Construction and interpretation of graphs

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14.1 Kick off with CAS

Solving simultaneous equations with CAS

Simultaneous equations are a set of two (or more) equations that contain the same variables. By solving simultaneous equations we are finding the solution(s) which satisfy all of the equations within the set.

To solve simultaneous equations graphically we can sketch graphs of the equations on the same set of axes. The intersection of the two graphs is the solution to the set of simultaneous equations.

1 Using CAS, sketch the graph of \( y = 4x - 3 \).
2 Using CAS, sketch the graph of \( y = 6 - 3x \) on the same set of axes as the graph in question 1.
3 Use CAS to find the point of intersection of both of the graphs you have sketched.
4 Confirm your solution to question 3 by using CAS to solve the pair of simultaneous equations \((y = 4x - 3 \text{ and } y = 6 - 3x)\) algebraically on a calculator page.
5 Use CAS to sketch the graphs of \( y = 2x + 5 \) and \( y = 2x - 1 \) on the same set of axes.
6 Is there a point of intersection between the two graphs you plotted in question 5? What is the relationship between the two equations?

Please refer to the Resources tab in the Prelims section of your eBookPLUS for a comprehensive step-by-step guide on how to use your CAS technology.
Constructing and interpreting straight-line graphs

Equations and graphs are used to study the relationship between two variables such as distance and speed, tax payable and income, or radioactivity and time.

If a relationship exists between the variables, one can be said to be a **function** of the other. A function can be described by a table, a rule or a graph. A function whose graph is a straight line is called a **linear function**.

The general equation of a straight line is \( y = mx + c \), where \( m \) is the gradient and \( c \) is the \( y \)-intercept.

The **\( y \)-intercept** is the value of \( y \) where the graph cuts the \( y \)-axis.

The **gradient** (or slope) of a straight line is denoted by \( m \) where

\[
    m = \frac{\text{rise}}{\text{run}} \quad \text{or} \quad m = \frac{y_2 - y_1}{x_2 - x_1},
\]

A graph that rises as \( x \) increases will have a positive gradient.

A graph that falls as \( x \) increases will have a negative gradient.

**WORKED EXAMPLE 1**

Consider the equation \( y = 3x + 4 \).

**a** What is the value of \( y \) when \( x = 2 \)?

**b** What is the value of \( x \) when \( y = 13 \)?

**c** Does the point \((2, 4)\) lie on the graph of \( y = 3x + 4 \)?

**d** Does the point \((5, 19)\) lie on the graph of \( y = 3x + 4 \)?

**e** State the value of the gradient and the \( y \)-intercept.

**THINK**

- **a** Substitute \( x = 2 \) into the equation.
- **b** Substitute \( y = 13 \) into the equation and solve for \( x \).

**WRITE**

- **a** When \( x = 2 \),
  
  \[ y = 3 \times 2 + 4 \]
  
  \[ = 10 \]

- **b** When \( y = 13 \),
  
  \[ 13 = 3x + 4 \]
  
  \[ 3x = 9 \]
  
  \[ x = 3 \]
In previous years of study you have learned that only one straight line can be drawn through two distinct points. Therefore, the graph of a straight line can be obtained by plotting and joining together any two points on the line. Thus, to sketch a straight-line graph by hand, we first need to find the coordinates of any two points on the line. This can be done in a number of different ways. The three most common methods are outlined as follows.

**Gradient–intercept method**

This method is used if the equation is in \( y = mx + c \) form.

1. The first point plotted is the \( y \)-intercept, given by the value of \( c \). Plot it on a set of axes.
2. The gradient is given by \( m = \frac{\text{rise}}{\text{run}} \). Write the gradient as a fraction and identify the values of the rise and the run.
3. To obtain the second point, start from the \( y \)-intercept and move up (or down) and across, as suggested by the gradient.
4. Join the two points together with a straight line and label the graph.

**WORKED EXAMPLE 2**

Two points \( A (1, 7) \) and \( B (3, 13) \) lie on the same line. Use the points \( A \) and \( B \) to calculate the gradient of the line.

<table>
<thead>
<tr>
<th>THINK</th>
<th>WRITE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Write the gradient formula and match points A and B with ((x_1, y_1)) and ((x_2, y_2)).</td>
<td>[ m = \frac{y_2 - y_1}{x_2 - x_1} ] where ((x_1, y_1) = (1, 7)) and ((x_2, y_2) = (3, 13))</td>
</tr>
</tbody>
</table>
| 2 Substitute the values into the formula and evaluate. | \[ m = \frac{13 - 7}{3 - 1} \]
|               | \[ = \frac{6}{2} \]
|               | \[ = 3 \] The gradient is 3. |

**Sketching straight-line graphs**

In previous years of study you have learned that only one straight line can be drawn through two distinct points. Therefore, the graph of a straight line can be obtained by plotting and joining together any two points on the line. Thus, to sketch a straight-line graph by hand, we first need to find the coordinates of any two points on the line. This can be done in a number of different ways. The three most common methods are outlined as follows.

**Gradient–intercept method**

This method is used if the equation is in \( y = mx + c \) form.

1. The first point plotted is the \( y \)-intercept, given by the value of \( c \). Plot it on a set of axes.
2. The gradient is given by \( m = \frac{\text{rise}}{\text{run}} \). Write the gradient as a fraction and identify the values of the rise and the run.
3. To obtain the second point, start from the \( y \)-intercept and move up (or down) and across, as suggested by the gradient.
4. Join the two points together with a straight line and label the graph.
**x- and y-intercept method**

This method is used if the equation is in $ax + by = c$ form, or if you are required to show both the $x$- and $y$-intercepts.

1. If a point is on the $y$-axis, its $x$-coordinate is 0. To find the $y$-intercept, substitute 0 for $x$ and solve the resultant equation.
2. If a point is on the $x$-axis, its $y$-coordinate is 0. To find the $x$-intercept, substitute 0 for $y$ and solve the resultant equation.
3. Plot the $x$-intercept and the $y$-intercept on a set of axes.
4. Join the two points together with a straight line and label the graph.

**Sketching a line over a required interval**

If a graph needs to be sketched between two given $x$-values, its end points must be shown. Since only two points are needed to sketch a line, we can obtain the coordinates of the end points and join them together. To construct a graph of a straight line between $a$ and $b$, follow these steps:

1. Rearrange the equation to make $y$ the subject.
2. Substitute each of the two given $x$-values (that is, $a$ and $b$) into the equation and find corresponding values of $y$.
3. Plot the end points on a set of axes.
4. Join the two points together with a straight line and label the graph.

Worked example 3 illustrates the use of these three methods.

**WORKED EXAMPLE 3**

Sketch the graph of each of the following equations.

- a $y = 3x + 4$
- b $y = \frac{-2}{3}x$
- c $2x + 3y = 6$
- d $x - y = 3$ between $x = -2$ and $x = 6$.

**THINK**

a 1 The equation is in $y = mx + c$ form, so use the gradient-intercept method. State the value of the $y$-intercept.

2 Write the gradient as a fraction and identify the values of the rise and the run.

3 Draw a set of axes and plot the $y$-intercept. To obtain the second point, start from the $y$-intercept and move up and across as suggested by the positive gradient; that is, 3 units up and 1 unit right. The second point is $(1, 7)$.

4 Join the two points with a straight line and label the graph.
**b 1** The equation is in \( y = mx + c \) form, so use the gradient-intercept method. State the value of the \( y \)-intercept. Since the value of \( c \) is 0, the line passes through the origin.

2 Write the gradient as a fraction and identify the values of the rise and the run. Note that the negative sign always belongs to the rise.

3 Draw a set of axes and plot the \( y \)-intercept. To obtain the second point, start from the origin and move down and across as suggested by the negative gradient; that is, 2 units down and 3 units right. The second point is \((3, -2)\).

4 Join the two points with a straight line and label the graph.

**c 1** The equation is in \( ax + by = c \) form, so find the \( x \)- and \( y \)-intercepts.

2 Find the \( y \)-intercept by substituting \( x = 0 \) into the equation and solving.

3 State the coordinates of the \( y \)-intercept.

4 Find the \( x \)-intercept by substituting \( y = 0 \) into the equation and solving.

5 State the coordinates of the \( x \)-intercept.

6 Plot the intercepts on a set of axes and join them together with a straight line. Label the graph.

**d 1** The graph needs to be sketched between two \( x \)-values. Rearrange the equation to make \( y \) the subject.

2 Substitute each of the two given \( x \)-values (that is, \(-2 \) and \( 6 \)) into the equation and find corresponding values of \( y \).

3 State the coordinates of the end points.
Applications of straight-line graphs

Many real-life situations involve variables whose relationship is linear and hence can be described by a linear rule and represented graphically by a straight line.

When modelling a linear relationship, remember that the \( y \)-intercept represents the value of the function when \( x = 0 \). In most situations, it represents the initial (or original) value of something, a fixed cost or fixed fall.

The gradient represents the rate of change of the \( y \)-value with respect to \( x \). It shows the change (increase or decrease) in \( y \), as \( x \) increases by 1 unit. For example, let the equation \( V = 1000 - 200t \) describe the volume (in litres) of water in a tub \( t \) minutes after a plug is removed. The \( y \)-intercept shows that the initial amount of water in the tub is 1000 litres. The gradient means that the volume of water decreases (since the gradient is negative) at a rate of 200 litres every minute.

\[
\begin{align*}
2 & \div 100 = 0.02 \\
Earnings &= retainer + commission \\
&= 150 + 2\% \times 40000 \\
&= 150 + 0.02 \times 40000 \\
&= 150 + 800 \\
&= 950 \\
\end{align*}
\]

Mikaela earns $950.
Extrapolation and interpolation

Two terms used widely in interpreting data are **extrapolation** and **interpolation**. Extrapolation means to examine the relationship between the variables by extending it beyond the data. Interpolation means to infer the relationship between distinct data points. Worked example 5 should make these meanings clear.

## WORKED EXAMPLE 5

The tension (measured in Newtons) in a spring is linearly related to the extension of the spring (measured in centimetres). Some values relating tension \( T \) and extension \( x \) are given:

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>50</td>
<td>125</td>
<td>175</td>
</tr>
</tbody>
</table>

---

b For each of the four values of sales, repeat the working of a opposite.

c Use the pattern seen in a and b to relate \( E \) and \( S \). Writing the relationship in words may assist in finding the equation.

d 1 Compare the equation relating earnings to sales with that of the general equation of a straight line, \( y = mx + c \). The gradient is the coefficient of the \( x \)-term, \( m \), and the \( y \)-intercept is the constant term, \( c \).

2 Interpret the meaning of the gradient and the \( y \)-intercept.

e Earnings depends on sales. Thus sales is the **explanatory** variable and goes on the horizontal axis. Do not crowd the axes by writing the sales values in thousands — use a legend and write only the number of multiples of ten thousand.

---

<table>
<thead>
<tr>
<th>Sales</th>
<th>Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>150</td>
</tr>
<tr>
<td>10000</td>
<td>350</td>
</tr>
<tr>
<td>20000</td>
<td>550</td>
</tr>
<tr>
<td>80000</td>
<td>1750</td>
</tr>
</tbody>
</table>

c \( Earnings = retainer + 2\% \text{ of sales} \)

\[
E = 150 + 0.02 \times S
\]

or \( E = 0.02S + 150 \)

d The gradient is 0.02 and the \( y \)-intercept is 150.

The gradient represents the commission rate. It shows that for every $100 worth of sales, Mikaela earns $2 in commission. The \( y \)-intercept represents the value of the retainer; that is, the amount Mikaela is paid if no sales are made.
a Plot these data.
b By extrapolating from the data, predict the tension when the extension is 10 cm.
c By interpolating, predict the tension when the extension is 4 cm.

**THINK**

a Plot the data on a set of axes that shows $x$-values up to 10. Draw a line through these points, extending it beyond the given values.

b Extrapolation involves inference beyond the data range. Use the graph to find $T$ when $x$ is 10.

c Interpolation involves inference between the data points. Use the graph to find $T$ when $x$ is 4.

**WRITE/DRAW**

a

\[
\begin{array}{c|c}
\text{Extension (cm)} & \text{Tension (Newtons)} \\
0 & 0 \\
2 & 50 \\
4 & 100 \\
6 & 150 \\
8 & 200 \\
10 & 250 \\
\end{array}
\]

b When $x = 10$, $T = 250$

When the spring is stretched 10 cm, the tension is 250 newtons.

c When $x = 4$, $T = 100$

When the spring is stretched 4 cm, the tension is 100 newtons.

---

**EXERCISE 14.2**

**Constructing and interpreting straight-line graphs**

1 **WE1** Consider the equation of the line $y = 2x + 5$.
   a What is the value of $y$ when $x = 4$?
   b What is the value of $x$ when $y = 17$?
   c Does the point $(3, 12)$ lie on the line?
   d State the value of the gradient and the $y$-intercept.

2 **WE2** Consider the equation $5x - 2y = 10$.
   a Find the value of $y$ when $x = 2$.
   b Find the value of $x$ when $y = 5$.
   c Does the point $(-8, -25)$ lie on the line?

3 **WE2** Two points $P (2, 5)$ and $Q (4, 13)$ lie on the same line.
   Calculate the gradient of the line.

4 The points $(5, -3)$ and $(-2, 4)$ lie on a line. Calculate the gradient of this line.
5 Sketch the graphs of the following functions.
   a \( y = 3x - 4 \)  
   b \( y - 3x = 12 \)  
   c \( y = \frac{3}{4}x \)  
   d \( 4x - 3y = 12 \) between \( x = -2 \) and \( x = 4 \)
6 Sketch the graph of the following functions.
   a \( 5y + x = 10 \)  
   b \( y = -x + 3 \)  
   c \( y = 3x \)  
   d \( 4x - 3y = 12 \) between \( x = -2 \) and \( x = 2 \)
7 Greta sells boats and she is paid $275 per week plus 3% of her weekly sales.
   a How much does Greta receive if the value of her weekly sales is $12000?  
   b Construct a table to show Greta’s weekly pay for the following values of weekly sales: $5000, $10000, $15000 and $20000.  
   c Write an equation that relates her weekly pay \( (P) \) and the value of her weekly sales \( (S) \).  
   d Draw a graph of the relationship between weekly pay and weekly sales. That is, draw a graph of \( P \) versus \( S \). Place \( S \) on the horizontal axis.  
   e State the value of the gradient and the \( y \)-intercept, and interpret their meaning.
8 Sheela, who works as a car salesperson, is paid a retainer of $200 a week and receives 3% commission on her sales.
   a Copy and complete this table.
<table>
<thead>
<tr>
<th>Sales</th>
<th>$0</th>
<th>$20000</th>
<th>$40000</th>
<th>$60000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
   b Write an equation that relates earnings \( (E) \) to sales \( (S) \).  
   c Draw the graph of earnings \( (E) \) versus sales \( (S) \) using the information in a.
9 For a farm water tank, the volume of water in the tank \( (V) \) is related linearly to the depth of water in the tank \( (D) \) for values of \( D \) greater than 450 mm.
<table>
<thead>
<tr>
<th>Depth (mm)</th>
<th>Volume (L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>2500</td>
</tr>
<tr>
<td>700</td>
<td>4500</td>
</tr>
<tr>
<td>10000</td>
<td>7500</td>
</tr>
</tbody>
</table>
   a Plot these data using suitable axes.  
   b By extrapolating from the data, predict the depth needed to give a volume of 13000 litres.  
   c By interpolating from the data, predict the volume when the depth of the water is 600 mm.
10 The distance covered (in metres) of an object over a period of time (in seconds) is shown in the table.
<table>
<thead>
<tr>
<th>Time (s)</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (m)</td>
<td>8</td>
<td>16</td>
<td>20</td>
<td>28</td>
</tr>
</tbody>
</table>
   a Plot these data.  
   b By extrapolating from the data, predict the distance covered when 9 seconds have elapsed.  
   c By interpolating, predict the distance covered when the time elapsed is 6 seconds.
11 Consider the equation of the line $y = -2x + 4$.
   a What is the value of $y$ when $x = 2$?
   b What is the value of $x$ when $y = -4$?
   c Does the point $(0, 4)$ lie on the line?
   d State the value of the gradient and the $y$-intercept.

12 Consider the equation of the line $2y + 3x = 12$.
   a What is the value of $y$ when $x = 2$?
   b What is the value of $x$ when $y = 6$?
   c Does the point $(3, 2)$ lie on the line?
   d State the value of the gradient and the $y$-intercept.

13 Consider the equation of the line $2y + x = 8$.
   a What is the value of $y$ when $x = 2$?
   b What is the value of $x$ when $y = 1$?
   c Does the point $(3, 2)$ lie on the line?
   d Using the two points whose coordinates were found in a and b, calculate the gradient.

14 Calculate the gradient of the line shown in each of the following.

15 Sketch the graphs of the following functions.
   a $3x + y = 12$
   b $y = -2x + 8$
   c $y = -2x$
   d $y = 2x + 1$ between $x = -2$ and $x = 2$

16 The graphs of the following lines are shown at right.
   i $y + x = 4$
   ii $y = x + 4$
   iii $y + x + 4 = 0$
   iv $y = x - 4$

   The best match between graphs P to S and equations i to iv is:
   A P → iv  Q → i  R → iii  S → ii
   B P → i  Q → iii  R → ii  S → iv
   C P → iv  Q → i  R → ii  S → iii
   D P → iv  Q → iii  R → i  S → ii
   E P → ii  Q → iv  R → i  S → iii
17 An object that falls freely due to gravity increases its speed ($S$) by 9.8 m/s each second. Assume that at the start the speed of the object was 0.
   a How fast is the object travelling after 3 seconds?
   b How long has the object been falling when its speed is 78.4 metres per second?
   c If $t$ stands for the number of seconds the object has been falling, write a formula that relates $S$ to $t$.
   d Draw a graph of $S$ versus $t$.

18 Visa car rentals charge $75 per day plus $15 per hundred kilometres.
   a How much would it cost to rent a car for one day if the car travelled 345 km?
   b The bill for one day’s rental came to $142.50. How many kilometres did the car travel?
   c Sketch a graph of the cost of renting the car for one day ($C$) versus the number of kilometres travelled ($d$).

19 A psychology experiment is testing the relationship between the number of errors ($n$) made by a child on a task and the time taken to complete the task in seconds ($t$). The results of five trials of the experiment are given in the table below.
   If the number of errors made is linearly related to the time taken, which experiment does not fit the pattern?

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Time taken (seconds)</th>
<th>Number of errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>30</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>45</td>
<td>6</td>
</tr>
<tr>
<td>C</td>
<td>36</td>
<td>9</td>
</tr>
<tr>
<td>D</td>
<td>48</td>
<td>7</td>
</tr>
<tr>
<td>E</td>
<td>42</td>
<td>8</td>
</tr>
</tbody>
</table>

20 Janine sells cosmetics at a department store. She knows she is paid a retainer plus commission on sales but she is not sure of the exact rates. For three weeks she records her wages and the value of product she sold during that week.

<table>
<thead>
<tr>
<th>Week</th>
<th>Sales</th>
<th>Wages</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1200</td>
<td>$494</td>
</tr>
<tr>
<td>2</td>
<td>$750</td>
<td>$440</td>
</tr>
<tr>
<td>3</td>
<td>$880</td>
<td>$455.60</td>
</tr>
</tbody>
</table>

   a In week 4 her sales totalled $1000. Predict her wage for that week.
   b Write a formula to calculate Janine’s weekly wage in terms of her weekly sales.

21 In a one-day international cricket match, each team bats for 50 overs. After 20 overs Australia’s score had reached 108 runs. If the target for Australia is 250 runs at the end of 50 overs:
   a how many runs per over does the team need to score for the remaining 30 overs?
   b Write an equation to relate the score ($S$) to the number of overs completed ($n$).
   c The graph of $S$ versus $n$ could be a straight line. What would be the gradient of that straight line?
22 Taxi hire charges are shown in the diagram.
   a Calculate the cost of travelling 15 km in a taxi.
   b If the taxi fare was $20.80, how far did the taxi travel?
   c Write an equation relating the fare \( F \) to the distance \( d \).
   d Draw a graph relating \( F \) to \( d \).
   e What is the slope of this graph?

14.3 Line segments and step functions

In this section we consider graphs which are not straight lines but are made from straight lines. We also consider graphs which are not straight lines but discrete sets of points.

WORKED EXAMPLE 6

When a real estate agent sells a property, he earns commission at the following rate:

1.5% on the first $20000
0.9% on the remainder.

a Calculate the commission earned on sales of:
   i $10 000
   ii $20 000
   iii $30 000
   iv $40 000.

b Draw a line segment graph of commission \( C \) versus the value of the sales \( S \) up to a sales value of $40 000.

c Give a reason for the difference of the slopes of the two segments.

THINK

a i For any amount up to and including $20 000, the commission is 1.5% of the amount.

ii The commission is 1.5% of the amount.

iii $30 000 is more than $20 000, so calculate 1.5% on $20 000 plus 0.9% on the remainder over $20 000; that is, $10 000.

WRITE/DRAW

a i Commission on $10 000
   = 1.5% of 10 000
   = 0.015 \times 10 000
   = $150

ii Commission on $20 000
   = 1.5% of 20 000
   = 0.015 \times 20 000
   = $300

iii Commission on $30 000
   = 1.5% of $20 000 + 0.9% of $10 000
   = 0.015 \times 20 000 + 0.009 \times 10 000
   = 300 + 90
   = $390
$40 000 is more than $20 000, so calculate 1.5% on $20 000 plus 0.9% on the remainder over $20 000; that is, $20 000.

**iv** Commission on $40 000

\[
= 1.5\% \text{ of } 20000 + 0.9\% \text{ of } 20000 \\
= 0.015 \times 20000 + 0.009 \times 20000 \\
= 300 + 180 \\
= $480
\]

**b** Since the commission rate changes when sales exceed $20 000, the graph will consist of two segments. One segment for all sales up to and including $20 000 and the other for sales of more than $20 000 and up to and including $40 000.

2 The starting point of the first segment is (0, 0) (since there is no commission when there are no sales). The end point of this segment is (20000, 300) (as was found in part **a (ii)**). Plot these points and join them with a straight line.

3 The second segment starts where the first segment ends. The end point of the second segment is (40 000, 480) (as was found in part **a (iv)**). Plot this end point and join it to the first segment.

**c** The slope of the line is determined by its gradient. In this case, the gradient is the rate of commission. When the sales exceed $20 000, the commission rate decreases — this results in a line segment that is less steep.

**WORKED EXAMPLE 7**

An electrician charges the rates shown.

**a** Calculate the charges for service calls of the following durations:
- 10 minutes
- 25 minutes
- 40 minutes
- 55 minutes
- 70 minutes
- 85 minutes

**b** Draw a step graph of charges ($C$) versus the time of the service call in minutes ($t$) for calls of up to 120 minutes.

$35 \text{ call-out fee plus } $30 \text{ per half hour or part thereof.}$
THINK

a ‘Part thereof’ means that whether the repairs take 3 minutes, 10 minutes or 30 minutes, you are charged for 30 minutes. For a service call up to 30 minutes, the electrician charges $35 call-out fee plus $30. For a service call over 30 minutes long and up to 60 minutes, the charge is $35 call-out fee plus 2 lots of $30. Finally, for a service call over 60 minutes and up to 90 minutes, the charge is $3 call-out fee plus 3 lots of $30.

WRITE/DRAW

<table>
<thead>
<tr>
<th>Time</th>
<th>Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 min</td>
<td>$35 + 30 = $65</td>
</tr>
<tr>
<td>25 min</td>
<td>$35 + 30 = $65</td>
</tr>
<tr>
<td>40 min</td>
<td>$35 + 2 x 30 = $95</td>
</tr>
<tr>
<td>55 min</td>
<td>$35 + 2 x 30 = $95</td>
</tr>
<tr>
<td>70 min</td>
<td>$35 + 3 x 30 = $125</td>
</tr>
<tr>
<td>85 min</td>
<td>$35 + 3 x 30 = $125</td>
</tr>
</tbody>
</table>

b Draw the graph. Place $t$ along the horizontal axis as it is the explanatory variable. The graph consists of four horizontal sections (steps). This reflects the fact that the fixed fee is charged for any amount of time up to and including 30 minutes; then a different fixed fee for any amount of time above 30 minutes and up to and including 60 minutes, and so on. Note that where the end point of the step is included in the step, it is shown as a full circle (closed circle), whereas if the end point is not included, it is shown as an empty circle (open circle).

In the previous example, the explanatory variable, time ($t$), was a continuous variable; that is, $t$ can take any values — 2, 15.5, 87 — and so on. In the next example we consider a situation where the explanatory variable can take whole number values only. A variable that is not continuous is called a discrete variable. If the data are discrete, the points on the graph are not joined together.

WORKED EXAMPLE 8

A bakery sells bread rolls for 50 cents each or $2.50 for 6.

a Calculate the cost of 4, 5, 6, 7, 11, 12, 13, 17, 18 and 19 bread rolls.

b Sketch this information on a graph for up to 24 rolls.

THINK

a Calculate the cost for each number of bread rolls. If the number of rolls is above 5, it can be formed by combining pack(s) of 6 rolls and single rolls. For example, 7 rolls can be bought as a pack of 6 plus 1 single roll; 17 rolls as 2 packs of 6 plus 5 single rolls etc.

WRITE/DRAW

<table>
<thead>
<tr>
<th>Number of rolls</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$4 \times 50c = $2.00</td>
</tr>
<tr>
<td>5</td>
<td>$5 \times 50c = $2.50</td>
</tr>
<tr>
<td>6</td>
<td>$2.50</td>
</tr>
<tr>
<td>7</td>
<td>$2.50 + 1 \times 50c = $3.00</td>
</tr>
<tr>
<td>11</td>
<td>$2.50 + 5 \times 50c = $5.00</td>
</tr>
<tr>
<td>12</td>
<td>$2 \times 2.50 = $5.00</td>
</tr>
<tr>
<td>13</td>
<td>$2 \times 2.50 + 1 \times 50c = $5.50</td>
</tr>
<tr>
<td>17</td>
<td>$2 \times 2.50 + 5 \times 50c = $7.50</td>
</tr>
<tr>
<td>18</td>
<td>$3 \times 2.50 = $7.50</td>
</tr>
<tr>
<td>19</td>
<td>$3 \times 2.50 + 1 \times 50c = $8.00</td>
</tr>
</tbody>
</table>
EXERCISE 14.3

### PRACTISE

**study on**

- Unit 4
- AOS M4
- Topic 1
- Concept 2
  - Segment graphs
    - Concept summary
    - Practice questions

- Unit 4
- AOS M4
- Topic 1
- Concept 3
  - Step graphs
    - Concept summary
    - Practice questions

---

**Line segments and step functions**

1. **WE6** Suppose a real estate agent is paid commission at the following rates:
   - 1.5% on the first $40,000 and 1% on the remainder.
   - a. Calculate the commission due on sales of $20,000, $30,000, $40,000, $50,000 and $60,000.
   - b. Draw a graph of commission (C) versus sales (S).
   - c. Give a reason for the difference of the slopes of the two segments.

2. Beena, who runs a real estate agency, sells a property and earns commission at the following rate:
   - 2% on the first $25,000
   - 0.8% on the remainder.
   - Calculate the commission earned by Beena on sales of:
     - a. $10,000
     - b. $25,000
     - c. $35,000
     - d. $45,000.
   - e. Construct a graph of commission (C) versus the value of sales (S).

3. **WE7** A plumber charges these rates:
   - $55 call out fee
   - $40 per half hour or part thereof.
   - a. Using this information, construct the graph of charges (C) versus time of the service call, in minutes (t), for calls of up to 120 minutes.
   - b. Calculate the charges for these service calls:
     - 12 minutes, 23 minutes, 44 minutes, 56 minutes, 73 minutes, 87 minutes.

4. A telephone company charges users at a rate of 25 cents for each completed 30 seconds. This implies a call of less than 30 seconds is free.
   - a. Copy and complete this table for the calls shown.
   - b. Construct the graph of cost versus length of call for calls of up to 120 seconds.

<table>
<thead>
<tr>
<th>Length of call (seconds)</th>
<th>Cost (cents)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td></td>
</tr>
<tr>
<td>105</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td></td>
</tr>
</tbody>
</table>
5. A country bakery sells buns for 40 cents each or 6 for $2.00.
   a. Using this information, construct a graph for buying up to 20 buns using the number of buns as the independent variable.
   b. Calculate the cost of 2, 4, 6, 8 and 10 buns.

6. The Explorindo Travel Company specialises in surfing tours of remote islands in Indonesia. They will take individuals but prefer to deal with groups of people. They have the following charges for a holiday package:
   - 1 person: $900
   - 2 people: $1650
   - Each extra person: $600
   a. Draw a table with the following headings and complete it for costs for 1, 2, 3, 6, 8 and 10 people.
   b. Draw the graph of total cost versus number of people. (Include only the number of people discussed in part a.)

This information relates to questions 7 and 8. The amount of electricity used around the home is measured in kilowatt hours (kWh). A light bulb left on for 10 hours will consume about 1 kWh of power. The local power supplier charges at the following rates.

<table>
<thead>
<tr>
<th>Power</th>
<th>Cost per kWh ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First 400 kWh</td>
<td>0.20</td>
</tr>
<tr>
<td>Next 1000 kWh</td>
<td>0.15</td>
</tr>
<tr>
<td>Remaining kWh</td>
<td>0.10</td>
</tr>
</tbody>
</table>

7. Copy and complete the following table by calculating the cost due for each of the consumptions:

<table>
<thead>
<tr>
<th>Consumption (kWh)</th>
<th>Power bill ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td></td>
</tr>
<tr>
<td>600</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td></td>
</tr>
</tbody>
</table>

8. Using the data in the table, draw a graph of power bill versus consumption.

9. An electrician charges at the following rates:
   - $45 call out fee plus $35 per half hour or part thereof.
   a. Calculate the charges for the following service calls:
      - 20 minutes, 30 minutes, 45 minutes, 60 minutes, 80 minutes and 90 minutes.
   b. Draw a graph of charges (C) versus time of the service call in minutes (t) for calls of up to 90 minutes.
10 A mobile phone company charges users a rate of 15 cents for each completed 20 seconds of the call. This means a call of less than 20 seconds is free.

a Draw a table with the following headings and complete it for calls of 10, 20, 30, 40, 50, 60, 70, 80, and 90 seconds.

<table>
<thead>
<tr>
<th>Length of call (seconds)</th>
<th>Cost (cents)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b Draw the graph of cost versus length of call for calls of up to 90 seconds.

11 Which of the following is clearly a discrete variable?

A the time taken for a phone call
B the number of CDs in a collection
C the commission earned on sales
D the power consumed by a hot-water jug
E a person’s weight

12 On a particular visit, a tradesperson charges the householder $100. It was noted he arrived at 11.30 am. Which of the following departure times could be correct?

A 11.38 am  B 11.48 am  C 12.05 pm  D 12.20 pm  E 12.35 pm

13 This is the 2011–12 tax table.

<table>
<thead>
<tr>
<th>Taxable income</th>
<th>Tax on this income</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–$6000</td>
<td>Nil</td>
</tr>
<tr>
<td>$6001–$37000</td>
<td>15c for each $1 over $6000</td>
</tr>
<tr>
<td>$37001–$80000</td>
<td>$4650 plus 30c for each $1 over $37000</td>
</tr>
<tr>
<td>$80001–$180000</td>
<td>$17550 plus 37c for each $1 over $80000</td>
</tr>
<tr>
<td>$180001 and over</td>
<td>$54550 plus 45c for each $1 over $180000</td>
</tr>
</tbody>
</table>

Use this tax table to calculate the amount of tax paid in 2012 by people with taxable incomes of:

a $4000  b $7000  c $20000  d $35000  

e $40000  f $60000  g $100000  h $20000.
14. **a** Using the values calculated in question 13 and any other values, draw the graph of income tax versus taxable income.
   **b** Use the graph to estimate the income tax payable on a taxable income of:
   1. $24,000
   2. $95,000.
   Confirm your answers with calculations.

15. This is the 2014–15 tax table.

<table>
<thead>
<tr>
<th>Taxable income</th>
<th>Tax on this income</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–$18,200</td>
<td>Nil</td>
</tr>
<tr>
<td>$18,201–$37,000</td>
<td>19c for each $1 over $18,200</td>
</tr>
<tr>
<td>$37,001–$80,000</td>
<td>$3572 plus 32.5c for each $1 over $37,000</td>
</tr>
<tr>
<td>$80,001–$180,000</td>
<td>$17,547 plus 37c for each $1 over $80,000</td>
</tr>
<tr>
<td>$180,001 and over</td>
<td>$54,547 plus 45c for each $1 over $180,000</td>
</tr>
</tbody>
</table>

Use this tax table to calculate the amount of tax paid in 2015 by people with taxable incomes of:

a. $4000  
b. $7000  
c. $20,000  
d. $35,000  
e. $40,000  
f. $60,000  
g. $100,000  
h. $200,000.

16. **a** Use the values calculated in question 15 to draw a graph of income tax versus taxable income.
   **b** Use the graph to estimate the income tax payable on a taxable income of:
   1. $24,000
   2. $95,000.
   Confirm your answers with calculations.
   **c** Compare your answers to questions 14b and 16b.

17. What situation could be described by the following graphs? In your response, clearly identify the variables used on both axes and explain how the graph represents that situation. For example, the first graph could measure the profit versus the number of units of production.

18. ‘Fix-it-fast’ is a photocopier repair service for schools in North-central Victoria. The call-out charge depends on the distance the repair person has to travel. The call-out fees for distances up to 85 km are shown on the following graph.

   **a** i What is the call-out fee for a distance of 40 km?
   ii What is the maximum distance travelled for a call-out fee of $50?

   A call-out fee of $125 is charged for schools at a distance of more than 85 km but less than 120 km.
   **b** Copy the shown graph and add this information to it.
Simultaneous equations and break-even point

In this topic, we have seen that when an equation such as \( y = 3x - 4 \) is graphed, it forms a straight line. Each point on the line will have an \( x \)- and a \( y \)-coordinate, and these values of \( x \) and \( y \) satisfy the equation. The reverse is also true. Each pair, \( x \) and \( y \), which satisfy the equation, will be the coordinates of a point on the line.

Solving linear simultaneous equations

If we draw the graph of \( y = 3x - 4 \) and \( y = x + 8 \) on the same set of axes, the result is shown at right. The point where the lines intersect (6, 14) is called the simultaneous solution because \( x = 6 \) and \( y = 14 \) satisfies both equations.

In this example, we found the simultaneous solution graphically; that is, from the graph. It is also possible to find the simultaneous solution algebraically, either by the substitution method or the elimination method.

WORKED EXAMPLE 9

Find the simultaneous solutions to these equations using algebraic methods.

a \( y = 3x - 4 \) and \( y = x + 8 \)

b \( 2x + 3y = 13 \) and \( x - 4y = 1 \)

THINK

a 1 Since both equations start with \( y = \), solve using the substitution method. Use equation [1] to substitute for \( y \) in equation [2].

2 Solve equation [3].

3 Find \( y \) by substituting \( x = 6 \) into one of the original equations.

4 State the solution.

b 1 If one of the equations was rearranged to a \( y = mx + c \) format, the substitution method could be used. However, it will be more efficient to use the elimination method in this case.

2 Multiply equation [2] by 2 so that the \( x \)-coefficients of both equations are the same.

WRITE

\[
\begin{align*}
\text{a} & \quad y = 3x - 4 & [1] \\
\text{a} & \quad y = x + 8 & [2] \\
\text{a} & \quad \text{Substitute } [1] \text{ into } [2] \\
\text{a} & \quad 3x - 4 = x + 8 & [3] \\
\text{a} & \quad 3x - x = 8 + 4 \\
\text{a} & \quad 2x = 12 \\
\text{a} & \quad x = 6 \\
\text{b} & \quad \text{Substitute } x = 6 \text{ into } [2]: \quad y = 6 + 8 \\
\text{b} & \quad \text{The solution is } (6, 14). \\
\text{b} & \quad [2] \times 2: \quad 2x - 8y = 2 & [3]
\end{align*}
\]
Break-even analysis

The aim of most businesses is to make a profit. The profit depends on the costs associated with the business (labour, raw materials and plant) and its revenue (the money it earns through sales). It represents the difference between the revenue and the costs.

\[ \text{Profit} = \text{revenue} - \text{costs} \]

It is evident that a profit will occur if the revenue exceeds the costs. However, if the costs exceed the revenue, a loss will result. Finally, if the costs equal the revenue, there will be neither a profit nor a loss. This is referred to as a **break-even point**.

The diagram shows the graph of a cost function and a revenue function, drawn on the same set of axes. The point of intersection of the two lines represents the point at which costs and revenue are equal; that is, the break-even point. To the left of the
break-even point, the cost line is above the revenue line. This means that the costs are higher than the revenue and will result in a loss. To the right of the break-even point, the cost line is below the revenue line. This means that costs are lower than the revenue and will result in a profit.

**WORKED EXAMPLE 11**

The cost associated with publishing a particular maths book \((C)\) is given by

\[ C = 12n + 24000 \]

where \(n\) represents the number of books. The revenue \((R)\) made from selling \(n\) books is given by \(R = 28n\). Both \(C\) and \(R\) are in dollars.

**a** Copy and complete the following table.

<table>
<thead>
<tr>
<th>Number of books ((n))</th>
<th>Costs ($)</th>
<th>Revenue ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**b** Sketch the graph of the costs \((C)\) versus number \((n)\), and the graph of revenue \((R)\) versus number \((n)\) on the same set of axes.

**c** How many books need to be published and sold so that the revenue equals the costs?

**d** State the coordinates of the break-even point and interpret its meaning.

**THINK**

**a** 1 Write the equations for the costs \((C)\) and the revenue \((R)\).

2 Calculate the value of \(C\) and \(R\) for each value of \(n\) given in the table.

**WRITE/DRAW**

When \(n = 500\),
\[ C = 12 \times 500 + 24000 = 30000 \quad R = 28 \times 500 = 14000 \]
When \(n = 1000\),
\[ C = 12 \times 1000 + 24000 = 36000 \quad R = 28 \times 1000 = 28000 \]
When \(n = 1500\),
\[ C = 12 \times 1500 + 24000 = 42000 \quad R = 28 \times 1500 = 42000 \]
When \(n = 2000\),
\[ C = 12 \times 2000 + 24000 = 48000 \quad R = 28 \times 2000 = 56000 \]

3 Use the results from the calculations in step 2 to complete the table.

<table>
<thead>
<tr>
<th>Number of books ((n))</th>
<th>Costs ($)</th>
<th>Revenue ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>30000</td>
<td>14000</td>
</tr>
<tr>
<td>1000</td>
<td>36000</td>
<td>28000</td>
</tr>
<tr>
<td>1500</td>
<td>42000</td>
<td>42000</td>
</tr>
<tr>
<td>2000</td>
<td>48000</td>
<td>56000</td>
</tr>
</tbody>
</table>
b The number of books \((n)\) is the explanatory variable, so place it on the horizontal axis. The graphs of the cost and revenue functions are straight lines, so they can be constructed by plotting any two points from the table then joining them with a straight line.

c From the table (or the graph), we can see that when \(n = 1500\), the costs and the revenue are both the same.

d The break-even point is the point where the costs equal the revenue. On the graph, it is the point of intersection of the two lines.

Note that the break-even point in the previous worked example could have been found using CAS or algebraically.

**EXERCISE 14.4**

**Simultaneous equations and break-even point**

1. Solve the following simultaneous equations.
   
   \[
   y = 10x - 7 \\
   y = 2x + 1
   \]

2. Solve the following simultaneous equations.
   
   \[
   6x - 11y = 2 \\
   5x - 9y = 1
   \]

3. Find the simultaneous solution of \(y = 2x - 3\) and \(y = -x\) using a graphical method.

4. Find the simultaneous solution of \(y = -2x - 3\) and \(y = x + 3\) using a graphical method.

5. The cost of manufacturing toys \((C)\) is related to the number of toys produced \((n)\), by the formula \(C = 600 + 3n\). The revenue \((R)\) made from selling \(n\) toys is \(R = 7n\). Both \(C\) and \(R\) are in dollars.
a Copy and complete the following table.

<table>
<thead>
<tr>
<th>Number of toys</th>
<th>Cost ($)</th>
<th>Revenue ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b Sketch the graph of cost ($C$) versus number ($n$) and the graph of revenue ($R$) versus number ($n$) on the same set of axes.

c How many toys need to be produced before revenue equals cost?

d State the coordinates of the break-even point and interpret its meaning.

6 The cost of manufacturing basketballs ($C$) is related to the number of basketballs produced ($n$) by the formula $C = 2800 + 4n$. The revenue ($R$) made from selling $n$ basketballs is $R = 14n$.

a Copy and complete the table below.

<table>
<thead>
<tr>
<th>Number of basketballs</th>
<th>Cost ($)</th>
<th>Revenue ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>250</td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>350</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b Using the information supplied, construct the graph of cost ($C$) versus number ($n$), and the graph of revenue ($R$) versus number ($n$), on the same set of axes.

c Using the graph, write the number of basketballs produced before revenue equals cost to ‘break even’.

7 Find the simultaneous solution, algebraically, to:

a $y = 4x - 4$

b $y = 3x - 4$

c $2y + x = 5$

8 Find the simultaneous solution of each pair of linear equations in question 7 using a graphical method.

9 Find the simultaneous solution of $y = 3x - 10$ and $x = 2y + 5$ using the substitution method.

10 Find the simultaneous solution of $3x - y = -8$ and $2x + 2y = 0$ using the elimination method.

11 Find the simultaneous solution of $y = x$ and $y = -x$ using a graphical method.

12 Find the simultaneous solution of $-3x + 2y = 1$ and $-2x + 3y = 9$ using the elimination method.

13 Consider the phone connection plans shown at left.

a Copy and complete the following table.

<table>
<thead>
<tr>
<th>Call time (minutes)</th>
<th>Cost ($) — Plan A</th>
<th>Cost ($) — Plan B</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
b Sketch the graph of cost versus time for each of the two plans on the same set of axes.

c How many minutes of calls would you need to make for both plans to cost the same?

14 The cost of manufacturing electronic components ($C$) is related to the number of components produced ($n$), by the formula $C = 6000 + 2.5n$. The revenue ($R$) made from selling $n$ components is $R = 4.5n - 8000$. Both $C$ and $R$ are in dollars.

a Copy and complete the following table.

<table>
<thead>
<tr>
<th>Number of components</th>
<th>Cost ($)</th>
<th>Revenue ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b Sketch the graph of cost ($C$) versus number ($n$) and the graph of revenue ($R$) versus number ($n$) on the same set of axes.

c How many components need to be produced before revenue equals cost?

15 A new employee, whose job it is to sell a software package, is offered two different salary plans by Minitech:

Plan A: $400 per week plus $25 for each package sold
Plan B: $150 per week plus $45 for each package sold.

a Copy and complete the following table.

<table>
<thead>
<tr>
<th>Number of packages sold</th>
<th>Salary ($) Plan A</th>
<th>Salary ($) Plan B</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b Sketch the graph of salary versus number of packages sold for both Plan A and Plan B on the same set of axes.

c How many packages need to be sold before Plan B is the better choice?

16 The statement that best matches the graph is:

A revenue exceeds costs when more than 50 units are sold
B revenue exceeds costs when less than 50 units are sold
C revenue exceeds costs when more than 1500 units are sold
D revenue exceeds costs when less than 1500 units are sold
E revenue exceeds costs when exactly 50 units are sold.

17 A factory producing mattresses finds that the equation linking cost in dollars ($C$) and the number of mattresses produced ($m$) is $C = 800m$. 
a What is the cost of producing 10 mattresses?
b How many mattresses could be produced for $124 000?

18 From question 17, if all mattresses are sold, the revenue in dollars \( R \) from the sale of \( m \) mattresses is \( R = 1800m \).

a Draw graphs of \( C \) and \( R \) on the same set of axes for \( 0 \leq m \leq 200 \).
b Determine the number of mattresses which would need to be sold for the factory to break even.
c From the two equations given, write a profit equation \( (P) \).
d Calculate the profit if 175 mattresses were produced and sold.

### Interpreting non-linear graphs

Why are graphs used so widely in papers, magazines, journals, in education, in government, and in sales and marketing? Graphs are used because they have the capacity to convey a significant amount of information effectively.

To get the most from a graph, a user needs to learn to read and interpret information presented graphically. In this section, we will look at non-linear graphs. A non-linear graph is a graph which is not a straight line.

**WORKED EXAMPLE 12**

The graph shows the distance a cyclist is from her home over a period of 120 minutes.

a How far from her home was the cyclist at the start of the time period?
b At what speed did the cyclist travel for the first 30 minutes?
c Describe the motion of the cyclist after 45 minutes.
d What was the cyclist’s furthest distance from home?
e When did the cyclist turn for home?
f How long did the cyclist take to get home on the way back?

**THINK**

a The start of the time period means \( t = 0 \) min.
The graph shows a distance of 6 km when \( t = 0 \).
b Use the relationship: \( \text{speed} = \frac{\text{distance}}{\text{time}} \).
The cyclist travels from 6 km to 12 km in 30 minutes or 0.5 hour.

**WRITE**

a The cyclist was 6 km from home at the start of the time period.
b Speed = \( \frac{\text{distance}}{\text{time}} \) 
\[ = \frac{6 \text{ km}}{0.5 \text{ h}} \]
\[ = 12 \text{ km/h} \]
The value of a car originally worth $30,000 decreases over time. The graph describes the value of the car as a percentage of its starting price.

a. What is the value of the car, in percentage terms, after 5 years?

b. The ‘half-life’ is the time taken for the value to decrease by half. How long does it take for the car to lose half its original value?

c. How long does it take for the car to fall in value from 50% of its original value to 25% of its original value?

d. Estimate, in dollar terms, the value of the car after 30 years.

**THINK**

a. Read directly from the graph.

b. ‘Half’ means 50%. Reading from the graph, it takes 10 years for the value of the car to fall to 50%.

c. From the graph, the car is worth 50% of the original price at $t = 10$ and it is worth 25% at $t = 20$. Note that the half-life is also the time taken to fall from 50% to 25%.

d. As the half-life is 10 years, use this to calculate the value after 30 years. That is, after 3 half-lives.

**WRITE**

a. The value of the car after 5 years is approximately 70% of its original value.

b. The time taken to fall in value by 50% (half-life) is 10 years.

c. Time taken to fall from 50% to 25% of original value

\[ = 20 - 10 \text{ years} \]

\[ = 10 \text{ years}. \]

d. Half-life $= 10$ years

Therefore, the value after:

- 10 years is 50% 
- 20 years is 25% 
- 30 years is 12.5%.

The value after 30 years

\[ = 12.5\% \times 30000 \]

\[ = 0.125 \times 30000 \]

\[ = 3750 \]
EXERCISE 14.5  Interpreting non-linear graphs

1. The graph shows the distance a skateboarder is from the skate park over a period of 30 minutes.
   a. How far from the skate park was the boarder at the start of the time period?
   b. At what speed did the boarder travel for the first 10 minutes?
   c. Describe the motion of the boarder after 15 minutes.
   d. What was the boarder’s furthest distance from the skate park?
   e. When did the boarder start travelling towards the skate park?
   f. How long did it take the boarder to get back to the skate park on the way back?

2. The graph at right shows the distance a runner is from the finish line over a period of 40 minutes.
   a. How far from the finish line was the runner at the start of the time period?
   b. At what speed did the runner travel for the first 15 minutes?
   c. Describe the motion of the runner after 25 minutes.
   d. What was the runner’s furthest distance from the finish line?
   e. When did the runner start travelling towards the finish line?
   f. How long did it take the runner to get back to the finish line on the way back?

3. The value of a ute originally worth $42,000 decreases over time. The graph describes the value of the car as a percentage of its starting price.
   a. What is the value of the ute, in percentage terms, after 5 years?
   b. The ‘half-life’ is the time taken for the value to decrease by half. How long does it take for the car to lose half its original value?
c How long does it take for the car to fall in value from 50% of its original value to 25% of its original value?
d Estimate, in dollar terms, the value of the car after 20 years.

4 The value of a boat originally worth $37,500 decreases over time. The graph drawn describes the value of the boat as a percentage of its starting price.
a What is the value of the boat, in percentage terms, after 3 years?
b The ‘half-life’ is the time taken for the value to decrease by half. How long does it take for the boat to lose half its original value?
c How long does it take for the boat to fall in value from 50% of its original value to 40% of its original value?
d Estimate, in dollar terms, the value of the boat after 15 years.

5 The graph shown gives the distance of a cyclist from her home over a period of 120 minutes.
a How far from her home was the cyclist at the start of the time period?
b At what speed did the cyclist travel for the first 30 minutes?
c Describe the motion of the cyclist after 7 minutes.
d What was the cyclist’s furthest distance from home?
e When did the cyclist begin to travel home?
f How long did the cyclist take to get home?
g What was the average speed of the cyclist on the journey home?

6 In an experiment, a ball is dropped from a height of 20 m. The height of the ball above ground level, as it bounces, is given in the graph.
a How long did it take the ball to reach the ground for the first time?
b How long was the ball in the air between the second and third bounce?
c What total distance is travelled by the ball in 7 seconds?
d Calculate the average speed of the ball for the first 2 seconds.
7 The value of a car, originally worth $32000, decreases over time. The graph describes the value of the car as a percentage of its original price.

![Graph showing the value of a car over time as a percentage of its original price.]

a What is the value of the car, as a percentage of its original price, after 6 years?
b The ‘half-life’ is the time taken for the value to decrease by half. How long does it take for the car to lose half its original value?
c How long does it take the car to fall in value from 50% of its original value to 25% of its original value?
d Estimate, in dollars, the value of the car after 27 years.

8 The depth of water across a sand bar varies according to the tide. This is shown in the graph.

![Graph showing the depth of water across a sand bar over time.]

a Approximately how long after high tide is the next low tide?
b How deep is the water at the first low tide?
c What is the lowest value in the depth of the water across the bar?
d What is the difference in the heights of the first and second high tide?
e If the first high tide is at 9.40 am Tuesday, at what time approximately will the first high tide occur on Wednesday?
f The water must be at least 1.5 m deep for Judy’s boat to travel across the bar. Between what times, after the first high tide, can the boat not travel across the bar?

9 The graph shown describes the distance from home of an object as it travels.

The point at which the object is moving with the greatest speed is:

A P  B Q  C R  D S  E T
10 Water is poured into a container at a constant rate. The depth of water in the vessel is described in the graph. From the options following, the vessel that best matches the rate at which the vessel fills with water is:

A  
B  
C  
D  
E

11 Carbon-14 is a radioactive element which breaks down over time. It has a half-life of about 6000 years; that is, every 6000 years, half of the material present breaks down.

a Copy and complete the table for the amount of Carbon-14 (C-14) present.

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>0</th>
<th>6000</th>
<th>12000</th>
<th>18000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount of Carbon-14</td>
<td>800 units</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b Plot the graph using these data.

c Use the graph to estimate the time taken for the amount of C-14 to fall to 600 units.

d Use extrapolation to estimate the amount of C-14 present after 24000 years.

12 The following table displays the profit a company makes from selling $x$ units of toothpaste.

<table>
<thead>
<tr>
<th>Units of toothpaste sold ($\times1000$)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit ($\times$ $\times1000$)</td>
<td>-5</td>
<td>-6</td>
<td>-5</td>
<td>-2</td>
<td>1</td>
<td>9</td>
</tr>
</tbody>
</table>

a Plot the graph using these data.

b Use your graph to estimate how many units of toothpaste need to be sold in order to break even.

13 Yasmin borrows $70000. She can pay a certain amount each month or half that value each fortnight. The graph shows the progress of the loan under both systems of payment.

a How much time does it take to pay off the loan using monthly repayments?

b How long does it take to pay off the loan using fortnightly repayments?

c Using monthly repayments, after how many years has the balance fallen to 50% of its original value?
The graph shows the volume of air in a person’s lungs during a cycle of breathing.

a. What is the maximum volume of the lungs?
b. When relaxed, how much air is contained in the lungs?
c. How much air is exhaled during the breathing cycle?

15 The graph at right shows the profits made by a manufacturing firm versus the number of units produced.

a. What profit is made when 400 units are produced?
b. What is the result if 0 units are produced?

16 a. Using the graph from question 15, how many units need to be produced to break even?
b. Can you suggest an explanation for the dip in profits in the region n = 500?

14.6 **Constructing non-linear relations and graphs**

Linear relationships between two variables are the simplest. However, there are many important non-linear relationships where the graphs are not a straight line. Some examples follow.

A rivet is accidentally dropped from a tall pylon under construction. The relationship between the distance the rivet has fallen (d), and the time taken (t) is 

\[ d = 5t^2 \]

The gas in a piston is compressed. The relationship between the pressure (p) and the volume (v), is

\[ p = 20v^{-1} \text{ or } p = \frac{20}{v}. \]
The mass of a cubic block of ice \( (m) \) is related to the length of the side of the block \( (s) \) by the formula \( m = 1000s^3 \).

**Graphical representation of relations of the form \( y = kx^n \)**

Given a non-linear equation of the form \( y = kx^n \), the easiest way to graph it is to use CAS. Alternatively, from the equation a table of values can be produced and this can be used to sketch a graph of the relationship. The general shape of the graphs of \( y = kx^n \) for \( n = -2, -1, 1, 2, 3 \) are given here.

\[
\begin{align*}
    y &= \frac{k}{x^2} (n = -2) \\
    y &= \frac{k}{x} (n = -1) \\
    y &= kx (n = 1) \\
    y &= kx^2 (n = 2) \\
    y &= kx^3 (n = 3)
\end{align*}
\]

The value of \( k \) in these relationships is known as the **constant of proportionality**.

**WORKED EXAMPLE 14**  
A research scientist has discovered that her data are related according to the equation \( y = 5x^2 \). Construct a table of values to draw the graph of the equation \( y = 5x^2 \) for \( x \) between 0 and 10.
Finding a non-linear relationship using linear graphs

If you know the equation, it is reasonably straightforward to produce a table of values. The reverse process (that is, discovering a formula for a non-linear relationship from a table of values) is more difficult.

If the relationship between \(x\) and \(y\) is of the form \(y = kx^n\), then plotting \(y\) against \(x^n\) will produce a straight line from the origin. The gradient of this line will equal the value of \(k\), the constant of proportionality. The following algorithm can be used for finding the rule of the relationship, if the value of \(n\) is known.

1. Plot \(y\) against \(x^n\). The result must be a straight line, coming from the origin.
2. Select any two points on the line to calculate the value of the gradient.
3. Since the gradient represents the value of \(k\), substitute it into \(y = kx^n\) to give the rule for the relationship.

Note that if the value of \(n\) is not known, plot \(y\) against \(x\) first. The shape of the graph will indicate the possible value(s) of \(n\). Test this value by plotting \(y\) against \(x^n\). If a straight line is produced, proceed as above; otherwise try a different value of \(n\).
2 Plot the values of $x^2$ on the horizontal axis and $y$ on the vertical axis and join with a smooth line. (A straight line from the origin confirms that the relationship is of the form $y = kx^2$.)

3 Choose two points on the line, say $(0, 0)$ and $(100, 300)$, and calculate the gradient.

4 The gradient represents the value of $k$ in $y = kx^2$. Replace $k$ with 3 to state the equation for the relationship.

Therefore, $y = 3x^2$.

### EXERCISE 14.6

**Constructing non-linear relations and graphs**

1 WE14 A statistician found that his data are related according to the equation $y = 3x^2$. Construct a table of values and draw the graph of the equation $y = 3x^2$ for values of $x$ between 0 and 10.

2 A researcher found that her data are related according to the equation $y = \frac{2}{x}$.

Construct a table of values and draw the graph of the equation $y = \frac{2}{x}$ for values of $x$ between 2 and 8.

3 WE15 By graphing $y$ against $x^2$ find the equation for the relationship between $x$ and $y$ if the equation is of the form $y = kx^2$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0</td>
<td>8</td>
<td>32</td>
<td>72</td>
<td>128</td>
<td>200</td>
</tr>
</tbody>
</table>

4 By graphing $y$ against $x^2$, find the equation for the relationship between $x$ and $y$ if the equation is of the form $y = kx^2$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0</td>
<td>10</td>
<td>40</td>
<td>90</td>
<td>160</td>
<td>250</td>
</tr>
</tbody>
</table>

5 Construct a table of values to draw the graphs of the following equations.

(Use values of $x$ from $-5$ to 5.)

- $a \ y = x^2$
- $b \ y = 2x^2$
- $c \ y = 2x^3$
- $d \ y = 0.5x^3$
- $e \ y = \frac{0.5}{x}$
- $f \ y = \frac{1}{x^2}$

Use CAS to check your answers.
6 Construct a table of values to draw the graphs of the following equations. (Use values of \( x \) from \(-5\) to \(5\).)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>( y = 0.5x^2 )</td>
<td>b</td>
<td>( y = x^3 )</td>
</tr>
<tr>
<td>c</td>
<td>( y = \frac{1}{x} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>( y = \frac{2}{x} )</td>
<td>e</td>
<td>( y = \frac{2}{x^2} )</td>
</tr>
<tr>
<td>f</td>
<td>( y = \frac{0.5}{x^2} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use CAS to check your answers.

7 Write the letters \(a\) to \(d\) in your book. Against each letter that identifies a graph, write the equation which best matches that graph.

\[
\begin{align*}
\text{a} & : y = x^2 \\
\text{b} & : y = 2x^2 \\
\text{c} & : y = x^3 \\
\text{d} & : y = \frac{3}{x} \\
\end{align*}
\]

8 By graphing \( y \) against \( x^3 \), find the equation of the relationship between \( x \) and \( y \), if the equation is of the form \( y = kx^3 \).

\[
\begin{array}{c|c|c|c|c|c}
\hline
x & 0 & 2 & 4 & 6 & 8 \\
\hline
y & 0 & 12 & 96 & 324 & 768 \\
\end{array}
\]

9 The Safety Council conducted research on the braking distance of vehicles and its relationship to the speed of the vehicle. The following data were obtained.

<table>
<thead>
<tr>
<th>Speed (( s )) (km/h)</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>80</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Braking distance (( d )) (metres)</td>
<td>7.5</td>
<td>16.9</td>
<td>30</td>
<td>53.3</td>
<td>83.3</td>
</tr>
</tbody>
</table>

a Plot \( d \) versus \( s^2 \).
b What is the equation relating \( d \) and \( s \)?

10

<table>
<thead>
<tr>
<th>x</th>
<th>0.1</th>
<th>0.4</th>
<th>1</th>
<th>2</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>200</td>
<td>50</td>
<td>20</td>
<td>10</td>
<td>4</td>
</tr>
</tbody>
</table>

a Plot these values on a set of axes.
b Plot \( y \) versus \( \frac{1}{x} \) and draw a straight line through the points.
c What is the slope of the line?
d Deduce the relationship between \( y \) and \( x \).

11 In a physics experiment a student measured the current (\( I \)) flowing through a resistor for different values of the resistance (\( R \)) and obtained the following data.

<table>
<thead>
<tr>
<th>Resistance (( R )) (ohms)</th>
<th>100</th>
<th>200</th>
<th>1000</th>
<th>1500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current (( I )) (milliamps)</td>
<td>300</td>
<td>150</td>
<td>30</td>
<td>20</td>
</tr>
</tbody>
</table>

a Plot values of \( I \) versus \( \frac{1}{R} \).
b Deduce a relationship between \( I \) and \( R \).
12 Examine the following table of values.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>1</td>
<td>8</td>
<td>27</td>
<td>64</td>
</tr>
</tbody>
</table>

a Plot these values on a set of axes.
b Plot y versus $x^3$ and draw a straight line through the points.
c What is the slope of the line?
d Deduce the relationship between y and x.

13 It is suspected that for male adults, mass ($m$) is related to height ($h$) by a formula like $m = kh^3$. Use the data in the table to find a relationship between $m$ and $h$.

<table>
<thead>
<tr>
<th>Mass (kg)</th>
<th>45</th>
<th>55</th>
<th>71</th>
<th>84</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (cm)</td>
<td>150</td>
<td>160</td>
<td>175</td>
<td>185</td>
</tr>
</tbody>
</table>

14 The intensity of light drops off as you move away from the source. The relationship is of the form $I = \frac{k}{r^2}$. If the intensity ($I$) is 50 when $r = 20$, find:
a the value of $k$
b the distance at which the intensity of light falls below 35.

15 Which of the following equations describes the relationship shown on the graph at right?
A $V = 10t^2$
B $V = 10t$
C $V = 5t^2$
D $V = 5t$
E $V^2 = 5t$

16 Which of the graphs shows the relationship $m = \frac{3}{p^2}$?

A

B

C

D

E
The Maths Quest Review is available in a customisable format for you to demonstrate your knowledge of this topic.

The Review contains:

- **Multiple-choice** questions — providing you with the opportunity to practise answering questions using CAS technology
- **Short-answer** questions — providing you with the opportunity to demonstrate the skills you have developed to efficiently answer questions using the most appropriate methods
- **Extended-response** questions — providing you with the opportunity to practise exam-style questions.

A summary of the key points covered in this topic is also available as a digital document.

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studyON is an interactive and highly visual online tool that helps you to clearly identify strengths and weaknesses prior to your exams. You can then confidently target areas of greatest need, enabling you to achieve your best results.
14 Answers

EXERCISE 14.2

1 a 13  b 6  c No  d $m = 2, c = 5$
2 a $y = 0$  b $x = 4$  c Yes
3 4 4
4 $-1$
5 a

\[ y = 3x - 4 \]

\[ y = 3x \]

\[ y = \frac{3}{2} x \]

\[ 4x - 3y = 12 \]

\[ (-2, -6) \]

b

\[ y = 3x + 12 \]

\[ (0, 3) \]

\[ (4, 3) \]

\[ (4, \frac{1}{2}) \]

\[ (10, 0) \]

c

d

\[ y = 3x - 4 \]

\[ y = 3x – 12 \]

\[ y = \frac{3}{2} x \]

\[ 4x - 3y = 42 \]

\[ (-2, -6) \]

7 a $\$635$

b

<table>
<thead>
<tr>
<th>Weekly sales</th>
<th>Weekly pay</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>$425</td>
</tr>
<tr>
<td>10000</td>
<td>$575</td>
</tr>
<tr>
<td>15000</td>
<td>$725</td>
</tr>
<tr>
<td>20000</td>
<td>$875</td>
</tr>
</tbody>
</table>

c

$P = 275 + 0.03S$

d

\[
P = \begin{cases} 
0 & \text{if } S \leq 5 \\
100 & \text{if } 5 < S \leq 10 \\
200 & \text{if } 10 < S \leq 15 \\
300 & \text{if } 15 < S \leq 20 \\
400 & \text{if } S > 20 
\end{cases}
\]

e $m = 0.03; y$-intercept $= 275; \text{retainer amount}$

8 a

<table>
<thead>
<tr>
<th>Sales</th>
<th>0</th>
<th>$$20000$</th>
<th>$$40000$</th>
<th>$$60000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings</td>
<td>$$200$</td>
<td>$$800$</td>
<td>$$1400$</td>
<td>$$2000$</td>
</tr>
</tbody>
</table>

b $E = 200 + 0.03S$
**Exercise 14.3**

<table>
<thead>
<tr>
<th>Sales ($)</th>
<th>Commission ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 000</td>
<td>300</td>
</tr>
<tr>
<td>30 000</td>
<td>450</td>
</tr>
<tr>
<td>40 000</td>
<td>600</td>
</tr>
<tr>
<td>50 000</td>
<td>700</td>
</tr>
<tr>
<td>60 000</td>
<td>800</td>
</tr>
</tbody>
</table>
Different commission rates mean different gradients.

2. a. $200  
   b. $500  
   c. $580  
   d. $660

e. Sales ($ × $10,000)

3. a. $215
   b. $195
   c. $175
   d. $155

4. a. Length of call (seconds)  |  cost (cents)
   |  15  |  0
   |  30  |  25
   |  45  |  25
   |  60  |  50
   |  75  |  50
   |  90  |  75
   |  105 |  75
   |  120 |  100

5. a. Number of buns  |  Cost
   |  2  |  $0.80
   |  4  |  $1.60
   |  6  |  $2.00
   |  8  |  $2.80
   |  10 |  $3.60

6. a. Number of people  |  Total cost
   |  1  |  $900
   |  2  |  $1650
   |  3  |  $2250
   |  6  |  $4050
   |  8  |  $5250
   |  10 |  $6450

7. Consumption (kWh)  |  Power bill
   |  200 |  $40
   |  400 |  $80
   |  600 |  $110
   |  1000|  $170
   |  1500|  $240
8

Power bill ($)

Consumption (kWh)

9 a

<table>
<thead>
<tr>
<th>Time of service (minutes)</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>$80</td>
</tr>
<tr>
<td>30</td>
<td>$80</td>
</tr>
<tr>
<td>45</td>
<td>$115</td>
</tr>
<tr>
<td>60</td>
<td>$115</td>
</tr>
<tr>
<td>80</td>
<td>$150</td>
</tr>
<tr>
<td>90</td>
<td>$150</td>
</tr>
</tbody>
</table>

b

10 a

<table>
<thead>
<tr>
<th>Length of call (seconds)</th>
<th>Cost (cents)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>30</td>
<td>15</td>
</tr>
<tr>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>50</td>
<td>30</td>
</tr>
<tr>
<td>60</td>
<td>45</td>
</tr>
<tr>
<td>70</td>
<td>45</td>
</tr>
<tr>
<td>80</td>
<td>60</td>
</tr>
<tr>
<td>90</td>
<td>60</td>
</tr>
</tbody>
</table>

11 B

12 B

13 a $0 b $150 c $2100 d $4350
e $5550 f $11 550 g $24950 h $63 550

14 a

<table>
<thead>
<tr>
<th>Taxable income ($ × 1000)</th>
<th>Income tax ($ × 1000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>60</td>
<td>35</td>
</tr>
<tr>
<td>80</td>
<td>40</td>
</tr>
<tr>
<td>100</td>
<td>45</td>
</tr>
</tbody>
</table>

15 a $0 b $0 c $342 d $3192 e $4547 f $11 047
g $24947 h $63 547

16 a

b i Approx $1100 ii Approx $23 000 c Answers will vary.

17 Answers will vary.

18 a i $75 ii 20 km

EXERCISE 14.4

1 (1, 3) 2 (–7, –4) 3 (1, –1) 4 (–2, 1)

5 a

<table>
<thead>
<tr>
<th>Number of toys</th>
<th>Cost ($)</th>
<th>Revenue ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>750</td>
<td>350</td>
</tr>
<tr>
<td>100</td>
<td>900</td>
<td>700</td>
</tr>
<tr>
<td>150</td>
<td>1050</td>
<td>1050</td>
</tr>
<tr>
<td>200</td>
<td>1200</td>
<td>1400</td>
</tr>
</tbody>
</table>
c 150

d (150, 1050). If 150 toys are produced, the cost and revenue is equal to $1050. No profit nor loss is made.

6 a

<table>
<thead>
<tr>
<th>Number of basketballs</th>
<th>Cost ($)</th>
<th>Revenue ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>$3400</td>
<td>$2100</td>
</tr>
<tr>
<td>200</td>
<td>$3600</td>
<td>$2800</td>
</tr>
<tr>
<td>250</td>
<td>$3800</td>
<td>$3500</td>
</tr>
<tr>
<td>300</td>
<td>$4000</td>
<td>$4200</td>
</tr>
<tr>
<td>350</td>
<td>$4200</td>
<td>$4900</td>
</tr>
</tbody>
</table>

b

Number of basketballs

6 c Number of basketballs \((n) = 280\) required to break even

7 a (2, 4)  
b (2, 2)  
c (1, 2)

8 a

y = 4x - 4  
y = 2x  

8 b

y = 3x - 4  
y = 2x - 2  

8 c

3y - 2x = 4  
y = 2x + 5  

9 (3, -1)  
10 (-2, 2)  
11 (0, 0)  
12 (3, 5)

13 a

<table>
<thead>
<tr>
<th>Call time (minutes)</th>
<th>Cost Plan A</th>
<th>Cost Plan B</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$21</td>
<td>$28</td>
</tr>
<tr>
<td>20</td>
<td>$27</td>
<td>$31</td>
</tr>
<tr>
<td>30</td>
<td>$33</td>
<td>$34</td>
</tr>
<tr>
<td>40</td>
<td>$39</td>
<td>$37</td>
</tr>
</tbody>
</table>

b

Plan A  
Plan B

14 a

<table>
<thead>
<tr>
<th>Number of components</th>
<th>Cost ($)</th>
<th>Revenue ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>18 500</td>
<td>14 500</td>
</tr>
<tr>
<td>10 000</td>
<td>31 000</td>
<td>37 000</td>
</tr>
<tr>
<td>15 000</td>
<td>43 500</td>
<td>59 500</td>
</tr>
<tr>
<td>20 000</td>
<td>56 000</td>
<td>82 000</td>
</tr>
</tbody>
</table>

b

Plan A  
Plan B

15 a

<table>
<thead>
<tr>
<th>Number of packages sold</th>
<th>Salary Plan A</th>
<th>Salary Plan B</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$525</td>
<td>$375</td>
</tr>
<tr>
<td>10</td>
<td>$650</td>
<td>$600</td>
</tr>
<tr>
<td>15</td>
<td>$775</td>
<td>$825</td>
</tr>
<tr>
<td>20</td>
<td>$900</td>
<td>$1050</td>
</tr>
</tbody>
</table>

b

Salary Plan A  
Salary Plan B

15 c

13
EXERCISE 14.5

1 a 1.0 km  
b 0.15 km/min  
c The runner is moving.  
d 2.5 km  
e 20 mins  
f 10 mins  
2 a 2 km  
b 0.4 km/min  
c The runner is moving towards the finish line.  
d 8 km  
e 20 mins  
f 20 mins  
3 a \approx 70\%  
b 9 years  
c 9 years  
4 a \approx 70\%  
b 8 years  
c 7 years  
5 a 2 km  
c Stationary (not moving)  
d 8 km  
e 90 min  
f 16 km/h  
6 a 2 s  
b 2 s  
c 50 m  
d 10 m/s  
7 a \approx 62\%  
b 9 years  
c 9 years  
8 a About 6 hours  
b 1.5 m  
c 0.5 m  
e 10.40 am  
f Between about 15 hours and 21 hours after the first high tide  
9 B  
10 C  
11 a

<table>
<thead>
<tr>
<th>Time (number of years)</th>
<th>Amount of Carbon-14 (units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>800</td>
</tr>
<tr>
<td>6000</td>
<td>400</td>
</tr>
<tr>
<td>12000</td>
<td>200</td>
</tr>
<tr>
<td>18000</td>
<td>100</td>
</tr>
</tbody>
</table>

EXERCISE 14.6

1 \( y = 3x^2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>12</td>
<td>48</td>
<td>108</td>
<td>192</td>
<td>300</td>
</tr>
</tbody>
</table>

2

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{4} )</td>
</tr>
</tbody>
</table>
3 \( y = 2x^2 \)
4 \( y = 2.5x^2 \)

5 a \( y = x^2 \)

6 a \( y = 0.5x^2 \)

b \( y = 2x^2 \)

b \( y = x^3 \)

c \( y = 2x^3 \)

c \( y = x^3 \)

d \( y = 0.5x^3 \)

d \( y = 0.5^x \)

e \( y = \frac{1}{x} \)

e \( y = \frac{1}{x} \)

f \( y = \frac{1}{x^2} \)

f \( y = \frac{0.5}{x^2} \)
7  
\[ a) y = \frac{3}{x} \]  
\[ b) y = x^3 \]  
\[ c) y = 2x^2 \]  
\[ d) y = x^2 \]  
\[ 8 y = 1.5x^3 \]  

9  
\[ a) \quad \text{Graph of } y = \frac{d}{x} \]  
\[ b) d = 0.0083x^2 \]  

10  
\[ a) \quad \text{Graph of } y = \frac{200}{x} \]  
\[ b) \quad \text{Graph of } y = 20 \]  
\[ c) \quad y = \frac{20}{x} \]  

11  
\[ a) \quad \text{Graph of } y = \frac{300}{x} \]  

<table>
<thead>
<tr>
<th>( R )</th>
<th>100</th>
<th>200</th>
<th>1000</th>
<th>1500</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{R} )</td>
<td>0.01</td>
<td>0.005</td>
<td>0.001</td>
<td>0.00067</td>
</tr>
<tr>
<td>( I )</td>
<td>300</td>
<td>150</td>
<td>30</td>
<td>20</td>
</tr>
</tbody>
</table>

12  
\[ b) \quad I = \frac{30000}{R} \]  

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^3 )</td>
<td>0</td>
<td>8</td>
<td>64</td>
<td>216</td>
<td>512</td>
</tr>
<tr>
<td>( y )</td>
<td>0</td>
<td>1</td>
<td>8</td>
<td>27</td>
<td>64</td>
</tr>
</tbody>
</table>

13  
\[ m = \frac{h}{75000} \]  

14  
\[ a) \quad k = 20000 \]  
\[ b) \quad 23.9 \]  
\[ 15 \text{ C} \]  
\[ 16 \text{ E} \]