the state can earn 10% per annum on its money?

33. **Depletion Problem** How much should Mr. Graff pay for a gold mine expected to yield an annual return of $20,000 and to have a life expectancy of 20 years, if he wants to have a 15% annual return on his investment and he can set up a sinking fund that earns 10% a year?

34. **Retirement Income** Mr. Doody, at age 70, is expected to live for 15 years. If he can invest at 12% per annum compounded monthly, how much does he need now to guarantee himself $300 every month for the next 15 years?

35. **Depletion Problem** An oil well is expected to yield an annual net return of $25,000 for the next 15 years, after which it will run dry. An investor wants a return on his investment of 20%. He can establish a sinking fund earning 10% annually. How much should he pay for the oil well?

36. **Saving for the Future** Hal deposited $100 in an account paying 9% per annum compounded monthly for 25 years. At the end of the 25 years Hal retires. What is the largest amount he may withdraw monthly for the next 35 years?

37. **Saving for College** Mr. Jones wants to save for his son’s college education. If he deposits $500 every 6 months at 6% compounded semiannually, how much will he have on hand at the end of 8 years?

38. How much money should be invested at 8% compounded quarterly in order to have $20,000 in 6 years?

39. Bill borrows $1000 at 5% compounded annually. He is able to establish a sinking fund to pay off the debt and interest in 7 years. The sinking fund will earn 8% compounded quarterly. What should be the size of the quarterly payments into the fund?

40. A man has $50,000 invested at 12% compounded quarterly at the time he retires. If he wishes to withdraw money every 3 months for the next 7 years, what will be the size of his withdrawals?

41. How large should monthly payments be to amortize a loan of $3000 borrowed at 12% compounded monthly for 2 years?
42. **Paying off School Bonds** A school board issues bonds in the amount of $20,000,000 to be retired in 25 years. How much must be paid into a sinking fund at 6% compounded annually to pay off the total amount due?

43. What effective rate of interest corresponds to a nominal rate of 9% compounded monthly?

44. If an amount was borrowed 5 years ago at 6% compounded quarterly, and $6000 is owed now, what was the original amount borrowed?

45. **Trust Fund Payouts** John is the beneficiary of a trust fund set up for him by his grandparents. If the trust fund amounts to $20,000 earning 8% compounded semiannually and he is to receive the money in equal semiannual installments for the next 15 years, how much will he receive each 6 months?

46. **IRA** Mike has $4000 in his IRA. At 6% annual interest compounded monthly, how much will he have at the end of 20 years?

47. **Retirement Funds** An employee gets paid at the end of each month and $60 is withheld from her paycheck for a retirement fund. The fund pays 1% per month (equivalent to 12% annually compounded monthly). What amount will be in the fund at the end of 30 months?

48. **$6000 is borrowed at 10% compounded semiannually. The amount is to be paid back in 5 years. If a sinking fund is established to repay the loan and interest in 5 years, and the fund earns 8% compounded quarterly, how much will have to be paid into the fund every 3 months?**

49. **Buying a Car** A student borrowed $4000 from a credit union toward purchasing a car. The interest rate on such a loan is 14% compounded quarterly, with payments due every quarter. The student wants to pay off the loan in 4 years. Find the quarterly payment.

50. **Leasing Decision** A firm can lease office furniture for 5 years (its useful life) for $20,000 a year, or it can purchase the furniture for $80,000. Which is a better choice if the firm can invest its money at 10% per annum?

51. **Leasing Decision** A company can lease 10 trucks for 4 years (their useful life) for $50,000 a year, or it can purchase them for $175,000. If the company can earn 5% per annum on its money, which choice is preferable, leasing or purchasing?

52. **Capital Expenditures** In an effort to increase productivity, a corporation decides to purchase a new piece of equipment. Two models are available; both reduce labor costs. Model A costs $50,000, saves $12,000 per year in labor costs, and has a useful life of 10 years. Model B costs $42,000, saves $10,000 annually in labor costs, and has a useful life of 8 years. If the time value of money is 10% per annum, what piece of equipment provides a better investment?

53. **Corporate Bonds** A bond has a face amount of $10,000 and matures in 8 years. The nominal rate of interest on the bond is 6.25%. At what price would the bond yield a true rate of interest of 6.5%?

54. **Treasury Notes** Determine the selling price, per $1000 maturity value, of a 20 year treasury note with a nominal interest rate of 5.0% and a true interest rate of 5.1%.

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**Chapter 5 Project**

**MORTGAGE POINTS: SHOULD YOU BUY THEM?**

When you are shopping for a house, you are usually also shopping for a mortgage. Banks and other lending institutions offer an amazing variety of mortgages. Mortgages may differ in their amount, their rates of interest, and their term (the time needed to pay off the mortgage). If you know the amount, the rate of interest, and the term, you can use the amortization formula from Section 5.4 to figure out your monthly payment, determine the total amount of interest you will pay, and even create a schedule of payments for the loan.

Another difference among mortgages concerns points.* A point is a fee equal to some percentage of the loan amount and is usually paid at the time of closing—when you pay your down payment and other fees and take possession of the house. Paying points sometimes lessens the total amount of a mortgage. Since paying points is considered to be prepaid interest on your mortgage, often the lender will offer a reduced rate for mortgages with points. The big question for the home buyer is whether to pay points. As we shall see, the

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*The discussion here pertains to a certain type of points called discount points. Other types of points behave differently.*
answer depends on the amount of the mortgage, how large a
down payment you can make, and the amount of time you
expect to spend in your house.

Suppose you are thinking of buying a house in Charlotte,
North Carolina, that costs $150,000. You plan to make a
down payment of $30,000, so you will need a mortgage of
$120,000. A local lender, Hartland Mortgage Centers, gives
you four quotes on 30-year, fixed rate mortgages. See Table 1.

**TABLE 1** Mortgage Rates and Points—February 2003

<table>
<thead>
<tr>
<th>Rate</th>
<th>Points</th>
<th>Fee Paid for Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.50</td>
<td>0.00</td>
<td>$ 0.00</td>
</tr>
<tr>
<td>5.38</td>
<td>0.50</td>
<td>$ 600.00</td>
</tr>
<tr>
<td>5.25</td>
<td>1.10</td>
<td>$1320.00</td>
</tr>
<tr>
<td>5.00</td>
<td>2.70</td>
<td>$3240.00</td>
</tr>
</tbody>
</table>

1. Compute your monthly payment for the loan with no
points.

2. Now consider the loan with 0.5 points. Compute the
monthly payment you would have in this case.

3. The loan with 0.5 points gives you a lower monthly pay-
ment. How much is the difference?

4. If you take the loan with 0.5 points, you will eventually
make back the $600 you spent for the points in reduced pay-
ments. How many months will it take for you to make back
your points fee?

5. If you plan to leave your house before this “break-even”
time, then which of the two loans makes better sense?

6. Compare the other two loan options given in the table
with the two you have already computed. What are the
monthly payments for those loans? How many months does
it take to regain the points fee in each of these cases?

7. Does it really make sense to compare the monthly pay-
ments as we have calculated them? Sometimes it does, but
consider the following situation. If you were willing to pay
$3240 for the points for the last loan, it is reasonable to
expect that you would also be willing to add that $3240 to the
down payment if you were going to take the loan with
no points. Likewise, you should be willing to add
$3240 – $600 = $2640 to the down payment for the loan
with 0.5 points and to add $3240 – $1320 = $1920 to the
down payment for the loan with 1.10 points. Recalculate
the monthly payment for each loan by filling in Table 2.

8. Now which mortgage gives you the lowest payment?

---

**MATHEMATICAL QUESTIONS FROM PROFESSIONAL EXAMS**

1. **CPA Exam** Which of the following should be used to cal-
culate the amount of the equal periodic payments that
could be equivalent to an outlay of $3000 at the time of
the last payment?

   (a) Amount of 1
   (b) Amount of an annuity of 1
   (c) Present value of an annuity of 1
   (d) Present value of 1

2. **CPA Exam** A businessman wants to withdraw $3000
   (including principal) from an investment fund at the end
   of each year for 5 years. How should he compute his
   required initial investment at the beginning of the first
   year if the fund earns 6% compounded annually?

   (a) $3000 times the amount of an annuity of $1 at 6% at
   the end of each year for 5 years
   (b) $3000 divided by the amount of an annuity of $1 at
   6% at the end of each year for 5 years
   (c) $3000 times the present value of an annuity of $1 at
   6% at the end of each year for 5 years
   (d) $3000 divided by the present value of an annuity of
   $1 at 6% at the end of each year for 5 years

3. **CPA Exam** A businesswoman wants to invest a certain
   sum of money at the end of each year for 5 years. The
   investment will earn 6% compounded annually. At the
   end of 5 years, she will need a total of $30,000 accumulat-
ed. How should she compute the required annual invest-
   ment?

   (a) $30,000 times the amount of an annuity of $1 at 6%
   at the end of each year for 5 years
   (b) $30,000 divided by the amount of an annuity of $1 at
   6% at the end of each year for 5 years

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4. CPA Exam  Shaid Corporation issued $2,000,000 of 6%, 10-year convertible bonds on June 1, 1993, at 98 plus accrued interest. The bonds were dated April 1, 1993, with interest payable April 1 and October 1. Bond discount is amortized semiannually on a straight-line basis.

On April 1, 1994, $500,000 of these bonds were converted into 500 shares of $20 par value common stock. Accrued interest was paid in cash at the time of conversion.

What was the effective interest rate on the bonds when they were issued?
(a) 6%    (b) Above 6%    (c) Below 6%    (d) Cannot be determined from the information given.

5. CPA Exam  What amount should be deposited in a bank today to grow to $1000 3 years from today?

(a) $1000.794  (b) $1000 \times 0.926 \times 3
(c) ($1000 \times 0.926) + ($1000 \times 0.857) + ($1000 \times 0.794)
(d) $1000 \times 0.794

6. CPA Exam  What amount should an individual have in her bank account today before withdrawal if she needs $2000 each year for 4 years with the first withdrawal to be made today and each subsequent withdrawal at 1-year intervals? (She is to have exactly a zero balance in her bank account after the fourth withdrawal.)

(a) $2000 + ($2000 \times 0.926) + ($2000 \times 0.857) + ($2000 \times 0.794)
(b) \frac{$2000}{0.735} \times 4
(c) ($2000 \times 0.926) + ($2000 \times 0.857) + ($2000 \times 0.794) + ($2000 \times 0.735)
(d) \frac{$2000}{0.926} \times 4

7. CPA Exam  If an individual put $3000 in a savings account today, what amount of cash would be available 2 years from today?

(a) $3000 \times 0.857  (b) $3000 \times 0.857 \times 2
(c) \frac{$3000}{0.857}  (d) \frac{$3000}{0.926} \times 2