TOPIC 7

Congruence

7.1 Overview

Why learn this?

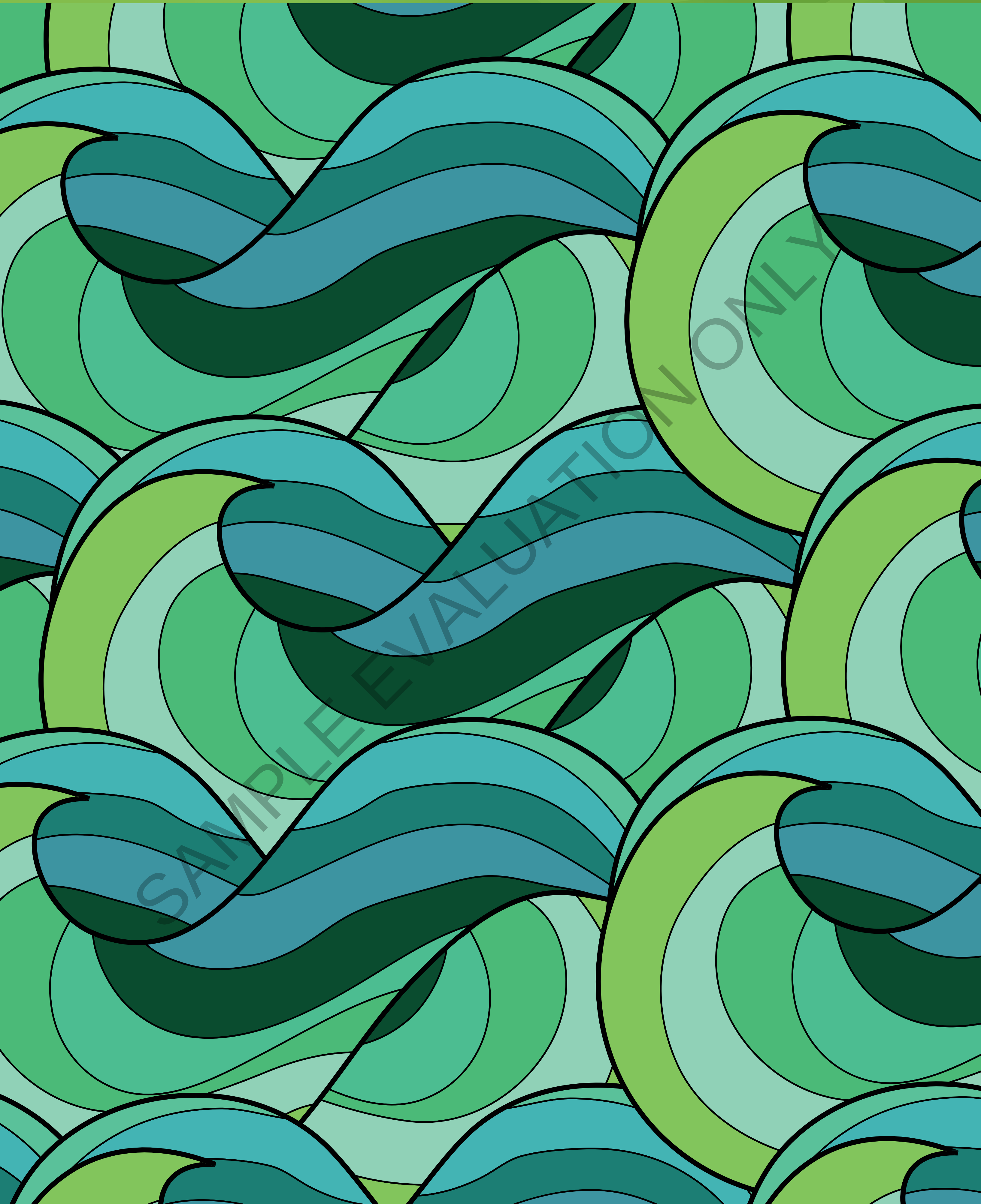
From backyard sheds and home extensions to architecture itself, the design of buildings is founded on congruence. Many structures, from the modern to the very old, were created by people who had the chance to read a chapter similar to this one. Grand structures such as Flinders St Station, and even the very house in which you are living, are all based on the principles of congruence.

What do you know?

assess on

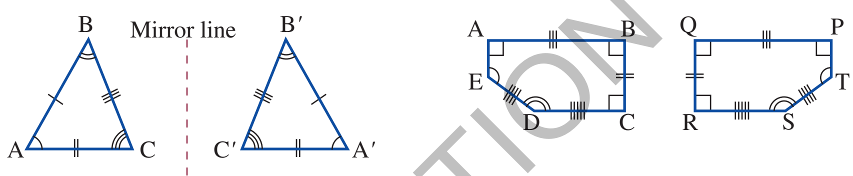
1. **THINK** List what you know about congruence and transformations. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of congruence and transformations.

Learning sequence



7.2 Congruent figures

- **Congruent figures** are identical in size and shape.
- The transformations of reflection, rotation and translation do not change the shape and size of a figure. The original and transformed figure are said to be congruent.
- Repeated transformations of a congruent shape or a group of congruent shapes so that an entire surface is covered with a regular pattern are called tessellations.
- The symbol used for congruence is ≡.
- When writing congruence statements, we name the vertices of the figures in corresponding (or matching) order.
- Sometimes you will need to rotate, reflect or translate the figures for them to be orientated the same way.
- If, through one or more translations, rotations and reflections, one shape can lie exactly on top of the other, then these two figures are congruent.
- The figures below are congruent. For the triangle, we can write ABC ≡ A′B′C′; for the pentagon, we can write ABCDE ≡ PQRST.



Note that equivalent sides and angles are labelled with the same symbol.

WORKED EXAMPLE 1

Select a pair of congruent shapes from the following set.

THINK

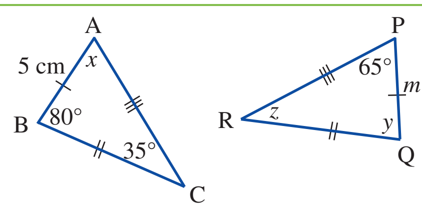
Figures **a**, **b** and **c** have the same shape (that is, a semicircle). Figure **d** is not a semicircle and thus is not congruent to any other figures. Figure **c** is larger than figures **a** and **b** and so is not congruent to any one of them. Figures **a** and **b** are identical in shape and size (**b** is a reflection of **a**) and therefore, are congruent to each other.

WRITE

Shape **a** ≡ Shape **b**

WORKED EXAMPLE 2

Find the value of the variables in the pair of congruent triangles at right.



THINK

1 Since ΔABC ≡ ΔPQR, the corresponding angles are equal in size. Corresponding angles are included between the sides of equal length. So, by looking at the markings on the sides of the triangles, we can conclude that:
∠BAC corresponds to ∠RPQ
∠ABC corresponds to ∠PQR
∠ACB corresponds to ∠PRQ.
So, match the variables with the corresponding angles whose sizes are given.

WRITE

$x = 65°$
$y = 80°$
$z = 35°$

2	In congruent triangles, the corresponding sides are equal in length. Using the markings on the sides of the triangles, observe that the unknown side PQ corresponds to side AB. State the value of the variable (which represents the length of PQ).	$m = 5$ cm

The principle of congruence is applied in the construction of many everyday objects. For example the frame of the bicycle shown is partly made up of two congruent triangles; the wheels are congruent circles.

Exercise 7.2 Congruent figures

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5, 7	Questions: 1–5, 7	Questions: 1–8

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REFLECTION
When we write a congruence statement, why are the vertices listed in corresponding order?

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SkillSHEET
Calculating angles in triangles
doc-6916

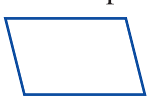
FLUENCY

1 Plot the following coordinates on a Cartesian plane: $(1, -1)$, $(1, 0)$, $(1, 1)$, $(2, 1)$, $(1, 2)$, $(1, 3)$, $(2, 3)$, $(3, 3)$. Join the points with straight lines to make the shape of the letter F.

- **a** Reflect this figure in the y-axis and describe the new location using the coordinate points.
- **b** Translate the original figure by three units to the right and two units down, and describe the new location using the coordinate points.
- **c** Rotate the original figure through 90° about the origin and describe the new location using the coordinate points.
- **d** Look at the transformations in parts **a**, **b** and **c**. Are the shapes congruent? Why?

2 **WE1** Select a pair of congruent shapes from each of the following sets.

a **i**

ii

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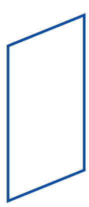
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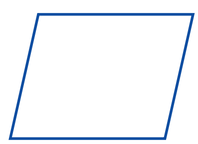

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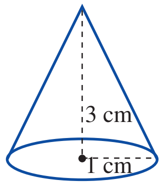
b **i**



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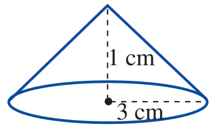

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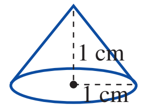

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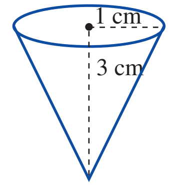


c **i**

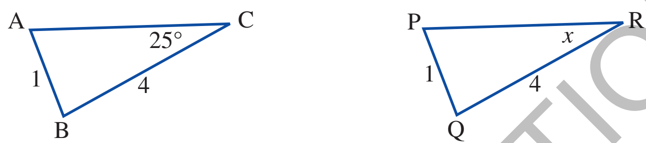
ii

iii

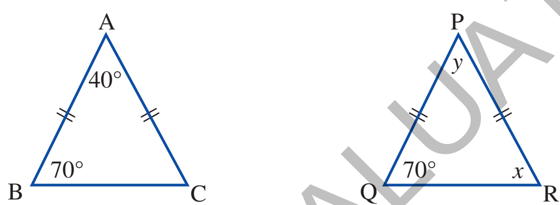
iv

3 **WE2** All of these pairs of triangles are congruent. Name the congruent triangles and find the value of the variables in each case. (*Remember:* The vertices must be listed in corresponding order.)

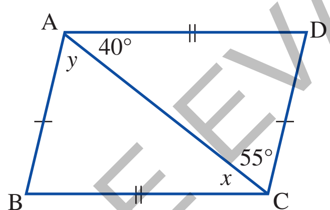
a



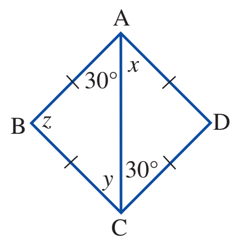
b



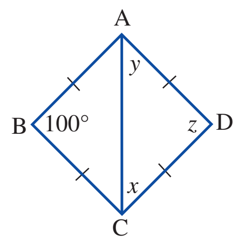
c



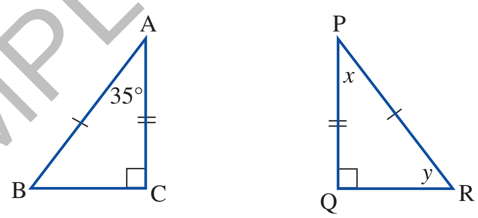
d



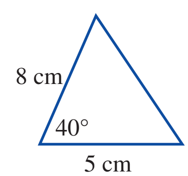
e



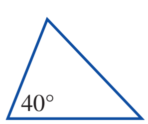
f



4 **MC** Which of the following is congruent to the triangle shown?



a



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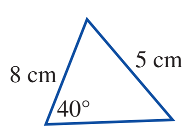

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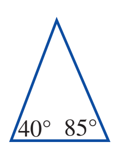
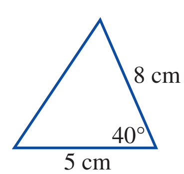


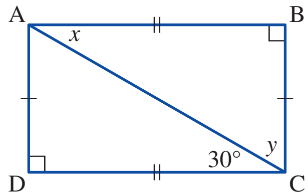
d



A **a** only **B** **a** and **b** **C** **d** only **D** **b** and **d**
E None of the above

UNDERSTANDING

5 Name the congruent triangles in each question and find the value of the variable(s).

a


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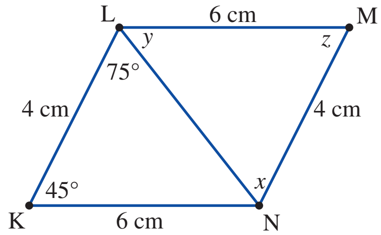

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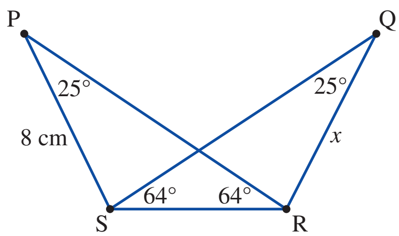

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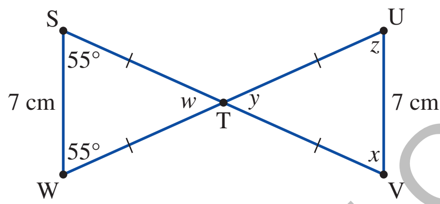

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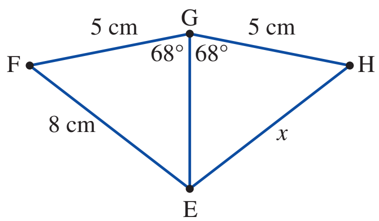

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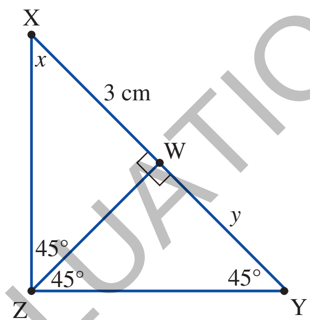

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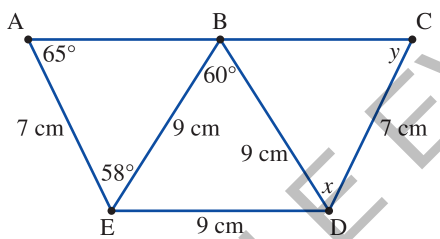
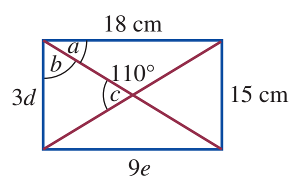


REASONING

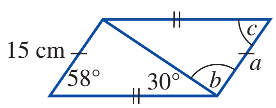
6 Give an example to show that triangles with two angles of equal size and a pair of non-corresponding sides of equal length may not be congruent.

PROBLEM SOLVING

7 Given that the four right-angled triangles in the figure below are congruent, find the values of the pronumerals.



8 Given that the right-hand triangle is congruent to the left-hand triangle in the figure below, find the values of the pronumerals.



CHALLENGE 7.1

Determine which regular polygons will tessellate on their own without any spaces or overlaps.

7.3 Triangle constructions

- Using a ruler, protractor and a pair of compasses, you can construct any triangle from three pieces of information.
- Sometimes, only one triangle can be drawn from the information. Sometimes more than one triangle can be drawn.

Constructing a triangle given three side lengths

- If the lengths of the three sides of a triangle are known, it can be constructed with the help of a ruler and a pair of compasses.

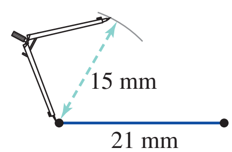
WORKED EXAMPLE 3

Using a ruler and a pair of compasses, construct a triangle with side lengths 15 mm, 20 mm and 21 mm.

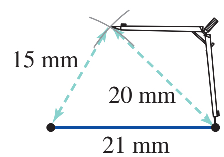
THINK / **DRAW**

1. Rule out the longest side (21 mm).

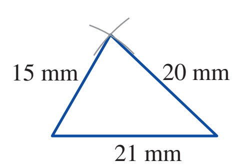
2. Open the compasses to the shortest side length (15 mm).
3. Draw an arc from one end of the 21 mm side.



4. Open the compasses to the length of the third side (20 mm) and draw an arc from the other end of the 21 mm side.



5. Join the point of intersection of the two arcs and the end points of the 21 mm side with lines.



Constructing a triangle given two angles and the side between them

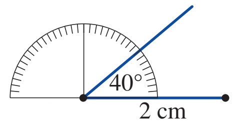
- If the size of any two angles of a triangle and the length of the side between these two angles are known, the triangle can be constructed with the aid of a ruler and a protractor.

WORKED EXAMPLE 4

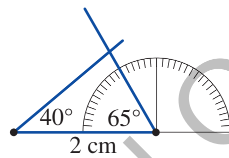
Use a ruler and protractor to construct a triangle with angles 40° and 65°, and the side between them of length 2 cm.

THINK | **DRAW**

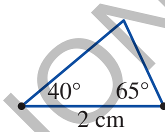
1. Rule a line of length 2 cm.
2. Place the centre of your protractor on one end point of the line and measure out a 40° angle. Draw a line so that it makes an angle of 40° with the 2 cm line.



3. Place the centre of your protractor on the other end point of the 2 cm line and measure an angle of 65°. Draw a line so that it makes a 65° angle with the 2 cm line.



4. If necessary, continue the lines until they intersect each other to form a triangle.



Constructing a triangle given two sides and the angle between them

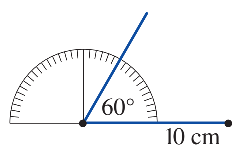
- The angle between two sides is called the **included angle**.
- If the length of two sides and the size of the included angle are known, the triangle can be constructed with a protractor and a ruler.

WORKED EXAMPLE 5

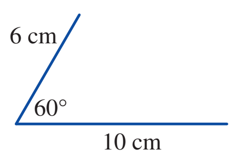
Use a ruler and protractor to construct a triangle with sides 6 cm and 10 cm long, and an angle between them of 60°.

THINK | **DRAW**

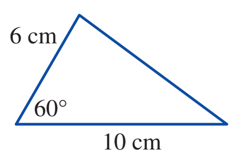
1. Rule a line 10 cm long.
2. Place the centre of your protractor on one end point of the line and mark an angle of 60°. *Note:* These figures have been reduced.



3. Join the 60° mark and the end point of the 10 cm side with the straight line. Extend the line until it is 6 cm long.



4. Join the end points of the two lines to complete the triangle.



assess on

Exercise 7.3 Triangle constructions

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 8	Questions: 1, 2a–d, 3a–d, 4–6, 8, 9	Questions: 1, 2e–h, 3e–h, 4–7, 10, 11

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REFLECTION
When you are constructing a triangle and you know two angles and a side, do you need to know the side between the angles?

FLUENCY

1 **WE3** Using a ruler and a pair of compasses, construct 2 congruent triangles with the following side lengths:
- a 7 cm, 6 cm, 4 cm
- b 5 cm, 4 cm, 5 cm
- c 6 cm, 5 cm, 3 cm
- d 6 cm, 6 cm, 6 cm
- e 7.5 cm, 4.5 cm, 6 cm
- f 2 cm, 6.5 cm, 5 cm
- g an equilateral triangle of side 3 cm
- h an equilateral triangle of side 4.5 cm.

2 **WE4** Use a ruler and protractor to construct these triangles:
- a angles 60° and 60° with the side between them 5 cm long
- b angles 50° and 50° with the side between them 6 cm long
- c angles 30° and 40° with the side between them 4 cm long
- d angles 60° and 45° with the side between them 3 cm long
- e angles 30° and 60° with the side between them 4 cm long
- f angles 65° and 60° with the side between them 3.5 cm long
- g angles 60° and 90° with the side between them 5 cm long
- h angles 60° and 36° with the side between them 4.5 cm long.

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3 **WE5** Use a ruler and protractor to construct the following triangles:
- a two sides 10 cm and 5 cm long, angle of 30° between them
- b two sides 8 cm and 3 cm long, angle of 45° between them
- c two sides 6 cm and 6 cm long, angle of 60° between them
- d two sides 4 cm and 5 cm long, angle of 90° between them
- e two sides 7 cm and 6 cm long, angle of 80° between them
- f two sides 9 cm and 3 cm long, angle of 110° between them
- g two sides 6 cm and 6 cm long, angle of 50° between them
- h two sides 5 cm and 4 cm long, angle of 120° between them.

UNDERSTANDING

4 a Use your ruler and compass to draw an isosceles triangle with two sides 5 cm long and one side 7 cm.
- b Use your protractor to measure the size of the largest angle.
- c Complete this sentence using one of the words below: This triangle is an __________ angled triangle.
  - i acute
  - ii right
  - iii obtuse

REASONING

5 **a** Construct a triangle with angles of 45°, 55° and 80°.

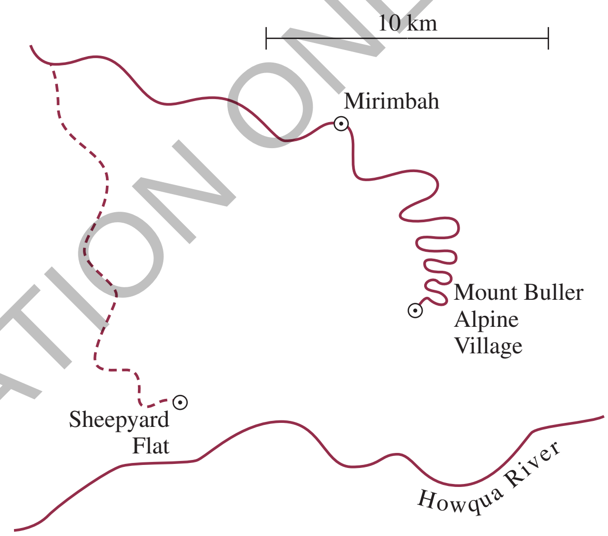
b How many different triangles can be constructed from this information? Justify your answer.

6 Construct a right-angled triangle with the longest side 13 cm and one of the other sides 5 cm.

7 Mike and Susan were constructing a triangle ABC where angle ABC = 30°, BC = 12 cm and AC = 8 cm. Mike's triangle was an acute-angled triangle and Susan's was an obtuse-angled triangle. Draw both triangles.

PROBLEM SOLVING

8 A bushwalker is injured and cannot walk. Before the battery of his mobile phone died, he reported that he was equidistant from the peak of Mount Buller and Sheepyard Flat where he parked his car. He estimated that he had walked 6 km and had crossed Howqua River before his fall. At rescue headquarters, the SES captain looked at a map. Sheepyard Flat is approximately 10 km from Mount Buller. Trace the map at right into your workbook and draw the point where the SES should send the rescue helicopter.



9 Use a ruler and protractor to construct triangles with the following features.

a Angles 30° and 90° with a 3-cm edge between them

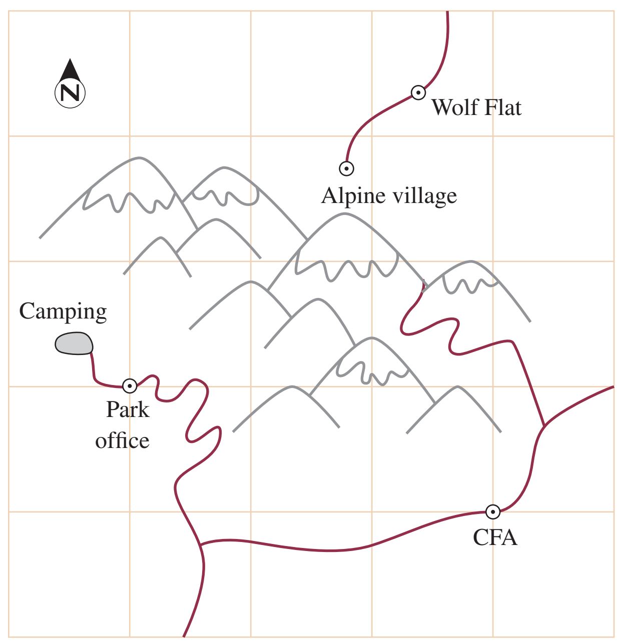
b Angles 45° and 45° with a 2.5-cm edge between them

c Angles 60° and 100° with a 40-mm edge between them

d Angles 22° and 33° with a 33-mm edge between them

e Isosceles triangle with adjacent angles of 57° and a 4-cm baseline

10 A forestry ranger informs the CFA that he sees smoke rising from behind a mountain range at 40° east of due north from his park office. The CFA chief sees the same smoke at an angle of 30° west of due north from her station. Trace the map at right into your workbook and mark the location of the fire.



11 Use a ruler and protractor to construct triangles with the following features.

a Edge lengths of 5.2 cm and 3 cm and an angle between them of 45°

b Edge lengths of 2.5 cm and 2.5 cm and an angle between them of 60°

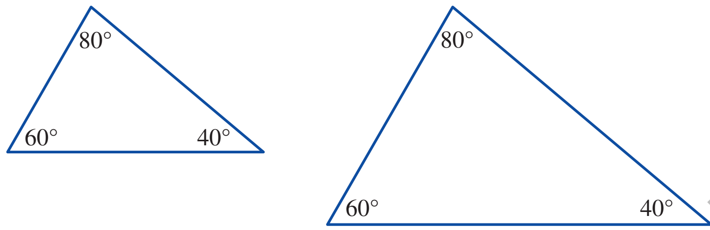
c Edge lengths of 28 mm and 40 mm and an angle between them of 120°

d Edge lengths of 63 mm and 33 mm and an angle between them of 135°

e A right-angled triangle with edge lengths of 4.5 cm and 2.5 cm creating the right angle

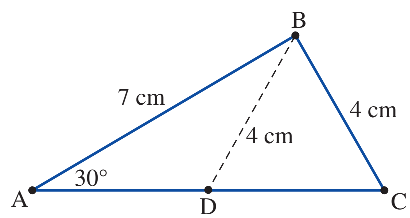
7.4 Congruent triangles

- From section 7.3 it can be seen that a triangle can be described with three pieces of information about side length and/or angle size.
- If three angles are known, an infinite number of triangles can be drawn. For example the triangles below have angle sizes of 60°, 40° and 80°.



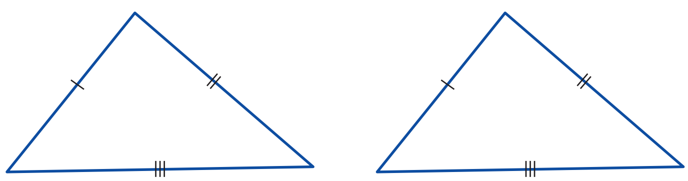
∴ If only three angles are known, this is not a test for congruency.

- There can be two possible triangles drawn from two sides and the non-included angle. In the diagram at right, angles ABC and ABD both have sides of 7 cm and 4 cm, and an angle of 30° between the 7-cm side and the unknown side.
  ∴ If two sides and the non-included angle are given, this is not a test for congruence.

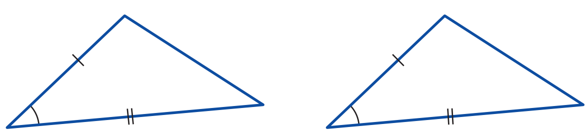


Congruency tests

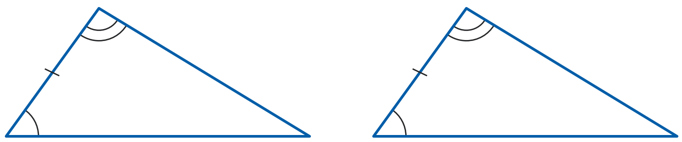
- There are four tests that can be used to demonstrate that two triangles are congruent.
- It is not enough for the diagrams to look the same; you need evidence!
- Two triangles are congruent if:
  – three sides of one triangle are equal to the three sides of the other (SSS)



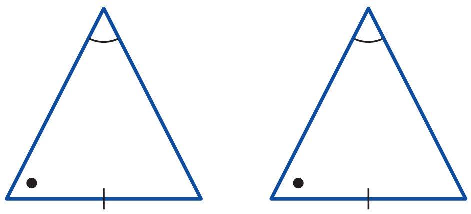
  – two sides and the included angle of one triangle are equal to two sides and the included angle of the other (SAS)



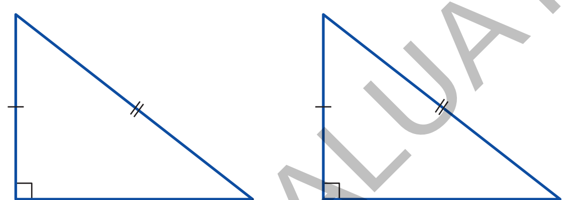
– two angles and a side of one triangle are equal to the two angles and the corresponding side of the other (AAS)



– a pair of corresponding angles and a non-included side are equal (ASA)

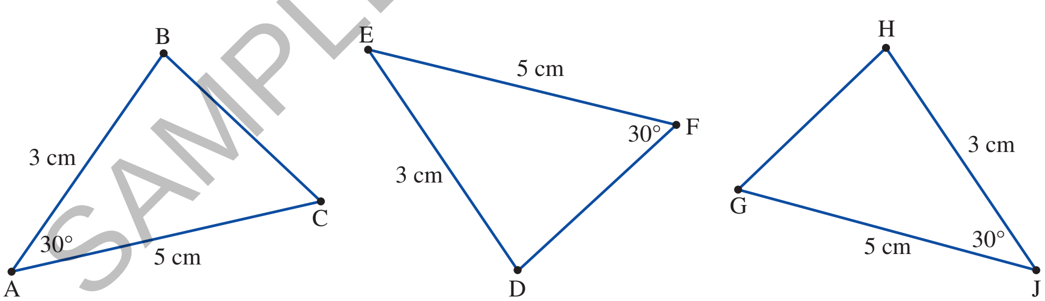


– the triangles are right-angled and the hypotenuse and a side of one are equal to the hypotenuse and a side of the other (RHS).



WORKED EXAMPLE 6

Which of the following triangles are definitely congruent? Give a reason for your answer.



THINK

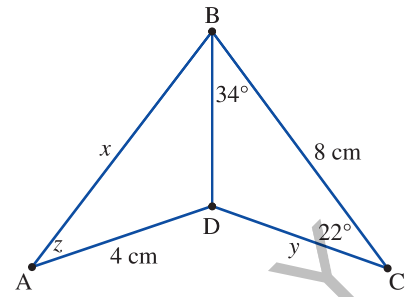
In all three triangles, two given sides are of equal length (3 cm and 5 cm). In triangles ABC and JHG, the included angle is 30°. In triangle AFD, 30° is not the included angle.

WRITE

$\Delta ABC \equiv \Delta JHG$ (SAS)

WORKED EXAMPLE 7

Given that ΔABD ≡ ΔCBD, find the value of the variables.



THINK

1. The corresponding sides are equal in length.
   ΔABD ≡ ΔCBD.
   AB and CD are unknown.
   AB corresponds to CB.
   CD corresponds to AD.
2. The corresponding angles are equal.
   ∠BAD is unknown.
   ∠BAD corresponds to ∠BCD.

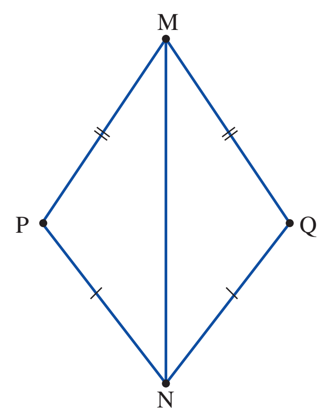
WRITE

ΔABD ≡ ΔCBD
AB = CB
$x = 8$ cm
CD = AD
$y = 4$ cm

∠BAD = ∠BCD
$z = 22°$

WORKED EXAMPLE 8

Prove that ΔMNP is congruent to ΔMNQ.



THINK

1. Study the diagram and state which sides and/or angles are equal.
2. Select the appropriate congruency test (in this case SSS, as the triangles have all corresponding sides congruent in length).

WRITE

NP = NQ (given)
PM = QM (given)
MN is common

ΔNPM ≡ ΔNQM (SSS)

Exercise 7.4 Congruent triangles

assess on

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1–3	Questions: 1–6, 9	Questions: 1–10

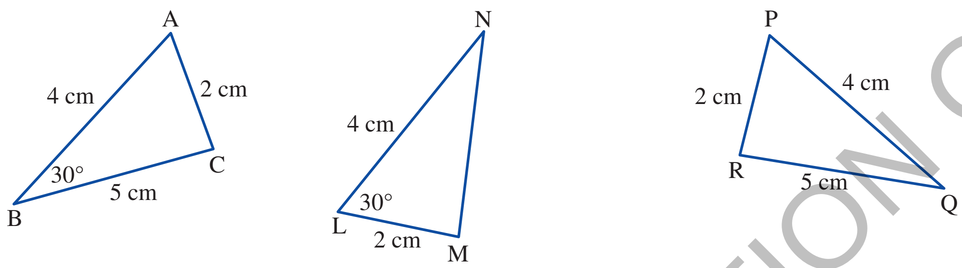
■■■ **Individual pathway interactivity** int-4426 eBook*plus*

REFLECTION
When using the congruency rule ASA, why is it not necessary for the side to be between the two angles?

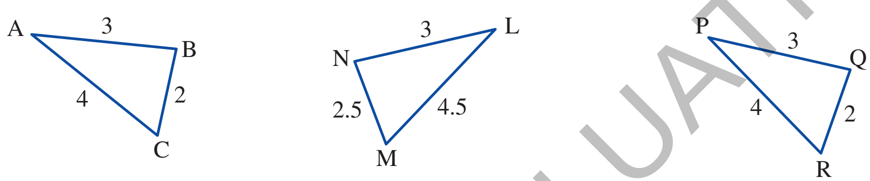
FLUENCY

1 WE6 In each part of the question, which of the triangles are congruent? Give a reason for your answer.

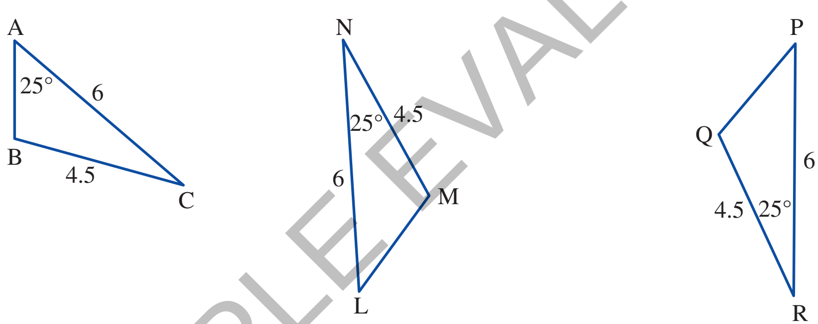
a



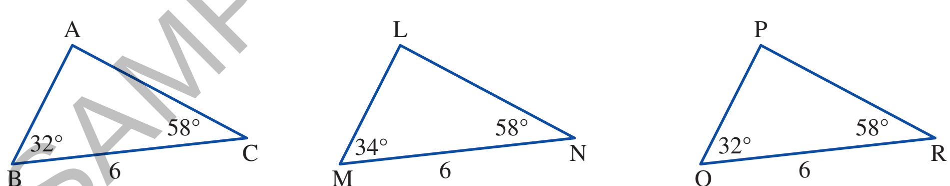
b



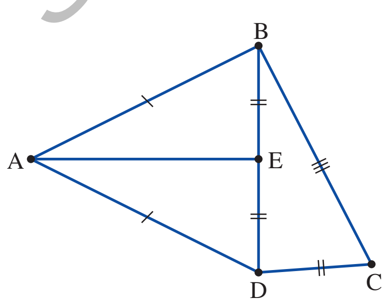
c



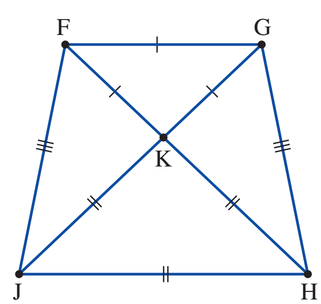
d



e

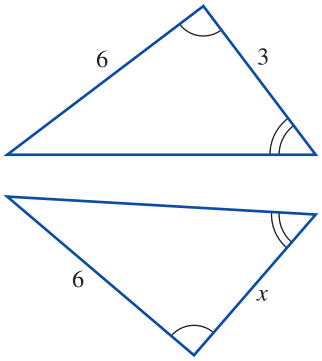


f

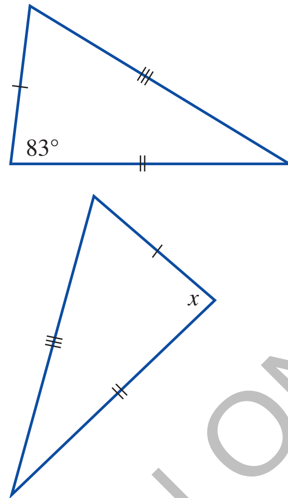


2 WE7 Find the values of the variables in each of the following pairs of congruent triangles. All side lengths are in centimetres.

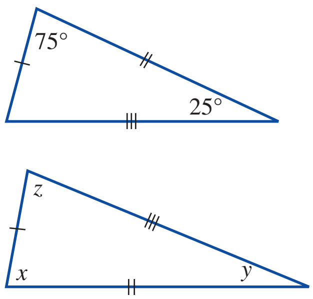
a



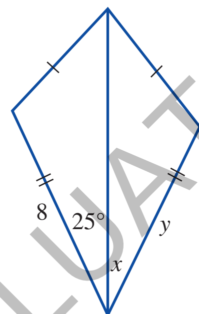
b



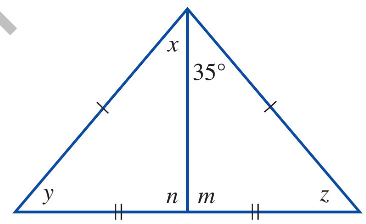
c



d

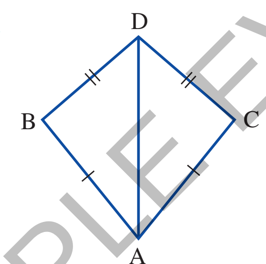


e



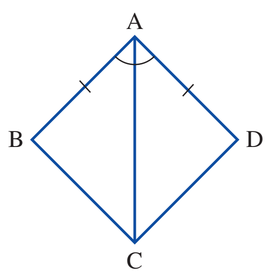
3 WE8 For each of the following, prove that:

a



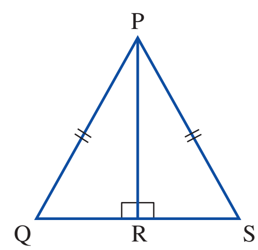
ΔDBA ≡ ΔDCA

b



ΔABC ≡ ΔADC

c



ΔPQR ≡ ΔPSR

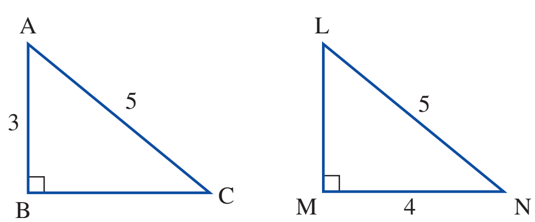
UNDERSTANDING

4 Draw an example of two different triangles that have two corresponding sides equal and one pair of equal angles.

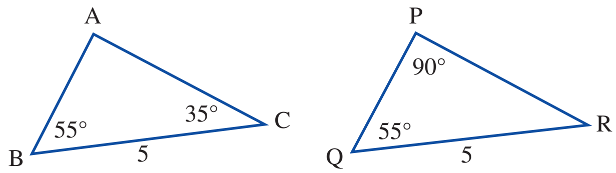
5 Draw an example of two different triangles that have three pairs of equal angles.

6 Are the following triangles congruent? Give a reason for your answer. *Hint:* Pythagoras' theorem (Topic 15) is needed for this question.

a



b



REASONING

7 In an equilateral triangle ABC, the midpoints of the sides are labelled D, E and F. Prove that triangle DEF is an equilateral triangle.

8 All triangles have three sides and three angles. Using mathematical reasoning, including examples, give the minimum information required to construct a unique triangle.

PROBLEM SOLVING

9 Prove that the construction of a diagonal in a parallelogram creates two congruent triangles.

10 From the right angle of a right-angled isosceles triangle, a straight line is that which bisects the hypotenuse. Prove that two congruent triangles are created.

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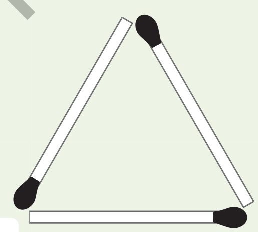
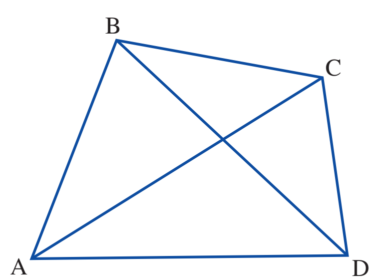
CHALLENGE 7.2

The figure shows how you can construct an equilateral triangle from three matchsticks. Show how you can construct five equilateral triangles using only nine matchsticks.



7.5 Quadrilaterals

Review of terms and rules



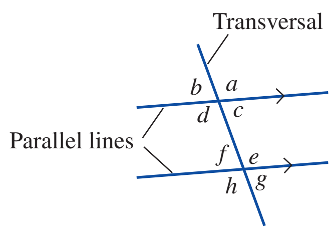
- The figure ABCD is a quadrilateral.
- A **quadrilateral** is a four-sided figure.
- The sum of the interior angles at the four vertices of a quadrilateral is 360°.
- **Opposite angles** do not have rays in common. ∠ABC and ∠ADC are opposite angles. ∠BCD and ∠BAD are also opposite angles.

- **Opposite sides** do not intersect. BC and AD are opposite sides. AB and CD are opposite sides.
- **Diagonals** connect opposite angles. AC is one diagonal and BD is the other.
- In some quadrilaterals, the *diagonals bisect each other*. In the diagram above, BD would bisect AC only if it divided AC into two equal sections. AC would bisect BD only if it divided it into two equal sections. In the previous diagram, the diagonals do not bisect each other.
- In some quadrilaterals, the *diagonals bisect the angles*. Diagonal BD bisects ∠ABC if ∠ABD and ∠DBC are congruent. In the diagram, above, BD does not bisect ∠ABC.
- There are several special quadrilaterals.
  – A **parallelogram** is a quadrilateral where the opposite pairs of sides are parallel.
  – A **rhombus** is a parallelogram in which all sides are equal. (Because it is a parallelogram, the properties of a parallelogram are also properties of a rhombus.)
  – A **rectangle** is a parallelogram in which all angles are 90°. (Because it is a parallelogram, the properties of a parallelogram are also properties of a rectangle.)
  – A **square** is a rectangle with all sides equal. (Because rectangles are parallelograms and a rhombus is a parallelogram with all sides congruent, a square is also a right-angled rhombus.)
  – A **kite** is a quadrilateral which has two pairs of adjacent sides that are equal.
  – A **trapezium** is a quadrilateral with one pair of parallel unequal sides.

Properties of quadrilaterals

- By considering triangles in quadrilaterals, the properties of quadrilaterals can be determined.

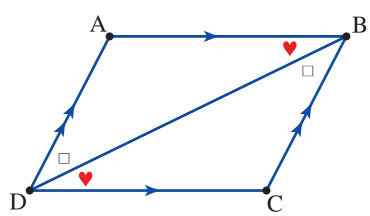
Angles and parallel lines



- Parallel lines are denoted by identical arrowheads.
- A straight line cutting a parallel line is called a **transversal**.
- **Corresponding angles** are equal in magnitude. For example, in the diagram, $a = e$, $c = g$, $b = f$ and $d = h$.
- **Vertically opposite angles** are equal in magnitude. For example, in the diagram, $a = d$, $b = c$, $f = g$ and $e = h$.
- **Alternate angles** are equal in magnitude. For example, in the diagram, $c = f$ and $d = e$.
- **Co-interior angles** (also known as allied angles) sum to 180°. For example, in the diagram, $c + e = 180°$ and $d + f = 180°$.
- Angles and parallel lines can be used to establish congruency of triangles in parallelograms.

WORKED EXAMPLE 9

When a diagonal is drawn in a parallelogram, two pairs of equal angles are formed, as shown (alternate angles).



a **State the congruency relationship between the two triangles formed, giving a reason.**

b **Hence write a relationship between:**

i **AB and DC** **ii** **DA and CB** **iii** **∠BAD and ∠BCD.**

c **What conclusion can be drawn about:**

i **opposite sides of a parallelogram?** **ii** **opposite angles of a parallelogram?**

THINK	WRITE
a There are two equal angles, and BD is in both triangles.	**a** ∠ABD = ∠CDB (given) ∠ADB = ∠CBD (given) BD is common to both ∴ ΔABD ≡ ΔCDB (ASA)
b **i** AB and DC are corresponding sides in the triangles.	**b** **i** AB = DC
ii DA and CB are corresponding sides in the triangles.	**ii** DA = CB
iii ∠BAD and ∠BCD are corresponding angles.	**iii** ∠BAD = ∠BCD
c **i** AB and DC are opposite sides. DA and CB are opposite sides.	**c** **i** AB = DC and DA = CB; therefore, opposite sides are equal.
ii ∠BAD and ∠BCD are opposite angles. ∠ABC and ∠ADC are opposite angles.	**ii** ∠BAD = ∠BCD ∠ABD = ∠BDC (given) ∠ADB = ∠CBD (given) ∴ ∠ABC = ∠ADC Opposite angles are equal.

Exercise 7.5 Quadrilaterals

assess on

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5, 9, 10	Questions: 1–6, 9, 10	Questions: 1–11

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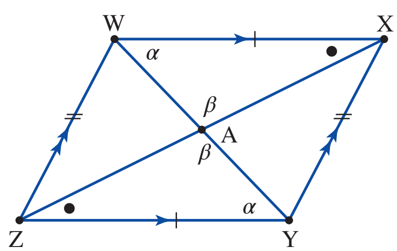
REFLECTION
Why is it important to be able to show congruency using mathematical reasoning?

FLUENCY

1 WE9a,b Consider the parallelogram WXYZ. From the worked example, we know that opposite sides are equal. Both diagonals are drawn giving 3 pairs of angles as shown (alternate angles and opposite angles).



a State the congruency relationship between ΔWXA and ΔYZA, giving a reason.

b Hence state the relationship between:

i WA and YA **ii** XA and ZA.

c Do the diagonals bisect each other?

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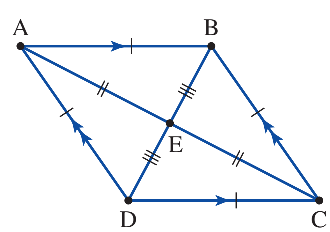
2 Consider the rhombus ABCD. Because the rhombus is also a parallelogram, we know that the diagonals bisect each other.



a State the congruency relationship between ΔABE and ΔBEC, giving a reason.

b Hence state the relationship between:

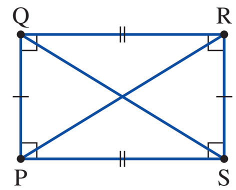
i ∠AEB and ∠CEB **ii** ∠ABE and ∠CBE.

c What is the magnitude of ∠AEB?

d Name all the triangles congruent to ΔABE.

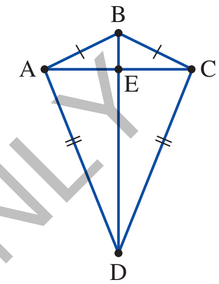
e Do the diagonals bisect the angles?

3 Consider the rectangle PQRS.



a Show that ΔPQR and ΔSRP are congruent.
b What can you say about the diagonals of a rectangle?

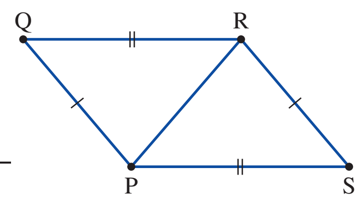
4 Consider the kite ABCD.



a Show that ΔABD and ΔCBD are congruent.
b What can you say about:
 i $\angle ABD$ and $\angle CBD$?
 ii $\angle BAD$ and $\angle BCD$?
 iii $\angle BDA$ and $\angle BDC$?
c Use part **b** to show that $\Delta ABD \equiv \Delta CBD$.
d What can you say about:
 i AE and CE?
 ii $\angle AEB$ and $\angle CEB$?
e Are ΔBEC and ΔDCE congruent?
f Use the above results to answer the following questions about kites.
 i Are the opposite sides equal?
 ii Are the opposite angles equal?
 iii Do the diagonals bisect each other?
 iv Do the diagonals intersect at 90°?
 v Do the diagonals bisect the angles?

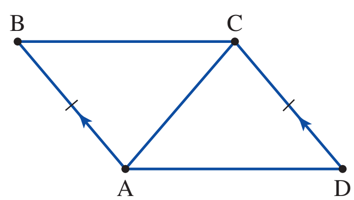
UNDERSTANDING

5 The quadrilateral PQRS has opposite sides that are equal.



a Show that the triangles are congruent.
b Complete the following.
 i $\angle QPR =$ ________ ii $\angle QRP =$ ________
c Complete the following.
 i QP is parallel to ________. ii QR is parallel to ________.
d What type of quadrilateral is PQRS?

6 In quadrilateral ABCD, AB and DC are both parallel and equal in length.

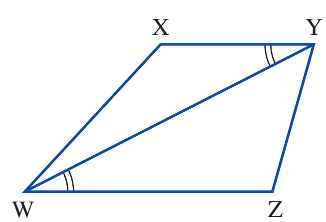


a Identify one pair of alternate angles.
b Show that the two triangles are congruent.
c Hence demonstrate that BC and AD are parallel.
d What type of quadrilateral is ABCD?

7 In a quadrilateral MNOP, use 2 pairs of adjacent triangles to show that the sum of the interior angles of a quadrilateral is 360°.

REASONING

8 WXYZ is a quadrilateral, where $\angle XYW = \angle YWZ$. Use mathematical reasoning to determine the type of quadrilateral.



PROBLEM SOLVING

9 The frame of the swing shown in the photograph is a trapezium. The diagonal sides make an angle of 73° with the ground. Evaluate the angle, x, that the horizontal crossbar makes with the sides.

10 The pattern for a storm sail for a sailboard is shown in the photograph. Calculate the size of each interior angle.

11 **a** Prove that a quadrilateral can have no more than one interior angle that is greater than 180°.

b If all interior angles are integers, what is the largest possible interior angle that a quadrilateral can have? Use your protractor to draw it.

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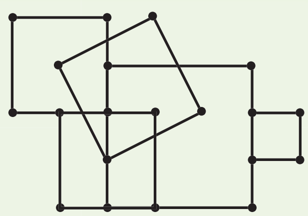
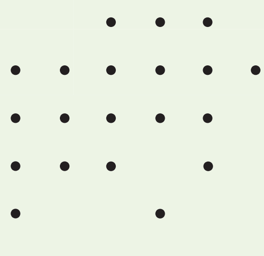
CHALLENGE 7.3



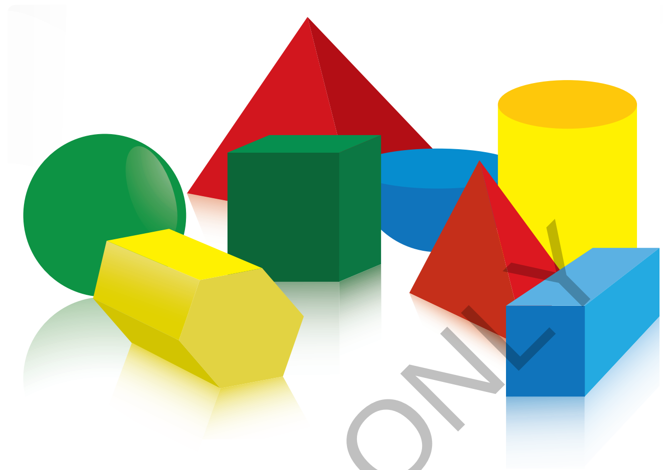
The diagram shows five squares drawn among a network of dots. The dots represent a corner of a square and no two squares share a corner, although some squares can share part of a side. Use the network of dots below to draw squares that follow the same rules — that is, with corners represented by the dots and no two squares sharing a corner. How many squares can be drawn?



7.6 Nets, polyhedra and Euler's rule

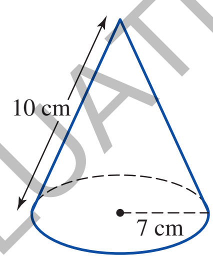
Nets

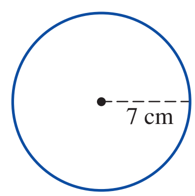
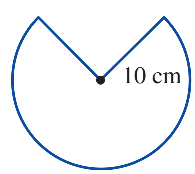
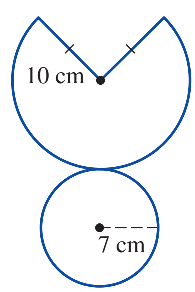
- The faces forming a solid can be drawn as plane shapes, which are joined across the edges to form the solid. The complete set of faces forming a solid is called its net. Note that for some figures, different nets can be drawn.



WORKED EXAMPLE 10

Draw a possible net for a cone, which has a radius of 7 cm and a slanting height of 10 cm.

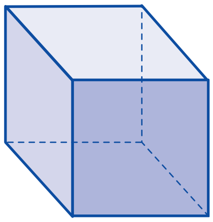


THINK	DRAW
1 The base of a cone is a circle of radius 7 cm.	
2 The other (slanted) part of the cone when split open will form a sector of a circle of radius 10 cm.	
3 If the two parts (from steps 1 and 2) are put together, the complete net of a cone is obtained.	

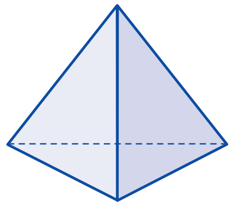
Polyhedron construction

- A 3-dimensional solid where each of the faces is a polygon is called a **polyhedron**. If all the faces are congruent, the solid is called a regular polyhedron or a Platonic solid.
- Platonic solids are shown below.

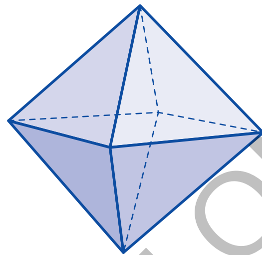
1. A **cube**, with 6 faces, each of which is a square.



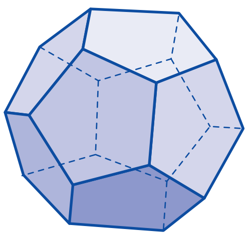
2. A **tetrahedron**, with 4 faces, each of which is an equilateral triangle.



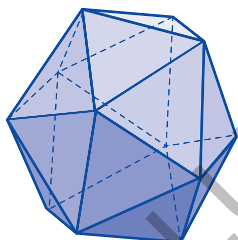
3. An **octahedron**, with 8 faces, each of which is an equilateral triangle.



4. A **dodecahedron**, with 12 faces, each of which is a regular pentagon.



5. An **icosahedron**, with 20 faces, each of which is an equilateral triangle.



- Construction of polyhedra is conveniently done from nets.

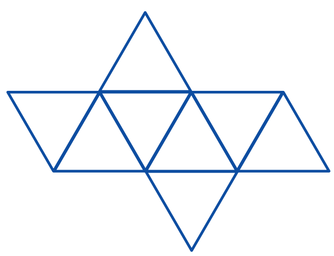
WORKED EXAMPLE 11

Draw a net of an octahedron.

THINK

An octahedron is a polyhedron with 8 faces, each of which is an equilateral triangle. So its net will consist of 8 equilateral triangles. Draw a possible net of an octahedron.

DRAW



Euler's rule

- **Euler's rule** shows the relationship between the number of **edges**, the number of **faces** and the number of vertices in any polyhedron. Note that a **vertex** (the singular of vertices) is a point or a corner of a shape where the straight edges meet.

Euler's rule states that for any polyhedron:
Number of faces (F) + number of vertices (V) − 2 = number of edges (E), or $F + V - 2 = E$.

assess on

Exercise 7.6 Nets, polyhedra and Euler's rule

INDIVIDUAL PATHWAYS

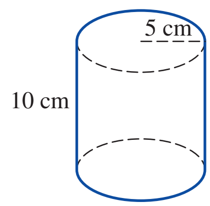
REFLECTION
When would you find it a useful skill to be able to construct 3D shapes?

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 13	Questions: 1–13	Questions: 1–14

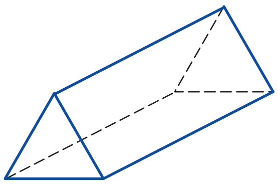
Individual pathway interactivity int-4428 eBook plus

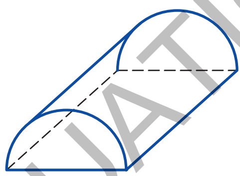
FLUENCY

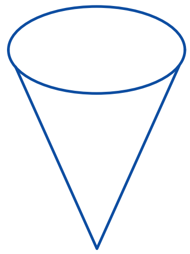
1 WE10 Draw a possible net for the cylinder shown below.



2 Draw a possible net for each of the following solids.

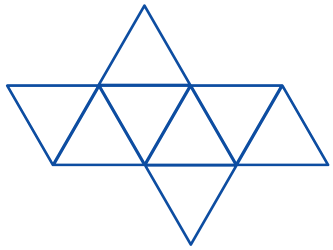
a 

b 

c 

3 WE11 Draw 2 different nets to form a cube.

4 Draw a net for the octahedron that is different from the one shown below.

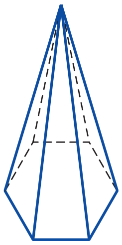


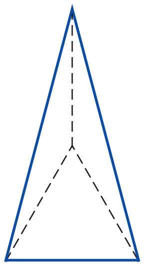
Cut out your net and fold it to form the octahedron.

5 Using nets generated from geometry software or elsewhere, construct:

a a cube
b a tetrahedron
c a dodecahedron.

6 Construct the pyramids shown in the figures below.

a 

b 

UNDERSTANDING

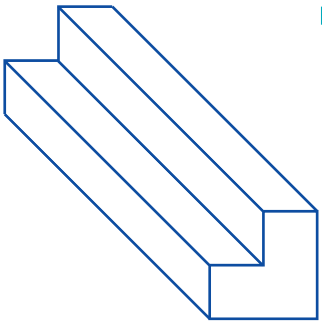
7 Draw some 3-dimensional shapes of your choice. How many of these shapes are polyhedra? Can you name them?

8 Verify Euler's rule for the following Platonic solids.

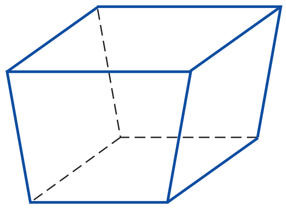
- **a** A cube
- **b** An octahedron

9 Show that Euler's rule holds true for these solids.

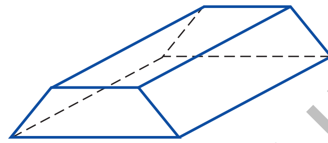
a



b

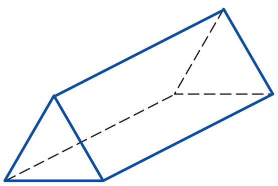


c

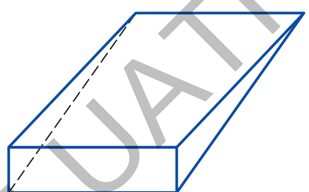


10 Make the prisms shown below. Verify Euler's rule for each shape.

a



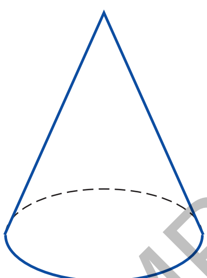
b



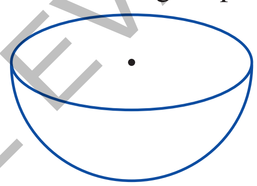
REASONING

11 Can you verify Euler's rule for the following shapes? Give reasons for your answer.

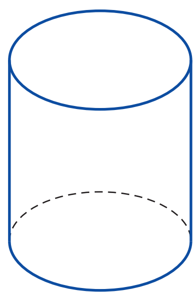
a



b



c

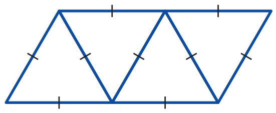


12 A spherical scoop of ice-cream sits on a cone. Draw this shape. Can you verify Euler's rule for this shape? Explain your reasoning.

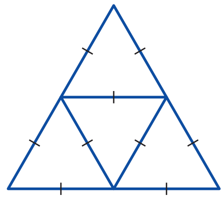
PROBLEM SOLVING

13 Which of the following compound shapes are nets that can be folded into a 3-D solid? You may wish to enlarge them onto paper, cut them out and fold them.

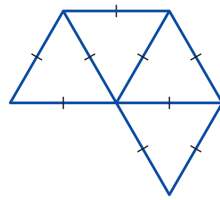
a



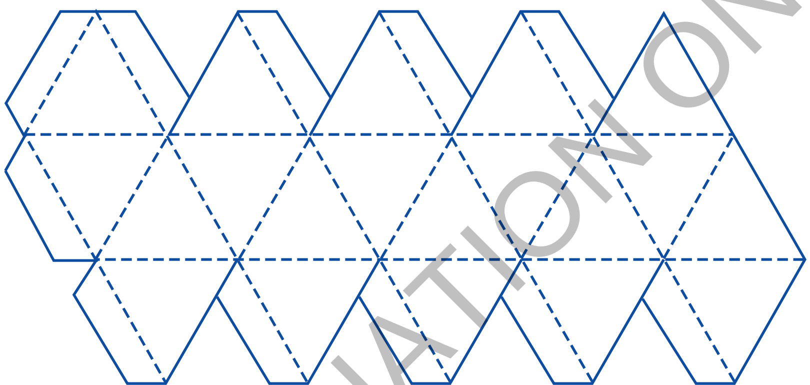
b



c



14 **a** The net below forms an icosahedron. Copy the net onto grid paper and fold it to make the icosahedron. You may wish to use the computer program called Poly to help you with the construction.



b Verify Euler's rule for this shape, showing your working.

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7.7 Review

www.jacplus.com.au

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

Download the Review questions document from the links found in your eBookPLUS.

eBook*plus*

Interactivities

Word search: **int-2627**

Crossword: **int-2628**

Sudoku: **int-3187**

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

alternate angles	kite	rhombus
co-interior angles	opposite angles	square
congruent	opposite sides	transversal
corresponding angles	parallelogram	trapezium
corresponding sides	quadrilateral	triangle
diagonals	rectangle	vertically opposite angles

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

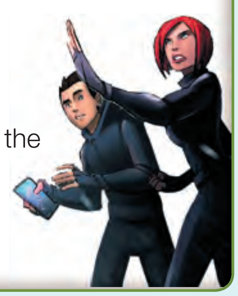
assess on

assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au

Link to SpyClass, an exciting online game combining a comic book–style story with problem-based learning in an immersive environment.

Join Jesse, Toby and Dan and help them to tackle some of the world's most dangerous criminals by using the knowledge you've gained through your study of mathematics.

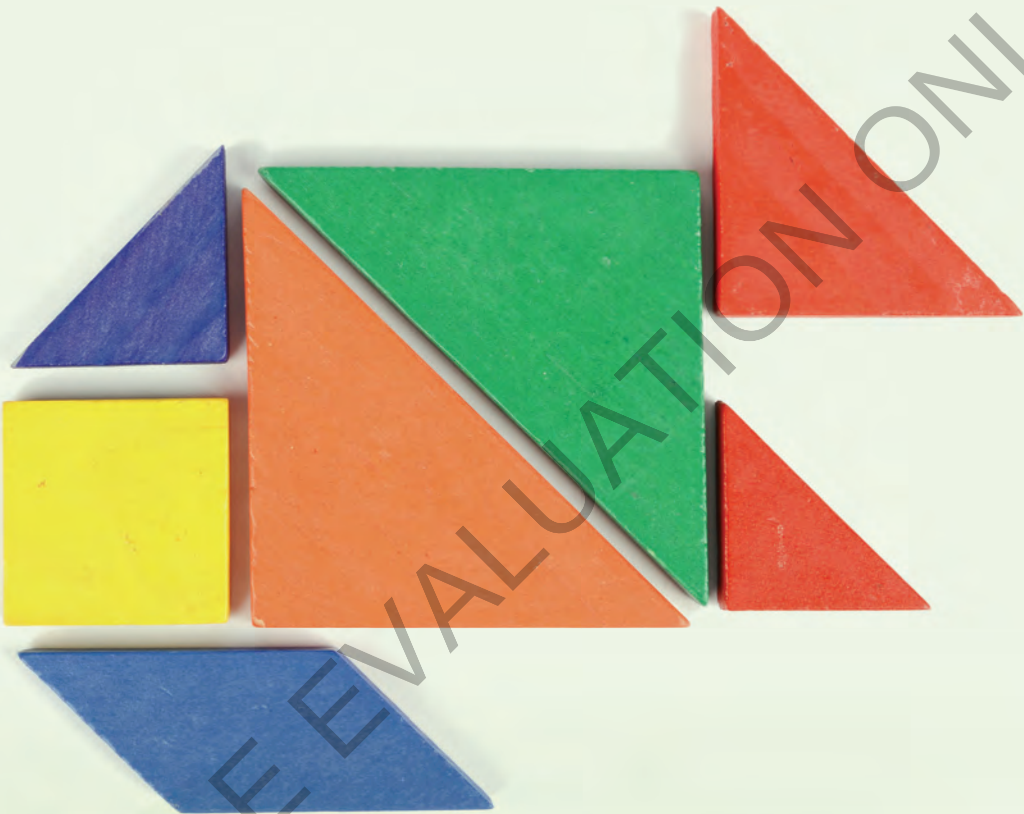


www.spyclass.com.au

INVESTIGATION

RICH TASK

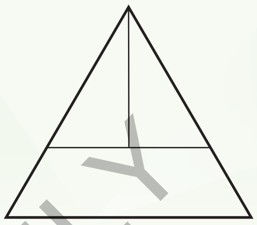
Puzzling constructions



How well do you know the properties of the different triangles and quadrilaterals? This activity will challenge you to create as many triangles and quadrilaterals as you can from three different types of puzzles.

Follow the instructions in this activity to create your three puzzles. Draw each shape onto a firm sheet of A4 paper. Carefully space the puzzles, so they will all fit onto the one sheet.

Three-piece triangle

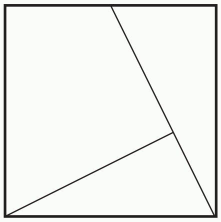


- Construct an equilateral triangle with side lengths of 10 cm. Mark the centre of each side.
- Lightly rule a line from each corner to the midpoint on the opposite side. These lines will intersect at one point.
- From this point, rule a line to the top corner. Rule another line parallel to the base side that goes through the centre point. Cut out the three shapes.

  1 Name the three shapes created in this puzzle. List the main properties of each shape.

  2 Use any number of pieces to construct as many triangles and quadrilaterals as you can. Draw each solution and list the main properties of each shape.

Three-piece square

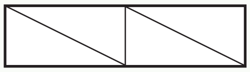


- Draw a square with side lengths of 10 cm. Mark the centre of the top and right sides.
- Rule a line from the centre of the top side to the bottom right corner.
- Rule another line from the bottom left corner towards the centre on the right side. Stop when you meet the previous line. Cut out the three shapes.

  3 Name the three shapes created in this puzzle. List the main properties of each shape.

  4 Repeat question **2** using the pieces from this puzzle.

Four-piece rectangle



- Draw a rectangle measuring 20 cm by 5 cm.
- Mark the centre of the long side and divide the rectangle into two.
- Rule a diagonal line in each smaller rectangle. Cut out the four shapes.

  5 What do you notice about the shapes you created? List their main properties.

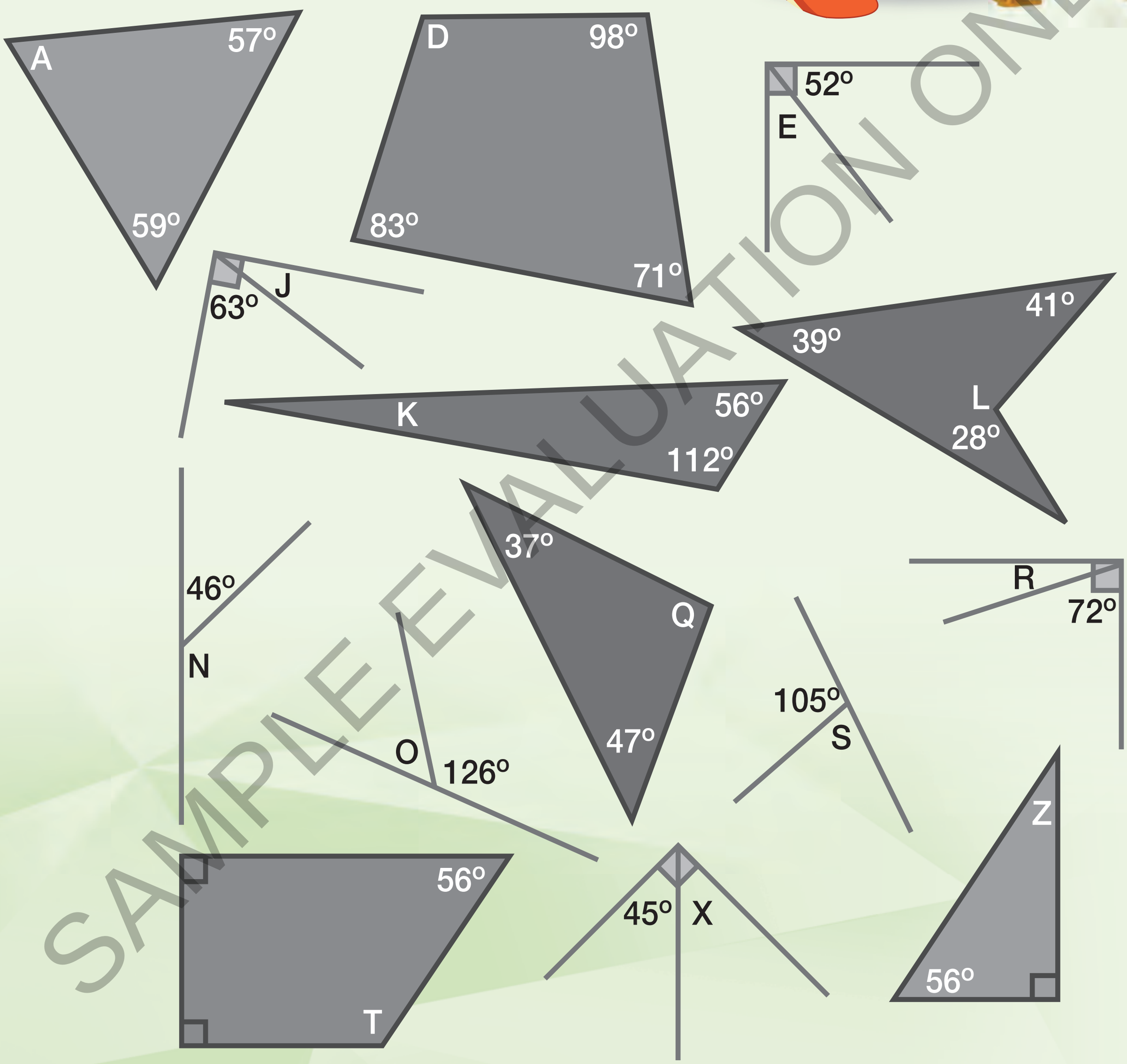
  6 Repeat question **2** using the pieces from this puzzle.

  7 What other types of polygons can you construct from each individual puzzle?

CODE PUZZLE

What did the pencil say to the eraser?

The size of the lettered angles gives the puzzle answer code.



64°	108°	108°	64°	252°	38°	124°	124°	38°	18°	124°	54°	:

75°	34°	27°	108°	252°	108°	75°	134°

45°	134°	124°	96°	96°	124°	12°	108°	96°

eBookplus Activities

7.2 Congruent figures

Digital docs

- SkillSHEET (doc-6915) Naming angles and shapes
- SkillSHEET (doc-6916) Calculating angles in triangles

Interactivities

- Transformations and the Cartesian plane (int-2368)
- IP interactivity 7.2 (int-4424) Congruent figures

7.3 Triangle constructions

Interactivities

- Random triangles (int-2369)
- IP interactivity 7.3 (int-4425) Triangle constructions

7.4 Congruent triangles

Digital doc

- WorkSHEET 7.1 (doc-2250)

Interactivity

- IP interactivity 7.4 (int-4426) Congruent triangles

7.5 Quadrilaterals

Digital docs

- SkillSHEET (doc-6917) Angles and parallel lines
- WorkSHEET 7.2 (doc-2251)

Interactivity

- IP interactivity 7.5 (int-4427) Quadrilaterals

7.6 Nets, polyhedra and Euler's rule

Interactivity

- IP interactivity 7.6 (int-4428) Nets, polyhedra and Euler's rule

7.7 Review

Interactivities

- Word search (int-2627)
- Crossword (int-2628)
- Sudoku (int-3187)

Digital docs

- Topic summary (doc-10759)
- Concept map (doc-10772)
- Topic review (Word doc-14936, PDF doc-14937)

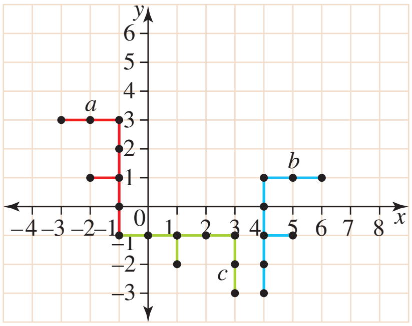
To access eBookPLUS activities, log on to www.jacplus.com.au

Answers

TOPIC 7 Congruence

7.2 Congruent figures

1 a–c



d Yes

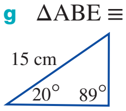
2 a i and iii **b** i and iv **c** ii and iv

3 a $\Delta ABC \equiv \Delta PQR$, $x = 25°$
b $\Delta ABC \equiv \Delta PQR$, $x = 70°$, $y = 40°$
c $\Delta ABC \equiv \Delta CDA$, $x = 40°$, $y = 55°$
d $\Delta ABC \equiv \Delta ADC$, $x = 30°$, $y = 30°$, $z = 120°$
e $\Delta ABC \equiv \Delta ADC$, $x = 40°$, $y = 40°$, $z = 100°$
f $\Delta ABC \equiv \Delta PRQ$, $x = 35°$, $y = 55°$

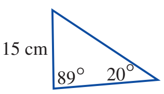
4 C

5 a $\Delta ABC \equiv \Delta ADC$ $x = 30°$ $y = 60°$
b $\Delta KLN \equiv \Delta MNL$ $x = 75°$ $y = 60°$ $z = 45°$
c $\Delta PSR \equiv \Delta QRS$ $x = 8$ cm
d $\Delta SWT \equiv \Delta UVT$ $w = y = 70°$ $x = z = 55°$
e $\Delta EFG \equiv \Delta EHG$ $x = 8$ cm
f $\Delta WXZ \equiv \Delta WYZ$ $x = 45°$ $y = 3$ cm
g $\Delta ABE \equiv \Delta CDB$ $x = 58°$ $y = 65°$

6



and



7 $a = 35°$, $b = 55°$, $c = 70°$, $d = 5$ cm, $e = 2$ cm

8 $a = 15$ cm, $b = 92°$, $c = 58°$

Challenge 7.1

Equilateral triangles, squares and regular hexagons

7.3 Triangle constructions

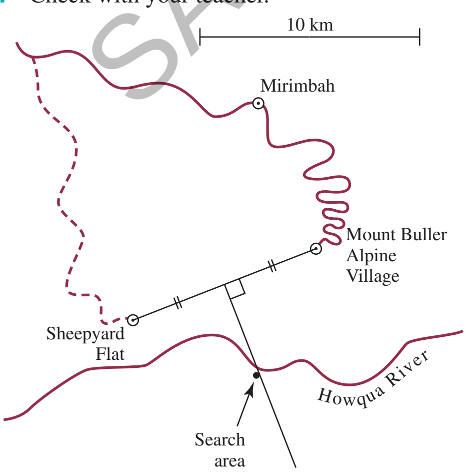
1 to **3** Check with your teacher.

4 a Check with your teacher.
b 89°
c i

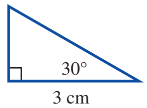
5 a Check with your teacher.
b 1 triangle

6, 7 Check with your teacher.

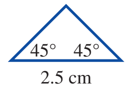
8



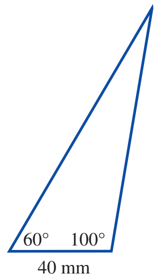
9 a



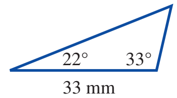
b



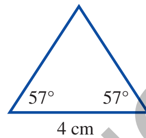
c



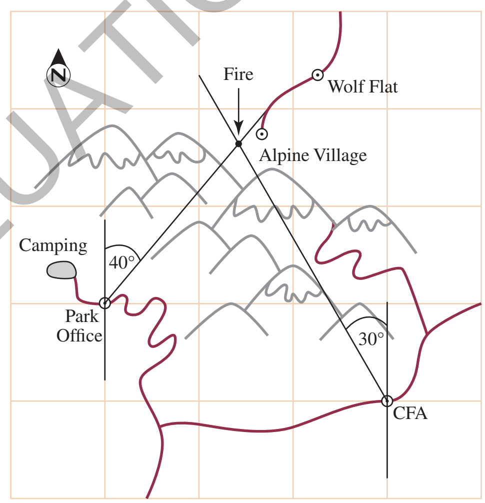
d



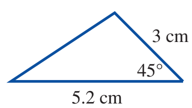
e



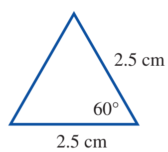
10



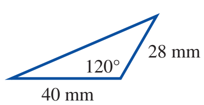
11 a



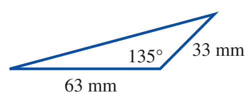
b



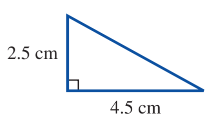
c



d



e



7.4 Congruent triangles

1 a $\Delta ABC \equiv \Delta PQR$ (SSS)
b $\Delta ABC \equiv \Delta PQR$ (SSS)
c $\Delta LMN \equiv \Delta PQR$ (SAS)
d $\Delta ABC \equiv \Delta PQR$ (ASA)
e $\Delta ABE \equiv \Delta ADE$ (SSS)
f $\Delta FKJ \equiv \Delta GKH$ (SSS)

2 **a** $x = 3$ cm
b $x = 83°$
c $x = 75°$ $y = 25°$ $z = 80°$
d $x = 25°$ $y = 8$ cm
e $x = 35°$ $y = z = 55°$ $n = m = 90°$

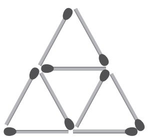
3 **a** AB = AC
BD = CD
AD is common.
$\Delta DBA \equiv \Delta DCA$ (SSS)
b AB = AD
AC is common.
$\angle BAC = \angle CAD$
$\Delta ABC \equiv \Delta ADC$ (SAS)
c PQ = PS
PR is common.
$\angle PRQ = \angle PRS = 90°$
$\Delta PQR = \Delta PSR$ (RHS)

4, 5 Check with your teacher.

6 **a** Congruent (SSS) **b** Congruent (ASA)

7 to **10** Check with your teacher.

Challenge 7.2



7.5 Quadrilaterals

1 **a** $\Delta WXA = \Delta YZA$ (ASA)
b **i** WA = YA
ii XA = ZA
c Diagonals bisect each other.

2 **a** $\Delta ABE = \Delta CBE$ (SSS)
b **i** $\angle AEB = \angle CEB$
ii $\angle ABE = \angle CBE$
c 90°
d ΔCBE, ΔADE, ΔCDE
e Diagonals bisect the angles.

3 **a** $\Delta PQR \equiv \Delta SRP$ (SAS)
b Diagonals of a rectangle are equal.

4 **a** $\Delta ABD \equiv \Delta CBD$ (SSS)
b **i** $\angle ABD = \angle CBD$
ii $\angle BAD = \angle BCD$
iii $\angle BDA = \angle BDC$
c $\Delta ABD \equiv \Delta CBD$ (SAS)
d **i** AE = CE
ii $\angle AEB = \angle CEB$
e No
f **i** No
ii No (one pair of opposite angles are equal)
iii No (one diagonal bisects the other)
iv Yes
v No (one diagonal bisects the unequal angles)

5 **a** $\Delta QPR \equiv \Delta SRP$ (SSS)
b **i** $\angle SRP$
ii $\angle SPR$
c **i** RS
ii PS
d Parallelogram

6 **a** $\angle BAC = \angle DCA$
b $\Delta ABC \equiv \Delta CDA$ (SAS)
c $\angle BCA = \angle DAC$ (alternate angles)
d Parallelogram

7 All quadrilaterals can be divided into two adjacent triangles. The interior angle sum of a triangle is 180°, so the interior angle sum of a quadrilateral is equal to 360° (180° × 2).

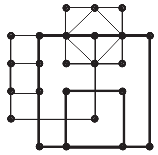
8 Trapezium

9 $x = 107°$

10 50°, 70°, 110°, 130°

11 **a** Check with your teacher. **b** 357°

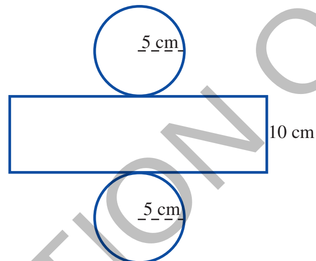
Challenge 7.3



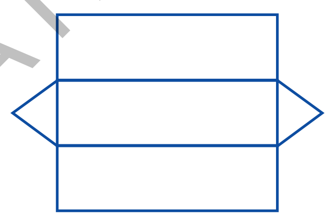
Six squares can be drawn.

7.6 Nets, polyhedra and Euler's rule

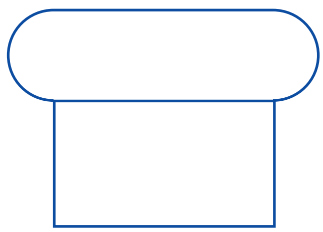
1



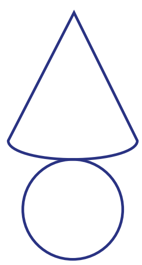
2 **a**



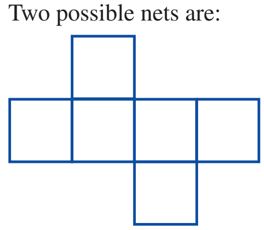
b



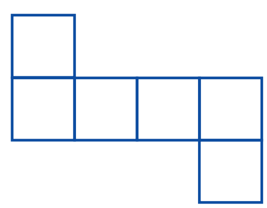
c



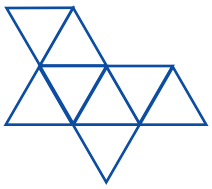
3 Two possible nets are:



or



4 A possible net is:



5 to **7** Check with your teacher.

8 **a** $E = 12, V = 8, F = 6, 8 + 6 - 2 = 12$

b $E = 12, V = 6, F = 8, 6 + 8 - 2 = 12$

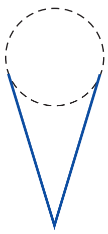
9 **a** $E = 18, V = 12, F = 8, 8 + 12 - 2 = 18$

b and **c** $E = 12, V = 8, F = 6, 6 + 8 - 2 = 12$

10 For each shape, $E = 9$, $V = 6$ and $F = 5$ where $5 + 6 - 2 = 9$.

11 No. Edges, vertices and faces can not be identified on a solid with curves.

12



No

13 **a** Yes

b Yes

c No

14 **a** Check with your teacher.

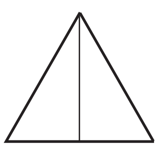
b 20 triangular faces, 30 edges and 12 vertices

$20 + 12 - 2 = 30$

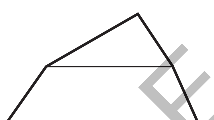
Investigation — Rich task

1 Two right-angled triangles and one trapezium with two parallel sides

2 Some possible solutions are shown; teacher to check for others.



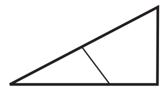
Equilateral triangle with two pieces; all angles = 60°



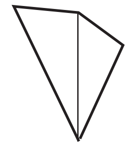
Irregular quadrilateral with two pieces; all sides have different lengths

3 One large and one smaller right-angled triangle and an irregular quadrilateral

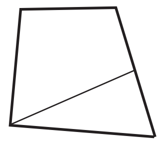
4 Some possible solutions are shown; teacher to check for others.



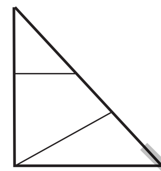
Right-angled triangle with two pieces



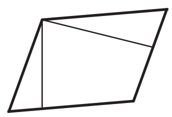
Irregular quadrilateral with two pieces



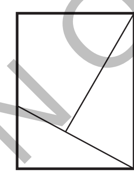
Trapezium with two pieces



Right-angled triangle with three pieces



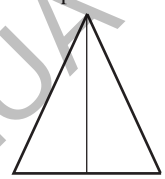
Parallelogram with three pieces



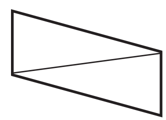
Rectangle with three pieces

5 Four identical right-angled triangles

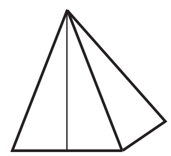
6 Some possible solutions are shown; teacher to check for others.



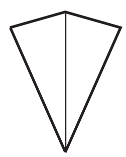
Isosceles triangle with two pieces



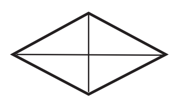
Parallelogram with two pieces



Irregular quadrilateral with three pieces



Kite with two pieces



Rhombus with all four pieces

7 Teacher to check.

Code puzzle

Take me to your ruler.