5

Trigonometric ratios and their applications

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5.1 Kick off with CAS

To come

Please refer to the Resources tab in the Prelims section of your eBookPLUS for a comprehensive step-by-step guide on how to use your CAS technology.
5.2 Trigonometry of right-angled triangles

Trigonometry, derived from the Greek words *trigon* (triangle) and *metron* (measurement), is the branch of mathematics that deals with the relationship between the sides and angles of a triangle. It involves finding unknown angles, side lengths and areas of triangles. The principles of trigonometry are used in many practical situations such as building, surveying, navigation and engineering. In previous years you will have studied the trigonometry of right-angled triangles. We will review this material before considering non–right-angled triangles.

![Trigonometric Ratios Diagram]

The symbol \( \theta \) (theta) is one of the many letters of the Greek alphabet used to represent the angle. Other symbols include \( \alpha \) (alpha), \( \beta \) (beta) and \( \gamma \) (gamma). Non-Greek letters may also be used.

Writing the mnemonic **SOH–CAH–TOA** each time we perform trigonometric calculations will help us to remember the ratios and solve the problem.

**Pythagoras’ theorem**

For specific problems it may be necessary to determine the side lengths of a right-angled triangle before calculating the trigonometric ratios. In this situation, Pythagoras’ theorem is used. Pythagoras’ theorem states:

\[
\text{In any right-angled triangle, } c^2 = a^2 + b^2. 
\]

**Proof of Pythagoras’ theorem**

Start with a square with side length \( a + b \).

\[
\text{Area 1} = (a + b)^2
\]

Divide this square into four congruent right-angled triangles, \( \Delta abc \), and a square of length, \( c \).

\[
\text{Area 2} = \text{Area of four triangles} + \text{one square} \\
= 4 \left( \frac{1}{2} ab \right) + c^2 \\
= 2ab + c^2 
\]
∴ Area 1 = Area 2

\[(a + b)^2 = 2ab + c^2\]
\[a^2 + 2ab + b^2 = 2ab + c^2\]
\[a^2 + b^2 = c^2\]

Pythagoras’ theorem states that the square of the hypotenuse is equal to the sum of the squares of the other two sides.

WORKED EXAMPLE 1

Determine the value of the pronumerals, correct to 2 decimal places.

THINK

a 1 Label the sides, relative to the marked angles.

2 Write what is given.

3 Write what is needed.

4 Determine which of the trigonometric ratios is required, using SOH–CAH–TOA.

5 Substitute the given values into the appropriate ratio.

6 Transpose the equation and solve for \(x\).

7 Round the answer to 2 decimal places.

b 1 Label the sides, relative to the marked angle.

2 Write what is given.

3 Write what is needed.

4 Determine which of the trigonometric ratios is required, using SOH–CAH–TOA.

5 Substitute the given values into the appropriate ratio.

6 Round the answer to 2 decimal places.
Find the angle $\theta$, giving the answer in degrees and minutes.

\[ \tan(\theta) = \frac{18}{12} \]

$\theta = \tan^{-1}\left(\frac{18}{12}\right) = 56^\circ 19'$

**Exact values**

Most of the trigonometric values that we will deal with in this chapter are only approximations. However, angles of 30°, 45° and 60° have exact values of sine, cosine and tangent.

Consider an equilateral triangle, ABC, of side length 2 units. If the triangle is perpendicularly bisected, then two congruent triangles, ABD and CBD, are obtained. From triangle ABD it can be seen that BD creates a right-angled triangle with angles of 60° and 30° and base length (AD) of 1 unit. The length of BD is obtained using Pythagoras’ theorem.

Using triangle ABD and the three trigonometric ratios, the following exact values are obtained:

\[
\begin{align*}
\sin(30^\circ) &= \frac{1}{2} & \sin(60^\circ) &= \frac{\sqrt{3}}{2} \\
\cos(30^\circ) &= \frac{\sqrt{3}}{2} & \cos(60^\circ) &= \frac{1}{2} \\
\tan(30^\circ) &= \frac{1}{\sqrt{3}} \text{ or } \frac{\sqrt{3}}{3} & \tan(60^\circ) &= \frac{\sqrt{3}}{1} \text{ or } \sqrt{3}
\end{align*}
\]
Consider a right-angled isosceles triangle EFG whose equal sides are of 1 unit. The hypotenuse EG is obtained by using Pythagoras’ theorem.

\[(EG)^2 = (EF)^2 + (FG)^2\]
\[= 1^2 + 1^2\]
\[= 2\]
\[EG = \sqrt{2}\]

Using triangle EFG and the three trigonometric ratios, the following exact values are obtained:

\[
\sin(45^\circ) = \frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2}
\]
\[
\cos(45^\circ) = \frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2}
\]
\[
\tan(45^\circ) = 1 \text{ or } 1
\]

WORKED EXAMPLE 3

Determine the height of the triangle shown in surd form.

THINK

1. Label the sides relative to the marked angle.

WRITE/DRAW

2. Write what is given.
3. Write what is needed.
4. Determine which of the trigonometric ratios is required, using SOH–CAH–TOA.
5. Substitute the given values into the appropriate ratio.
6. Substitute exact values where appropriate.
7. Transpose the equation to find the required value.
8. State the answer.

Have: angle and adjacent \((A)\) side

Need: opposite \((O)\) side

\[
\tan(\theta) = \frac{O}{A}
\]

\[
\tan(60^\circ) = \frac{h}{8}
\]

\[\sqrt{3} = \frac{h}{8}\]

\[h = 8\sqrt{3}\]

The triangle’s height is \(8\sqrt{3}\) cm.
**Trigonometry of right-angled triangles**

1. **WE1** Find the value of the pronumerals, correct to 2 decimal places.
   a) \[ x \] 
   b) \[ x = 7.5 \]
   c) \[ x = 17 \]
   d) \[ x = 684 \]

2. **WE1** Find the value of \( x \) and \( y \), correct to 2 decimal places.
   a) \[ x = 1.03 \]
   b) \[ x = 3.85 \]
   c) \[ x = 504 \]
   d) \[ x = 17 \]

3. **WE2** Find the angle \( \theta \), giving the answer in degrees and minutes.
   a) \[ \theta = 10^\circ 7 \]
   b) \[ \theta = 5^\circ 3 \]
   c) \[ \theta = 28^\circ 0 \]
   d) \[ \theta = 2.1^\circ 6 \]

4. **WE2** Find the angle \( \theta \), giving the answer in degrees and minutes.
   a) \[ \theta = 11.7^\circ 4.2 \]
   b) \[ \theta = 48^\circ 0 \]
   c) \[ \theta = 53.2^\circ 78.1 \]
   d) \[ \theta = 3.26^\circ 1.74 \]

5. **WE3** An isosceles triangle has a base of 12 cm and equal angles of 30°. Find, in the simplest surd form:
   a) the height of the triangle
   b) the area of the triangle
   c) the perimeter of the triangle.

6. **WE3** Find the perimeter of the composite shape below, in surd form. The length measurements are in metres.

7. **CONSOLIDATE** A ladder 6.5 m long rests against a vertical wall and makes an angle of 50° to the horizontal ground.
   a) How high up the wall does the ladder reach?
   b) If the ladder needs to reach 1 m higher what angle should it make to the ground, to the nearest minute?
8 A road 400 m long goes straight up a slope. If the road rises 50 m vertically, what is the angle that the road makes with the horizontal?

9 An ice-cream cone has a diameter of 6 cm and a sloping edge of 15 cm. Find the angle at the bottom of the cone.

10 A vertical flagpole is supported by a wire attached from the top of the pole to the horizontal ground, 4 m from the base of the pole. Joanne measures the angle the wire makes with the ground as 65°. How tall is the flagpole?

11 A stepladder stands on a floor, with its feet 1.5 m apart. If the angle formed by the legs is 55°, how high above the floor is the top of the ladder?

12 The angle formed by the diagonal of a rectangle and one of its shorter sides is 60°. If the diagonal is 8 cm long, find the dimensions of the rectangle, in surd form.

13 In the figure below, find the value of the pronumerals, correct to 2 decimal places.

14 In the figure at right, find the value of the pronumeral $x$, correct to 2 decimal places.

15 An advertising balloon is attached to a rope 120 m long. The rope makes an angle of 75° to level ground. How high above the ground is the balloon?

16 An isosceles triangle has sides of 17 cm, 20 cm and 20 cm. Find the magnitude of the angles.

17 A garden bed in the shape of a trapezium is shown below. What volume of garden mulch is needed to cover it to a depth of 15 cm?

18 A ladder 10 m long rests against a vertical wall at an angle of 55° to the horizontal. It slides down the wall, so that it now makes an angle of 48° with the horizontal.
   a Through what vertical distance did the top of the ladder slide?
   b Does the foot of the ladder move through the same distance? Justify your answer.

### 5.3 Elevation, depression and bearings

Trigonometry is especially useful for measuring distances and heights that are difficult or impractical to access. For example, two important applications of right-angled triangles are:

1. angles of elevation and depression
2. bearings.

#### Angles of elevation and depression

Angles of elevation and depression are employed when dealing with directions that require us to look up and down respectively. An *angle of elevation* is the angle between the horizontal and an object that is *higher* than the observer (for example, the top of a mountain or flagpole).
An *angle of depression* is the angle between the horizontal and an object that is *lower* than the observer (for example, a boat at sea when the observer is on a cliff). Unless otherwise stated, the angle of elevation or depression is measured and drawn from the horizontal.

**Angles of elevation and depression are each measured from the horizontal.**

When solving problems involving angles of elevation and depression, it is always best to draw a diagram. The angle of elevation is equal to the angle of depression because they are *alternate ‘Z’ angles*. 

**Bearings**

Bearings measure the direction of one object from another. There are two systems used for describing bearings.

*True bearings* are measured in a *clockwise* direction, starting from north (0° T).

*Conventional* or *compass bearings* are measured: first, relative to north or south, and second, relative to east or west.
The two systems are interchangeable. For example, a bearing of 240° T is the same as S60° W.

When solving questions involving direction, always start with a diagram showing the basic compass points: north, south, east and west.

**WORKED EXAMPLE 5** A ship sails 40 km in a direction of N52° W. How far west of the starting point is it?

**THINK**
1. Draw a diagram of the situation, labelling each of the compass points and the given information.

**WRITE/DRAW**

2. Write what is given for the triangle.
   - Have: angle and hypotenuse
3. Write what is needed for the triangle.
   - Need: opposite side
4. Determine which of the trigonometric ratios is required (SOH–CAH–TOA).
   - \( \sin(\theta) = \frac{O}{H} \)
5. Substitute the given values into the appropriate ratio.
   - \( \sin(52°) = \frac{x}{40} \)
6. Transpose the equation and solve for \( x \).
   - \( 40 \times \sin(52°) = x \)
    - \( x = 40 \times \sin(52°) \)
    - \( = 31.52 \)
7. Round the answer to 2 decimal places.
8. Answer the question.
   - The ship is 31.52 km west of the starting point.

**WORKED EXAMPLE 6** A ship sails 10 km east, then 4 km south. What is its bearing from its starting point?

**THINK**
1. Draw a diagram of the situation, labelling each of the compass points and the given information.

**WRITE/DRAW**

2. Write what is given for the triangle.
   - Have: angle and hypotenuse
3. Write what is needed for the triangle.
   - Need: opposite side
4. Determine which of the trigonometric ratios is required (SOH–CAH–TOA).
   - \( \sin(\theta) = \frac{O}{H} \)
5. Substitute the given values into the appropriate ratio.
   - \( \sin(\theta) = \frac{4}{10} \)
6. Transpose the equation and solve for \( \theta \).
   - \( \theta = \sin^{-1}\left(\frac{4}{10}\right) \)
   - \( \theta = 23.58° \)
2 Write what is given for the triangle. Have: adjacent and opposite sides
3 Write what is needed for the triangle. Need: angle
4 Determine which of the trigonometric ratios is required (SOH–CAH–TOA).
5 Substitute the given values into the appropriate ratio.
6 Transpose the equation and solve for \(\theta\), using the inverse tan function.
\[ \theta = \tan^{-1}\left(\frac{4}{10}\right) \]
7 Convert the angle to degrees and minutes.
\[ = 21.80140949^\circ = 21^\circ48' \]
8 Express the angle in bearings form. The bearing of the ship was initially 0° T; it has since rotated through an angle of 90° and an additional angle of 21°48’. To obtain the final bearing these values are added.
\[ \text{Bearing} = 90° + 21°48' \]
\[ = 111°48' T \]
9 Answer the question. The bearing of the ship from its starting point is 111°48’ T.

### Elevation, depression and bearings

1. From a vertical fire tower 60 m high, the angle of depression to a fire is 6°. How far away, to the nearest metre, is the fire?
2. A person stands 20 m from the base of a building and measures the angle of elevation to the top of the building as 55°. If the person is 1.7 m tall, how high, to the nearest metre, is the building?
3. A pair of kayakers paddle 1800 m on a bearing of N20°E. How far north of their starting point are they, to the nearest metre?
4. A ship sails 230 km on a bearing of S20°W. How far west of its starting point has it travelled, correct to the nearest kilometre?
5. A ship sails 20 km south, then 8 km west. What is its bearing from the starting point?
6. A cross-country competitor runs 2 km west, then due north for 3 km. What is the true bearing of the runner from the starting point?
7. Express the following conventional bearings as true bearings, and the true bearings in conventional form.
   - a) N35°W
   - b) S47°W
   - c) N58°E
   - d) S17°E
   - e) 246° T
   - f) 107° T
   - g) 321° T
   - h) 074° T
8 a A bearing of S30°E is the same as:
   A 030° T   B 120° T   C 150° T   D 210° T   E 240° T
b A bearing of 280° T is the same as:
   A N10°W   B S10°W   C S80°W   D N80°W   E N10°E

9 An observer on a cliff top 57 m high observes a ship at sea. The angle of
depression to the ship is 15°. The ship sails towards the cliff, and the angle of
depression is then 25°. How far, to the nearest metre, did the ship sail between
sightings?

10 A new skyscraper is proposed for the Melbourne Docklands region. It is to be
500 m tall. What would be the angle of depression, in degrees and minutes,
from the top of the building to the island on Albert Park Lake, which is
4.2 km away?

11 From a rescue helicopter 2500 m above the ocean, the angles of depression to two
shipwreck survivors are 48° (survivor 1) and 35° (survivor 2).
a Draw a labelled diagram that represents the situation.
b Calculate how far apart the two survivors are.

12 A lookout tower has been erected on top of a mountain. At a distance of
5.8 km, the angle of elevation from the ground to the base of the tower is
15.7°, and the angle of elevation to the observation deck (on the top of the
tower) is 15.9°. How high, to the nearest metre, is the observation deck above
the top of the mountain?

13 From point A on level ground, the angle of elevation to the top of a building
50 m high is 45°. From point B on the ground and in line with A and the foot
of the building, the angle of elevation to the top of the building is 60°. Find, in
simplest surd form, the distance from A to B.

14 A yacht race consists of four legs. The first three legs are 4 km due east, then
5 km south, followed by 2 km due west.
a How long is the final leg, if the race finishes at the starting point?
b On what bearing must the final leg be sailed?

15 Two hikers set out from the same campsite. One walks 7 km in the direction
043° T and the other walks 10 km in the direction 133° T.
a What is the distance between the two hikers?
b What is the bearing of the first hiker from the second?

16 A ship sails 30 km on a bearing of 220° T, then 20 km on a bearing of
250° T. Find:
a how far south of the original position it is
b how far west of the original position it is
c the true bearing of the ship from its original position, to the nearest degree.

17 The town of Bracknaw is due west of Arley. Chris, in an ultralight plane, starts
at a third town, Champaon, which is due north of Bracknaw, and flies directly
towards Arley at a speed of 40 km/h in a direction of 110° T. She reaches Arley in
3 hours. Find:
a the distance between Arley and Bracknaw
b the time to complete the journey from Champaon to Bracknaw, via Arley, if she
increases her speed to 45 km/h between Arley and Bracknaw.
18 A bird flying at 50 m above the ground was observed at noon from my front door at an angle of elevation of 5°. Two minutes later its angle of elevation was 4°.

a If the bird was flying straight and level, find the horizontal distance of the bird:
   i from my doorway at noon
   ii from my doorway at 12.02 pm.

b Hence, find:
   i the distance travelled by the bird in the two minutes
   ii its speed of flight in km/h.

5.4 The sine rule

When working with non–right-angled triangles, it is usual to label the angles A, B and C, and the sides a, b and c, so that side a is the side opposite angle A, side b is the side opposite angle B and side c is the side opposite angle C.

In a non–right-angled triangle, a perpendicular line, h, can be drawn from the angle B to side b.

Using triangle ABD, we obtain \( \sin(A) = \frac{h}{c} \). Using triangle CBD, we obtain \( \sin(C) = \frac{h}{a} \).

Transposing each equation to make h the subject, we obtain 
\( h = c \times \sin(A) \) and \( h = a \times \sin(C) \). Equate to get 
\( c \times \sin(A) = a \times \sin(C) \).

Transpose to get
\[
\frac{c}{\sin(C)} = \frac{a}{\sin(A)}
\]

In a similar way, if a perpendicular line is drawn from angle A to side a, we get
\[
\frac{b}{\sin(B)} = \frac{c}{\sin(C)}
\]

From this, the sine rule can be stated.

- In any triangle ABC: \( \frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \)

Notes
1. When using this rule, depending on the values given, any combination of the two equalities may be used to solve a particular triangle.
2. To solve a triangle means to find all unknown side lengths and angles.

The sine rule can be used to solve non–right-angled triangles if we are given:
1. two angles and one side length
2. two side lengths and an angle opposite one of these side lengths.
Worked Example 7

In the triangle ABC, \( a = 4 \text{ m}, b = 7 \text{ m} \) and \( B = 80^\circ \). Find \( A \), \( C \) and \( c \).

**THINK**

1. Draw a labelled diagram of the triangle ABC and fill in the given information.

2. Check that one of the criteria for the sine rule has been satisfied.

3. Write the sine rule to find \( A \).

4. Substitute the known values into the rule.

5. Transpose the equation to make \( \sin (A) \) the subject.

6. Evaluate.

7. Round the answer to degrees and minutes.

8. Determine the value of angle \( C \) using the fact that the angle sum of any triangle is \( 180^\circ \).

9. Write the sine rule to find \( c \).

10. Substitute the known values into the rule.

11. Transpose the equation to make \( c \) the subject.

12. Evaluate. Round the answer to 2 decimal places and include the appropriate unit.

**WRITE/DRAW**

The sine rule can be used since two side lengths and an angle opposite one of these side lengths have been given.

To find angle \( A \):

\[
\frac{a}{\sin (A)} = \frac{b}{\sin (B)}
\]

\[
\frac{4}{\sin (A)} = \frac{7}{\sin (80^\circ)}
\]

\[
\sin (A) = \frac{4 \times \sin (80^\circ)}{7}
\]

\[
A = \sin^{-1} \left( \frac{4 \times \sin (80^\circ)}{7} \right)
\]

\[
= \sin^{-1} (0.562747287)
\]

\[
= 34.24600471^\circ
\]

\[
= 34^\circ 15'
\]

To find side length \( c \):

\[
\frac{c}{\sin (C)} = \frac{b}{\sin (B)}
\]

\[
\frac{c}{\sin (65^\circ 45')} = \frac{7}{\sin (80^\circ)}
\]

\[
c = \frac{7 \times \sin (65^\circ 45')}{\sin (80^\circ)}
\]

\[
= \frac{7 \times 0.911762043}{0.984807753}
\]

\[
= 6.382334305
\]

\[
= 6.480792099
\]

\[
= 6.48 \text{ m}
\]
The ambiguous case

When using the sine rule there is one important issue to consider. If we are given two side lengths and an angle opposite one of these side lengths, then two different triangles may be drawn. For example, if \( a = 10 \), \( c = 6 \) and \( C = 30^\circ \), two possible triangles could be created.

In the first case, angle \( A \) is an acute angle, while in the second case, angle \( A \) is an obtuse angle. The two values for \( A \) will add to \( 180^\circ \).

The ambiguous case does not work for every example. It would be useful to know, before commencing a question, whether or not the ambiguous case exists and, if so, to then find both sets of solutions.

The ambiguous case exists if \( C \) is an acute angle and \( a > c > a \times \sin (C) \), or any equivalent statement; for example, if \( B \) is an acute angle and \( a > b > a \times \sin (B) \), and so on.

In the triangle \( ABC \), \( a = 10 \) m, \( c = 6 \) m and \( C = 30^\circ \).

a Show that the ambiguous case exists.

b Find two possible values of \( A \), and hence two possible values of \( B \) and \( b \).

**THINK**

a 1 Check that the conditions for an ambiguous case exist, i.e. that \( C \) is an acute angle and that \( a > c > a \times \sin (C) \).

2 State the answer.

Case 1

b 1 Draw a labelled diagram of the triangle \( ABC \) and fill in the given information.

2 Write the sine rule to find \( A \).

3 Substitute the known values into the rule.
4 Transpose the equation to make \( \sin (A) \) the subject.

\[
10 \times \sin (30°) = 6 \times \sin (A)
\]

\[
\sin (A) = \frac{10 \times \sin (30°)}{6}
\]

\[
A = \sin^{-1} \left( \frac{10 \times \sin (30°)}{6} \right)
\]

\[
A = 56°27'
\]

5 Evaluate angle \( A \), in degrees and minutes.

\[
B = 180° - (30° + 56°27')
\]

\[
B = 93°33'
\]

6 Determine the value of angle \( B \), using the fact that the angle sum of any triangle is 180°.

To find side length \( b \):

\[
\frac{b}{\sin (B)} = \frac{c}{\sin (C)}
\]

\[
\frac{b}{\sin (93°33')} = \frac{6}{\sin (30°)}
\]

\[
b = \frac{6 \times \sin (93°33')}{\sin (30°)}
\]

\[
b = 11.98 \text{ m}
\]

7 Write the sine rule to find \( b \).

8 Substitute the known values into the rule.

Case 2

c 1 Draw a labelled diagram of the triangle ABC and fill in the given information.

2 Write the alternative value for angle \( A \). Subtract the value obtained for \( A \) in Case 1 from 180°.

3 Determine the alternative value of angle \( B \), using the fact that the angle sum of any triangle is 180°.

4 Write the sine rule to find the alternative \( b \).

5 Substitute the known values into the rule.

6 Transpose the equation to make \( b \) the subject and evaluate.

To find the alternative angle \( A \):

If \( \sin (A) = 0.8333 \), then \( A \) could also be:

\[
A = 180° - 56°27'
\]

\[
A = 123°33'
\]

\[
B = 180° - (30° + 123°33')
\]

\[
B = 26°27'
\]

To find side length \( b \):

\[
\frac{b}{\sin (B)} = \frac{c}{\sin (C)}
\]

\[
\frac{b}{\sin (26°27')} = \frac{6}{\sin (30°)}
\]

\[
b = \frac{6 \times \sin (26°27')}{\sin (30°)}
\]

\[
b = 5.35 \text{ m}
\]
Hence, for Worked example 8 there were two possible solutions as shown by the diagram below.

**The sine rule**

1. **WE7** In the triangle ABC, \( a = 10 \), \( b = 12 \) and \( B = 58^\circ \). Find \( A \), \( C \) and \( c \).
2. **WE7** In the triangle ABC, \( c = 17.35 \), \( a = 26.82 \) and \( A = 101^\circ 47' \). Find \( B \) and \( b \).
3. **WE8** In the triangle ABC, \( a = 10 \), \( c = 8 \) and \( C = 50^\circ \). Find two possible values of \( A \) and hence two possible values of \( b \).
4. **WE8** In the triangle ABC, \( a = 20 \), \( b = 12 \) and \( B = 35^\circ \). Find two possible values for the perimeter of the triangle.
5. In the triangle ABC, \( c = 27 \), \( C = 42^\circ \) and \( A = 105^\circ \). Find \( B \), \( a \) and \( b \).
6. In the triangle ABC, \( a = 7 \), \( c = 5 \) and \( A = 68^\circ \). Find the perimeter of the triangle.
7. Find all unknown sides and angles for the triangle ABC, given \( a = 32 \), \( b = 51 \) and \( A = 28^\circ \).
8. Find the perimeter of the triangle ABC if \( a = 7.8 \), \( b = 6.2 \) and \( A = 50^\circ \).
9. In a triangle ABC, \( A = 40^\circ \), \( C = 80^\circ \) and \( c = 3 \). The value of \( b \) is:
   - A 2.64
   - B 2.86
   - C 14
   - D 4.38
   - E 4.60
10. Find all unknown sides and angles for the triangle ABC, given \( A = 27^\circ \), \( B = 43^\circ \) and \( c = 6.4 \).
11. Find all unknown sides and angles for the triangle ABC, given \( A = 25^\circ \), \( b = 17 \) and \( a = 13 \).
12. To calculate the height of a building, Kevin measures the angle of elevation to the top as \( 48^\circ \). He then walks 18 m closer to the building and measures the angle of elevation as \( 64^\circ \). How high is the building?
13. A river has parallel banks that run directly east–west. From the river bank Kylie takes a bearing to a tree on the river bank on the opposite side. The bearing is \( 047^\circ \) T. She then walks 10 m due east and takes a second bearing to the tree. This is \( 305^\circ \) T. Find:
   - a her distance from the second measuring point to the tree
   - b the width of the river, to the nearest metre.
14. A ship sails on a bearing of S20°W for 14 km, then changes direction and sails for 20 km and drops anchor. Its bearing from the starting point is now N65°W.
   - a How far is it from the starting point?
   - b On what bearing did it sail the 20 km leg?
15. **a** A cross-country runner runs at 8 km/h on a bearing of 150° T for 45 minutes, then changes direction to a bearing of 053° T and runs for 80 minutes until he is due east of the starting point.
   - i How far was the second part of the run?
   - ii What was his speed for this section?
   - iii How far does he need to run to get back to the starting point?
b From a fire tower, A, a fire is spotted on a bearing of N42°E. From a second tower, B, the fire is on a bearing of N12°W. The two fire towers are 23 km apart, and A is N63°W of B. How far is the fire from each tower?

16 A cliff is 37 m high. The rock slopes outward at an angle of 50° to the horizontal, then cuts back at an angle of 25° to the vertical, meeting the ground directly below the top of the cliff.

Carol wishes to abseil from the top of the cliff to the ground as shown in the diagram below right. Her climbing rope is 45 m long, and she needs 2 m to secure it to a tree at the top of the cliff. Will the rope be long enough to allow her to reach the ground?

5.5 The cosine rule

In any non–right-angled triangle ABC, a perpendicular line can be drawn from angle B to side b. Let D be the point where the perpendicular line meets side b, and the length of the perpendicular line be h. Let the length AD = x units. The perpendicular line creates two right-angled triangles, ADB and CDB.

Using triangle ADB and Pythagoras’ theorem, we obtain:
\[ c^2 = h^2 + x^2 \]  \[ \text{[1]} \]

Using triangle CDB and Pythagoras’ theorem, we obtain:
\[ a^2 = h^2 + (b - x)^2 \]  \[ \text{[2]} \]

Expanding the brackets in equation [2]:
\[ a^2 = h^2 + b^2 - 2bx + x^2 \]

Rearranging equation [2] and using \( c^2 = h^2 + x^2 \) from equation [1]:
\[ a^2 = h^2 + x^2 + b^2 - 2bx \]
\[ = c^2 + b^2 - 2bx \]
\[ = b^2 + c^2 - 2bc \times \cos (A) \]

From triangle ABD, \( x = c \times \cos (A) \), therefore \( a^2 = b^2 + c^2 - 2bc \times \cos (A) \) becomes
\[ a^2 = b^2 + c^2 - 2bc \times \cos (A) \]

This is called the cosine rule and is a generalisation of Pythagoras’ theorem.
In a similar way, if the perpendicular line was drawn from angle \( A \) to side \( a \) or from angle \( C \) to side \( c \), the two right-angled triangles would give \( c^2 = a^2 + b^2 - 2ab \cos (C) \) and \( b^2 = a^2 + c^2 - 2ac \cos (B) \) respectively. From this, the cosine rule can be stated:

\[
\begin{align*}
\text{In any triangle } ABC, \\
a^2 &= b^2 + c^2 - 2bc \cos (A) \\
b^2 &= a^2 + c^2 - 2ac \cos (B) \\
c^2 &= a^2 + b^2 - 2ab \cos (C)
\end{align*}
\]

The cosine rule can be used to solve non-right-angled triangles if we are given:
1. three sides of the triangle
2. two sides of the triangle and the included angle (the angle between the given sides).

**WORKED EXAMPLE 9** Find the third side of triangle ABC given \( a = 6 \), \( c = 10 \) and \( B = 76^\circ \), correct to 2 decimal places.

**THINK**
1. Draw a labelled diagram of the triangle ABC and fill in the given information.

2. Check that one of the criteria for the cosine rule has been satisfied.

3. Write the appropriate cosine rule to find side \( b \).

4. Substitute the given values into the rule.

5. Evaluate.

6. Round the answer to 2 decimal places.

**WRITE/DRAW**

\[
\begin{align*}
\text{Yes, the cosine rule can be used since two side lengths and the included angle have been given.} \\
\text{To find side } b: \\
b^2 &= a^2 + c^2 - 2ac \cos (B) \\
&= 6^2 + 10^2 - 2 \times 6 \times 10 \times \cos (76^\circ) \\
&= 36 + 100 - 120 \times 0.241921895 \\
&= 106.9693725 \\
b &= \sqrt{106.9693725} \\
&\approx 10.34 \text{ correct to 2 decimal places}
\end{align*}
\]

**Note:** Once the third side has been found, the sine rule could be used to find other angles if necessary.

If three sides of a triangle are known, an angle could be found by transposing the cosine rule to make \( \cos A \), \( \cos B \) or \( \cos C \) the subject.

\[
\begin{align*}
\cos (A) &= \frac{b^2 + c^2 - a^2}{2bc} \\
\cos (B) &= \frac{a^2 + c^2 - b^2}{2ac} \\
\cos (C) &= \frac{a^2 + b^2 - c^2}{2ab}
\end{align*}
\]
Find the smallest angle in the triangle with sides 4 cm, 7 cm and 9 cm.

THINK
1 Draw a labelled diagram of the triangle, call it ABC and fill in the given information.

Note: The smallest angle will be opposite the smallest side.

2 Check that one of the criteria for the cosine rule has been satisfied.

3 Write the appropriate cosine rule to find angle A.

4 Substitute the given values into the rearranged rule.

5 Evaluate.

6 Transpose the equation to make A the subject by taking the inverse cos of both sides.

7 Round the answer to degrees and minutes.

WRITE/DRAW

Let $a = 4$

$b = 7$

$c = 9$

The cosine rule can be used since three side lengths have been given.

$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$

$= \frac{7^2 + 9^2 - 4^2}{2 \times 7 \times 9}$

$= \frac{49 + 81 - 16}{126}$

$= \frac{114}{126}$

$A = \cos^{-1}\left(\frac{114}{126}\right)$

$= 25.2087653^\circ$

$= 25^\circ13'$

Two rowers set out from the same point. One rows N70°E for 2000 m and the other rows S15°W for 1800 m. How far apart are the two rowers?

THINK
1 Draw a labelled diagram of the triangle, call it ABC and fill in the given information.
2 Check that one of the criteria for the cosine rule has been satisfied.

The cosine rule can be used since two side lengths and the included angle have been given.

3 Write the appropriate cosine rule to find side $c$.

To find side $c$:

$$c^2 = a^2 + b^2 - 2ab \cos (C)$$

4 Substitute the given values into the rule.

$$= 2000^2 + 1800^2 - 2 \times 2000 \times 1800 \times \cos (125^\circ)$$

5 Evaluate.

$$= 4000000 + 3240000 - 7200000$$

$$\times -0.573 \ 576 \ 436$$

$$= 11369750.342$$

$$c = \sqrt{11369750.342}$$

$$= 3371.906 \ 04$$

6 Round the answer to 2 decimal places.

$$= 3371.91$$

7 Answer the question.

The rowers are 3371.91 m apart.

---

**EXERCISE 5.5**

**The cosine rule**

1. Find the third side of triangle ABC given $a = 3.4$, $b = 7.8$ and $C = 80^\circ$.
2. In triangle ABC, $b = 64.5 \text{ cm}$, $c = 38.1 \text{ cm}$ and $A = 58^\circ34'$. Find the third side, $a$.
3. Find the smallest angle in the triangle with sides 6 cm, 4 cm and 8 cm.
4. In triangle ABC, $a = 356$, $b = 207$ and $c = 296$. Find the smallest angle.
5. Two rowers set out from the same point. One rows N30$^\circ$E for 1500 m and the other rows S40$^\circ$E for 1200 m. How far apart are the two rowers?
6. Two rowers set out from the same point. One rows 16.2 km on a bearing of 053$^\circ$T and the other rows 31.6 km on a bearing of 117$^\circ$T. How far apart are the two rowers?
7. In triangle ABC, $a = 17$, $c = 10$ and $B = 115^\circ$. Find $b$, and hence find $A$ and $C$.
8. In triangle ABC, $a = 23.6$, $b = 17.3$ and $c = 26.4$. Find the size of all the angles.
9. In triangle DEF, $d = 3 \text{ cm}$, $e = 7 \text{ cm}$ and $F = 60^\circ$. Find $f$ in exact form.
10. Maria cycles 12 km in a direction of N68$^\circ$W, then 7 km in a direction of N34$^\circ$E.
    a. How far is she from her starting point?
    b. What is the bearing of the starting point from her finishing point?
11. A garden bed is in the shape of a triangle with sides of length 3 m, 4.5 m and 5.2 m.
    a. Calculate the smallest angle.
    b. Hence, find the area of the garden. (*Hint: Draw a diagram with the longest length as the base of the triangle.*)
12. A hockey goal is 3 m wide. When Sophie is 7 m from one post and 5.2 m from the other, she shoots for goal. Within what angle, to the nearest degree, must the shot be made if it is to score a goal?
13 An advertising balloon is attached to two ropes 120 m and 100 m long. The ropes are anchored to level ground 35 m apart. How high can the balloon fly?

14 A plane flies N70°E for 80 km, then on a bearing of S10°W for 150 km.
   a How far is the plane from its starting point?
   b What direction is the plane from its starting point?

15 A plane takes off at 10:00 am from an airfield and flies at 120 km/h on a bearing of N35°W. A second plane takes off at 10:05 am from the same airfield and flies on a bearing of S80°E at a speed of 90 km/h. How far apart are the planes at 10:25 am?

16 Three circles of radii 5 cm, 6 cm and 8 cm are positioned so that they just touch one another. Their centres form the vertices of a triangle. Find the largest angle in the triangle.

17 For the given shape at near right, determine:
   a the length of the diagonal
   b the magnitude (size) of angle \(\theta\)
   c the length of \(x\).

18 From the top of a vertical cliff 68 m high, an observer notices a yacht at sea. The angle of depression to the yacht is 47°. The yacht sails directly away from the cliff, and after 10 minutes the angle of depression is 15°. How fast does the yacht sail?

5.6 Arcs, sectors and segments

Radian measurement

In all of the trigonometry tasks covered so far, the unit for measuring angles has been the degree. There is another commonly used measurement for angles: the radian. This is used in situations involving length and areas associated with circles.

Consider the unit circle, a circle with a radius of 1 unit. OP is the radius.

If OP is rotated \(\theta^\circ\) anticlockwise, the point P traces a path along the circumference of the circle to a new point, \(P_1\).

The arc length \(PP_1\) is a radian measurement, symbolised by \(\theta\).

Note: 1° is equivalent to the angle in degrees formed when the length of \(PP_1\) is 1 unit; in other words, when the arc is the same length as the radius.

If the length OP is rotated 180°, the point P traces out half the circumference. Since the circle has a radius of 1 unit, and \(C = 2\pi r\), the arc \(PP_1\) has a length of \(\pi\).
The relationship between degrees and radians is thus established.

\[ 180^\circ = \pi \text{rad} \]

This relationship will be used to convert from one system to another. Rearranging the basic conversion factor gives:

\[ 180^\circ = \pi \text{rad} \]
\[ 1^\circ = \frac{\pi \text{rad}}{180} \]

To convert an angle in degrees to radian measure, multiply by \( \frac{\pi}{180} \).

Also, since \( \pi \text{rad} = 180^\circ \), it follows that \( 1^\circ = \frac{180^\circ}{\pi} \).

To convert an angle in radian measure to degrees, multiply by \( \frac{180^\circ}{\pi} \).

Where possible, it is common to have radian values with \( \pi \) in them. It is usual to write radians without any symbol, but degrees must always have a symbol. For example, an angle of \( 25^\circ \) must have the degree symbol written, but an angle of \( 1.5 \) is understood to be \( 1.5 \) radians.

**WORKED EXAMPLE 12**

**a** Convert \( 135^\circ \) to radian measure, expressing the answer in terms of \( \pi \).

**b** Convert the radian measurement \( \frac{4\pi}{5} \) to degrees.

**THINK**

**a** 1 To convert an angle in degrees to radian measure, multiply the angle by \( \frac{\pi}{180} \).

2 Simplify, leaving the answer in terms of \( \pi \).

**b** 1 To convert radian measure to an angle in degrees, multiply the angle by \( \frac{180}{\pi} \).

2 Simplify.

**WRITE**

**a** \( 135^\circ = 135^\circ \times \frac{\pi}{180^\circ} \)

\[ = \frac{135\pi}{180} \]

\[ = \frac{3\pi}{4} \]

**b** \( \frac{4\pi}{5} = \frac{4\pi}{5} \times \frac{180^\circ}{\pi} \)

\[ = \frac{720^\circ}{5} \]

\[ = 144^\circ \]

*Note: \( \pi \) cancels out.*
If the calculation does not simplify easily, write the answers in degrees and minutes, or radians to 4 decimal places. If angles are given in degrees and minutes, convert to degrees only before converting to radians.

**Arc length**

An arc is a section of the circumference of a circle. The length of an arc is proportional to the angle subtended at the centre. For example, an angle of 90° will create an arc that is \( \frac{1}{4} \) the circumference.

We have already defined an arc length as equivalent to \( \theta \) radians if the circle has a radius of 1 unit.

Therefore, a simple dilation of the unit circle will enable us to calculate the arc length for any sized circle, as long as the angle is expressed in radians.

If the radius is dilated by a factor of \( r \), the arc length is also dilated by a factor of \( r \).

Therefore, \( l = r \theta \), where \( l \) represents the arc length, \( r \) represents the radius and \( \theta \) represents an angle measured in radians.

---

**WORKED EXAMPLE 13**

Find the length of the arc that subtends an angle of 75° at the centre of a circle with radius 8 cm.

**THINK**

1. Draw a diagram representing the situation and label it with the given values.

2. Convert the angle from 75° to radian measure by multiplying the angle by \( \frac{\pi}{180°} \).

3. Evaluate to 4 decimal places.

4. Write the rule for the length of the arc.

5. Substitute the values into the formula.

6. Evaluate to 2 decimal places and include the appropriate unit.

**WRITE/DRAW**

\[
\begin{align*}
75° &= 75° \times \frac{\pi}{180°} \\
&= \frac{75\pi}{180} \\
&= 1.3090 \\
l &= r \theta \\
&= 8 \times 1.3090 \\
&= 10.4720 \\
&= 10.47 \text{ cm}
\end{align*}
\]

*Note: In order to use the formula for the length of the arc, the angle must be in radian measure.*
Area of a sector

In the diagram at right, the shaded area is the \textit{minor} sector AOB, and the unshaded area is the \textit{major} sector AOB. The area of the sector is proportional to the arc length. For example, an area of \(\frac{1}{4}\) of the circle contains an arc that is \(\frac{1}{4}\) of the circumference.

Thus, in any circle: \[ \frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{arc length}}{\text{circumference of circle}} \]

\[ \frac{A}{\pi r^2} = \frac{r \theta}{2 \pi r} \] where \(\theta\) is measured in radians.

\[ A = \frac{r \theta \times \pi r^2}{2 \pi r} = \frac{1}{2} r^2 \theta \]

The area of a sector is: \[ A = \frac{1}{2} r^2 \theta \]

\[ \text{Think} \]

1. Convert the angle from \(107^\circ\) to radian measure by multiplying the angle by \(\frac{\pi}{180^\circ}\).

\[ 107^\circ = 107^\circ \times \frac{\pi}{180^\circ} = \frac{107\pi}{180} \]

\[ \text{Write} \]

\[ 107^\circ = \frac{107\pi}{180} \]
2. Evaluate to 4 decimal places. 

3. Write the rule for the area of a sector. 

4. Substitute the values into the formula. 

5. Transpose the equation to make \( r^2 \) the subject. 

6. Take the square root of both sides of the equation. 

7. Evaluate to 2 decimal places and include the appropriate unit.

---

**Area of a segment**

A segment is that part of a sector bounded by the arc and the chord.

As can be seen from the diagram at right:

Area of segment = area of sector – area of triangle

\[
A = \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin (\theta^\circ)
\]

\[
= \frac{1}{2} r^2 (\theta - \sin (\theta^\circ))
\]

*Note:* \( \theta \) is in radians and \( \theta^\circ \) is in degrees.

---

**The area of a segment:** 

\[
A = \frac{1}{2} r^2 (\theta - \sin (\theta^\circ))
\]

---

**WORKED EXAMPLE 16**

Find the area of the segment in a circle of radius 5 cm, subtended by an angle of 40°.

**THINK**

1. Convert the angle from 40° to radian measure by multiplying the angle by \( \frac{\pi}{180^\circ} \).

2. Evaluate to 4 decimal places.

3. Write the rule for the area of a segment.

4. Identify each of the variables.

5. Substitute the values into the formula.

6. Evaluate.

7. Round to 2 decimal places and include the appropriate unit.

**WRITE**

\[
40^\circ = 40^\circ \times \frac{\pi}{180^\circ}
\]

\[
= \frac{40\pi}{180}
\]

\[
= 0.6981
\]

\[
A = \frac{1}{2} r^2 (\theta - \sin (\theta^\circ))
\]

\[
r = 5, \ \theta = 0.6981, \ \theta^\circ = 40^\circ
\]

\[
A = \frac{1}{2} \times 5^2 (0.6981 - \sin (40^\circ))
\]

\[
= \frac{1}{2} \times 25 \times 0.0553
\]

\[
= 0.69125
\]

\[
= 0.69 \text{ cm}^2
\]
EXERCISE 5.6  **Arcs, sectors and segments**

1. **Convert the following angles to radian measure, expressing answers in terms of \( \pi \).**
   - a. 30°
   - b. 60°
   - c. 120°
   - d. 150°
   - e. 225°
   - f. 270°
   - g. 315°
   - h. 480°
   - i. 72°
   - j. 200°

2. **Convert the following radian measurements into degrees.**
   - a. \( \frac{\pi}{4} \)
   - b. \( \frac{3\pi}{2} \)
   - c. \( \frac{7\pi}{6} \)
   - d. \( \frac{5\pi}{3} \)
   - e. \( \frac{7\pi}{12} \)
   - f. \( \frac{17\pi}{6} \)
   - g. \( \frac{\pi}{12} \)
   - h. \( \frac{13\pi}{10} \)
   - i. \( \frac{11\pi}{8} \)
   - j. \( \frac{8\pi}{10} \)

3. **Find the length of the arc that subtends an angle of 65° at the centre of a circle of radius 14 cm.**

4. **Find the length of the arc that subtends an angle of 153° at the centre of a circle of radius 75 mm.**

5. **Find the angle subtended by a 20 cm arc in a circle of radius 75 cm:**
   - a. in radians
   - b. in degrees.

6. **Find the angle subtended by an 8 cm arc in a circle of radius 5 cm:**
   - a. in radians
   - b. in degrees.

7. **A sector has an area of 825 cm\(^2\) and subtends an angle of 70°. What is the radius of the circle?**

8. **A sector has an area of 309 cm\(^2\) and subtends an angle of 106°. What is the radius of the circle?**

9. **Find the area of the segment in a circle of radius 25 cm subtended by an angle of 100°.**

10. **Find the area of the segment of a circle of radius 4.7 m that subtends an angle of 85°20′ at the centre.**

11. **Convert the following angles in degrees to radians, giving answers to 4 decimal places.**
    - a. 27°
    - b. 109°
    - c. 243°
    - d. 351°
    - e. 7°
    - f. 63°42′
    - g. 138°21′
    - h. 274°8′
    - i. 326°53′
    - j. 47°2′

12. **Convert the following radian measurements into degrees and minutes.**
    - a. 2.345
    - b. 0.6103
    - c. 1
    - d. 1.61
    - e. 3.592
    - f. 7.25
    - g. 0.182
    - h. 5.8402
    - i. 4.073
    - j. 6.167

13. **Find the length of the arc that subtends an angle of 135° at the centre of a circle of radius 10 cm. Leave the answer in terms of \( \pi \).**

14. **An arc of a circle is 27.8 cm long and subtends an angle of 205° at the centre of the circle. What is the radius of the circle?**

15. **An arc of length 8 cm is marked out on the circumference of a circle of radius 13 cm. What angle does the arc subtend at the centre of the circle?**

16. **The minute hand of a clock is 35 cm long. How far does the tip of the hand travel in 20 minutes?**
17 A child’s swing is suspended by a rope 3 m long. What is the length of the arc it travels if it swings through an angle of 42°?

18 Find the area of the sector of a circle of radius 6 cm with an angle of 100°. Write your answer in terms of π.

19 A garden bed is in the form of a sector of a circle of radius 4 m. The arc of the sector is 5 m long. Find:
   a the area of the garden bed
   b the volume of mulch needed to cover the bed to a depth of 10 cm.

20 A sector whose angle is 150° is cut from a circular piece of cardboard whose radius is 12 cm. The two straight edges of the sector are joined so as to form a cone.
   a What is the surface area of the cone?
   b What is the radius of the cone?

21 Two irrigation sprinklers spread water in circular paths with radii of 7 m and 4 m. If the sprinklers are 10 m apart, find the area of crop that receives water from both sprinklers.

22 Two circles of radii 3 cm and 4 cm have their centres 5 cm apart. Find the area of the intersection of the two circles.
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A summary of the key points covered in this topic is also available as a digital document.

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**Units 1 & 2**

**Trigonometric ratios and their applications**

**AOS #**

**Topic 5**

**Concept #**

**Sit Topic test**
5 Answers

EXERCISE 5.2
1 a 6.43
   b 11.89
   c 24.99
   d 354.05
2 a 4.14
   b 18.11
   c 445.90
   d $x = 21.14, y = 27.13$
3 a $44^\circ 26'$
   b $67^\circ 23'$
   c $44^\circ 25'$
   d $17^\circ 10'$
4 a $68^\circ 58'$
   b $38^\circ 41'$
   c $47^\circ 4'$
   d $61^\circ 55'$
5 a $2\sqrt{3}$ cm
   b $12\sqrt{3}$ cm²
   c $12 + 8\sqrt{3}$ cm
6 $26\sqrt{3} + 54$ m
7 a 4.98 m
   b $66^\circ 56'$
8 $7^\circ 11'$
9 $23^\circ 4'$
10 8.58 m
11 1.44 m
12 4 and $4\sqrt{3}$
13 $a = 14.90, b = 20.05$
14 $x = 13.39$
15 115.91 m
16 $54^\circ 51', 64^\circ 51', 50^\circ 18'$
17 10.91 m
18 a 0.76 m
   b No, the foot of the ladder moves through a distance of 0.95 m.

EXERCISE 5.3
1 571 m
2 30 m
3 1691 m
4 79 km
5 $201^\circ 48'$ T
6 326.3° T
7 a $325^\circ$ T
   b $227^\circ$ T
   c $058^\circ$ T
   d $163^\circ$ T
   e $S66^\circ W$
   f $S73^\circ E$
   g $N39^\circ W$
   h $N74^\circ E$
8 a C
   b D
9 91 m
10 $6^\circ 47'$
11 a
   b 1319.36 m
12 $22$ m
13 $50 - \frac{50\sqrt{3}}{3}$ m
14 a $5.39$ km
   b $N21^\circ 48'$ W
15 a $12.2$ km
   b $348^\circ$ T or $N12^\circ$ W
16 a $29.82$ km
   b $38.08$ km
   c $232^\circ$ T
17 a $112.76$ km
   b 5 hours 30 minutes
18 a i $571.5$ m
    ii $715$ m
   b i $143.5$ m
    ii $4.31$ km/h

EXERCISE 5.4
1 $44^\circ 58', 77^\circ 2', 13.79$
2 $39^\circ 18', 38^\circ 55', 17.21$
3 $A = 73^\circ 15', b = 8.73; or A = 106^\circ 45', b = 4.12$
4 51.9 or 44.86
5 $33^\circ, 38.98, 21.98$
6 19.12
7 $B = 48^\circ 26', C = 103^\circ 34', c = 66.26; \text{ or } B = 131^\circ 34', C = 20^\circ 26', c = 23.8$
8 24.17
9 A
10 $C = 110^\circ, a = 3.09, b = 4.64$
11 $B = 33^\circ 33', C = 121^\circ 27', c = 26.24; \text{ or } B = 146^\circ 27', C = 8^\circ 33', c = 4.57$
12 43.62 m
13 a 6.97 m
b 4 m
14 a 13.11 km
b N20$^\circ$47$'$W
15 a i 8.63 km
ii 6.48 km/h
iii 9.90 km
b 22.09 km from A and 27.46 km from B
16 Yes, she needs 43 m altogether.

EXERCISE 5.5
1 7.95
2 55.22 cm
3 28$^\circ$57$'$
4 35$^\circ$32$'$
5 2218 m
6 28.5 km
7 23.08, 41$^\circ$53$'$, 23$^\circ$7$'$
8 $A = 61^\circ 15', B = 40^\circ, C = 78^\circ 45'$
9 $\sqrt{37}$ cm
10 a 12.57 km
b S35$^\circ$1'E
11 a 35$^\circ$6$'$
b 6.73 m$^2$
12 23$^\circ$
13 89.12 m
14 a 130 km
b S22$^\circ$12'E
15 74.3 km
16 70$^\circ$49$'$
17 a 8.89 m
b 76$^\circ$59$'$
c $x = 10.07$ m
18 1.14 km/h

EXERCISE 5.6
1 a $\frac{\pi}{6}$
b $\frac{\pi}{3}$
c $\frac{2\pi}{3}$
2 $\frac{5\pi}{6}$
e $\frac{5\pi}{4}$
3 $\frac{3\pi}{2}$
g $\frac{7\pi}{4}$
h $\frac{8\pi}{3}$
i $\frac{2\pi}{5}$
j $\frac{10\pi}{9}$
2 a 45$^\circ$
b 270$^\circ$
c 210$^\circ$
d 300$^\circ$
e 105$^\circ$
f 510$^\circ$
g 15$^\circ$
h 234$^\circ$
i 247.5$^\circ$
j 1440$^\circ$
3 15.88 cm
4 200.28 mm
5 a 0.2667c
b 15$^\circ$17$'$
6 a 1.6$^c$
b 91$^\circ$40$'$
7 36.75 cm
8 18.28 cm
9 237.66 cm$^2$
10 5.44 m$^2$
11 a 0.4712
b 1.9024
c 4.2412
d 6.1261
e 0.1222
f 1.1118
g 2.4147
h 4.7845
i 5.7052
j 0.8209
12 a 134$^\circ$22$'$
b 34$^\circ$58$'$
c 57$^\circ$18$'$
d 92$^\circ$15$'$
e 205$^\circ$48$'$
f  415°24′
g  10°26′
h  334°37′
i  233°22′
j  353°21′

\[
\frac{15\pi}{2}
\]

14 7.77 cm
15 35°16′
16 73.3 cm
17 2.20 m
18 \( A = 10\pi \text{ cm}^2 \)
19 \( a \quad 10 \text{ m}^2 \)
   \( b \quad 1 \text{ m}^3 \)
20 \( a \quad 188.5 \text{ cm}^2 \)
   \( b \quad 5 \text{ cm} \)
21 2.95 m²
22 6.64 cm²