1

Linear relations
and equations

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1.1 Kick off with CAS

Please refer to the Resources tab in the Prelims section of your eBookPLUS for a comprehensive step-by-step guide on how to use your CAS technology.
1.2 Linear relations

Identifying linear relations

A linear relation is a relationship between two variables that when plotted gives a straight line. Many real-life situations can be described by linear relations, such as water being added to a tank at a constant rate, or money being saved when the same amount of money is deposited into a bank at regular time intervals.

When a linear relation is expressed as an equation, the highest power of both variables in the equation is 1.

WORKED EXAMPLE

Identify which of the following equations are linear.

a\[ y = 4x + 1 \]
b\[ b = c^2 - 5c + 6 \]
c\[ y = \sqrt{x} \]
d\[ m^2 = 6(n - 10) \]
e\[ d = \frac{3t + 8}{7} \]
f\[ y = 5^x \]

THINK

a 1 Identify the variables.
2 Write the power of each variable.
3 Check if the equation is linear.

b 1 Identify the two variables.
2 Write the power of each variable.
3 Check if the equation is linear.

c 1 Identify the two variables.
2 Write the power of each variable.
   Note: A square root is a power of \( \frac{1}{2} \).
3 Check if the equation is linear.

WRITE

a  y and \( x \)
   y has a power of 1.
   \( x \) has a power of 1.

   Since both variables have a power of 1, this is a linear equation.

b  \( b \) and \( c \)
   \( b \) has a power of 1.
   \( c \) has a power of 2.

   \( c \) has a power of 2, so this is not a linear equation.

c  y and \( x \)
   y has a power of 1.
   \( x \) has a power of \( \frac{1}{2} \).

   \( x \) has a power of \( \frac{1}{2} \), so this is not a linear equation.

d  m and \( n \)
   m has a power of 2.
   \( n \) has a power of 1.

   m has a power of 2, so this is not a linear equation.
RULES FOR LINEAR RELATIONS

Rules define or describe relationships between two or more variables. Rules for linear relations can be found by determining the first common difference between the terms of the rule.

Consider the number pattern 4, 7, 10 and 13. This pattern is formed by adding 3s (the first common difference is 3). If each number in the pattern is assigned a term number as shown in the table, then the expression to represent the first common difference is $3n$ (i.e. $3 \times n$).

<table>
<thead>
<tr>
<th>Term number, $n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3n$</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
</tr>
</tbody>
</table>

Each term in the number pattern is 1 greater than $3n$, so the rule for this number pattern is $3n + 1$.

If a rule has an equals sign, it is described as an equation. For example, $3n + 1$ is referred to as an expression, but if we define the term number as $t$, then $t = 3n + 1$ is an equation.

WORKED EXAMPLE 2

Find the equations for the linear relations formed by the following number patterns.

a  3, 7, 11, 15

b  8, 5, 2, −1

THINK

a  1 Determine the first common difference.

2 Write the first common difference as an expression using the term number $n$.

3 Substitute any term number into $4n$ and evaluate.

4 Check the actual term number against the one found.

5 Add or subtract a number that would result in the actual term number.

WRITE

a  $7 - 3 = 4$

$15 - 11 = 4$

$4n$

$n = 3$

$4 \times 3 = 12$

The actual 3rd term is 11.

$12 - 1 = 11$
6 Write the equation for the linear relation.

\[ t = 4n - 1 \]

b 1 Determine the first common difference.

\[ \begin{align*}
5 - 8 &= -3 \\
2 - 5 &= -3 
\end{align*} \]

2 Write the first common difference as an expression using the term number \( n \).

\[ -3n \]

3 Substitute any term number into \(-3n\) and evaluate.

\[ n = 2 \]

\[ -3 \times 2 = -6 \]

4 Check the actual term number against the one found.

The actual 2nd term is 5.

5 Add or subtract a number that would result in the actual term number.

\[-6 + 11 = 5\]

6 Write the equation for the linear relation.

\[ t = -3n + 11 \]

*Note:* It is good practice to substitute a second term number into your equation to check that your answer is correct.

### Transposing linear equations

If we are given a linear equation between two variables, we are able to transpose this relationship. That is, we can change the equation so that the variable on the right-hand side of the equation becomes the stand-alone variable on the left-hand side of the equation.

### WORKED EXAMPLE 3

Transpose the linear equation \( y = 4x + 7 \) to make \( x \) the subject of the equation.

**THINK**

1. Isolate the variable on the right-hand side of the equation (by subtracting 7 from both sides).

2. Divide both sides of the equation by the coefficient of the variable on the right-hand side (in this case 4).

3. Transpose the relation by interchanging the left-hand side and the right-hand side.

**WRITE**

\[ \begin{align*}
y - 7 &= 4x + 7 - 7 \\
y - 7 &= 4x \\
\frac{y - 7}{4} &= \frac{4x}{4} \\
\frac{y - 7}{4} &= x \\
x &= \frac{y - 7}{4}
\end{align*} \]

### EXERCISE 1.2 Linear relations

1 Identify which of the following equations are linear.

- \( a \) \( y^2 = 7x + 1 \)
- \( b \) \( t = 7x^3 - 6x \)
- \( c \) \( y = 3(x + 2) \)
- \( d \) \( m = 2^{x+1} \)
- \( e \) \( 4x + 5y - 9 = 0 \)
Bethany was asked to identify which equations from a list were linear. The following table shows her responses.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Bethany’s response</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 4x + 1 )</td>
<td>Yes</td>
</tr>
<tr>
<td>( y^2 = 5x - 2 )</td>
<td>Yes</td>
</tr>
<tr>
<td>( y + 6x = 7 )</td>
<td>Yes</td>
</tr>
<tr>
<td>( y = x^2 - 5x )</td>
<td>No</td>
</tr>
<tr>
<td>( t = 6d^2 - 9 )</td>
<td>No</td>
</tr>
<tr>
<td>( m^3 = n + 8 )</td>
<td>Yes</td>
</tr>
</tbody>
</table>

a. Insert another column into the table and add your responses identifying which of the equations are linear.

b. Provide advice to Bethany to help her to correctly identify linear equations.

Find the equations for the linear relations formed by the following number patterns.

**a** 2, 6, 10, 14, 18, …

**b** 4, 4.5, 5, 5.5, 6, …

Jars of vegetables are stacked in ten rows. There are 8 jars in the third row and 5 jars in the sixth row. The number of jars in any row can be represented by a linear relation.

a. Find the first common difference.

b. Find an equation that will express the number of jars in any of the ten rows.

c. Determine the total number of jars of vegetables.

Transpose the linear equation \( y = 6x - 3 \) to make \( x \) the subject of the equation.

Transpose the linear equation \( 6y = 3x + 1 \) to make \( x \) the subject of the equation.

Identify which of the following are linear equations.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Samson’s response</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 5x + 6 )</td>
<td>Yes, linear</td>
</tr>
<tr>
<td>( y^2 = 6x - 1 )</td>
<td>Yes, linear</td>
</tr>
<tr>
<td>( y = x^2 + 4 )</td>
<td>Not linear</td>
</tr>
<tr>
<td>( y^3 = 7(x + 3) )</td>
<td>Yes, linear</td>
</tr>
<tr>
<td>( y = \frac{1}{2}x + 6 )</td>
<td>Yes, linear</td>
</tr>
<tr>
<td>( \sqrt{y} = 4x + 2 )</td>
<td>Yes, linear</td>
</tr>
<tr>
<td>( y^2 + 5x^3 + 9 = 0 )</td>
<td>Not linear</td>
</tr>
<tr>
<td>( 10y - 11x = 12 )</td>
<td>Yes, linear</td>
</tr>
</tbody>
</table>
9 A number pattern is formed by multiplying the previous term by 1.5. The first term is 2.
   a Find the next four terms in the number pattern.
   b Could this number pattern be represented by a linear equation? Justify your answer.

10 Find equations for the linear relations formed by the following number patterns.
   a \(3, 7, 11, 15, 19, \ldots\)   b \(7, 10, 13, 16, 19, \ldots\)
   c \(12, 9, 6, 3, 0, -3, \ldots\)   d \(13, 7, 1, -5, -11, \ldots\)
   e \(-12, -14, -16, -18, -20, \ldots\)

11 Consider the following number pattern: \(1.2, 2.0, 2.8, 3.6, 4.4, \ldots\)
   a Find the first common difference.
   b Could this number pattern be represented by a linear equation? Justify your answer.

12 Transpose the following linear equations to make \(x\) the subject.
   a \(y = 2x + 5\)   b \(3y = 6x + 8\)
   c \(p = 5x - 6\)

13 Water is leaking from a water tank at a linear rate. The amount of water, in litres, is measured at the start of each day. At the end of the first day there are 950 litres in the tank, and at the end of the third day there are 850 litres in the tank.
   a Complete the following table.

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount of water</td>
<td>950</td>
<td></td>
<td>850</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
   b Determine the amount of water that was initially in the tank (i.e. at day 0).
   c Determine an equation that finds the amount of water, \(w\), in litres, at the end of any day, \(d\).

14 At the start of the year Yolanda has $1500 in her bank account. At the end of each month she deposits an additional $250.
   a How much, in dollars, does Yolanda have in her bank account at the end of March?
   b Find an equation that determines the amount of money, \(A\), Yolanda has in her bank account at the end of each month, \(m\).
   c At the start of the following year, Yolanda deposits an additional $100 each month. How does this change the equation found in part b?

15 On the first day of Sal’s hiking trip, she walks halfway into a forest. On each day after the first, she walks exactly half the distance she walked the previous day. Could the distance travelled by Sal each day be described by a linear equation? Justify your answer.
16 Anton is a runner who has a goal to run a total of 350 km over 5 weeks to raise money for charity.

   a If each week he runs 10 km more than he did on the previous week, how far does he run in week 3?
   b Find an equation that determines the distance Anton runs each week.

17 Using a spreadsheet, determine an equation that describes the number pattern shown in the table below.

<table>
<thead>
<tr>
<th>Term number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>−4</td>
<td>−2</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

18 The terms in a number sequence are found by multiplying the term number, \( n \), by 4 and then subtracting 1. The first term of the sequence is 3.

   a Find an equation that determines the terms in the sequence.
   b Using a spreadsheet, calculator or otherwise, find the first 10 terms of the sequence.
   c Show that the first common difference is 4.

1.3 Solving linear equations

Solving linear equations with one variable

To solve linear equations with one variable, all operations performed on the variable need to be identified in order, and then the opposite operations need to be performed in reverse order.

In practical problems, solving linear equations can answer everyday questions such as the time required to have a certain amount in the bank, the time taken to travel a certain distance, or the number of participants needed to raise a certain amount of money for charity.

WORKED EXAMPLE 4

Solve the following linear equations to find the unknowns.

a \( 5x = 12 \)

b \( 8t + 11 = 20 \)

c \( 12 = 4(n - 3) \)

d \( \frac{4x - 2}{3} = 5 \)

THINK

a 1 Identify the operations performed on the unknown.

2 Write the opposite operation.

3 Perform the opposite operation on both sides of the equation.

WRITE

a \( 5x = 5 \times x \)

So the operation is \( \times 5 \).

The opposite operation is \( \div 5 \).

Step 1: \( \div 5 \)

\( 5x = 12 \)

\( \frac{5x}{5} = \frac{12}{5} \)

\( x = \frac{12}{5} \)
4 Write the answer in its simplest form.

b 1 Identify the operations performed in order on the unknown.

2 Write the opposite operations.

3 Perform the opposite operations in reverse order on both sides of the equation, one operation at a time.

\[ x = \frac{12}{5} \]

b \[ 8t + 11 \]
The operations are \( \times 8, + 11 \).
\[ \div 8, - 11 \]

Step 1: \(-11\)
\[ 8t + 11 = 20 \]
\[ 8t + 11 - 11 = 20 - 11 \]
\[ 8t = 9 \]
Step 2: \(\div 8\)
\[ 8t = 9 \]
\[ \frac{8t}{8} = \frac{9}{8} \]
\[ t = \frac{9}{8} \]

4 Write the answer in its simplest form.

c 1 Identify the operations performed in order on the unknown. (Remember operations in brackets are performed first.)

2 Write the opposite operations.

3 Perform the opposite operations on both sides of the equation in reverse order, one operation at a time.

\[ 4(n - 3) \]
The operations are \(-3, \times 4\).
\[ + 3, \div 4 \]
Step 1: \(\div 4\)
\[ 12 = 4(n - 3) \]
\[ \frac{12}{4} = \frac{4(n - 3)}{4} \]
\[ 3 = n - 3 \]
Step 2: \(+ 3\)
\[ 3 = n - 3 \]
\[ 3 + 3 = n - 3 + 3 \]
\[ 6 = n \]
\[ n = 6 \]

d 1 Identify the operations performed in order on the unknown.

2 Write the opposite operations.

3 Perform the opposite operations on both sides of the equation in reverse order, one operation at a time.

\[ \frac{4x - 2}{3} \]
The operations are \(\times 4, - 2, \div 3\).
\[ \div 4, + 2, \times 3 \]
Step 1: \(\times 3\)
\[ \frac{4x - 2}{3} = 5 \]
\[ 3 \times \frac{4x - 2}{3} = 5 \times 3 \]
\[ 4x - 2 = 15 \]
Step 2: $+ 2$
$4x - 2 = 15$
$4x - 2 + 2 = 15 + 2$
$4x = 17$

Step 3: $\div 4$
$4x = 17$
$\frac{4x}{4} = \frac{17}{4}$
$x = \frac{17}{4}$

4 Write the answer in its simplest form.

Substituting into linear equations
If we are given a linear equation between two variables and we are given the value of one of the variables, we can substitute this into the equation to determine the other value.

WORKED EXAMPLE 5 Substitute $x = 3$ into the linear equation $y = 2x + 5$ to determine the value of $y$.

THINK
1 Substitute the variable ($x$) with the given value.
2 Equate the right-hand side of the equation.

WRITE
$y = 2(3) + 5$
$y = 6 + 5$
$y = 11$

Literal linear equations
A literal equation is an equation that includes several pronumerals or variables. Literal equations often represent real-life situations.
The equation $y = mx + c$ is an example of a linear literal equation that represents the general form of a straight line.
To solve literal linear equations, you need to isolate the variable you are trying to solve for.

WORKED EXAMPLE 6 Solve the linear literal equation $y = mx + c$ for $x$.

THINK
1 Isolate the terms containing the variable you want to solve on one side of the equation.
2 Divide by the coefficient of the variable you want to solve for.
3 Transpose the equation.

WRITE
$y - c = mx$
$\frac{y - c}{m} = x$
$x = \frac{y - c}{m}$
**EXERCISE 1.3  Solving linear equations**

**PRACTISE**

1. **WE4** Solve the following linear equations to find the unknowns.
   a) \(2(x + 1) = 8\)  
   b) \(n - 12 = -2\)  
   c) \(4d - 7 = 11\)  
   d) \(\frac{x + 1}{2} = 9\)

2. **a** Write the operations in order that have been performed on the unknowns in the following linear equations.
   i) \(10 = 4a + 3\)  
   ii) \(3(x + 2) = 12\)  
   iii) \(\frac{s + 1}{2} = 7\)  
   iv) \(16 = 2(3c - 9)\)

   **b** Find the exact values of the unknowns in part a by solving the equations.
   Show all of the steps involved.

3. **WE5** Substitute \(x = 5\) into the equation \(y = 5 - 6x\) to determine the value of \(y\).

4. Substitute \(x = -3\) into the equation \(y = 3x + 3\) to determine the value of \(y\).

5. **WE6** Solve the literal linear equation \(px - q = r\) for \(x\).

6. Solve the literal linear equation \(C = \pi d\) for \(d\).

7. Find the exact values of the unknowns in the following linear equations.
   a) \(14 = 5 - x\)  
   b) \(\frac{4(3y - 1)}{5} = -2\)  
   c) \(\frac{2(3 - x)}{3} = 5\)

8. Solve the following literal linear equations for the pronumerals given in brackets.
   a) \(v = u + at\) \(\text{ (a)}\)  
   b) \(xy - k = m\) \(\text{ (x)}\)  
   c) \(\frac{x}{p} - r = s\) \(\text{ (x)}\)

9. The equation \(w = 10t + 120\) represents the amount of water in a tank, \(w\) (in litres), at any time, \(t\) (in minutes). Find the time, in minutes, that it takes for the tank to have the following amounts of water.
   a) 450 litres  
   b) 1200 litres

10. Yorx was asked to solve the linear equation \(5w - 13 = 12\). His solution is shown.
    
    **Step 1.** \(\times 5, -13\)
    **Step 2.** Opposite operations \(\div 5, +13\)
    **Step 3.** \(5w - 13 = 12\)
    \(\frac{5w - 13}{5} = \frac{12}{5}\)
    \(w - 13 = 2.4\)
    **Step 4.** \(w - 13 + 13 = 2.4 + 13\)
    \(w = 15.4\)

    **a** Show that Yorx’s answer is incorrect by finding the value of \(w\).
    **b** What advice would you give to Yorx so that he can solve linear equations correctly?

11. The literal linear equation \(F = 1.8(K - 273) + 32\) converts the temperature in Kelvin \((K)\) to Fahrenheit \((F)\). Solve the equation for \(K\) to give the formula for converting the temperature in Fahrenheit to Kelvin.
12 Consider the linear equation \( y = \frac{3x + 1}{4} \). Find the value of \( x \) for the following \( y \)-values.

a 2   b −3   c \( \frac{1}{2} \)   d 10

13 The distance travelled, \( d \) (in kilometres), at any time \( t \) (in hours) can be found using the equation \( d = 95t \). Find the time in hours that it takes to travel the following distances. Write your answers correct to the nearest minute.

a 190 km   b 250 km   c 65 km   d 356.5 km   e 50 000 m

14 The amount, \( A \), in dollars in a bank account at the end of any month, \( m \), can found using the equation \( A = 150m + 400 \).

a How many months would it take to have the following amounts of money in the bank account?
   i $1750   ii $3200

b How many years would it take to have $10000 in the bank account? Write your answer correct to the nearest month.

15 The temperature, \( C \), in degrees Celsius can be found using the equation

\[ C = \frac{5(F - 32)}{9} \]  

where \( F \) is the temperature in degrees Fahrenheit. Nora needs to set her oven at 190°C, but her oven’s temperature is measured in Fahrenheit.

a Write the operations performed on the variable \( F \).

b Write the order in which the operations need to be performed to find the value of \( F \).

c Determine the temperature in Fahrenheit that Nora should set her oven to.

16 The equation that determines the surface area of a cylinder with a radius of 3.5 cm is \( A = 3.5\pi(3.5 + h) \).

Determine the height in cm of cylinders with radii of 3.5 cm and the following surface areas. Write your answers correct to 2 decimal places.

a 200 cm²  b 240 cm²  c 270 cm²

17 Using a calculator, spreadsheet or otherwise, solve the following equations to find the unknowns. Express your answer in exact form.

a \( \frac{2 - 5x}{8} = \frac{3}{5} \)  b \( \frac{6(3y - 2)}{11} = \frac{5}{9} \)

\[ c \left( \frac{4x}{5} - \frac{3}{7} \right) + 8 = 2 \]  d \( \frac{7x + 6}{9} + \frac{3x}{10} = \frac{4}{5} \)
The height of a plant can be found using the equation \( h = \frac{2(3t + 15)}{3} \), where \( h \) is the height in cm and \( t \) is time in weeks.

a. Using a calculator, spreadsheet or otherwise, determine the time the plant takes to grow to the following heights. Write your answers to the nearest week.

i. 20 cm
ii. 30 cm
iii. 35 cm
iv. 50 cm

b. How high is the plant initially?

When the plant reaches 60 cm it is given additional plant food. The plant’s growth each week for the next 4 weeks is found using the equation \( g = t + 2 \), where \( g \) is the growth each week in cm and \( t \) is the time in weeks since additional plant food was given.

c. Using a spreadsheet, determine the height of the plant in cm for the next 4 weeks.

### Developing linear equations

**Developing linear equations from word descriptions**

To write a worded statement as a linear equation, we must first identify the unknown and choose a pronumeral to represent it. We can then use the information given in the statement to write a linear equation in terms of the pronumeral.

The linear equation can then be solved as before, and we can use the result to answer the original question using words.

#### WORKED EXAMPLE 7

Cans of soft drinks are sold at SupaSave in packs of 12 costing $5.40. Form and solve a linear equation to determine the price of 1 can of soft drink.

**THINK**

1. Identify the unknown and choose a pronumeral to represent it.

   - \( S \) = price of a can of soft drink

2. Use the given information to write an equation in terms of the pronumeral.

   - \( 12S = 5.4 \)

3. Solve the equation.

   - \( \frac{12S}{12} = \frac{5.4}{12} \)
   - \( S = 0.45 \)

4. Interpret the solution in terms of the original problem.

   - The price of 1 can of soft drink is $0.45 or 45 cents.
WORD PROBLEMS WITH MORE THAN ONE UNKNOWN
In some instances a word problem might contain more than one unknown. If we are able to express both unknowns in terms of the same pronumeral, we can create a linear equation as before and solve it to determine the value of both unknowns.

WORKED EXAMPLE 8
Georgina is counting the number of insects and spiders she can find in her back garden. All insects have 6 legs and all spiders have 8 legs. In total, Georgina finds 43 bugs with a total of 290 legs. Form a linear equation to determine exactly how many insects and spiders Georgina found.

THINK
1 Identify one of the unknowns and choose a pronumeral to represent it.
Let \( s \) = the number of spiders.
2 Define the other unknown in terms of this pronumeral.
Let \( 43 - s \) = the number of insects.
3 Write expressions for the total numbers of spiders’ legs and insects’ legs.
Total number of spiders legs = \( 8s \)
Total number of insects legs = \( 6(43 - s) \)
\[= 258 - 6s \]
4 Create an equation for the total number of legs of both types of creature.
\[8s + (258 - 6s) = 290\]
5 Solve the equation.
\[8s + 258 - 6s = 290\]
\[8s - 6s = 290 - 258\]
\[2s = 32\]
\[s = 16\]
6 Substitute this value back into the second equation to determine the other unknown.
The number of insects = \( 43 - 16 \)
\[= 27\]
7 Answer the question using words.
Georgina found 27 insects and 16 spiders.

Tables of values
Tables of values can be generated from formulas by entering given values of one variable into the formula. These tables of values can be used to solve problems and to draw graphs representing these situations (as covered in more detail in chapter 10).
The amount of water that is filling a tank is found by the rule \( W = 100t + 20 \), where \( W \) is the amount of water in the tank in litres and \( t \) is the time in hours.

a Generate a table of values that shows the amount of water, \( W \), in the tank every hour for the first 8 hours (i.e. \( t = 0, 1, 2, 3, \ldots, 8 \)).

b Using your table, how long in hours will it take for there to be over 700 litres in the tank?

**THINK**

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W )</td>
<td>20</td>
<td>120</td>
<td>220</td>
<td>320</td>
<td>420</td>
<td>520</td>
<td>620</td>
<td>720</td>
<td>820</td>
</tr>
</tbody>
</table>

**WRITE**

a  
\[
\begin{align*}
  t = 0: & \quad W = 100(0) + 20 = 20 \\
  t = 1: & \quad W = 100(1) + 20 = 120 \\
  t = 2: & \quad W = 100(2) + 20 = 220 \\
  t = 3: & \quad W = 100(3) + 20 = 320 \\
  t = 4: & \quad W = 100(4) + 20 = 420 \\
  t = 5: & \quad W = 100(5) + 20 = 520 \\
  t = 6: & \quad W = 100(6) + 20 = 620 \\
  t = 7: & \quad W = 100(7) + 20 = 720 \\
  t = 8: & \quad W = 100(8) + 20 = 820
\end{align*}
\]

b  

b  

\[
\begin{align*}
  t = 7 & \quad \text{It will take 7 hours for there to be over 700 litres of water in the tank.}
\end{align*}
\]

**Linear relations defined recursively**

Many sequences of numbers are obtained by following rules that define a relationship between any one term and the previous term. Such a relationship is known as a recurrence relation.

A term in such a sequence is defined as \( t_n \), with \( n \) denoting the place in the sequence. The term \( t_{n-1} \) is the previous term in the sequence.
If the recurrence relation is of a linear nature — that is, there is a common difference \((d)\) between each term in the sequence — then we can define the recurrence relation as:

\[
t_n = t_{n-1} + d, \quad t_1 = a
\]

This means that the first term in the sequence is \(a\), and each subsequent term is found by adding \(d\) to the previous term.

**WORKED EXAMPLE 10** A linear recurrence relation is given by the formula \(t_n = t_{n-1} + 6, \quad t_1 = 5\). Write the first six terms of the sequence.

<table>
<thead>
<tr>
<th>THINK</th>
<th>WRITE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Calculate the value for (t_2) by substituting the value for (t_1) into the formula. Then use this to calculate the value for (t_3) and so on.</td>
<td>(t_2 = t_1 + 6) (= 5 + 6) (= 11) (t_3 = t_2 + 6) (= 11 + 6) (= 17)</td>
</tr>
<tr>
<td>2 State the answer.</td>
<td>The first six values are 5, 11, 17, 23, 29 and 35.</td>
</tr>
</tbody>
</table>

**WORKED EXAMPLE 11** The weekly rent on an inner-city apartment increases by $10 every year. In a certain year the weekly rent is $310.

- **a** Model this situation by setting up a linear recurrence relation between the weekly rental prices in consecutive years.
- **b** Find the weekly rent for the first six years.
- **c** Find an expression for the weekly rent \((r)\) in the \(n\)th year.

<table>
<thead>
<tr>
<th>THINK</th>
<th>WRITE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong> Write the values of (a) and (d) in the generalised linear recurrence relation formula.</td>
<td>(a \quad a = 310, \quad d = 10)</td>
</tr>
<tr>
<td><strong>b</strong> Substitute these values into the generalised linear recurrence relation formula.</td>
<td>(t_n = t_{n-1} + 10, \quad t_1 = 310)</td>
</tr>
<tr>
<td><strong>b</strong> Substitute (n = 2, n = 3, n = 4, n = 5) and (n = 6) into the recurrence relation.</td>
<td>(t_2 = t_1 + 10) (= 310 + 10) (= 320) (t_3 = t_2 + 10) (= 320 + 10) (= 330)</td>
</tr>
<tr>
<td></td>
<td>(t_4 = t_3 + 10) (= 330 + 10) (= 340) (t_5 = t_4 + 10) (= 340 + 10) (= 350)</td>
</tr>
<tr>
<td></td>
<td>(t_6 = t_5 + 10) (= 350 + 10) (= 360)</td>
</tr>
</tbody>
</table>
### Developing linear equations

#### EXERCISE 1.4

1. **WE7** Artists’ pencils at the local art supply store sell in packets of 8 for $17.92. Form and solve a linear equation to determine the price of 1 artists’ pencil.

2. Natasha is trying to determine which type of cupcake is the best value for money. The three options Natasha is considering are:
   - 4 red velvet cupcakes for $9.36
   - 3 chocolate delight cupcakes for $7.41
   - 5 caramel surprise cupcakes for $11.80.
   Form and solve linear equations for each type of cupcake to determine which has the cheapest price per cupcake.

3. **WE8** Fredo is buying a large bunch of flowers for his mother in advance of Mother’s Day. He picks out a bunch of roses and lilies, with each rose costing $6.20 and each lily costing $4.70. In total he picks out 19 flowers and pays $98.30. Form a linear equation to determine exactly how many roses and lilies Fredo bought.

4. Miriam has a sweet tooth, and her favourite sweets are strawberry twists and chocolate ripples. The local sweet shop sells both as part of their pick and mix selection, so Miriam fills a bag with them. Each strawberry twist weighs 5 g and each chocolate ripple weighs 9 g. In Miriam’s bag there are 28 sweets, weighing a total of 188 g. Determine the number of each type of sweet that Miriam bought by forming and solving a linear equation.

5. **WE9** Libby enjoys riding along Beach Road on a Sunday morning. She rides at a constant speed of 0.4 kilometres per minute.
   a. Generate a table of values that shows how far Libby has travelled for each of the first 10 minutes of her journey.
   b. One Sunday Libby stops and meets a friend 3 kilometres into her journey. Between what minutes does Libby stop?
6 Tommy is saving for a remote-controlled car that is priced at $49. He has $20 in his piggy bank. Tommy saves $3 of his pocket money every week and puts it in his piggy bank. The amount of money in dollars, $M$, in his piggy bank after $w$ weeks can be found using the rule $M = 3w + 20$.

a Generate a table of values that shows the amount of money, $M$, in Tommy’s piggy bank every week for the 12 weeks (i.e. $w = 0, 1, 2, 3, \ldots, 12$).

b Using your table, how many weeks will it take for Tommy to have saved enough money to purchase the remote-controlled car?

7 A linear recurrence relation is given by the formula $t_n = t_{n-1} - 6$, $t_1 = 12$. Write the first six terms of the sequence.

8 A linear recurrence relation is given by the formula $t_n = t_{n-1} + 3.2$, $t_1 = -5.8$. Write the first six terms of the sequence.

9 Jake is a stamp collector. He notices that the value of the rarest stamp in his collection increases by $25 each year. Jake purchased the stamp for$450.

a Set up a recurrence relation between the yearly values of Jake’s rarest stamp.

b Find the value of the stamp for each year over the first 8 years.

c Find an expression for the stamp’s value ($v$) in the $n$th year.

10 Juliet is a zoologist and has been monitoring the population of a species of wild lemur in Madagascar over a number of years. Much to her dismay, she finds that on average the population decreases by 13 each year. In her first year of monitoring, the population was 313.

a Set up a recurrence relation between the yearly populations of the lemurs.

b Find the population of the lemurs for each year over the first 7 years.

c Find an expression for the population of lemurs ($l$) in the $n$th year.

11 Three is added to a number and the result is then divided by four, giving an answer of nine. Find the number.

12 The sides in one pair of sides of a parallelogram are each 3 times the length of a side in the other pair. Find the side lengths if the perimeter of the parallelogram is 84 cm.

13 Fred is saving for a holiday and decides to deposit $40 in his bank account each week. At the start of his saving scheme he has$150 in his account.

a Set up a recurrence relation between the amounts in Fred’s account on consecutive weeks.

b Use the recurrence relation to construct a table of values detailing how much Fred will have in his account after each of the first 8 weeks.

c The holiday Fred wants to go on will cost $720 dollars. How many weeks will it take Fred to save up enough money to pay for his holiday?

14 One week Jordan bought a bag of his favourite fruit and nut mix at the local market. The next week he saw that the bag was on sale for 20% off the previously marked price. Jordan purchased two more bags at the reduced price. Jordan spent$20.54 in total for the three bags. Find the original price of a bag of fruit and nut mix.
15 Six times the sum of four plus a number is equal to one hundred and twenty-six. Find the number.

16 Sabrina is a landscape gardener and has been commissioned to work on a rectangular piece of garden. The length of the garden is 6 metres longer than the width, and the perimeter of the garden is 64 m. Find the parameters of the garden.

17 Yuri is doing his weekly grocery shop and is buying both carrots and potatoes. He calculates that the average weight of a carrot is 60 g and the average weight of a potato is 125 g. Furthermore, he calculates that the average weight of the carrots and potatoes that he purchases is 86 g. If Yuri’s shopping weighed 1.29 kg in total, how many of each did he purchase?

18 Ho has a water tank in his back garden that can hold up to 750 L in water. At the start of a rainy day (at 0:00) there is 165 L in the tank, and after a heavy day’s rain (at 24:00) there is 201 L in the tank.
   a Assuming that the rain fell consistently during the 24-hour period, set up a linear equation to represent the amount of rain in the tank at any point during the day.
   b Generate a table of values that shows how much water is in the tank after every 2 hours of the 24-hour period.
   c At what time of day did the amount of water in the tank reach 192 L?

19 A large fish tank is being filled with water. After 1 minute the height of the water is 2 cm and after 4 minutes the height of the water is 6 cm. The height of the water, \( h \), in cm after \( t \) minutes can be modelled by a linear equation.
   a Construct a recurrence relation between consecutive minutes of the height of water in the fish tank.
   b Determine the height of the water in the fish tank after each of the first five minutes.
   c Was the fish tank empty of water before being filled? Justify your answer by using calculations.

20 Michelle and Lydia live 325 km apart. On a Sunday they decide to drive to each other’s respective towns. They pass each other after 2.5 hours. If Michelle drives an average of 10 km/h faster than Lydia, calculate the speed at which they are both travelling.

21 Jett is starting up a small business selling handmade surfboard covers online. The start-up cost is $250. He calculates that each cover will cost $14.50 to make. The rule that finds the cost, \( C \), to make \( n \) covers is \( C = 14.50n + 250 \).
   a Using a spreadsheet, calculator or otherwise, generate a table of values to determine the cost of producing 10 to 20 surfboard covers.
b If Jett sells the covers for $32.95, use your table of values to determine the revenue for selling 10 to 20 surfboard covers.

c The profit Jett makes is the difference between his selling price and the cost price. Explain how the profit Jett makes can be calculated using the table of values already constructed in parts a and b.

d Using your explanation in part c and your table of values, determine the profits made by Jett if he sells 10 to 20 surfboard covers.

22 Benito decides to set up a market stall selling fruit based energy drinks. He has to pay $300 for the pitch of his stall on a particular day. The ingredients for each energy drink total $1.35, and he sells each energy drink for $4.50.

a If the cost of making $m$ energy drinks is $c_m$, write a recurrence relation for the cost of making the energy drinks.

b If the income from selling $n$ energy drinks is $s_n$, write a recurrence relation for the income from selling the energy drinks.

c Using a spreadsheet, calculator or otherwise, determine the minimum number of energy drinks Benito needs to sell to make a profit.

d Generate a table of values showing the profit/loss for selling up to 120 energy drinks in multiples of 10 (i.e. 0, 10, 20, ..., 120).

1.5 Simultaneous linear equations

Solving simultaneous equations graphically

Simultaneous equations are sets of equations that can solved at the same time. They often represent practical problems that have two or more unknowns. For example, you can use simultaneous equations to find the costs of individual apples and oranges when different amounts of each were bought.

Solving simultaneous equations gives the set of values that is common to all of the equations. If these equations are presented graphically, then the set of values common to all equations is the point of intersection.

To solve a set of simultaneous equations graphically, the equations must be sketched on the same set of axes and the point of intersection must be found. If the equations do not intersect then there is no solution for the simultaneous equations.

WORKED EXAMPLE 12

The following equations represent a pair of simultaneous equations.

\[ y = 2x + 4 \quad \text{and} \quad y = 3x + 3 \]

Using a calculator, spreadsheet or otherwise, sketch both graphs on the same set of axes and solve the equations.
Solving simultaneous equations using substitution

Simultaneous equations can also be solved algebraically. One algebraic method is known as substitution. This method requires one of the equations to be substituted into the other by replacing one of the variables. The second equation is then solved and the value of one of the variables is found. The substitution method is often used when one or both of the equations are written with variables on either side of the equals sign; for example, \( c = 12b - 15 \) and \( 2c + 3b = -3 \), or \( y = 4x + 6 \) and \( y = 6x + 2 \).

**WORKED EXAMPLE 13**

Solve the following pairs of simultaneous equations using substitution;

\[ a \quad c = 12b - 15 \text{ and } 2c + 3b = -3 \]

\[ b \quad y = 4x + 6 \text{ and } y = 6x + 2 \]

\[ c \quad 3x + 2y = -1 \text{ and } y = x - 8 \]

**THINK**

1 Identify which variable will be substituted into the other equation.

2 Substitute the variable into the equation.

**WRITE**

\[ a \quad c = 12b - 15 \]

\[ 2c + 3b = -3 \]

\[ 2(12b - 15) + 3b = -3 \]
3 Expand and simplify the left-hand side, and solve the equation for the unknown variable.

\[ 24b - 30 + 3b = -3 \]
\[ 27b - 30 = -3 \]
\[ 27b = -3 + 30 \]
\[ 27b = 27 \]
\[ b = 1 \]

4 Substitute the value for the unknown back into one of the equations.

\[ c = 12b - 15 \]
\[ = 12(1) - 15 \]
\[ = -3 \]

5 Answer the question.

The solution is \( b = 1 \) and \( c = -3 \).

### b

1 Both equations are in the form \( y = \), so let them equal each other.

2 Move all of the pronumerals to one side.

\[ 4x - 4x + 6 = 6x - 4x + 2 \]
\[ 6 = 6x - 4x + 2 \]
\[ 6 = 2x + 2 \]
\[ 6 - 2 = 2x + 2 - 2 \]
\[ 4 = 2x \]
\[ 4 = \frac{2x}{2} \]
\[ 2 = x \]

3 Solve for the unknown.

\[ y = 4x + 6 \]
\[ = 4 \times 2 + 6 \]
\[ = 8 + 6 \]
\[ = 14 \]

The solution is \( x = 2 \) and \( y = 14 \).

### c

1 Both equations are not in the form \( y = \), so substitute one into the other.

2 Expand and simplify the equation.

\[ 3x + 2(x - 8) = -1 \]
\[ 3x + 2x - 16 = -1 \]
\[ 5x - 16 = -1 \]
\[ 5x - 16 + 16 = -1 + 16 \]
\[ 5x = 15 \]
\[ x = 3 \]

3 Solve for the unknown.

\[ y = x - 8 \]
\[ = 3 - 8 \]
\[ = -5 \]

The solution is \( x = 3 \) and \( y = -5 \).

---

**Solving simultaneous equations using elimination**

Solving simultaneous equations using elimination requires the equations to be added or subtracted so that one of the pronumerals is eliminated or removed. Simultaneous equations that have both pronumerals on the same side are often solved using elimination. For example, \( 3x + y = 5 \) and \( 4x - y = 2 \) both have \( x \) and \( y \) on the same side of the equation, so they can be solved with this method.
### WORKED EXAMPLE 14

Solve the following pairs of simultaneous equations using elimination:

<table>
<thead>
<tr>
<th>a</th>
<th>3x + y = 5 and 4x − y = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>2a + b = 7 and a + b = 5</td>
</tr>
<tr>
<td>c</td>
<td>3c + 4d = 5 and 2c + 3d = 4</td>
</tr>
</tbody>
</table>

#### WRITE

| 3x + y = 5  | [1] |
| 4x − y = 2  | [2] |

Select y.

The coefficients of y are 1 and −1.

[1] + [2]:

\[
3x + 4x + y − y = 5 + 2 \\
7x = 7 \\
x = 1
\]

3x + y = 5

3(1) + y = 5

3 + y = 5

3 − 3 + y = 5 − 3

y = 2

The solution is \( x = 1 \) and \( y = 2 \).

| 2a + b = 7  | [1] |
| a + b = 5   | [2] |

Select b.

The coefficients of b are both 1.

[1] − [2]:

\[
2a − a + b − b = 7 − 5 \\
a = 2
\]

\[
a + b = 5 \\
2 + b = 5
\]

a = 2

b = 5 − 2

b = 3

The solution is \( a = 2 \) and \( b = 3 \).
**c 1** Write the simultaneous equations with one on top of the other.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$3c + 4d = 5$</td>
<td>$2c + 3d = 4$</td>
</tr>
</tbody>
</table>

**c 2** Select one pronumeral to be eliminated.

Select $c$.

**c 3** Check the coefficients of the pronumeral being eliminated.

The coefficients of $c$ are 3 and 2.

**c 4** If the coefficients are different numbers, then multiply them both by another number, so they both have the same coefficient value.

- $3 \times 2 = 6$
- $2 \times 3 = 6$

**c 5** Multiply the equations (all terms in each equation) by the numbers selected in step 4.

- $[1] \times 2$: $6c + 8d = 10$
- $[2] \times 3$: $6c + 9d = 12$

**c 6** Check the sign of each coefficient for the selected pronumeral.

Both coefficients of $c$ are positive 6.

**c 7** If the signs are the same, then subtract one equation from the other and simplify.

- $-d = -2$

**c 8** Solve the equation for the unknown.

$d = 2$

**c 9** Substitute the pronumeral back into one of the equations.

- $2c + 3d = 4$
- $2c + 3(2) = 4$

**c 10** Solve the equation to find the value of the other pronumeral.

- $2c + 6 = 4$
- $2c + 6 - 6 = 4 - 6$
- $2c = -2$
- $\frac{2c}{2} = \frac{-2}{2}$
- $c = -1$

**c 11** Answer the question.

The solution is $c = -1$ and $d = 2$.

---

**EXERCISE 1.5 Simultaneous equations**

<table>
<thead>
<tr>
<th><strong>PRACTISE</strong></th>
</tr>
</thead>
</table>

1 WE12 The following equations represent a pair of simultaneous equations.

$y = 5x + 1$ and $y = 2x - 5$

Using a calculator, spreadsheet or otherwise, sketch both graphs on the same set of axes and solve the equations.

2 Using a calculator, spreadsheet or otherwise, sketch and solve the following three simultaneous equations.

$y = 3x + 7$, $y = 2x + 8$ and $y = -2x + 12$

3 WE13 Solve the following pair of simultaneous equations using substitution.

**a** $y = 2x + 1$ and $2y - x = -1$

**b** $m = 2n + 5$ and $m = 4n - 1$

**c** $2x - y = 5$ and $y = x + 1$
4 Find the solutions to the following pairs of simultaneous equations using substitution.
   \[a \quad 2(x + 1) + y = 5 \quad \text{and} \quad y = x - 6\]
   \[b \quad \frac{x + 5}{2} + 2y = 11 \quad \text{and} \quad y = 6x - 2\]

5 Solve the following pairs of simultaneous equations using the method of elimination.
   \[a \quad 3x + y = 5 \quad \text{and} \quad 4x - y = 2\]
   \[b \quad 2a + b = 7 \quad \text{and} \quad a + b = 5\]
   \[c \quad 3c + 4d = 5 \quad \text{and} \quad 2c + 3d = 4\]

6 Consider the following pair of simultaneous equations:
   \[ax - 3y = -16 \quad \text{and} \quad 3x + y = -2\]
   If \(y = 4\), find the values of \(a\) and \(x\).

7 Which one of the following pairs of simultaneous equations would best be solved using the substitution method?
   \[a \quad 4y - 5x = 7 \quad \text{and} \quad 3x + 2y = 1\]
   \[b \quad 3c + 8d = 19 \quad \text{and} \quad 2c - d = 6\]
   \[c \quad 12x + 6y = 15 \quad \text{and} \quad 9x - y = 13\]
   \[d \quad 3(t + 3) - 4s = 14 \quad \text{and} \quad 2(s - 4) + t = -11\]
   \[e \quad n = 9m + 12 \quad \text{and} \quad 3m + 2n = 7\]

8 A pair of simultaneous equations is solved graphically as shown in the diagram. From the diagram, determine the solution for this pair of simultaneous equations.

9 Using a calculator, spreadsheet or otherwise, solve the following pairs of simultaneous equations graphically.
   \[a \quad y = 4x + 1 \quad \text{and} \quad y = 3x - 1\]
   \[b \quad y = x - 5 \quad \text{and} \quad y = -3x + 3\]
   \[c \quad y = 3(x - 1) \quad \text{and} \quad y = 2(2x + 1)\]
   \[d \quad y = \frac{x}{2} - 1 \quad \text{and} \quad y = \frac{x}{2} + 4\]
   \[e \quad y = 3x + 4, \quad y = 2x + 3 \quad \text{and} \quad y = -x\]

10 Using the substitution method, solve the following pairs of simultaneous equations
   \[a \quad y = 2x + 5 \quad \text{and} \quad y = 3x - 2\]
   \[b \quad y = 5x - 2 \quad \text{and} \quad y = 7x + 2\]
   \[c \quad y = 2(3x + 1) \quad \text{and} \quad y = 4(2x - 3)\]
   \[d \quad y = 5x - 9 \quad \text{and} \quad 3x - 5y = 1\]
   \[e \quad 3(2x + 1) + y = -19 \quad \text{and} \quad y = x - 1\]
   \[f \quad \frac{3x + 5}{2} + 2y = 2 \quad \text{and} \quad y = x - 2\]

11 A student chose to solve the following pair of simultaneous equations using the elimination method.
   \[3x + y = 8 \quad \text{and} \quad 2x - y = 7\]
   \[a \quad \text{Explain why this student’s method would be the most appropriate method for this pair of simultaneous equations.}\]
   \[b \quad \text{Show how these equations would be solved using this method.}\]
12 Using the elimination method, solve the following pairs of simultaneous equations

- a) \(4x + y = 6\) and \(x - y = 4\)
- b) \(x + y = 7\) and \(x - 2y = -5\)
- c) \(2x - y = -5\) and \(x - 3y = -10\)
- d) \(4x + 3y = 29\) and \(2x + y = 13\)
- e) \(5x - 7y = -33\) and \(4x + 3y = 8\)
- f) \(\frac{x}{2} + y = 7\) and \(3x + \frac{y}{2} = 20\)

13 The first step when solving the following pair of simultaneous equations using the elimination method is:

\[
\begin{align*}
2x + y &= 3 \quad [1] \\
3x - y &= 2 \quad [2]
\end{align*}
\]

A) equations [1] and [2] should be added together
B) both equations should be multiplied by 2
C) equation [1] should be subtracted from equation [2]
D) equation [1] should be multiplied by 2 and equation [2] should be multiplied by 3
E) equation [2] should be subtracted from equation [1]

14 Brendon and Marcia were each asked to solve the following pair of simultaneous equations.

\[
\begin{align*}
3x + 4y &= 17 \quad [1] \\
4x - 2y &= 19 \quad [2]
\end{align*}
\]

Marcia decided to use the elimination method. Her solution steps were:

Step 1. equation [1] \(\times 4\):
\[
12x + 16y = 68
\]
Equation [2] \(\times 3\):
\[
12x - 6y = 57
\]
Step 2. [1] + [2]:
\[
10y = 125
\]
Step 3. \(y = 12.5\)
Step 4. substitute \(y = 12.5\) into [1]:
\[
3x + 4 (12.5) = 17
\]
Step 5. solve for \(x\):
\[
3x = 17 - 50
\]
\[
3x = -33
\]
\[
x = -11
\]
Step 6. The solution is \(x = -11\) and \(y = 12.5\).

a) Marcia has made an error in step 2. Explain where she has made her error, and hence correct her mistake.

b) Using the correction you made in part a, find the correct solution to this pair of simultaneous equations.

Brendon decided to eliminate \(y\) instead of \(x\).

c) Using Brendon’s method of eliminating \(y\) first, show all the appropriate steps involved to reach a solution.
15 In a ball game, a player can throw the ball into the net to score a goal or place the ball over the line to score a behind. The scores in a game between the Rockets and the Comets were as follows.
Rockets: 6 goals 12 behinds, total score 54
Comets: 7 goals 5 behinds, total score 45
The two simultaneous equations that can represent this information are shown.
Rockets: $6x + 12y = 54$
Comets: $7x + 5y = 45$

a By solving the two simultaneous equations, determine the number of points that are awarded for a goal and a behind.
b Using the results from part a, determine the scores for the game between the Jetts, who scored 4 goals and 10 behinds, and the Meteorites, who scored 6 goals and 9 behinds.

16 Mick and Minnie both work part time at an ice-cream shop. The simultaneous equations shown represent the number of hours Mick ($x$) and Minnie ($y$) work each week.
Equation 1: Total number of hours worked by Minnie and Mick: $x + y = 15$
Equation 2: Number of hours worked by Minnie in terms of Mick’s hours: $y = 2x$
a Explain why substitution would be the best method to use to solve these equations.
b Using substitution, determine the number of hours worked by Mick and Minnie each week.
To ensure that he has time to do his Mathematics homework, Mick changes the number of hours he works each week. He now works $\frac{1}{3}$ of the number of hours worked by Minnie. An equation that can be used to represent this information is $x = \frac{y}{3}$.
c Using a calculator, spreadsheet or otherwise, find the number of hours worked by Mick, given that the total number of hours that Mick and Minnie work does not change.

17 Using a calculator, spreadsheet or otherwise, solve the following pairs of simultaneous equations. Write your answers correct to 2 decimal places.

a $y = 5x + 6$ and $3x + 2y = 7$
b $4(x + 6) = y - 6$ and $2(y + 3) = x - 9$
c $6x + 5y = 8.95$, $y = 3x - 1.36$ and $2x + 3y = 4.17$

18 Consider the following pairs of graphs.
i $y_1 = 5x - 4$ and $y_2 = 6x + 8$
ii $y_1 = -3x - 5$ and $y_2 = 3x + 1$
iii $y_1 = 2x + 6$ and $y_2 = 2x - 4$
iv $y_1 = -x + 3$, $y_2 = x + 5$ and $y_3 = 2x + 6$

a Where possible, find the point of intersection for each pair of equations using any method.
b Are there solutions for all pair of equations? If not, for which pair of equations is there no solution, and why is this?

1.6 Problem solving with simultaneous equations

Setting up simultaneous equations

The solutions to a set of simultaneous equations satisfy all equations that were used. Simultaneous equations can be used to solve problems involving two or more variables or unknowns, such as the cost of 1 kilogram of apples and bananas, or the number of adults and children attending a show.

At a fruit shop, 2 kg of apples and 3 kg of bananas cost $13.16, and 3 kg of apples and 2 kg of bananas cost $13.74. Represent this information in the form of a pair of simultaneous equations.

**THINK**

1 Identify the two variables.

2 Select two pronumerals to represent these variables. Define the variables.

3 Identify the key information and rewrite it using the pronumerals selected.

4 Construct two equations using the information.

**WRITE**

The cost of 1 kg of apples and the cost of 1 kg of bananas

$a =$ cost of 1 kg of apples

$b =$ cost of 1 kg of bananas

2 kg of apples can be written as $2a$.

3 kg of bananas can be written as $3b$.

$2a + 3b = 13.16$

$3a + 2b = 13.74$

**Break-even points**

A break-even point is a point where the costs equal the selling price. It is also the point where there is zero profit. For example, if the equation $C = 45 + 3r$ represents the production cost to sell $r$ shirts, and the equation $R = 14r$ represents the revenue from selling the shirts for $14 each, then the break-even point is the number of shirts that need to be sold to cover all costs. To find the break-even point, the equations for cost and revenue are solved simultaneously.
Santo sells shirts for $25. The revenue, \( R \), for selling \( n \) shirts is represented by the equation \( R = 25n \). The cost to make \( n \) shirts is represented by the equation \( C = 2200 + 3n \).

a  Solve the equations simultaneously to determine the break-even point.

b  Determine the profit or loss, in dollars, for the following shirt orders.

i  75 shirts

ii  220 shirts

### THINK

a  1 Write the two equations.

b  i  1 Write the two equations.

### WRITE

a  \[
\begin{align*}
C &= 2200 + 3n \\
R &= 25n
\end{align*}
\]

2 Equate the equations \( (R = C) \).

3 Solve for the unknown.

\[
\begin{align*}
2200 + 3n &= 25n \\
2200 + 3n - 3n &= 25n - 3n \\
2200 &= 22n \\
\frac{2200}{22} &= n \\
100 &= n
\end{align*}
\]

4 Answer the question in the context of the problem.

The break-even point is \((100, 2500)\). Therefore, 100 shirts need to be sold to cover the production cost, which is $2500.

b i  1 Write the two equations.

2 Substitute the given value into both equations.

3 Determine the profit/loss.

\[
\begin{align*}
\text{Profit/loss} &= R - C \\
&= 1875 - 2425 \\
&= -550
\end{align*}
\]

Since the answer is negative, it means that Santo lost $550 (i.e. selling 75 shirts did not cover the cost to produce the shirts).

ii  1 Write the two equations.

2 Substitute the given value into both equations.
3 Determine the profit/loss. Profit/loss = \( R - C \)
\[ = 5500 - 2860 \]
\[ = 2640 \]
Since the answer is positive, it means that Santo made $2640 profit from selling 220 shirts.

### EXERCISE 1.6

**Problem solving with simultaneous equations**

**PRACTISE**

1. **WE15** Mary bought 4 donuts and 3 cupcakes for $10.55, and Sharon bought 2 donuts and 4 cupcakes for $9.90. Letting \( d \) represent the cost of a donut and \( c \) represent the cost of a cupcake, set up a pair of simultaneous equations to represent this information.

2. A pair of simultaneous equations representing the number of adults and children attending the zoo is shown below.
   - Equation 1: \( a + c = 350 \)
   - Equation 2: \( 25a + 15c = 6650 \)
   - a) By solving the pair of simultaneous equations, determine the total number of adults and children attending the zoo.
   - b) In the context of this problem, what does equation 2 represent?

3. **WE16** Yolanda sells handmade bracelets at a market for $12.50.
   - The revenue, \( R \), for selling \( n \) bracelets is represented by the equation \( R = 12.50n \).
   - The cost to make \( n \) bracelets is represented by the equation \( C = 80 + 4.50n \).
   - a) i) By solving the equations simultaneously, determine the break-even point.
   - ii) In the context of this problem, what does the break-even point mean?
   - b) Determine the profit or loss, in dollars, if Yolanda sells:
   - i) 8 bracelets
   - ii) 13 bracelets.

4. The entry fee for a charity fun run event is $18. It costs event organisers $2550 for the hire of the tent and $3 per entry for administration. Any profit will be donated to local charities.
   - An equation to represent the revenue for the entry fee is \( R = an \), where \( R \) is the total amount collected in entry fees, in dollars, and \( n \) is the number of entries.
a Write an equation for the value of $a$.
The equation that represents the cost for the event is $C = 2550 + bn$.
b Write an equation for the value of $b$.
c By solving the equations simultaneously, determine the number of entries needed to break even.
d A total of 310 entries are received for this charity event. Show that the organisers will be able to donate $2100 to local charities.
e Determine the number of entries needed to donate $5010 to local charities.

A school group travelled to the city by bus and returned by train. The two equations show the adult, $a$, and student, $s$, ticket prices to travel on the bus and train.

Bus: $3.5a + 1.5s = 42.50$
Train: $4.75a + 2.25s = 61.75$

a Write the cost of a student bus ticket, $s$, and an adult bus ticket, $a$.
b What is the most suitable method to solve these two simultaneous equations?
c Using your method in part b, solve the simultaneous equations and hence determine the number of adults and the number of students in the school group.

The following pair of simultaneous equations represents the number of adult and concession tickets sold and the respective ticket prices for the premier screening of the blockbuster *Aliens Attack*.

Equation 1: $a + c = 544$
Equation 2: $19.50a + 14.50c = 9013$

a What are the costs, in dollars, of an adult ticket, $a$, and a concession ticket, $c$?
b In the context of this problem, what does equation 1 represent?
c By solving the simultaneous equations, determine how many adult and concession tickets were sold for the premier.

Charlotte has a babysitting service and charges $12.50 per hour. After Charlotte calculated her set-up and travel costs, she constructed the cost equation $C = 45 + 2.50h$, where $C$ represents the cost in dollars and $h$ represents the hours Charlotte babysits for.

a Write an equation that represents the revenue, $R$, earned by Charlotte in terms of number of hours, $h$.
b By solving the equations simultaneously, determine the number of hours Charlotte needs to babysit to cover her costs (that is, the break-even point).
c In one week, Charlotte had four babysitting jobs as shown in the table.

<table>
<thead>
<tr>
<th>Babysitting job</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of hours, ( h )</td>
<td>5</td>
<td>3.5</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

i Determine whether Charlotte made a profit or loss for each individual babysitting job.
ii Did Charlotte make a profit this week? Justify your answer using calculations.
e One week Charlotte made a $50 profit. Determine the total number of hours she babysat for.

8 Trudi and Mia work part time at the local supermarket after school. The following table shows the number of hours worked for both Trudi and Mia and the total wages, in dollars, paid over two weeks.

<table>
<thead>
<tr>
<th>Week</th>
<th>Trudi’s hours worked</th>
<th>Mia’s hours worked</th>
<th>Total wages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>15</td>
<td>12</td>
<td>$400.50</td>
</tr>
<tr>
<td>Week 2</td>
<td>9</td>
<td>13</td>
<td>$328.75</td>
</tr>
</tbody>
</table>

a Construct two equations to represent the number of hours worked by Trudi and Mia and the total wages paid for each week. Write your equations using the pronumerals \( t \) for Trudi and \( m \) for Mia.
b In the context of this problem, what do \( t \) and \( m \) represent?
c By solving the pair of simultaneous equations, find the values of \( t \) and \( m \).

9 Brendan uses carrots and apples to make his special homemade fruit juice. One week he buys 5 kg of carrots and 4 kg of apples for $31.55. The next week he buys 4 kg of carrots and 3 kg of apples for $24.65.

a Set up two simultaneous equations to represent the cost of carrots, \( x \), in dollars per kilogram, and the cost of apples, \( y \), in dollars per kilogram.
b By solving the simultaneous equations, determine how much Brendan spends on 1 kg each of carrots and apples.
c Determine the amount Brendan spends the following week when he buys 2 kg of carrots and 1.5 kg of apples. Write your answer to the nearest 5 cents.

10 The table shows the number of 100-g serves of strawberries and grapes and the total kilojoule intake.

<table>
<thead>
<tr>
<th>Fruit</th>
<th>Serves per 100 g</th>
<th>Total kilojoules</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strawberry, ( s )</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Grapes, ( g )</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

a Construct two equations to represent the number of serves of strawberries, \( s \), and grapes, \( g \), and the total kilojoules using the pronumerals shown.
b By solving the pair of simultaneous equations constructed in part a, determine
the number of kilojoules (kJ) for a 100-g serve of strawberries.

c Freda eats only strawberries and grapes for dessert. Her total kilojoule intake
for dessert is 720 kJ. Determine the number of 100-g serves of strawberries and
grapes she had for dessert.

11 Two budget car hire companies offer the following deals for hiring a medium size
family car.

<table>
<thead>
<tr>
<th>Car company</th>
<th>Deal</th>
</tr>
</thead>
<tbody>
<tr>
<td>FreeWheels</td>
<td>$75 plus $1.10 per kilometre travelled</td>
</tr>
<tr>
<td>GetThere</td>
<td>$90 plus $0.90 per kilometre travelled</td>
</tr>
</tbody>
</table>

a Construct two equations to represent the deals for each car hire company.
Write your equations in terms of cost, $C$, and kilometres travelled, $k$.

b By solving the two equations simultaneously, determine the value of $k$ at which
the cost of hiring a car will be the same.

c Rex and Jan hire a car for the weekend. They expect to travel a distance of
250 km over the weekend. Which car hire company should they use and why?
Justify your answer using calculations.

12 The following table shows the number of boxes of three types of cereal bought
each week for a school camp, as well as the total cost for each week.

<table>
<thead>
<tr>
<th>Cereal</th>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn Pops, $c$</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Rice Crunch, $r$</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Muesli, $m$</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Total cost, $</td>
<td>$ 27.45</td>
<td>24.25</td>
<td>36.35</td>
</tr>
</tbody>
</table>

Wen is the cook at the camp. She decides to work out the cost of each box
of cereal using simultaneous equations. She incorrectly sets up the following
equations:

\[
2c + c + 3c = 27.45 \\
3r + 2r + 4r = 29.20 \\
m + 2m + m = 36.35
\]

a Explain why these simultaneous equations will not determine the cost of each
box of cereal.

b Write the correct simultaneous equations.

c Using a calculator, spreadsheet or otherwise, solve the three simultaneous
equations, and hence write the total cost for cereal for week 4’s order of
3 boxes of Corn Pops, 2 boxes of Rice Crunch and 2 boxes of muesli.

13 Sally and Nem decide to sell cups of lemonade
from their front yard to the neighbourhood
children. The cost to make the lemonade using
their own lemons can be represented using
the equation $C = 0.25n + 2$, where $C$ is the
cost in dollars and $n$ is the number of cups of
lemonade sold.
a If they sell cups of lemonade for 50 cents, write an equation to represent the selling price, \( S \), for \( n \) number of cups of lemonade.

b By solving two simultaneous equations, determine the number of cups of lemonade Sally and Nem need to sell in order to break even (i.e. cover their costs).

c Sally and Nem increase their selling price. If they make a $7 profit for selling 20 cups of lemonade, what is the new selling price?

14 The CotX T-Shirt Company produces T-shirts at a cost of $7.50 each after an initial set-up cost of $810.

a Determine the cost to produce 100 T-shirts.

b Using a calculator, spreadsheet or otherwise, complete the following table that shows the cost of producing T-shirts.

<table>
<thead>
<tr>
<th>( n )</th>
<th>0</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d Write an equation that represents the cost, \( C \), to produce \( n \) T-shirts.

e CotX sells each T-shirt for $25.50. Write an equation that represent the amount of sales, \( S \), in dollars for selling \( n \) T-shirts.

f By solving two simultaneous equations, determine the number of T-shirts that must be sold for CotX to break even.

g If CotX needs to make a profit of $5000, determine the number of T-shirts they will need to sell to achieve this outcome.

15 There are three types of fruit for sale at the market: starfruit, \( s \), mango, \( m \), and papaya, \( p \). The following table shows the amount of fruit bought and the amount paid in dollars.

<table>
<thead>
<tr>
<th>Starfruit, ( s )</th>
<th>Mango, ( m )</th>
<th>Papaya, ( p )</th>
<th>Total cost, $</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3</td>
<td>4</td>
<td>19.40</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>5</td>
<td>17.50</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>6</td>
<td>24.60</td>
</tr>
</tbody>
</table>

a Using the pronumerals \( s \), \( m \) and \( p \), represent this information with three equations.

b Using a calculator, spreadsheet or otherwise, find the cost of one starfruit, one mango and one papaya.

c Using your answer from part \( b \), determine the cost of 2 starfruit, 4 mangoes and 4 papayas.
16 The Comet Cinema offers four types of tickets to the movies: adult, concession, senior and member. The table below shows the number and types of tickets bought to see four different movies and the total amount of tickets sales in dollars.

<table>
<thead>
<tr>
<th>Movie</th>
<th>Adult, $a$</th>
<th>Concession, $c$</th>
<th>Seniors, $s$</th>
<th>Members, $m$</th>
<th>Total sales, $$</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Wizard Boy</em></td>
<td>24</td>
<td>52</td>
<td>12</td>
<td>15</td>
<td>1071.00</td>
</tr>
<tr>
<td><em>Champions</em></td>
<td>35</td>
<td>8</td>
<td>45</td>
<td>27</td>
<td>1105.50</td>
</tr>
<tr>
<td><em>Pixies on Ice</em></td>
<td>20</td>
<td>55</td>
<td>9</td>
<td>6</td>
<td>961.50</td>
</tr>
<tr>
<td><em>Horror Nite</em></td>
<td>35</td>
<td>15</td>
<td>7</td>
<td>13</td>
<td>777.00</td>
</tr>
</tbody>
</table>

- **a** Represent this information in four simultaneous equations, using the pronumerals given in the table.
- **b** Using a calculator, spreadsheet or otherwise determine the cost, in dollars, for each of the four different movie tickets.
- **c** The blockbuster movie *Love Hurts* took the following tickets sales: 77 adults, 30 concessions, 15 seniors and 45 members. Using your values from part **b**:
  - **i** write the expression that represents this information
  - **ii** determine the total ticket sales in dollars and cents.
**1.7 Review**

The Maths Quest Review is available in a customisable format for you to demonstrate your knowledge of this topic.

The Review contains:
- **Short-answer** questions — providing you with the opportunity to demonstrate the skills you have developed to efficiently answer questions using the most appropriate methods
- **Multiple-choice** questions — providing you with the opportunity to practise answering questions using CAS technology
- **Extended-response** questions — providing you with the opportunity to practise exam-style questions.

A summary of the key points covered in this topic is also available as a digital document.

**REVIEW QUESTIONS**

Download the Review questions document from the links found in the Resources section of your eBookPLUS.

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**Interactivities**

A comprehensive set of relevant interactivities to bring difficult mathematical concepts to life can be found in the Resources section of your eBookPLUS.

**studyON**

studyON is an interactive and highly visual online tool that helps you to clearly identify strengths and weaknesses prior to your exams. You can then confidently target areas of greatest need, enabling you to achieve your best results.

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**Sit Topic test**
1 Answers

EXERCISE 1.2

1 a Non-linear  
   b Non-linear  
   c Linear

d Non-linear  

2 a

<table>
<thead>
<tr>
<th>Equation</th>
<th>Bethany’s response</th>
<th>Correct response</th>
</tr>
</thead>
<tbody>
<tr>
<td>y = 4x + 1</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>y = 5x - 2</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>y = 6x = 7</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>y = x² - 5x</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>t = 6d² - 9</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>m³ = n + 8</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

b Bethany should look at both variables (numbers or letters). Both variables need to have a highest power of 1.

3 a 4n - 2  b 0.5n + 3.5

4 a -1  b -n + 11  c 55 jars

5 x = \( \frac{y + 3}{6} \)

6 x = \( \frac{2y - 1}{3} \)

7 a Linear  b Non-linear  c Linear

d Linear  e Non-linear  f Non-linear

g Non-linear

8 a Yes, as the power of \( x \) is 1.
   b The power of both variables in a linear relation must be 1.

9 a 3, 4.5, 6.75, 10.125  b No, as there is no common first difference.

d 4n - 1  e 5n + 4  c -3n + 15

d -6n + 19  e -2n - 10

11 a 0.8 
   b Yes, as it has a first common difference.

12 a \( x = \frac{y - 5}{2} \)  b \( x = \frac{y}{2} - \frac{4}{3} \)  c \( x = \frac{p + 6}{5} \)

13 a

<table>
<thead>
<tr>
<th>Day</th>
<th>Amount of water</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>950</td>
</tr>
</tbody>
</table>

b 1000 L  c \( w = -50d + 1000 \)

14 a $2250 
   b \( A = 1500 + 250m \)
   c This changes the equation to \( A = 4500 + 350m \).

15 No, because her distance each day is half the previous distance, so there is no common difference.

16 a 70 km  
   b 10w + 40, where \( w \) = number of weeks

17 2n - 6

18 a \( t = 4n - 1 \)
   b 3, 7, 11, 15, 19, 23, 27, 31, 35, 39
   c 7 - 3 = 4
   11 - 7 = 4
   15 - 11 = 4 
   d 3, 6, 9, 12, 15, and so on ...

EXERCISE 1.3

1 a \( x = 3 \)  b \( n = 10 \)  c \( d = 4.5 \)  d \( x = 17 \)

2 a i \( i \times 4, +3 \)  ii \( +2, \times 3 \)
   iii \( +1, \times 2 \)  iv \( \times 3, \times 9, \times 2 \)

b i \( a = \frac{5}{3} \)  ii \( x = 2 \)
   iii \( x = 15 \)  iv \( \frac{c}{3} = \frac{1}{3} \)

3 y = \( 2x - 1 \)

4 y = \( -6 \)

5 x = \( \frac{r + q}{p} \)

6 d = \( \frac{C}{\pi} \)

7 a \( x = -9 \)  b \( y = -0.5 \)  c \( x = -4.5 \)

8 a \( a = \frac{v - u}{t} \)  b \( x = \frac{m + k}{y} \)  c \( p(r + s) \)

9 a 33 minutes  b 108 minutes

10 a \( w = 5 \)
   b Operations need to be performed in reverse order.

11 \( K = \frac{F - 32}{1.8} + 273 \)

12 a \( x = \frac{7}{3} \)  b \( x = -\frac{13}{3} \)  c \( x = \frac{1}{3} \)  d \( x = 13 \)

13 a 2 hours  
   b 2 hours 38 minutes  
   c 41 mins  
   d 3 hours 45 minutes  
   e 32 minutes

14 a i 9 months  ii 19 months (= 18.67 months)

b 5 years, 4 months

15 a \( -32, \times 5, +9 \)
   b \( \times 9, +5, +32 \)
   c 374°F

16 a i 14.69 cm  ii 18.33 cm  iii 21.06 cm
\[ a \quad x = \frac{-14}{3} \]
\[ b \quad y = \frac{163}{162} \]
\[ c \quad x = \frac{-195}{28} \]
\[ d \quad x = \frac{12}{97} \]

\[ 18 \quad a \]
\[ i \quad 5 \text{ weeks} \]
\[ ii \quad 10 \text{ weeks} \]
\[ iii \quad 13 \text{ weeks} \]
\[ iv \quad 20 \text{ weeks} \]
\[ b \quad 10 \text{ cm} \]
\[ c \quad 63 \text{ cm}, 67 \text{ cm}, 72 \text{ cm}, 78 \text{ cm} \]

**EXERCISE 1.4**

1. $2.24
2. The red velvet cupcakes are the cheapest per cupcake.
3. 6 roses and 13 lilies
4. 16 strawberry twists and 12 chocolate ripples
5. 

<table>
<thead>
<tr>
<th>Minute</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (km)</td>
<td>0.4</td>
<td>0.8</td>
<td>1.2</td>
<td>1.6</td>
<td>2</td>
<td>2.4</td>
<td>2.8</td>
<td>3.2</td>
<td>3.6</td>
<td>4</td>
</tr>
</tbody>
</table>

6. 

<table>
<thead>
<tr>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money in piggy ban</td>
<td>23</td>
<td>26</td>
<td>29</td>
<td>32</td>
<td>35</td>
<td>38</td>
<td>41</td>
<td>44</td>
<td>47</td>
<td>50</td>
<td>53</td>
<td>56</td>
</tr>
</tbody>
</table>

7. 10 weeks
8. 12, 6, 0, −6, −12, −18
9. 

\[ t_n = t_{n-1} + 25, \quad t_1 = 450 \]

\[ b \quad $450, $475, $500, $525, $550, $575, $600, $625 \]

\[ c \quad v = 425 + 25n \]

**21a**

<table>
<thead>
<tr>
<th>Number of boards</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td>395</td>
<td>409.50</td>
<td>424</td>
<td>438.50</td>
<td>453</td>
<td>467.50</td>
<td>482</td>
<td>496.50</td>
<td>511</td>
<td>525.50</td>
<td>540</td>
</tr>
</tbody>
</table>

**21b**

<table>
<thead>
<tr>
<th>Number of boards</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue ($)</td>
<td>329.50</td>
<td>362.45</td>
<td>395.40</td>
<td>428.35</td>
<td>461.30</td>
<td>494.25</td>
<td>527.20</td>
<td>560.15</td>
<td>593.10</td>
<td>626.05</td>
<td>659</td>
</tr>
</tbody>
</table>

**21d**

<table>
<thead>
<tr>
<th>Number of boards</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit ($)</td>
<td>−65.50</td>
<td>−47.05</td>
<td>−28.60</td>
<td>−10.15</td>
<td>8.30</td>
<td>26.75</td>
<td>45.20</td>
<td>63.65</td>
<td>82.10</td>
<td>100.55</td>
<td>119</td>
</tr>
</tbody>
</table>
6. \( a = 2 \) and \( x = 2 \)

7. E

8. \( x = -2, y = 5 \)

9. \( a \ x = -2, y = -7 \)
   \( b \ x = 2, y = -3 \)
   \( c \ x = -5, y = -18 \)
   \( d \) No solution
   \( e \ x = -1, y = 1 \)

10. \( a \ x = 7, y = 19 \)
    \( b \ x = -2, y = -12 \)
    \( c \ x = 7, y = 44 \)
    \( d \ x = 2, y = 1 \)
    \( e \ x = -3, y = -4 \)
    \( f \ x = 1, y = -1 \)

11. a. Both unknowns are on the same side.
    b. Add the two equations and solve for \( x \), then substitute \( x \) into one of the equations to solve for \( y \).

12. \( a \ x = 2, y = -2 \)
    \( b \ x = 3, y = 4 \)
    \( c \ x = -1, y = 3 \)
    \( d \ x = 5, y = 3 \)
    \( e \ x = -1, y = 4 \)
    \( f \ x = 6, y = 4 \)

13. A

14. a. Marcia added the equations together instead of subtracting. The correct result for step 2 is \( 22y = 11 \).
    b. \( x = 5, y = 1 \)
    c. Check with your teacher.

15. a. Goal = 5 points, behind = 2 points
    b. Jets 40 points, Meteorites 42 points

16. a. The equation has unknowns on each side of the equals sign.
    b. Mick works 5 hours and Minnie works 10 hours.
    c. 3 hours 45 minutes (3.75 hours)

17. \( a \ x = -0.38, y = 4.08 \)
    \( b \ x = -10.71, y = -12.86 \)
    \( c \ x = 0.75, y = 0.89 \)

18. a. \( i \ (-12, -64) \)
    \( ii \ (-1, -2) \)
    \( iii \) No solution
    \( iv \ (-1, 4) \)
    b. No, the graphs in part iii are parallel (have the same gradient).

**EXERCISE 1.6**

1. \( 4d + 3c = 10.55 \) and \( 2d + 4c = 9.90 \)

2. a. 140 adults and 210 children
    b. The cost of an adult’s ticket is $25, the cost of a children’s ticket costs $15, and the total ticket sales is $6650.

3. a. \( i \) 10
    \( ii \) Yolanda needs to sell 10 bracelets to cover her costs.
    b. \( i \) $16 loss
    \( ii \) $24 profit

4. a. \( a = 18 \)
    \( b \ b = 3 \)
    \( c \) 170 entries
    d. \( R = 5580, C = 3480, P = 2100 \)
    \( e \) 504 entries

5. a. \( s = 1.50, a = 3.50 \)
    b. Elimination method
    c. 4 adults and 19 students
6 a $a = $19.50, $c = $14.50
   b The total number of tickets sold (both adult and concession)
   c 225 adult tickets and 319 concession tickets
7 a $R = 12.50h$
   b 4.5 hours
   c i Charlotte made a profit for jobs 1 and 4, and a loss for jobs 2 and 3.
      ii Yes, she made $15 profit.
         \[(25 + 5 - (10 + 5)) = 30 - 15 = $15\]
   d 9.5 hours
8 a \[15r + 12m = 400.50\] and \[9t + 13m = 328.75\]
   b \(t\) represents the hourly rate earned by Trudi and \(m\)
      represents the hourly rate earned by Mia.
   c \(t = $14.50, m = $15.25\)
9 a \(5x + 4y = 31.55\) and \(4x + 3y = 24.65\)
   b \(x = $3.95, y = $2.95\)
   c $12.35
10 a \(3s + 2g = 1000\) and \(4s + 3g = 1430\)
   b 140 kJ
   c One 100-g serve of strawberries and two 100-g serves of grapes
11 a \(C = 75 + 1.10k\) and \(C = 90 + 0.90k\)
   b \(k = 75\) km
   c \(C_{Freewheels} = $350, C_{GetThere} = $315\). They should use GetThere.
12 a The cost is for the three different types of cereal, but
   the equations only include one type of cereal.
   b \(2c + 3r + m = 27.45\)
      \(c + 2r + 2m = 24.25\)
      \(3c + 4r + m = 36.35\)
   c $34.15
13 a \(S = 0.5n\)
   b 8 cups of lemonade
   c 70 cents
14 a $1560$
   b See table at foot of the page.*
   c \(C = 810 + 7.5n\)
   d \(C ($) = 25.50n\)
15 a \(5x + 3m + 4p = 19.40\)
   b \(4s + 2m + 5p = 17.50\)
   c \(3x + 5m + 6p = 24.60\)
   d \(x = $1.25, m = $2.25, p = $1.60\)
16 a \(24a + 52c + 12s + 15m = 1071\)
   b \(35a + 8c + 45s + 27m = 1105.5\)
   c \(20a + 55c + 9s + 6m = 961.5\)
   d \(35a + 15c + 7s + 13m = 777\)
   e \(S = 25.50n\)
   f 45 T-shirts
17 a \(24a + 52c + 12s + 15m = 1071\)
   b \(35a + 8c + 45s + 27m = 1105.5\)
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