Interest and depreciation

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eBook plus
6.1 Kick off with CAS

Calculating interest with CAS

A recursion relation links one term in a series to the previous or next term in the same series.

We can use recursion equations to model financial situations, such as the amount of interest accrued in a bank account for a number of consecutive years.

1 $3000 was placed into a bank account earning simple interest at a rate of 5% per annum.
   a Using CAS, define the recurrence relation $V_{n+1} = V_n + 150$, $V_0 = 3000$ that represents this investment.
   b Use your recurrence relation defined in part a to complete the following table which represents the amount of money in the bank account after $n$ years.
   c Use CAS to plot a graph of the amount of money in the bank after the first 5 years (using the figures in the table).

2 $3000 was placed into a bank account earning compound interest at a rate of 5% per annum.
   a Using CAS, define the recurrence relation $V_{n+1} = 1.05V_n$, $V_0 = 3000$ that represents this investment.
   b Use your recurrence relation defined in part a to complete the following table which represents the amount of money in the bank account after $n$ years.
   c Use CAS to plot a graph of the amount of money in the bank after the first 5 years (using the figures in the table).

3 Comment on the shape of the two graphs drawn in questions 1 and 2.
6.2 Simple interest

People often wish to buy goods and services that they cannot afford to pay for at the time, but which they are confident they can pay for over a period of time. The options open to these people include paying by credit card (usually at a very high interest rate), lay-by (where the goods are paid off over a period of time with no interest charged but no access to or use of the goods until the last payment is made), hire-purchase, or a loan from the bank.

The last two options usually attract what is called simple interest. This is the amount of money charged by the financial institution for the use of its money. It is calculated as a percentage of the money borrowed multiplied by the number of periods (usually years) over which the money is borrowed.

As an example, Monica wishes to purchase a television for $550, but does not currently have the cash to pay for it. She makes an agreement to borrow the money from a bank at 12% p.a. (per year) simple interest and pay it back over a period of 5 years. The amount of interest Monica will be charged on top of the $550 is:

\[ \text{Interest} = 550 \times 12\% \times 5 \text{ years} \]

\[ = 330 \]

Therefore, Monica is really paying $550 + $330 = $880 for the television.

Besides taking out a loan, you can also make an investment. One type of investment is depositing money into an account with a bank or financial institution for a period of time. The financial institution uses your money and in return adds interest to the account at the end of the period.

Simple interest can be represented by a first-order linear recurrence relation, where \( V_n \) represents the value of the investment after \( n \) time periods, \( V_0 \) is the initial (or starting) amount and \( d \) is the amount of interest earned per period:

\[
V_{n+1} = V_n + d,
\]

\[
d = \frac{V_0 \times r}{100},
\]

where \( V_n \) represents the value of the investment after \( n \) time periods, \( d \) is the amount of interest earned per period, \( V_0 \) is the initial (or starting) amount and \( r \) is the interest rate.

You can also calculate the total amount of a simple interest loan or investment by using:

\[
\text{Total amount of loan or investment} = \text{initial amount or principal} + \text{interest}
\]

\[
V_n = V_0 + I
\]

Simple interest is the percentage of the amount borrowed or invested multiplied by the number of time periods (usually years). The amount is added to the principal either as payment for the use of the money borrowed or as return on money invested.
\[
I = \frac{V_0rn}{100}
\]

\(I\) = simple interest charged or earned ($)
\(V_0\) = principal (money invested or loaned) ($)
\(r\) = rate of interest per period (% per period)
\(n\) = the number of periods (years, months, days) over which the agreement operates

In the case of simple interest, the total value of investment increases by the same amount per period. Therefore, if the values of the investment at the end of each time period are plotted, a straight line graph is formed.

*Hint:* The interest rate, \(r\), and time period, \(n\), must be stated and calculated in the same time terms; for example:

1. 4% per annum for 18 months must be calculated over \(1\frac{1}{2}\) years, as the interest rate period is stated in years (per annum);
2. 1% per month for 2 years must be calculated over 24 months, as the interest rate period is stated in months.

**WORKED EXAMPLE 1**

$325 is invested in a simple interest account for 5 years at 3% p.a. (per year).

\(\text{a}\) Set up a recurrence relation to find the value of the investment after \(n\) years.

\(\text{b}\) Use the recurrence relation from part \(\text{a}\) to find the value of the investment at the end of each of the first 5 years.

**THINK**

\(\text{a}\) 1. Write the formula to calculate the amount of interest earned per period \((d)\).
2. List the values of \(V_0\) and \(r\).
3. Substitute into the formula and evaluate.
4. Use the values of \(d\) and \(V_0\) to set up your recurrence relation.

\(\text{b}\) 1. Set up a table to find the value of the investment for up to \(n = 5\).
2. Use the recurrence relation from part \(\text{a}\) to complete the table.

**WRITE**

\(\text{a}\)

\[d = \frac{V_0 \times r}{100}\]

\(V_0 = 325\), \(r = 3\)

\[d = \frac{325 \times 3}{100} = 9.75\]

\(V_{n+1} = V_n + 9.75, V_0 = 325\)

\(\text{b}\)

<table>
<thead>
<tr>
<th>(n + 1)</th>
<th>(V_n) ($)</th>
<th>(V_{n+1}) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>325</td>
<td>325 + 9.75 = 334.75</td>
</tr>
<tr>
<td>2</td>
<td>334.75</td>
<td>334.75 + 9.75 = 344.50</td>
</tr>
<tr>
<td>3</td>
<td>344.50</td>
<td>344.50 + 9.75 = 354.25</td>
</tr>
<tr>
<td>4</td>
<td>354.25</td>
<td>354.25 + 9.75 = 364</td>
</tr>
<tr>
<td>5</td>
<td>364</td>
<td>364 + 9.75 = 373.75</td>
</tr>
</tbody>
</table>
3 Write your answer. The value of the investment at the end of each of the first 5 years is: $334.75, $344.50, $354.25, $364 and $373.75.

**WORKED EXAMPLE 2**

Jan invested $210 with a building society in a fixed deposit account that paid 8% p.a. simple interest for 18 months.

a How much did she receive after the 18 months?

b Represent the account balance for each of the 18 months graphically.

**THINK**

a 1 Write the simple interest formula.

2 List the values of $V_0$, $r$ and $n$.
   Check that $r$ and $n$ are in the same time terms. Need to convert 18 months into years.

3 Substitute the values of the pronumerals into the formula and evaluate.

4 Write the answer.

5 Add the interest to the principal (total amount received).

6 Write your answer.

**WRITE/DRAW**

a $I = \frac{V_0 r n}{100}$

$V_0 = 210$

$r = 8\% \text{ per year}$

$n = 18 \text{ months} = 1\frac{1}{2} \text{ years}$

$I = \frac{210 \times 8 \times 1.5}{100}$

$= 25.2$

The interest charged is $25.20.

$V_n = V_0 + I$

$= 210 + 25.20$

$= 235.20$

Total amount received at the end of 18 months is $235.20.

b Increase per month $= \frac{25.20}{18} = $1.40

**Finding $V_0$, $r$ and $n$**

In many cases we may wish to find the principal, interest rate or period of a loan. In these situations it is necessary to rearrange or transpose the simple interest formula after (or before) substitution, as the following example illustrates.
WORKED EXAMPLE 3  A bank offers 9% p.a. simple interest on an investment. At the end of 4 years the total interest earned was $215. How much was invested?

THINK

1 Write the simple interest formula. 

\[ I = \frac{V_0 \times r \times n}{100} \]

2 List the values of \( I \), \( r \) and \( n \). Check that \( r \) and \( n \) are in the same time terms.

\( I = \$215 \)

\( r = 9\% \text{ per year} \)

\( n = 4 \text{ years} \)

3 Substitute into the formula.

\( 215 = \frac{V_0 \times 9 \times 4}{100} \)

4 Make \( V_0 \) the subject by multiplying both sides by 100 and dividing both sides by \((9 \times 4)\).

\[ V_0 = \frac{215 \times 100}{9 \times 4} = 597.22 \]

5 Write your answer in the correct units. 

The amount invested was $597.22.

Transposed simple interest formula

It may be easier to use the transposed formula when finding \( V_0 \), \( r \) or \( n \).

To find the principal:

\[ V_0 = \frac{100 \times I}{r \times n} \]

To find the interest rate:

\[ r = \frac{100 \times I}{V_0 \times n} \]

To find the period of the loan or investment:

\[ n = \frac{100 \times I}{V_0 \times r} \]

WORKED EXAMPLE 4  When $720 is invested for 36 months it earns $205.20 simple interest. Find the yearly interest rate.

THINK

1 Write the simple interest formula, where rate is the subject.

\[ r = \frac{100 \times I}{V_0 \times n} \]

2 List the values of \( V_0 \), \( I \) and \( n \). \( n \) must be expressed in years so that \( r \) can be evaluated in \% per year.

\( V_0 = \$720 \)

\( I = \$205.20 \)

\( n = 36 \text{ months} = 3 \text{ years} \)

3 Substitute into the formula and evaluate.

\[ r = \frac{100 \times 205.20}{720 \times 3} = 9.5 \]

4 Write your answer.

The interest rate offered is 9.5\% per annum.
WORKED EXAMPLE 5

An amount of $255 was invested at 8.5% p.a. How long will it take, to the nearest year, to earn $86.70 in interest?

THINK

1. Write the simple interest formula, where time is the subject.

2. Substitute the values of $V_0$, $I$, and $r$. The rate, $r$, is expressed per annum so time, $n$, will be evaluated in the same time terms, namely years.

3. Substitute into the formula and evaluate.

4. Write your answer.

WRITE

\[ n = \frac{100 \times I}{V_0 \times r} \]

\[ V_0 = $255 \]

\[ I = $86.70 \]

\[ r = 8.5\% \text{ p.a.} \]

\[ n = \frac{100 \times 86.70}{255 \times 8.5} \]

\[ = 4 \]

It will take 4 years.

EXERCISE 6.2 Simple Interest

PRACTISE

1. \( \text{WE1} \) $1020 is invested in a simple interest account for 5 years at 8.5% p.a.
   a. Set up a recurrence relation to find the value of the investment after $n$ years.
   b. Use the recurrence relation from part a to find the value of the investment at the end of the first 5 years.

2. $713 is invested in a simple interest account for 5 years at 6.75% p.a.
   a. Set up a recurrence relation to find the value of the investment after $n$ years.
   b. Use the recurrence relation from part a to find the value of the investment at the end of the first 5 years.

3. a. \( \text{WE2} \) Find the amount to which the investment has grown if $1020 is invested at $12\frac{1}{2}$% p.a. for 2 years.
   b. Represent the account balance for each of the first 12 months graphically.

4. a. Find the amount to which the investment has grown if $713 is invested at $6\frac{3}{4}$% p.a. for 7 years.
   b. Represent the account balance for each of the first 12 months graphically.

5. \( \text{WE3} \) Find the principal invested if simple interest is 7% p.a., earning a total of $1232 interest over 4 years.

6. Find the principal invested if simple interest is 8% p.a., earning a total of $651 interest over 18 months.

7. \( \text{WE4} \) Find the interest rate offered, expressed in % p.a., for an investment of $5000 earning a total of $1250 interest for 4 years.

8. Find the interest rate offered, expressed in % p.a., for a loan of $150 with a $20 interest charge for 2 months.

9. \( \text{WE5} \) Find the period of time (to the nearest month) for which the principal was invested or borrowed.
   Loan of $6000 at simple interest of 7% p.a. with an interest charge of $630.
10 Find the period of time (to the nearest month) for which the principal was invested or borrowed.
   Loan of $100 at simple interest of 24% p.a. with an interest charge of $6.

11 Find the value of the following investments at the end of each year of the investment by using a recurrence relation.
   a $680 for 4 years at 5% p.a.
   b $210 for 3 years at 9% p.a.

12 Find the interest charged or earned on the following loans and investments:
   a $690 loaned at 12% p.a. simple interest for 15 months
   b $7500 invested for 3 years at 1% per month simple interest
   c $25000 borrowed for 13 weeks at 0.1% per week simple interest
   d $250 invested at $\frac{3}{4}$% per month for 2$\frac{1}{2}$ years.

13 Find the amount to which each investment has grown after the investment periods shown in the following situations:
   a $300 invested at 10% p.a. simple interest for 24 months
   b $750 invested for 3 years at 1% per month simple interest
   c $20000 invested for 3 years and 6 months at 11% p.a. simple interest.

14 Silvio invested the $1500 he won in Lotto with an insurance company bond that pays 12$\frac{1}{2}$% p.a. simple interest provided he keeps the bond for 5 years.
   a What is Silvio’s total return from the bond at the end of the 5 years?
   b Represent the balance at the end of each year graphically.

15 Jill and John decide to borrow money to improve their boat, but cannot agree which loan is the better value. They would like to borrow $2550. Jill goes to the Big-4 Bank and finds that they will lend her the money at 11$\frac{1}{2}$% p.a. simple interest for 3 years. John finds that the Friendly Building Society will lend the $2550 to them at 1% per month simple interest for the 3 years.
   a Which institution offers the better rate over the 3 years?
   b Explain why.

16 The value of a simple interest investment at the end of year 2 is $3377. At the end of year 3 the investment is worth $3530.50.
   Use a recurrence relation to work out how much was invested.

17 Find the interest rate offered. Express rates in % per annum.
   Loan of $10000, with a $2000 total interest charge, for 2 years.

18 Find the period of time (to the nearest month) for which the principal was invested or borrowed.
   Investment of $1000 at simple interest of 5% p.a. earning $50.
19 a Lennie earned $576 in interest when she invested in a fund paying 9.5% simple interest for 4 years. How much did Lennie invest originally?

b Lennie’s sister Lisa also earned $576 interest at 9% simple interest, but she had to invest it for only 3 years. What was Lisa’s initial investment?

20 James needed to earn $225 in one year. He invested $2500 in an account earning simple interest at a rate of 4.5% p.a. paid monthly. How many months will it take James to achieve his aim?

21 Carol has $3000 to invest. Her aim is to earn $450 in interest at a rate of 5% p.a. Over what term would she invest?

22 A loan of $1000 is taken over 5 years. The simple interest is calculated monthly. The total amount repaid for this loan is $1800. The simple interest rate per year on this loan is closest to:

A 8.9%  
D 5%  
B 16%  
E 11.1%  
C 36%

### Compound interest tables

As you have seen in simple interest calculations, the amount present at the start does not change throughout the life of the investment. Interest is added at the end.

For investments, if interest is added to the initial amount (principal) at the end of an interest-bearing period, then both the interest and the principal earn further interest during the next period, which in turn is added to the balance. This process continues for the life of the investment. The interest is said to be compounded.

The result is that the balance of the account increases at regular intervals and so too does the interest earned.

Let the starting amount be \( V_n \). Then the amount at the start of the next compounding period is \( V_{n+1} \).

Consider $1000 invested for 4 years at an interest rate of 12% p.a. with interest compounded annually (added on each year). What will be the final balance of this account?

<table>
<thead>
<tr>
<th>Time period ((n + 1))</th>
<th>( V_n ) ($)</th>
<th>Interest ($)</th>
<th>( V_{n+1} ) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>12% of 1000 = 120</td>
<td>1000 + 120 = 1120</td>
</tr>
<tr>
<td>2</td>
<td>1120</td>
<td>12% of 1120 = 134.40</td>
<td>1120 + 134.40 = 1254.40</td>
</tr>
<tr>
<td>3</td>
<td>1254.40</td>
<td>12% of 1254.40 = 150.53</td>
<td>1254.40 + 150.53 = 1404.93</td>
</tr>
<tr>
<td>4</td>
<td>1404.93</td>
<td>12% of 1404.93 = 168.59</td>
<td>1404.93 + 168.59 = 1573.52</td>
</tr>
<tr>
<td>5</td>
<td>1573.52</td>
<td>12% of 1573.52 = 188.82</td>
<td>1573.52 + 188.82 = 1762.34</td>
</tr>
</tbody>
</table>

So the balance after 5 years is $1762.34.

During the total period of an investment, interest may be compounded many times, so a formula has been derived to make calculations easier.
In the above example the principal is increased by 12% each year. That is, the end of year balance $= 1.12$ of the start of the year balance.

Now let us look at how this growth or compounding factor of $1.12$ is applied in the example.

<table>
<thead>
<tr>
<th>Time period</th>
<th>Balance ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1120 = 1000 \times 1.12 = 1000(1.12)^1$</td>
</tr>
<tr>
<td>2</td>
<td>$1254.40 = 1120 \times 1.12 = 1000 \times 1.12 \times 1.12 = 1000(1.12)^2$</td>
</tr>
<tr>
<td>3</td>
<td>$1404.93 = 1254.40 \times 1.12 = 1000 \times 1.12 \times 1.12 \times 1.12 = 1000(1.12)^3$</td>
</tr>
<tr>
<td>4</td>
<td>$1573.52 = 1404.93 \times 1.12 = 1000 \times 1.12 \times 1.12 \times 1.12 \times 1.12 = 1000(1.12)^4$</td>
</tr>
<tr>
<td>5</td>
<td>$1762.34 = 1573.52 \times 1.12 = 1000 \times 1.12 \times 1.12 \times 1.12 \times 1.12 \times 1.12 = 1000(1.12)^5$</td>
</tr>
</tbody>
</table>

If this investment continued for $n$ years the final balance would be:

$$1000(1.12)^n = 1000 \left(1 + \frac{12}{100}\right)^n.$$

**WORKED EXAMPLE 6**  
Laura invested $2500 for 5 years at an interest rate of 8% p.a. with interest compounded annually. Complete the table by calculating the values A, B, C, D, E and F.

<table>
<thead>
<tr>
<th>Time period $(n + 1)$</th>
<th>$V_n$ ($)</th>
<th>Interest ($)</th>
<th>$V_{n+1}$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2500</td>
<td>A% of 2500 = 200</td>
<td>2700</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>8% of C = 216</td>
<td>D</td>
</tr>
<tr>
<td>3</td>
<td>2916</td>
<td>8% of 2916 = 233.28</td>
<td>3149.28</td>
</tr>
<tr>
<td>4</td>
<td>3149.28</td>
<td>8% of 3149.28 = 251.94</td>
<td>E</td>
</tr>
<tr>
<td>5</td>
<td>F</td>
<td>8% of 3401.22 = 272.10</td>
<td>3673.32</td>
</tr>
</tbody>
</table>

**THINK**  
A: The percentage interest per annum earned on the investment.  
B: The principal at the start of the second year is the balance at the end of the first.  
C: 8% interest is earned on the principal at the start of the second year.  
D: The balance is the sum of the principal at the start of the year and the interest earned.  
E: The final balance is the sum of the principal at the start of the 4th year and the interest earned.  
F: The principle at the start of the fifth year is the balance at the end of the fourth year.

**WRITE**  
A = 8  
B = 2700  
C = B = 2700  
D = 2700 + 216 = 2916  
E = 3149.28 + 251.94 = 3401.22  
F = 3401.22
**EXERCISE 6.3**  

**PRACTISE**  

**Compound interest tables**

1. Fraser invested $7500 for 4 years at an interest rate of 6% p.a. with interest compounded annually. Complete the table by calculating the values A, B, C, D, E and F.

<table>
<thead>
<tr>
<th>Time period $(n + 1)$</th>
<th>$V_n$ ($)</th>
<th>Interest ($)</th>
<th>$V_{n+1}$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7500</td>
<td>A% of 7500 = 450</td>
<td>7950</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>6% of C = 477</td>
<td>D</td>
</tr>
<tr>
<td>3</td>
<td>8427</td>
<td>6% of 8427 = 505.62</td>
<td>8932.62</td>
</tr>
<tr>
<td>4</td>
<td>8932.62</td>
<td>6% of 8932.62 = 535.96</td>
<td>E</td>
</tr>
<tr>
<td>5</td>
<td>F</td>
<td>6% of 9468.58 = 568.11</td>
<td>10</td>
</tr>
</tbody>
</table>

2. Bob invested $3250 for 4 years at an interest rate of 7.5% p.a. with interest compounded annually. Complete the table by calculating the values A, B, C, D, E and F.

<table>
<thead>
<tr>
<th>Time period $(n + 1)$</th>
<th>$V_n$ ($)</th>
<th>Interest ($)</th>
<th>$V_{n+1}$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3250</td>
<td>A% of 3250 = 243.75</td>
<td>3493.75</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>7.5% of C = 262.03</td>
<td>D</td>
</tr>
<tr>
<td>3</td>
<td>3755.78</td>
<td>7.5% of 3755.78 = 281.68</td>
<td>4037.46</td>
</tr>
<tr>
<td>4</td>
<td>4037.46</td>
<td>7.5% of 4037.46 = 302.81</td>
<td>E</td>
</tr>
<tr>
<td>5</td>
<td>F</td>
<td>7.5% of 4340.27 = 325.52</td>
<td>4665.79</td>
</tr>
</tbody>
</table>

The following information relates to questions 3 to 6. An investment of $3000 was made over 3 years at an interest rate of 5% p.a. with interest compounding annually.

3. The interest earned in the first year is:
   - A $50  
   - B $100  
   - C $150  
   - D $200  
   - E $300

4. The balance at the end of the first year is:
   - A $3000  
   - B $3050  
   - C $3100  
   - D $3150  
   - E $3200

5. The principal at the start of the second year is:
   - A $3300  
   - B $3200  
   - C $3150  
   - D $3250  
   - E $3100

6. The interest earned during the second year is:
   - A $100  
   - B $125  
   - C $150  
   - D $157.50  
   - E $172.50

7. If $4500 is invested for 10 years at 12% p.a. with interest compounding annually and the interest earned in the third year was $677.38, then the interest earned in the fourth year is closest to:
   - A $600  
   - B $625  
   - C $650  
   - D $677  
   - E $759
8 Declan invested $5750 for 4 years at an interest rate of 8% p.a. with interest compounded annually. Complete the table by calculating the values A, B, C, D, E and F.

<table>
<thead>
<tr>
<th>Time period ((n + 1))</th>
<th>(V_n) ($)</th>
<th>Interest ($)</th>
<th>(V_{n + 1}) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5750</td>
<td>A% of 5750 = 460</td>
<td>6210</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>8% of C = 496.80</td>
<td>D</td>
</tr>
<tr>
<td>3</td>
<td>6706.80</td>
<td>8% of 6706.80 = 536.54</td>
<td>7243.34</td>
</tr>
<tr>
<td>4</td>
<td>7243.34</td>
<td>8% of 7243.34 = 579.47</td>
<td>E</td>
</tr>
<tr>
<td>5</td>
<td>F</td>
<td>8% of 7822.81 = 625.82</td>
<td>8448.63</td>
</tr>
</tbody>
</table>

9 Alex invested $12000 for 4 years at an interest rate of 7.5% p.a. with interest compounded annually. Complete the table by calculating the values A, B, C, D, E and F.

<table>
<thead>
<tr>
<th>Time period ((n + 1))</th>
<th>(V_n) ($)</th>
<th>Interest ($)</th>
<th>(V_{n + 1}) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12000</td>
<td>7.5% of 12000 = 900</td>
<td>12900</td>
</tr>
<tr>
<td>2</td>
<td>12900</td>
<td>7.5% of 12900 = A</td>
<td>B</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td>7.5% of 13867.50 = 1040.06</td>
<td>14907.56</td>
</tr>
<tr>
<td>4</td>
<td>14907.56</td>
<td>7.5% of 14907.56 = 1118.07</td>
<td>D</td>
</tr>
<tr>
<td>5</td>
<td>E</td>
<td>7.5% of 16025.63 = 1201.92</td>
<td>F</td>
</tr>
</tbody>
</table>

10 Sarina invested $5500 for 3 years at an interest rate of 6.5% p.a. with interest compounded annually. Complete the table shown.

<table>
<thead>
<tr>
<th>Time period ((n + 1))</th>
<th>(V_n) ($)</th>
<th>Interest ($)</th>
<th>(V_{n + 1}) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5500</td>
<td>6.5% of 5500 =</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

11 Fredrick invested $15000 for 5 years at an interest rate of 4.25% p.a. with interest compounded annually. Complete the table shown to find the value of his investment after 5 years.

<table>
<thead>
<tr>
<th>Time period</th>
<th>Balance ($)</th>
</tr>
</thead>
</table>
| 1           | 15000 \times 1.0425 = 15000(1.0425)^1  
= 15637.5  |
| 2           | 15637.5 \times 1.0425 = 15000 \times 1.0425 \times 1.0425  
= 15000(1.0425)^2  
= 16302.09  |
| 3           |             |
| 4           |             |
| 5           |             |
12 Joel invested $25000 for 5 years at an interest rate of 6.5% p.a. with interest compounded annually. Complete the table shown to find the value of his investment after 5 years.

<table>
<thead>
<tr>
<th>Time period</th>
<th>Balance ($)</th>
</tr>
</thead>
</table>
| 1           | 25000 × 1.065 = 25000(1.065)
= 26625       |
| 2           | 26625 × 1.065 = 25000 × 1.065 × 1.065
= 25000(1.065)
= 28355.63   |
| 3           |             |
| 4           |             |
| 5           |             |

13 An investment of $10000 was made for 5 years at an interest rate of 5% p.a. with interest compounded quarterly. Complete the table shown to find the value of the investment after 5 years.

<table>
<thead>
<tr>
<th>Time period</th>
<th>Balance ($)</th>
</tr>
</thead>
</table>
| 1           | 10000(1 + 0.05)
= 10509.45   |
| 2           | 10000(1 + 0.05)
= 11044.86   |
| 3           |             |
| 4           |             |
| 5           |             |

14 An investment of $8500 was made for 5 years at an interest rate of 7.5% p.a. with interest compounded monthly. Complete the table shown to find the value of the investment after 5 years.

<table>
<thead>
<tr>
<th>Time period</th>
<th>Balance ($)</th>
</tr>
</thead>
</table>
| 1           | 8500(1 + 0.075)
= 9159.88    |
| 2           |             |
| 3           |             |
| 4           |             |
| 5           |             |

6.4 Compound interest formula

From the previous exercise we saw that we could write the value of the investment in terms of its previous value. This can be expressed as the recurrence relation:
\[ V_{n+1} = V_n R \]
where \( V_{n+1} \) is the amount of the investment 1 time period after \( V_n \), \( R \) is the growth or compounding factor \( \left( 1 + \frac{r}{100} \right) \) and \( r \) is interest rate per period.

This pattern can be expanded further to write the value of the investment in terms of the initial investment. This is known as the compound interest formula.

\[ V_n = V_0 R^n \quad \text{where} \quad V_n = \text{final or total amount (\$)} \]
\[ V_0 = \text{principal (\$)} \]

\[ R = \text{growth or compounding factor } \left( 1 + \frac{r}{100} \right) \]
\[ r = \text{interest rate per period} \]
\[ n = \text{number of interest-bearing periods} \]

Note that the compound interest formula gives the total amount in an account, not just the interest earned as in the simple interest formula.

To find the total interest compounded, \( I \):

\[ I = V_n - V_0 \quad \text{where} \quad V_n = \text{final or total amount (\$)} \]
\[ V_0 = \text{principal (\$)} \]

If compound interest is used, the value of the investment at the end of each period grows by an increasing amount. Therefore, when plotted, the values of the investment at the end of each period form an exponential curve.

Now let us consider how the formula is used.

**WORKED EXAMPLE**

$5000 is invested for 4 years at 6.5\% \text{ p.a.}, \text{ interest compounded annually}.

**a** Generate the compound interest formula for this investment.

**b** Find the amount in the balance after 4 years and the interest earned over this period.

**THINK**

1. Write the compound interest formula.
2. List the values of \( n, r \) and \( P \).

**WRITE**

\[ a \quad V_n = V_0 \left( 1 + \frac{r}{100} \right)^n \]
\[ n = 4 \]
\[ r = 6.5 \]
\[ V_0 = 5000 \]
In Worked example 7, interest was compounded annually. However, in many cases the interest is compounded more often than once a year, for example, quarterly (every 3 months), weekly, or daily. In these situations \( n \) and \( r \) still have their usual meanings and we calculate them as follows.

- **Number of interest periods,** \( n = \) number of years \( \times \) number of interest periods per year
- **Interest rate per period,** \( r = \) nominal interest rate per annum \( \div \) number of interest periods per year

**Note:** Nominal interest rate per annum is simply the annual interest rate advertised by a financial institution.

### WORKED EXAMPLE 8

If $3200 is invested for 5 years at 6% p.a., interest compounded quarterly:

- **a** find the number of interest-bearing periods, \( n \)
- **b** find the interest rate per period, \( r \)
- **c** find the balance of the account after 5 years
- **d** graphically represent the balance at the end of each quarter for 5 years. Describe the shape of the graph.

**THINK**

- **a** Calculate \( n \).
- **b** Convert % p.a. to % per quarter to match the time over which the interest is calculated.

**WRITE/DRAW**

- **a** \( n = 5 \) (years) \( \times \) 4 (quarters) 
  \[ = 20 \]
- **b** \( r\% = \frac{6\% \text{ p.a.}}{4} \) 
  \[ = 1.5\% \text{ per quarter} \] 
  \[ r = 1.5 \]
c 1 Write the compound interest formula.
   2 List the values of $V_0$, $r$ and $n$.
   3 Substitute into the formula.
   4 Simplify.
   5 Evaluate correct to 2 decimal places.
   6 Write your answer.

d 1 Using CAS, find the balance at the end of each quarter and
   plot these values on the set of axes. (The first point is (0, 3200),
   which represents the principal.)

2 Comment on the shape of the graph.

The situation often arises where we require a certain amount of money by a future
date. It may be to pay for a holiday or to finance the purchase of a car. It is then
necessary to know what principal should be invested now in order that it will increase
in value to the desired final balance within the time available.

WORKED EXAMPLE 9

Find the principal that will grow to $4000 in 6 years, if interest is added quarterly at 6.5% p.a.

THINK

1 Calculate $n$ (there are 4 quarters in a year).
   \[ n = 6 \times 4 = 24 \]

2 Calculate $r$.
   \[ r = \frac{6.5}{4} = 1.625 \]

3 List the value of $V_{24}$.
   \[ V_{24} = 4000 \]

4 Write the compound interest formula, substitute and simplify.
   \[ V_{24} = V_0 \left(1 + \frac{r}{100}\right)^n \]
   \[ 4000 = V_0 \left(1 + \frac{1.625}{100}\right)^{24} \]
   \[ 4000 = V_0(1.01625)^{24} \]

WRITE
5 Transpose to isolate $V_0$. 

\[ V_0 = \frac{4000}{(1.01625)^{24}} = 2716.73 \]

6 Evaluate correct to 2 decimal places.

7 Write a summary statement. $2716.73$ would need to be invested.

**EXERCISE 6.4**

**Compound interest formula**

1 **WE7** Find the amount in the account and interest earned after $2500 is invested for 5 years at 7.5% p.a., interest compounded annually.

2 Find the amount in the account and interest earned after $6750 is invested for 7 years at 5.25% p.a., interest compounded annually.

3 **WE8** If $4200 is invested for 3 years at 7% p.a., interest compounded quarterly:
   a find the number of interest-bearing periods, $n$
   b find the interest rate per period, $r$
   c find the balance of the account after 3 years
   d graphically represent the balance at the end of each quarter for 3 years. Describe the shape of the graph.

4 If $7500 is invested for 2 years at 5.5% p.a., interest compounded monthly:
   a find the number of interest-bearing periods, $n$
   b find the interest rate per period, $r$
   c find the balance of the account after 2 years
   d graphically represent the balance at the end of each month for 2 years. Describe the shape of the graph.

5 **WE9** Find the principal that will grow to $5000 in 5 years, if interest is added quarterly at 7.5% p.a.

6 Find the principal that will grow to $6300 in 7 years, if interest is added monthly at 5.5% p.a.

For the following questions, use the compound interest formula to calculate the answer, then check your answer using Finance Solver on your CAS.

7 Use the compound interest formula to find the amount, $V_n$, when:
   a $V_0 = 500, n = 2, r = 8$
   b $V_0 = 1000, n = 4, r = 13$
   c $V_0 = 3600, n = 3, r = 7.5$
   d $V_0 = 2915, n = 5, r = 5.25$.

8 Using a recurrence relation, find: i the balance, and ii the interest earned (interest compounded annually) after:
   a $2000 is invested for 1 year at 7.5% p.a.
   b $2000 is invested for 2 years at 7.5% p.a.
   c $2000 is invested for 6 years at 7.5% p.a.

9 Find the number of interest-bearing periods, $n$, if interest is compounded:
   a annually for 5 years
   b quarterly for 5 years
   c semi-annually for 4 years
   d monthly for 6 years
   e 6-monthly for $4\frac{1}{2}$ years
   f quarterly for 3 years and 9 months.
10 Find the interest rate per period, \( r \), if the annual rate is:
   a) 6% and interest is compounded quarterly
   b) 4% and interest is compounded half-yearly
   c) 18% and interest is compounded monthly
   d) 7% and interest is compounded quarterly.

11 $1500 is invested for 2 years into an account paying 8% p.a. Find the balance if:
   a) interest is compounded yearly
   b) interest is compounded quarterly
   c) interest is compounded monthly
   d) interest is compounded weekly.
   e) Compare your answers to parts a–d

12 Use the recurrence relation \( V_{n+1} = 1.045 V_n \) to answer the following questions.
   a) If the balance in an account after 1 year is $2612.50, what will the balance be after 3 years?
   b) If the balance in an account after 2 years is $4368.10, what will the balance be after 5 years?
   c) If the balance in an account after 2 years is $6552.15, what was the initial investment?

13 Find the amount that accrues in an account which pays compound interest at a nominal rate of:
   a) 7% p.a. if $2600 is invested for 3 years (compounded monthly)
   b) 8% p.a. if $3500 is invested for 4 years (compounded monthly)
   c) 11% p.a. if $960 is invested for \( 5\frac{1}{2} \) years (compounded fortnightly)
   d) 7.3% p.a. if $2370 is invested for 5 years (compounded weekly)
   e) 15.25% p.a. if $4605 is invested for 2 years (compounded daily).

14 The greatest return is likely to be made if interest is compounded:
   A) annually     B) semi-annually     C) quarterly
   D) monthly     E) fortnightly

15 If $12000 is invested for \( 4\frac{1}{4} \) years at 6.75% p.a., compounded fortnightly, the amount of interest that would accrue would be closest to:
   A) $3600     B) $4200     C) $5000
   D) $12100     E) $16300

16 Use the compound interest formula to find the principal, \( V_0 \), when:
   a) \( V_n = $5000, r = 9, n = 4 \)
   b) \( V_n = $2600, r = 8.2, n = 3 \)
   c) \( V_n = $3550, r = 1.5, n = 12 \)
   d) \( V_n = $6661.15, r = 0.8, n = 36 \)

17 Find the principal that will grow to:
   a) $3000 in 4 years, if interest is compounded 6-monthly at 9.5% p.a.
   b) $2000 in 3 years, if interest is compounded quarterly at 9% p.a.
   c) $5600 in \( 5\frac{1}{4} \) years, if interest is compounded quarterly at 8.7% p.a.
   d) $10000 in \( 4\frac{1}{4} \) years, if interest is compounded monthly at 15% p.a.

18 Find the interest accrued in each case in question 17.
Finding rate or time for compound interest

Sometimes we know how much we can afford to invest as well as the amount we want to have at a future date. Using the compound interest formula we can calculate the interest rate that is needed to increase the value of our investment to the amount we desire. This allows us to ‘shop around’ various financial institutions for an account which provides the interest rate we want.

We must first find the interest rate per period, \( r \), and convert this to the corresponding nominal rate per annum.

**WORKED EXAMPLE 10**

Find the interest rate per annum (correct to 2 decimal places) that would enable an investment of \$3000 to grow to \$4000 over 2 years if interest is compounded quarterly.

**THINK**

1. List the values of \( V_n \), \( V_0 \) and \( n \). For this example, \( n \) needs to represent quarters of a year and therefore \( r \) will be evaluated in \% per quarter.

2. Write the compound interest formula and substitute the known values.

3. Divide \( V_n \) by \( V_0 \).

4. Obtain \( R \) to the power of 1, that is, raise both sides to the power of \( \frac{1}{8} \).

5. Replace \( R \) with \( 1 + \frac{r}{100} \).

6. Isolate \( r \) and evaluate.

7. Multiply \( r \) by the number of interest periods per year to get the annual rate (correct to 2 decimal places).

8. Write your answer.

**WRITE**

\[
V_n = V_0 R^n \\
4000 = 3000 R^8 \\
\frac{4000}{3000} = R^8 \\
\left(\frac{4}{3}\right)^{\frac{1}{8}} = (R^8)^{\frac{1}{8}} = R \\
\frac{4}{3} = 1 + \frac{r}{100} \\
\frac{r}{100} = \frac{4}{3} - 1 \\
= 0.0366146 \\
r = 3.66146 \\
r\% = 3.66146\% \text{ per quarter} \\
\text{Annual rate} = r\% \text{ per quarter } \times 4 \\
= 3.66146\% \text{ per quarter } \times 4 \\
= 14.65\% \text{ per annum} \\
\text{Interest rate of 14.65\% p.a. is required, correct to 2 decimal places.}
\]

Note: Worked example 10 requires a number of operations to find the solution. This is one of the reasons why most financial institutions use finance software for efficient and error-free calculations. Your CAS has a finance function called Finance Solver. This can be used for compound interest calculations as shown in the worked examples in this section. Finance Solver will also be used extensively in the remaining topics of this topic.
Finding time in compound interest

We have seen how the compound interest formula, \( V_n = V_0 R^n \), where \( R = 1 + \frac{r}{100} \) can be manipulated to solve situations where \( V_n, V_0 \) and \( r \) were unknown.

To find \( n \), the number of interest-bearing periods, that is, to find the time period of an investment, is more difficult. Possible methods to solve for \( n \) include:
1. trial and error
2. logarithms
3. using Finance Solver on CAS.

The value obtained for \( n \) may be a whole number, but it is more likely to be a decimal. That is, the time required will lie somewhere between two consecutive integers. The smaller of the two integers represents insufficient time for the investment to amount to the balance desired; the larger integer represents more than enough time.

In practice, if this is the case, an investor may choose to:
   a. withdraw funds as soon as the final balance is reached, in which case a fee may be imposed for early withdrawal
   b. withdraw funds at the first integral value of \( n \) after the final balance is reached; that is, when the investment matures.

In this section, we will use Finance Solver to calculate the time period of an investment.

**WORKED EXAMPLE 11** How long it will take $2000 to amount to $3500 at 8% p.a. with interest compounded annually?

**THINK**
1. State the values of \( V_n, V_0 \) and \( r \).
2. Use the Finance Solver on CAS to enter the following values:
   - \( n \) (N:) = unknown
   - \( r \) (I(%):) = 8
   - \( V_0 \) (PV:) = −2000
   - \( V_n \) (FV:) = 3500
   - PpY: = 1
   - CpY: = 1
3. Solve for \( n \).
4. Interest is compounded annually, so \( n \) represents years. Raise \( n \) to the next whole year.
5. Write your answer.

**WRITE**
- \( V_n = 3500, V_0 = 2000 \) and \( r = 8\% \) p.a.
- \( n = 7.27 \) years
- As the interest is compounded annually, \( n = 8 \) years.
- It will take 8 years for $2000 to amount to $3500.

**WORKED EXAMPLE 12** Calculate the number of interest-bearing periods, \( n \), required and hence the time it will take $3600 to amount to $5100 at a rate of 7% p.a., with interest compounded quarterly.
Finding rate or time for compound interest

1. We10 Find the interest rate per annum (correct to 2 decimal places) that would enable an investment of $4000 to grow to $5000 over 2 years if interest is compounded quarterly.

2. We11 Find the interest rate per annum (correct to 2 decimal places) that would enable an investment of $7500 to grow to $10000 over 3 years if interest is compounded monthly.

3. We12 Calculate the number of interest-bearing periods, \( n \), required and hence the time it will take $4700 to amount to $6100 at a rate of 9% p.a., with interest compounded quarterly.

4. We13 Calculate the number of interest-bearing periods, \( n \), required and hence the time it will take $3800 to amount to $6300 at a rate of 15% p.a., with interest compounded quarterly.

5. Lillian wishes to have $24000 in a bank account after 6 years so that she can buy a new car. The account pays interest at 15.5% p.a. compounded quarterly. The amount (correct to the nearest dollar) that Lillian should deposit in the account now, if she is to reach her target, is:

   - A $3720
   - B $9637
   - C $10109
   - D $12117
   - E $22320

6. We14 Find the interest rates per annum (correct to 2 decimal places) that would enable investments of:
   - a $2000 to grow to $3000 over 3 years if interest is compounded 6-monthly
   - b $12000 to grow to $15000 over 4 years (interest compounded quarterly).
9 Find the interest rates per annum (correct to 2 decimal places) that would enable investments of:
   a $25000 to grow to $40000 over 2.5 years (compounded monthly)
   b $43000 to grow to $60000 over 4.5 years (compounded fortnightly)
   c $1400 to grow to $1950 over 2 years (compounded weekly).
10 What is the minimum interest rate per annum (compounded quarterly) needed for $2300 to grow to at least $3200 in 4 years?
11 Use Finance Solver on CAS to find out how long it will take (with interest compounded annually) for:
   a $2000 to amount to $3173.75 at 8% p.a.
   b $9250 to amount to $16565.34 at 6% p.a.
12 Use Finance Solver on CAS to find out how long it will take (with interest compounded annually) for:
   a $850 to amount to $1000 at 7% p.a.
   b $12000 to amount to $20500 at 13.25% p.a.
13 Calculate the number of interest-bearing periods, \( n \), required, and hence the time in more meaningful terms when:
   a \( V_n = 2100, V_0 = 1200, r = 3\% \) per half-year
   b \( V_n = 13500, V_0 = 8300, r = 2.5\% \) per quarter
   c \( V_n = 16900, V_0 = 9600, r = 1\% \) per month.
14 Calculate the number of interest-bearing periods, \( n \), required, and hence the time in more meaningful terms when:
   a \( V_n = 24000, V_0 = 16750, r = 0.25\% \) per fortnight
   b $7800 is to amount to $10000 at a rate of 8% p.a. (compounded quarterly)
   c $800 is to amount to $1900 at a rate of 11% p.a. (compounded quarterly).
15 Wanda has invested $1600 in an account at a rate of 10.4% p.a., interest compounded quarterly. How long will it take to reach $2200?
16 What will be the least number of interest periods, \( n \), required for $6470 to grow to at least $9000 in an account with interest paid at 6.5% p.a. and compounded half-yearly?
   A 10       B 11       C 12       D 20       E 22
17 About how long would it take for:
   a $1400 to accrue $300 interest at 8% p.a., interest compounded monthly?
   b $800 to accrue $4400 interest at 9.6% p.a., interest compounded fortnightly?
18 Jennifer and Dawn each want to save $15000 for a car. Jennifer has $11000 to invest in an account with her bank which pays 8% p.a., interest compounded quarterly. Dawn’s credit union has offered her 11% p.a., interest compounded quarterly.
   a How long will it take Jennifer to reach her target?
   b How much will Dawn need to invest in order to reach her target at the same time as Jennifer? Assume their accounts were opened at the same time.
6.6 Flat rate depreciation

Many items such as antiques, jewellery or real estate increase in value (appreciate or increase in capital gain) with time. On the other hand, items such as computers, vehicles or machinery decrease in value (depreciate) with time as a result of wear and tear, advances in technology or a lack of demand for those specific items. The estimated loss in value of assets is called depreciation. Each financial year a business will set aside money equal to the depreciation of an item in order to cover the cost of the eventual replacement of that item. The estimated value of an item at any point in time is called its future value (or book value).

When the value becomes zero, the item is said to be written off. At the end of an item’s useful or effective life (as a contributor to a company’s income) its future value is then called its scrap value.

Future value = cost price − total depreciation to that time
When book value = $0, then the item is said to be written off.
Scrap value is the book value of an item at the end of its useful life.

There are 3 methods by which depreciation can be calculated. They are:
1. flat rate depreciation
2. reducing balance depreciation
3. unit cost depreciation.

Flat rate (straight line) depreciation

If an item depreciates by the flat rate method, then its value decreases by a fixed amount each unit time interval, generally each year. This depreciation value may be expressed in dollars or as a percentage of the original cost price.

This method of depreciation may also be referred to as prime cost depreciation. Since the depreciation is the same for each unit time interval, the flat rate method is an example of straight line (linear) decay. The relationship can be represented by the recurrence relation:

\[ V_{n+1} = V_n - d \]

where \( V_n \) is the value of the asset after \( n \) depreciating periods and \( d \) is the depreciation each time period.

We can also look at the future value of an asset after \( n \) periods of depreciation, which can be calculated by:

\[ V_n = V_0 - nd \]

We can use this relationship to analyse flat rate depreciation or we can use a depreciation schedule (table) which can then be used to draw a graph of future value against time. The schedule displays the future value after each unit time interval, that is:

<table>
<thead>
<tr>
<th>Time, ( n )</th>
<th>Depreciation, ( d )</th>
<th>Future value, ( V_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fast Word Printing Company bought a new printing press for $15 000 and chose to depreciate it by the flat rate method. The depreciation was 15% of the prime cost price each year and its useful life was 5 years.

a Find the annual depreciation.

b Set up a recurrence relation to represent the depreciation.

c Draw a depreciation schedule for the useful life of the press and use it to draw a graph of book value against time.

d Generate the relationship between the book value and time and use it to find the scrap value.

**THINK**

a 1 State the cost price.

2 Find the depreciation rate as 15% of the prime cost price.

3 Write your answer.

b Write the recurrence relation for flat rate depreciation and substitute in the value for $d$.

c 1 Draw a depreciation schedule for 0–5 years, using depreciation of 2250 each year and a starting value of $15 000.

2 Draw a graph of the tabled values for future value against time.

<table>
<thead>
<tr>
<th>Time, $n$ (years)</th>
<th>Depreciation, $d$ ($)</th>
<th>Future value, $V_n$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15 000</td>
<td>15 000</td>
</tr>
<tr>
<td>1</td>
<td>2250</td>
<td>12 750</td>
</tr>
<tr>
<td>2</td>
<td>2250</td>
<td>10 500</td>
</tr>
<tr>
<td>3</td>
<td>2250</td>
<td>8 250</td>
</tr>
<tr>
<td>4</td>
<td>2250</td>
<td>6 000</td>
</tr>
<tr>
<td>5</td>
<td>2250</td>
<td>3 750</td>
</tr>
</tbody>
</table>

**WRITE/DRAW**

a $V_0 = 15 000$

d $d = V_0 \times \frac{r}{100}$

$= 15 000 \times \frac{15}{100}$

$= 2250$

Annual depreciation is $2250.

b $V_{n+1} = V_n - d$

$c V_{n+1} = V_n - 2250$

<table>
<thead>
<tr>
<th>Time, $n$ (years)</th>
<th>Depreciation, $d$ ($)</th>
<th>Future value, $V_n$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15 000</td>
<td>15 000</td>
</tr>
<tr>
<td>1</td>
<td>2250</td>
<td>12 750</td>
</tr>
<tr>
<td>2</td>
<td>2250</td>
<td>10 500</td>
</tr>
<tr>
<td>3</td>
<td>2250</td>
<td>8 250</td>
</tr>
<tr>
<td>4</td>
<td>2250</td>
<td>6 000</td>
</tr>
<tr>
<td>5</td>
<td>2250</td>
<td>3 750</td>
</tr>
</tbody>
</table>
1. **Set up the equation:**
   \[ V_n = V_0 - nd. \]
   State \( d \) and \( V_0 \).

2. **The press is scrapped after 5 years so substitute \( n = 5 \) into the equation.**

3. **Write your answer.**
   The scrap value is $3750.

The depreciation schedule gives the scrap value, as can be seen in Worked example 13. So too does a graph of book value against time, since it is only drawn for the item’s useful life and its end point is the scrap value.

Businesses need to keep records of depreciation for tax purposes on a year-to-year basis. What if an individual wants to investigate the rate at which an item has depreciated over many years? An example is the rate at which a private car has depreciated. If a straight line depreciation model is chosen, then the following example demonstrates its application.

**WORKED EXAMPLE 14**

Jarrod bought his car 5 years ago for $15,000. Its current market value is $7,500. Assuming straight line depreciation, find:

a. the car’s annual depreciation rate
b. the relationship between the future value and time, and use it to find when the car will have a value of $3,000.

**THINK**

a. 1. Find the total depreciation over the 5 years and thus the rate of depreciation.

2. Write your answer.

b. 1. Set up the future value equation.

2. Use the equation and substitute \( V_n = $3,000 \) and transpose the equation to find \( n \).

**WRITE**

a. Total depreciation = cost price − current value
   \[ = 15,000 - 7,500 \]
   \[ = 7,500 \]
   Rate of depreciation = \[ \frac{\text{total depreciation}}{\text{number of years}} \]
   \[ = \frac{7,500}{5 \text{ years}} \]
   \[ = $1,500 \text{ per year} \]
   The annual depreciation rate is $1,500.

b. \[ V_n = V_0 - nd \]
   \[ V_n = 15,000 - 1500n \]

When \( V_n = 3,000 \),
\[ 3,000 = 15,000 - 1500n \]
\[ -1500n = 3,000 - 15,000 \]
\[ -1500n = -12,000 \]
\[ n = \frac{-12,000}{-1500} \]
\[ = 8 \]
We13 A mining company bought a vehicle for $25000 and chose to depreciate it by the flat rate method. The depreciation was 15% of the cost price each year and its useful life was 5 years.

a Find the annual depreciation.

b Set up a recurrence relation to represent the depreciation.

c Draw a depreciation schedule for the item’s useful life and draw a graph of future value against time.

d Find the relationship between future value and time. Use it to find the scrap value.

We14 Write the future value functions for the following items.

a A $30000 car that is depreciated by 20% p.a. flat rate

b A $2000 display unit depreciated by 10% of its prime cost

c A $6000 piano depreciated by $500 p.a.

3 For the situations described below, and using a straight line depreciation model, find:

i the annual rate of depreciation

ii the relationship between the future value and time and use it to find at what age the item will be written off, that is, have a value of $0.

a A car purchased for $50000 with a current value of $25000; it is now 5 years old.

b A stereo unit bought for $850 seven years ago; it now has a current value of $150.

c A refrigerator with a current value of $285 bought 10 years ago for $1235

4 A second-hand car is currently on sale for $22500. Its value is expected to depreciate by $3200 per year.

a Write an equation that will predict the car’s price \( V_n \) in the future \( n \).

b Work out the expected value of the car after 5 years.

5 All Clean carpet cleaners bought a cleaner for $10000 and chose to depreciate it by the flat rate method. The depreciation was 15% of the cost price each year and its useful life was 5 years.

a Find the annual depreciation.

b Draw a depreciation schedule for the item’s useful life and draw a graph of future value against time.

For the situations outlined in questions 6 and 7:

a draw a depreciation schedule for the item’s useful life and draw a graph of future value against time

b find the relationship between future value and time. Use it to find the scrap value.

6 A farming company chose to depreciate a tractor by the prime cost method and the annual depreciation was $4000. The tractor was purchased for $45000 and its useful life was 10 years.

The depreciation equation for the car is \( V_n = 15000 - 1500n \). The future value will reach $3000 when the car is 8 years old.
A winery chose to depreciate a corking machine, that cost $13,500 when new, by the prime cost method. The annual depreciation was $2000 and its useful life was 6 years.

For the situations outlined in questions 8 and 9:

a. find the annual depreciation
b. set up a recurrence relation to represent the depreciation
c. draw a depreciation schedule for the item’s useful life and draw a graph of future value against time
d. generate the relationship between future value and time. Use it to find how long it will take for the item to reach its scrap value.

8. Machinery is bought for $7750 and depreciated by the flat rate method. The depreciation is 20% of the cost price each year and its scrap value is $1550.

9. An excavation company buys a digger for $92,000 and depreciates it by the flat rate method. The depreciation is 15% of the cost price per year and its scrap value is $9200.

10. Each of the following graphs represents the flat rate depreciation of four particular items. In each case determine:

i. the cost price of the item
ii. the annual depreciation
iii. the time taken for the item to reach its scrap value or to be written off.

11. A 1-tonne truck, bought for $31,000, was depreciated using the flat rate method. If the scrap value of $5000 was reached after 5 years, the annual depreciation would be:

A. $31
B. $1000
C. $5200
D. $6200
E. $26,000
12 The depreciation of a piece of machinery is given by the equation, $V_n = 6000 - 450n$. The machinery will have a future value of $2400 after:

- A 7 years
- C 9 years
- D 10 years

13 The depreciation of a computer is given by the equation, $V_n = 3450 - 280n$. After how many years will the computer have a future value of $1770?

14 Listed below are the depreciation equations for 5 different items. Which item would be written off in the least amount of time?

- A $V_n = 7000 - 650n$
- B $V_n = 7000 - 750n$
- C $V_n = 6000 - 650n$
- D $V_n = 6000 - 750n$
- E $V_n = 6000 - 850n$

15 A business buys two different photocopiers at the same time. One costs $2200 and is to be depreciated by $225 per annum. It also has a scrap value of $400. The other costs $3600 and is to be depreciated by $310 per annum. This one has a scrap value of $500.

- a Which machine would need to be replaced first?
- b How much later would the other machine need to be replaced?

16 A car valued at $20000 was bought 5 years ago for $45000. The straight line depreciation model is represented by:

- A $V_n = 45000 - 20000n$
- B $V_n = 45000 - 5000n$
- C $V_n = 45000 - 4000n$
- D $V_n = 20000n$
- E $V_n = 45000 - 25000n$

6.7 Reducing balance depreciation

If an item depreciates by the reducing balance depreciation method then its value decreases by a fixed rate each unit time interval, generally each year. This rate is a percentage of the previous value of the item. Reducing balance depreciation is also known as diminishing value depreciation.

Reducing balance depreciation can be expressed by the recurrence relation:

$$V_{n+1} = RV_n$$

where $V_n$ is the value of the asset after $n$ depreciating periods and $R = 1 - \frac{r}{100}$, where $r$ is the depreciation rate.

WORKED EXAMPLE 15

Suppose the new $15000 printing press considered in Worked example 13 was depreciated by the reducing balance method at a rate of 20% p.a. of the previous value.

- a Generate a depreciation schedule using a recurrence relation for the first 5 years of work for the press.
- b What is the future value after 5 years?
- c Draw a graph of future value against time.
THINK

a 1 Calculate the value of $R$.

WRITE/DRAW

a  $R = 1 - \frac{r}{100}$
   $= 1 - \frac{20}{100}$
   $= 0.8$

2 Write the recurrence relation for reducing balance depreciation and substitute in the known information.

3 Use the recurrence relation to calculate the future value for the first 5 years (up to $n = 5$).

4 Draw the depreciation schedule.

b State the future value after 5 years from the depreciation schedule.

c Draw a graph of the future value against time.

<table>
<thead>
<tr>
<th>Time, $n$ (years)</th>
<th>Future value, $V_n$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15000</td>
</tr>
<tr>
<td>1</td>
<td>12000</td>
</tr>
<tr>
<td>2</td>
<td>9600</td>
</tr>
<tr>
<td>3</td>
<td>7680</td>
</tr>
<tr>
<td>4</td>
<td>6144</td>
</tr>
<tr>
<td>5</td>
<td>4915.20</td>
</tr>
</tbody>
</table>

b The future value of the press after 5 years will be $4915.20$.
It is clear from the graph and the schedule that the reducing balance depreciation results in greater depreciation during the early stages of the asset’s life (the future value drops more quickly at the start since the annual depreciation falls from $3000 in year 1 to $1228.80 in year 4).

Currently, the Australian Taxation Office allows depreciation of an asset as a tax deduction. This means that the annual depreciation reduces the amount of tax paid by a business in that year. The higher the depreciation, the greater the tax benefit. Therefore, depreciating an asset by the reducing balance method allows a greater tax benefit for a business in the beginning of an asset’s life rather than towards the end. In contrast, flat rate depreciation remains constant throughout the asset’s life. People have a choice as to whether they depreciate an item by the flat rate or reducing balance methods, but once a method is applied to an article it cannot be changed for the life of that article. The percentage depreciation rates, which are set by the Australian Taxation Office, vary from one item to another but for each item the rate applied for the reducing balance method is greater than that for the flat rate method.

Let us compare depreciation for both methods.

WORKED EXAMPLE 16

A transport business has bought a new bus for $60,000. The business has the choice of depreciating the bus by a flat rate of 20% of the cost price each year or by 30% of the previous value each year.

a Generate depreciation schedules using both methods for a life of 5 years.

b Draw graphs of the future value against time for both methods on the same set of axes.

c After how many years does the reducing balance future value become greater than the flat rate future value?

**THINK**

a 1 Calculate the flat rate depreciation per year.

2 Generate a flat rate depreciation schedule for 0–5 years.

**WRITE/DRAW**

a \[ d = 20\% \text{ of } 60000 \]

\[ = 12000 \text{ per year} \]

<table>
<thead>
<tr>
<th>Time, ( n ) (years)</th>
<th>Depreciation, ( d ) ($)</th>
<th>Future value, ( V_n ) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>—</td>
<td>60000</td>
</tr>
<tr>
<td>1</td>
<td>12000</td>
<td>48000</td>
</tr>
<tr>
<td>2</td>
<td>12000</td>
<td>36000</td>
</tr>
<tr>
<td>3</td>
<td>12000</td>
<td>24000</td>
</tr>
<tr>
<td>4</td>
<td>12000</td>
<td>12000</td>
</tr>
<tr>
<td>5</td>
<td>12000</td>
<td>0</td>
</tr>
</tbody>
</table>
3 Generate a reducing balance depreciation schedule. Annual depreciation is 30% of the previous value, so to calculate the future value multiply the previous value by 0.7. Continue to calculate the future value for a period of 5 years.

<table>
<thead>
<tr>
<th>Time, ( n ) (years)</th>
<th>Future value, ( V_n ) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>60000</td>
</tr>
<tr>
<td>1</td>
<td>( 60000 \times 0.7 = 42000 )</td>
</tr>
<tr>
<td>2</td>
<td>( 42000 \times 0.7 = 29400 )</td>
</tr>
<tr>
<td>3</td>
<td>( 29400 \times 0.7 = 20580 )</td>
</tr>
<tr>
<td>4</td>
<td>( 20580 \times 0.7 = 14406 )</td>
</tr>
<tr>
<td>5</td>
<td>( 14406 \times 0.7 = 10084.20 )</td>
</tr>
</tbody>
</table>

b) Draw graphs using values for \( V_n \) and \( n \) from the schedules. In this instance the blue line is the flat rate value and the pink curve is the reducing balance value.

c) Look at the graph to see when the reducing balance curve lies above the flat rate line. State the first whole year after this point of intersection.

c) The future value for the reducing balance method is greater than that of the flat rate method after 4 years.

We can write a general formula for reducing balance depreciation which is similar to the compound interest formula, except that the rate is subtracted rather than added to 1.

**The reducing balance depreciation formula is:**

\[
V_n = V_0 R^n
\]

\( V_n \) = book value after time, \( n \)

\( R \) = rate of depreciation \( = 1 - \frac{r}{100} \)

\( V_0 \) = cost price

\( n \) = time since purchase

That is, given the cost price and depreciation rate we can find the future value (including scrap value) of an article at any time after purchase.

Let us now see how we can use this formula.

**WORKED EXAMPLE 17**

The printing press from Worked example 13 was depreciated by the reducing balance method at 20% p.a. What will be the future value and total depreciation of the press after 5 years if it cost $15 000 new?

**THINK**

1. State \( V_0, r \) and \( n \).
2. Calculate the value of \( R \).

**WRITE**

\( V_0 = 15000, r = 20, n = 5 \)

\[ R = 1 - \frac{20}{100} = 0.8 \]
A photocopier purchased for $8000 depreciates by 25% p.a. by the reducing balance method. If the photocopier has a scrap value of $1200, how long will it be before this value is reached?

THINK
1 State the values of \(V_n\), \(V_0\) and \(r\).
2 Calculate the value of \(R\).
3 Substitute the values of the pronumerals into the formula and simplify.
4 Use CAS to find the value of \(n\).
5 Interest is compounded annually, so \(n\) represents years. Raise \(n\) to the next whole year.
6 Write a summary statement.

WRITE
\[ V_n = V_0 R^n \]
\[ V_5 = 15000(0.75)^5 \]
\[ = 4915.2 \]

Total depreciation is:
\[ V_0 - V_n \]
\[ = 15000 - 4915.2 \]
\[ = 10084.8 \]

The future value of the press after 5 years will be $4915.20 and its total depreciation will be $10084.80.

**Effective life**
The situation may arise where the scrap value is known and we want to know how long it will be before an item reaches this value; that is, its useful or **effective life**. So, in the reducing balance formula \(V_n = V_0 R^n\), \(n\) is needed.

**WORKED EXAMPLE 18**
A photocopier purchased for $8000 depreciates by 25% p.a. by the reducing balance method. If the photocopier has a scrap value of $1200, how long will it be before this value is reached?

**THINK**
1 State the values of \(V_n\), \(V_0\) and \(r\).
2 Calculate the value of \(R\).
3 Substitute the values of the pronumerals into the formula and simplify.
4 Use CAS to find the value of \(n\).
5 Interest is compounded annually, so \(n\) represents years. Raise \(n\) to the next whole year.
6 Write your answer.

**WRITE**
\[ V_n = V_0 R^n \]
\[ V_5 = 15000(0.75)^5 \]
\[ = 4915.2 \]

Total depreciation is:
\[ V_0 - V_5 \]
\[ = 15000 - 4915.2 \]
\[ = 10084.8 \]

The future value of the press after 5 years will be $4915.20 and its total depreciation will be $10084.80.

**Reducing balance depreciation**

**EXERCISE 6.7**

**PRACTISE**

1 **WE15** A laptop was bought new for $1500 and it depreciates by the reducing balance method at a rate of 17% p.a. of the previous book value.

a Generate a depreciation schedule using a recurrence relation for the first 5 years.
b What is the future value after 5 years?
c Draw a graph of future value against time.
2 A road bike was bought new for $3300 and it depreciates by the reducing balance method at a rate of 13% p.a. of the previous value.

a Generate a depreciation schedule using a recurrence relation for the first 5 years.
b What is the future value after 5 years?
c Draw a graph of future value against time.

3 A taxi company bought a new car for $29,990. The company has the choice of depreciating the car by a flat rate of 20% of the cost price each year or by 27% of the previous value each year.

a Generate depreciation schedules using both methods for a life of 5 years.
b Draw graphs of the future value against time for both methods on the same axis.
c After how many years does the reducing balance future value become greater than the flat rate future value?

4 A sailing rental company bought a new catamaran for $23,000. The company has the choice of depreciating the catamaran by a flat rate of 20% of the cost price each year or by 28% of the previous value each year.

a Generate depreciation schedules using both methods for a life of 5 years.
b Draw graphs of the future value against time for both methods on the same axis.
c After how many years does the reducing balance future value become greater than the flat rate future value?

5 Using the reducing balance formula, find $V_n$ (correct to 2 decimal places) given $V_0 = 45,000, r = 15, n = 6$.

6 Using the reducing balance formula, find $V_n$ (correct to 2 decimal places) given $V_0 = 2675, r = 22.5, n = 5$.

7 Use the Finance Solver to find $n$ (correct to 2 decimal places), given $V_n = 900, V_0 = 4500, r = 25$.

8 Use the Finance Solver to find $n$ (correct to 2 decimal places), given $V_n = 1500, V_0 = 7600, r = 15$.

9 A farming company chose to depreciate its new $60,000 bulldozer by the reducing balance method at a rate of 20% p.a. of the previous value.

a Write the recurrence relation that represents this depreciation.
b Draw a depreciation schedule for the first 5 years of the bulldozer’s life.
c What is its future value after 5 years?
d Draw a graph of future value against time.

10 A retail store chose to depreciate its new $4000 computer by the reducing balance method at a rate of 40% p.a. of the previous value.

a Write the recurrence relation that represents this depreciation.
b Draw a depreciation schedule for the first 5 years of the computer’s life.
c What is its future value after 5 years?
d Draw a graph of future value against time.
11 A café buys a cash register for $550. The owner has the choice of depreciating the register by the flat rate method (at 20% of the cost price each year) or the reducing balance method (at 30% of the previous value each year).

a Draw depreciation schedules for both methods for a life of 5 years.

b Draw graphs of future value against time for both methods on the same set of axes.

c After how many years does the reducing balance future value become greater than the flat rate future value?

12 Speedy Cabs taxi service has bought a new taxi for $30000. The company has the choice of depreciating the taxi by the flat rate method (at 33 1/3% of the cost price each year) or the diminishing value method (at 50% of the previous value each year).

a Draw depreciation schedules for both methods for 3 years.

b Draw graphs of future value against time for both methods on the same set of axes.

c After how many years does the reducing balance future value become greater than the flat rate future value?

13 Using the reducing balance formula, find $V_n$ (correct to 2 decimal places) given:

a $V_0 = 20000$, $r = 20$, $n = 4$

b $V_0 = 30000$, $r = 25$, $n = 4$.

Check your answers using CAS.

14 A refrigerator costing $1200 new is depreciated by the reducing balance method at 20% a year. After 4 years its future value will be:

A $240  
B $491.52  
C $960  
D $1105  
E $2488.32

15 The items below are depreciated by the reducing balance method at 25% p.a. What will be the future value and total depreciation of:

a a TV after 8 years, if it cost $1150 new

b a photocopier after 4 years, if it cost $3740 new

c carpets after 6 years, if they cost $7320 new?

16 The items below are depreciated at 30% p.a. by the reducing balance method. What will be the future value and total depreciation of:

a a lawn mower after 5 years, if it cost $685 new

b a truck after 4 years, if it cost $32500 new

c a washing machine after 3 years, if it cost $1075 new?

17 After 7 years, a new $3000 photocopier, which devalues by 25% of its value each year, will have depreciated by:

A $400.45  
B $750  
C $2250  
D $2599.55  
E $2750

18 New office furniture valued at $17500 is subjected to reducing balance depreciation of 20% p.a. and will reach its scrap value in 15 years. The scrap value will be:

A less than $300  
B between $300 and $400  
C between $400 and $500  
D between $500 and $600  
E between $600 and $700
A new chainsaw bought for $1250 has a useful life of only 3 years. If it depreciates annually by a 60% reducing balance rate, its scrap value will be:

A $0  B $60  C $80  D $250  E $270

Use Finance Solver to find \( n \) (correct to 2 decimal places), given:

a. \( V_n = 3000, V_0 = 40000, r = 20 \)

b. \( V_n = 500, V_0 = 3000, r = 30 \)

**Unit cost depreciation**

The flat rate and reducing balance deprecations of an item are based on the age of the item. With the **unit cost method**, the depreciation is based on the possible maximum output (units) of the item. For instance, the useful life of a truck could be expressed in terms of the distance travelled rather than a fixed number of years — for example, 120,000 kilometres rather than 6 years. The actual depreciation of the truck for the financial year would be a measure of the number of kilometres travelled. (The value of the truck decreases by a certain amount for each kilometre travelled.)

The future value over time using unit cost depreciation can be expressed by the recurrence relation:

\[
V_{n+1} = V_n - d
\]

where \( V_n \) is the value of the asset after \( n \) outputs and \( d \) is the depreciation per output.

### WORKED EXAMPLE 19

A motorbike purchased for $12,000 depreciates at a rate of $14 per 100 km driven.

a. Set up a recurrence relation to represent the depreciation.

b. Use the recurrence relation to generate a depreciation schedule for the future value of the bike after it has been driven for 100 km, 200 km, 300 km, 400 km and 500 km.

#### THINK

a. **1** Write the recurrence relation for unit cost depreciation as well as the known information.

  
  b. Substitute the values into the recurrence relation.

  
  b. **1** Use the recurrence relation to calculate the future value for the first 5 outputs (up to 500 km).

#### WRITE

a. \[
V_{n+1} = V_n - d
\]

\[
V_0 = 12000, d = 14
\]

\[
V_{n+1} = V_n - 14, V_0 = 12000
\]

b. \[
V_1 = V_0 - 14
\]

\[
= 12000 - 14
\]

\[
= 11986
\]

\[
V_2 = V_1 - 14
\]

\[
= 11986 - 14
\]

\[
= 11972
\]

\[
V_3 = V_2 - 14
\]

\[
= 11972 - 14
\]

\[
= 11958
\]
A taxi is bought for $31000 and it depreciates by 28.4 cents per kilometre driven. In one year the car is driven 15614 km. Find:

a the annual depreciation for this particular year

b its useful life if its scrap value is $12000.

THINK

a 1 Depreciation amount
   = distance travelled × rate

2 Write a summary statement.
   Annual depreciation for the year is $4434.38.

b 1 Total depreciation
   = cost price − scrap value

   = \frac{\text{distance travelled}}{\text{rate of depreciation}}

   \text{where rate of depreciation}
   = 28.4 \text{ cents/km}
   = 0.284 \text{ per km}

2 State your answer.
   The taxi has a useful life of 66901 km.

WORKED EXAMPLE

A photocopier purchased for $10800 depreciates at a rate of 20 cents for every 100 copies made. In its first year of use 500000 copies were made and in its second year, 550000. Find:

a the depreciation each year

b the future value at the end of the second year.
### THINK

**a** To find the depreciation, identify the rate and number of copies made.

Express the rate of 20 cents per 100 copies in a simpler form of dollars per 100 copies, that is, $0.20 per 100 copies or \( \frac{0.20}{100 \text{ copies}} \).

**b** Future value = cost price − total depreciation

### WRITE

**a** Depreciation = copies made \( \times \) rate

\[
\text{depreciation}_{1\text{st year}} = 500000 \times \frac{0.20}{100 \text{ copies}} = $1000
\]

Depreciation in the first year is $1000.

\[
\text{depreciation}_{2\text{nd year}} = 550000 \times \frac{0.20}{100 \text{ copies}} = $1100
\]

Depreciation in the second year is $1100.

**b** Total depreciation after 2 years

\[
= 1000 + 1100 = $2100
\]

Book value = 10 800 − 2100

\[
= $8700
\]

The future value after \( n \) outputs using unit cost depreciation can be expressed as:

\[
V_n = V_0 - nd
\]

where \( V_n \) is the value of the asset after \( n \) outputs and \( d \) is the depreciation per output.

### WORKED EXAMPLE 22

The initial cost of a vehicle was $27 850 and its scrap value is $5050. If the vehicle needs to be replaced after travelling 80 000 km (useful life):

**a** find the depreciation rate (depreciation (\( \$ \)) per km)

**b** find the amount of depreciation in a year when 16 497 km were travelled

**c** set up an equation to determine the value of the car after travelling \( n \) km

**d** find the future value after it has been used for a total of 60 000 km

**e** set up a schedule table listing future value for every 20 000 km.

### THINK

**1** To find the depreciation rate, first find the total depreciation.

Total amount of depreciation = cost price − scrap value

**2** Find the rate of depreciation.

It is common to express rates in cents per use if less than a dollar.

### WRITE

**a** Total amount of depreciation

\[
= 27850 - 5050
\]

\[
= $22800
\]

Depreciation rate = \( \frac{\text{total depreciation}}{\text{total distance travelled}} \)

\[
= \frac{22800}{80000}
\]

\[
= 0.285 \text{ per km}
\]

\[
= 28.5 \text{ cents per km}
\]
b Find the amount of depreciation using the rate calculated.

Amount of depreciation is always expressed in dollars.

\[ \text{Amount of depreciation} = \text{amount of use} \times \text{rate of depreciation} \]
\[ = 16497 \times 28.5 \]
\[ = 470165 \text{ cents} \]
\[ = \$4701.65 \]

c 1 Write the equation for unit cost depreciation after \( n \) outputs as well as the known information.

\[ V_n = V_0 - nd \]
\[ V_0 = 27850, \ d = 0.285 \]

2 Substitute the values into the equation.

\[ V_n = 27850 - 0.285d \]

2 Use the equation from part c to find the future value when \( n = 60000 \).

\[ V_{60000} = 27850 - 0.285 \times 60000 \]
\[ = 27850 - 17100 \]
\[ = 10750 \]

The future value after the car has been used for 60000 km is \$10750.

e Calculate the future value for every 20000 km of use and summarise in a table.

<table>
<thead>
<tr>
<th>Use, ( n ) (km)</th>
<th>Future value, ( V_n ) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$27850</td>
</tr>
</tbody>
</table>
| 20000            | \[ V_1 = 27850 - (20000 \times 0.285) \]
|                  | \[ = 22150 \]             |
| 40000            | \[ V_2 = 27850 - (20000 \times 0.285 \times 2) \]
|                  | \[ = 16450 \]             |
| 60000            | \[ V_3 = 27850 - (20000 \times 0.285 \times 3) \]
|                  | \[ = 10750 \]             |
| 80000            | \[ V_4 = 27850 - (20000 \times 0.285 \times 4) \]
|                  | \[ = 5050 \]              |

### Unit cost depreciation

1. A washing machine purchased for $800 depreciates at a rate of 35 cents per wash.
   a. Set up a recurrence relation to represent the depreciation.
   b. Use the recurrence relation to generate a depreciation schedule for the future value of the washing machine after each of the first 5 washes.

2. An air conditioning unit purchased for $1400 depreciates at a rate of 65 cents per hour of use.
   a. Set up a recurrence relation to represent the depreciation.
   b. Use the recurrence relation to generate a depreciation schedule for the future value of the air conditioning unit at the end of each of the first 5 hours of use.

3. Below are the depreciation details for a vehicle. Find:
   a. the annual depreciation in the first year
   b. the useful life (km).

<table>
<thead>
<tr>
<th>Purchase price ($)</th>
<th>Scrap value ($)</th>
<th>Rate of depreciation (cents/km)</th>
<th>Distance travelled in first year (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>29600</td>
<td>12000</td>
<td>28.5</td>
<td>14000</td>
</tr>
</tbody>
</table>
4 A taxi was purchased for $42000 and depreciates by 25 cents per km driven. During its first year the taxi travelled 64000 km; during its second year it travelled 56000 km. Find:
   a the depreciation in each of the first 2 years
   b how far the car had travelled if its total depreciation was $20000.

5 A photocopier is bought for $8600 and it depreciates at a rate of 22 cents for every 100 copies made. In its first year of use, 400000 copies are made and in its second year, 480000 copies are made. Find:
   a the depreciation for each year
   b the future value at the end of the second year.

6 A printing machine was purchased for $38000 and depreciated at a rate of $1.50 per million pages printed. In its first year 385 million pages were printed and 496 million pages were printed in its second year. Find:
   a the depreciation for each year
   b the future value at the end of the second year.

7 A delivery service purchases a van for $30000 and it is expected that the van will be written off after travelling 200000 km. It is estimated that the van will travel 1600 km each week.
   a Find the depreciation rate (charge per km).
   b Find how long it will take for the van to be written off.
   c Set up an equation to determine the value of the van after travelling $n$ kilometres.
   d Find the distance travelled for the van to depreciate by $13800.
   e Find its future value after it has travelled 160000 kilometres.
   f Set up a schedule table for the value of the van for every 20000 kilometres.

8 A car bought for $28395 depreciates at a rate of 23.6 cents for every km travelled. Copy and complete the table below.

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>Distance travelled, $n$ (km)</th>
<th>Depreciation ($)</th>
<th>Future value at end of year, $V_n$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13290</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>15650</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>14175</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>9674</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>16588</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9 Below are depreciation details for 2 vehicles. In each case find:
   i the annual depreciation in the first year
   ii the useful life (km).

<table>
<thead>
<tr>
<th>Purchase price ($)</th>
<th>Scrap value ($)</th>
<th>Rate of depreciation (cents/km)</th>
<th>Distance travelled in first year (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a 25000</td>
<td>10000</td>
<td>26</td>
<td>12600</td>
</tr>
<tr>
<td>b 21400</td>
<td>8000</td>
<td>21.6</td>
<td>13700</td>
</tr>
</tbody>
</table>

In each situation in questions 10 to 12, find:
   a the annual depreciation in the first year
   b the item’s useful life.
10 A company buys a $32,000 car which depreciates at a rate of 23 cents per km driven. It covers 15,340 km in the first year and has a scrap value of $9,500.

11 A new taxi is worth $29,500 and it depreciates at 27.2 cents per km travelled. In its first year of use it travelled 28,461 km. Its scrap value was $8,200.

12 A photocopier purchased for $7,200 depreciates at a rate of $1.50 per 1000 copies made. In its first year of use, 620,000 copies were made and in its second year, 540,000 were made. Find:
   a the depreciation for each year
   b the future value at the end of the second year.

13 A photocopier bought for $11,300 depreciates at a rate of 2.5 cents for every 10 copies made.
   a Set up a recurrence relation to represent the depreciation.
   b Copy and complete the table below.

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>Outputs, $n$ (10 copies)</th>
<th>Annual depreciation ($)</th>
<th>Future value at end of year, $V_n$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>42,500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>37,620</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>29,104</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>38,562</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

14 A corking machine bought for $14,750 depreciates at a rate of $2.50 for every 100 bottles corked.
   a Set up a recurrence relation to represent the depreciation.
   b Copy and complete the table below.

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>Outputs, $n$ (100 bottles corked)</th>
<th>Depreciation ($)</th>
<th>Future value at end of year, $V_n$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>425</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>467</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>382.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>430.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

15 A vehicle is bought for $25,900 and it depreciates at a rate of 21.6 cents per km driven. After its first year of use, in which it travels 13,690 km, the future value of the vehicle is closest to:
   A $1000   B $3000   C $20,000   D $23,000   E $25,000

In each situation in questions 16 and 17, find:
   a the depreciation for each year
   b the future value at the end of the second year.
16 A company van is purchased for $32600 and it depreciates at a rate of 24.8 cents per km driven. In its first year of use the van travels 15620 km and it travels 16045 km in its second year.

17 A taxi is bought for $35099 and it depreciates at a rate of 29.2 cents per km driven. It travels 21216 km in its first year of use and 19950 km in its second year.

18 A car is bought for $35000 and a scrap value of $10000 is set for it. The following three options for depreciating the car are available:
   i flat rate of 10% of the purchase price each year
   ii 20% p.a. of the reducing balance
   iii 25 cents per km driven (the car travels an average of 10000 km per year).

   a Which method will enable the car to reach its scrap value soonest?
   b If the car is used in a business the annual depreciation can be claimed as a tax deduction. What would the tax deduction be in the first year of use for each of the depreciation methods?
   c How would your answers to part b vary for the 5th year of use?

19 A machine which was bought for $8500 was depreciated at the rate of 2 cents per unit produced. By the time the book value had decreased to $2000, the number of units produced would be:

   A 75000
   B 100000
   C 125000
   D 300000
   E 325000

20 An $8500 machine depreciates by 2 cents per unit. By the time the machine had depreciated by $5000, it would have produced:

   A 275000 units
   B 250000 units
   C 225000 units
   D 175000 units
   E 150000 units
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The Review contains:
- **Multiple-choice** questions — providing you with the opportunity to practise answering questions using CAS technology
- **Short-answer** questions — providing you with the opportunity to demonstrate the skills you have developed to efficiently answer questions using the most appropriate methods
- **Extended-response** questions — providing you with the opportunity to practise exam-style questions.

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**studyON**

studyON is an interactive and highly visual online tool that helps you to clearly identify strengths and weaknesses prior to your exams. You can then confidently target areas of greatest need, enabling you to achieve your best results.
6 Answers

EXERCISE 6.2
1 a $V_{n+1} = V_n + 86.7, V_0 = 1020$
  b $\$1106.70, \$1193.40, \$1280.10, \$1366.80$ and $\$1453.50$
2 a $V_{n+1} = V_n + 48.13, V_0 = 713$
  b $\$761.13, \$809.26, \$857.39, \$905.52$ and $\$953.65$
3 a $\$1275$

EXERCISE 6.3
1 A = 6
  B = 7950
  C = 7950
  D = 8427
  E = 9468.58
  F = 9468.58
2 A = 7.5
  B = 3493.75
  C = 3493.75
  D = 3755.78
  E = 4340.27
  F = 4340.27
3 C
4 D
5 C
6 D
7 E
8 A = 8%
  B = 6210
  C = 6210
  D = 6706.80
  E = 7882.81
  F = 7882.81
9 A = 967.50
  B = 13867.50
  C = 13867.50
  D = 16025.63
  E = 16025.63
  F = 17227.55

EXERCISE 6.4

<table>
<thead>
<tr>
<th>Time period</th>
<th>Principal, P ($)</th>
<th>Interest ($)</th>
<th>Balance ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5500</td>
<td>357.50</td>
<td>5857.50</td>
</tr>
<tr>
<td>2</td>
<td>5857.50</td>
<td>380.74</td>
<td>6238.24</td>
</tr>
<tr>
<td>3</td>
<td>6238.24</td>
<td>405.49</td>
<td>6643.72</td>
</tr>
<tr>
<td>4</td>
<td>6643.72</td>
<td>431.84</td>
<td>7075.56</td>
</tr>
<tr>
<td>5</td>
<td>7075.56</td>
<td>459.91</td>
<td>7535.47</td>
</tr>
</tbody>
</table>

15 a The Big-4 Bank offers the better rate.
  b The Big-4 Bank charges 11\% p.a. for a loan while The Friendly Building Society charges 12\% (= 12 \times 1\% per month).

16 $\$3070$
17 10\%
18 1 year
Despite it being difficult to see on the graph, the graph is exponential.

<table>
<thead>
<tr>
<th>Time period</th>
<th>Balance ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15637.50</td>
</tr>
<tr>
<td>2</td>
<td>16302.09</td>
</tr>
<tr>
<td>3</td>
<td>16994.93</td>
</tr>
<tr>
<td>4</td>
<td>17717.22</td>
</tr>
<tr>
<td>5</td>
<td>18470.20</td>
</tr>
</tbody>
</table>

Despite it being difficult to see on the graph, the graph is exponential.

<table>
<thead>
<tr>
<th>Time period</th>
<th>Balance ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26625</td>
</tr>
<tr>
<td>2</td>
<td>28355.63</td>
</tr>
<tr>
<td>3</td>
<td>30198.74</td>
</tr>
<tr>
<td>4</td>
<td>32161.66</td>
</tr>
<tr>
<td>5</td>
<td>34252.17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time period</th>
<th>Balance ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10509.45</td>
</tr>
<tr>
<td>2</td>
<td>11044.86</td>
</tr>
<tr>
<td>3</td>
<td>11607.55</td>
</tr>
<tr>
<td>4</td>
<td>12198.90</td>
</tr>
<tr>
<td>5</td>
<td>12820.37</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time period</th>
<th>Balance ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9159.88</td>
</tr>
<tr>
<td>2</td>
<td>9870.98</td>
</tr>
<tr>
<td>3</td>
<td>10637.29</td>
</tr>
<tr>
<td>4</td>
<td>11463.09</td>
</tr>
<tr>
<td>5</td>
<td>12353.00</td>
</tr>
</tbody>
</table>

**EXERCISE 6.4**

1. $3589.07
2. $9657.36
3. a 12
   b 1.75%
   c $5172.05
   d

4. a 24
   b 0.4583%

5. $3448.40
6. $4290.73
7. a $583.20
   b 0.4583%
   c $4472.27
   d $3764.86

8. a i $2150
   b i $2311.25
   c i $3086.60
   d
9. a 5
   b 20
   c 8
   d 72
   e 9
   f 15

10. a 1.5%
    b 2%
    c 1.5%
    d 1.75%

11. a $1749.60
    b $1757.49
    c $1759.33
    d $1760.24

12. a $2852.92
    b $4984.72
    c $6000

13. a $605.60
    b $1314.84
    c $795.77
    d $1043.10
    e $1641.82

14. E

15. B

16. a $3542.13
    b $2052.54
    c $2969.18
    d $3764.86

17. a $2069.61
    b $1759.33
    c $1757.49
    d $1760.24

18. a $930.39
    b $468.67
    c $2035.90

**EXERCISE 6.5**

1. 11.31%
2. 9.63%
3. Approx. 6 years
4. Approx. 5 years
5. Approx. 3 years
6. Approx. 3.5 years
7 B
8 a 13.98%  b 5.62%
9 a 18.95%  b 7.41%  c 16.59%
10 8.34%, that is, D
11 a 6 years  b 10 years
12 a 3 years  b 5 years
13 a 19, 9\(\frac{1}{2}\) years  b 20, 5 years  c 57, 4\(\frac{3}{4}\) years
14 a 145, 5 years 15 fortnights  b 13, 3\(\frac{1}{2}\) years  c 32, 8 years
15 13, 3\(\frac{1}{2}\) years
16 B
17 a \(n = 30, 2\frac{1}{2}\) years  b \(n = 119, 4\) years 15 fortnights
18 a 4 years  b 9718.11

EXERCISE 6.6
1 a $3750  
   b \(V_{n+1} = V_n - 3750\)
   c 

2 a \(V_n = 30000 - 6000n\)
   b \(V_n = 2000 - 200n\)
   c \(V_n = 6000 - 500n\)
3 a i $5000 per year  ii 10 years old
   b i $100 per year  ii 8.5 years old
   c i $95 per year  ii 13 years old
4 a \(V_n = 22500 - 3200n\)
   b $6500
5 a $1500
6 a 

b \(V_n = 45000 - 4000n\), scrap value is $5000

EXERCISE 6.7
1 a 

\begin{array}{|c|}
\hline
\text{Time, } n \text{ (years)} & \text{Future value, } V_n \text{ ($)} \\
\hline
0 & 1500 \\
1 & 1245.00 \\
2 & 1033.35 \\
3 & 857.68 \\
4 & 711.87 \\
5 & 590.86 \\
\hline
\end{array}

b $590.86  
   c 

12 B
13 6
14 E
15 a The cheaper machine  b 2 years
16 B
2 a

<table>
<thead>
<tr>
<th>Time, (n) (years)</th>
<th>Future value, (V_n) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3300.00</td>
</tr>
<tr>
<td>1</td>
<td>2871.00</td>
</tr>
<tr>
<td>2</td>
<td>2497.77</td>
</tr>
<tr>
<td>3</td>
<td>2173.06</td>
</tr>
<tr>
<td>4</td>
<td>1890.56</td>
</tr>
<tr>
<td>5</td>
<td>1644.79</td>
</tr>
</tbody>
</table>

b $1644.79

c

The future value for the reducing balance method is greater than the flat rate method by 4 years.

3 a

<table>
<thead>
<tr>
<th>Time, (n) (years)</th>
<th>Depreciation ($)</th>
<th>Future value, (V_n) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>—</td>
<td>29990</td>
</tr>
<tr>
<td>1</td>
<td>5998</td>
<td>23992</td>
</tr>
<tr>
<td>2</td>
<td>5998</td>
<td>17994</td>
</tr>
<tr>
<td>3</td>
<td>5998</td>
<td>11996</td>
</tr>
<tr>
<td>4</td>
<td>5998</td>
<td>5998</td>
</tr>
<tr>
<td>5</td>
<td>5998</td>
<td>0</td>
</tr>
</tbody>
</table>

b

c

The future value for the reducing balance method is greater than the flat rate method by 4 years.

4 a

<table>
<thead>
<tr>
<th>Time, (n) (years)</th>
<th>Depreciation ($)</th>
<th>Future value, (V_n) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-</td>
<td>23000</td>
</tr>
<tr>
<td>1</td>
<td>4600</td>
<td>18400</td>
</tr>
<tr>
<td>2</td>
<td>4600</td>
<td>13800</td>
</tr>
<tr>
<td>3</td>
<td>4600</td>
<td>9200</td>
</tr>
<tr>
<td>4</td>
<td>4600</td>
<td>4600</td>
</tr>
<tr>
<td>5</td>
<td>4600</td>
<td>0</td>
</tr>
</tbody>
</table>

b

c

The future value for the reducing balance method is greater than the flat rate method by 4 years.

5 $16971.73

6 $747.88

7 5.59

8 9.98

9 a \(V_{n+1} = 0.8V_n\)

b

<table>
<thead>
<tr>
<th>Time, (n) (years)</th>
<th>Future value, (V_n) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>60000.00</td>
</tr>
<tr>
<td>1</td>
<td>48000.00</td>
</tr>
<tr>
<td>2</td>
<td>38400.00</td>
</tr>
<tr>
<td>3</td>
<td>30720.00</td>
</tr>
<tr>
<td>4</td>
<td>24576.00</td>
</tr>
<tr>
<td>5</td>
<td>19660.80</td>
</tr>
</tbody>
</table>

c $19660.80
10 a \[ V_{n+1} = 0.6V_n \]

b

<table>
<thead>
<tr>
<th>Time, ( n ) (years)</th>
<th>Depreciation ($)</th>
<th>Future value, ( V_n ) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>—</td>
<td>4000.00</td>
</tr>
<tr>
<td>1</td>
<td>1600</td>
<td>2400.00</td>
</tr>
<tr>
<td>2</td>
<td>960</td>
<td>1440.00</td>
</tr>
<tr>
<td>3</td>
<td>576</td>
<td>864.00</td>
</tr>
<tr>
<td>4</td>
<td>345.60</td>
<td>518.40</td>
</tr>
<tr>
<td>5</td>
<td>207.36</td>
<td>311.04</td>
</tr>
</tbody>
</table>

c $311.04

d

11 a

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>Flat rate</th>
<th>Reducing balance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>—</td>
<td>550</td>
</tr>
<tr>
<td>1</td>
<td>110</td>
<td>440</td>
</tr>
<tr>
<td>2</td>
<td>110</td>
<td>330</td>
</tr>
<tr>
<td>3</td>
<td>110</td>
<td>220</td>
</tr>
<tr>
<td>4</td>
<td>110</td>
<td>110</td>
</tr>
<tr>
<td>5</td>
<td>110</td>
<td>0</td>
</tr>
</tbody>
</table>

b

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>Dep. ($)</th>
<th>Future value ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>—</td>
<td>550</td>
</tr>
<tr>
<td>1</td>
<td>115.50</td>
<td>269.50</td>
</tr>
<tr>
<td>2</td>
<td>80.85</td>
<td>188.65</td>
</tr>
<tr>
<td>3</td>
<td>56.60</td>
<td>132.05</td>
</tr>
<tr>
<td>4</td>
<td>39.62</td>
<td>92.43</td>
</tr>
</tbody>
</table>

c 4 years

d

12 a

<table>
<thead>
<tr>
<th>Flat rate</th>
<th>Reducing balance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Time (years)</td>
<td>Dep. ($)</td>
</tr>
<tr>
<td>0</td>
<td>—</td>
</tr>
<tr>
<td>1</td>
<td>10000</td>
</tr>
<tr>
<td>2</td>
<td>10000</td>
</tr>
<tr>
<td>3</td>
<td>10000</td>
</tr>
</tbody>
</table>

c 3 years

d

EXERCISE 6.8

1 a \[ V_{n+1} = V_n - 0.35n \]

b

<table>
<thead>
<tr>
<th>Number of washes (( n ))</th>
<th>Future value, ( V_n ) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>799.65</td>
</tr>
<tr>
<td>2</td>
<td>799.30</td>
</tr>
<tr>
<td>3</td>
<td>798.95</td>
</tr>
<tr>
<td>4</td>
<td>798.60</td>
</tr>
<tr>
<td>5</td>
<td>798.25</td>
</tr>
</tbody>
</table>

c 4 years

2 a \[ V_{n+1} = V_n - 0.65n \]

b

<table>
<thead>
<tr>
<th>Hours of use (( n ))</th>
<th>Future value, ( V_n ) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1399.35</td>
</tr>
<tr>
<td>2</td>
<td>1398.70</td>
</tr>
<tr>
<td>3</td>
<td>1398.05</td>
</tr>
<tr>
<td>4</td>
<td>1397.40</td>
</tr>
<tr>
<td>5</td>
<td>1396.75</td>
</tr>
</tbody>
</table>
3 a $3990
   b 61754 km
4 a $16000, $14000
   b 80000 km
5 a $880, $1056
   b $6664
6 a $577.50, $744
   b $36678.50
7 a 15 cents per km
   b 2 years 21 weeks
   c $V_n = 30000 − 0.15n$
   d 92000 km
   e $6000

<table>
<thead>
<tr>
<th>Distance, $n$ (km)</th>
<th>Value ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30000</td>
</tr>
<tr>
<td>20000</td>
<td>27000</td>
</tr>
<tr>
<td>40000</td>
<td>24000</td>
</tr>
<tr>
<td>60000</td>
<td>21000</td>
</tr>
<tr>
<td>80000</td>
<td>18000</td>
</tr>
<tr>
<td>100000</td>
<td>15000</td>
</tr>
<tr>
<td>120000</td>
<td>12000</td>
</tr>
<tr>
<td>140000</td>
<td>9000</td>
</tr>
<tr>
<td>160000</td>
<td>6000</td>
</tr>
<tr>
<td>180000</td>
<td>3000</td>
</tr>
<tr>
<td>200000</td>
<td>0</td>
</tr>
</tbody>
</table>

8

<table>
<thead>
<tr>
<th>Depreciation ($)</th>
<th>Future value at end of year, $V_n$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3136.44</td>
<td>25258.56</td>
</tr>
<tr>
<td>3693.40</td>
<td>21565.16</td>
</tr>
<tr>
<td>3345.30</td>
<td>18219.86</td>
</tr>
<tr>
<td>2283.06</td>
<td>15936.80</td>
</tr>
<tr>
<td>3914.77</td>
<td>12022.03</td>
</tr>
</tbody>
</table>

9 a i $3276
   ii 57692 km
   b i $2959.20
      ii 62037 km
10 a $3528.20
    b 97826 km

11 a $7741.39
    b 78309 km
12 a $930, $810
    b $5460
13 a $V_{n+1} = V_n − 0.025$

<table>
<thead>
<tr>
<th>Annual depreciation ($)</th>
<th>Future value at end of year, $V_n$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>875</td>
<td>10425</td>
</tr>
<tr>
<td>1062.50</td>
<td>9362.50</td>
</tr>
<tr>
<td>940.50</td>
<td>8422</td>
</tr>
<tr>
<td>727.60</td>
<td>7694.40</td>
</tr>
<tr>
<td>964.05</td>
<td>6730.35</td>
</tr>
</tbody>
</table>

14 a $V_{n+1} = V_n − 2.5$

<table>
<thead>
<tr>
<th>Depreciation ($)</th>
<th>Future value at end of year, $V_n$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>13750</td>
</tr>
<tr>
<td>1062.50</td>
<td>12687.50</td>
</tr>
<tr>
<td>1167.50</td>
<td>11520</td>
</tr>
<tr>
<td>956.25</td>
<td>10563.75</td>
</tr>
<tr>
<td>1076.50</td>
<td>9487.25</td>
</tr>
</tbody>
</table>

15 D
16 a $3873.76, $3979.16
    b $24747.08
17 a $6195.07, $5825.40
    b $23078.53
18 a Reducing balance depreciation (6 years compared to 7+ years and 10 years)
    b i $3500
       ii $7000
       iii $2500
    c i $3500 (same)
       ii $2867.20 ($4132.80 less)
       iii $2500 (same)
19 E
20 B