TOPIC 6

Pythagoras and trigonometry

6.1 Overview

Why learn this?

Pythagoras was a great mathematician and philosopher who lived in the 6th century BCE. He is best known for the theorem that bears his name. It concerns the relationship between the lengths of the sides in a right-angled triangle. Geometry and trigonometry are branches of mathematics where Pythagoras' theorem is still widely applied. Trigonometry is a branch of mathematics that allows us to relate the side lengths of triangles to angles. Combining trigonometry with Pythagoras' theorem allows us to solve many problems involving triangles.

What do you know?

assess on

1 **THINK** List what you know about trigonometry. Use a thinking tool such as a concept map to show your list.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, create a thinking tool such as a large concept map to show your class's knowledge of trigonometry.

Learning sequence

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The story of mathematics:
Secret society

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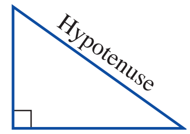
6.2 Pythagoras’ theorem

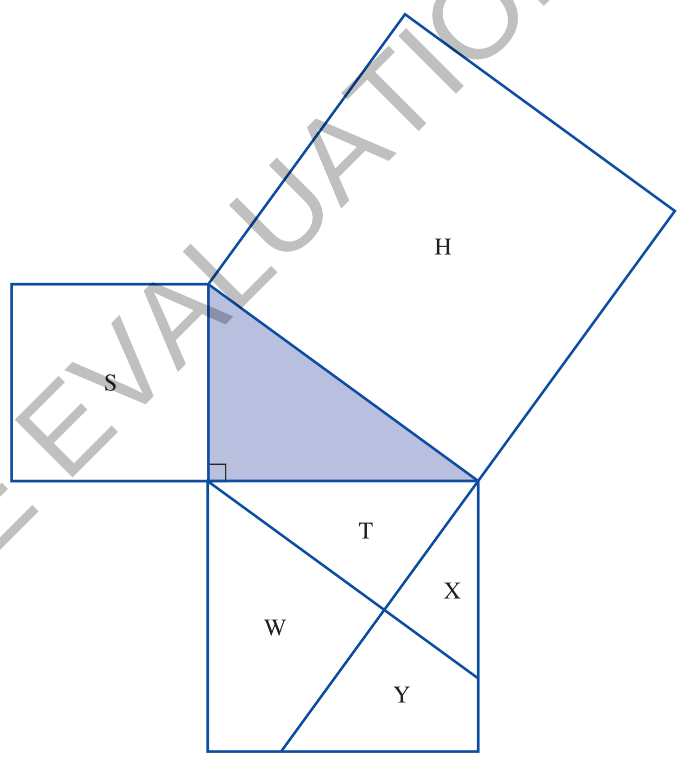
Right-angled triangles



- Triangles can be classified according to their side lengths or their angles.
- Equilateral, isosceles and scalene triangles are classified by the length of their sides.
- A **right-angled triangle** is classified by the type of angle it contains.
- The longest side of a right-angled triangle is opposite the right angle and is called the **hypotenuse**.
- In all triangles the longest side is opposite the largest angle.
- Pythagoras (580–501 BC) was a Greek mathematician who explored the relationship between the lengths of the sides of right-angled triangles.
- The relationship he described, and has been credited with discovering over 2500 years ago, is known as **Pythagoras’ theorem**.

Activity

1. The diagram below shows a right-angled triangle with squares drawn on each of its sides. Copy the figure onto a piece of paper.



2. Cut around the perimeter of the figure, then cut off the two smaller squares from the base and height of the right-angled triangle. Cut the larger of these two squares into the four pieces shown.
3. You should now have a right-angled triangle with the square H attached to its hypotenuse and five smaller pieces (S, T, W, X and Y). Rearrange these five pieces to fit exactly in the area H.
4. Since the squares from the two shorter sides of the right-angled triangle fit exactly into the square on the hypotenuse, what can you conclude?

Pythagoras’ theorem

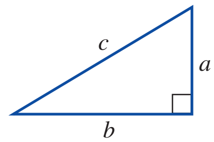
- Pythagoras investigated the relationship between the lengths of the sides of a right-angled triangle. This is now known as Pythagoras’ theorem.

- The theorem states that:

In any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. The rule is:

$$c^2 = a^2 + b^2$$

where *a* and *b* are the two shorter sides and *c* is the hypotenuse.



- If the lengths of two sides of a right-angled triangle are given, then Pythagoras' theorem enables the length of the unknown side to be found.

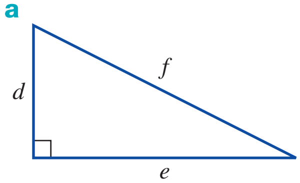
WORKED EXAMPLE 1

For the right-angled triangles shown at right:

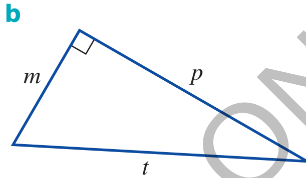
i **state which side is the hypotenuse**

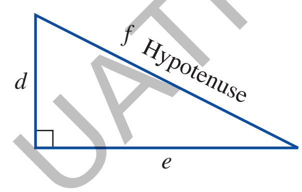
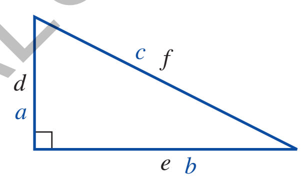
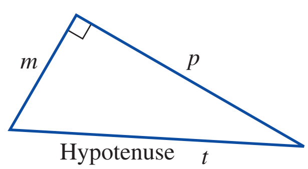
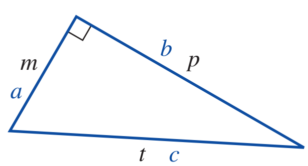
ii **write Pythagoras' theorem.**

a



b



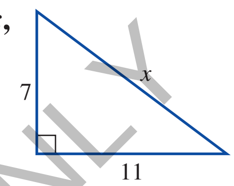
THINK	WRITE/DRAW
a **i** The hypotenuse is always opposite the right angle.	Side f is opposite the right angle. Therefore, side f is the hypotenuse. 
ii If the triangle is labelled as usual with a, b and c, as shown in blue, Pythagoras' theorem can be written and then the letters can be replaced with their new values (names).	 $c = f$; $a = d$; $b = e$ $c^2 = a^2 + b^2$ $f^2 = d^2 + e^2$
b **i** The hypotenuse is opposite the right angle.	 Side t is opposite the right angle. Therefore, side t is the hypotenuse.
ii If the triangle is labelled as usual with a, b and c, as shown in blue, Pythagoras' theorem can be written and then the letters can be replaced with their new values (names).	 $c = t$; $b = p$; $a = m$ $c^2 = a^2 + b^2$ $t^2 = m^2 + p^2$

Finding the hypotenuse

- To calculate the length of the hypotenuse given the length of the two other shorter sides, a and b, substitute the values of a and b into the rule $c^2 = a^2 + b^2$ and solve for c.
- Remember that the hypotenuse is the longest side, so the value of c must be *greater than* that of either a or b.

WORKED EXAMPLE 2

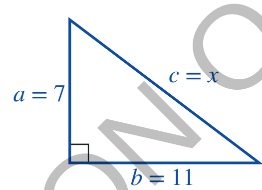
For the triangle at right, calculate the length of the hypotenuse, x, correct to 1 decimal place.



THINK

1. Copy the diagram and label the sides a, b and c. Remember to label the hypotenuse as c.
2. Write Pythagoras' theorem.
3. Substitute the values of a, b and c into this rule and simplify.
4. Calculate x by taking the square root of 170. Round the answer correct to 1 decimal place.

WRITE/DRAW



$c^2 = a^2 + b^2$

$$\begin{aligned} x^2 &= 7^2 + 11^2 \\ &= 49 + 121 \\ &= 170 \end{aligned}$$

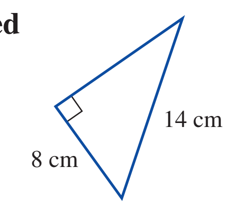
$x = \sqrt{170}$
$x = 13.0$

Finding the length of a shorter side

- When given both the hypotenuse and one other shorter side, either a or b, rearrange Pythagoras' theorem to find the value of the unknown shorter side, e.g., $a^2 = c^2 - b^2$ or $b^2 = c^2 - a^2$.
- Remember that the value of the unknown shorter side, either a or b, should be *less than* the value of the hypotenuse.

WORKED EXAMPLE 3

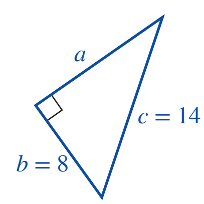
Calculate the length, correct to 1 decimal place, of the unmarked side of the triangle at right.



THINK

1. Copy the diagram and label the sides a, b and c. Remember to label the hypotenuse as c.

WRITE/DRAW



2	Write Pythagoras' theorem for a shorter side.	$a^2 = c^2 - b^2$
3	Substitute the values of b and c into this rule and simplify.	$a^2 = 14^2 - 8^2$ $= 196 - 64$ $= 132$
4	Find a by taking the square root of 132. Round to 1 decimal place.	$a = \sqrt{132}$ $= 11.5\text{ cm}$

Practical problems

- When using Pythagoras' theorem to solve practical problems, draw a right-angled triangle to represent the problem.
  – Identify the unknown variable by reading what the question is asking.
  – Identify the known values and substitute these into Pythagoras' theorem.
  – Solve for the unknown value.
  – Use the result to write the answer as a complete sentence.

WORKED EXAMPLE 4

Calculate the value of the missing side length. Give your answer correct to 1 decimal place.

THINK		WRITE
1	• Identify the length of the hypotenuse and the two shorter sides. • Check that the units for all measurements are the same. Convert 84 mm to centimetres by dividing by 10.	$c = 12\text{ cm}$ $a = 84\text{ mm} = 8.4\text{ cm}$ Let x = the length of the missing side. $b = x$
2	Substitute the values into the equation and simplify.	$c^2 = a^2 + b^2$ $12^2 = (8.4)^2 + x^2$ $144 = 70.56 + x^2$
3	Solve the equation for x by: • subtracting 70.56 from both sides, as shown in red • taking the square root of both sides.	$144 - 70.56 = 70.56 - 70.56 + x^2$ $73.44 = x^2$ $\sqrt{73.44} = \sqrt{x^2}$ $\pm 8.5697 \approx x$

4 Answer the question.
- Possible values for x are 8.5697 and −8.5697. It is not possible to have a length of −8.5697.
- Round the answer to 1 decimal place.
- Include the appropriate units in the answer. The calculations were carried out in centimetres. The answer will be in centimetres.

$x \approx 8.5697$ is the only valid solution as x is the length of a side in a triangle. It is not possible to have a side with a negative length.

Therefore, the missing side length is 8.6 cm (correct to 1 decimal place).

Pythagorean triads

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Interactivity Pythagorean triples **int-2765**

- A **Pythagorean triad** is a group of any three whole numbers that satisfy Pythagoras' theorem. For example, {5, 12, 13} and {7, 24, 25} are Pythagorean triads.
  $13^2 = 5^2 + 12^2$; $25^2 = 24^2 + 7^2$
- Pythagorean triads are useful when solving problems using Pythagoras' theorem. If two known side lengths in a triangle belong to a triad, the length of the third side can be stated without performing any calculations.
- Some well known Pythagorean triads are:
  {3, 4, 5}; {5, 12, 13}; {8, 15, 17}; {7, 24, 25}.

WORKED EXAMPLE 5

Determine whether the following sets of numbers are Pythagorean triads.

a {9, 10, 14} **b {33, 65, 56}**

THINK

WRITE

a 1 Pythagorean triads satisfy Pythagoras' theorem. Substitute the values into the equation $c^2 = a^2 + b^2$ and determine whether the equation is true. Remember, c is the longest side.

a $c^2 = a^2 + b^2$

LHS $= c^2$
$= 14^2$
$= 196$

RHS $= a^2 + b^2$
$= 9^2 + 10^2$
$= 81 + 100$
$= 181$

2 State your conclusion.

Since LHS ≠ RHS, the set {9, 10, 14} is not a Pythagorean triad.

b 1 Pythagorean triads satisfy Pythagoras' theorem. Substitute the values into the equation $c^2 = a^2 + b^2$ and determine whether the equation is true. Remember, c is the longest side.

b $c^2 = a^2 + b^2$

LHS $= 65^2$
$= 4225$

RHS $= 33^2 + 56^2$
$= 1089 + 3136$
$= 4225$

2 State your conclusion.

Since LHS = RHS, the set {33, 65, 56} is a Pythagorean triad.

- If each term in a triad is multiplied by the same number, the result is also a triad. For example, if we multiply each number in {5, 12, 13} by 2, the result {10, 24, 26} is also a triad.

- Builders and gardeners use multiples of the Pythagorean triad {3, 4, 5} to ensure that walls and floors are at right angles.

Exercise 6.2 Pythagoras' theorem

assess on

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–11, 13, 16	Questions: 1–14, 16	Questions: 1–17

Individual pathway interactivity int-4472 eBookplus

REFLECTION
What is the quickest way to identify whether a triangle is right-angled or not?

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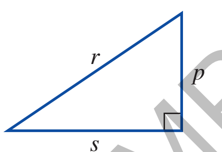
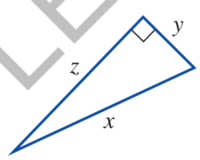
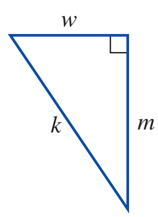
Digital docs
SkillSHEET Rearranging formulas doc-11429
SkillSHEET Converting units of length doc-11430
SkillSHEET Rounding to a given number of decimal places doc-11428

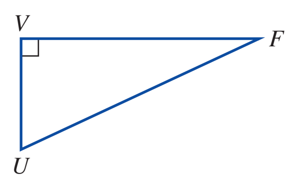
FLUENCY

1 WE1 For the right-angled triangles shown below:
  i state which side is the hypotenuse
  ii write Pythagoras' theorem.

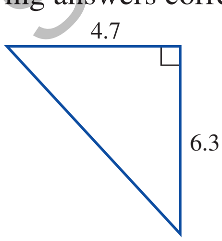
a


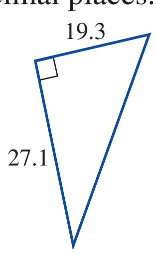
b


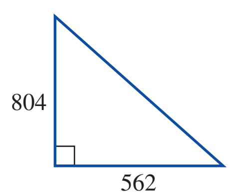
c


d


2 WE2 For each of the following triangles, calculate the length of the hypotenuse, giving answers correct to 2 decimal places.

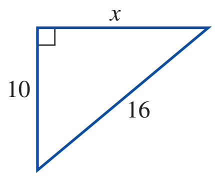
a


b


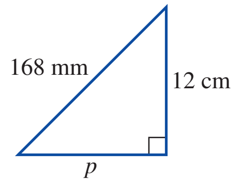
c


3 **WE3** Calculate the value of the pronumeral in each of the following triangles. Give your answers correct to 2 decimal places.

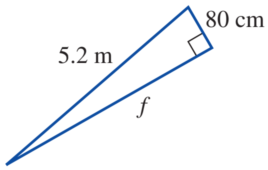
a



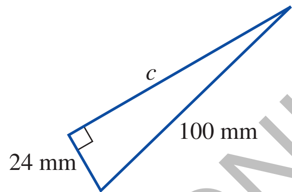
b



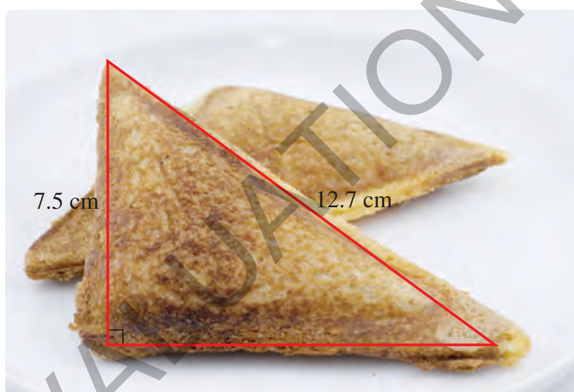
c



d



4 **WE4** Calculate the value of the missing side length. Give your answer correct to 1 decimal place.



5 **WE5** Determine whether the following sets of numbers are Pythagorean triads.

a {2, 5, 6} **b** {7, 10, 12} **c** {18, 24, 30}
d {72, 78, 30} **e** {13, 8, 15} **f** {50, 40, 30}

UNDERSTANDING

6 In a right-angled triangle, the two shortest sides are 4.2 cm and 3.8 cm.

a Draw a sketch of the triangle.
b Calculate the length of the hypotenuse correct to 2 decimal places.

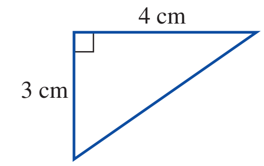
7 A right-angled triangle has a hypotenuse of 124 mm and another side of 8.5 cm.

a Draw a sketch of the triangle.
b Calculate the length of the third side. Give your answer in millimetres correct to 2 decimal places.

8 **MC** Which of the following sets is formed from the triad {21, 20, 29}?

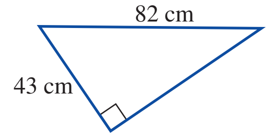
A {95, 100, 125}
B {105, 145, 100}
C {84, 80, 87}
D {105, 80, 87}
E {215, 205, 295}

9 MC What is the length of the hypotenuse in this triangle?



A 25 cm B 50 cm
C 50 mm D 500 mm
E 2500 mm

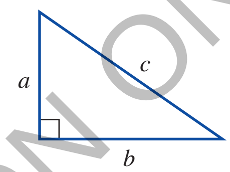
10 MC What is the length of the third side in this triangle?



A 48.75 cm B 0.698 m
C 0.926 m D 92.6 cm
E 69.8 mm

11 A ladder that is 7 metres long leans up against a vertical wall. The top of the ladder reaches 6.5 m up the wall. How far from the wall is the foot of the ladder, correct to 2 decimal places?

12 Two sides of a right-angled triangle are given. Find the third side in the units specified. The diagram shows how each triangle is to be labelled. Give your answers correct to 2 decimal places.
Remember: c is always the hypotenuse.



a $a = 37$ cm, $c = 180$ cm; find b in cm.
b $a = 856$ mm, $b = 1200$ mm; find c in cm.
c $b = 4950$ m, $c = 5.6$ km; find a in km.
d $a = 125\,600$ mm, $c = 450$ m; find b in m.

REASONING

13 An art student is trying to hang her newest painting on an existing hook in a wall. She leans a 1.2-m ladder against the wall so that the distance between the foot of the ladder and the wall is 80 cm.
a Draw a sketch showing the ladder leaning on the wall.
b How far up the wall does the ladder reach, correct to 2 decimal places?
c The student climbs the ladder to check whether she can reach the hook from the step at the very top of the ladder. Once she extends her arm, the distance from her feet to her fingertips is 1.7 m. If the hook is 2.5 m above the floor, will the student reach it from the top step?

14 A rectangular park is 260 m by 480 m. Danny usually trains by running 5 circuits around the edge of the park. After heavy rain, two adjacent sides are too muddy to run along, so he runs a triangular path along the other two sides and the diagonal. Danny does 5 circuits of this path for training. Show that Danny runs about 970 metres less than his usual training session.

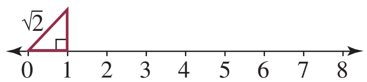
15 An adventure water park has hired Sally to build part of a ramp for a new water slide. She builds a ramp that is 12 m long and rises to a height of 250 cm. To meet the regulations, the ramp must have a gradient between 0.1 and 0.25. Show that the ramp Sally has built is within the regulations.

PROBLEM SOLVING

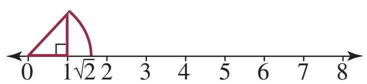
16 a The smallest number of four Pythagorean triads is given below. Find the middle number and, hence, find the third number.
i 9 ii 11 iii 13 iv 29
b What do you notice about the triads formed in part a?

17 We know that it is possible to find the exact square root of some numbers but not others. For example, we can find $\sqrt{4}$ exactly but not $\sqrt{3}$ or $\sqrt{5}$. Our calculators can find decimal approximations of these, but because they cannot be found exactly they are called irrational numbers. There is a method, however, of showing their exact location on a number line.

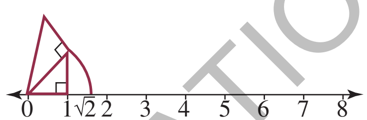
a Using graph paper, draw a right-angled triangle with two equal sides of length 1 cm as shown below.



b Using Pythagoras' theorem, we know that the longest side of this triangle is $\sqrt{2}$ units. Place the compass point at zero and make an arc that will show the location of $\sqrt{2}$ on the number line.



c Draw another right-angled triangle using the longest side of the first triangle as one side, and make the other side 1 cm in length.



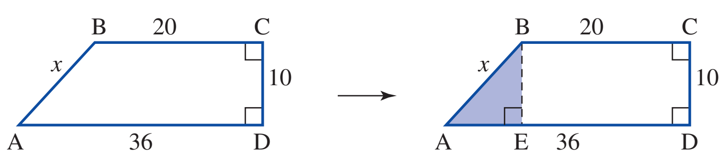
d The longest side of this triangle will have a length of $\sqrt{3}$ units. Draw an arc to find the location of $\sqrt{3}$ on the number line.

e Repeat steps **c** and **d** to draw triangles that will have sides of length $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$ units and so on.

6.3 Applications of Pythagoras' theorem

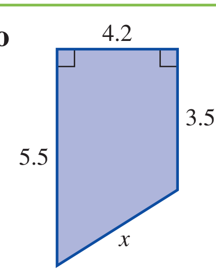
Using composite shapes to solve problems

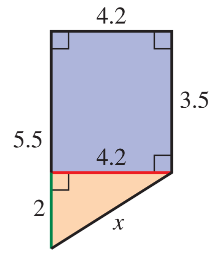
- Dividing a composite shape into simpler shapes creates shapes that have known properties. For example, to calculate the value of x in the trapezium shown, a vertical line can be added to create a right-angled triangle and a rectangle. The length of x can be found using Pythagoras' theorem.



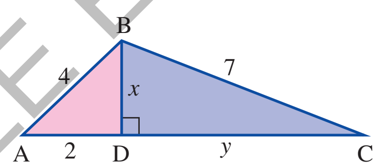
WORKED EXAMPLE 6

Calculate the value of x in the diagram at right, correct to 1 decimal place.



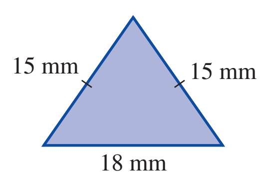
THINK	WRITE/DRAW
1 Divide the shape into smaller shapes that create a right-angled triangle, as shown in red.	
2 x is the hypotenuse of a right-angled triangle. To use Pythagoras' theorem, the length of the side shown in green must be known. This length can be calculated as the difference between the long and short vertical edges of the trapezium: $5.5 - 3.5 = 2$.	
3 Substitute the values of the side lengths into Pythagoras' theorem.	$c^2 = a^2 + b^2$ $x^2 = 2^2 + (4.2)^2$ $x^2 = 4 + 17.64$ $x^2 = 21.64$ $\sqrt{x^2} = \sqrt{21.64}$ $x \approx \pm\, 4.651\,88$
4 Answer the question. • Possible values for x are 4.651 88 and −4.651 88. It is not possible for x to have a length of negative value. • Round the answer to 1 decimal place. • Include units if appropriate. There are no units.	$x \approx 4.651\,88$ is the only valid solution, as x is the side length of a triangle. Therefore, $x = 4.7$ (correct to 1 decimal place).

- To find the value of y in the irregular triangle shown, the triangle can be split into two right-angled triangles: ΔABD (pink) and ΔBDC (purple). There is enough information to calculate the missing side length from ΔABD. This newly calculated length can be used to find the value of y.

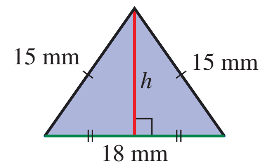


WORKED EXAMPLE 7

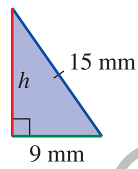
a **Calculate the perpendicular height of the isosceles triangle whose equal sides are each 15 mm long and whose third side is 18 mm long.**
b **Calculate the area of the triangle.**

THINK	WRITE/DRAW
a **1** Draw the triangle and label all side lengths as described.	**a** 

2 Draw an additional line to represent the height of the triangle, as shown in red, and label it appropriately.
Since the triangle is an isosceles triangle, h bisects the base of 18 mm, as shown in green, and creates 2 right-angled triangles.



3 Focus on one right-angled triangle containing the height.



4 Substitute the values from this right-angled triangle into Pythagoras' theorem to calculate the value of h.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 15^2 &= h^2 + 9^2 \\ 225 &= h^2 + 81 \\ 225 - 81 &= h^2 + 81 - 81 \\ 144 &= h^2 \\ \sqrt{144} &= \sqrt{h^2} \\ \pm 12 &= h \end{aligned}$$

5 Answer the question.
- Possible values for h are 12 and −12. It is not possible for h to have a length of negative value.
- Include units if appropriate.

$h = 12$ is the only valid solution, since h represents the height of a triangle.
The height of the triangle is 12 mm.

b **1** Write the formula for the area of a triangle.
In this case the base is 18 mm and the height is 12 mm.

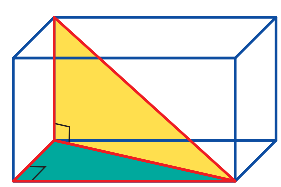
b $$\begin{aligned} A_\Delta &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2} \times 18 \times 12 \\ &= 108 \end{aligned}$$

2 Answer the question. Include units if appropriate.

The area of the triangle is 108 mm^2.

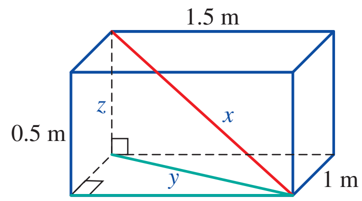
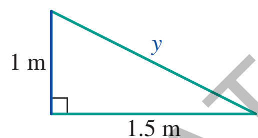
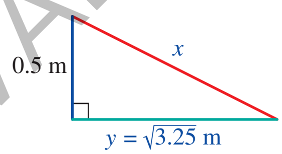
Pythagoras' theorem in 3-D

- Pythagoras' theorem can also be used to solve problems in three dimensions.
- In 3-D problems, often more than one right-angled triangle exists.
- It is often helpful to redraw sections of the 3-D diagram in 2 dimensions, allowing the right angles to be seen with greater clarity.



WORKED EXAMPLE 8

Calculate the maximum length of a metal rod that would fit into a rectangular crate with dimensions 1 m × 1.5 m × 0.5 m.

THINK	WRITE/DRAW
1 Draw a diagram of a rectangular box with a rod in it, labelling the dimensions.	
2 • Draw in a right-angled triangle that has the metal rod as one of the sides, as shown in red. The length of y in this right-angled triangle is not known. • Draw in another right-angled triangle, as shown in green, to calculate the length of y.	
3 • Calculate the length of y using Pythagoras' theorem. • Calculate the exact value of y.	 $c^2 = a^2 + b^2$ $y^2 = 1.5^2 + 1^2$ $y^2 = 3.25$ $y = \sqrt{3.25}$
4 Draw a right-angled triangle containing the rod and use Pythagoras' theorem to calculate the length of the rod (x). *Note:* z = height of the crate = 0.5 m	 $c^2 = a^2 + b^2$ $x^2 = (\sqrt{3.25})^2 + 0.5^2$ $x^2 = 3.25 + 0.25$ $x^2 = 3.5$ $x = \sqrt{3.5}$ ≈ 1.87
5 Answer the question.	The maximum length of the metal rod is 1.87 m (correct to 2 decimal places).

Exercise 6.3 Applications of Pythagoras' theorem

assess on

INDIVIDUAL PATHWAYS

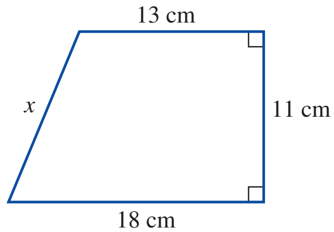
■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1–8, 10, 12, 14, 16, 19	Questions: 1–12, 14–16, 19, 20	Questions: 1–20

■■■ **Individual pathway interactivity** int-4475 eBook*plus*

REFLECTION
Which simple shapes should you try to divide composite shapes up into?

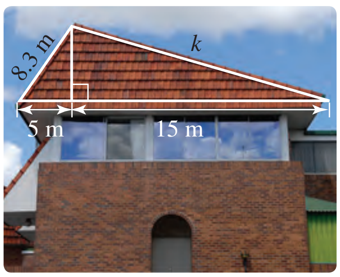
FLUENCY

1 WE6 Calculate the length of the side x in the figure below, correct to 2 decimal places.

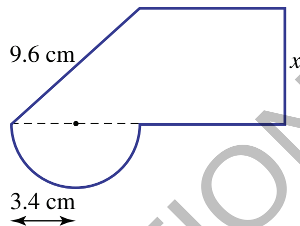


2 Calculate the values of the pronumerals in each of the following. Give your answers correct to 2 decimal places.

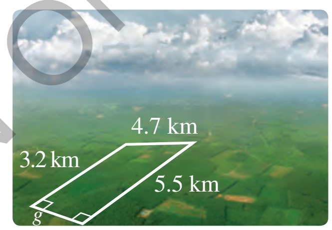
a



b

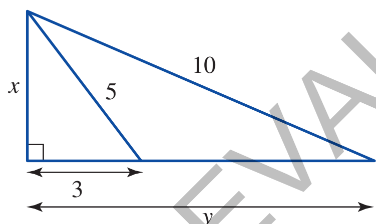


c

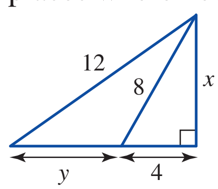


3 For each of the following diagrams, calculate the lengths of the sides marked x and y. Give your answer correct to 2 decimal places where necessary.

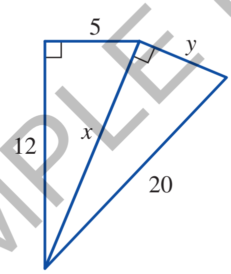
a



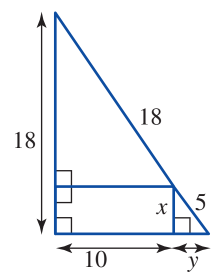
b



c

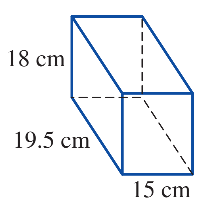


d

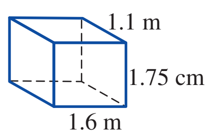


4 WE8 Calculate the length of the longest metal rod that can fit diagonally into each of the boxes shown below. Give your answers correct to 2 decimal places.

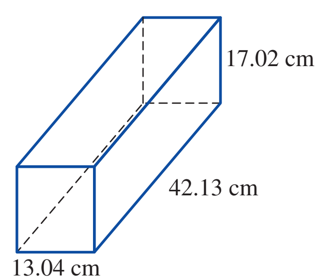
a



b



c



UNDERSTANDING

5 WE7 The height of an isosceles triangle is 3.4 mm and its equal sides are twice as long.
- a Sketch the triangle, showing all the information given.
- b Calculate the length of the third side to 2 decimal places.
- c Calculate the area of the triangle to 2 decimal places.

6 The side length of an equilateral triangle is 1 m. Calculate:
- a the height of the triangle in metres to 2 decimal places
- b the area of the triangle in m^2 to 3 decimal places.

7 MC The longest length object that can fit into a box with dimensions 42 cm × 60 cm × 13 cm is:

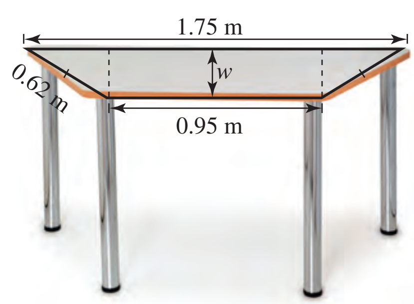
A 74.5 cm B 60 cm C 73.2 cm
D 5533 cm E 74 cm

8 A friend wants to pack an umbrella into her suitcase. If the suitcase measures 89 cm × 21 cm × 44 cm, will her 1-m umbrella fit in? Give the length (correct to 3 decimal places) of the longest object that will fit in the suitcase.

9 A friend is packing his lunch into his lunchbox and wants to fit in a 22-cm-long straw. If the lunchbox is 15 cm × 5 cm × 8 cm, will it fit in? What is the length (correct to 2 decimal places) of the longest object that will fit into the lunchbox?

10 A cylindrical pipe has a length of 2.4 m and an internal diameter of 30 cm. What is the largest object that could fit in the pipe? Give your answer correct to the nearest cm.

11 A classroom contains 20 identical desks like the one shown at right.
- a Calculate the width of each desktop (labelled w) correct to 2 decimal places.
- b Calculate the area of each desktop correct to 2 decimal places.
- c If the top surface of each desk is to be painted, what is the total area that needs to be painted correct to 2 decimal places?
- d What is the cost of the paint needed to finish the job if all desks are to be given two coats of fresh paint and paint is sold in 1-litre containers that cost $29.95 each and give coverage of 12 m^2 per litre?



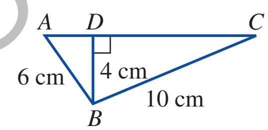
12 A hobby knife has a blade in the shape of a right-angled trapezium with the sloping edge 2 cm long and parallel sides 32 mm and 48 mm long. Calculate the width of the blade and, hence, the area.

13 A cylindrical pen holder has an internal height of 16 cm. If the diameter of the pen holder is 8.5 cm and the width of the pen holder is 2 mm, what is the largest pen that could fit completely inside the holder? Give your answer correct to the nearest mm.

REASONING

14 Explain in your own words how a 2-D right-angled triangle can be seen in a 3-D figure.

15 Consider the figure shown at right.

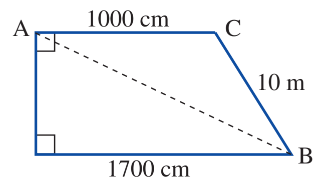


- **a** In which order will you need to calculate the lengths of the following sides: AD, AC and DC?
- **b** Calculate the lengths of the sides in part **a** to 2 decimal places.
- **c** Is the triangle ABC right-angled? Use calculations to justify your answer.

16 Katie goes on a hike and walks 2.5 km north, then 3.1 km east. She then walks 1 km north and 2 km west. She then walks in a straight line back to her starting point. Show that she walks a total distance of 12.27 km.

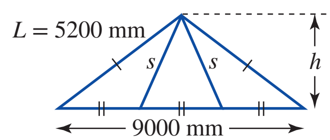
17 **a** Show that the distance AB in the plan of the paddock at right is 18.44 metres.

- **b** Prove that the angle ∠ACB is not a right angle.



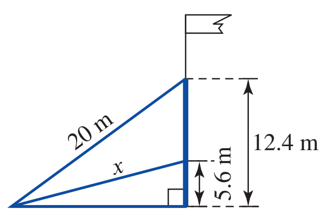
18 The diagram at right shows the cross-section through a roof.

- **a** Calculate the height of the roof, h, to the nearest millimetre.
- **b** The longer supports, L, are 5200 mm long. Show that the shorter supports, s, are 2193 mm shorter than the longer supports.

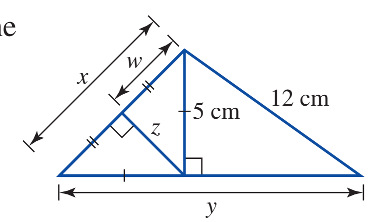


PROBLEM SOLVING

19 A flagpole is attached to the ground by two wires as shown in the diagram at right. Use the information from the diagram to calculate the length of the lower wire, x, to 1 decimal place.



20 Calculate the values of the pronumerals w, x, y and z in the diagram at right.



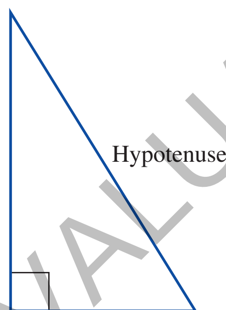
6.4 What is trigonometry?

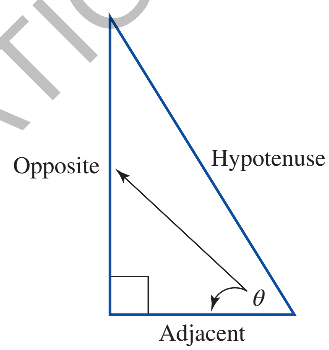
- The word **trigonometry** is derived from the Greek words *trigonon* (triangle) and *metron* (measurement). Thus, it literally means 'to measure a triangle'.
- Trigonometry deals with the relationship between the sides and the angles of a triangle.
- Modern day uses of trigonometry include surveying land, architecture, measuring distances and determining heights of inaccessible objects.
- In this section relationships between the sides and angles of a right-angled triangle will be explored.

Naming the sides of a right-angled triangle

- The longest side of a right-angled triangle (the side opposite the right angle) is called the hypotenuse.
- In order to name the remaining two sides another angle, called the 'reference angle', must be added to the diagram.

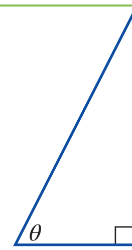
  The side that is across from the reference angle, θ, is called the **opposite side**, and the remaining side (the side next to the reference angle) is called the **adjacent side**.
- *Note:* If there is no reference angle marked, only the hypotenuse can be named.

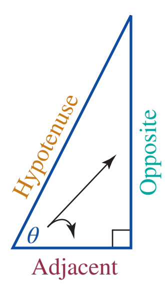




WORKED EXAMPLE 9

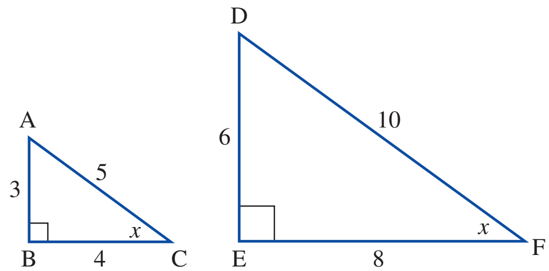
Label the sides of the right-angled triangle shown using the words hypotenuse, adjacent and opposite.



THINK	WRITE/DRAW
1 The hypotenuse is opposite the right angle.	
2 Label the side next to angle θ as 'adjacent' and the side opposite angle θ as 'opposite'.	

Similar right-angled triangles

- Consider the two right-angled triangles shown below. The second triangle (ΔDEF) is an enlargement of the first, using a scale factor of 2. Therefore, the triangles are similar (ΔABC ~ ΔDEF), and ∠BCA = ∠EFD = x.



- For ΔABC, $\frac{\text{opposite side}}{\text{adjacent side}} = \frac{3}{4}$, and for ΔDEF, $\frac{\text{opposite side}}{\text{adjacent side}} = \frac{6}{8} = \frac{3}{4}$.
- Now complete the table below.

	ΔABC	**ΔDEF**
$\frac{\textbf{Opposite side}}{\textbf{hypotenuse}}$		
$\frac{\textbf{Adjacent side}}{\textbf{hypotenuse}}$		

In similar right-angled triangles, the ratios of corresponding sides are equal.

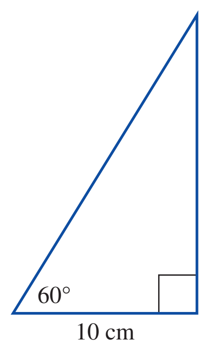
Experiment

A. Using a protractor and ruler, carefully measure and draw a right-angled triangle of base 10 cm and angle of 60° as shown in the diagram.

Measure the length of the other two sides to the nearest mm, and mark these lengths on the diagram as well.

Use your measurements to calculate these ratios correct to 2 decimal places:

$\frac{\text{opposite}}{\text{adjacent}} =$, $\frac{\text{opposite}}{\text{hypotenuse}} =$, $\frac{\text{adjacent}}{\text{hypotenuse}} =$



B. Draw another triangle, similar to the one in part A (all angles the same), making the base length anything that you choose, and measuring the length of all the sides.

Once again, calculate the three ratios correct to 2 decimal places:

$\frac{\text{opposite}}{\text{adjacent}} =$, $\frac{\text{opposite}}{\text{hypotenuse}} =$, $\frac{\text{adjacent}}{\text{hypotenuse}} =$

C. Compare your results with the rest of the class. What conclusions can you draw?

Trigonometric ratios

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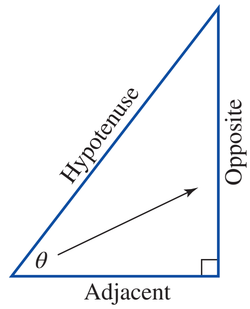
Interactivity
Investigation: Trigonometric ratios
int-0744

- Trigonometry is based upon the ratios between pairs of side lengths, and each one is given a special name as follows.
  In any right-angled triangle:

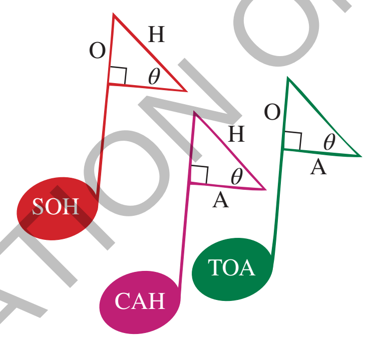
$$\textbf{sine}\,(\theta) = \frac{\textbf{opposite}}{\textbf{hypotenuse}}$$

$$\textbf{cosine}\,(\theta) = \frac{\textbf{adjacent}}{\textbf{hypotenuse}}$$

$$\textbf{tangent}\,(\theta) = \frac{\textbf{opposite}}{\textbf{adjacent}}$$

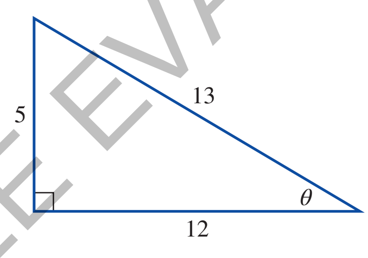


- These rules are abbreviated to:
  $\sin(\theta) = \dfrac{\text{O}}{\text{H}}$, $\cos(\theta) = \dfrac{\text{A}}{\text{H}}$ and $\tan(\theta) = \dfrac{\text{O}}{\text{A}}$.
- The following mnemonic can be used to help remember the trigonometric ratios.
  SOH – CAH – TOA



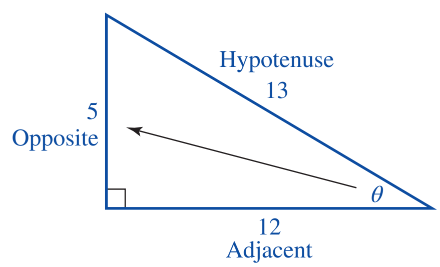
WORKED EXAMPLE 10

For this triangle, write the equations for the sine, cosine and tangent ratios of the given angle.



THINK | **WRITE/DRAW**

1 Label the sides of the triangle.



2 Write the trigonometric ratios.

$$\sin(\theta) = \frac{\text{O}}{\text{H}},\ \cos(\theta) = \frac{\text{A}}{\text{H}},\ \tan(\theta) = \frac{\text{O}}{\text{A}}$$

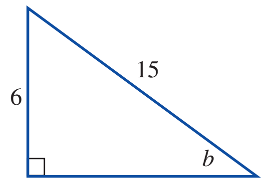
3 Substitute the values of A, O and H into each formula.

$$\sin(\theta) = \frac{5}{13},\ \cos(\theta) = \frac{12}{13},\ \tan(\theta) = \frac{5}{12}$$

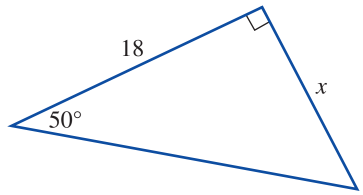
WORKED EXAMPLE 11

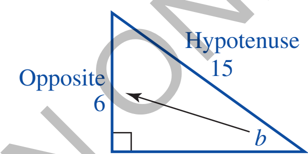
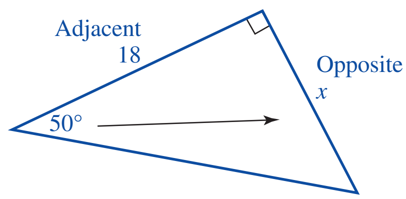
Write the trigonometric ratio that relates the two given sides and the reference angle in each of the following triangles.

a



b



THINK		WRITE/DRAW
a	**1** Label the given sides.	**a** 
	2 We are given O and H. These are used in SOH. Write the ratio.	$\sin(\theta) = \dfrac{\text{O}}{\text{H}}$
	3 Substitute the values of the pronumerals into the ratio.	$\sin(b) = \dfrac{6}{15}$
	4 Simplify the fraction.	$\sin(b) = \dfrac{2}{5}$
b	**1** Label the given sides.	**b** 
	2 We are given A and O. These are used in TOA. Write the ratio.	$\tan(\theta) = \dfrac{\text{O}}{\text{A}}$
	3 Substitute the values of the angle and the pronumerals into the ratio.	$\tan(50°) = \dfrac{x}{18}$

assess on

Exercise 6.4 What is trigonometry?

INDIVIDUAL PATHWAYS

REFLECTION
Why does sin(30°) = cos(60°)?

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8	Questions: 1–11	Questions: 1–12

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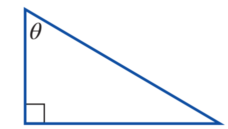
FLUENCY

1 **WE9** Label the sides of the following right-angled triangles using the words hypotenuse, adjacent and opposite.

a



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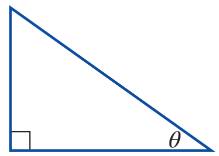

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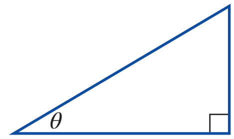

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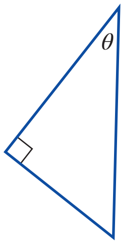

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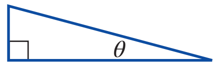

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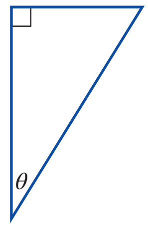
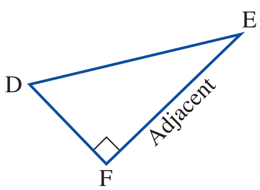


2 Label the hypotenuse, adjacent and opposite sides, and reference angle θ, where appropriate, in each of the following right-angled triangles.

a



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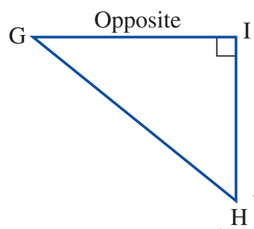

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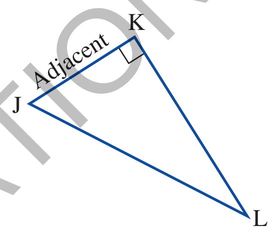
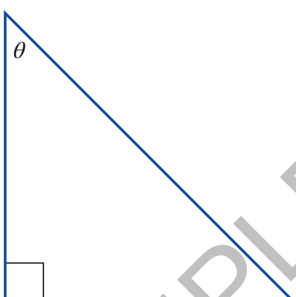
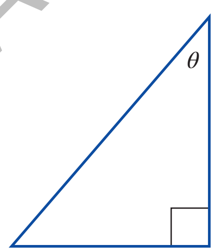


3 For each triangle below, carefully measure θ correct to the nearest degree, then carefully measure each side correct to the nearest mm. Use this information to copy and complete the table below.

a

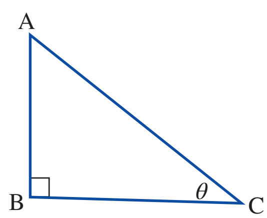


b



	$\boldsymbol{\theta}$	O	A	H	sin($\boldsymbol{\theta}$)	cos($\boldsymbol{\theta}$)	tan($\boldsymbol{\theta}$)
a							
b							

4 **MC** Which alternative correctly names the sides and angle of the triangle at right?



A $\angle C = \theta$, AB = adjacent side, AC = hypotenuse, AC = opposite side

B $\angle C = \theta$, AB = opposite side, AC = hypotenuse, AC = adjacent side

C $\angle A = \theta$, AB = opposite side, AC = hypotenuse, BC = adjacent side

D $\angle C = \theta$, AB = opposite side, AC = hypotenuse, BC = adjacent side

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Measuring angles with a protractor
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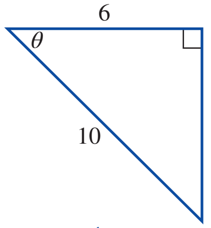
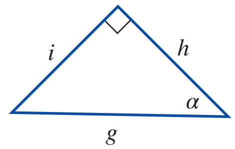
5 WE10 For each of the following triangles, write the expressions for ratios of each of the given angles:

i sine **ii** cosine **iii** tangent.

a



b



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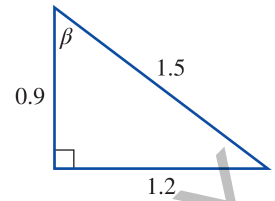

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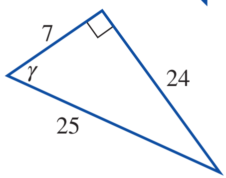

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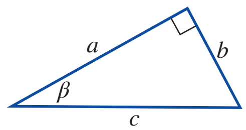

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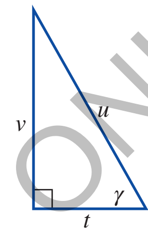
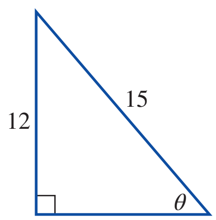


6 WE11 Write the trigonometric ratio that relates the two given sides and the reference angle in each of the following triangles.

a



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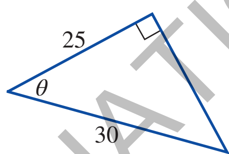

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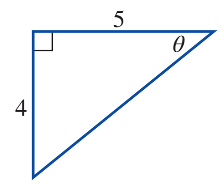

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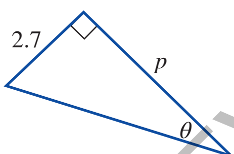

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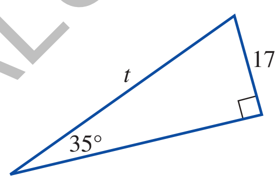

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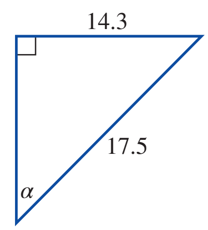

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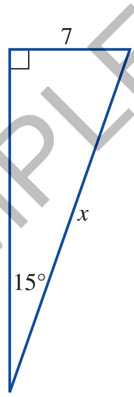

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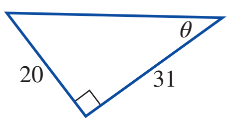
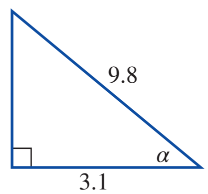
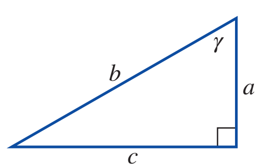


i



UNDERSTANDING

7 MC **a** What is the correct trigonometric ratio for the triangle shown at right?



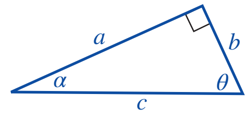
A $\tan(\gamma) = \dfrac{a}{c}$ **B** $\sin(\gamma) = \dfrac{c}{a}$

C $\cos(\gamma) = \dfrac{c}{b}$ **D** $\sin(\gamma) = \dfrac{c}{b}$

b Which trigonometric ratio for the triangle shown at right is incorrect?

A $\sin(\alpha) = \dfrac{b}{c}$ **B** $\sin(\alpha) = \dfrac{a}{c}$

C $\cos(\alpha) = \dfrac{a}{c}$ **D** $\tan(\alpha) = \dfrac{b}{a}$



REASONING

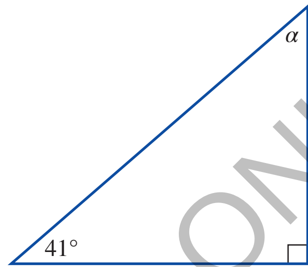
8 Consider the right-angled triangle shown at right.

a Label each of the sides using the letters O, A, H with respect to the 41° angle.

b Measure the side lengths (to the nearest millimetre).

c Determine the value of each trigonometric ratio. (Where applicable, answers should be given correct to 2 decimal places.)

i $\sin(41°)$ **ii** $\cos(41°)$ **iii** $\tan(41°)$



d What is the value of the unknown angle, α?

e Determine the value of each of these trigonometric ratios, correct to 2 decimal places.

i $\sin(\alpha)$ **ii** $\cos(\alpha)$ **iii** $\tan(\alpha)$

(*Hint:* First re-label the sides of the triangle with respect to angle α.)

f What do you notice about the relationship between $\sin(41°)$ and $\cos(\alpha)$?

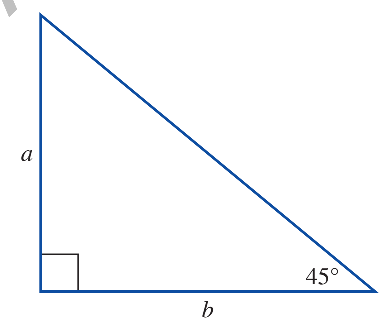
g What do you notice about the relationship between $\sin(\alpha)$ and $\cos(41°)$?

h Make a general statement about the two angles.

9 Given the triangle shown:

a why does $a = b$?

b what would the value of $\tan(45°)$ be?



PROBLEM SOLVING

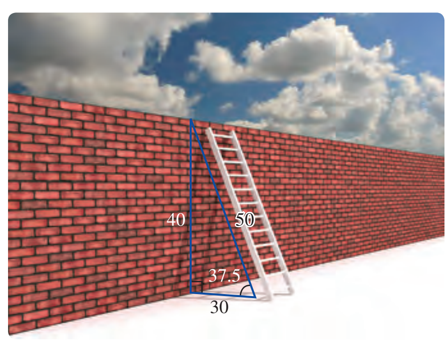
10 If a right-angled triangle has side lengths m, $(m + n)$ and $(m - n)$, which one of the lengths is the hypotenuse? Explain your reasoning.

11 A ladder leans on a wall as shown. Use the information from the diagram to answer the following questions. In relation to the angle given, what part of the image represents:

a the adjacent side

b the hypotenuse

c the opposite side?



12 Use sketches of right-angled triangles to investigate the following.

a As the acute angle increases in size, what happens to the ratio of the length of the opposite side to the length of the hypotenuse in any right-angled triangle?

b As the acute angle increases in size, what happens to the other two ratios (i.e. the ratio of the length of the adjacent side to the length of the hypotenuse and that of the opposite side to the adjacent)?

c What is the largest possible value for:

i $\sin(\theta)$ **ii** $\cos(\theta)$ **iii** $\tan(\theta)$?

6.5 Calculating unknown side lengths

Values of trigonometric ratios

- The values of trigonometric ratios can be found using a calculator.
- Each calculator has several modes. For the following calculations, your calculator must be in degree mode.

WORKED EXAMPLE 12 TI CASIO

Evaluate each of the following, giving answers correct to 4 decimal places.

a $\sin(53°)$ **b** $\cos(31°)$ **c** $\tan(79°)$

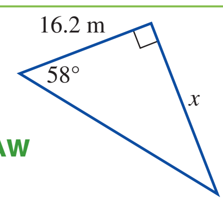
THINK	WRITE
a 1 Set the calculator to degree mode. Write the first 5 decimal places.	**a** $\sin(53°) = 0.798\ 63$
2 Round correct to 4 decimal places.	≈ 0.7986
b 1 Write the first 5 decimal places.	**b** $\cos(31°) = 0.857\ 16$
2 Round correct to 4 decimal places.	≈ 0.8572
c 1 Write the first 5 decimal places.	**c** $\tan(79°) = 5.144\ 55$
2 Round correct to 4 decimal places.	≈ 5.1446

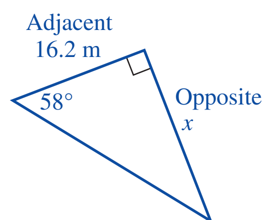
Finding side lengths

If a reference angle and any side length of a right-angled triangle are known, it is possible to find the other sides using trigonometry.

WORKED EXAMPLE 13 TI CASIO

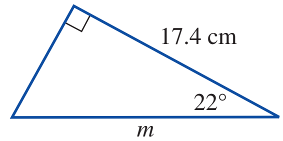
Use the appropriate trigonometric ratio to find the length of the unknown side in the triangle shown. Give your answer correct to 2 decimal places.



THINK	WRITE/DRAW
1 Label the given sides.	
2 These sides are used in TOA. Write the ratio.	$\tan(\theta) = \dfrac{\text{O}}{\text{A}}$
3 Substitute the values of θ, O and A into the tangent ratio.	$\tan(58°) = \dfrac{x}{16.2}$
4 Solve the equation for x.	$16.2 \times \tan(58°) = x$ $x = 16.2\tan(58°)$
5 Calculate the value of x to 3 decimal places, then round the answer to 2 decimal places.	$x = 25.925$ $x \approx 25.93$ m

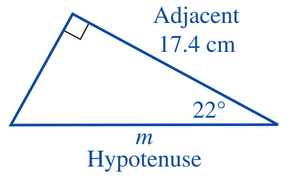
WORKED EXAMPLE 14 TI CASIO

Find the length of the side marked *m* in the triangle at right. Give your answer correct to 2 decimal places.



THINK | **WRITE/DRAW**

1 Label the given sides.



2 These sides are used in CAH. Write the ratio.

$\cos(\theta) = \dfrac{A}{H}$

3 Substitute the values of θ, A and H into the cosine ratio.

$\cos(22°) = \dfrac{17.4}{m}$

4 Solve for m:
- Multiply both sides by m.
- Divide both sides by cos(22°).

$m\cos(22°) = 17.4$

$m = \dfrac{17.4}{\cos(22°)}$

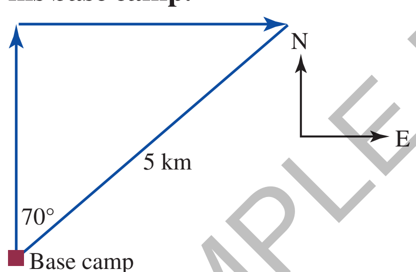
5 Calculate the value of m to 3 decimal places, then round the answer to 2 decimal places.

$m = 18.766$

$m \approx 18.77$ cm

WORKED EXAMPLE 15

Benjamin set out on a bushwalking expedition. Using a compass, he set off on a course N 70°E (or 070°T) and travelled a distance of 5 km from his base camp.



a How far east has he travelled?

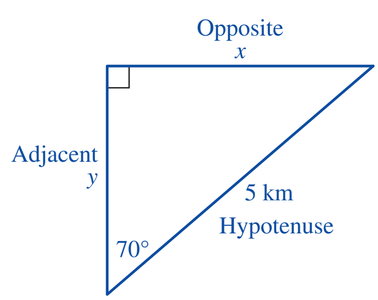
b How far north has he travelled from the base camp?

Give answers correct to 2 decimal places.

THINK | **WRITE/DRAW**

a 1 Label the easterly distance x.
Label the northerly distance y.
Label the sides of the triangle:
Hypotenuse, Opposite, Adjacent.

a



2 To calculate the value of x, use the sides of the triangle: $x = \text{O}$, $5 = \text{H}$. These are used in SOH. Write the ratio. $\sin(\theta) = \dfrac{\text{O}}{\text{H}}$

3 Substitute the values of the angle and the pronumerals into the sine ratio. $\sin(70°) = \dfrac{x}{5}$

4 Make x the subject of the equation. $x = 5\sin(70°)$

5 Evaluate x to 3 decimal places, using a calculator. $= 4.698$

6 Round to 2 decimal places. ≈ 4.70 km

7 Answer the question in sentence form. Benjamin has travelled 4.70 km east of the base camp.

b 1 To calculate the value of y, use the sides: $y = \text{A}$, $5 = \text{H}$. These are used in CAH. Write the ratio. **b** $\cos(\theta) = \dfrac{\text{A}}{\text{H}}$

2 Substitute the values of the angle and the pronumerals into the cosine ratio. $\cos(70°) = \dfrac{y}{5}$

3 Make y the subject of the equation. $y = 5\cos(70°)$

4 Evaluate y using a calculator. $= 1.710$

5 Round the answer to 2 decimal places. ≈ 1.71 km

6 Answer the question in sentence form. Benjamin has travelled 1.71 km north of the base camp.

Exercise 6.5 Calculating unknown side lengths

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–3, 4a–c, 5a–c, 6a–f, 7–9, 11, 12	Questions: 1–3, 4b–d, 5b–d, 6d–i, 7–10, 12–15	Questions: 1–3, 4d–f, 5d–f, 6g–i, 7–10, 11, 13–17

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REFLECTION
What does sin(60°) actually mean?

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SkillSHEET Solving equations of the type $a = \dfrac{b}{x}$ to find x doc-10833
SkillSHEET Rearranging formulas doc-10834

FLUENCY

1 **WE12** **a** Evaluate the following correct to 4 decimal places.
 i $\sin(55°)$ ii $\sin(11.6°)$

b Copy and complete the table below. Use your calculator to find each value of $\sin(\theta)$ correct to 2 decimal places.

θ	0°	15°	30°	45°	60°	75°	90°
$\sin(\theta)$							

c Summarise the trend in these values.

2 **a** Evaluate the following correct to 4 decimal places.
 i $\cos(38°)$ ii $\cos(53.71°)$

b Copy and complete the table below. Use your calculator to find each value of cos(θ) correct to 2 decimal places.

$\boldsymbol{\theta}$	0°	15°	30°	45°	60°	75°	90°
cos($\boldsymbol{\theta}$)							

c Summarise the trend in these values.

3 **a** Evaluate the following correct to 4 decimal places.

i tan(18°) **ii** tan(51.9°)

b Copy and complete the table below. Use your calculator to find each value of tan(θ) correct to 2 decimal places.

$\boldsymbol{\theta}$	0°	15°	30°	45°	60°	75°	90°
tan($\boldsymbol{\theta}$)							

c Find the value of tan(89°) and tan(89.9°).

d What do you notice about these results?

4 **WE13** Use the appropriate trigonometric ratios to find the length of the unknown side in each of the triangles shown. Give the answers correct to 2 decimal places.

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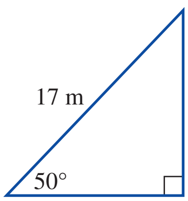
a



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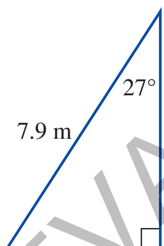

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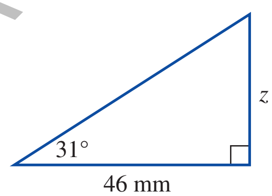

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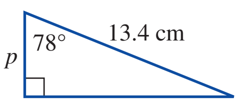

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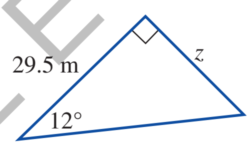

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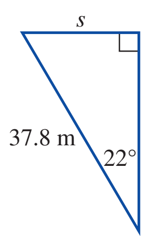
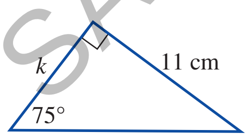


5 **WE14** Use the appropriate trigonometric ratio to find the length of the unknown side in each of the triangles shown. Give the answers correct to 2 decimal places.

a



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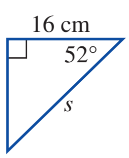

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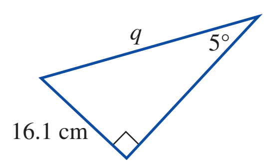

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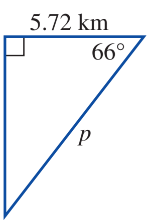

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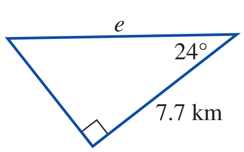

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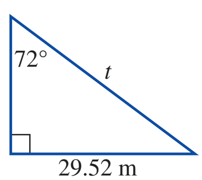
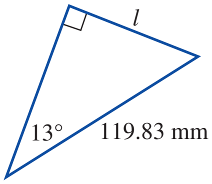


6 Find the length of the unknown side in each of the following triangles, correct to 2 decimal places. (*Note:* In some cases the unknown will be in the numerator and in other cases it will be in the denominator.)

a


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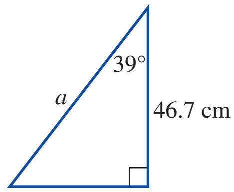

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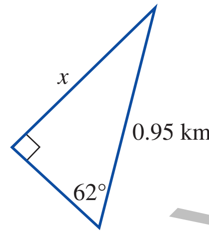

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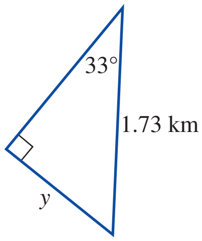

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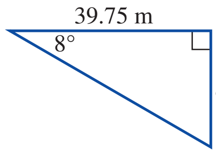

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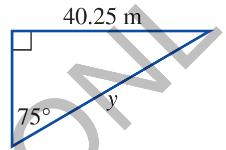

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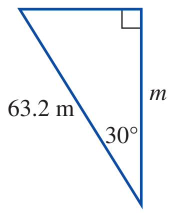

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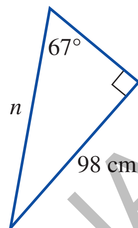

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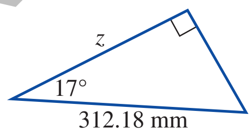
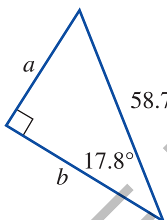


7 Find the lengths of the unknown sides in the triangles shown, correct to 2 decimal places.

a


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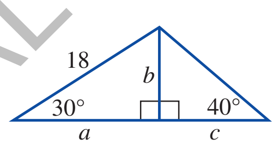

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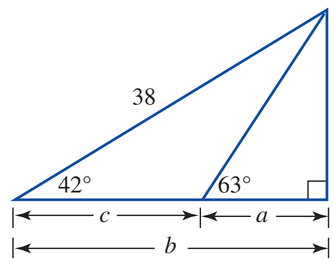
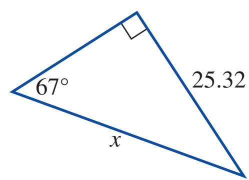


UNDERSTANDING

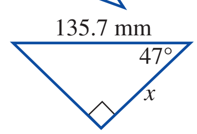
8 MC **a** The value of x correct to 2 decimal places is:

A 59.65 **B** 23.31 **C** 64.80 **D** 27.51



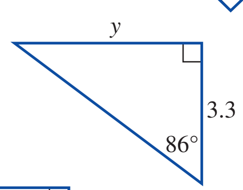
b The value of x correct to 2 decimal places is:

A 99.24 mm **B** 92.55 mm **C** 185.55 mm **D** 198.97 mm



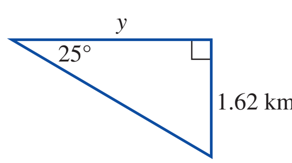
c The value of y correct to 2 decimal places is:

A 47.19 **B** 7.94 **C** 1.37 **D** 0.23



d The value of y correct to 2 decimal places is:

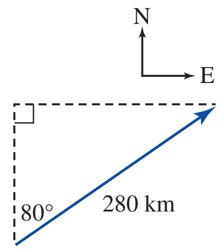
A 0.76 km **B** 1.79 km
C 3.83 km **D** 3.47 km



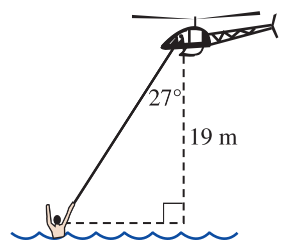
9 WE15 A ship that was to travel due north veered off course and travelled N 80°E (or 080°T) for a distance of 280 km, as shown in the diagram.

a How far east had the ship travelled?

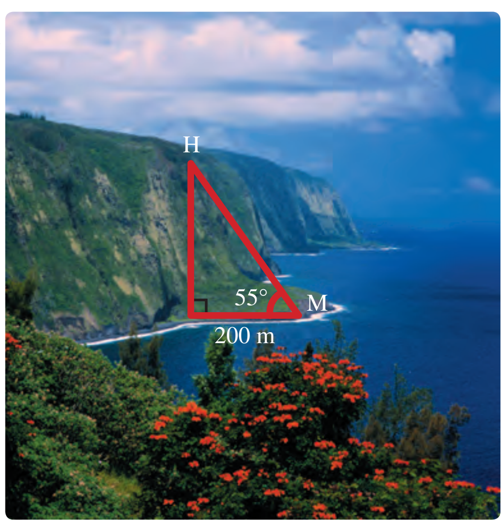
b How far north had the ship travelled?



10 A rescue helicopter spots a missing surfer drifting out to sea on his damaged board. The helicopter descends vertically to a height of 19 m above sea level and drops down an emergency rope, which the surfer grips. Due to the wind the rope swings at an angle of 27° to the vertical, as shown in the diagram. What is the length of the rope?

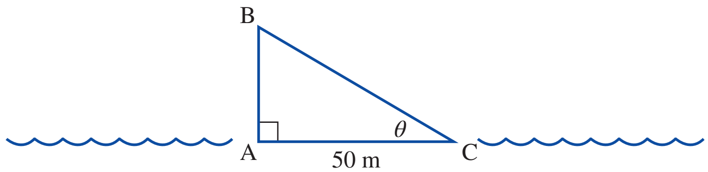


11 Walking along the coastline, Michelle (M) looks up through an angle of 55° and sees her friend Helen (H) on top of the cliff at the lookout point. How high is the cliff if Michelle is 200 m from its base? (Assume both girls are the same height.)



REASONING

12 One method for determining the distance across a body of water is shown in the diagram below.



The required distance is AB. A surveyor moves at right angles 50 m to point C and uses a tool called a transit to measure the angle θ (∠ACB).

a If $\theta = 12.3°$, show that the length AB is 10.90 m.

b Show that a value of $\theta = 63.44°$ gives a length of AB = 100 m.

c Find a rule that can be used to calculate the length AC.

13 Using a diagram, explain why $\sin(70°) = \cos(20°)$ and $\cos(70°) = \sin(20°)$. In general, $\sin(\theta)$ will be equal to which cosine?

PROBLEM SOLVING

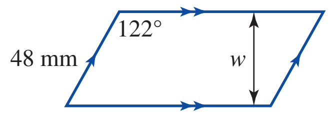
14 Calculate the value of the pronumeral in each of the following triangles.

a

b

c

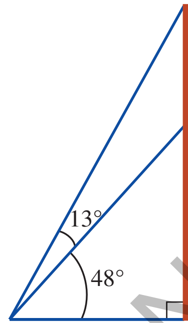
15 A tile is in the shape of a parallelogram with measurements as shown. Calculate the width of the tile, w, to the nearest mm.



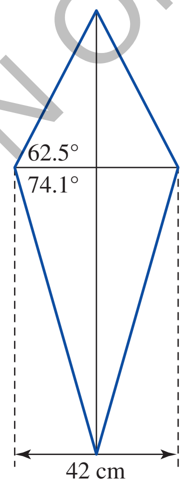
16 A pole is supported by two wires as shown. If the length of the lower wire is 4.3 m, calculate to 1 decimal place:

a the length of the top wire

b the height of the pole.



17 The frame of a kite is built from 6 wooden rods as shown. Calculate the total length of wood used to make the frame of the kite to the nearest metre.



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CHALLENGE 6.1

A track in a national park has four legs from its starting point to its return. The first leg travels due east. The second, third and fourth legs follow paths southwest, due north and finally due east back to the starting point. If the distances of the first two legs are 1.5 km and 3.5 km respectively, what distances are the third and fourth legs?

6.6 Calculating unknown angles

Inverse trigonometric ratios

- We have seen that $\sin(30°) = 0.5$; therefore, 30° is the inverse sine of 0.5. This is written as $\sin^{-1}(0.5) = 30°$.
- The expression $\sin^{-1}(x)$ is read as 'the inverse sine of x'.
  The expression $\cos^{-1}(x)$ is read as 'the inverse cosine of x'.
- The expression $\tan^{-1}(x)$ is read as 'the inverse tangent of x'.

Experiment

1. Use your calculator to find $\sin(30°)$, then find the inverse sine of the answer. Choose another angle and do the same thing.
2. Now find $\cos(30°)$ and then find the inverse cosine ($\cos^{-1}$) of the answer. Choose another angle and do the same thing.

3. Lastly, find $\tan(45°)$ and then find the inverse tangent ($\tan^{-1}$) of the answer. Try this with other angles.

- The fact that sin and $\sin^{-1}$ cancel each other out is useful in solving equations such as:

$$\sin(\theta) = 0.3 \quad \text{(Take the inverse sine of both sides.)}$$
$$\sin^{-1}(\sin(\theta)) = \sin^{-1}(0.3)$$
$$\theta = \sin^{-1}(0.3)$$
$$\sin^{-1}(x) = 15° \quad \text{(Take the sine of both sides.)}$$
$$\sin(\sin^{-1}(x)) = \sin(15°)$$
$$x = \sin(15°)$$

Similarly, $\cos(\theta) = 0.522$ means that

$$\theta = \cos^{-1}(0.522)$$

and $\tan(\theta) = 1.25$ means that

$$\theta = \tan^{-1}(1.25).$$

WORKED EXAMPLE 16 TI CASIO

Evaluate $\cos^{-1}(0.3678)$, correct to the nearest degree.

THINK	WRITE
1 Set your calculator to degree mode.	$\cos^{-1}(0.3678) = 68.4$
2 Round the answer to the nearest whole number and include the degree symbol.	$\approx 68°$

WORKED EXAMPLE 17

Determine the size of angle θ in each of the following. Give answers correct to the nearest degree.
a $\sin(\theta) = 0.6543$ **b $\tan(\theta) = 1.745$**

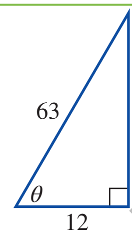
THINK	WRITE
a 1 θ is the inverse sine of 0.6543.	a $\sin(\theta) = 0.6543$ $\theta = \sin^{-1}(0.6543)$
2 Calculate and record the answer.	$= 40.8$
3 Round the answer to the nearest degree.	$\approx 41°$
b 1 θ is the inverse tangent of 1.745.	b $\tan(\theta) = 1.745$ $\theta = \tan^{-1}(1.745)$
2 Use the inverse tangent function on a calculator. Record the number shown.	$= 60.18$
3 Round the answer to the nearest degree.	$\approx 60°$

Finding the angle when 2 sides are known

If the lengths of any 2 sides of a right-angled triangle are known, it is possible to find an angle using inverse sine, inverse cosine or inverse tangent.

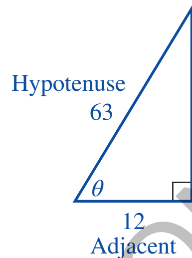
WORKED EXAMPLE 18

Determine the value of θ in the triangle at right. Give your answer correct to the nearest degree.



THINK / **WRITE/DRAW**

1 Label the given sides. These are used in CAH. Write the ratio.



$$\cos(\theta) = \frac{A}{H}$$

2 Substitute the given values into the cosine ratio.

$$\cos(\theta) = \frac{12}{63}$$

3 θ is the inverse cosine of $\frac{12}{63}$.

$$\theta = \cos^{-1}\left(\frac{12}{63}\right)$$

4 Evaluate.

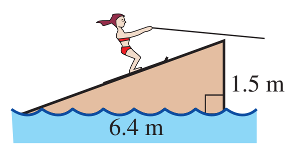
$$= 79.0$$

5 Round the answer to the nearest degree.

$$\approx 79°$$

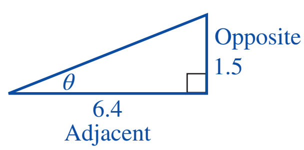
WORKED EXAMPLE 19

Roberta enjoys water skiing and is about to try a new ramp on the Hawkesbury River. The inclined ramp rises 1.5 m above the water level and spans a horizontal distance of 6.4 m. What is the magnitude (size) of the angle that the ramp makes with the water? Give the answer correct to the nearest degree.



THINK / **WRITE/DRAW**

1 Draw a simple diagram, showing the known lengths and the angle to be found.



2 Label the given sides. These are used in TOA. Write the ratio.

$$\tan(\theta) = \frac{O}{A}$$

3 Substitute the values of the pronumerals into the tangent ratio.

$$\tan(\theta) = \frac{1.5}{6.4}$$

4 θ is the inverse inverse tangent of $\frac{1.5}{6.4}$.

$$\theta = \tan^{-1}\left(\frac{1.5}{6.4}\right)$$

5	Evaluate.	$= 13.19$
6	Round the answer to the nearest degree.	$\approx 13°$
7	Write the answer in words.	The ramp makes an angle of 13° with the water.

assess on

Exercise 6.6 Calculating unknown angles

INDIVIDUAL PATHWAYS

REFLECTION
Why does $\cos(0°) = 1$?

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–c, 2, 3a–f, 4–10	Questions: 1d–f, 2, 3d–h, 4–11	Questions: 1g–i, 2, 3e–i, 4–13

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SkillSHEET
Rounding angles to the nearest degree
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FLUENCY

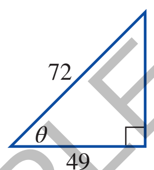
1 **WE16** Evaluate each of the following, correct to the nearest degree.

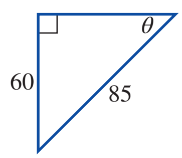
a $\sin^{-1}(0.6294)$ **b** $\cos^{-1}(0.3110)$ **c** $\tan^{-1}(0.7409)$
d $\tan^{-1}(1.3061)$ **e** $\sin^{-1}(0.9357)$ **f** $\cos^{-1}(0.3275)$
g $\cos^{-1}(0.1928)$ **h** $\tan^{-1}(4.1966)$ **i** $\sin^{-1}(0.2554)$

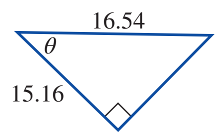
2 **WE17** Determine the size of the angle in each of the following. Give answers correct to the nearest degree.

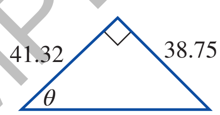
a $\sin(\theta) = 0.3214$ **b** $\sin(\theta) = 0.6752$ **c** $\sin(\beta) = 0.8235$
d $\cos(\beta) = 0.9351$ **e** $\cos(\alpha) = 0.6529$ **f** $\cos(\alpha) = 0.1722$
g $\tan(\theta) = 0.7065$ **h** $\tan(a) = 1$ **i** $\tan(b) = 0.876$
j $\sin(c) = 0.3936$ **k** $\cos(\theta) = 0.5241$ **l** $\tan(\alpha) = 5.6214$

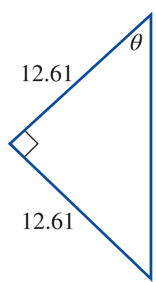
3 **WE18** Determine the value of θ in each of the following triangles. Give answers correct to the nearest degree.

a


b


c


d


e


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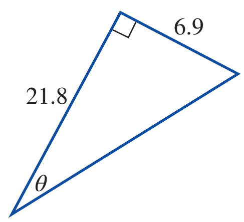

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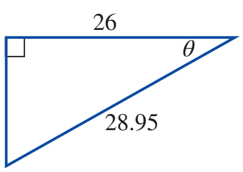

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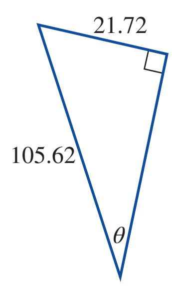

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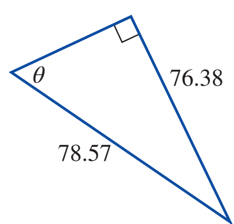


4 MC **a** If $\cos(\theta) = 0.8752$, the value of θ correct to 2 decimal places is:

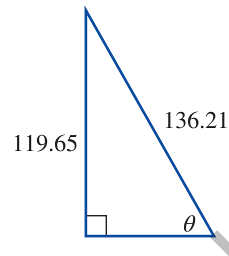
A 61.07° B 41.19° C 25.84° D 28.93°

b If $\sin(\theta) = 0.5530$, the value of θ correct to 2 decimal places is:

A 56.43° B 33.57° C 28.94° D 36.87°

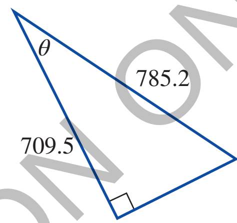
c The value of θ in the triangle shown, correct to 2 decimal places, is:

A 41.30°
B 28.55°
C 48.70°
D 61.45°



d The value of θ in the triangle shown, correct to 2 decimal places, is:

A 42.10°
B 64.63°
C 25.37°
D 47.90°



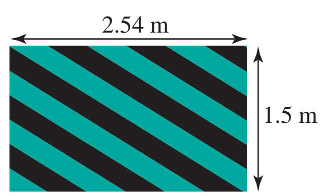
5 **a** Copy and fill in the table below.

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$y = \cos^{-1}(x)$	90°					60°					0°

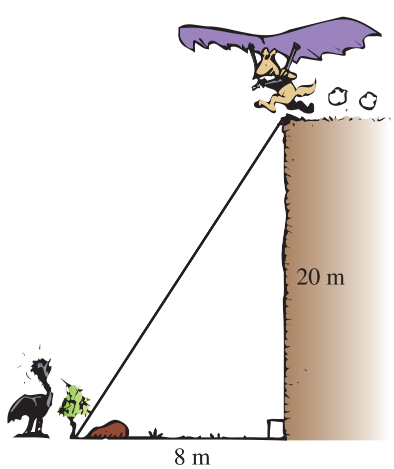
b Plot the above table on graph paper or with a spreadsheet or suitable calculator.

UNDERSTANDING

6 A piece of fabric measuring 2.54 m by 1.5 m has a design consisting of parallel diagonal stripes. What angle does each diagonal make with the length of the fabric? Give your answer correct to 2 decimal places.



7 WE19 Danny Dingo is perched on top of a cliff 20 m high watching an emu feeding 8 m from the base of the cliff. Danny has purchased a flying contraption, which he hopes will help him capture the emu. At what angle to the cliff must he swoop to catch his prey? Give your answer correct to 2 decimal places.



REASONING

8 Jenny and Les are camping with friends Mark and Susie. Both couples have a 2-m-high tent. The top of each 2-m tent pole is to be tied with a piece of rope that will be used to keep the pole upright. So that the rope doesn't trip a passerby, Jenny and Les decide that the angle between the rope and the ground should be 60°. Answer the following questions, correct to 2 decimal places.

a Find the length of the rope needed from the top of the tent pole to the ground to support their tent pole.

b Further down the camping ground, Mark and Susie also set up their tent. However, they want to use a piece of rope that they know is in the range of 2 to 3 metres in length.

i Explain why the rope will have to be greater than 2 metres in length.

ii Show that the minimum angle the rope will make with the ground will be 41.8°.

9 Safety guidelines for wheelchair access ramps used to state that the gradient had to be in the ratio 1 : 20.

a Using this ratio, show that the angle that the ramp had to make with the horizontal is closest to 3°.

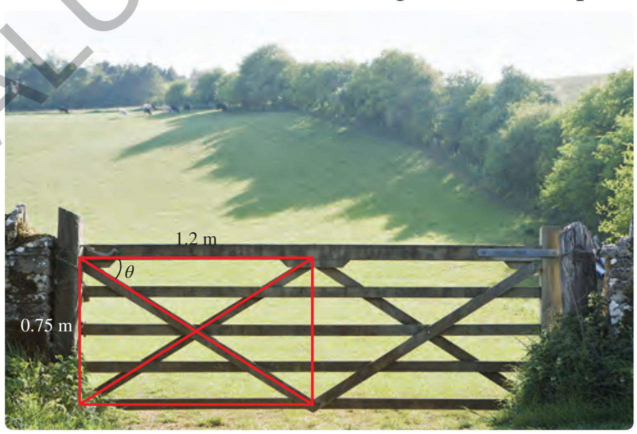
b New regulations have changed the ratio of the gradient, so the angle the ramp must make with the horizontal is now closest to 6°. Explain why, using this angle size, the new ratio could be 1 to 9.5.

PROBLEM SOLVING

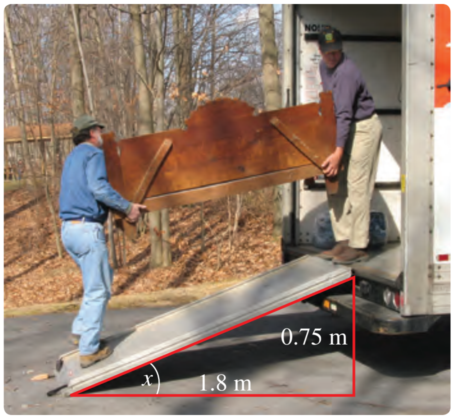
10 Calculate the value of the pronumeral in each of the following to 2 decimal places.

a

b



c



11 A family is building a patio extension to their house. One section of the patio will have a gable roof. A similar structure is pictured with the planned post heights and span shown. To allow more light in, the family wants the peak (highest point) of the gable roof to be at least 5 m above deck level. According to building regulations, the slope of the roof (i.e. the angle that the sloping edge makes with the horizontal) must be 22°.

a Use trigonometry to calculate whether the roof would be high enough if the angle was 22°.

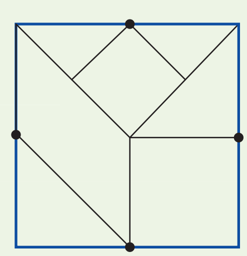
b Use trigonometry to calculate the size of the obtuse angle formed at the peak of the roof.

12 Use the formulas $\sin(\theta) = \dfrac{o}{h}$ and $\cos(\theta) = \dfrac{a}{h}$ to prove that $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$.

CHALLENGE 6.2

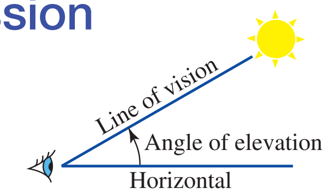
1 A square-based prism has a height twice its base length. What angle does the diagonal of the prism make with the diagonal of the base?

2 Seven smaller shapes are created inside a square with side length 10 cm as shown in the diagram. If the dots represent the midpoints of the square's sides, find the dimensions of each shape.

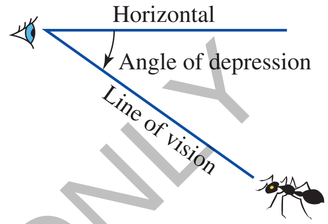


6.7 Angles of elevation and depression

- When looking up towards an object, an **angle of elevation** is the angle between the horizontal line and the line of vision.



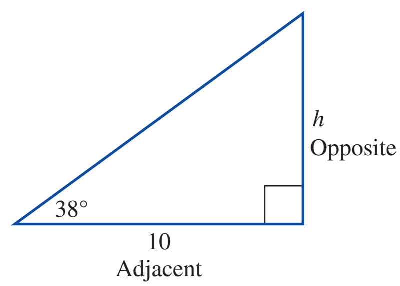
- When looking down at an object, an **angle of depression** is the angle between the horizontal line and the line of vision.



- Angles of elevation and depression are measured from horizontal lines.

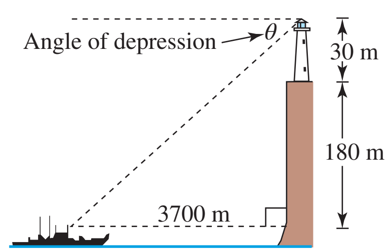
WORKED EXAMPLE 20

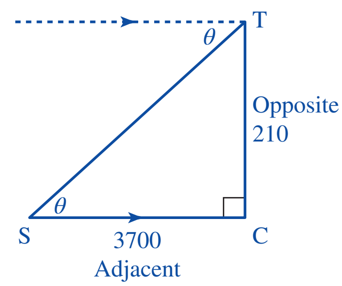
At a point 10 m from the base of a tree, the angle of elevation of the treetop is 38°. How tall is the tree to the nearest centimetre?

THINK	WRITE/DRAW
1 Draw a simple diagram. The angle of elevation is 38° from the horizontal.	
2 Label the given sides of the triangle. These sides are used in TOA. Write the ratio.	$\tan(38°) = \frac{h}{10}$
3 Multiply both sides by 10.	$10\tan(38°) = h$
4 Calculate correct to 3 decimal places.	$h = 7.812$
5 Round to 2 decimal places.	≈ 7.81
6 Write the answer in words.	The tree is 7.81 m tall.

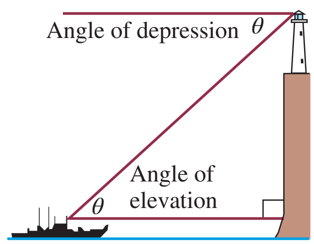
WORKED EXAMPLE 21

A lighthouse, 30 m tall, is built on top of a cliff that is 180 m high. Find the angle of depression (θ) of a ship from the top of the lighthouse if the ship is 3700 m from the bottom of the cliff.

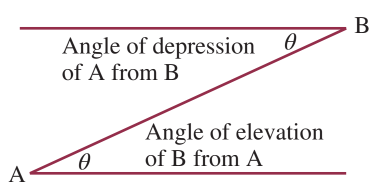


	THINK	WRITE/DRAW
1	Draw a simple diagram to represent the situation. The height of the triangle is $180 + 30 = 210$ m. Draw a horizontal line from the top of the triangle and mark the angle of depression, θ. Also mark the alternate angle.	
2	Label the triangle. These sides are used in TOA. Write the ratio.	$\tan(\theta) = \dfrac{O}{A}$
3	Substitute the given values into the ratio.	$\tan(\theta) = \dfrac{210}{3700}$
4	θ is the inverse tangent of $\dfrac{210}{3700}$.	$\theta = \tan^{-1}\left(\dfrac{210}{3700}\right)$
5	Evaluate.	$= 3.2$
6	Round the answer to the nearest degree.	$\approx 3°$
7	Write the answer in words.	The angle of depression of the ship from the top of the lighthouse is 3°.

- *Note:* In Worked example 21, the angle of depression from the top of the lighthouse to the ship is equal to the angle of elevation from the ship to the top of the lighthouse. This is because the angle of depression and the angle of elevation are alternate (or 'Z') angles.



- This can be generalised as follows:
For any two objects, A and B, the angle of elevation of B, as seen from A, is equal to the angle of depression of A, as seen from B.



Exercise 6.7 Angles of elevation and depression

assess on

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6, 8, 10, 12	Questions: 1–6, 8, 11–14	Questions: 1–3, 6, 7, 9–16

Individual pathway interactivity int-4501 eBook*plus*

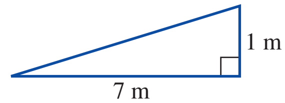
REFLECTION
Why does the angle of elevation have the same value as the angle of depression?

eBook*plus*

Digital doc
SkillSHEET
Drawing a diagram from given directions
doc-10837

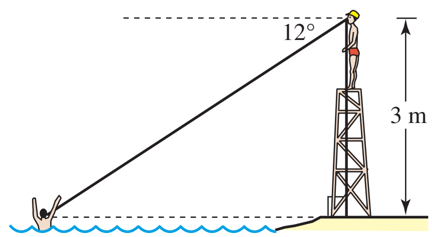
FLUENCY

1 **WE20** Building specifications require the angle of elevation of any ramp constructed for public use to be less than 3°.

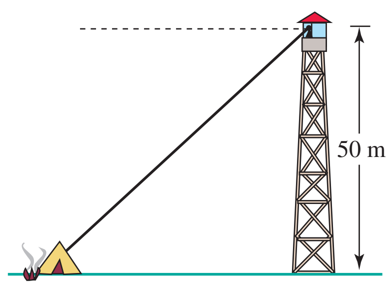


Ramps being constructed at a new shopping centre are each made in the ratio 7 m horizontal length to 1 m vertical height. Find the angle of elevation of these ramps and, hence, decide whether they meet building specifications.

2 A lifesaver standing on his tower 3 m above the ground spots a swimmer experiencing difficulty. The angle of depression of the swimmer from the lifesaver is 12°. How far is the swimmer from the lifesaver's tower? (Give your answer correct to 2 decimal places.)



3 From the top of a lookout 50 m above the ground, the angle of depression of a camp site that is level with the base of the lookout is 37°. How far is the camp site from the base of the lookout?



UNDERSTANDING

4 From a rescue helicopter 80 m above the ocean, the angles of depression of two shipwreck survivors are 40° and 60° respectively. If the two sailors and the helicopter are in line with each other:

   a draw a labelled diagram to represent the situation

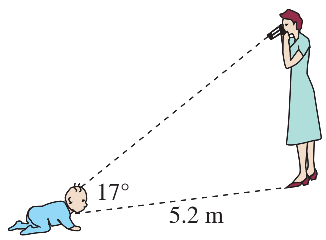
   b calculate the distance between the two sailors, to the nearest metre.

5 The angle of elevation of the top of a tree from a point on the ground, 60 m from the tree, is 35°.

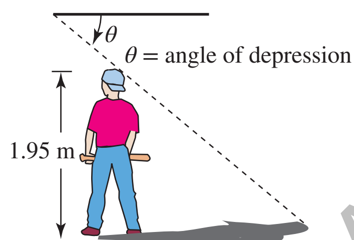
   a Draw a labelled diagram to represent the situation.

   b Find the height of the tree to the nearest metre.

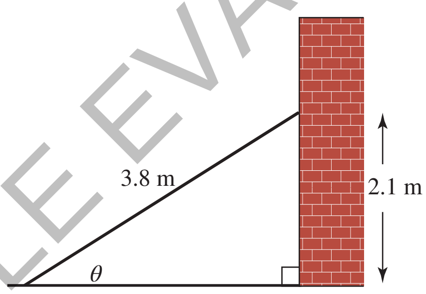
6 Miriam, an avid camerawoman from Perth, wants to record her daughter Alexandra's first attempts at crawling. As Alexandra lies on the floor and looks up at her mother, the angle of elevation is 17°. If Alexandra is 5.2 m away from her mother, how tall is Miriam? Give your answer correct to 1 decimal place.



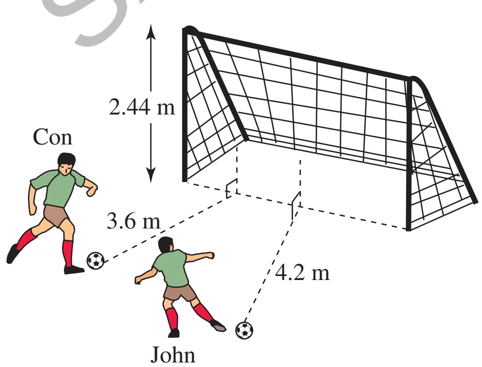
7 WE21 Stan, who is 1.95 m tall, measures the length of the shadow he casts along the ground as 0.98 m. Find the angle of depression of the sun's rays to the nearest degree.



8 What angle does a 3.8-m ladder make with the ground if it reaches 2.1 m up the wall? How far is the foot of the ladder from the wall? (Give your answers to the nearest degree and the nearest metre.)

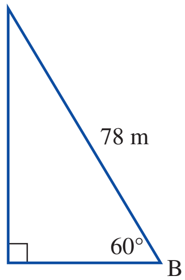


9 Con and John are practising shots on goal. Con is 3.6 m away from the goal and John is 4.2 m away, as shown in the diagram. If the height of the goal post is 2.44 m, what is the maximum angle of elevation, to the nearest degree, that each can kick the ball in order to score a goal?

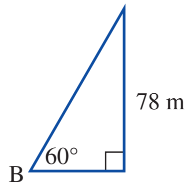


10 **MC** The angle of elevation of the top of a lighthouse tower 78 m tall, from a point B on the same level as the base of the tower, is 60°. The correct diagram for this information is:

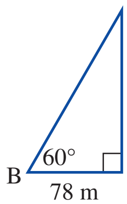
A



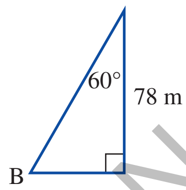
B



C



D

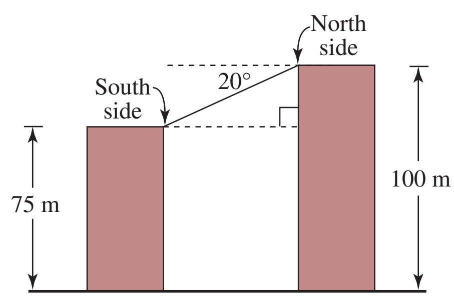


REASONING

11 Lifesaver Sami spots some dolphins playing near a marker at sea directly in front of him. He is sitting in a tower that is situated 10 m from the water's edge and he is 4 m above sea level. The marker is 20 m from the water's edge.

- **a** Draw a diagram to represent this information.
- **b** Show that the angle of depression of Sami's view of the dolphins, correct to 1 decimal place, is 7.6°.
- **c** As the dolphins swim towards Sami, would the angle of depression increase or decrease? Justify your answer in terms of the tangent ratios.

12 Two buildings are 100 m and 75 m high. From the top of the north side of the taller building, the angle of depression to the top of the south side of the smaller building is 20°, as shown below. Show that the horizontal distance between the north side of the taller building and the south side of the smaller building is closest to 69 metres.



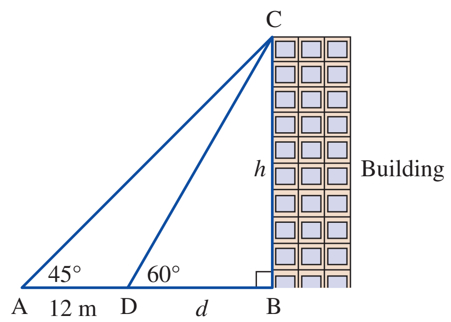
PROBLEM SOLVING

13 Rouka was hiking in the mountains when she spotted an eagle sitting up in a tree. The angle of elevation of her view of the eagle was 35°. She then walked 20 metres towards the tree and her angle of elevation was 50°. The height of the eagle from the ground was 35.5 metres.

- **a** Draw a labelled diagram to represent this information.
- **b** Determine how tall Rouka is, if her eyes are 9 cm from the top of her head. Write your answer in metres, correct to the nearest centimetre.

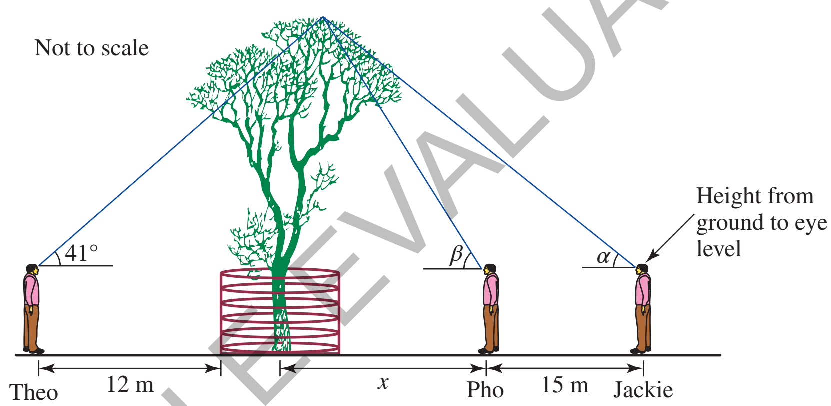
14 A lookout in a lighthouse tower can see two ships approaching the coast. Their angles of depression are 25° and 30°. If the ships are 100 m apart, show that the height of the lighthouse, to the nearest metre, is 242 metres.

15 At a certain distance away, the angle of elevation to the top of a building is 60°. From 12 m further back, the angle of elevation is 45° as shown in the diagram below.



Show that the height of the building is 28.4 metres.

16 A tall gum tree stands in a courtyard in the middle of some office buildings. Three Year 9 students, Jackie, Pho and Theo measure the angle of elevation from three different positions. They are unable to measure the distance to the base of the tree because of the steel tree guard around the base. The diagram below shows the angles of elevation and the distances measured.



a Show that $x = \dfrac{15 \tan \alpha}{\tan \beta - \tan \alpha}$, where x is the distance, in metres, from the base of the tree to Pho's position.

b The girls estimate the tree to be 15 m taller than them. Pho measured the angle of elevation to be 72°. What should Jackie have measured her angle of elevation to be, if these measurements are assumed to be correct? Write your answer to the nearest degree.

c Theo did some calculations and determined that the tree was only about 10.4 m taller than them. Jackie claims that Theo's calculation of 10.4 m is incorrect.

i Is Jackie's claim correct? Show how Theo calculated a height of 10.4 m.

ii If the height of the tree was actually 15 metres above the height of the students, determine the horizontal distance Theo should have used in his calculations. Write your answer to the nearest centimetre.

eBook*plus*

Digital doc
WorkSHEET 6.2
doc-10838

eBook*plus* ONLINE ONLY

6.8 Review

www.jacplus.com.au

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

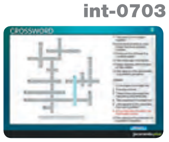
Download the Review questions document from the links found in your eBookPLUS.

eBook*plus*

Interactivities

Word search
int-0889

Crossword
int-0703



Sudoku
int-3206

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

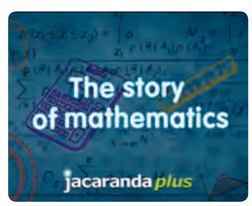
angle of depression	inverse	sine ratio
angle of elevation	opposite	tangent ratio
adjacent	Pythagoras' theorem	trigonometric inverses
cosine ratio	Pythagorean triad	trigonometric ratios
hypotenuse	right-angled triangle	

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au

The story of mathematics is an exclusive Jacaranda video series that explores the history of mathematics and how it helped shape the world we live in today.



Secret society (eles-1693) delves into the world of Pythagoras and his followers, known as the Pythagoreans. It highlights the structure of the society in which they lived and how the Pythagoreans came to influence our world today.

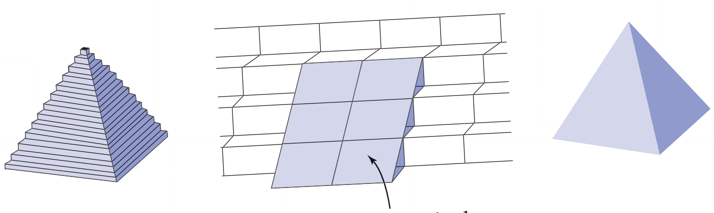
RICH TASK

The Great Pyramid of Giza

The Great Pyramid of Giza was built over four and a half thousand years ago. It was constructed using approximately 2 300 000 rectangular granite blocks and took over 20 years to complete. When built, its dimensions measured 230 m at the base and its vertical height was 146.5 m.

1 Each side of the pyramid has a triangular face. Use the dimensions given and Pythagoras' theorem to calculate the height of each triangle. Give your answer correct to 2 decimal places.

2 Special finishing blocks were added to the ends of each row of the pyramid to give each triangular face a smooth and flat finish. Calculate the area of each face of the pyramid.

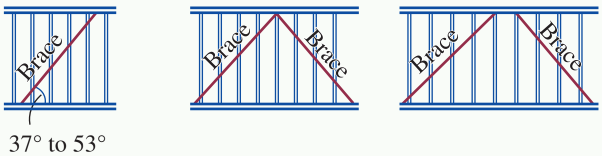


3 The edge of the pyramid joins two faces from the ground to the tip of the pyramid. Use Pythagoras' theorem to calculate the length of the edge.

Wall braces

In the building industry, wall frames are strengthened with the use of braces. These braces run between the top and bottom horizontal sections of the frame.

Industry standards stipulate that the acute angle the brace makes with the horizontal sections lies in the range 37° to 53°. Sometimes, more than one brace may be required if the frame is a long one.



1 Cut thin strips of cardboard and arrange them in the shape of a rectangle to represent a rectangular frame. Pin the corners to hold them together. Notice that the frame moves out of shape easily. Attach a brace according to the angle stipulation of the building industry. Write a brief comment to describe what effect the brace had on the frame.
2 Investigate what happens to the length of the brace required as the acute angle with the base increases from 37° to 53°.
3 Use your finding from question **2** to state which angle requires the shortest brace and which angle requires the longest brace.

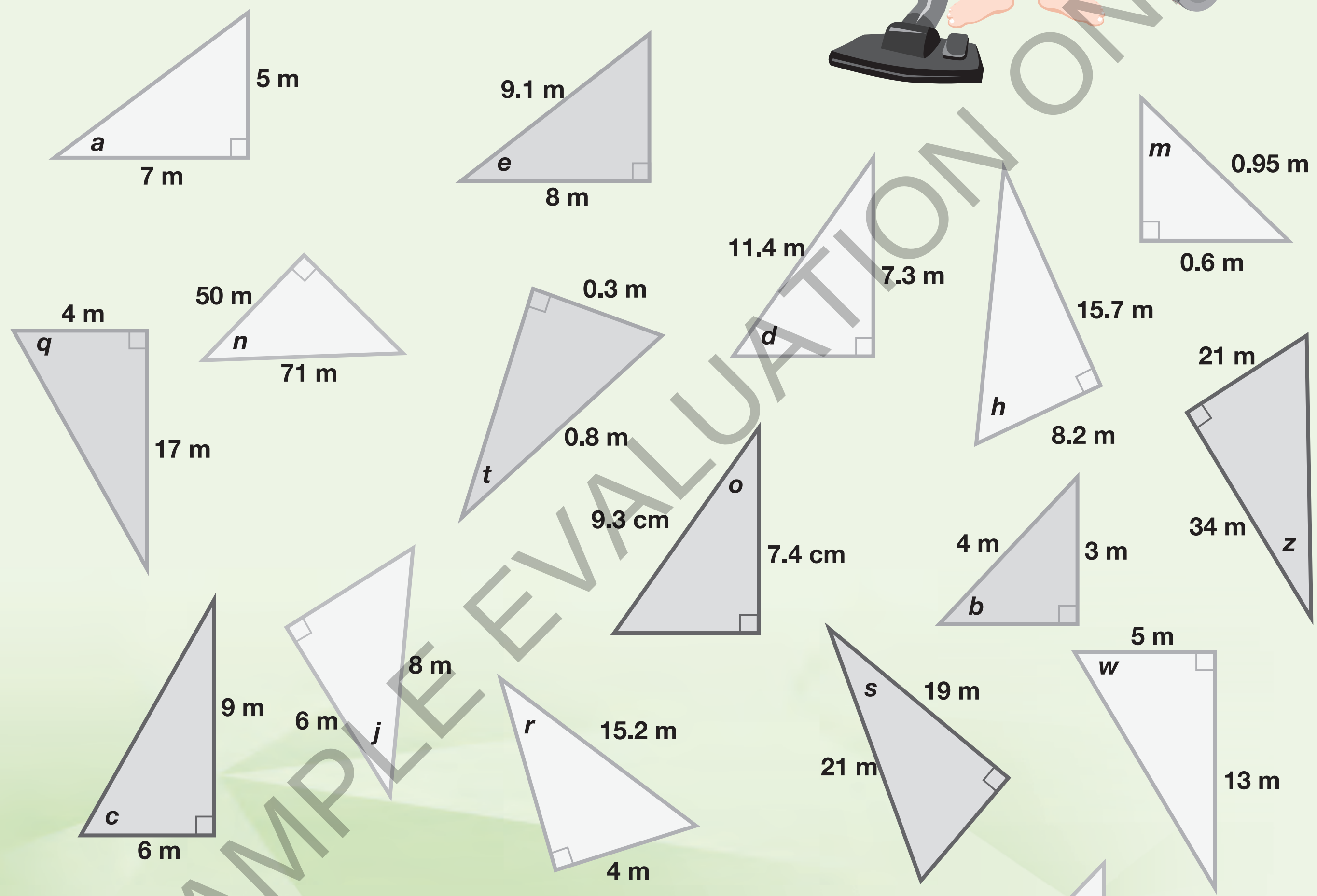
Most contemporary houses are constructed with a ceiling height of 2.4 metres; that is, the height of the walls from the floor to the ceiling. Use this fact to assist in your calculations for the following questions.

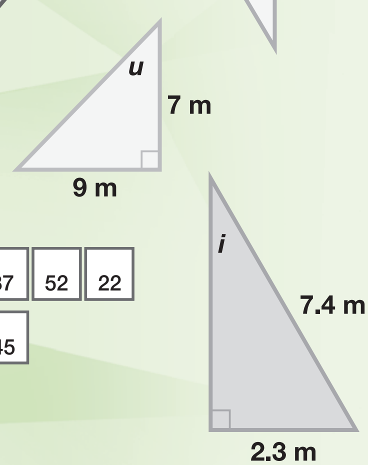
4 Assume you have a section of a wall that is 3.5 metres long. What would be the length of the longest brace possible? Draw a diagram and show your working to support your answer in the space below.
5 What would be the minimum wall length in which two braces were required? Show your working, along with a diagram, in the space provided.
6 Some older houses have ceilings over 2.4 metres. Repeat questions **4** and **5** for a frame with a height of 3 metres. Draw diagrams and show your workings to support your answers in the space below.
7 Take the measurements of a wall without windows in your school or at home. Draw a scale drawing of the frame on a separate sheet of paper and show the positions in which a brace or braces might lie. Calculate the length and angle of each brace.

CODE PUZZLE

What does it mean?

The values of lettered angles to the nearest degree give the puzzle's answer code.





Vacuum cleaner:

36	49	15	37	37	39						
69	18	22	62	36	25	22	37	39	36	56	62

Dust:

39	52	40	69	18	22	62	22	62	28						
41	52	18	56	28	25	77	52	28	28	32	28	40	37	52	22

Egg:

36	49	18	15	40	25	62	37	39	28	22	37	69	45

Fodder:

22	62	28	39	36	45	69	62	37				
39	36	15	15	18	28	40	39	52	40	40	28	15

eBookplus Activities

6.1 Overview

Video

- The story of mathematics: Secret society (eles-1693)

6.2 Pythagoras' theorem

Digital docs

- SkillSHEET (doc-11429): Rearranging formulas
- SkillSHEET (doc-11430): Converting units of length
- SkillSHEET (doc-11428): Rounding to a given number of decimal places

Interactivities

- Pythagorean triples (int-2765)
- IP interactivity (int-4472): Working with different units

6.3 Applications of Pythagoras' theorem

Interactivity

- IP interactivity (int-4475): Pythagoras in 3-D

6.4 What is trigonometry?

Digital docs

- SkillSHEET (doc-10830): Rounding to a given number of decimal places
- SkillSHEET (doc-10831): Measuring angles with a protractor

Interactivities

- Investigation: Trigonometric ratios (int-0744)
- IP interactivity 6.2 (int-4498): What is trigonometry?

6.5 Calculating unknown side lengths

eLesson

- Using an inclinometer (eles-0116)

Digital docs

- SkillSHEET (doc-10832): Solving equations of the type $a = \frac{x}{b}$ to find x
- SkillSHEET (doc-10833): Solving equations of the type $a = \frac{b}{x}$ to find x
- SkillSHEET (doc-10834): Rearranging formulas
- WorkSHEET 6.1 (doc-10835): Trigonometry

Interactivity

- IP interactivity 6.3 (int-4499): Calculating unknown side lengths

6.6 Calculating unknown angles

Digital doc

- SkillSHEET (doc-10836): Rounding angles to the nearest degree

Interactivity

- IP interactivity 6.4 (int-4500): Calculating unknown angles

6.7 Angles of elevation and depression

Digital docs

- SkillSHEET (doc-10837): Drawing a diagram from given directions
- WorkSHEET 6.2 (doc-10838): Trigonometry using elevation and depression

Interactivity

- IP interactivity 6.5 (int-4501): Angles of elevation and depression

6.8 Review

Interactivities

- Word search (int-0889)
- Crossword (int-0703)
- Sudoku (int-3206)

Digital docs

- Topic summary (doc-10784)
- Concept map (doc-10797)

To access eBookPLUS activities, log on to www.jacplus.com.au

Answers

TOPIC 6 PYTHAGORAS AND TRIGONOMETRY

Exercise 6.2 Pythagoras' theorem

1 a i r **ii** $r^2 = p^2 + s^2$
b i x **ii** $x^2 = y^2 + z^2$
c i k **ii** $k^2 = m^2 + w^2$
d i FU **ii** $(FU)^2 = (VU)^2 + (VF)^2$

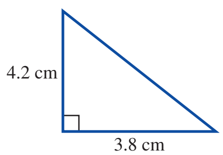
2 a 7.86 **b** 33.27 **c** 980.95

3 a $x = 12.49$ **b** $p = 11.76$ cm
c $f = 5.14$ m **d** $c = 97.08$ mm

4 10.2 cm

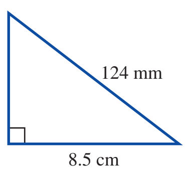
5 a No **b** No **c** Yes
d Yes **e** No **f** Yes

6 a



b 5.66 cm

7 a



b 90.28 mm

8 B

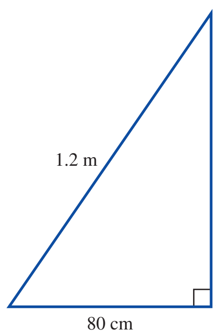
9 C

10 B

11 2.60 m

12 a 176.16 cm **b** 147.40 cm
c 2.62 km **d** 432.12 m

13 a



b 89.44 cm
c Yes, she will reach the hook from the top step.

14 Answers will vary.

15 The horizontal distance is 11.74 m, so the gradient is 0.21, which is within the limits.

16 a i {9, 40, 41} **ii** {11, 60, 61}
iii {13, 84, 85} **iv** {29, 420, 421}
b The middle number and the large number are one number apart.

17 Check with your teacher.

Exercise 6.3 Applications of Pythagoras' theorem

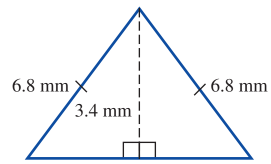
1 a 12.08 cm

2 a $k = 16.40$ m **b** $x = 6.78$ cm **c** $g = 4.10$ km

3 a $x = 4, y = 9.17$ **b** $x = 6.93, y = 5.80$
c $x = 13, y = 15.20$ **d** $x = 2.52, y = 4.32$

4 a 30.48 cm **b** 2.61 cm **c** 47.27 cm

5 a



b 11.78 mm
c 20.02 mm^2

6 a 0.87 m **b** 0.433 m^2

7 E

8 Yes, 1.015 m

9 No, 17.72 cm

10 2.42 m

11 a $w = 0.47$ m **b** 0.64 m^2
c 12.79 m^2 **d** \$89.85

12 12 mm; 480 mm^2

13 17.9 cm

14 Even though a problem may be represented in 3-D, right-angled triangles in 2-D can often be found within the problem. This can be done by drawing a cross-section of the shape or by looking at individual faces of the shape.

15 a AD, DC, AC
b AD = 4.47 cm, DC = 9.17 cm, AC = 13.64 cm
c The triangle ABC is not right-angled because $(AB)^2 + (BC)^2 \neq (AC)^2$.

16 Answers will vary.

17 Answers will vary.

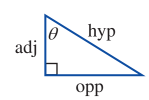
18 a 2606 mm **b** Answers will vary.

19 16.7 m

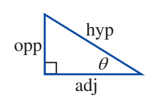
20 $w = 3.536$ m, $x = 7.071$ cm, $y = 15.909$ cm, $z = 3.536$ cm

Exercise 6.4 What is trigonometry?

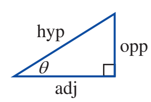
1 a



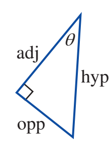
b



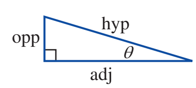
c



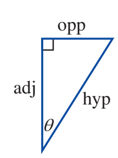
d



e



f



2 a DE = hyp DF = opp $\angle E = \theta$
b GH = hyp IH = adj $\angle H = \theta$
c JL = hyp KL = opp $\angle J = \theta$

3

	θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
a	45°	0.71	0.71	1.00
b	35°	0.57	0.82	0.70

4 D

5 a i $\sin(\theta) = \frac{4}{5}$ **ii** $\cos(\theta) = \frac{3}{5}$ **iii** $\tan(\theta) = \frac{4}{3}$
b i $\sin(\alpha) = \frac{i}{g}$ **ii** $\cos(\alpha) = \frac{h}{g}$ **iii** $\tan(\alpha) = \frac{i}{h}$

c **i** $\sin(\beta) = 0.8$ **ii** $\cos(\beta) = 0.6$ **iii** $\tan(\beta) = 1.3$

d **i** $\sin(\gamma) = \frac{24}{25}$ **ii** $\cos(\gamma) = \frac{7}{25}$ **iii** $\tan(\gamma) = \frac{24}{7}$

e **i** $\sin(\beta) = \frac{b}{c}$ **ii** $\cos(\beta) = \frac{a}{c}$ **iii** $\tan(\beta) = \frac{b}{a}$

f **i** $\sin(\gamma) = \frac{v}{u}$ **ii** $\cos(\gamma) = \frac{t}{u}$ **iii** $\tan(\gamma) = \frac{v}{t}$

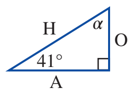
6 **a** $\sin(\theta) = \frac{12}{15} = \frac{4}{5}$ **b** $\cos(\theta) = \frac{25}{30} = \frac{5}{6}$ **c** $\tan(\theta) = \frac{4}{5}$

d $\tan(\theta) = \frac{2.7}{p}$ **e** $\sin(35°) = \frac{17}{t}$ **f** $\sin(\alpha) = \frac{14.3}{17.5}$

g $\sin(15°) = \frac{7}{x}$ **h** $\tan(\theta) = \frac{20}{31}$ **i** $\cos(\alpha) = \frac{3.1}{9.8}$

7 **a** D **b** B

8 **a**



b O = 34 mm, A = 39 mm, H = 52 mm
c **i** $\sin(41°) = 0.66$ **ii** $\cos(41°) = 0.75$ **iii** $\tan(41°) = 0.87$
d $\alpha = 49°$
e **i** $\sin(49°) = 0.75$ **ii** $\cos(49°) = 0.66$ **iii** $\tan(49°) = 1.15$
f They are equal. **g** They are equal.
h The sine of an angle is equal to the cosine of its complement.

9 **a** The missing angle is also 45°, so the triangle is an isosceles triangle, therefore $a = b$.
b 1

10 Provided n is a positive value, $(m + n)$ would be the hypotenuse, as it has a greater value than both m and $(m - n)$.

11 **a** Ground **b** Ladder **c** Brick wall

12 **a** The ratio of the length of the opposite side to the length of the hypotenuse will increase.
b The ratio of the length of the adjacent side will decrease, and the ratio of the opposite side to the adjacent will increase.
c **i** 1 **ii** 1 **iii** ∞

Exercise 6.5 Calculating unknown side lengths

1 **a** **i** 0.8192 **ii** 0.2011

b

θ	0°	15°	30°	45°	60°	75°	90°
$\sin(\theta)$	0	0.26	0.50	0.71	0.87	0.97	1.00

c As θ increases, so does $\sin(\theta)$, starting at 0 and increasing to 1.

2 **a** **i** 0.7880 **ii** 0.5919

b

θ	0°	15°	30°	45°	60°	75°	90°
$\cos(\theta)$	1.00	0.97	0.87	0.71	0.50	0.26	0

c As θ increases, $\cos(\theta)$ decreases, starting at 1 and decreasing to 0.

3 **a** **i** 0.3249 **ii** 1.2753

b

θ	0°	15°	30°	45°	60°	75°	90°
$\tan(\theta)$	0	0.27	0.58	1.00	1.73	3.73	Undefined

c $\tan(89°) = 57.29$, $\tan(89.9°) = 572.96$
d As θ increases, $\tan(\theta)$ increases, starting at 0 and becoming very large. There is no value for $\tan(90°)$.

4 **a** 13.02 m **b** 7.04 m **c** 27.64 mm
d 2.79 cm **e** 6.27 m **f** 14.16 m

5 **a** 2.95 cm **b** 25.99 cm **c** 184.73 cm
d 14.06 km **e** 8.43 km **f** 31.04 m

6 **a** 26.96 mm **b** 60.09 cm **c** 0.84 km
d 0.94 km **e** 5.59 m **f** 41.67 m
g 54.73 m **h** 106.46 cm **i** 298.54 mm

7 **a** $a = 17.95$, $b = 55.92$
b $a = 15.59$, $b = 9.00$, $c = 10.73$
c $a = 12.96$, $b = 28.24$, $c = 15.28$

8 **a** D **b** B **c** A **d** D

9 **a** 275.75 km **b** 48.62 km

10 21.32 m

11 285.63 m

12 **a, b** Answers will vary.
c $AC = \frac{AB}{\tan(\theta)}$

13 Answers will vary.

14 **a** $x = 12.87$ m **b** $h = 3.00$ m **c** $x = 2.60$ m

15 $w = 41$ mm

16 **a** 5.9 m **b** 5.2 m

17 4 m

Challenge 6.1

2.47 km; 0.97 km

Exercise 6.6 Calculating unknown angles

1 **a** 39° **b** 72° **c** 37° **d** 53° **e** 69°
f 71° **g** 79° **h** 77° **i** 15°

2 **a** 19° **b** 42° **c** 55° **d** 21° **e** 49°
f 80° **g** 35° **h** 45° **i** 41° **j** 23°
k 58° **l** 80°

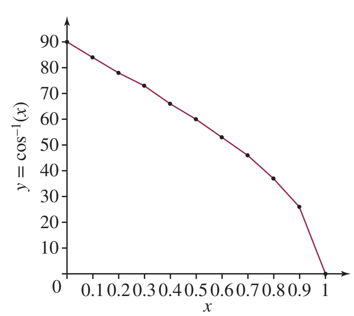
3 **a** 47° **b** 45° **c** 24° **d** 43° **e** 45°
f 18° **g** 26° **h** 12° **i** 76°

4 **a** D **b** B
c D **d** C

5 **a**

x	$y = \cos^{-1}(x)$
0.0	90°
0.1	84°
0.2	78°
0.3	73°
0.4	66°
0.5	60°
0.6	53°
0.7	46°
0.8	37°
0.9	26°
1.0	0°

b



6 30.56°

7 21.80°

8 **a** 2.31 m
b Answers will vary.

9 Answers will vary.

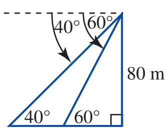
10 **a** $\theta = 26°$
b $\theta = 32.01°$
c $x = 6.41°$
11 **a** The roof would not be high enough.
b 136°
12 Answers will vary.

Challenge 6.2

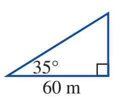
1 54.74°
2 Large square: 5 cm × 5 cm
Large triangles: 5 cm × 5 cm
Small square: $\frac{5\sqrt{2}}{2}$ cm × $\frac{5\sqrt{2}}{2}$ cm
Small triangles: $\frac{5\sqrt{2}}{2}$ cm × $\frac{5\sqrt{2}}{2}$ cm
Parallelogram: $5\sqrt{2}$ cm × 5 cm

Exercise 6.7 Angles of elevation and depression

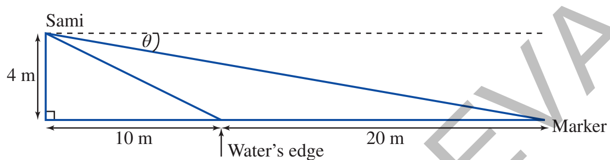
1 8.13°. No, the ramps do not meet specifications.
2 14.11 m
3 66.35 m
4 **a**



b 49 m
5 **a**

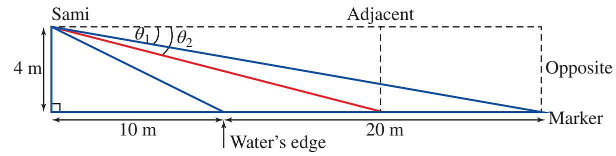


b 42 m
6 1.6 m **7** 63° **8** 34°, 3 m
9 Con: 34°, John: 30°
10 B
11 **a**



b $\tan(\theta) = \frac{4}{30}$
$\theta = \tan^{-1}\left(\frac{4}{30}\right)$
$\theta \approx 7.595$
$\approx 7.6°$

c



As the dolphins swim towards Sami, the adjacent length decreases and the opposite remains unchanged.
$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$
Therefore, θ will increase as the adjacent length decreases.
If the dolphins are at the water's edge,
$\tan(\theta) = 10$
$\theta = \tan^{-1}\left(\frac{4}{10}\right)$
$\theta \approx 21.801$
$\approx 21.8°$

12 Answers will vary.
13 **a** Answers will vary. **b** 1.64 m
14 Answers will vary.
15 Answers will vary.
16 **a** Answers will vary. **b** 37°
c **i** Yes **ii** 17.26 m

Investigation — Rich task

The Great Pyramid of Giza

1 186.25 m
2 21 418.75 m^2
3 218.89 m

Wall braces

1 Answers will vary.
2 Answers will vary.
3 53° requires the shortest brace and 37° requires the longest brace.
4 4.24 m
5 3.62 m
6 4.61 m; 4.52 m
7 Answers will vary.

Code puzzle

A broom with a stomach
Mud with the juice squeezed out
A bird's home town
The man who married mudder

projectsplus

Learning or earning?

Searchlight ID: Pro-0086

Scenario

Year 9 Students at Progressive High School have seen the paper *Are young people learning or earning*? produced by the Australian Bureau of Statistics. They start the week discussing their thoughts and then decide to do their own research. Simon decides that he will use this research to demonstrate to his parents that life is very different to when they were his age and that he is capable of undertaking a part-time job and keeping up with his studies.

Task

Read the article, summarise it and design a survey to research the topic. Your findings will reflect the work–study balance of the students in your school and demographic. You will produce a presentation for your parents outlining the article and detailing their research and the conclusions you have drawn.

Process

- Open the ProjectsPLUS application for this chapter in your eBookPLUS. Watch the introductory video lesson, click the 'Start Project' button and then set up your project group. You can complete this project individually or invite other members of your class to form a group. Save your settings and the project will be launched.
- Navigate to your *Media Centre*. Read the articles and complete the task below. Answer the questions provided in the Are Young People Learning or Earning file in the Media Centre. Summarise the thoughts of the article in 200 words or less.
- Wordle is a site that creates an image of the words in an article according to the frequency of their usage. Use the **Wordle** weblink in your eBookPLUS and select the **create** tab. Copy the summary of your article into the text box. Keep selecting the **randomise** button until you are happy with the result. Print your final choice. Take a screenprint of your final choice. Take it into **Paint** for use as a slide in your presentation. Save your Paint file.
- Use **Surveymonkey** to survey 100 students at your school to determine who has part-time jobs and how long they work at these each week. You are able to ask only 10 questions per survey. Record your 10 questions in Word. When planning your survey think carefully about the types of questions you want to ask. For example, do you want to know why they have a part-time job?

Your 100 students need to represent the school as a population. How will you make your sample representative of this? Will it be a random sample or a stratified sample? Explain. What about the gender balance? Justify your decision. How will you notify the chosen students that they need to do the survey? What instructions will you give to the students completing your survey? How long will they have to complete it? What will you do to ensure they have all completed it? Complete the survey table provided in the Media Centre. Type your instructions to each person completing the survey. Copy this into the Survey instructions template in the Media Centre.

- **Analysis.** Record the results from your survey in a frequency distribution table. Use the Results table provided in the Media Centre. Write a paragraph summarising your findings. Include mean, median, mode and range for each year group. What percentages of the students have part-time jobs? Is there a trend as the students get older (are more or less students working)? Include a statement as to why you still want to get a part-time job. Represent your findings in a frequency histogram and frequency polygon. Use the Excel template for your results and include your graphs on that sheet. If you were to do this research again, is there anything that you would change? Why? Are there any better resources for your research? What is the mean, median and mode and range of your results?
- Research. Visit the Media Centre in your eBookPLUS and open the **Suburb statistics labour force by age** weblink. Type in your postcode. Follow the prompts to download the labour force statistics by age, and by sex for your postcode, and save. If you live in an urban area repeat for a rural region, if you live in a rural region, repeat for an urban area. Save the Excel files columns for the rural and urban postcodes. Use Excel to create a column graph with a series of columns. See Sample spreadsheet file. Include the graph in your Prezi file with an explanation of what you did.
- Visit the Media Centre and download the Prezi sample and the Prezi planning template to help you prepare your presentation. Your Media Centre also includes images that can help to liven up your presentation. As you arrange your images on your Prezi page make them form a large circle so that they flow smoothly when they are linked and presented.
- Use the Prezi template to develop your presentation. Remember that you are trying to convince your parents that you should be able to undertake a part-time job. Make sure you include all the results of your research, and that your presentation will grab their attention. To include tables in Prezi you need to take them into paint and save the file as a jpeg in order to upload them. Use Word to type up your dialogue to your parents when you present your case (200–500 words).

SUGGESTED SOFTWARE

- ProjectsPLUS
- Microsoft Word
- PowerPoint, Prezi, Keynote or other presentation software
- Microsoft Excel
- Surveymonkey
- Wordle