

7

Shape and measurement

- 7.1 Kick off with CAS
- 7.2 Pythagoras' theorem
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7.1 Kick off with CAS

Exploring area and volume with CAS

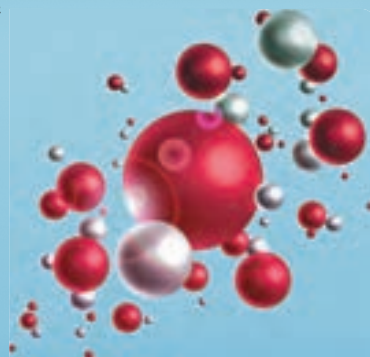
The area of a circle is defined by the formula $A = \pi r^2$, where r is the length of the radius of the circle.

Areas of circles and other shapes will be studied in more detail in this topic.

- 1 Using CAS, define and save the formula for the area of a circle.
- 2 Use your formula to calculate the areas of circles with the following radii.
 - a $r = 3$
 - b $r = 7$
 - c $r = 12$
 - d $r = 15$
- 3 Using CAS, sketch a graph plotting the area of a circle (y-axis) against the radius of a circle (x-axis).

The volume of a sphere is defined by the formula $V = \frac{4\pi r^3}{3}$.

- 4 Using CAS, calculate the volumes of spheres with the following radii.
 - a $r = 3$
 - b $r = 7$
 - c $r = 12$
 - d $r = 15$
- 5 Using CAS, sketch a graph plotting the volume of a sphere (y-axis) against the radius of a sphere (x-axis).
- 6 Comment on the differences and similarities between the two graphs you plotted in questions 3 and 5.



7.2 Pythagoras' theorem

Review of Pythagoras' theorem

study on

Unit 1

AOS 4

Topic 7

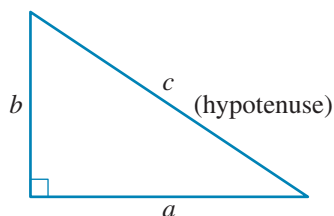
Concept 1

Pythagoras' theorem

Summary screen and practice questions

Even though the theorem that describes the relationship between the side lengths of right-angled triangles bears the name of the famous Greek mathematician Pythagoras, who is thought to have lived around 550 BC, evidence exists in some of humanity's earliest relics that it was known and used much earlier than that.

The side lengths of any right-angled triangle are related according to the rule $a^2 + b^2 = c^2$, where c represents the hypotenuse (the longest side), and a and b represent the other two side lengths.



The hypotenuse is always the side length that is opposite the right angle. Pythagoras' theorem can be used to find an unknown side length when any two side lengths are known.

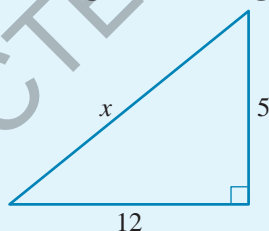


Statue of Pythagoras on the island of Samos

WORKED EXAMPLE 1

1

Find the unknown side length in the right-angled triangle shown.



THINK

- 1 Identify that the triangle is right-angled so Pythagoras' theorem can be applied.
- 2 Identify which of the side lengths is the hypotenuse.
- 3 Substitute the known values into the theorem and simplify.
- 4 Take the square root of both sides to obtain the value of x .

WRITE

$$a^2 + b^2 = c^2$$

x is opposite the right angle, so it is the hypotenuse; $a = 12$ and $b = 5$.

$$12^2 + 5^2 = x^2$$

$$144 + 25 = x^2$$

$$169 = x^2$$

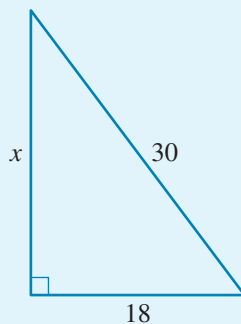
$$\sqrt{169} = x$$

$$13 = x$$

If the length of the hypotenuse and one other side length are known, the other side length can be found by subtracting the square of the known side length from the square of the hypotenuse.

WORKED
EXAMPLE 2

Find the length of the unknown side in the right-angled triangle shown.



THINK

- 1 Identify that the triangle is right-angled so Pythagoras' theorem can be applied.
- 2 Identify which of the side lengths is the hypotenuse.
- 3 Substitute the known values into the theorem and simplify.
- 4 Take the square root of both sides to obtain the value of x .

WRITE

$$a^2 + b^2 = c^2$$

30 is opposite the right angle, so it is the hypotenuse; $a = x$ and $b = 18$.

$$\begin{aligned} x^2 + 18^2 &= 30^2 \\ x^2 &= 30^2 - 18^2 \\ &= 576 \\ x &= \sqrt{576} \\ &= 24 \end{aligned}$$

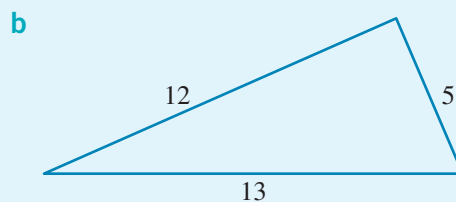
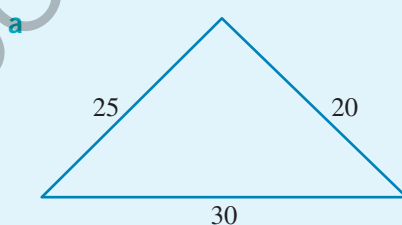
Pythagorean triads

Any group of three numbers that satisfies Pythagoras' theorem is referred to as a *Pythagorean triad*. For example, because $6^2 + 8^2 = 10^2$, the numbers 6, 8 and 10 form a Pythagorean triad.

Demonstrating that three numbers form a Pythagorean triad is a way of showing that a triangle with these side lengths is right-angled.

WORKED
EXAMPLE 3

Demonstrate whether the given triangles are right-angled.



THINK

- a
- 1 Write the rule.
 - 2 Substitute the two smaller numbers into the left-hand side of the rule and evaluate it.
 - 3 Substitute the largest number into the right-hand side of the rule and evaluate it.

WRITE

a For any right-angled triangle, $a^2 + b^2 = c^2$.

$$\begin{aligned} a^2 + b^2 &= 20^2 + 25^2 \\ &= 1025 \\ c^2 &= 30^2 \\ &= 900 \end{aligned}$$

4 Identify whether $a^2 + b^2 = c^2$.

$$a^2 + b^2 = 1025 \text{ and } c^2 = 900$$

Therefore, $a^2 + b^2 \neq c^2$.

5 State the final answer.

As $a^2 + b^2 \neq c^2$, this is not a right-angled triangle.

b 1 Write the rule.

b For any right-angled triangle, $a^2 + b^2 = c^2$.

2 Substitute the two smaller numbers into the left-hand side of the rule and evaluate it.

$$\begin{aligned} a^2 + b^2 &= 12^2 + 5^2 \\ &= 169 \end{aligned}$$

3 Substitute the largest number into the right-hand side of the rule and evaluate it.

$$\begin{aligned} c^2 &= 13^2 \\ &= 169 \end{aligned}$$

4 Identify whether $a^2 + b^2 = c^2$.

$$a^2 + b^2 = 169 \text{ and } c^2 = 169$$

Therefore, $a^2 + b^2 = c^2$.

5 State the final answer.

As $a^2 + b^2 = c^2$, this is a right-angled triangle.

Pythagoras' theorem in three dimensions

Many three-dimensional objects contain right-angled triangles that can be modelled with two-dimensional drawings. Using this method we can calculate missing side lengths of three-dimensional objects.

WORKED
EXAMPLE

4

Calculate the maximum length of a metal rod that would fit into a rectangular crate with dimensions 1 m \times 1.5 m \times 0.5 m.

THINK

1 Draw a diagram of a rectangular box with a rod in it, labelling the dimensions.

2 • Draw in a right-angled triangle that has the metal rod as one of the sides, as shown in pink.

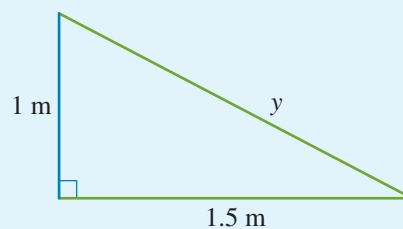
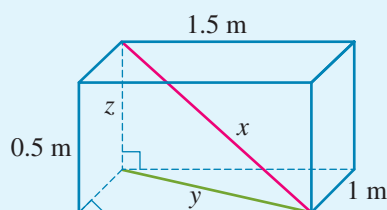
The length of y in this right-angled triangle is not known.

• Draw in another right-angled triangle, as shown in green, to calculate the length of y .

3 • Calculate the length of y using Pythagoras' theorem.

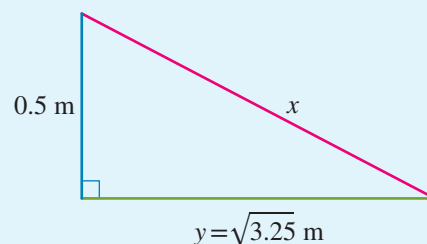
• Calculate the exact value of y .

WRITE/DRAW



$$\begin{aligned} c^2 &= a^2 + b^2 \\ y^2 &= 1.5^2 + 1^2 \\ &= 3.25 \\ y &= \sqrt{3.25} \end{aligned}$$

- 4 Draw the right-angled triangle containing the rod and use Pythagoras' theorem to calculate the length of the rod (x).



$$\begin{aligned} c^2 &= a^2 + b^2 \\ x^2 &= (\sqrt{3.25})^2 + 0.5^2 \\ &= 3.25 + 0.25 \\ &= 3.5 \\ x &= \sqrt{3.5} \\ &\approx 1.87 \end{aligned}$$

- 5 Answer the question.

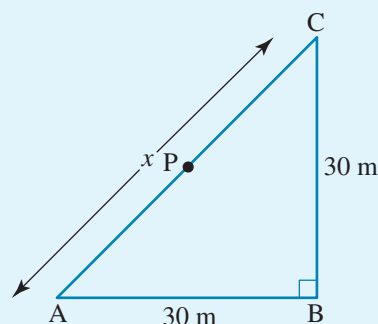
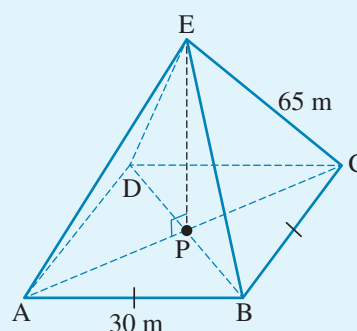
The maximum length of the metal rod is 1.87 m (correct to 2 decimal places).

WORKED EXAMPLE 5 A square pyramid has a base length of 30 metres and a slant height of 65 metres. Determine the height of the pyramid, giving your answer correct to 1 decimal place.

THINK

- 1 Draw a diagram to represent the situation. Add a point in the centre of the diagram below the apex of the pyramid.
- 2 Determine the diagonal distance across the base of the pyramid by using Pythagoras' theorem.

WRITE/DRAW

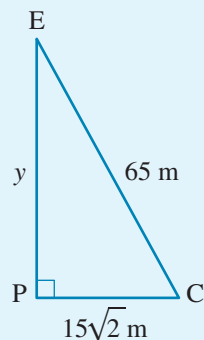


$$\begin{aligned} c^2 &= a^2 + b^2 \\ x^2 &= 30^2 + 30^2 \\ &= 900 + 900 \\ &= 1800 \\ x &= \sqrt{1800} \\ &= \sqrt{900 \times 2} \\ &= 30\sqrt{2} \end{aligned}$$

- 3 Calculate the distance from one of the corners on the base of the pyramid to the centre of the base of the pyramid.

$$\begin{aligned} AP &= \frac{1}{2}AC \\ &= \frac{1}{2} \times 30\sqrt{2} \\ &= 15\sqrt{2} \end{aligned}$$

- 4 Draw the triangle that contains the height of the pyramid and the distance from one of the corners on the base of the pyramid to the centre of the base of the pyramid.



- 5 Use Pythagoras' theorem to calculate the height of the pyramid, rounding your answer to 1 decimal place.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 65^2 &= y^2 + (15\sqrt{2})^2 \\ 4225 &= y^2 + 450 \\ y^2 &= 4225 - 450 \\ &= 3775 \\ y &= \sqrt{3775} \\ &= 61.4 \end{aligned}$$

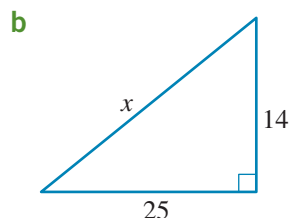
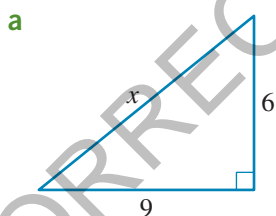
- 6 State the answer.

The height of the pyramid is 61.4 metres.

EXERCISE 7.2 Pythagoras' theorem

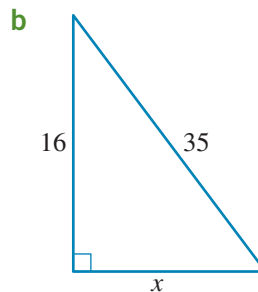
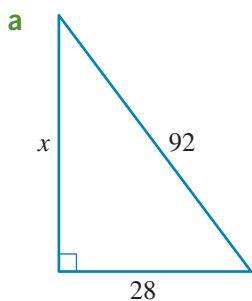
PRACTISE

- 1 **WE1** Find the unknown side length in the right-angled triangles shown, giving your answers correct to 1 decimal place.



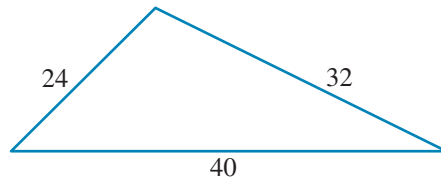
- 2 Show that a right-angled triangle with side lengths of 40 cm and 96 cm will have a hypotenuse of length 104 cm.

- 3 **WE2** Find the length of the unknown side in the right-angled triangles shown, giving your answers correct to 1 decimal place.

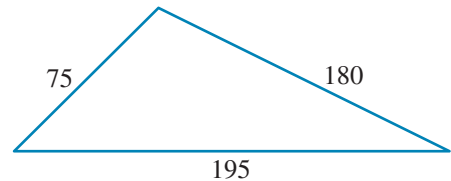


- 4 Show that if a right-angled triangle has a hypotenuse of 24 cm and a side length of 19.2 cm, the third side length will be 14.4 cm.
- 5 **WE3** Demonstrate whether the given triangles are right-angled.

a



b



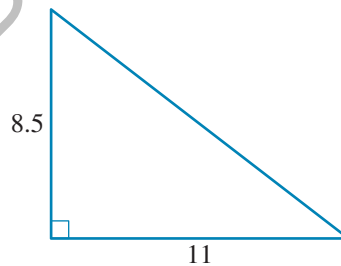
- 6 Demonstrate whether a triangle with side lengths of 1.5 cm, 2 cm and 2.5 cm is right-angled.
- 7 **WE4** Calculate the maximum length of a metal rod that would fit into a rectangular crate with dimensions 1.2 m \times 83 cm \times 55 cm.
- 8 Determine whether a metal rod of length 2.8 metres would be able to fit into a rectangular crate with dimensions 2.3 m \times 1.2 m \times 0.8 m.
- 9 **WE5** A square pyramid has a base length of 25 metres and a slant height of 45 metres. Determine the height of the pyramid, giving your answer correct to 1 decimal place.
- 10 Determine which of the following square pyramids has the greatest height.
 Pyramid 1: base length of 18 metres and slant height of 30 metres
 Pyramid 2: base length of 22 metres and slant height of 28 metres
- 11 Evaluate the unknown side lengths in the right-angled triangles shown, giving your answers correct to 2 decimal places.

CONSOLIDATE

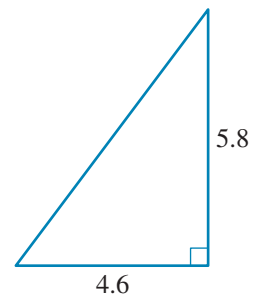
a



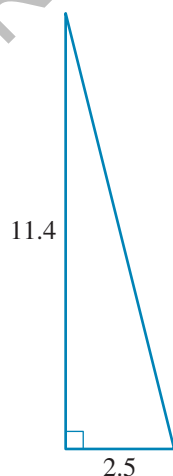
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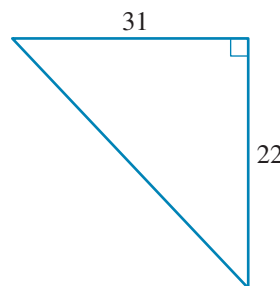
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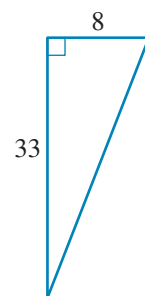
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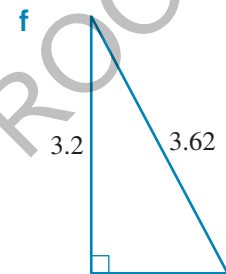
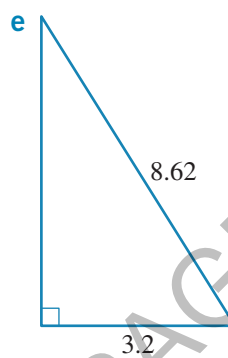
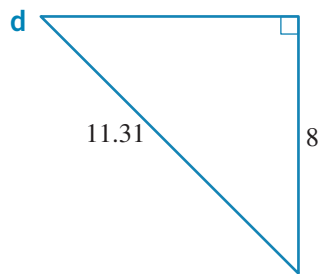
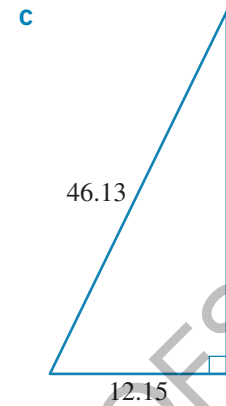
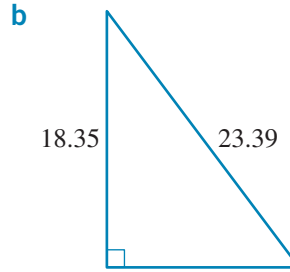
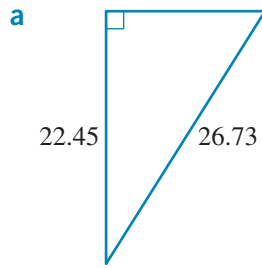
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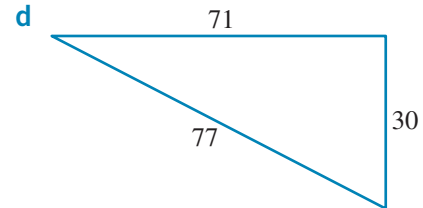
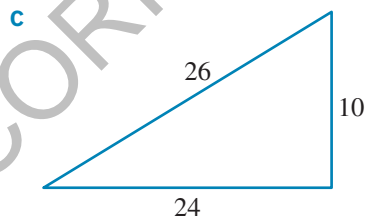
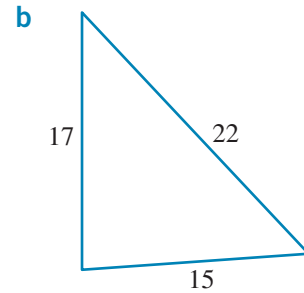
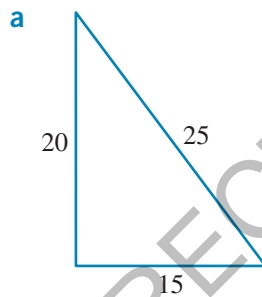
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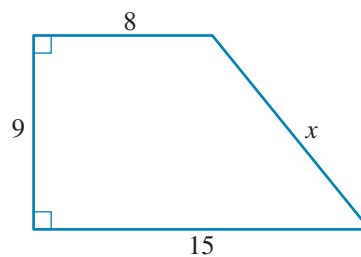
- 12** Evaluate the length of the unknown side in each of the right-angled triangles shown. Give your answers correct to 2 decimal places.



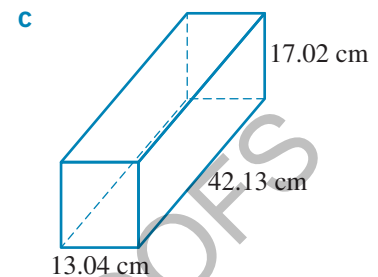
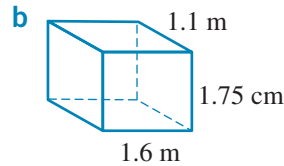
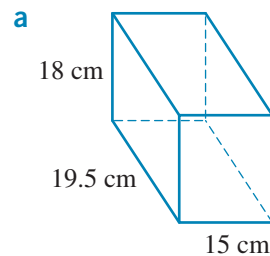
- 13** Demonstrate whether the given triangles are right-angled.



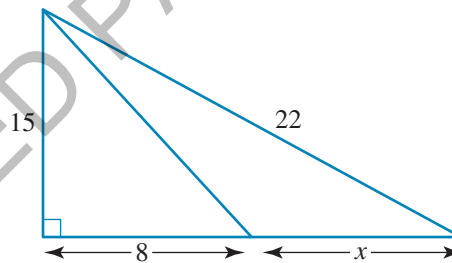
- 14** Calculate the length of the unknown side in the following diagram, giving your answers correct to 2 decimal places.



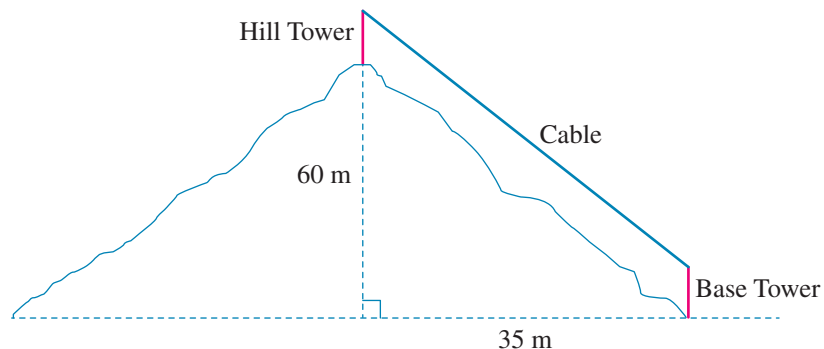
- 15** Find two possible values for x that would produce a Pythagorean triad with the two numbers listed. Give your answers correct to 2 decimal places.
- a** $x, 5, 8$ **b** $x, 64, 36$ **c** $x, 15, 21$
d $x, 33, 34$ **e** $x, 6, 10$ **f** $x, 15, 36$
- 16** Calculate the length of the longest metal rod that can fit diagonally into the boxes shown below.



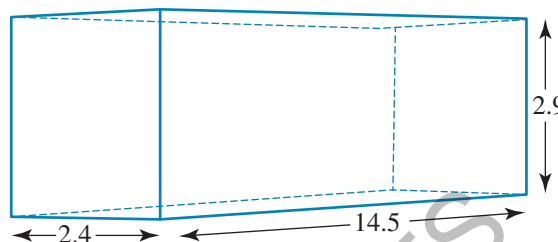
- 17** Calculate, correct to 2 decimal places, the lengths of the shorter sides of right-angled isosceles triangles that have a hypotenuse of length:
- a** 20 cm **b** 48 cm **c** 5.5 cm **d** 166 cm.
- 18** A friend wants to pack an umbrella into her suitcase. If the suitcase measures $89 \text{ cm} \times 21 \text{ cm} \times 44 \text{ cm}$, will her 1-m umbrella fit in? Give the length of the longest object that will fit in the suitcase.
- 19** Find the value of x , correct to 2 decimal places, in the following diagram.



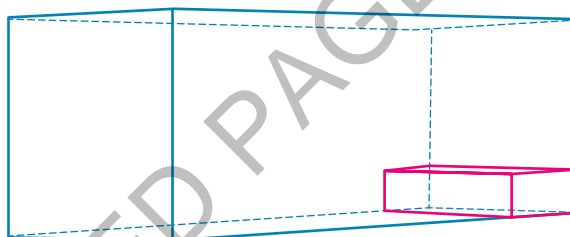
- 20** A cable joins the top of two vertical towers that are 12 metres high. One of the towers is at the bottom of a hill and the other is at the top. The horizontal distance between the towers is 35 metres and the vertical height of the base of the upper tower is 60 metres above ground level. What is the minimum length of cable required to join the top of the towers? Give your answer correct to 2 decimal places.



- 21 A semi-trailer carries a container that has the following internal dimensions: length 14.5 m, width 2.4 m and height 2.9 m. Give your answers to the following questions correct to 2 decimal places.



- Calculate the length of the longest object that can be placed on the floor of the container.
- Calculate the length of the longest object that can be placed in the container if only one end is placed on the floor.
- If a rectangular box with length 2.4 m, width 1.2 m and height 0.8 m is placed on the floor at one end so that it fits across the width of the container, calculate the length of the longest object that can now be placed inside if it touches the floor adjacent to the box.



- 22 An ultralight aircraft is flying at an altitude of 1000 metres and a horizontal distance of 10 kilometres from its landing point.

- If the aircraft travels in a straight line from its current position to its landing point, how far does it travel correct to the nearest metre? (Assume the ground is level.)
- If the aircraft maintained the same altitude for a further 4 kilometres, what would be the straight-line distance from the new position to the same landing point, correct to the nearest metre?
- From the original starting point the pilot mistakenly follows a direct line to a point on the ground that is 2.5 kilometres short of the correct landing point. He realises his mistake when he is at an altitude of 400 metres and a horizontal distance of 5.5 kilometres from the correct landing point. He then follows a straight-line path to the correct landing point. Calculate the total distance travelled by the aircraft from its starting point to the correct landing point, correct to the nearest metre.



7.3 Perimeter and area I

Units of length and area

study on

Unit 1

AOS 4

Topic 7

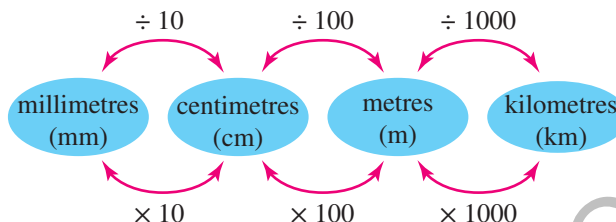
Concept 2

Perimeter and area I

Summary screen and practice questions

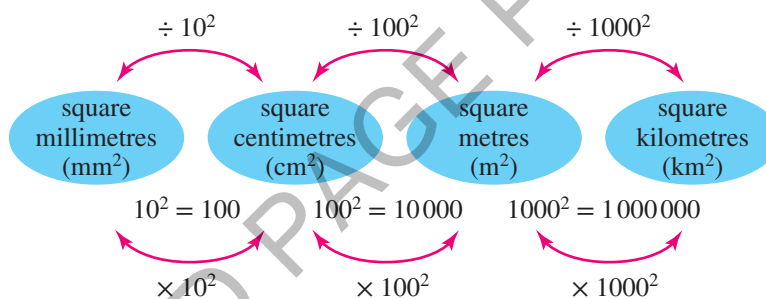
Units of length are used to describe the distance between any two points.

The standard unit of length in the metric system is the metre. The most commonly used units of length are the millimetre (mm), centimetre (cm), metre (m) and kilometre (km). These are related as shown in the following diagram.



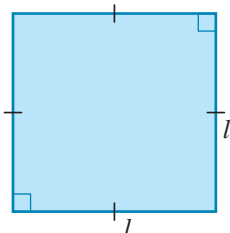
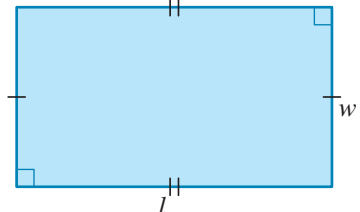
Units of area are named by the side length of the square that encloses that amount of space. For example, a square metre is the amount of space enclosed by a square with a side length of 1 metre.

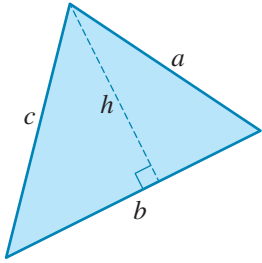
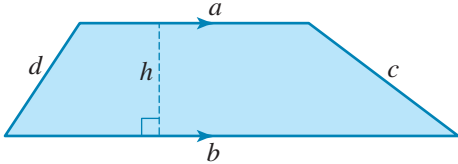
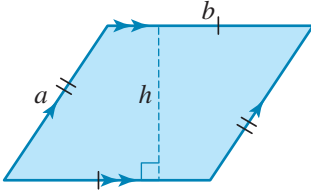
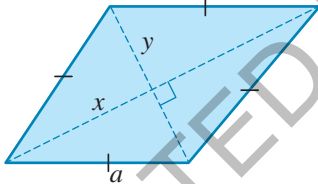
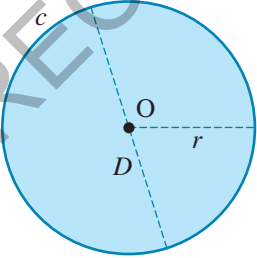
The most common units of area are related as shown in the following diagram:



Perimeter and area of standard shapes

You should now be familiar with the methods and units of measurement used for calculating the perimeter (distance around an object) and area (two-dimensional space taken up by an object) of standard polygons and other shapes. These are summarised in the following table.

Shape	Perimeter and area
<p>Square</p> 	<p>Perimeter: $P = 4l$ Area: $A = l^2$</p>
<p>Rectangle</p> 	<p>Perimeter: $P = 2l + 2w$ Area: $A = lw$</p>

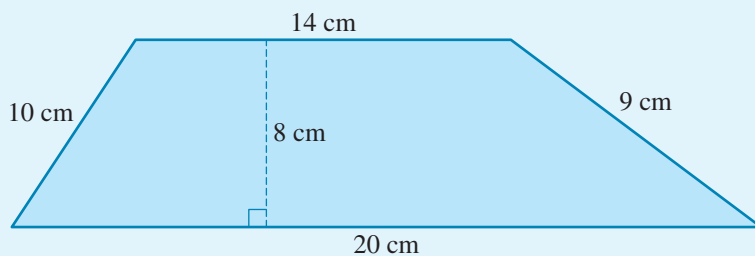
Shape	Perimeter and area
Triangle 	Perimeter: $P = a + b + c$ Area: $A = \frac{1}{2}bh$
Trapezium 	Perimeter: $P = a + b + c + d$ Area: $A = \frac{1}{2}(a + b)h$
Parallelogram 	Perimeter: $P = 2a + 2b$ Area: $A = bh$
Rhombus 	Perimeter: $P = 4a$ Area: $A = \frac{1}{2}xy$
Circle 	Circumference (perimeter): $C = 2\pi r = \pi D$ Area: $A = \pi r^2$

Note: The approximate value of π is 3.14. However, when calculating circumference and area using π , always use the π button on your calculator and make rounding off to the required number of decimal places your final step.

WORKED EXAMPLE

6

Calculate the perimeter and area of the shape shown in the diagram.



THINK

- 1 Identify the shape.
- 2 Identify the components for the perimeter formula and evaluate.
- 3 State the perimeter including the units.
- 4 Identify the components for the area formula and evaluate.
- 5 State the area and give the units.

WRITE

Trapezium

$$P = 10 + 20 + 14 + 9$$

$$= 53$$

$$P = 53 \text{ cm}$$

$$A = \frac{1}{2}(a + b)h$$

$$= \frac{1}{2}(20 + 14)8$$

$$= \frac{1}{2} \times 34 \times 8$$

$$= 136$$

$$A = 136 \text{ cm}^2$$

Heron's formula

Heron's formula is a way of calculating the area of the triangle if you are given all three side lengths. It is named after Hero of Alexandria, who was a Greek engineer and mathematician.

Step 1: Calculate s , the value of half of the perimeter of the triangle:

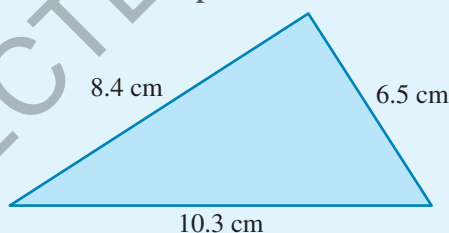
$$s = \frac{a + b + c}{2}$$

Step 2: Use the following formula to calculate the area of the triangle:

$$A = \sqrt{s(s - a)(s - b)(s - c)}$$

**WORKED
EXAMPLE****7**

Use Heron's formula to calculate the area of the following triangle. Give your answer correct to 1 decimal place.

**THINK**

- 1 Calculate the value of s .
- 2 Use Heron's formula to calculate the area of the triangle correct to 1 decimal place.
- 3 State the area and give the units.

WRITE

$$s = \frac{a + b + c}{2}$$

$$= \frac{6.5 + 8.4 + 10.3}{2}$$

$$= \frac{25.2}{2}$$

$$= 12.6$$

$$A = \sqrt{s(s - a)(s - b)(s - c)}$$

$$= \sqrt{12.6(12.6 - 6.5)(12.6 - 8.4)(12.6 - 10.3)}$$

$$= \sqrt{12.6 \times 6.1 \times 4.2 \times 2.3}$$

$$= \sqrt{742.4676}$$

$$= 27.2$$

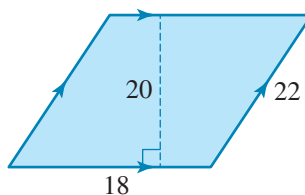
$$A = 27.2 \text{ cm}^2$$

EXERCISE 7.3 Perimeter and area I

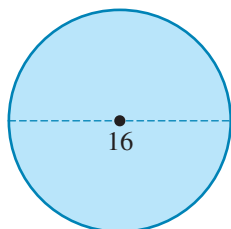
PRACTISE

In the following questions, assume all measurements are in centimetres unless otherwise indicated.

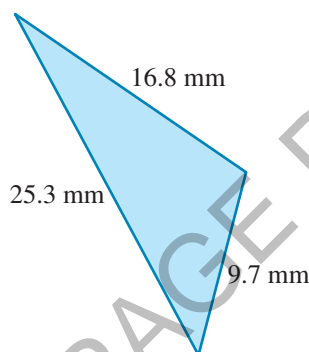
- 1 **WE6** Calculate the perimeter and area of the shape shown in the diagram.



- 2 Calculate the circumference and area of the shape shown in the diagram, giving your final answers correct to 2 decimal places.



- 3 **WE7** Use Heron's formula to calculate the area of the following triangle.



- 4 Use Heron's formula to determine which of the following triangles has the largest area.

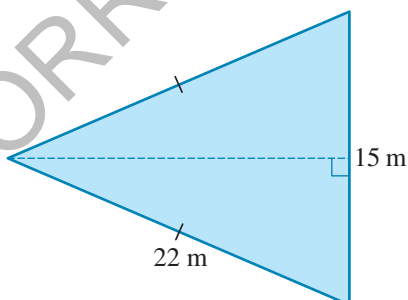
Triangle 1: side lengths of 10.6 cm, 13.5 cm and 16.2 cm

Triangle 2: side lengths of 10.8 cm, 14.2 cm and 24.6 cm

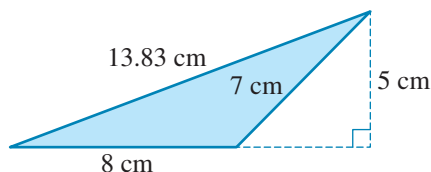
Triangle 3: side lengths of 12.1 cm, 12.6 cm and 12.7 cm

- 5 Calculate the perimeter and area of each of the following shapes, giving answers correct to 2 decimal places where appropriate.

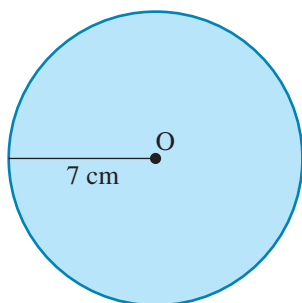
a



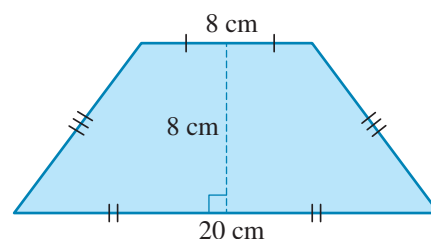
b



c

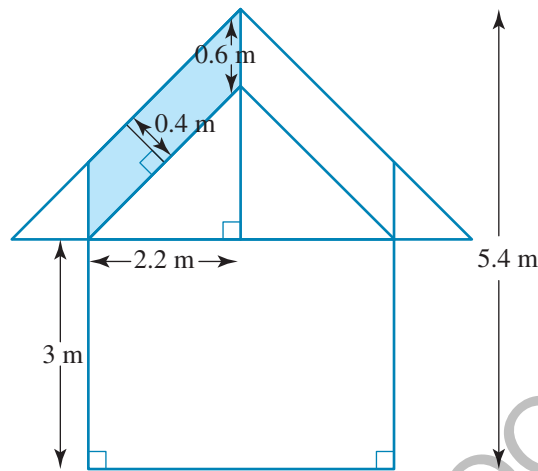


d

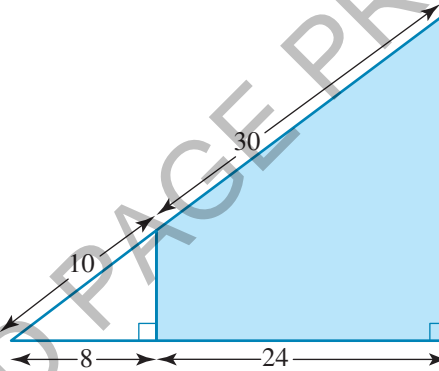


CONSOLIDATE

- 6 Calculate the area of the shaded region shown in the diagram, giving your answer correct to 2 decimal places.



- 7 Calculate the perimeter of the large triangle and hence find the shaded area.

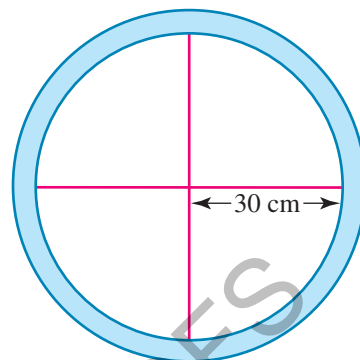


- 8 Correct to 2 decimal places, calculate the circumference and area of:
- a circle of radius 5 cm
 - a circle of diameter 18 cm.
- 9 Calculate the perimeter and area of a parallelogram with side lengths of 12 cm and 22 cm, and a perpendicular distance of 16 cm between the short sides.
- 10 Calculate the area of a rhombus with diagonals of 11.63 cm and 5.81 cm.
- 11 A circle has an area of 3140 cm^2 . What is its radius correct to 2 decimal places?
- 12 The Bayview Council wants to use the triangular park beside the beach to host a special Anzac Day barbecue. However, council rules stipulate that public areas can be used for such purposes only if the area chosen is over 350 m^2 in size. The sides of the triangular park measure 23 metres, 28 metres and 32 metres.



Calculate the area of the park using Heron's formula and determine whether it is of a suitable size to host the barbecue.

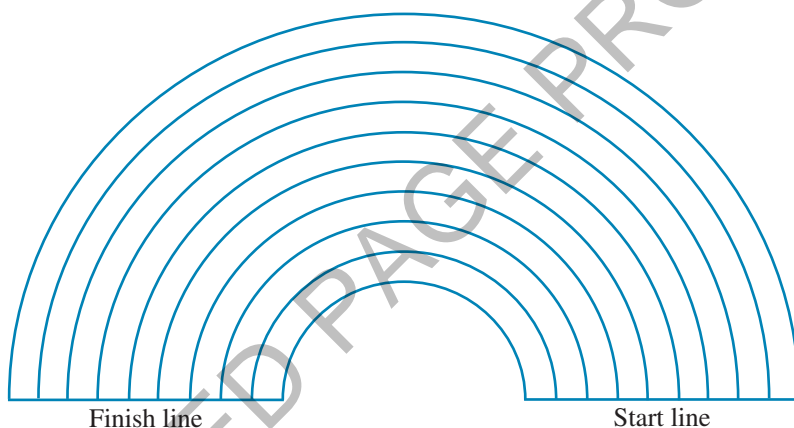
- 13** A rectangle has a side length that is twice as long as its width. If it has an area of 968 cm^2 , find the length of its diagonal correct to 2 decimal places.
- 14** A window consists of a circular metal frame 2 cm wide and two straight pieces of metal that divide the inner region into four equal segments, as shown in the diagram.



- a** If the window has an inner radius of 30 cm, calculate, correct to 2 decimal places:
- the outer circumference of the window
 - the total area of the circular metal frame.
- b** If the area of the metal frame is increased by 10% by reducing the size of the inner radius, calculate the circumference of the new inner circle.

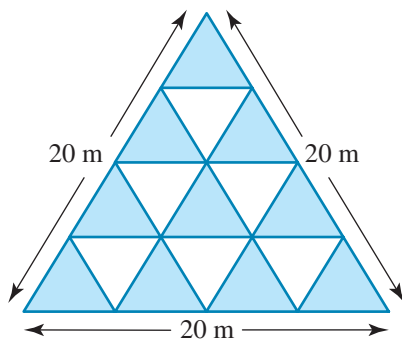
MASTER

- 15** A semicircular section of a running track consists of eight lanes that are 1.2 m wide.

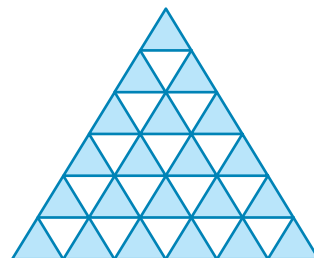


If the innermost line of the first lane has a total length of 100 m:

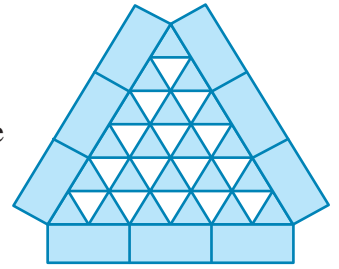
- a** how much further will someone in lane 8 run around the curve from the start line to the finish line?
- b** what is the total area of the curved section of the track?
- 16** A paved area of a garden courtyard forms an equilateral triangle with a side length of 20 m. It is paved using a series of identically sized blue and white triangular pavers as shown in the diagram.



- a** Calculate the total area of the paving correct to 2 decimal places.
- b** If the pattern is continued by adding two more rows of pavers, calculate the new perimeter and area of the paving correct to 2 decimal places.



- c After the additional two rows are added, the architects decide to add two rows of rectangular pavers. Each rectangular paver has a length that is twice the side length of a triangular paver, and a width that is half the side length of a triangular paver. If this was done on each side of the triangular paved area, calculate the perimeter and area of the paving.



7.4 Perimeter and area II

Composite shapes

study on

Unit 1

AOS 4

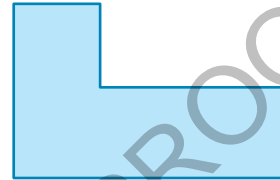
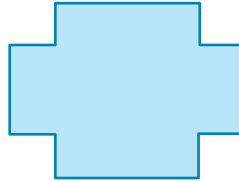
Topic 7

Concept 3

Perimeter and area II

Summary screen and practice questions

Many objects are not standard shapes but are combinations of them. For example:

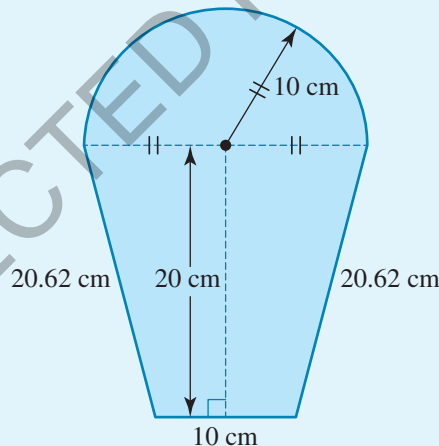


To find the areas of these shapes, split them up into standard shapes, calculate the individual areas of these standard shapes and sum the answers together.

To find the perimeters of these shapes, it is often easiest to calculate each individual side length and to then calculate the total, rather than applying any specific formula.

WORKED EXAMPLE 8

Calculate the area of the object shown.



THINK

- 1 Identify the given information.
- 2 Find the area of each component of the shape.

WRITE

The shape is a combination of a trapezium and a semicircle.

$$\begin{aligned}\text{Area of trapezium: } A &= \frac{1}{2}(a + b)h \\ &= \frac{1}{2}(10 + 20)20 \\ &= 300 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of semicircle: } A &= \frac{1}{2}\pi r^2 \\ &= \frac{1}{2}\pi(10)^2 \\ &= 157.08 \text{ cm}^2\end{aligned}$$

3 Sum the areas of the components. Total area: $300 + 157.08 = 457.08$

4 State the answer. The area of the shape is 457.08 cm^2 .

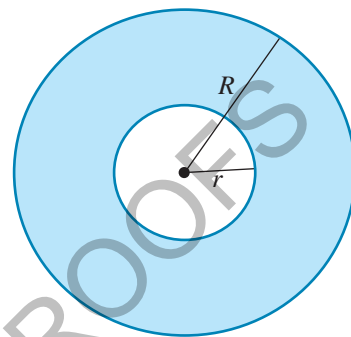
Some composite shapes do have specific formulas.

Annulus

The area between two circles with the same centre is known as an annulus. It is calculated by subtracting the area of the inner circle from the area of the outer circle.

Area of annulus = area of outer circle
– area of inner circle

$$A = \pi R^2 - \pi r^2 \\ = \pi(R^2 - r^2)$$



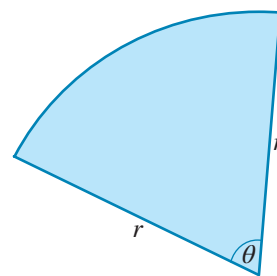
Sectors

Sectors are fractions of a circle. Because there are 360 degrees in a whole circle, the area of the sector can be found using

$A = \frac{\theta}{360} \times \pi r^2$, where θ is the angle between the two radii that form the sector.

The perimeter of a sector is a fraction of the circumference of the related circle plus two radii:

$$P = \left(\frac{\theta}{360} \times 2\pi r \right) + 2r \\ = 2r \left(\frac{\theta}{360} \pi + 1 \right)$$



$$\text{Area of an annulus} = \pi(R^2 - r^2)$$

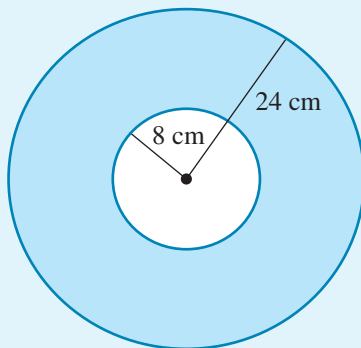
$$\text{Area of a sector} = \frac{\theta}{360} \times \pi r^2$$

$$\text{Perimeter of a sector} = 2r \left(\frac{\theta}{360} \pi + 1 \right)$$

WORKED
EXAMPLE

9

Calculate the annulus shown in the diagram.



THINK

- 1 Identify the given information.
- 2 Substitute the information into the formula and simplify.
- 3 State the answer.

WRITE

The area shown is an annulus.
 The radius of the outer circle is 24 cm.
 The radius of the inner circle is 8 cm.

$$\begin{aligned}
 A &= \pi R^2 - \pi r^2 \\
 &= \pi(R^2 - r^2) \\
 &= \pi(24^2 - 8^2) \\
 &= 512\pi \\
 &\approx 1608.5
 \end{aligned}$$

The shaded area is 1608.5 cm².

Applications

Calculations for perimeter and area have many and varied applications, including building and construction, painting and decorating, real estate, surveying and engineering.

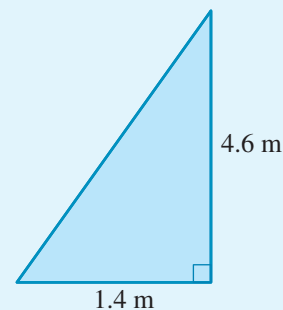
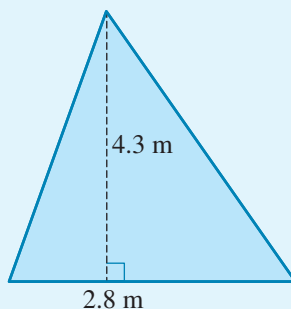
When dealing with these problems it is often useful to draw and/or redraw diagrams.

**WORKED
EXAMPLE 10**

Calculate the total area of the sails on a yacht correct to 2 decimal places, if the apex of one sail is 4.3 m above its base length of 2.8 m, and the apex of the other sail is 4.6 m above its base of length of 1.4 m.

**THINK**

- 1 Draw a diagram of the given information.

WRITE/DRAW

- 2 Identify the formulas required from the given information.

For each sail, use the formula for area of a triangle:

$$A = \frac{1}{2}bh.$$

- 3 Substitute the information into the required formulas for each area and simplify.

$$\begin{aligned}\text{Sail 1: } A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 2.8 \times 4.3 \\ &= 6.02\end{aligned}$$

$$\begin{aligned}\text{Sail 2: } A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 1.4 \times 4.6 \\ &= 3.22\end{aligned}$$

- 4 Add the areas of each of the required parts.

$$\begin{aligned}\text{Area of sail} &= 6.02 + 3.22 \\ &= 9.24\end{aligned}$$

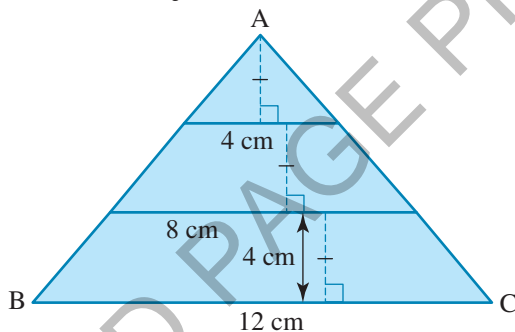
- 5 State the answer.

The total area of the sails is 9.24 m^2 .

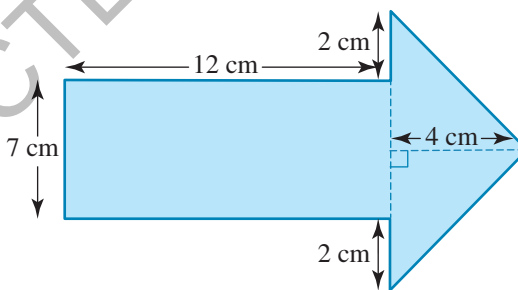
EXERCISE 7.4 Perimeter and area II

PRACTISE

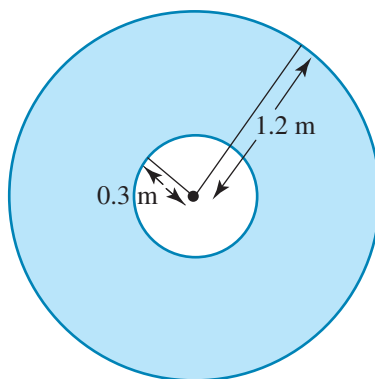
- 1 **WE8** Calculate the area of the object shown.



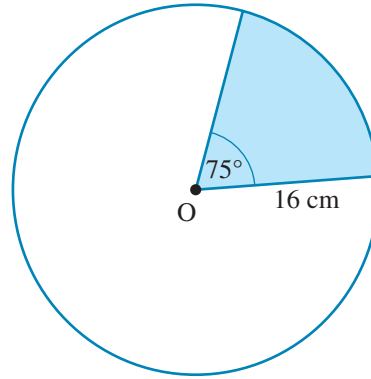
- 2 Calculate the perimeter and area of the object shown.



- 3 **WE9** Calculate the shaded area shown in the diagram correct to 2 decimal places.



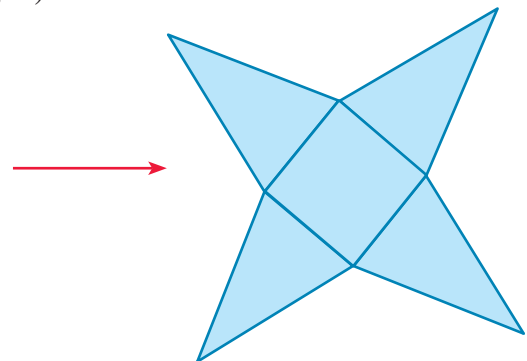
- 4 Calculate the area and perimeter of the shaded region shown in the diagram correct to 2 decimal places.



- 5 **WE10** Calculate the area of glass in a table that consists of three glass circles. The largest circle has a diameter of 68 cm. The diameters of the other two circles are 6 cm and 10 cm less than the diameter of the largest circle. Give your answer correct to 2 decimal places.

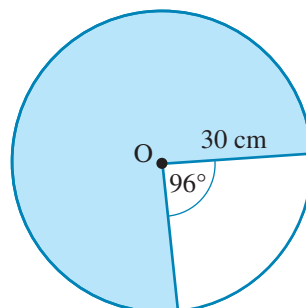


- 6 Part of the floor of an ancient Roman building was tiled in a pattern in which four identical triangles form a square with their bases. If the triangles have a base length of 12 cm and a height of 18 cm, calculate the perimeter and area they enclose, correct to 2 decimal places. (That is, calculate the perimeter and area of the shaded region shown on the right.)

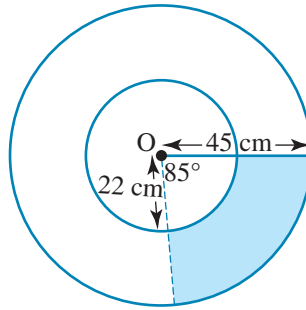


CONSOLIDATE

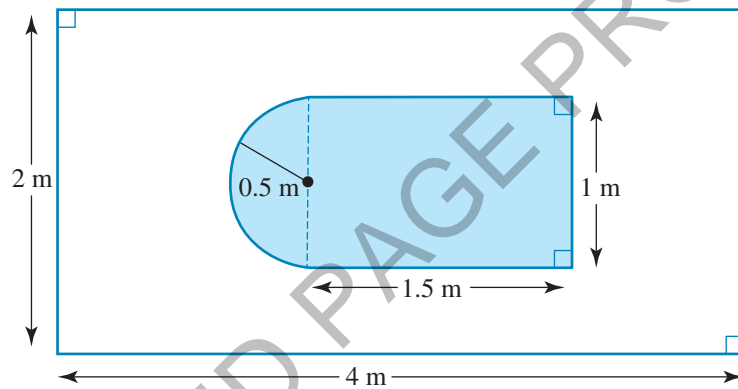
- 7 Calculate the area and perimeter of the shaded region shown in the diagram correct to 2 decimal places.



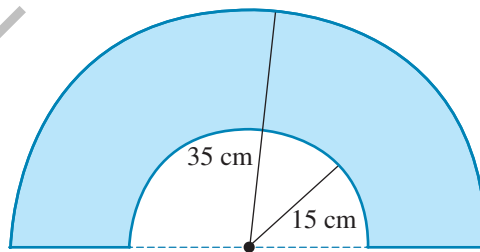
- 8 Calculate the area and perimeter of the shaded region shown in the diagram correct to 2 decimal places.



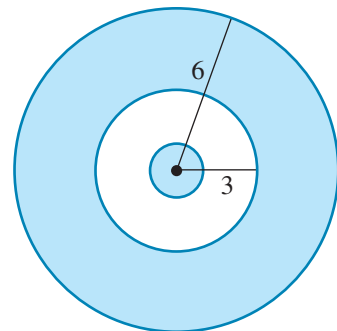
- 9 A circle of radius 8 cm is cut out from a square of side length 20 cm. How much of the area of the square remains? Give your answer correct to 2 decimal places.
- 10 a Calculate the perimeter of the shaded area inside the rectangle shown in the diagram correct to 2 decimal places.



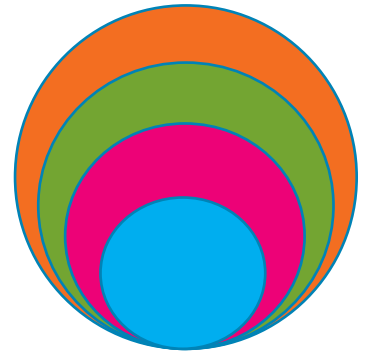
- b If the darker area inside the rectangle is removed, what area remains?
- 11 Calculate the perimeter and area of the shaded region in the half-annulus formed by 2 semicircles shown in the diagram. Give your answers correct to the nearest whole number.



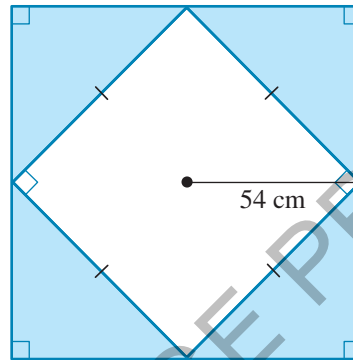
- 12 The area of the inner circle in the diagram shown is $\frac{1}{9}$ that of the annulus formed by the two outer circles. Calculate the area of the inner circle correct to 2 decimal places given that the units are in centimetres.



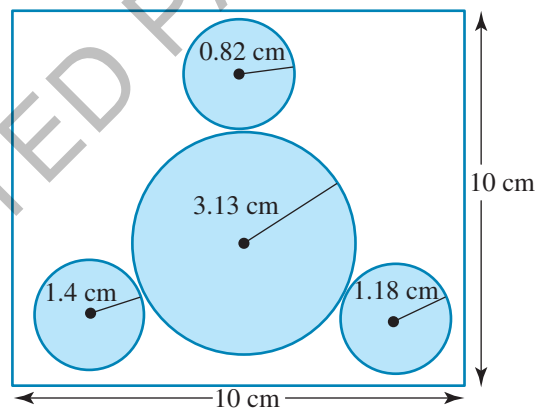
- 13** In the diagram the smallest circle has a diameter of 5 cm and the others of of 5 cm and the others have diameters that are progressively 2 cm longer than the one immediately before. Calculate the area that is shaded green, correct to 2 decimal places.



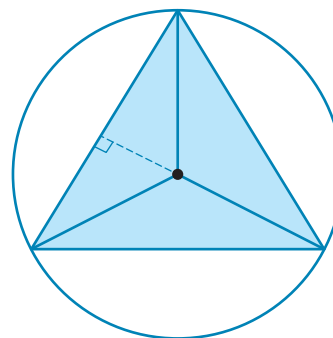
- 14 a** Calculate the shaded area in the diagram.



- b** Calculate the unshaded area inside the square shown in the diagram, giving your answer correct to 2 decimal places.

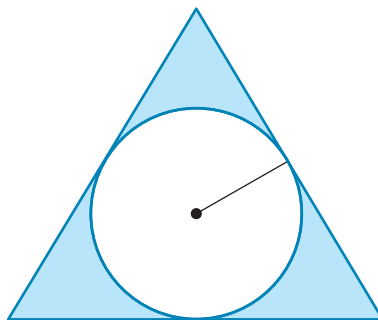


- 15** The vertices of an equilateral triangle of side length 2 metres touch the edge of a circle of radius 1.16 metres, as shown in the diagram. Calculate the area of the unshaded region.



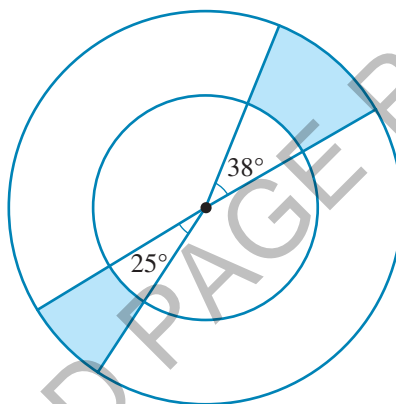
- 16** A circle of radius 0.58 metres sits inside an equilateral triangle of side length 2 metres so that it touches the edges of the triangle at three points. If the circle

represents an area of the triangle to be removed, how much area would remain once this was done?

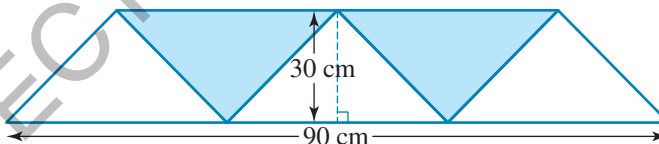


MASTER

- 17 An annulus has an inner radius of 20 cm and an outer radius of 35 cm. Two sectors are to be removed. If one sector has an angle at the centre of 38° and the other has an angle of 25° , what area remains? Give your answer correct to 2 decimal places.



- 18 A trapezium is divided into five identical triangles of equal size with dimensions as shown in the diagram. Find the area and perimeter of the shaded region.



7.5 Volume

study on

Unit 1

AOS 4

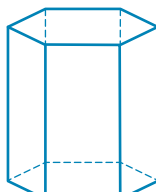
Topic 7

Concept 4

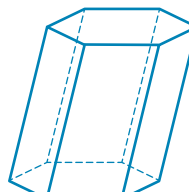
Volume

Summary screen and practice questions

The amount of space that is taken up by any solid or three-dimensional object is known as its volume. Many standard objects have formulas that can be used to calculate their volume. If the centre point of the top of the solid is directly above the centre point of its base, the object is called a 'right solid'. If the centre point of the top is not directly above the centre point of the base, the object is an 'oblique solid'.



Right solid



Oblique solid

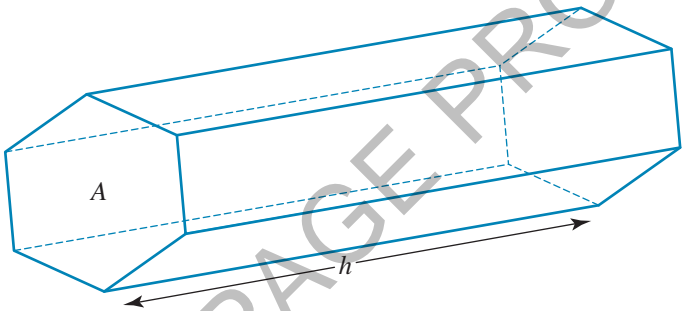
Note: For an oblique solid, the height, h , is the distance between the top and the base, not the length of one of the sides. (For a right solid, the distance between the top and the base equals the side length.)

Volume is expressed in cubic units of measurement, such as cubic metres (m^3) or cubic centimetres (cm^3). When calculations involve the amount of fluid that the object can contain, the units are commonly litres (L) or millilitres (mL).

To convert cubic centimetres to millilitres, use $1 \text{ cm}^3 = 1 \text{ mL}$.

Prisms

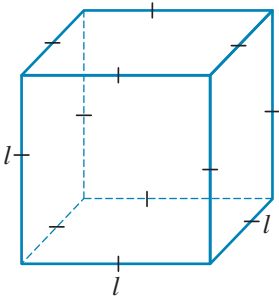
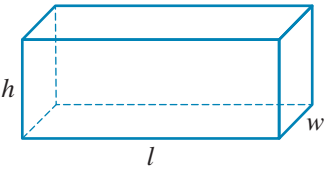
If a solid object has identical ends that are joined by flat surfaces, and the object’s cross-section is the same along its length, the object is a prism. The volume of a prism is calculated by taking the product of the base area and its height (or length).

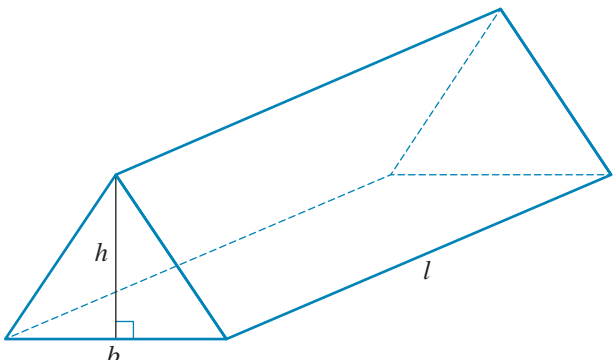


$V = A \times h$ (A = base area)

Common prisms

The formulas for calculating the volume of some of the most common prisms are summarised in the following table.

Prism	Volume
<div>Cube</div> <div></div>	$V = A \times h$ $= (l \times l) \times l$ $= l^3$
<div>Rectangular prism</div> <div></div>	$V = A \times h$ $= (l \times w) \times h$ $= l \times w \times h$

Prism	Volume
Triangular prism 	$V = A \times h$ $= \left(\frac{1}{2}bh\right) \times l$ $= \frac{1}{2}bhl$

Note: These formulas apply to both right prisms and oblique prisms, as long as you remember that the height of an oblique prism is its perpendicular height (the distance between the top and the base).

WORKED EXAMPLE 11 Calculate the volume of a triangular prism with length $l = 12$ cm, triangle base $b = 6$ cm and triangle height $h = 4$ cm.

THINK

- 1 Identify the given information.
- 2 Substitute the information into the appropriate formula for the solid object and evaluate.
- 3 State the answer.

WRITE

Triangular prism, $l = 12$ cm, $b = 6$ cm, $h = 4$ cm

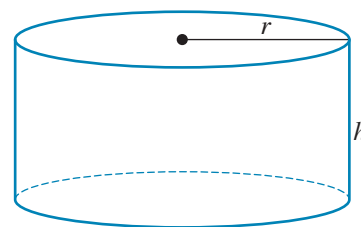
$$\begin{aligned}
 V &= \frac{1}{2}bhl \\
 &= \frac{1}{2} \times 6 \times 4 \times 12 \\
 &= 144
 \end{aligned}$$

The volume is 144 cm^3 .

Cylinders

A cylinder is a solid object with ends that are identical circles and a cross-section that is the same along its length (like a prism). As a result it has a curved surface along its length.

As for prisms, the volume of a cylinder is calculated by taking the product of the base area and the height:



$$\begin{aligned}
 V &= \text{Base area} \times \text{height} \\
 &= \pi r^2 h
 \end{aligned}$$

WORKED EXAMPLE 12 Calculate the volume of a cylinder of radius 10 cm and height 15 cm.

THINK

- 1 Identify the given information.

WRITE

Cylinder, $r = 10$ cm, $h = 15$ cm

- 2 Substitute the information into the appropriate formula for the solid object and evaluate.

$$\begin{aligned}V &= \pi r^2 h \\&= \pi \times 10 \times 10 \times 15 \\&= 4712.39\end{aligned}$$

- 3 State the answer.

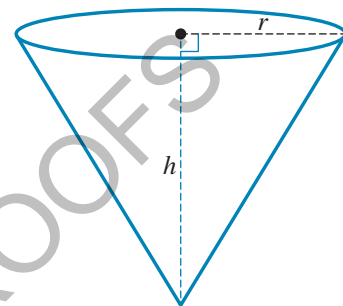
The volume is 4712.39 cm^3 .

Cones and pyramids

Cones

A cone is a solid object that is similar to a cylinder in that it has one end that is circular, but different in that at the other end it has a single vertex.

It can be shown that if you have a cone and a cylinder with identical circular bases and heights, the volume of the cylinder will be three times the volume of the cone. (The proof of this is beyond the scope of this course.) The volume of a cone can therefore be calculated by using the formula for a comparable cylinder and dividing by three.



$$\begin{aligned}V &= \frac{1}{3} \times \text{base area} \times \text{height} \\&= \frac{1}{3} \pi r^2 h\end{aligned}$$

WORKED EXAMPLE 13

Calculate the volume of a cone of radius 20 cm and a height of 36 cm.

THINK

- 1 Identify the given information.
- 2 Substitute the information into the appropriate formula for the solid object and evaluate.

WRITE

Given: a cone with $r = 20 \text{ cm}$ and $h = 36 \text{ cm}$

$$\begin{aligned}V &= \frac{1}{3} \pi r^2 h \\&= \frac{1}{3} \times \pi \times 20 \times 20 \times 36 \\&= 15\,079.6\end{aligned}$$

- 3 State the answer.

The volume is $15\,079.6 \text{ cm}^3$.

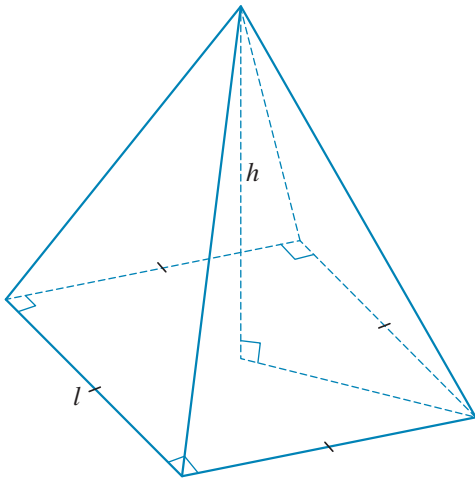
Pyramids

A pyramid is a solid object whose base is a polygon and whose sides are triangles that meet at a single point. The most famous examples are the pyramids of Ancient Egypt, which were built as tombs for the pharaohs.



A pyramid is named after the shape of its base. For example, a hexagonal pyramid has a hexagon as its base polygon. The most common pyramids are square pyramids and triangular pyramids. As with cones, the volume of a pyramid can be calculated by using the formula of a comparable prism and dividing by three.

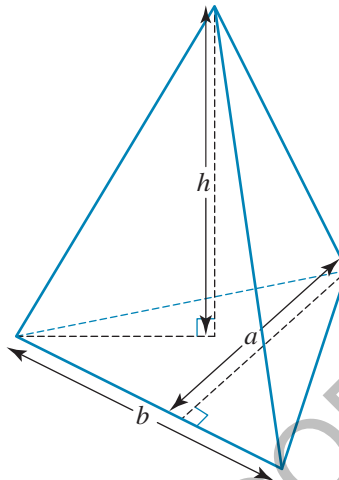
Square pyramid



$$V = \frac{1}{3} \times \text{base area} \times \text{height}$$

$$= \frac{1}{3} l^2 h$$

Triangular pyramid



$$V = \frac{1}{3} \times \text{base area} \times \text{height}$$

$$= \frac{1}{3} \left(\frac{1}{2} ab \right) h$$

$$= \frac{1}{6} abh$$

WORKED EXAMPLE 14 Calculate the volume of a pyramid that is 75 cm tall and has a rectangular base with dimensions 45 cm by 38 cm.

THINK

- 1 Identify the given information.
- 2 Substitute the information into the appropriate formula for the solid object and evaluate.
- 3 State the answer.

WRITE

Given: a pyramid with a rectangular base of 45×38 cm and a height of 75 cm

$$V = \frac{1}{3} lwh$$

$$= \frac{1}{3} \times 45 \times 38 \times 75$$

$$= 42\,750$$

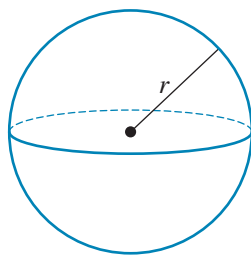
The volume is $42\,750 \text{ cm}^3$.

Spheres

A sphere is a solid object that has a curved surface such that every point on the surface is the same distance (the radius of the sphere) from a central point.



The formula for calculating the volume of a sphere has been attributed to the ancient Greek mathematician Archimedes.



$$V = \frac{4}{3}\pi r^3$$

WORKED EXAMPLE 15 Calculate the volume of a sphere of radius 63 cm.

THINK

- 1 Identify the given information.
- 2 Substitute the information into the appropriate formula for the solid object and evaluate.
- 3 State the answer.

WRITE

Given: a sphere of $r = 63$ cm

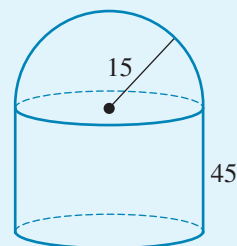
$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times \pi \times 63 \times 63 \times 63 \\ &= 1\,047\,394.4 \end{aligned}$$

The volume is $1\,047\,394.4 \text{ cm}^3$.

Volumes of composite shapes

As with calculations for perimeter and area, when a solid object is composed of two or more standard shapes, we need to identify each part and add their volumes to evaluate the overall volume.

WORKED EXAMPLE 16 Calculate the volume of an object that is composed of a hemisphere (half a sphere) of radius 15 cm that sits on top of a cylinder of height 45 cm.



THINK

- 1 Identify the given information.
- 2 Substitute the information into the appropriate formula for each component of the solid object and evaluate.

WRITE

Given: a hemisphere with $r = 15$ cm and a cylinder of $r = 15$ cm and $h = 45$ cm

$$\begin{aligned} \text{Hemisphere: } V &= \frac{1}{2} \left(\frac{4}{3}\pi r^3 \right) \\ &= \frac{1}{2} \times \left(\frac{4}{3} \times \pi \times 15 \times 15 \times 15 \right) \\ &= 7068.58 \end{aligned}$$

$$\begin{aligned} \text{Cylinder: } V &= \pi r^2 h \\ &= \pi \times 15 \times 15 \times 45 \\ &= 31\,808.6 \end{aligned}$$

3 Add the volume of each component.

$$\begin{aligned}\text{Composite object: } V &= 7068.58 + 31\,808.6 \\ &= 38\,877.2\end{aligned}$$

4 State the answer.

The volume is $38\,877.2\text{ cm}^3$.

EXERCISE 7.5 Volume

PRACTISE

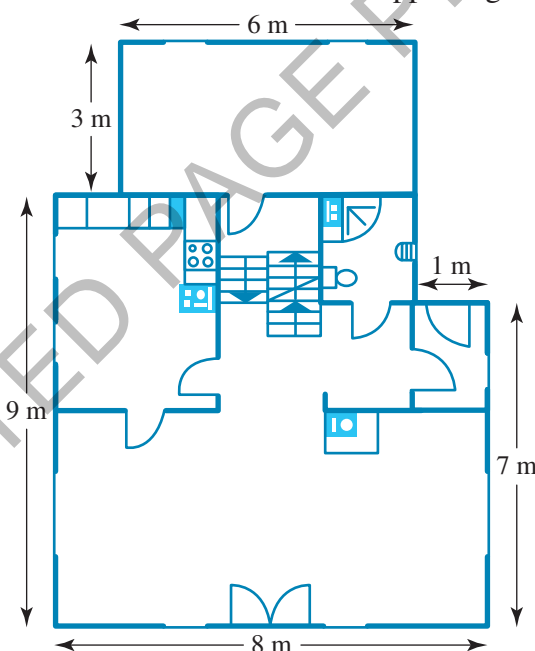
- 1 **WE11** Calculate the volume of a triangular prism with length $l = 2.5\text{ m}$, triangle base $b = 0.6\text{ m}$ and triangle height $h = 0.8\text{ m}$.
- 2 Giving answers correct to the nearest cubic centimetre, calculate the volume of a prism that has:
 - a a base area of 200 cm^2 and a height of 1.025 m
 - b a rectangular base 25.25 cm by 12.65 cm and a length of 0.42 m
 - c a right-angled triangular base with one side length of 48 cm , a hypotenuse of 73 cm and a length of 96 cm
 - d a height of 1.05 m and a trapezium-shaped base with parallel sides that are 25 cm and 40 cm long and 15 cm apart.
- 3 **WE12** Giving answers correct to the nearest cubic centimetre, calculate the volume of a cylinder of radius 22.5 cm and a height of 35.4 cm .
- 4 Calculate the volume of a cylinder that has:
 - a a base circumference of 314 cm and a height of 0.625 m , giving your answer correct to the nearest cubic centimetre
 - b a height of 425 cm and a radius that is three-quarters of its height, giving your answer correct to the nearest cubic metre.
- 5 **WE13** Calculate the volume of a cone of radius 30 cm and a height of 42 cm correct to 1 decimal place.
- 6 Calculate the volume of a cone that has:
 - a a base circumference of 628 cm and a height of 0.72 m
 - b a height of 0.36 cm and a radius that is two-thirds of its height.
- 7 **WE14** Calculate the volume of a pyramid that is 2.025 m tall and has a rectangular base with dimensions 1.05 m by 0.0745 m .
- 8 Calculate the volume of a pyramid that has:
 - a a base area of 366 cm^2 and a height of 1.875 m
 - b a rectangular base 18.45 cm by 26.55 cm and a length of 0.96 m
 - c a height of 3.6 m and a triangular base with one side length of 1.2 m that is a perpendicular distance of 0.6 m from its apex.
- 9 **WE15** Calculate the volume of a sphere of radius 0.27 m .
- 10 Calculate the radius of a sphere that has:
 - a a volume of $248\,398.88\text{ cm}^3$
 - b a volume of 4.187 m^3 .
- 11 **WE16** Calculate the volume of an object that is composed of a hemisphere (half a sphere) of radius 1.5 m that sits on top of a cylinder of height 2.1 m .

CONSOLIDATE

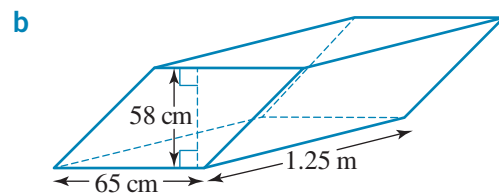
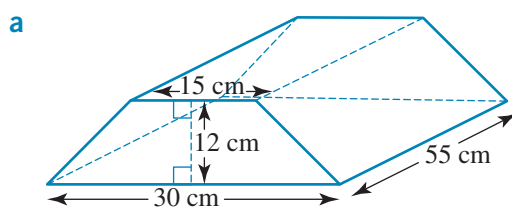
- 12** Calculate the volume of an object that is composed of:
- a** a square pyramid of height 48 cm that sits on top of a cube of side length 34 cm
 - b** a cone of height 75 cm that sits on top of a 60-cm-tall cylinder of radius 16 cm.
- 13** The Gold Medal Pool Company sells three types of above-ground swimming pools, with base shapes that are cubic, rectangular or circular. Use the information in the table to list the volumes of each type in order from largest to smallest, giving your answers in litres.

Type	Depth	Base dimensions
Square pool	1.2 m	Length: 3 m
Rectangular pool	1.2 m	Length: 4.1 m Width: 2.25 m
Circular pool	1.2 m	Diameter: 3.3 m

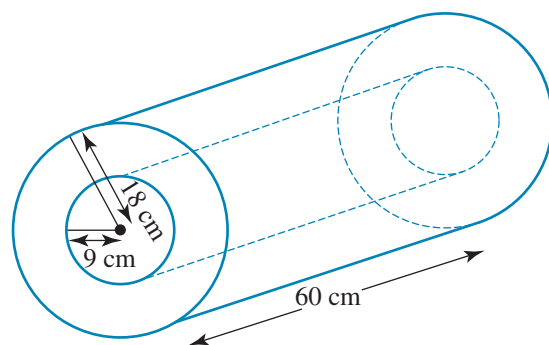
- 14** A builder uses the floor plan of the house he is building to calculate the amount of concrete he needs to order for the foundations supporting the brick walls.



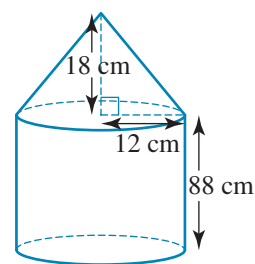
- a** The foundation needs to go around the perimeter of the house with a width of 600 mm and a depth of 1050 mm. How many cubic metres of concrete are required?
 - b** The builder also wants to order the concrete required to pour a rectangular slab 3 m by 4 m to a depth of 600 mm. How many cubic metres of extra concrete should he order?
- 15** Calculate the volumes of the solid objects shown in the following diagrams.



c



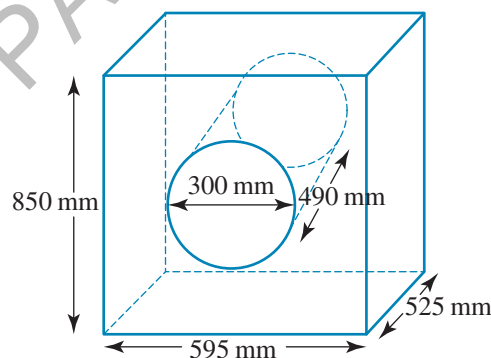
d



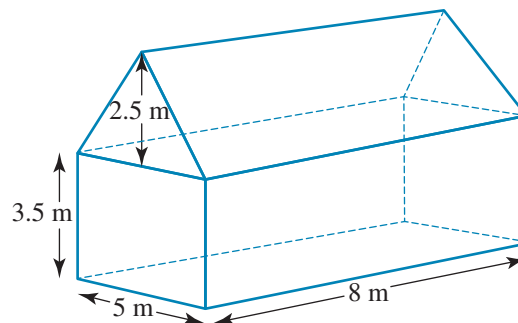
- 16** A company manufactures skylights in the shape of a cylinder with a hemispherical lid.

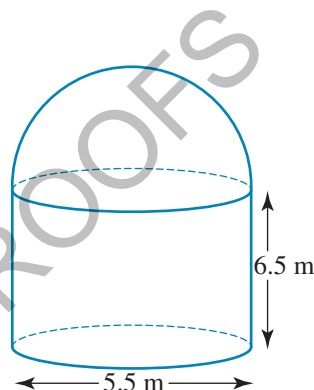
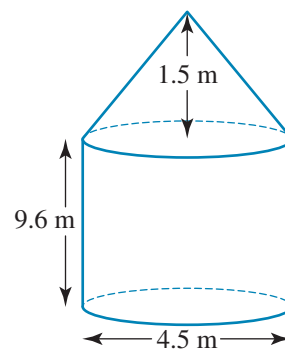
When they are fitted onto a house, three-quarters of the length of the cylinder is below the roof. If the cylinder is 1.5 metres long and has a radius of 30 cm, calculate the volume of the skylight that is above the roof and the volume that is below it. Give your answers correct to the nearest cubic centimetre.

- 17** The outer shape of a washing machine is a rectangular prism with a height of 850 mm, a width of 595 mm and a depth of 525 mm. Inside the machine, clothes are washed in a cylindrical stainless steel drum that has a diameter of 300 mm and a length of 490 mm.



- a** What is the maximum volume of water, in litres, that the stainless steel drum can hold?
- b** Calculate the volume of the washing machine, in cubic metres, after subtracting the volume of the stainless steel drum.
- 18** The diagram shows the dimensions for a proposed house extension. Calculate the volume of insulation required in the roof if it takes up an eighth of the overall roof space.
- 19** A wheat farmer needs to purchase a new grain silo and has the choice of two sizes. One is cylindrical with a conical top, and the other is cylindrical with a hemispherical top. Use the dimensions shown in the diagrams to determine which silo holds the greatest volume of wheat and by how much.





- 20** A tank holding liquid petroleum gas (LPG) is cylindrical in shape with hemispherical ends. If the tank is 8.7 metres from the top of the hemisphere at one end to the top of the hemisphere at the other, and the cylindrical part of the tank has a diameter of 1.76 metres, calculate the volume of the tank to the nearest litre.



- 21** The glass pyramid in the courtyard of the Louvre Museum in Paris has a height of 22 m and a square base with side lengths of 35 m.



- a** What is the volume of the glass pyramid in cubic metres?
- b** A second glass pyramid at the Louvre Museum is called the Inverted Pyramid as it hangs upside down from the ceiling. If its dimensions are one-third of those of the larger glass pyramid, what is its volume in cubic centimetres?



- 22** Tennis balls are spherical with a diameter of 6.7 cm. They are sold in packs of four in cylindrical canisters whose internal dimensions are 26.95 cm long with a diameter that is 5 mm greater than that of a ball. The canisters are packed vertically in rectangular boxes; each box is 27 cm high and will fit exactly eight canisters along its length and exactly four along its width.
- a** Calculate the volume of free space that is in a canister containing four tennis balls.
- b** Calculate the volume of free space that is in a rectangular box packed full of canisters.

- 23** Using a calculator, spreadsheet or otherwise, compare the volumes of cylinders that are 50 cm tall but have different radii.
- Tabulate the results for cylinders with radii of 10, 20, 40, 80, 160 and 320 cm.
 - Graph your results.
 - Use your graph to estimate the volume of a cylinder that is 50 cm tall and has a radius of:
 - 50 cm
 - 80 cm.
- 24** Using a calculator, spreadsheet or otherwise, compare the volumes of square pyramids that are 20 m tall but have different base lengths.
- Tabulate the results for pyramids with base lengths of 5, 10, 15, 20, 25 and 30 m.
 - Graph your results.
 - Use your graph to estimate the volume of a square pyramid that is 20 m tall and a base length of:
 - 9 m
 - 14 m.

7.6 Surface area

Surface area

study on

Unit 1

AOS 4

Topic 7

Concept 5

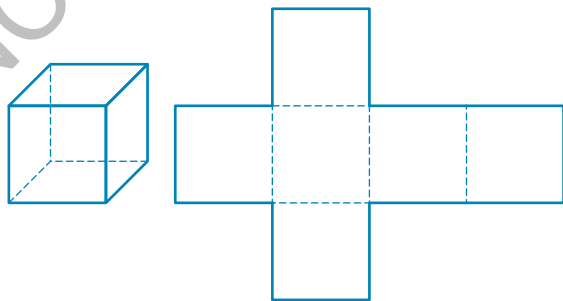
Surface area

Summary screen
and practice
questions

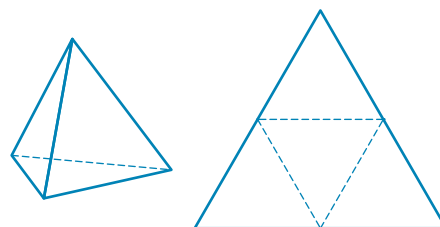
The surface area of a solid object is equal to the combined total of the areas of each individual surface that forms it. Some objects have specific formulas for the calculation of the total surface area, whereas others require the calculation of each individual surface in turn. Surface area is particularly important in design and construction when considering how much material is required to make a solid object. In manufacturing it could be important to make an object with the smallest amount of material that is capable of holding a particular volume. Surface area is also important in aerodynamics, as the greater the surface area, the greater the potential air resistance or drag.

Nets

The net of a solid object is like a pattern or plan for its construction. Each surface of the object is included in its net. Therefore, the net can be used to calculate the total surface area of the object. For example, the net of a cube will have six squares, whereas the net of a triangular pyramid (or tetrahedron) will have four triangles.



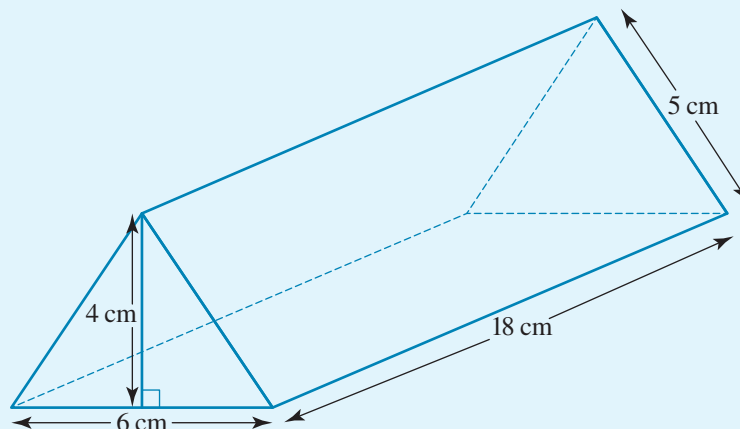
The net of a cube



The net of a tetrahedron

WORKED
EXAMPLE 17

Calculate the surface area of the prism shown by first drawing its net.

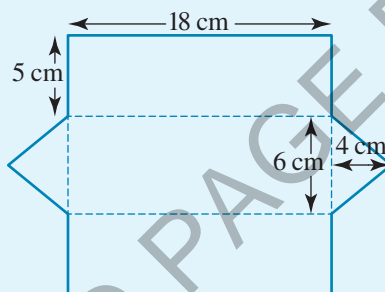


THINK

- 1 Identify the prism and each surface in it.
- 2 Redraw the given diagram as a net, making sure to check that each surface is present.

WRITE/DRAW

The triangular prism consists of two identical triangular ends, two identical rectangular sides and one rectangular base.



- 3 Calculate the area of each surface identified in the net.

Triangular ends:

$$\begin{aligned} A &= 2 \times \left(\frac{1}{2}bh \right) \\ &= 2 \times \left(\frac{1}{2} \times 6 \times 4 \right) \\ &= 24 \end{aligned}$$

Rectangular sides:

$$\begin{aligned} A &= 2 \times (lw) \\ &= 2 \times (18 \times 5) \\ &= 180 \end{aligned}$$

Rectangular base:

$$\begin{aligned} A &= lw \\ &= 18 \times 6 \\ &= 108 \end{aligned}$$

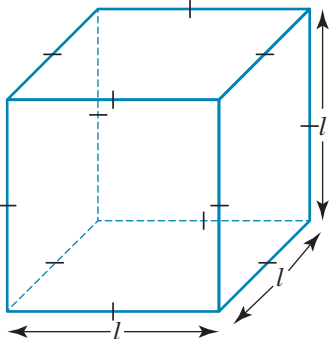
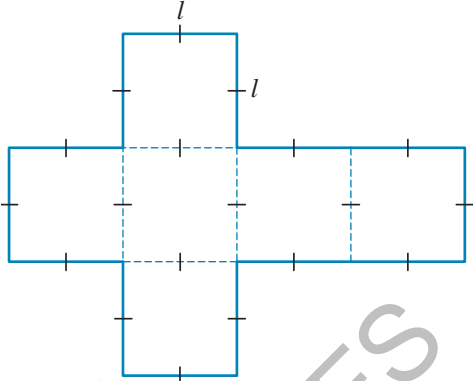
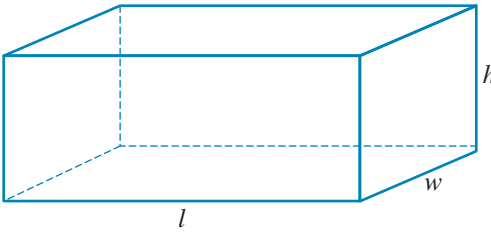
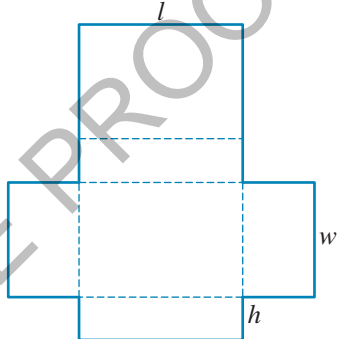
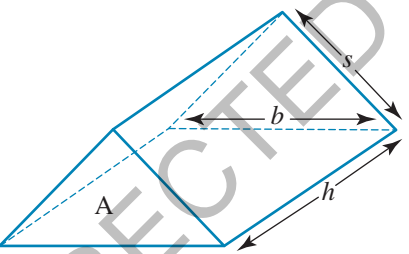
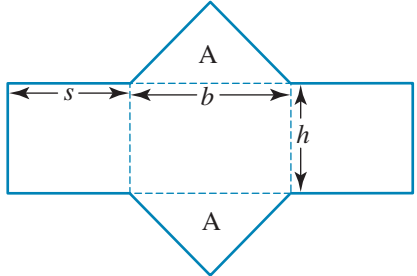
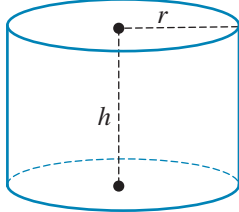
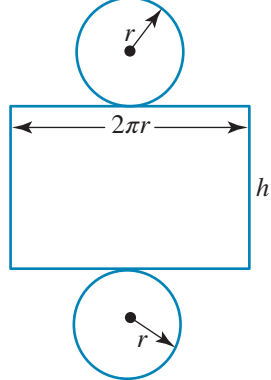
- 4 Add the component areas and state the answer.

Total surface area:

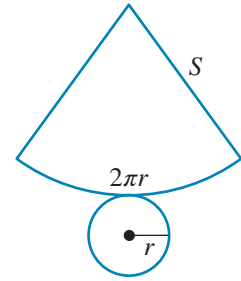
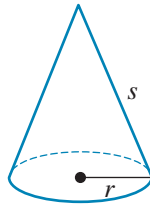
$$\begin{aligned} &= 24 + 180 + 108 \\ &= 312 \text{ cm}^2 \end{aligned}$$

Surface area formulas

The surface area formulas for common solid objects are summarised in the following table.

Object	Surface area
<p>Cube</p> 	 $SA = 6l^2$
<p>Rectangular prism</p> 	 $SA = 2lw + 2lh + 2wh$
<p>Triangular prism</p> 	 $SA = 2A + 2hs + bh$ <p>(where A is the area of the triangular end)</p>
<p>Cylinder</p> 	 $SA = 2\pi r^2 + 2\pi rh$ $= 2\pi r(r + h)$

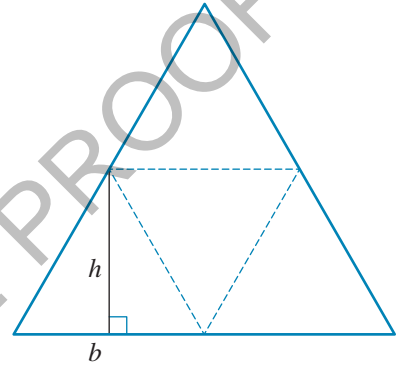
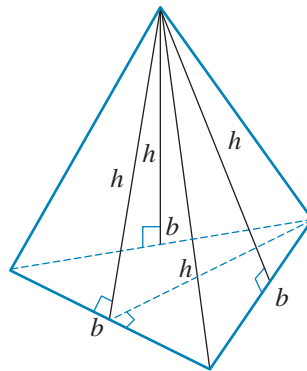
Cone



$$\begin{aligned} SA &= \pi r s + \pi r^2 \\ &= \pi r (s + r) \end{aligned}$$

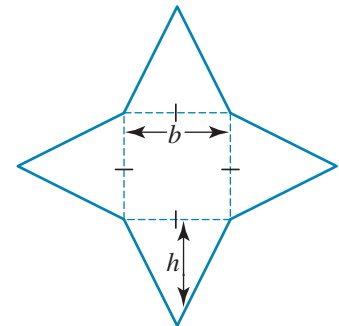
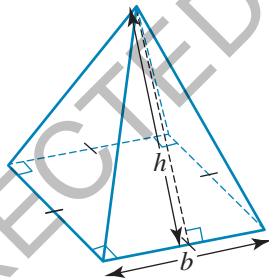
(including the circular base)

Tetrahedron



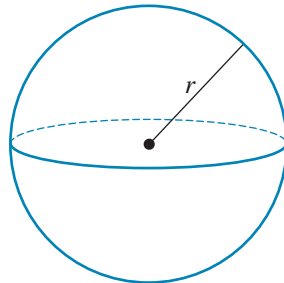
$$SA = 4 \times \left(\frac{1}{2} b h \right)$$

Square pyramid



$$SA = 4 \times \left(\frac{1}{2} b h \right) + b^2$$

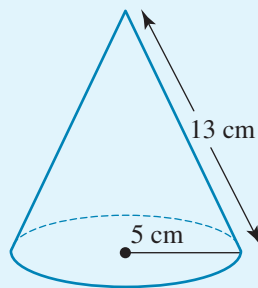
Sphere



$$SA = 4\pi r^2$$

WORKED
EXAMPLE 18

Calculate the surface area of the object shown by selecting an appropriate formula.



THINK

- 1 Identify the object and the appropriate formula.
- 2 Substitute the given values into the formula and evaluate.
- 3 State the final answer.

WRITE

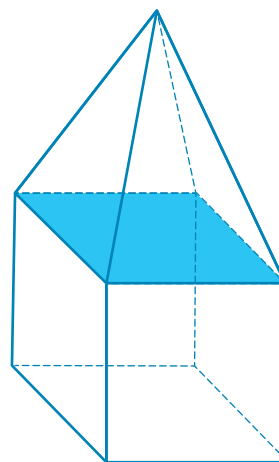
Given the object is a cone, the formula is $SA = \pi rs + \pi r^2$.

$$\begin{aligned} SA &= \pi rs + \pi r^2 \\ &= \pi r(s + r) \\ &= \pi \times 5(5 + 13) \\ &= \pi \times 90 \\ &= 282.7 \end{aligned}$$

The surface area of the cone is 282.7 cm^2 .

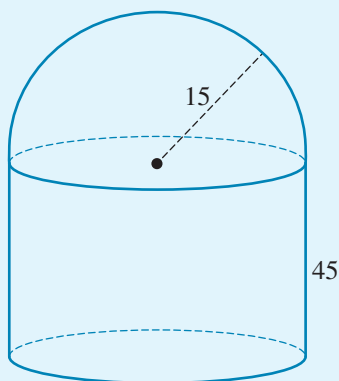
Surface areas of composite objects

For composite solids, be careful to include only those surfaces that form the outer part of the object. For example, if an object consisted of a pyramid on top of a cube, the surfaces of each object that are internal would not be included.



WORKED
EXAMPLE 19

Calculate the surface area of the object shown.



THINK

- 1 Identify the components of the composite object.
- 2 Substitute the given values into the formula for each surface of the object and evaluate.
- 3 Add the area of each surface to obtain the total surface area.
- 4 State the answer.

WRITE

The object consists of a hemisphere that sits on top of a cylinder.

Hemisphere:

$$\begin{aligned}
 SA &= \frac{1}{2}(4\pi r^2) \\
 &= \frac{1}{2}(4 \times \pi \times 15^2) \\
 &= 1413.72
 \end{aligned}$$

Cylinder (no top):

$$\begin{aligned}
 SA &= \pi r^2 + 2\pi rh \\
 &= \pi \times 15^2 + 2 \times \pi \times 15 \times 45 \\
 &= 4948.01
 \end{aligned}$$

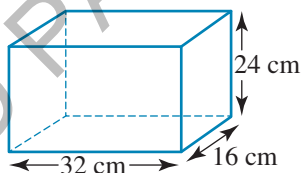
Total surface area:

$$\begin{aligned}
 SA &= 1413.72 + 4948.01 \\
 &= 6361.73
 \end{aligned}$$

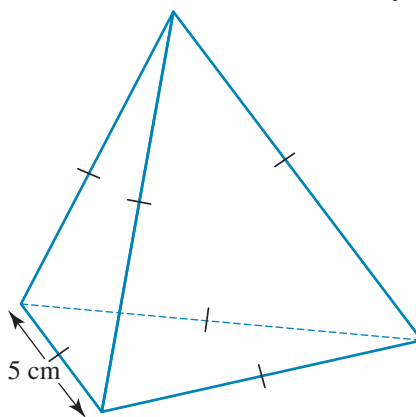
The total surface area of the object is 6361.73 cm².

EXERCISE 7.6 Surface area**PRACTISE**

- 1 **WE17** Calculate the surface area of the prism shown by first drawing its net.

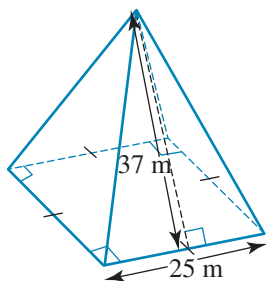


- 2 Calculate the surface area of the tetrahedron shown by first drawing its net.

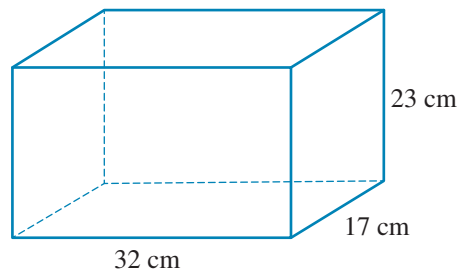


- 3 **WE18** Calculate the surface areas of the objects shown by selecting appropriate formulas.

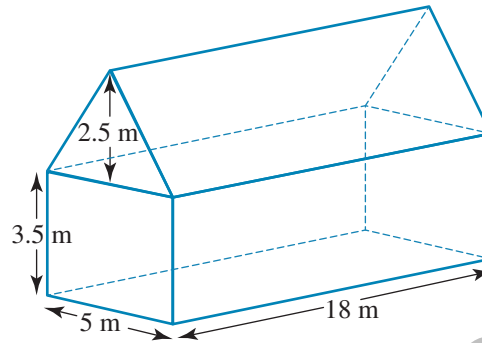
a



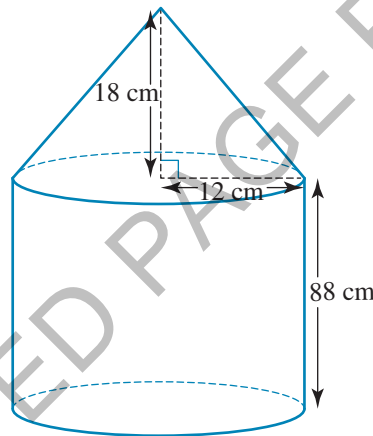
b



- 4 Calculate the surface area of:
- a a pyramid formed by four equilateral triangles with a side length of 12 cm
 - b a sphere with a radius of 98 cm
 - c a cylinder with a radius of 15 cm and a height of 22 cm
 - d a cone with a radius of 12.5 cm and a slant height of 27.2 cm.
- 5 **WE19** Calculate the surface area of the object shown.



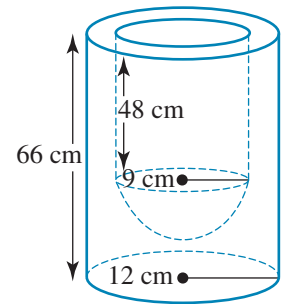
- 6 Calculate the surface area of the object shown.



CONSOLIDATE

- 7 Calculate the total surface area of:
- a a rectangular prism with dimensions 8 cm by 12 cm by 5 cm
 - b a cylinder with a base diameter of 18 cm and a height of 20 cm
 - c a square pyramid with a base length of 15 cm and a vertical height of 18 cm
 - d a sphere of radius 10 cm.
- 8 A prism is 25 cm high and has a trapezoidal base whose parallel sides are 8 cm and 12 cm long respectively, and are 10 cm apart.
- a Draw the net of the prism.
 - b Calculate the total surface area of the prism.
- 9 A hemispherical glass ornament sits on a circular base that has a circumference of 31.4 cm.
- a Calculate its total surface area to the nearest square centimetre.
 - b If an artist attaches it to an 8-cm-tall cylindrical stand with the same circumference, what is the new total surface area of the combined object that is created? Give your answer to the nearest square centimetre.
- 10 An ice-cream shop sells two types of cones. One is 6.5 cm tall with a radius of 2.2 cm. The other is 7.5 cm with a radius of 1.7 cm. Which cone (not including any ice-cream) has the greater surface area and by how much?

- 11** A cylindrical plastic vase is 66 cm high and has a radius of 12 cm. The centre has been hollowed out so that there is a cylindrical space with a radius of 9 cm that goes to a depth of 48 cm and ends in a hemisphere, as shown in the diagram.

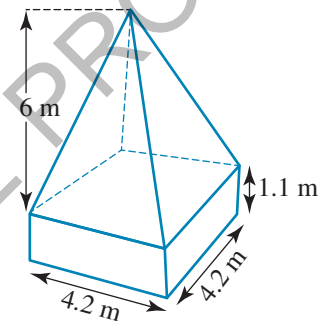


Giving your answers to the nearest square centimetre:

- a** calculate the area of the external surfaces of the vase
- b** calculate the area of the internal surface of the vase.

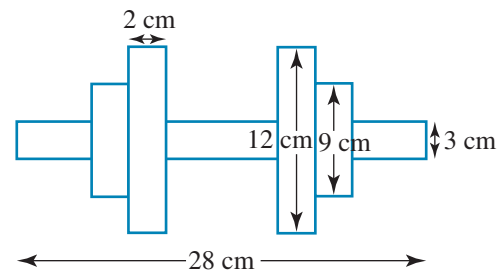
- 12** The top of a church tower is in the shape of a square pyramid that sits on top of a rectangular prism base that is 1.1 m high. The pyramid is 6 m high with a base length of 4.2 m.

Calculate the total external surface area of the top of the church tower if the base of the prism forms the ceiling of a balcony. Give your answer correct to 2 decimal places.



- 13** A dumbbell consists of a cylindrical tube that is 28 cm long with a diameter of 3 cm, and two pairs of cylindrical discs that are held in place by two locks. The larger discs have a diameter of 12 cm and a width of 2 cm, and the smaller discs are the same thickness with a diameter of 9 cm.

Calculate the total area of the exposed surfaces of the discs when they are held in position as shown in the diagram. Give your answer to the nearest square centimetre.

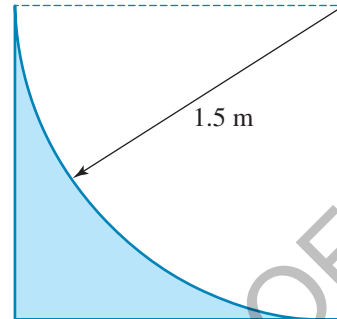


- 14** A staircase has a section of red carpet down its centre strip. Each of the nine steps is 16 cm high, 25 cm deep and 120 cm wide. The red carpet is 80 cm wide and extends from the back of the uppermost step to a point 65 cm beyond the base of the lower step.

- a** What is the area of the red carpet?
- b** If all areas of the front and top of the stairs that are not covered by the carpet are to be painted white, what is the area to be painted?



- 15 A rectangular swimming pool is 12.5 m long, 4.3 m wide and 1.5 m deep. If all internal surfaces are to be tiled, calculate the total area of tiles required.
- 16 A quarter-pipe skateboard ramp has a curved surface that is one-quarter of a cylinder with a radius of 1.5 m. If the surface of the ramp is 2.4 m wide, calculate the total surface area of the front, back and sides.



MASTER

- 17 Using a calculator, spreadsheet or otherwise, compare the surface areas of cones that have a slant height of 120 cm but different radii.
- Tabulate the results for cones with radii of 15, 30, 60, 120 and 240 cm.
 - Graph your results.
 - Use your graph to estimate the surface area of a cone that has a slant height of 120 cm and a radius of:
 - 100 cm
 - 200 cm.
- 18 Using a calculator, spreadsheet or otherwise, compare the surface areas of cylinders that have a height that is twice the length of their radius.
- Tabulate the results for cylinders with radii of 5, 10, 15, 20, 25 and 30 cm.
 - Graph your results.
 - Use your graph to estimate the surface area of one of these cylinders that has a height of:
 - 27 cm
 - 13 cm.



The Maths Quest Review is available in a customisable format for you to demonstrate your knowledge of this topic.

The Review contains:

- **Short-answer** questions — providing you with the opportunity to demonstrate the skills you have developed to efficiently answer questions using the most appropriate methods
- **Multiple-choice** questions — providing you with the opportunity to practise answering questions using CAS technology

- **Extended-response** questions — providing you with the opportunity to practise exam-style questions.

A summary of the key points covered in this topic is also available as a digital document.

REVIEW QUESTIONS

Download the Review questions document from the links found in the Resources section of your eBookPLUS.

Activities

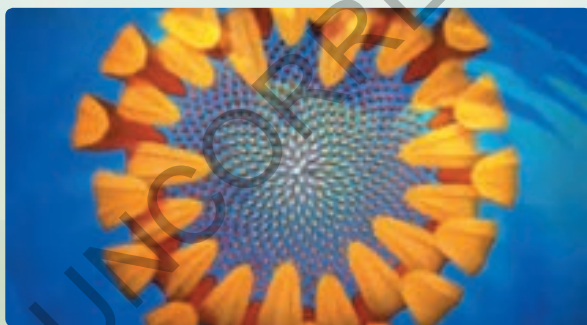
To access eBookPLUS activities, log on to



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Interactivities

A comprehensive set of relevant interactivities to bring difficult mathematical concepts to life can be found in the Resources section of your eBookPLUS.



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studyON is an interactive and highly visual online tool that helps you to clearly identify strengths and weaknesses prior to your exams. You can then confidently target areas of greatest need, enabling you to achieve your best results.

studyon

Unit 4	<Topic title to go here>
AOS 20	
Topic 1	
Concept 3	Sit Topic test



7 Answers

EXERCISE 7.2

- 1 a 10.8
b 28.7
- 2 $40^2 + 96^2 = 104^2$
- 3 a 87.6
b 31.1
- 4 $24^2 - 19.2^2 = 14.4^2$
- 5 a $24^2 + 32^2 = 40^2$
b $180^2 + 75^2 = 195^2$
- 6 $1.5^2 + 2^2 = 2.5^2$
- 7 1.56 m
- 8 No, the maximum length rod that could fit would be 2.71 m long.
- 9 41.4 m
- 10 Pyramid 1 has the greatest height.
- 11 a 6.71
b 13.90
c 7.40
d 11.67
e 38.01
f 33.96
- 12 a 14.51
b 14.50
c 44.50
d 7.99
e 8.00
f 1.69
- 13 a The triangle is right-angled.
b The triangle is not right-angled.
c The triangle is right-angled.
d The triangle is not right-angled.
- 14 11.40
- 15 a 9.43, 6.24
b 73.43, 52.92
c 25.81, 14.70
d 47.38, 8.19
e 11.66, 8.00
f 39.00, 32.73
- 16 a 30.48 cm
b 2.61 cm
c 47.27 cm

- 17 a 14.14 cm
b 33.94 cm
c 3.89 cm
d 117.38 cm
- 18 Yes, 1.015 m
- 19 8.09
- 20 69.46 m
- 21 a 14.70 m
b 14.98 m
c 13.82 m
- 22 a 10 050 m
b 6083 m
c 10 054 m

EXERCISE 7.3

- 1 Perimeter = 80 cm, area = 360 cm^2
- 2 Circumference = 50.27 cm,
area = 201.06 cm^2
- 3 47.8 mm^2
- 4 Triangle 1 has the largest area.
- 5 a Perimeter = 59 m, area = 155.16 m^2
b Perimeter = 28.83 m, area = 20 m^2
c Perimeter = 43.98 cm, area = 153.94 cm^2
d Perimeter = 48 cm, area = 112 cm^2
- 6 Area = 1.14 m^2
- 7 Perimeter = 96 cm, area = 360 cm^2
- 8 a Circumference = 31.42 cm, area = 78.54 cm^2
b Circumference = 56.55 cm, area = 254.47 cm^2
- 9 Perimeter = 68 cm, area = 192 cm^2
- 10 33.79 cm^2
- 11 31.61 cm
- 12 The area of the park is 313.8 m^2 , so it is not of a suitable size to host the barbecue.
- 13 49.19 cm
- 14 a i 201.06 cm ii 389.56 cm^2
b 187.19 cm
- 15 a 26.39 m
b 1104.76 m^2
- 16 a 173.21 m^2
b Perimeter = 90 m, area = 389.71 m^2
c Perimeter = 120 m, area = 839.7 m^2

EXERCISE 7.4

- 1 72 cm^2
- 2 Perimeter = 48.6 cm , area = 106 cm^2
- 3 4.24 cm^2
- 4 Perimeter = 52.94 cm , area = 167.55 cm^2
- 5 9292.83 cm^2
- 6 Perimeter = 151.79 cm , area = 576 cm^2
- 7 Perimeter = 198.23 cm , area = 2073.45 cm^2
- 8 Perimeter = 80.12 cm , area = 1143.06 cm^2
- 9 198.94 cm^2
- 10 a 5.57 m
b 6.11 m^2
- 11 Perimeter = 197 cm , area = 1570 cm^2
- 12 9.42 cm^2
- 13 25.13 cm^2
- 14 a 5832 cm^2
b 56.58 cm^2
- 15 2.50 m^2
- 16 0.68 m^2
- 17 2138.25 cm^2
- 18 Area = 900 cm^2 , perimeter = 194.16 cm

EXERCISE 7.5

- 1 0.6 m^3
- 2 a $20\,500 \text{ cm}^3$
b $13\,415 \text{ cm}^3$
c $126\,720 \text{ cm}^3$
d $51\,188 \text{ cm}^3$
- 3 $56\,301 \text{ cm}^3$
- 4 a $490\,376 \text{ cm}^3$
b 136 m^3
- 5 $39\,584.1 \text{ cm}^3$
- 6 a $753\,218 \text{ cm}^3$
b 0.022 cm^3
- 7 0.0528 m^3
- 8 a $22\,875 \text{ cm}^3$
b $15\,675.12 \text{ cm}^3$
c 0.432 m^3
- 9 0.0824 m^3
- 10 a 39 cm
b 1 m
- 11 21.9 m^3
- 12 a $57\,800 \text{ cm}^3$
b $68\,361.1 \text{ cm}^3$

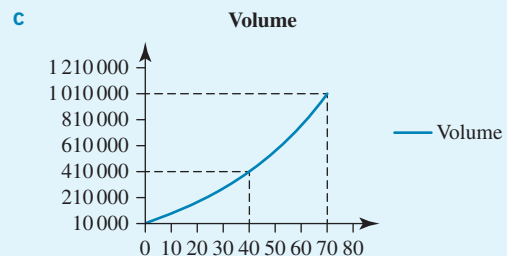
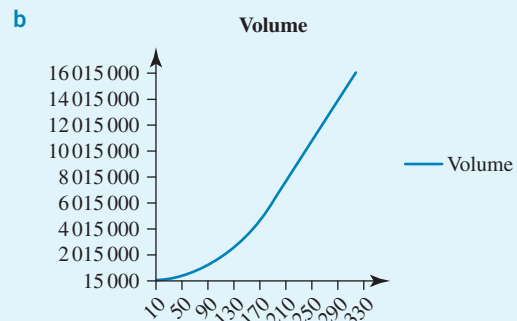
13 Rectangular pool: $11\,070 \text{ litres}$

Square pool: $10\,800 \text{ litres}$

Circular pool: $10\,263.58 \text{ litres}$

- 14 a 25.2 m^3
b 7.2 m^3
- 15 a $14\,850 \text{ cm}^3$
b $471\,250 \text{ cm}^3$
c $45\,804.4 \text{ cm}^3$
d $42\,524.6 \text{ cm}^3$
- 16 Volume above = $162\,577 \text{ cm}^3$,
volume below = $318\,086 \text{ cm}^3$
- 17 a 34.64 litres
b 0.231 m^3
- 18 6.25 m^3
- 19 The hemispherical-topped silo holds 37.35 m^3 more.
- 20 $19\,738.52 \text{ litres}$
- 21 a 8983.3 m^3
b $332\,716\,049.4 \text{ cm}^3$
- 22 a 467.35 cm^3
b 9677.11 cm^3
- 23 a

	A	B
	Cylinder	
1	radius	Volume
2	10	15 708
3	20	62 832
4	40	251 327
5	80	1 005 310
6	160	4 021 239
7	320	16 084 954

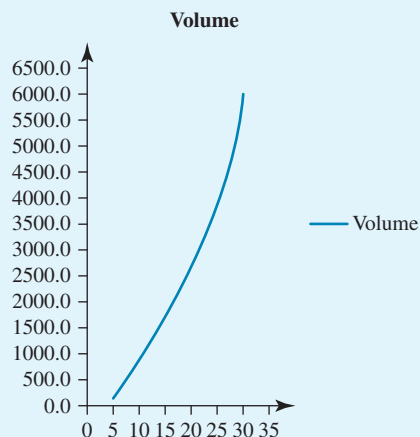


- i Approximately $400\,000 \text{ cm}^3$
- ii Approximately $1\,005\,000 \text{ cm}^3$

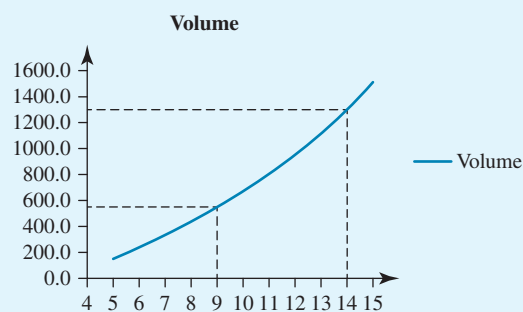
24 a

	A	B
1	Base length	Volume
2	5	166.7
3	10	666.7
4	15	1500.0
5	20	2666.7
6	25	4166.7
7	30	6000.0

b

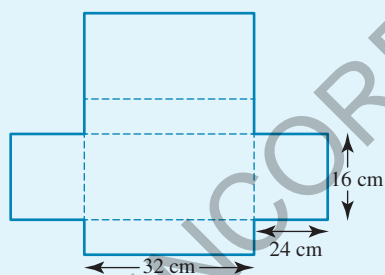


c

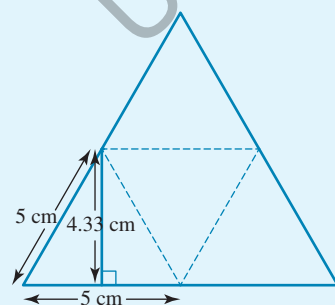
i Approximately 550 cm^3 ii Approximately 1300 cm^3

EXERCISE 7.6

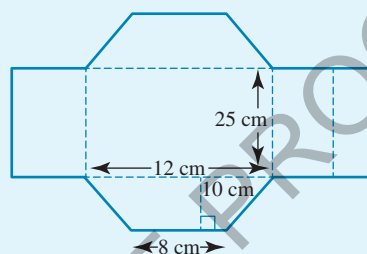
1

 3328 cm^2

2

 43.3 cm^2 3 a 2475 m^2 b 3342 cm^2 4 a 249.4 cm^2 b $120\,687.42 \text{ cm}^2$ c 3487.17 cm^2 d 1588.225 cm^2 5 390.94 m^2 6 7902.99 cm^2 7 a 392 cm^2 b 1639.91 cm^2 c 810 cm^2 d 1256.6 cm^2

8 a

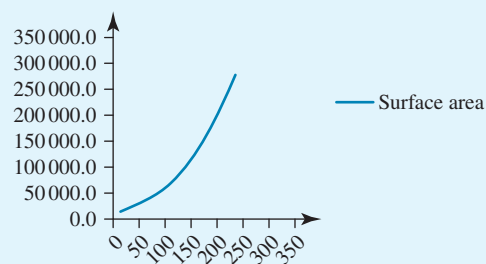
b 1210 cm^2 9 a 236 cm^2 b 487 cm^2 10 The cone with height 6.5 cm and radius 2.2 cm has the greater surface area by 6.34 cm^2 .11 a 5627 cm^2 b 3223 cm^2 12 89.54 m^2 13 688 cm^2 14 a $37\,320 \text{ cm}^2$ b $14\,760 \text{ cm}^2$ 15 104.15 m^2 16 10.22 m^2

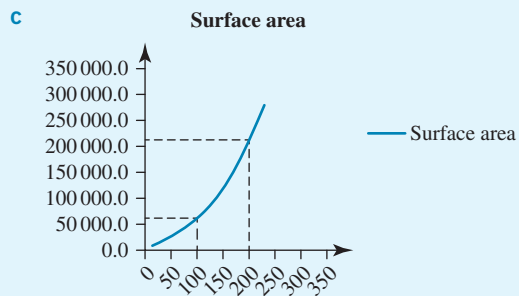
17 a

	A	B
1	Radius	Surface area
2	15	6361.7
3	30	14\,137.2
4	60	33\,929.2
5	120	90\,477.9
6	240	271\,433.6

b

Surface area



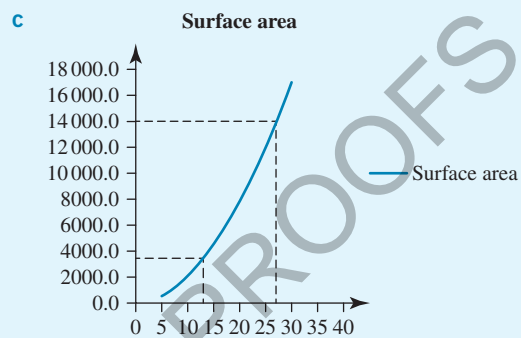
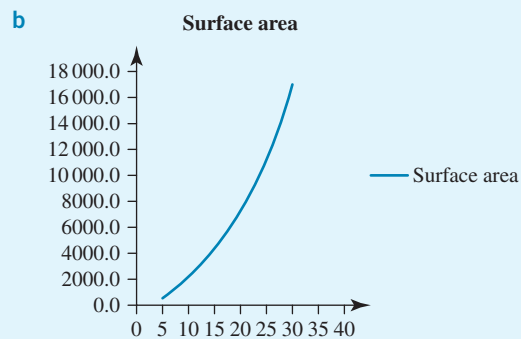


i Approximately 60 000 cm²

ii Approximately 210 000 cm²

18 a

	A	B
1	Radius	Surface area
2	5	471.24
3	10	1884.96
4	15	4241.15
5	20	7539.82
6	25	11 780.97
7	30	16 964.60



i Approximately 14 000 cm²

ii Approximately 3000 cm²