TOPIC 15

Pythagoras' theorem

15.1 Overview

Why learn this?

Pythagoras was a famous mathematician who lived about 2500 years ago. He is credited with being the first person to prove that in any right-angled triangle there is a special relationship between the squares of the three sides. The theorem has an important role to play in everyday life because right-angled triangles occur in construction, navigation, planning, design and packaging. Pythagoras' theorem is one of the great geometrical theorems.
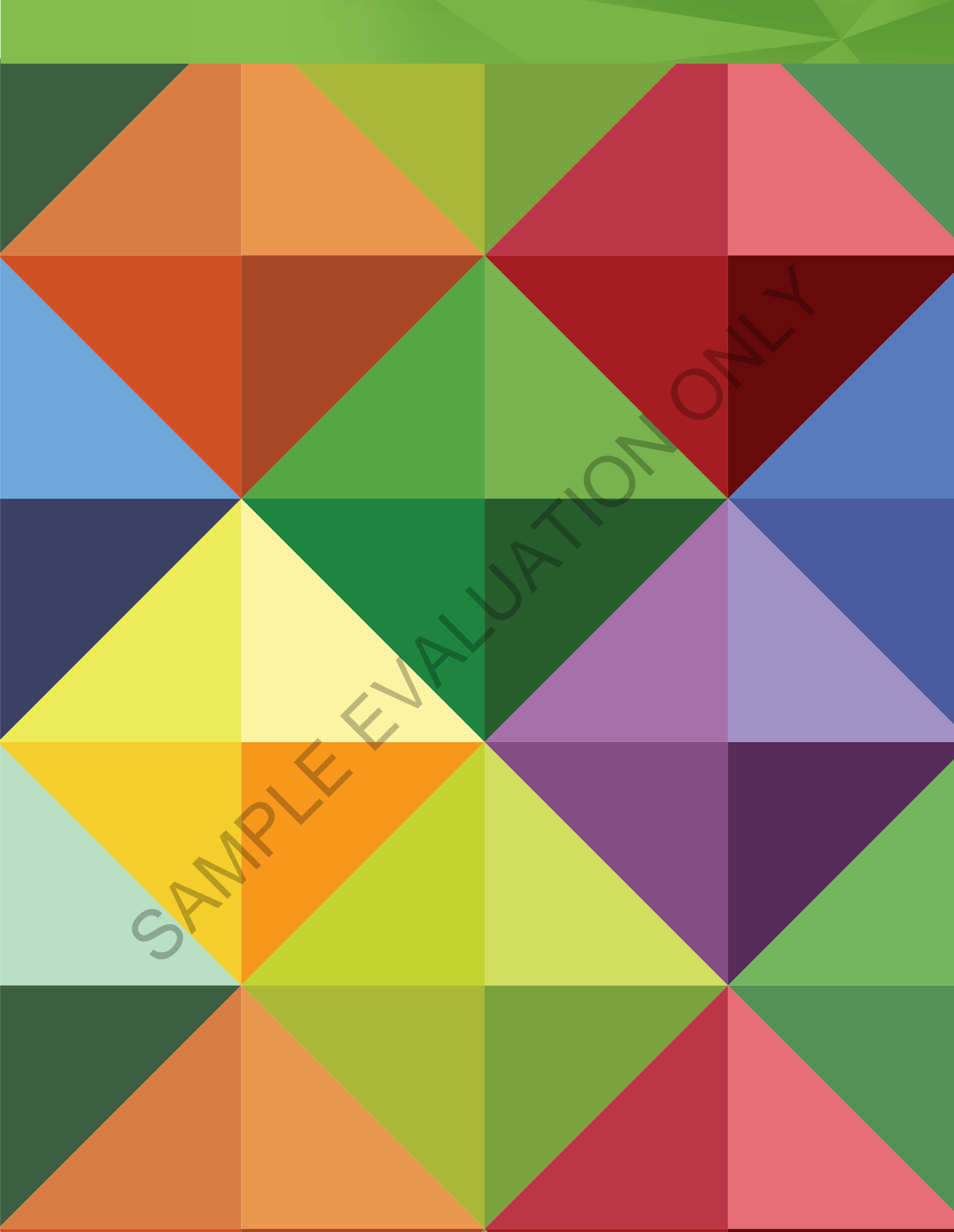
What do you know?

assess on

1 **THINK** List what you know about Pythagoras' theorem. Use a thinking tool such as a concept map to show your list.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of Pythagoras' theorem.

Learning sequence
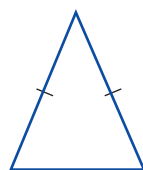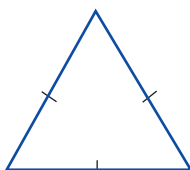
15.2 Right-angled triangles

- This topic investigates one of the most important ideas in geometry related to **right-angled triangles**. Triangles can be classified according to the length of their sides. They can be classified as either equilateral, isosceles or scalene. Triangles can be classified by their angles as well. A triangle with one right angle is said to be a right-angled triangle.
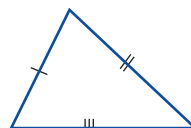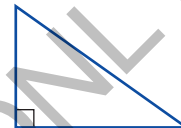
Types of triangles

Isosceles   Equilateral   Scalene   Right-angled

- In any triangle, the longest side is opposite the largest angle; so in a right-angled triangle, the longest side is opposite the right angle. This side has a special name. It is called the **hypotenuse**.

- The theorem, or rule, that you will learn more about was named after an ancient Greek mathematician, called Pythagoras (580–501 BC). It will enable you to solve all kinds of practical problems related to right-angled triangles.
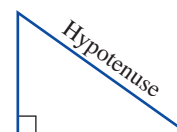
Exercise 15.2   Right-angled triangles

INDIVIDUAL PATHWAYS

REFLECTION
How will you remember the meaning of the pronumerals *a*, *b* and *c* in Pythagoras' theorem?

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1–6, 8, 10	Questions: 1–5, 7, 8, 10, 11	Questions: 1–5, 7–11

■ ■ ■ **Individual pathway interactivity**   int-4469   **eBook plus**

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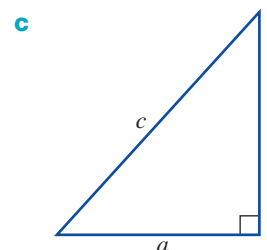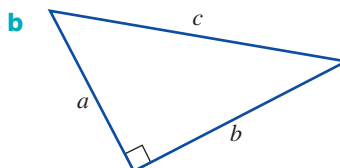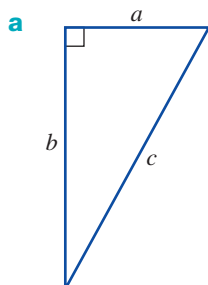**Digital docs**
SkillSHEET
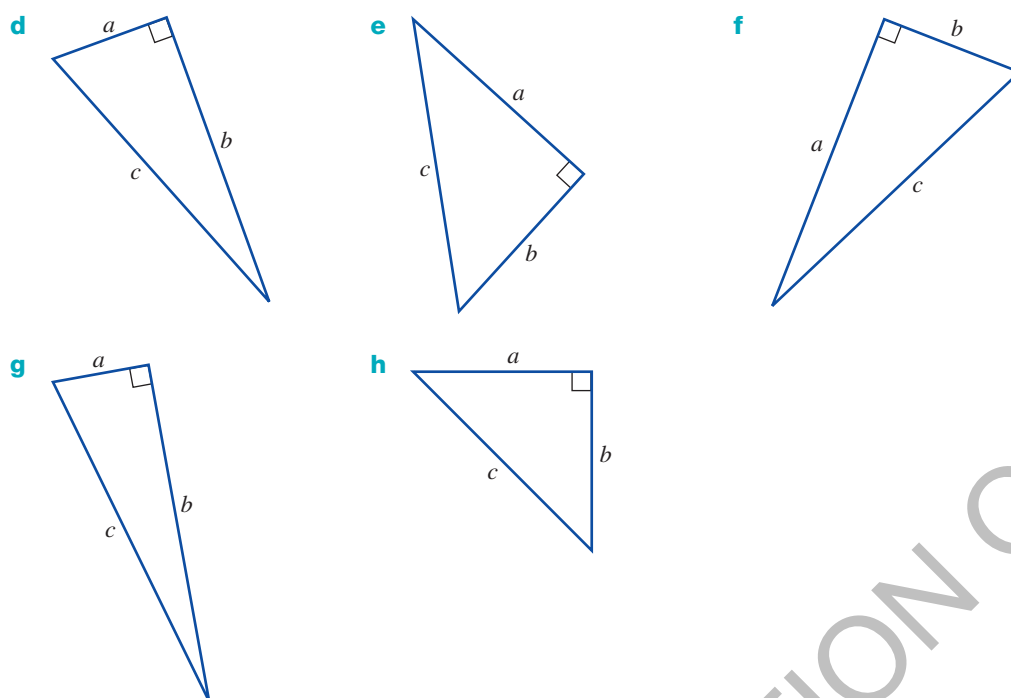Measuring lengths and angles
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SkillSHEET
Finding the square of a number
doc-11426

FLUENCY

1 For each of the following triangles, the lengths of the sides are recorded in the table that follows. Note that the hypotenuse is always marked as *c*. Complete the table.

d

e

f

g

h

Lengths (mm)	a	b	c	d	e	f	g	h
a	19	24	27	16	31	41	13	24
b	35	32	30	40	25	19	45	24
c	40	40	40	43	40	45	47	34
a^2								
b^2								
$a^2 + b^2$								
c^2								

2 What do you notice about the results in the table in question **1**?

3 On a sheet of paper, carefully draw 6 right-angled triangles of different sizes. You will need to use a protractor, set square or template to make sure the triangles are right-angled. Carefully measure the sides of each triangle, and complete a table like the one in question **1**. What do you notice about these results?

4 Now draw some triangles that are not right-angled, measure their sides and complete the same kind of table. Remember to label the longest side as c. What do you notice this time?

5 Explain why it is not possible to have a triangle with two right angles in it.

6 For this activity you will need graph paper, coloured pencils, glue and scissors.

   a On a sheet of graph paper, draw a right-angled triangle with a base of 4 cm and a height of 3 cm.

   b Carefully draw a square on each of the three sides of the triangle and mark a grid on each so that the square on the base is divided into 16 small squares, while the square on the height is divided into 9 small squares.

c Colour the square on the base, and the square on the height in different colours as shown at right so that you can still see the grid lines.

d Carefully cut out the two coloured squares from the triangle.

e Now stick the larger of the coloured squares on the uncoloured square of the triangle (the square on the hypotenuse).

f Using the grid lines as a guide, see if you can cut the smaller square up and fit it on the remaining space. The two coloured squares should have exactly covered the third square.

g What do you notice about the hypotenuse and the other two sides of a right-angled triangle?

3 cm

4 cm
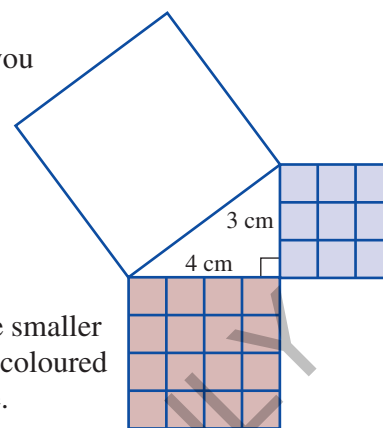
UNDERSTANDING

7 A right-angled triangle is a special triangle, but it has some properties that are common to all triangles. One of these common properties is that the sum of the lengths of any two sides must be greater than the length of the third side.

Using a ruler and compass, attempt to construct a triangle whose side lengths are 7 cm, 4 cm and 12 cm. What do you find?

REASONING

8 What is the sum of the two smaller angles in a right-angled triangle? Justify your answer.

9 If the two smaller angles in a right-angled triangle are equal in size, make a statement about the triangle. What size is each of the smaller angles?

PROBLEM SOLVING

10 Can you draw an equilateral triangle that has a right angle? Explain.

11 a Draw four triangles with a 30° angle like the one shown at right, ensuring that the length of a is different for each.

c

b

30°

a
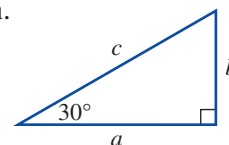
b For each triangle you have drawn in part a, find the value of:

i $\dfrac{b}{a}$ ii $\dfrac{b}{c}$ iii $\dfrac{a}{c}$

c What do you notice about your answers to part b?

d Repeat questions a to c for another four right-angled triangles using an angle other than 30°.
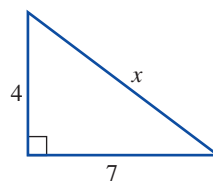
15.3 Finding the hypotenuse

• Pythagoras was perhaps the first mathematician to recognise and investigate a very important property of right-angled triangles. The theorem, or rule, named after him states that:

In any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. The rule is written as $c^2 = a^2 + b^2$ where a and b are the two shorter sides and c is the hypotenuse.

- The hypotenuse is the longest side of a right-angled triangle and is always the side that is opposite the right angle.
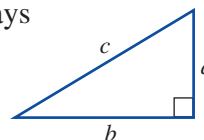
- Pythagoras' theorem gives us a way of finding the length of the third side in a triangle, if we know the lengths of the two other sides.
- We are able to find the length of the hypotenuse when we are given the length of the two shorter sides by substituting into the formula $c^2 = a^2 + b^2$ to calculate the hypotenuse c.
- Calculations with Pythagoras' theorem often result in a number under the root sign $(\sqrt{})$ that is not a square number. In such case answers may be left unsimplified, and this is called exact (surd) form. A calculator may be used to find an approximate answer, usually given to a specified number of decimal places.

WORKED EXAMPLE 1

For the triangle at right, find the length of the hypotenuse, x, correct to 1 decimal place.
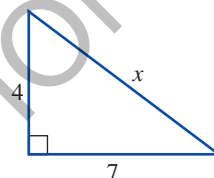
THINK

1 Copy the diagram and label the sides a, b and c. Remember to label the hypotenuse as c.

2 Write Pythagoras' theorem.

3 Substitute the values of a, b and c into this rule and simplify.

4 Calculate x by taking the square root of 65. As 65 is not a square number, a calculator can be used to find an approximate value for x. Round your answer correct to 1 decimal place.

WRITE/DRAW
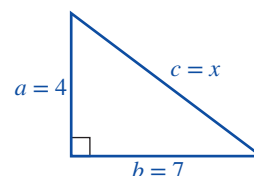
$c^2 = a^2 + b^2$

$x^2 = 4^2 + 7^2$
$= 16 + 49$
$= 65$

$x = \sqrt{65}$ This is the exact (surd) form.
$x \approx 8.1$

- The $\approx$ symbol means 'is approximately equal to'. An alternative symbol, $\doteqdot$, can also be used.
- In many cases we are able to use Pythagoras' theorem to solve practical problems. We can model the problem by drawing a diagram, and use Pythagoras' theorem to solve the right-angled triangle. We then use the result to give a worded answer.

WORKED EXAMPLE 2

A fire is on the twelfth floor of a building. A child needs to be rescued from a window that is 20 metres above ground level. If the rescue ladder can be placed no closer than 6 m from the foot of the building, what is the minimum length ladder needed to make the rescue? Leave your answer in exact form.

THINK

1. Draw a diagram and label the sides a, b and c. Remember to label the hypotenuse as c.

2. Write Pythagoras' theorem.

3. Substitute the values of a, b and c into this rule and simplify.

4. Calculate x by taking the square root of 436.

5. Give a worded answer.

WRITE/DRAW

$$a = 20 \text{ m}, \quad c = x, \quad b = 6 \text{ m}$$
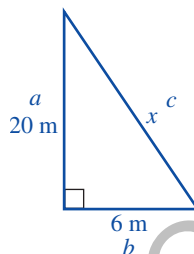
$$c^2 = a^2 + b^2$$

$$x^2 = 20^2 + 6^2$$
$$= 400 + 36$$
$$= 436$$

$$x = \sqrt{436}$$

The ladder needs to be $\sqrt{436}$ metres long.

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assess on

Exercise 15.3 Finding the hypotenuse

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions:	Questions:	Questions:
1–5, 9	1–6, 9, 10	1–11

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Digital docs
SkillSHEET
Finding the square root of a number
doc-11427
SkillSHEET
Rounding to a given number of decimal places
doc-11428

FLUENCY

1. **WE1** For the following triangles, find the length of the hypotenuse, x, correct to 1 decimal place where necessary.
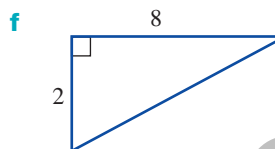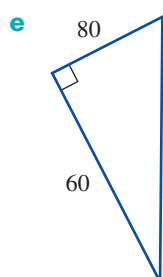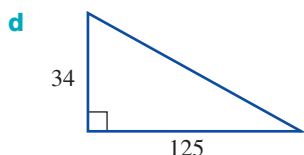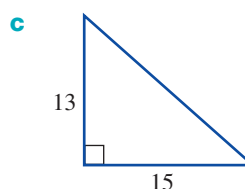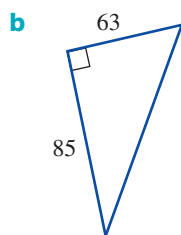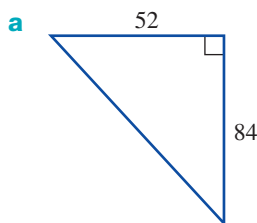
a. 3, x, 4

b. 5, x, 12

c. x, 7, 24

d. x, 17.5, 12.2

e. 896, x, 742

f. 1.3, 6.2, x

2 For each of the following triangles, find the length of the hypotenuse. Leave your answers in exact form.

a

52

84

b

63

85

c

13

15

d

34

125

e

80

60

f

8

2

UNDERSTANDING

3 A right-angled triangle has a base of 5 cm and a perpendicular height of 11 cm. Find the length of the hypotenuse. Leave your answers in exact form.

4 Find the lengths, correct to 2 decimal places, of the diagonals of squares that have side lengths of:

  a 12 cm   **b** 20 mm   **c** 4.9 cm

5 What is the length, correct to 2 decimal places, of the diagonal of a rectangle whose sides are:

  a 10 cm and 8 cm?   **b** 620 cm and 400 cm?   **c** 17 cm and 3 cm?

6 An isosceles triangle has a base of 30 cm and a perpendicular height of 10 cm. Find the length of the two equal sides of the isosceles triangle. Give your answer correct to 2 decimal places.

REASONING

7 A right-angled triangle has a perpendicular height of 17.2 cm, and a base that is half the height. Find the length of the hypotenuse, correct to 2 decimal places.

8 **WE2** A ladder leans against a vertical wall. The foot of the ladder is 1.2 m from the wall, and the top of the ladder reaches 4.5 m up the wall. How long is the ladder? Give your answer correct to 2 decimal places.

9 Wally is installing a watering system in his garden. The pipe is to go all around the edge of the rectangular garden, and have a branch diagonally across the garden. The garden measures 5 m by 7.2 m. If the pipe costs $2.40 per metre (or part thereof), what will be the total cost of the pipe? Show your working.

PROBLEM SOLVING

10 a Calculate the volume of the prism.

  b Find the length of the sloping edge of the cross-section.

6 cm

7 cm

9 cm
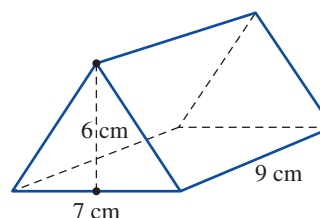
11 A ladder rests against a vertical wall with the top of the ladder being 7 m above the ground. If the bottom of the ladder was moved 1 m further away from the foot of the wall, the top of the ladder would rest against the foot of the wall. What is the length of the ladder?
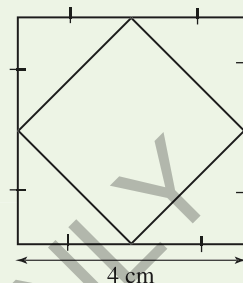
12 A right-angled isosceles triangle has an area of 200 cm^2. Find the area of the semi-circle which sits on the hypotenuse.

CHALLENGE 15.1

A smaller square is drawn inside a larger square as shown in the diagram.

Use Pythagoras' theorem to determine the side length of the smaller square and hence the area of the smaller square.

4 cm

15.4 Finding a shorter side

- Sometimes a question will give you the length of the hypotenuse and ask you to find one of the shorter sides. In such examples, we need to rearrange Pythagoras' formula.
- Given that $c^2 = a^2 + b^2$, we can rewrite this as:

$$a^2 = c^2 - b^2$$

or $b^2 = c^2 - a^2.$

WORKED EXAMPLE 3

Find the length of the unmarked side of the triangle at right. Leave your answer in exact form.
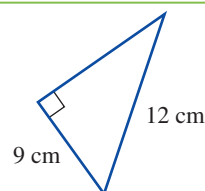
12 cm

9 cm
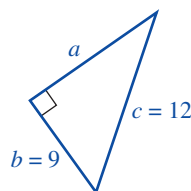
THINK	WRITE/DRAW
1 Copy the diagram and label the sides a, b and c. Remember to label the hypotenuse as c.	a $c = 12$ $b = 9$
2 Write Pythagoras' theorem for a shorter side.	$a^2 = c^2 - b^2$
3 Substitute the values of a, b and c into this rule and simplify.	$a^2 = 12^2 - 9^2$ $= 144 - 81$ $= 63$
4 Find a by taking the square root of 63.	$a = \sqrt{63}$ cm

- Practical problems may also involve you being required to find the shorter side of a right-angled triangle.

WORKED EXAMPLE 4

A ladder that is 4.5 m long leans up against a vertical wall. The foot of the ladder is 1.2 m from the wall. How far up the wall does the ladder reach? Give your answer correct to 2 decimal places.

THINK

WRITE/DRAW

1. Draw a diagram and label the sides a, b and c. Remember to label the hypotenuse as c.

$c = 4.5$ m

a

$b = 1.2$ m

2. Write Pythagoras' theorem for a shorter side.

$$a^2 = c^2 - b^2$$

3. Substitute the values of a, b and c into this rule and simplify.

$$a^2 = 4.5^2 - 1.2^2$$
$$= 20.25 - 1.44$$
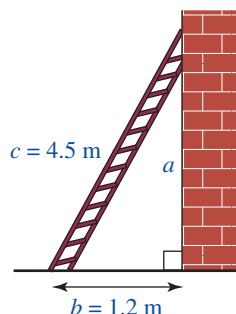$$= 18.81$$

4. Find a by taking the square root of 18.81. Round to 2 decimal places.

$$a = \sqrt{18.81}$$
$$\approx 4.34 \text{ m}$$

5. Give a written answer.

The ladder will reach a height of 4.34 m up the wall.

- Some questions will require you to decide which method is needed to solve the problem. A diagram will help you decide whether you are finding the hypotenuse or one of the shorter sides.

Exercise 15.4 Finding a shorter side

assess on

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions:	Questions:	Questions:
1–6, 8, 10, 14	1–10, 13, 14	1–15

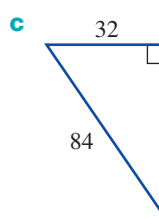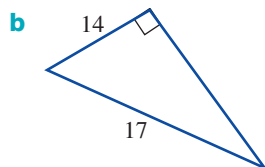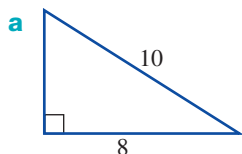■ ■ ■ **Individual pathway interactivity**   int-4471   eBook plus

REFLECTION
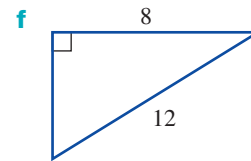What should you look out for when using Pythagoras' theorem to calculate the value of missing sides?

FLUENCY

1 **WE3** Find the length of the unmarked side in each of the following triangles. Leave your answer in exact form.

a

10

8

b 14

17

c 32

84

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Rearranging formulas
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d

382

457

e 1

4

f 8

12
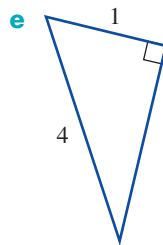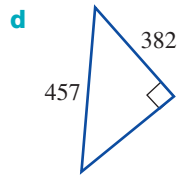
2 Find the value of the pronumeral, correct to 2 decimal places.

a a

30.1

47.2

b 1.98

2.56

b

c 8.4

c

17.52

d 0.28

d

0.67

e 2870

1920

e

f 468

f

114
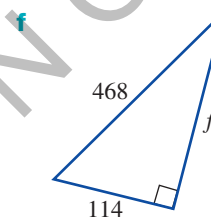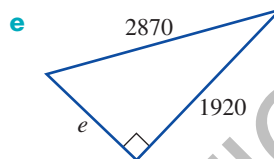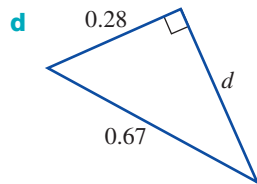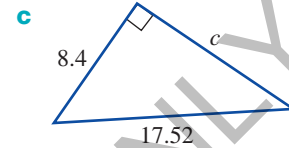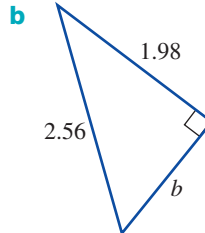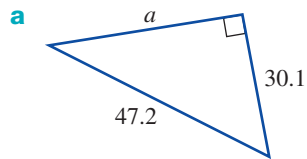
UNDERSTANDING

For questions **3** to **13**, give your answers correct to 2 decimal places.

3 The diagonal of the rectangular sign at right is 34 cm. If the height of this sign is 25 cm, find the width.

4 The diagonal of a rectangle is 120 cm. One side has a length of 70 cm. Find:

a the length of the other side

b the perimeter of the rectangle

c the area of the rectangle.

5 An equilateral triangle has sides of length 20 cm. Find the height of the triangle.

6 The roundabout sign shown at right is in the form of an equilateral triangle. Find the height of the sign and, hence, find its area.

7 **WE4** A ladder that is 7 metres long leans up against a vertical wall. The top of the ladder reaches 6.5 m up the wall. How far from the wall is the foot of the ladder?
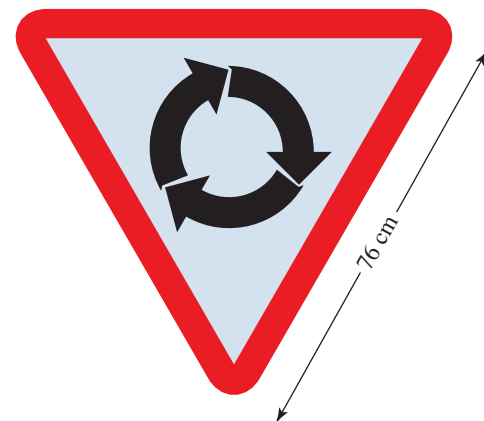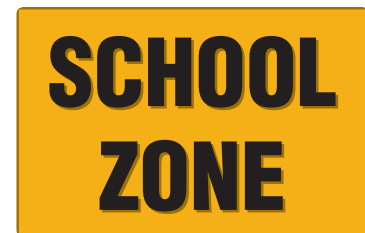
8 A tent pole that is 1.5 m high is to be supported by ropes attached to the top. Each rope is 2 m long. How far from the base of the pole can each rope be pegged?

SCHOOL ZONE

76 cm

9 Ben's dog 'Macca' has wandered onto a frozen pond, and is too frightened to walk back. Ben estimates that the dog is 3.5 m from the edge of the pond. He finds a plank, 4 m long, and thinks he can use it to rescue Macca. The pond is surrounded by a bank that is 1 m high. Ben uses the plank to make a ramp for Macca to walk up. Will he be able to rescue his dog?

REASONING

10 A kite is attached to a string 15 m long. Sam holds the end of the string 1 m above the ground, and the horizontal distance of the kite from Sam is 8 m as shown at right. How far above the ground is the kite? Show your working.

11 Penny, the carpenter, is building a roof for a new house. The roof has a gable end in the form of an isosceles triangle, with a base of 6 m and sloping sides of 7.5 m. She decides to put 5 evenly spaced vertical strips of wood as decoration on the gable as shown below. How many metres of this decorative wood does she need? Show your working.

7.5 m 7.5 m

6 m
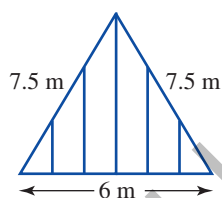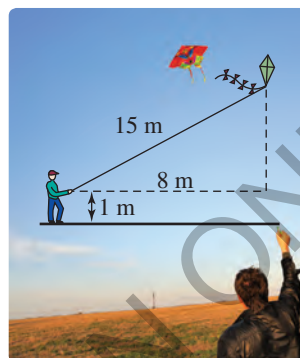
12 The size of a rectangular television screen is given by the length of its diagonal. The television below has a 92-cm screen. What is the length of the shorter side? Show your working.

80 cm

PROBLEM SOLVING

13 An art student is trying to hang her newest painting on an existing hook in a wall. She leans a 1.2-m ladder against the wall so that the distance between the foot of the ladder and the wall is 80 cm.

   a Draw a sketch showing the ladder leaning on the wall.

   b How far up the wall does the ladder reach?

   c The student climbs the ladder to check whether she can reach the hook from the step at the very top of the ladder. Once she extends her arm, the distance from her feet to her fingertips is 1.7 m. If the hook is 2.5 m above the floor, will the student reach it from the top step?

14 a A rectangle has two sides of length 3 cm and diagonals of length 5 cm. Calculate the length of the other two sides.

b It is known that the diagonals of this rectangle bisect each other at the point of intersection. Using this fact, check (using appropriate calculations) whether the diagonals are perpendicular to each other.

15.5 Working with different units

- When we use Pythagoras' theorem, we are usually working with a practical situation where measurements have been given. In any calculation, it is essential that all of the measurements are in the same units (for example, cm).
- Do you remember the relationship between the units of length? Here is a reminder:

10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km

To change to a larger unit, divide by the conversion factor.
To change to a smaller unit, multiply by the conversion factor.

For example, to change kilometres to metres, multiply by 1000.
To change centimetres to metres, divide by 100.
The following chart shows all the conversions of length that you are likely to need.

×1000   ×100   ×10

kilometres (km)   metres (m)   centimetres (cm)   millimetres (mm)

÷1000   ÷100   ÷10
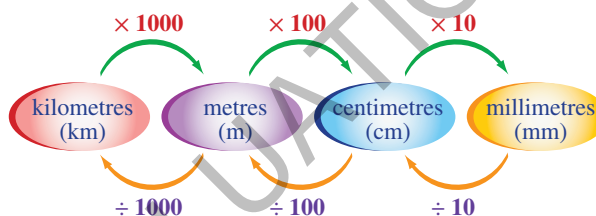
- When using Pythagoras' theorem, always check the units given for each measurement. If necessary, convert all measurements to the same units before using the rule.

WORKED EXAMPLE 5

Find the length, in mm, of the hypotenuse of a right-angled triangle if the 2 shorter sides are 7 cm and 12 cm. Give your answer correct to 1 decimal place.

THINK

1. Draw a diagram and label the sides a, b and c. Remember to label the hypotenuse as c.
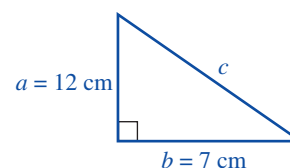
2. Check that all measurements are in the same units. They are.

3. Write Pythagoras' theorem for the hypotenuse.

4. Substitute the values of a and b into this rule and simplify.

WRITE/DRAW

$a = 12$ cm, c, $b = 7$ cm

$c^2 = a^2 + b^2$

$c^2 = 12^2 + 7^2$
$= 144 + 49$
$= 193$

5 Find c by taking the square root. Give the units in the answer.

$c = \sqrt{193}$

≈ 13.89 cm

6 Check the units required in the answer and convert if necessary.

$= 13.89 \times 10$ mm

$= 138.9$ mm

WORKED EXAMPLE 6

The hypotenuse and one other side of a right-angled triangle are 450 cm and 3.4 m respectively. Find the length of the third side, in cm, correct to the nearest whole number.

THINK

1 Draw a diagram and label the sides a, b and c. Remember to label the hypotenuse as c.

2 Check that all measurements are in the same units. They are different, so convert 3.4 m into cm.

3 Write Pythagoras' theorem for a shorter side.

4 Substitute the values of b and c into this rule and simplify.

5 Find a by taking the square root of 86 900. Round to the nearest whole number.

6 Give a worded answer.

WRITE/DRAW
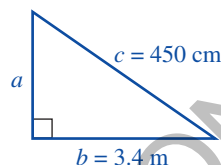
$3.4 \text{ m} = 3.4 \times 100 \text{ cm}$
$= 340 \text{ cm}$

$a^2 = c^2 - b^2$

$a^2 = 450^2 - 340^2$
$= 202\,500 - 115\,600$
$= 86\,900$

$a = \sqrt{86\,900}$
≈ 295

The third side will be approximately 295 cm long.

Exercise 15.5 Working with different units

assess on

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions:	Questions:	Questions:
1–8, 10, 11, 14, 17	1, 2a–e, 3a–e, 4a–d, 5–12, 14, 17	1, 2e–h, 3e–h, 4c–f, 5–18

REFLECTION
How do you remember whether to multiply or divide when converting units?

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FLUENCY

Where appropriate, give answers correct to 2 decimal places.

1 **WE5** Find the length, in mm, of the hypotenuse of a right-angled triangle, if the two shorter sides are 5 cm and 12 cm.

2 Find the length of the hypotenuse of the following right-angled triangles, giving the answer in the units specified.

   a Sides 456 mm and 320 mm, hypotenuse in cm.

   b Sides 12.4 mm and 2.7 cm, hypotenuse in mm.

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Digital doc
SkillSHEET
Converting units of length
doc-11430

    c Sides 32 m and 4750 cm, hypotenuse in m.

    d Sides 2590 mm and 1.7 m, hypotenuse in mm.

    e Sides 604 cm and 249 cm, hypotenuse in m.

    f Sides 4.06 km and 4060 m, hypotenuse in km.

    g Sides 8364 mm and 577 cm, hypotenuse in m.

    h Sides 1.5 km and 2780 m, hypotenuse in km.

3 `WE6` The hypotenuse and one other side of a right-angled triangle are given for each case below. Find the length of the third side in the units specified.

    a Sides 46 cm and 25 cm, third side in mm.

    b Sides 843 mm and 1047 mm, third side in cm.

    c Sides 4500 m and 3850 m, third side in km.

    d Sides 20.3 cm and 123 mm, third side in cm.

    e Sides 6420 mm and 8.4 m, third side in cm.

    f Sides 0.358 km and 2640 m, third side in m.

    g Sides 491 mm and 10.8 cm, third side in mm.

    h Sides 379 000 m and 82 700 m, third side in km.

4 Two sides of a right-angled triangle are given. Find the third side in the units specified. The diagram shows how each triangle is to be labelled. *Remember: c is always the hypotenuse.*

    a $a = 37$ cm, $c = 180$ cm, find b in cm.

    b $a = 856$ mm, $b = 1200$ mm, find c in cm.

    c $b = 4950$ m, $c = 5.6$ km, find a in km.

    d $a = 125\,600$ mm, $c = 450$ m, find b in m.

    e $a = 0.0641$ km, $b = 0.153$ km, find c in m.

    f $a = 639\,700$ cm, $b = 2.34$ km, find c in m.

5 `MC` *Note*: There may be more than one correct answer.

    a What is the length of the hypotenuse in this triangle?

        A 25 cm   **B** 50 cm

   **C** 50 mm   **D** 500 mm

    b What is the length of the third side in this triangle?

        A 0.698 m   **B** 69.8 cm

        C 6.98 cm   **D** 69.8 mm

    c The most accurate measure for the length of the third side in the triangle at right is:

        A 4.83 m   **B** 23.3 cm

        C 2330 mm   **D** 4826 mm

    d What is the length of the third side in this triangle?

        A 34.71 m   **B** 5.89 m

        C 1722 cm   **D** 4.4 m

6 A rectangle measures 35 mm by 4.2 cm. Find the length of the diagonal in mm.
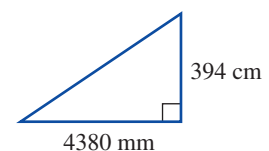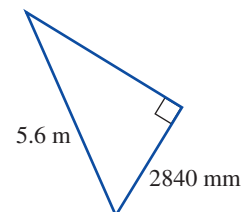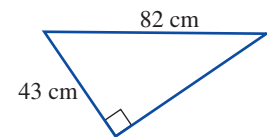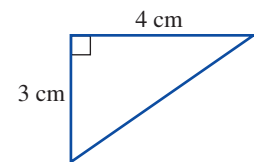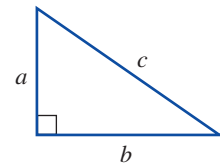
7 A sheet of A4 paper measures 210 mm by 297 mm. Find the length of the diagonal in centimetres.

8 A rectangular envelope has a length of 21 cm and a diagonal measuring 350 mm. Find:

a the width of the envelope   **b** the area of the envelope.

UNDERSTANDING

9 A right-angled triangle has a hypotenuse of 47.3 cm and one other side of 0.308 m. Find the area of the triangle.

10 A horse race is 1.2 km. The track is straight, and 35 m wide. How much further than 1.2 km will a horse run if it starts on the outside and finishes on the inside as shown?
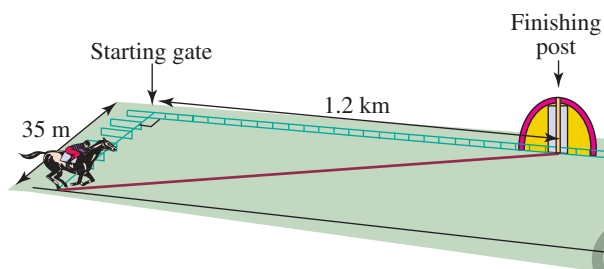
11 A ramp is 9 metres long, and rises to a height of 250 cm. What is the horizontal distance, in metres, between the bottom and the top of the ramp?

12 Sarah is making a gate, which has to be 1200 mm wide. It must be braced with a diagonal strut made of a different type of timber. She has only 2 m of this kind of timber available. What is the maximum height of the gate that she can make?

REASONING

13 A rectangular park is 260 m by 480 m. Danny usually trains by running around the edge of the park. After heavy rain, two adjacent sides are too muddy to run along, so he runs a triangular path along the other two sides and the diagonal. Danny does 5 circuits of this path for training. How far does he run? Give your answer in km.

14 A swimming pool is 50 m by 25 m. Peter is bored by his usual training routine, and decides to swim the diagonal of the pool. How many diagonals must he swim to complete his normal distance of 1.2 km?

15 A hiker walks 4.5 km west, then 3.8 km south. How far in metres is she from her starting point?

16 A square has a diagonal of 10 cm. What is the length of each side in mm?

PROBLEM SOLVING

17 The triangular sides of a wheat field measure 2.4 km, 2500 m and 700 000 mm. Is the triangle right-angled? Show your working.

18 The sides of a rectangle measure a cm by $5a$ mm. The diagonal measures $\dfrac{b}{10}$ m. Find the relationship between a and b.

15.6 Composite shapes

• In all of the exercises so far, we have been working with only one right-angled triangle. Many situations involve more complex diagrams, where the right-angled triangle is not as obvious. A neat diagram is essential for these questions.

WORKED EXAMPLE 7

Find the length of the side, x. Give your answer correct to 2 decimal places.

8 cm
x
4 cm
10 cm
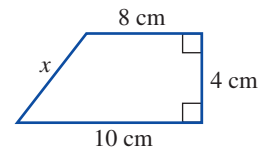
THINK

1. Copy the diagram. On the diagram, create a right-angled triangle and use the given measurements to work out the lengths of 2 sides.

2. Label the sides of your right-angled triangle as a, b and c. Remember to label the hypotenuse as c.

3. Check that all measurements are in the same units. They are the same.

4. Write Pythagoras' theorem for the hypotenuse.

5. Substitute the values of a, b and c into this rule and simplify.

6. Find x by taking the square root of 20. Round your answer correct to 2 decimal places.
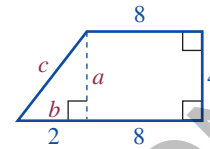
WRITE/DRAW

8
c a 4
b
2   8

$c^2 = a^2 + b^2$

$x^2 = 4^2 + 2^2$
$\quad = 16 + 4$
$\quad = 20$

$x = \sqrt{20}$
$\quad \approx 4.47 \text{ cm}$

- Some situations will involve shapes that contain more than one triangle. In this case it is a good idea to split the diagram into separate right-angled triangles first.

WORKED EXAMPLE 8

For the diagram at right, find the length of the sides marked x and y to 2 decimal places.

15   6 x
2
y
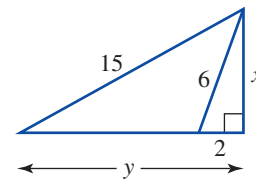
THINK

1. Copy the diagram.

2. Find and draw any right-angled triangles contained in the diagram. Label their sides.

3. To find an unknown side in a right-angled triangle, we need to know 2 sides, so find x first.

WRITE/DRAW

15   c a x
b
y

6   a
c x
b
2
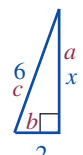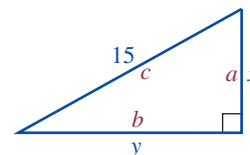
4 For the triangle containing x, write down Pythagoras' theorem for a shorter side. Use it to find x.

$$a^2 = c^2 - b^2$$
$$x^2 = 6^2 - 2^2$$
$$= 36 - 4$$
$$= 32$$
$$x = \sqrt{32}$$
$$\approx 5.66$$

5 We now know 2 sides for the other triangle because we can substitute $x = 5.66$.

6 For the triangle containing y, write down Pythagoras' theorem for a shorter side and use it to find y.

$$b^2 = c^2 - a^2$$
$$y^2 = 15^2 - 5.66^2$$
$$= 225 - 32$$
$$= 193$$
$$y = \sqrt{193}$$
$$\approx 13.89$$

Exercise 15.6 Composite shapes

assess on

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions:	Questions:	Questions:
1–8, 13	1–10, 13, 14	1–15

REFLECTION
What do you look for when dividing composite shapes?

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Where appropriate, give answers correct to 2 decimal places.

FLUENCY

1 **WE7** Find the length of the side x in the figure at right.
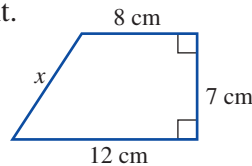
2 For the following diagrams, find the length of the sides marked x.

a

b

c
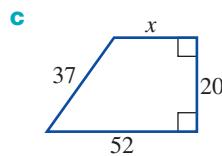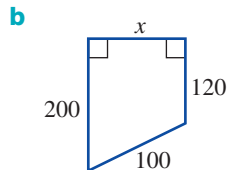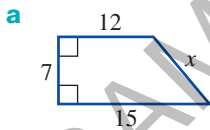
3 **WE8** For each of the following diagrams, find the length of the sides marked x and y.

a

b

c

d
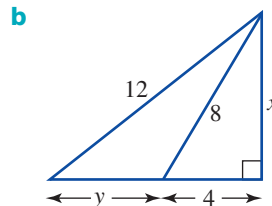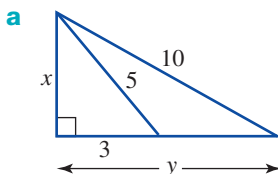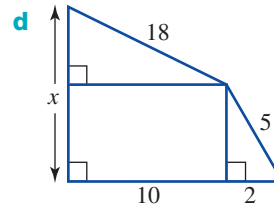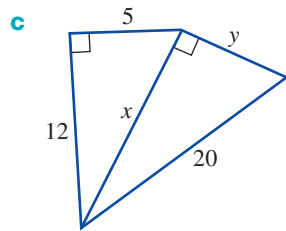
4 **MC** **a** The length of the diagonal of the rectangle at right is:

A 9.7 **B** 19

C 13.9 **D** 12.2

b The area of the rectangle at right is:

A 12.1 **B** 84.9

C 98 **D** 169.7

c The value of x in this shape is:

A 24 **B** 26

C 38.4 **D** 10

d What is the value of x in this figure?

A 5.4 **B** 8.6

C 10.1 **D** 10.3
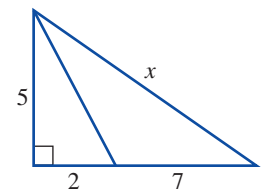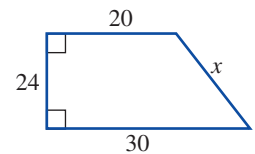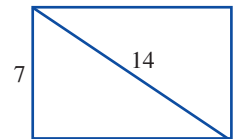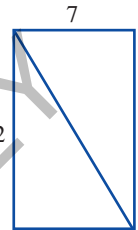
UNDERSTANDING

5 A garden bed is in the shape of a right-angled trapezium with the sloping edge 2.0 m, and parallel sides of 3.2 m and 4.8 m. Find the width of the garden and, hence, the area.

6 Two buildings, 10 and 18 m high, are directly opposite each other on either side of a European street that is 6 m wide (as shown in the following figure). What is the distance between the top of the two buildings?

7 Jess paddles a canoe 1700 m to the west, then 450 m south, and then 900 m to the east. She then stops for a rest. How far is she from her starting point?

8 A yacht race starts and finishes at A and consists of 6 legs; AB, BC, CA, AE, EC, CA, in that order as shown in the figure at right. If AB = 4 km, BC = 3 km and CE = 3 km, find:

a AC

b AE

c the total length of the race.
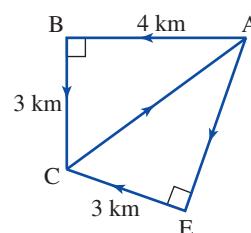
9 A painter uses a trestle to stand on in order to paint a ceiling. It consists of 2 stepladders connected by a 4 m long plank. The inner feet of the 2 stepladders are 3 m apart, and each ladder has sloping sides of 2.5 m. How high off the ground is the plank?
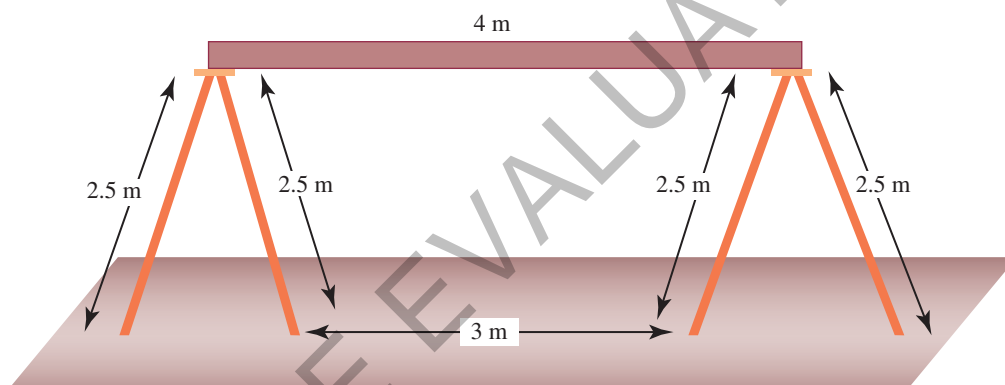
10 A feature wall in a garden is in the shape of a trapezium, with parallel sides of 6.5 m and 4.7 m. The wall is 3.2 m high. It is to have fairy lights around the perimeter (except for the base). How many metres of lighting are required?
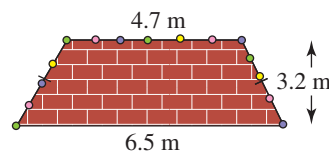
REASONING

11 A rectangular gate is 3.2 m long and 1.6 m high, and consists of three horizontal beams, and five vertical beams as shown in the diagram at right. Each section is braced with diagonals. How much timber is needed for the gate?

12 Find the distance AB in the following plan of the paddock. Distances are in metres.

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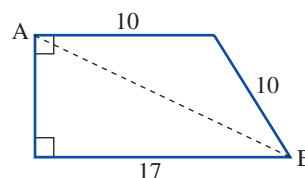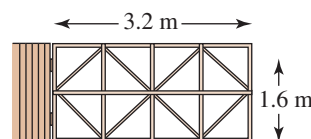
PROBLEM SOLVING

13 The front door shown is 1 m wide and 2.2 m high and has four identical glass panels, each 76 cm long and 15 cm wide.

   a Calculate the total area of the glass panels.

   b The door is to be painted inside and outside. Calculate the total area to be painted.

   c Two coats of paint are needed on each side of the door. If the paint is sold in 1-L tins at $24.95 per litre and each litre covers 8 m^2 of the surface, calculate the total cost of painting the door.
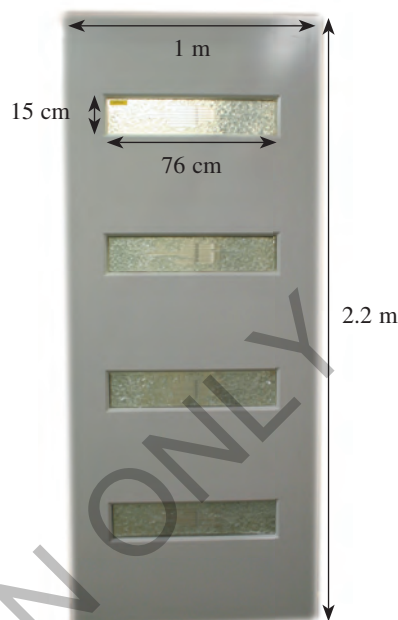
14 The leadlight panel shown depicts a sunrise over the mountains. The mountain is represented by a green triangle 45 cm high. The yellow sun is represented by a section of a circle with an 18-cm radius. There are 10 yellow sunrays in the shape of isosceles triangles with a base of 3 cm and a height of 12 cm. The sky is blue. Calculate the area of the leadlight panel made of:

   a green glass

   b yellow glass

   c blue glass

15 On each of the four sides of a square an equilateral triangle is drawn, the final figure resembling a four pointed star. If the sides of the square are $\dfrac{a}{b}$ cm long, what is the area of the complete figure?
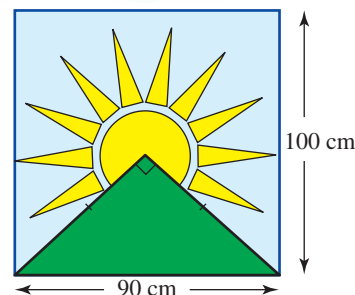
15.7 Pythagorean triads

• Pythagorean triads, or triads for short, are 3 integers (whole numbers) that satisfy Pythagoras' theorem. This means that the 3 numbers could be the sides of a right-angled triangle.

• We test Pythagorean triads by first calculating c^2, then calculating $a^2 + b^2$. If both calculations yield the same result, the original 3 numbers form a Pythagorean triad.

• The numbers 3, 4 and 5 form a Pythagorean triad because $3^2 + 4^2 = 25$ and $5^2 = 25$. If we multiply each of these 3 numbers by another number, the resulting numbers will also form a Pythagorean triad. For example, if we multiply 3, 4 and 5 by 5, we get 15, 20 and 25. These 3 numbers satisfy Pythagoras' theorem.

Check: $c^2 = 25^2 = 625$

           $a^2 + b^2 = 15^2 + 20^2 = 625$

so $c^2 = a^2 + b^2$

• Algebra can be used to find sets of numbers that are Pythagorean triads. There are two ways in which this can be done.

Method 1

• Start with any odd number, and make this the shortest length of the triangle.

- Use the formula $M = \dfrac{s^2 - 1}{2}$ where $S =$ shortest side and $M =$ middle side, to calculate the middle side of the triangle.
- You now have two sides of the triangle and can use $c^2 = a^2 + b^2$ to calculate the third side (hypotenuse, c).

WORKED EXAMPLE 9

If the smallest number of a Pythagorean triad is 7, find the middle number and, hence, find the third number.

THINK	WRITE
1 Write down the given information, and the formula to find the middle side, M.	$S = 7$ $$M = \frac{S^2 - 1}{2}$$
2 Substitute $S = 7$ into the formula and evaluate.	$$= \frac{7^2 - 1}{2}$$ $$= \frac{49 - 1}{2}$$ $$= \frac{48}{2}$$ $$= 24$$ The middle number is 24.
3 Use Pythagoras' theorem to find the third number.	$c^2 = a^2 + b^2$ $\quad = 7^2 + 24^2$ $\quad = 49 + 576$ $\quad = 625$ $c = \sqrt{625}$ $\quad = 25$ The third number is 25.
4 State the solution.	The Pythagorean triad required is 7, 24, 25.

- Why do you think this rule works only when the smallest side is an odd number?

Method 2

- This method takes any 2 numbers, and applies a set of rules to them, in order to generate Pythagorean triads. Select 2 numbers, x and y, with the following rules:
  Rule 1 The number chosen for x must be larger than y; that is $x > y$.
  Rule 2 One number must be odd and the other even, to find the first triad.
  Rule 3 The numbers chosen for x and y must have no common factors. For example, we could not choose 6 and 9 since they have a common factor of 3.
- The Pythagorean triad is given by $2xy$, $x^2 - y^2$, $x^2 + y^2$. Additional triads can be found by multiplying each of these side lengths by a common multiple.

WORKED EXAMPLE 10

Find a Pythagorean triad using the values $x = 7$ and $y = 2$.

THINK	WRITE
1 Write down the values given in the question.	$x = 7, y = 2$
2 Substitute these values into the expression for each term.	$2xy = 2 \times 7 \times 2$ $\quad = 28$ $x^2 - y^2 = 7^2 - 2^2$ $\quad = 49 - 4$ $\quad = 45$ $x^2 + y^2 = 7^2 + 2^2$ $\quad = 49 + 4$ $\quad = 53$
3 Check that the numbers obtained satisfy Pythagoras' theorem. Does $c^2 = a^2 + b^2$?	$c^2 = 53^2$ $\quad = 2809$ $a^2 + b^2 = 28^2 + 45^2$ $\quad = 784 + 2025$ $\quad = 2809$ Therefore, $c^2 = a^2 + b^2$.
4 State the solution.	The Pythagorean triad required is 28, 45, 53.

assess on

Exercise 15.7 Pythagorean triads

INDIVIDUAL PATHWAYS

REFLECTION
What strategies could you use to memorise as many common triads as possible?

■ PRACTISE Questions: 1–9	■ CONSOLIDATE Questions: 1–10, 12	■ MASTER Questions: 1–13

■ ■ ■ **Individual pathway interactivity**   int-4474   eBook *plus*

eBook *plus*

Interactivity
Pythagorean triples
int-2765

FLUENCY

1 Use Pythagoras' theorem to decide which of these triangles are right-angled.
   a 6, 8, 10   **b** 5, 12, 13   **c** 4, 5, 6
   d 24, 7, 25   **e** 16, 20, 12   **f** 14, 16, 30

2 Use the triads below to create three other triads, and check that they satisfy Pythagoras' theorem.
   a 3, 4, 5   **b** 5, 12, 13

3 **WE9** The smallest numbers of four Pythagorean triads are given below. Find the middle number and, hence, find the third number.
   a 9   **b** 11   **c** 13   **d** 29

4 What do you notice about the triads in question **3**?

5 **WE10** For each of the following, find a Pythagorean triad using the given values of x and y. (In each case check that the 3 numbers found satisfy Pythagoras' theorem.)
   a $x = 6$ and $y = 1$ **b** $x = 7$ and $y = 4$ **c** $x = 8$ and $y = 3$
   d $x = 11$ and $y = 6$ **e** $x = 14$ and $y = 9$ **f** $x = 15$ and $y = 2$

UNDERSTANDING

6 Are the following triangles right-angled? Use Pythagoras' theorem to check.

a
10, 12, 14

b
11, 7, 13
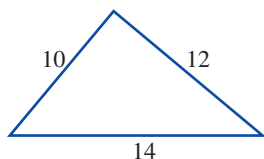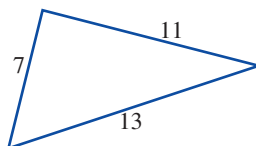
7 Use Pythagoras' theorem to prove that the following triangle is not right-angled.

Triangle A, sides 6.5, 7, base B 8 C
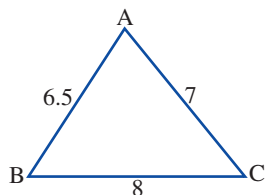
8 a Does the method used in question **5** work if both x and y are even?
  b Does it work if both x and y are odd?
  Try some examples of your own.

REASONING

9 If 32, x, 68 is a Pythagorean triad, what is the value of x? Show your working.
10 Find a Pythagorean triad in which the smallest number is 33. Explain how you reached the solution.

PROBLEM SOLVING

11 Prove that $(a^2 - b^2)$, $2ab$, $(a^2 + b^2)$ form a Pythagorean triad.
12 The lengths of the sides of a particular triangle are a cm, $2a$ cm and $3a$ cm. Prove that the triad formed is not a Pythagorean triad.
13 a If $(p - q)$, p, $(p + q)$ form a Pythagorean triad, find a relationship between p and q.
  b If $p = 8$, find the Pythagorean triad.

eBook_plus_

Digital doc
WorkSHEET 15.2
doc-11433

CHALLENGE 15.2
In a right-angled triangle the smallest angle is two-thirds the size of the other angle, which is not a right angle. What is the size of the smallest angle?

15.8 Pythagoras in 3-D

• Many real-life situations involve 3-dimensional (3-D) shapes: shapes with length, width and height. Some common 3-D shapes used in this section include boxes and pyramids.
• The important thing about 3-D shapes is that in a diagram, right angles may not look like right angles, so it is important to redraw sections of the diagram in 2 dimensions, where the right angles can be seen accurately.
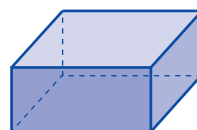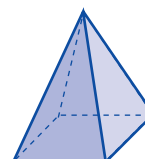
Box   Pyramid

WORKED EXAMPLE 11

Find the length AG in this box.

THINK

WRITE/DRAW
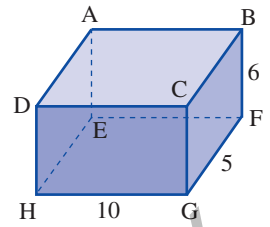
1 Draw the diagram in 3-D.

2 Draw, in 2-D, a right-angled triangle that contains AG and label the sides. Only 1 side is known, so we need to find another right-angled triangle to use.

3 Draw EFGH in 2-D and show the diagonal EG. Label the side EG as x.

4 Use Pythagoras' theorem to find EG.

$$c^2 = a^2 + b^2$$
$$x^2 = 5^2 + 10^2$$
$$= 25 + 100$$
$$= 125$$
$$x = \sqrt{125}$$
$$\approx 11.18$$

5 Place this information on triangle AEG. Label the side AG as y.

6 Solve this triangle for AG.
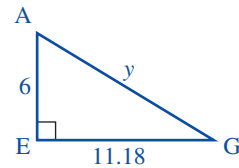
$$c^2 = a^2 + b^2$$
$$y^2 = 6^2 + 11.18^2$$
$$= 36 + 125$$
$$= 161$$
$$y = \sqrt{161}$$
$$\approx 12.69$$

7 Write your answer.

The length of AG is 12.69.

Exercise 15.8 Pythagoras in 3-D

assess on

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions:	Questions:	Questions:
1–6, 10	1–8, 10	1–11

■ ■ ■ **Individual pathway interactivity** int-4475 eBook*plus*

Where appropriate, give answers correct to 2 decimal places.

FLUENCY

1 **WE11** Find **i** length EG and hence **ii** length AG.

a

b

c
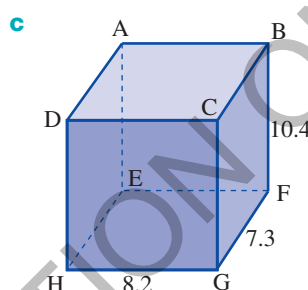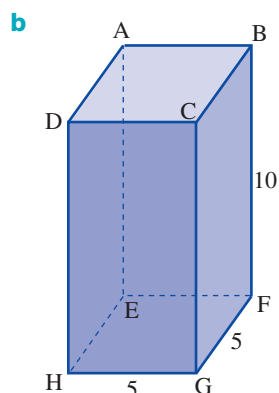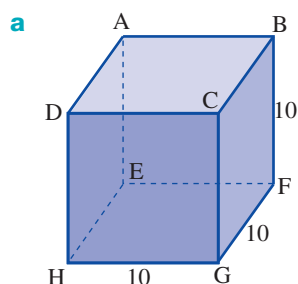
2 Find BD and, hence, the height of the pyramid at right.
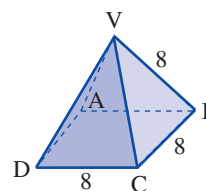
3 The sloping side of a cone is 10 cm and the height is 8 cm. Find the radius, r, of the base.
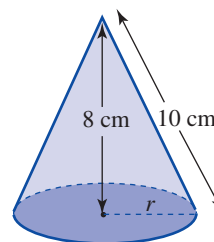
4 An ice-cream cone has a diameter across the top of 6 cm, and sloping side of 13 cm. How deep is the cone?
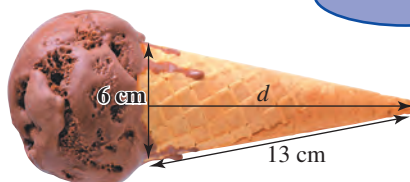
5 Jodie travels to Bolivia, taking with her a suitcase as shown in the photograph. She buys a carved walking stick 1.2 m long. Will she be able to fit it in her suitcase for the flight home?

UNDERSTANDING

6 A desk tidy is shaped like a cylinder, height 18 cm and diameter 10 cm as shown. A pencil that is 24 cm long rests inside. What length of the pencil is above the top of the cylinder?

18 cm

←10 cm→
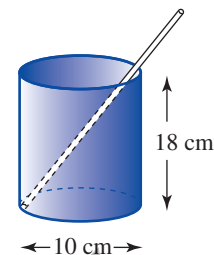
7 A 10 m high flagpole is in the corner of a rectangular park that measures 240 m by 150 m.

10 m

240 m A

150 m

B

  a Find:

    i the length of the diagonal of the park

    ii the distance from A to the top of the pole

    iii the distance from B to the top of the pole.

  b A bird flies from the top of the pole to the centre of the park. How far does it fly?
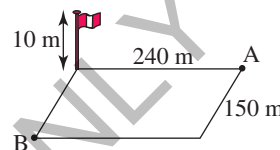
REASONING

8 The total height of the shape at right is 15 cm. Find the length of the sloping side of the pyramid.

15 cm

6 cm

14 cm

14 cm
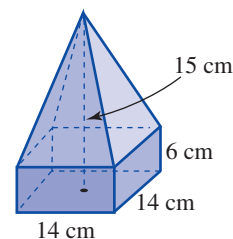
9 A tent is in the shape of a triangular prism, with a height of 120 cm as shown at right. The width across the base of the door is 1 m, and the tent is 2.3 m long.

    Find the length of each sloping side, in metres. Then find the area of fabric used in the construction of the sloping rectangles which form the sides.

120 cm

2.3 m

←1 m→
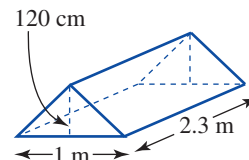
PROBLEM SOLVING

10 A wooden plank of the greatest possible length is placed inside a garden shed. Use the diagram to calculate:

  a the length of the diagonal of the floor of the shed

  b the length of the plank of wood to 1 decimal place.

1.8 m

R

90 cm

2.4 m

11 A square-based pyramid has a height of h m and a base side length of b m.

  a Find the length of the slant height in terms of h and b.

  b If the volume of a pyramid is $\frac{1}{3}$ of the volume of the prism into which it snugly fits, find the volume of the pyramid in terms of h and b.

SAMPLE EVALUATION ONLY
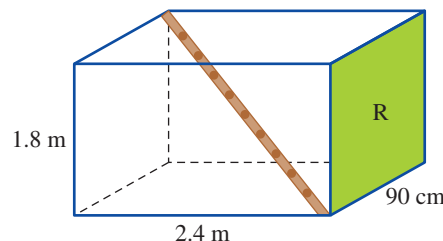
eBook*plus* ONLINE ONLY

15.9 Review

www.jacplus.com.au

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:
- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

Download the Review questions document from the links found in your eBookPLUS.

eBook*plus*

Interactivities
Word search
int-3385

Crossword
int-3386

Sudoku
int-3194

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

3 dimensions	Pythagoras' theorem	right-angled triangle
composite shape	Pythagorean triads	solve
hypotenuse	right angle	units

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

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RICH TASK

Are these walls at right angles?

Builders often use what is called a 'builder's square' when pegging out the foundations of a building to ensure that adjacent walls are at right angles. Even an error of just 1 or 2 degrees could mean that walls at the opposite end of a building may not end up in line. A builder's square uses the properties of Pythagoras' theorem. The diagram below shows how a builder's square is constructed. The hypotenuse side, c, acts as a brace to keep the two adjacent sides, a and b, in the correct position.
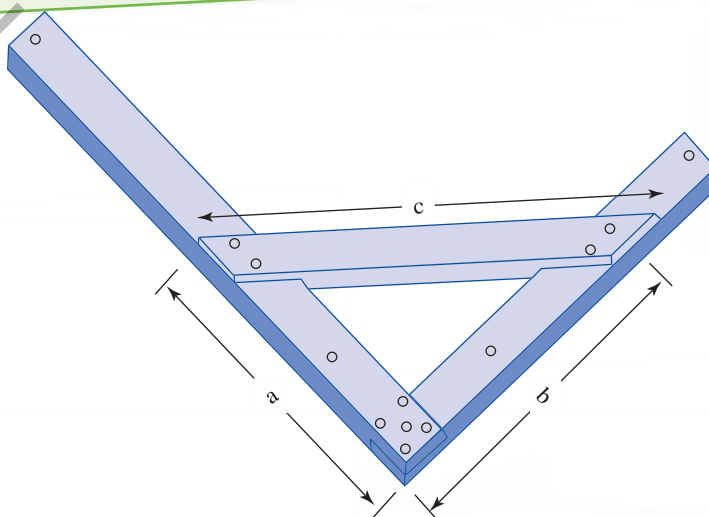
We will investigate the use of the builder's square. Cut two thin strips of paper to represent the arms a and b as shown in the diagram above. Join these two strips at one point with a pin to make the shape of the builder's square. The length of c can be obtained by measuring the distance from the end of a to the end of b. Complete questions 1 to 4, and record your results in the table on the following page.

1 Open the arms so that they make an angle of 90°. Use a protractor to measure the angle. Carefully measure the strips to obtain the values for *a*, *b* and *c* in millimetres. Record your results in the first row of the table.

2 Open the arms so that they make an angle less than 90°. Use a protractor to measure the angle. Carefully measure the strips to obtain the values for *a*, *b* and *c* in millimetres. Complete the second row of the table.

3 Repeat question **2** with the arms opened up to an angle greater than 90°. Complete the third row of the table.

4 Change the lengths of *a* and *b* by constructing a new builder's square. Repeat questions **1** to **3** and complete the last three rows of the table.

Length of a	Length of b	Length of c	$a^2 + b^2$	c^2	Angle

5 Consider the last three columns of the table. Using your results, what conclusions can you draw?

6 Construct a new builder's square and open it to an angle that you estimate to be 90°. Take measurements of *a*, *b* and *c* and record them below. What conclusions would you draw regarding your angle estimate? Check by measuring your angle with a protractor.

7 From your investigations, write a paragraph outlining how the measurements of the lengths of *a*, *b* and *c* on a builder's square can be used by the builder to determine whether the angle between adjacent walls would be equal to, greater than or less than a right angle.

CODE PUZZLE

What is Bagheera in *The Jungle Book*?

Calculate the lengths of the lettered sides in the triangles. Place the lengths with their letters above them, in ascending order in the boxes below. The letters will spell out the puzzle's answer.

Give lengths accurate to 2 decimal places.

6 m
R
9 m

130 cm
B
1 m

A
155 cm
180 cm

6 m 7 m
T

1.2 m
L
1.5 m

E
520 cm
170 cm

A
2 m
4 m

6.31 m
7.2 m
N

C
6.2 m
5.8 m

13 cm
12.98 cm
A

H
3 m
567 cm

4 m 3 m
P

K
578 cm
621 cm
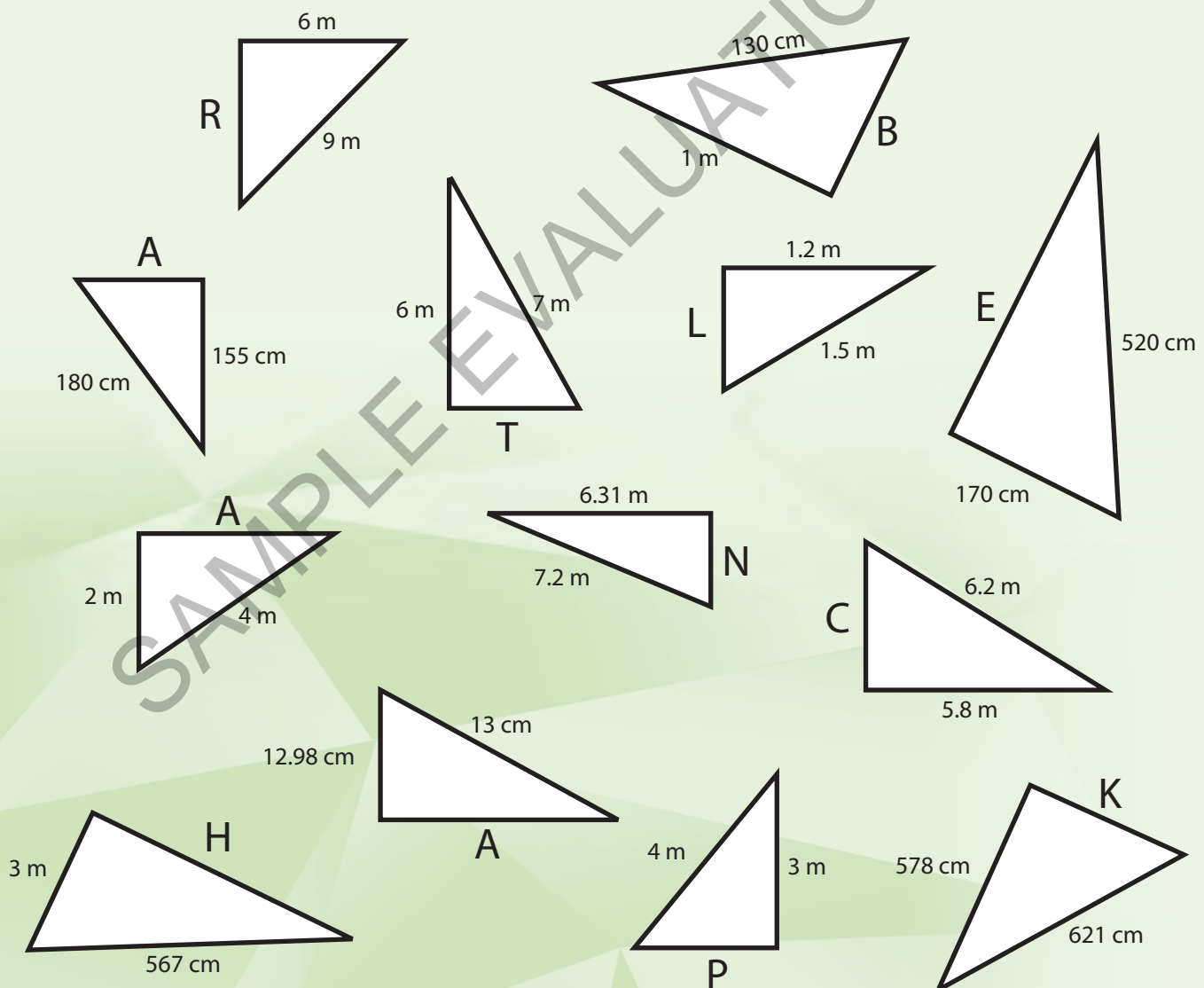
eBook*plus* Activities

15.2 Right-angled triangles
Digital docs
- SkillSHEET (doc-11425) Measuring lengths and angles
- SkillSHEET (doc-11426) Finding the square of a number

Interactivity
- IP interactivity 15.2 (int-4469) Right-angled triangles

15.3 Finding the hypotenuse
Digital docs
- SkillSHEET (doc-11427) Finding the square root of a number
- SkillSHEET (doc-11428) Rounding to a given number of decimal places

Interactivity
- IP interactivity 15.3 (int-4470) Finding the hypotenuse

15.4 Finding a shorter side
Digital docs
- SkillSHEET (doc-11429) Rearranging formulas
- WorkSHEET 15.1 (doc-11431)

Interactivity
- IP interactivity 15.4 (int-4471) Finding a shorter side

15.5 Working with different units
Digital doc
- SkillSHEET (doc-11430) Converting units of length

Interactivity
- IP interactivity 15.5 (int-4472) Working with different units

15.6 Composite shapes
Interactivity
- IP interactivity 15.6 (int-4473) Composite shapes

15.7 Pythagorean triads
Interactivities
- Pythagorean triples (int-2765)
- IP interactivity 15.6 (int-4474) Sketching linear graphs

15.8 Pythagoras in 3-D
Interactivity
- IP interactivity 15.8 (int-4475) Pythagoras in 3-D

Digital doc
- WorkSHEET 15.2 (doc-11433)

15.9 Review
Interactivities
- Word search (int-3385)
- Crossword (int-3386)
- Sudoku (int-3194)

Digital docs
- Topic summary (doc-10766)
- Concept map (doc-10779)
- Topic review (Word doc-14950, PDF doc-14951)

To access eBookPLUS activities, log on to **www.jacplus.com.au**

Answers

TOPIC 15 Pythagoras' theorem

15.2 Right-angled triangles

1

	a	b	c	d	e	f	g	h
a	19	24	27	16	31	41	13	24
b	35	32	30	40	25	19	45	24
c	40	40	40	43	40	45	47	34
a^2	361	576	729	256	961	1681	169	576
b^2	1225	1024	900	1600	625	361	2025	576
$a^2 + b^2$	1586	1600	1629	1856	1586	2042	2194	1152
c^2	1600	1600	1600	1849	1600	2025	2209	1156

2 $a^2 + b^2 \approx c^2$

3 $a^2 + b^2 = c^2$

4 $a^2 + b^2 \neq c^2$

5 The angle sum of a triangle is 180°. If a triangle contained two right angles, the third angle would be 0°, which would mean it was a straight line rather than a triangle.

6 g The square on the hypotenuse is equal to the sum of the squares on the two other sides.

7 It is not a possible triangle because $7 + 4 < 12$.

8 90°. The right angle is 90° and the angle sum of a triangle is 180°. Therefore the sum of the other two angles must equal 90°.

9 The triangle is isosceles; the smaller angles are 45°.

10 It is not possible as all angles are 60° in an equilateral triangle.

11 a Answers will vary.

 b **i** 0.577

  ii 0.5

  iii 0.866

 c The values are the same for all the triangles.

15.3 Finding the hypotenuse

1 a $x = 5$ **b** $x = 13$ **c** $x = 25$

 d $x = 21.3$ **e** $x = 1163.3$ **f** $x = 6.3$

2 a $\sqrt{9760}$ **b** $\sqrt{11\,194}$ **c** $\sqrt{394}$

 d $\sqrt{16\,781}$ **e** 100 **f** $\sqrt{68}$

3 $\sqrt{146}$ cm

4 a 16.97 cm **b** 28.28 mm **c** 6.93 cm

5 a 12.81 cm **b** 737.83 cm **c** 17.26 cm

6 18.03 cm

7 19.23 cm

8 4.66 m

9 $81.60

10 a 189 cm³ **b** 6.95 cm

11 25 m

12 314.16 cm²

Challenge 15.1
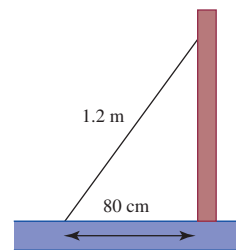
Side length $\approx$ 2.83 cm; area = 8 cm²

15.4 Finding a shorter side

1 a 6 **b** $\sqrt{93}$ **c** $\sqrt{6032}$

 d $\sqrt{62\,925}$ **e** $\sqrt{15}$ **f** $\sqrt{80}$

2 a 36.36 **b** 1.62 **c** 15.37

 d 0.61 **e** 2133.19 **f** 453.90

3 23.04 cm

4 a 97.47 cm **b** 334.94 cm **c** 6822.90 cm²

5 17.32 cm **6** 65.82 cm; 2501.16 cm²

7 2.60 m **8** 1.32 m

9 Yes **10** 13.69 m

11 20.61 m **12** 45.43 cm

13 a

 b 89.44 cm

 c Yes, she will reach the hook from the top step.

14 a 4 cm

 b The diagonals are perpendicular to each other because they create right-angled triangles.

15.5 Working with different units

1 130 mm

2 a 55.71 cm **b** 29.71 mm **c** 57.27 m

 d 3098.08 mm **e** 6.53 m **f** 5.74 km

 g 10.16 m **h** 3.16 km

3 a 386.13 mm **b** 62.09 cm **c** 2.33 km

 d 16.15 cm **e** 541.70 cm **f** 2615.61 m

 g 478.97 mm **h** 369.87 km

4 a 176.16 cm **b** 147.40 cm **c** 2.62 km

 d 432.12 m **e** 165.88 m **f** 6811.55 m

5 a C **b** A, B **c** D **d** B

6 54.67 mm **7** 36.37 cm

8 a 28 cm **b** 588 cm²

9 552.86 cm²

10 0.51 m **11** 8.65 m

12 1600 mm **13** 6.43 km

14 21.46 diagonals, so would need to complete 22

15 5889.82 m **16** 70.71 mm

17 Yes **18** $a^2 = 80b^2$

15.6 Composite shapes

1 8.06 cm

2 a $x = 7.62$ **b** $x = 60$ **c** $x = 20.87$

3 a $x = 4, y = 9.17$ **b** $x = 6.93, y = 5.80$

 c $x = 13, y = 15.20$ **d** $x = 19.55$

4 a C **b** B **c** B **d** D

5 1.2 m; 4.8 m²

6 10 m **7** 917.88 m

8 a 5 km **b** 4 km **c** 24 m

9 2.45 m **10** 11.35 m

11 26.65 m **12** 18.44 m

13 a 4560 cm² or 0.456 m²

 b 34 880 cm² or 3.488 m²

 c The paint job would need 1 tin of paint, costing $24.95. The cost of the paint used would be $21.76.

14 a 2025 cm² **b** 943.41 cm² **c** 6031.59 cm²

15 $(1 + \sqrt{3})\dfrac{a^2}{b^2}$

15.7 Pythagorean triads

1 Yes: **a**, **b**, **d**, **e** No: **c**, **f**

2 Answers will vary. Possible answers:

 a 6, 8, 10; 9, 12, 15; 12, 16, 20

 b 10, 24, 26; 15, 36, 39; 20, 48, 52

3 a 40, 41 **b** 60, 61 **c** 84, 85 **d** 420, 421

4 The two larger numbers are consecutive numbers.

5 a 12, 35, 37   **b** 33, 56, 65   **c** 48, 55, 73
   d 85, 132, 157   **e** 115, 252, 277   **f** 60, 221, 229

6 a $10^2 + 12^2 \neq 14^2$ No   **b** $7^2 + 11^2 \neq 13^2$ No

7 $6.5^2 + 7^2 = 91.25$
          $8^2 = 64$

  Therefore, the triangle is not right-angled.

8 a Yes   **b** Yes

9 60

10 The following Pythagorean triads can be found: 33, 44, 55; 33, 56, 65; 33, 180, 183; 33, 544, 545. Explanations will vary.

11, **12** Check with your teacher.

13 a $p = 4q$ **b** 6, 8, 10

Challenge 15.2
36°

15.8 Pythagoras in 3-D
1 a i 14.14   **ii** 17.32
  b i 7.07   **ii** 12.25
  c i 10.98   **ii** 15.12

2 11.31, 5.66

3 6 cm

4 12.65 cm

5 No: maximum stick can only be 115 cm long.

6 3.41 cm

7 a i 283.02 m   **ii** 240.21 m   **iii** 150.33 m
  b 141.86 m

8 13.38 cm

9 1.3 m, 5.98 m²

10 a 2.563 m   **b** 3.1 m

11 a $\sqrt{h^2 + \dfrac{b^2}{4}}$

   b $\frac{1}{3}b^2 h$

Investigation — Rich task
1 to **4** Answers will vary. Teacher to check.

5 When the angle is equal to 90°, $c^2 = a^2 + b^2$.
For an angle greater than 90°, $c^2 > a^2 + b^2$.
For an angle smaller than 90°, $c^2 < a^2 + b^2$.

6 If $c^2 = a^2 + b^2$, then the angle is equal to 90°. Check by measuring the angle.

7 Paragraph explanation of the results in question **5**; teacher to check.

Code puzzle
A black panther