In this chapter we introduce the basic concepts and laws that are fundamental to circuit analysis. These laws are Ohm’s law, Kirchhoff’s current law (KCL), and Kirchhoff’s voltage law (KVL). We cannot overemphasize the importance of these three laws because they will be used extensively throughout our entire study of circuit analysis. The reader who masters their use quickly will not only find the material in this text easy to learn, but will be well positioned to grasp subsequent topics in the field of electrical engineering.

As a general rule, most of our activities will be confined to analysis—that is, to the determination of a specific voltage, current, or power somewhere in a network. The techniques we introduce have wide application in circuit analysis, even though we will discuss them within the framework of simple networks.

Our approach here is to begin with the simplest passive element, the resistor, and the mathematical relationship that exists between the voltage across it and the current through it, as specified by Ohm’s law. As we build our confidence and proficiency by successfully analyzing some elementary circuits, we will introduce other techniques, such as voltage division and current division, that will accelerate our work.

In this chapter we introduce circuits containing dependent sources, which are used to model active devices such as transistors. Thus, our study of circuit analysis provides a natural introduction to many topics in the area of electronics.

Finally, we present a real-world application to indicate the usefulness of circuit analysis, and then we briefly introduce the topic of circuit design in an elementary fashion. In future chapters these topics will be revisited often to present some fascinating examples that describe problems we encounter in our everyday lives.
2.1 Ohm’s Law

Ohm’s law is named for the German physicist Georg Simon Ohm, who is credited with establishing the voltage–current relationship for resistance. As a result of his pioneering work, the unit of resistance bears his name.

*Ohm’s law states that the voltage across a resistance is directly proportional to the current flowing through it.* The resistance, measured in ohms, is the constant of proportionality between the voltage and current.

A circuit element whose electrical characteristic is primarily resistive is called a resistor and is represented by the symbol shown in Fig. 2.1a. A resistor is a physical device that can be purchased in certain standard values in an electronic parts store. These resistors, which find use in a variety of electrical applications, are normally carbon composition or wirewound. In addition, resistors can be fabricated using thick oxide or thin metal films for use in hybrid circuits, or they can be diffused in semiconductor integrated circuits. Some typical discrete resistors are shown in Fig. 2.1b.

The mathematical relationship of Ohm’s law is illustrated by the equation

\[ v(t) = R \times i(t), \text{ where } R \geq 0 \]  

or equivalently, by the voltage–current characteristic shown in Fig. 2.2a. Note carefully the relationship between the polarity of the voltage and the direction of the current. In addition, note that we have tacitly assumed that the resistor has a constant value and therefore that the voltage–current characteristic is linear.

The symbol \( \Omega \) is used to represent ohms, and therefore,

\[ 1 \ \Omega = 1 \ V/A \]
Although in our analysis we will always assume that the resistors are linear and are thus described by a straight-line characteristic that passes through the origin, it is important that readers realize that some very useful and practical elements do exist that exhibit a nonlinear resistance characteristic; that is, the voltage–current relationship is not a straight line.

The light bulb from the flashlight in Chapter 1 is an example of an element that exhibits a nonlinear characteristic. A typical characteristic for a light bulb is shown in Fig. 2.2b.

Since a resistor is a passive element, the proper current–voltage relationship is illustrated in Fig. 2.1a. The power supplied to the terminals is absorbed by the resistor. Note that the charge moves from the higher to the lower potential as it passes through the resistor and the energy absorbed is dissipated by the resistor in the form of heat. As indicated in Chapter 1, the rate of energy dissipation is the instantaneous power, and therefore

\[ p(t) = v(t)i(t) \]  \hspace{1cm} \text{(2.2)}

which, using Eq. (2.1), can be written as

\[ p(t) = Ri^2(t) = \frac{v^2(t)}{R} \]  \hspace{1cm} \text{(2.3)}

This equation illustrates that the power is a nonlinear function of either current or voltage and that it is always a positive quantity.

Conductance, represented by the symbol \( G \), is another quantity with wide application in circuit analysis. By definition, conductance is the reciprocal of resistance; that is,

\[ G = \frac{1}{R} \]  \hspace{1cm} \text{(2.4)}

The unit of conductance is the siemens, and the relationship between units is

\[ 1 \text{ S} = 1 \text{ A/V} \]

Using Eq. (2.4), we can write two additional expressions,

\[ i(t) = Gv(t) \]  \hspace{1cm} \text{(2.5)}

and

\[ p(t) = \frac{i^2(t)}{G} = Gv^2(t) \]  \hspace{1cm} \text{(2.6)}

Equation (2.5) is another expression of Ohm’s law.

Two specific values of resistance, and therefore conductance, are very important: \( R = 0 \) and \( R = \infty \).
In examining the two cases, consider the network in Fig. 2.3a. The variable resistance symbol is used to describe a resistor such as the volume control on a radio or television set. As the resistance is decreased and becomes smaller and smaller, we finally reach a point where the resistance is zero and the circuit is reduced to that shown in Fig. 2.3b; that is, the resistance can be replaced by a short circuit. On the other hand, if the resistance is increased and becomes larger and larger, we finally reach a point where it is essentially infinite and the resistance can be replaced by an open circuit, as shown in Fig. 2.3c. Note that in the case of a short circuit where \( R = 0 \),

\[
 v(t) = Ri(t) = 0
\]

Therefore, \( v(t) = 0 \), although the current could theoretically be any value. In the open-circuit case where \( R = \infty \),

\[
 i(t) = \frac{v(t)}{R} = 0
\]

Therefore, the current is zero regardless of the value of the voltage across the open terminals.

**Example 2.1**

In the circuit in Fig. 2.4a, determine the current and the power absorbed by the resistor.

**SOLUTION** Using Eq. (2.1), we find the current to be

\[
 I = \frac{V}{R} = \frac{12}{2k} = 6 \, mA
\]

Note that because many of the resistors employed in our analysis are in k\( \Omega \), we will use k in the equations in place of 1000. The power absorbed by the resistor is given by Eq. (2.2) or (2.3) as

\[
 P = VI = (12)(6 \times 10^{-3}) = 0.072 \, W
\]

\[
 = I^2R = (6 \times 10^{-3})^2(2k) = 0.072 \, W
\]

\[
 = \frac{V^2}{R} = \frac{(12)^2}{2k} = 0.072 \, W
\]
Example 2.2

The power absorbed by the 10-kΩ resistor in Fig. 2.4b is 3.6 mW. Determine the voltage and the current in the circuit.

**SOLUTION** Using the power relationship, we can determine either of the unknowns.

\[ \frac{V^2}{R} = P \]
\[ V_S^2 = (3.6 \times 10^{-3})(10k) \]
\[ V_S = 6 \text{ V} \]

and

\[ I^2R = P \]
\[ I^2 = (3.6 \times 10^{-3})/10k \]
\[ I = 0.6 \text{ mA} \]

Furthermore, once \( V_S \) is determined, \( I \) could be obtained by Ohm’s law, and likewise once \( I \) is known, then Ohm’s law could be used to derive the value of \( V_S \). Note carefully that the equations for power involve the terms \( I^2 \) and \( V_S^2 \). Therefore, \( I = -0.6 \text{ mA} \) and \( V_S = -6 \text{ V} \) also satisfy the mathematical equations and, in this case, the direction of both the voltage and current is reversed.

Example 2.3

Given the circuit in Fig. 2.4c, we wish to find the value of the voltage source and the power absorbed by the resistance.

**SOLUTION** The voltage is

\[ V_S = I/G = (0.5 \times 10^{-3})/(50 \times 10^{-6}) = 10 \text{ V} \]

The power absorbed is then

\[ P = I^2/G = (0.5 \times 10^{-3})^2/(50 \times 10^{-6}) = 5 \text{ mW} \]

Or we could simply note that

\[ R = 1/G = 20 \text{ kΩ} \]

and therefore

\[ V_S = IR = (0.5 \times 10^{-3})(20k) = 10 \text{ V} \]

and the power could be determined using \( P = I^2R = V_S^2/R = V_SI \).

Example 2.4

Given the network in Fig. 2.4d, we wish to find \( R \) and \( V_S \).

**SOLUTION** Using the power relationship, we find that

\[ R = P/I^2 = (80 \times 10^{-3})/(4 \times 10^{-3})^2 = 5 \text{ kΩ} \]

The voltage can now be derived using Ohm’s law as

\[ V_S = IR = (4 \times 10^{-3})(5k) = 20 \text{ V} \]

The voltage could also be obtained from the remaining power relationships in Eqs. (2.2) and (2.3).
Before leaving this initial discussion of circuits containing sources and a single resistor, it is important to note a phenomenon that we will find to be true in circuits containing many sources and resistors. The presence of a voltage source between a pair of terminals tells us precisely what the voltage is between the two terminals regardless of what is happening in the balance of the network. What we do not know is the current in the voltage source. We must apply circuit analysis to the entire network to determine this current. Likewise, the presence of a current source connected between two terminals specifies the exact value of the current through the source between the terminals. What we do not know is the value of the voltage across the current source. This value must be calculated by applying circuit analysis to the entire network. Furthermore, it is worth emphasizing that when applying Ohm’s law, the relationship \( V = IR \) specifies a relationship between the voltage directly across a resistor \( R \) and the current that is present in this resistor. Ohm’s law does not apply when the voltage is present in one part of the network and the current exists in another. This is a common mistake made by students who try to apply \( V = IR \) to a resistor \( R \) in the middle of the network while using a \( V \) at some other location in the network.

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**LEARNING EXTENSIONS**

**E2.1** Given the circuits in Fig. E2.1, find (a) the current \( I \) and the power absorbed by the resistor in Fig. E2.1a, and (b) the voltage across the current source and the power supplied by the source in Fig. E2.1b.

**ANSWER:**
- (a) \( I = 0.3 \text{ mA} \), \( P = 3.6 \text{ mW} \);
- (b) \( V_S = 3.6 \text{ V} \), \( P = 2.16 \text{ mW} \).

**Figure E2.1**

- (a) \( 12 \text{ V} \)
- (b) \( 0.6 \text{ mA} \)

**E2.2** Given the circuits in Fig. E2.2, find (a) \( R \) and \( V_S \) in the circuit in Fig. E2.2a, and (b) find \( I \) and \( R \) in the circuit in Fig. E2.2b.

**ANSWER:**
- (a) \( R = 10 \text{ k}\Omega \), \( V_S = 4 \text{ V} \);
- (b) \( I = 20.8 \text{ mA} \), \( R = 576 \text{ \Omega} \).

**Figure E2.2**

- (a) \( 0.4 \text{ mA} \)
- (b) \( 12 \text{ V} \), \( P = 1.6 \text{ mW} \)

---

### 2.2 Kirchhoff’s Laws

The previous circuits that we have considered have all contained a single resistor and were analyzed using Ohm’s law. At this point we begin to expand our capabilities to handle more complicated networks that result from an interconnection of two or more of these simple elements. We will assume that the interconnection is performed by electrical conductors (wires) that have zero resistance—that is, perfect conductors. Because the wires have zero resistance, the energy in the circuit is in essence lumped in each element, and we employ the term *lumped-parameter circuit* to describe the network.

To aid us in our discussion, we will define a number of terms that will be employed throughout our analysis. As will be our approach throughout this text, we will use examples to illustrate the concepts and define the appropriate terms. For example, the circuit shown...
in Fig. 2.5a will be used to describe the terms node, loop, and branch. A node is simply a point of connection of two or more circuit elements. The reader is cautioned to note that, although one node can be spread out with perfect conductors, it is still only one node. This is illustrated in Fig. 2.5b where the circuit has been redrawn. Node 5 consists of the entire bottom connector of the circuit.

If we start at some point in the circuit and move along perfect conductors in any direction until we encounter a circuit element, the total path we cover represents a single node. Therefore, we can assume that a node is one end of a circuit element together with all the perfect conductors that are attached to it. Examining the circuit, we note that there are numerous paths through it. A loop is simply any closed path through the circuit in which no node is encountered more than once. For example, starting from node 1, one loop would contain the elements $R_1, v_1, R_5$, and $i_1$; another loop would contain $R_2, v_1, v_2, R_4$, and $i_1$; and so on. However, the path $R_1, v_1, R_5, v_2, R_4$, and $i_1$ is not a loop because we have encountered node 3 twice. Finally, a branch is a portion of a circuit containing only a single element and the nodes at each end of the element. The circuit in Fig. 2.5 contains eight branches.

Given the previous definitions, we are now in a position to consider Kirchhoff’s laws, named after German scientist Gustav Robert Kirchhoff. These two laws are quite simple but extremely important. We will not attempt to prove them because the proofs are beyond our current level of understanding. However, we will demonstrate their usefulness and attempt to make the reader proficient in their use. The first law is Kirchhoff’s current law (KCL), which states that the algebraic sum of the currents entering any node is zero. In mathematical form the law appears as

$$\sum_{j=1}^{N} i_j(t) = 0$$

where $i_j(t)$ is the $j$th current entering the node through branch $j$ and $N$ is the number of branches connected to the node. To understand the use of this law, consider node 3 shown in Fig. 2.5. Applying Kirchhoff’s current law to this node yields

$$i_2(t) - i_4(t) + i_5(t) - i_7(t) = 0$$

We have assumed that the algebraic signs of the currents entering the node are positive and, therefore, that the signs of the currents leaving the node are negative.

If we multiply the foregoing equation by $-1$, we obtain the expression

$$-i_2(t) + i_4(t) - i_5(t) + i_7(t) = 0$$

which simply states that the algebraic sum of the currents leaving a node is zero. Alternatively, we can write the equation as

$$i_2(t) + i_5(t) = i_4(t) + i_7(t)$$

which states that the sum of the currents entering a node is equal to the sum of the currents leaving the node. Both of these italicized expressions are alternative forms of Kirchhoff’s current law.
Once again it must be emphasized that the latter statement means that the sum of the variables that have been defined entering the node is equal to the sum of the variables that have been defined leaving the node, not the actual currents. For example, \( i_j(t) \) may be defined entering the node, but if its actual value is negative, there will be positive charge leaving the node.

Note carefully that Kirchhoff’s current law states that the algebraic sum of the currents either entering or leaving a node must be zero. We now begin to see why we stated in Chapter 1 that it is critically important to specify both the magnitude and the direction of a current. Recall that current is charge in motion. Based on our background in physics, charges cannot be stored at a node. In other words, if we have a number of charges entering a node, then an equal number must be leaving that same node. Kirchhoff’s current law is based on this principle of conservation of charge.

**Example 2.5**

Let us write KCL for every node in the network in Fig. 2.5 assuming that the currents leaving the node are positive.

**SOLUTION**

The KCL equations for nodes 1 through 5 are

\[
-i_1(t) + i_3(t) \quad + \quad i_6(t) = 0
\]

\[
i_2(t) - i_4(t) + i_5(t) = 0
\]

\[
-i_5(t) + i_3(t) + i_7(t) = 0
\]

\[
-i_4(t) + i_1(t) = 0
\]

Note carefully that if we add the first four equations, we obtain the fifth equation. What does this tell us? Recall that this means that this set of equations is not linearly independent. We can show that the first four equations are, however, linearly independent. Store this idea in memory because it will become very important when we learn how to write the equations necessary to solve for all the currents and voltages in a network in the following chapter.

**Example 2.6**

The network in Fig. 2.5 is represented by the topological diagram shown in Fig. 2.6. We wish to find the unknown currents in the network.

**Figure 2.6**

Topological diagram for the circuit in Fig. 2.5.

**SOLUTION**

Assuming the currents leaving the node are positive, the KCL equations for nodes 1 through 4 are

\[
-i_1(t) + 0.06 + 0.02 = 0
\]

\[
i_1 - i_4 + i_6 = 0
\]

\[
-0.06 + i_4 - i_1 + 0.04 = 0
\]

\[
-0.02 + i_6 - 0.03 = 0
\]
The first equation yields \( I_1 \) and the last equation yields \( I_4 \). Knowing \( I_4 \), we can immediately obtain \( I_5 \) from the third equation. Then the values of \( I_1 \) and \( I_4 \) yield the value of \( I_6 \) from the second equation. The results are \( I_1 = 80 \text{ mA} \), \( I_4 = 70 \text{ mA} \), \( I_5 = 50 \text{ mA} \), and \( I_6 = -10 \text{ mA} \).

As indicated earlier, dependent or controlled sources are very important because we encounter them when analyzing circuits containing active elements such as transistors. The following example presents a circuit containing a current-controlled current source.

**Example 2.7**

Let us write the KCL equations for the circuit shown in Fig. 2.7.

**SOLUTION**

The KCL equations for nodes 1 through 4 follow.

\[
\begin{align*}
i_1(t) + i_2(t) - i_5(t) &= 0 \\
-i_2(t) + i_4(t) - 50i_2(t) &= 0 \\
-i_1(t) + 50i_2(t) + i_4(t) &= 0 \\
i_5(t) - i_3(t) - i_6(t) &= 0
\end{align*}
\]

If we added the first three equations, we would obtain the negative of the fourth. What does this tell us about the set of equations?

Finally, it is possible to generalize Kirchhoff’s current law to include a closed surface. By a closed surface we mean some set of elements completely contained within the surface that are interconnected. Since the current entering each element within the surface is equal to that leaving the element (i.e., the element stores no net charge), it follows that the current entering an interconnection of elements is equal to that leaving the interconnection. Therefore, Kirchhoff’s current law can also be stated as follows: *The algebraic sum of the currents entering any closed surface is zero.*

**Example 2.8**

Let us find \( I_5 \) and \( I_4 \) in the network represented by the topological diagram in Fig. 2.6.

**SOLUTION**

This diagram is redrawn in Fig. 2.8; node 1 is enclosed in surface 1, and nodes 3 and 4 are enclosed in surface 2. A quick review of the previous example indicates that we derived a value for \( I_5 \) from the value of \( I_4 \). However, \( I_4 \) is now completely enclosed in surface 2. If we apply KCL to surface 2, assuming the currents out of the surface are positive, we obtain

\[
I_4 - 0.06 - 0.02 - 0.03 + 0.04 = 0
\]

or

\[
I_4 = 70 \text{ mA}
\]

which we obtained without any knowledge of \( I_5 \). Likewise for surface 1, what goes in must come out and, therefore, \( I_5 = 80 \text{ mA} \). The reader is encouraged to cut the network in Fig. 2.6 into two pieces in any fashion and show that KCL is always satisfied at the boundaries.
Given the networks in Fig. E2.3, find (a) $I_1$ in Fig. E2.3a and (b) $I_T$ in Fig. E2.3b.

**Answer:**
(a) $I_1 = -50 \text{ mA}$;  
(b) $I_T = 70 \text{ mA}$.

Find (a) $I_1$ in the network in Fig. E2.4a and (b) $I_1$ and $I_2$ in the circuit in Fig. E2.4b.

**Answer:**
(a) $I_1 = 6 \text{ mA}$;  
(b) $I_1 = 8 \text{ mA}$ and $I_2 = 5 \text{ mA}$.

Find the current $i_x$ in the circuits in Fig. E2.5.

**Answer:**
(a) $i_x = 4 \text{ mA}$;  
(b) $i_x = 12 \text{ mA}$.

Kirchhoff’s second law, called Kirchhoff’s voltage law (KVL), states that the algebraic sum of the voltages around any loop is zero. As was the case with Kirchhoff’s current law, we will defer the proof of this law and concentrate on understanding how to apply it. Once again the reader is
cautioned to remember that we are dealing only with lumped-parameter circuits. These circuits are conservative, meaning that the work required to move a unit charge around any loop is zero.

In Chapter 1, we related voltage to the difference in energy levels within a circuit and talked about the energy conversion process in a flashlight. Because of this relationship between voltage and energy, Kirchhoff’s voltage law is based on the conservation of energy.

Recall that in Kirchhoff’s current law, the algebraic sign was required to keep track of whether the currents were entering or leaving a node. In Kirchhoff’s voltage law, the algebraic sign is used to keep track of the voltage polarity. In other words, as we traverse the circuit, it is necessary to sum to zero the increases and decreases in energy level. Therefore, it is important we keep track of whether the energy level is increasing or decreasing as we go through each element.

In applying KVL, we must traverse any loop in the circuit and sum to zero the increases and decreases in energy level. At this point, we have a decision to make. Do we want to consider a decrease in energy level as positive or negative? We will adopt a policy of considering a decrease in energy level as positive and an increase in energy level as negative. As we move around a loop, we encounter the plus sign first for a decrease in energy level and a negative sign first for an increase in energy level.

**Example 2.9**

Consider the circuit shown in Fig. 2.9. If \( V_{R_1} \) and \( V_{R_2} \) are known quantities, let us find \( V_{R_3} \).

![Figure 2.9 Circuit used to illustrate KVL.](image)

**SOLUTION** Starting at point \( a \) in the network and traversing it in a clockwise direction, we obtain the equation

\[
+V_{R_1} - 5 + V_{R_2} - 15 + V_{R_3} - 30 = 0
\]

which can be written as

\[
+V_{R_1} + V_{R_2} + V_{R_3} = 5 + 15 + 30 = 50
\]

Now suppose that \( V_{R_1} \) and \( V_{R_2} \) are known to be 18 V and 12 V, respectively. Then \( V_{R_3} = 20 \) V.

**Example 2.10**

Consider the network in Fig. 2.10.

![Figure 2.10 Circuit used to explain KVL.](image)

Let us demonstrate that only two of the three possible loop equations are linearly independent.
SOLUTION  Note that this network has three closed paths: the left loop, right loop, and outer loop. Applying our policy for writing KVL equations and traversing the left loop starting at point \( a \), we obtain

\[ V_R + V_{R_1} - 16 - 24 = 0 \]

The corresponding equation for the right loop starting at point \( b \) is

\[ V_{R_1} + V_{R_2} + 8 + 16 - V_R = 0 \]

The equation for the outer loop starting at point \( a \) is

\[ V_R + V_{R_1} + V_{R_2} + 8 - 24 = 0 \]

Note that if we add the first two equations, we obtain the third equation. Therefore, as we indicated in Example 2.5, the three equations are not linearly independent. Once again, we will address this issue in the next chapter and demonstrate that we need only the first two equations to solve for the voltages in the circuit.

Finally, we employ the convention \( V_{ab} \) to indicate the voltage of point \( a \) with respect to point \( b \): that is, the variable for the voltage between point \( a \) and point \( b \), with point \( a \) considered positive relative to point \( b \). Since the potential is measured between two points, it is convenient to use an arrow between the two points, with the head of the arrow located at the positive node. Note that the double-subscript notation, the \( + \) and \( - \) notation, and the single-headed arrow notation are all the same if the head of the arrow is pointing toward the positive terminal and the first subscript in the double-subscript notation. All of these equivalent forms for labeling voltages are shown in Fig. 2.11. The usefulness of the arrow notation stems from the fact that we may want to label the voltage between two points that are far apart in a network. In this case, the other notations are often confusing.

Figure 2.11  Equivalent forms for labeling voltage.

\[ V_x = V_{ab} \]

\[ V_x = V_a \]

\[ V_x = V_o \]

\[ V_x = V_{ab} - V_o \]

Example 2.11

Consider the network in Fig. 2.12a. Let us apply KVL to determine the voltage between two points. Specifically, in terms of the double-subscript notation, let us find \( V_{ae} \) and \( V_{ec} \).

Figure 2.12  Network used in Example 2.11.

SOLUTION  The circuit is redrawn in Fig. 2.12b. Since points \( a \) and \( e \) as well as \( e \) and \( c \) are not physically close, the arrow notation is very useful. Our approach to determining the unknown voltage is to apply KVL with the unknown voltage in the closed path. Therefore, to
determine $V_w$ we can use the path $aefa$ or $abcdea$. The equations for the two paths in which $V_w$ is the only unknown are:

$V_w + 10 - 24 = 0$

and

$16 - 12 + 4 + 6 - V_w = 0$

Note that both equations yield $V_w = 14$ V. Even before calculating $V_w$, we could calculate $V_e$ using the path $cdec$ or $cefabc$. However, since $V_w$ is now known, we can also use the path $ceabc$. KVL for each of these paths is:

$4 + 6 + V_e = 0$

$-V_w + 10 - 24 + 16 - 12 = 0$

$-V_w - V_e + 16 - 12 = 0$

Each of these equations yields $V_e = -10$ V.

In general, the mathematical representation of Kirchhoff’s voltage law is

$$
\sum_{j=1}^{N} v_j(t) = 0
$$

where $v_j(t)$ is the voltage across the $j$th branch (with the proper reference direction) in a loop containing $N$ voltages. This expression is analogous to Eq. (2.7) for Kirchhoff’s current law.

**Example 2.12**

Given the network in Fig. 2.13 containing a dependent source, let us write the KVL equations for the two closed paths $abda$ and $bcdb$.

**SOLUTION** The two KVL equations are:

$V_{R1} + V_{R3} - V_S = 0$

$20V_{R1} + V_{R3} - V_S = 0$

**Learning Extensions**

**E2.6** Find $V_{af}$ and $V_{eb}$ in the network in Fig. E2.6.

**Answer:** $V_{af} = 26$ V, $V_{eb} = 10$ V.
2.3 Single-Loop Circuits

**VOLTAGE DIVISION** At this point we can begin to apply the laws we have presented earlier to the analysis of simple circuits. To begin, we examine what is perhaps the simplest circuit—a single closed path, or loop, of elements.

Applying KCL to every node in a single-loop circuit reveals that the same current flows through all elements. We say that these elements are connected in series because they carry the same current. We will apply Kirchhoff’s voltage law and Ohm’s law to the circuit to determine various quantities in the circuit.

Our approach will be to begin with a simple circuit and then generalize the analysis to more complicated ones. The circuit shown in Fig. 2.15 will serve as a basis for discussion. This circuit consists of an independent voltage source that is in series with two resistors. We have assumed that the current flows in a clockwise direction. If this assumption is correct, the solution of the equations that yields the current will produce a positive value. If the current is actually flowing in the opposite direction, the value of the current variable will simply be negative, indicating that the current is flowing in a direction opposite to that assumed. We have also made voltage polarity assignments for \( v_{R_1} \) and \( v_{R_2} \). These assignments have been made using the convention employed in our discussion of Ohm’s law and our choice for the direction of \( i(t) \)—that is, the convention shown in Fig. 2.14a.
Applying Kirchhoff’s voltage law to this circuit yields

\[-v(t) + v_R + v_{R_2} = 0\]

or

\[v(t) = v_R + v_{R_2}\]

However, from Ohm’s law we know that

\[v_R = R_1i(t)\]
\[v_{R_2} = R_2i(t)\]

Therefore,

\[v(t) = R_1i(t) + R_2i(t)\]

Solving the equation for \(i(t)\) yields

\[i(t) = \frac{v(t)}{R_1 + R_2}\]  \[2.9\]

Knowing the current, we can now apply Ohm’s law to determine the voltage across each resistor:

\[v_R = R_1i(t)\]
\[= R_1\left(\frac{v(t)}{R_1 + R_2}\right)\]  \[2.10\]
\[= \frac{R_1}{R_1 + R_2}v(t)\]

Similarly,

\[v_{R_2} = \frac{R_2}{R_1 + R_2}v(t)\]  \[2.11\]

Though simple, Eqs. (2.10) and (2.11) are very important because they describe the operation of what is called a voltage divider. In other words, the source voltage \(v(t)\) is divided between the resistors \(R_1\) and \(R_2\) in direct proportion to their resistances.

In essence, if we are interested in the voltage across the resistor \(R_1\), we bypass the calculation of the current \(i(t)\) and simply multiply the input voltage \(v(t)\) by the ratio

\[\frac{R_1}{R_1 + R_2}\]

As illustrated in Eq. (2.10), we are using the current in the calculation, but not explicitly.

Note that the equations satisfy Kirchhoff’s voltage law, since

\[-v(t) + \frac{R_1}{R_1 + R_2}v(t) + \frac{R_2}{R_1 + R_2}v(t) = 0\]

**Example 2.13**

Consider the circuit shown in Fig. 2.16. The circuit is identical to Fig. 2.15 except that \(R_1\) is a variable resistor such as the volume control for a radio or television set. Suppose that \(V_S = 9\ V\), \(R_1 = 90\ k\Omega\), and \(R_2 = 30\ k\Omega\).

![Voltage-divider circuit.](image)
CHAPTER 2 RESISTIVE CIRCUITS

Let us examine the change in both the voltage across $R_2$ and the power absorbed in this resistor as $R_1$ is changed from 90 kΩ to 15 kΩ.

**SOLUTION** Since this is a voltage-divider circuit, the voltage $V_2$ can be obtained directly as

$$V_2 = \frac{R_2}{R_1 + R_2} \cdot V_S$$

$$= \left[ \frac{30k}{90k + 30k} \right] V_S$$

$$= 2.25 \text{ V}$$

Now suppose that the variable resistor is changed from 90 kΩ to 15 kΩ. Then

$$V_2 = \left[ \frac{30k}{30k + 15k} \right] V_S$$

$$= 6 \text{ V}$$

The direct voltage-divider calculation is equivalent to determining the current $I$ and then using Ohm’s law to find $V_2$. Note that the larger voltage is across the larger resistance. This voltage-divider concept and the simple circuit we have employed to describe it are very useful because, as will be shown later, more complicated circuits can be reduced to this form.

Finally, let us determine the instantaneous power absorbed by the resistor $R_2$ under the two conditions $R_1 = 90 \text{ kΩ}$ and $R_1 = 15 \text{ kΩ}$. For the case $R_1 = 90 \text{ kΩ}$, the power absorbed by $R_2$ is

$$P_2 = I^2 R_2 = \left( \frac{9}{120k} \right)^2 (30k)$$

$$= 0.169 \text{ mW}$$

In the second case

$$P_2 = \left( \frac{9}{45k} \right)^2 (30k)$$

$$= 1.2 \text{ mW}$$

The current in the first case is 75 μA, and in the second case it is 200 μA. Since the power absorbed is a function of the square of the current, the power absorbed in the two cases is quite different.

Let us now demonstrate the practical utility of this simple voltage-divider network.

**Example 2.14**

Consider the circuit in Fig. 2.17a, which is an approximation of a high-voltage dc transmission facility. We have assumed that the bottom portion of the transmission line is a perfect conductor and will justify this assumption in the next chapter. The load can be represented by a resistor of value 183.5 Ω. Therefore, the equivalent circuit of this network is shown in Fig. 2.17b.

![Diagram](a) (b)

*Figure 2.17 A high-voltage dc transmission facility.*

Let us determine both the power delivered to the load and the power losses in the line.
SOLUTION  Using voltage division, the load voltage is

\[ V_{\text{load}} = \left[ \frac{-183.5}{183.5 + 16.5} \right] 400k \]
\[ = 367 \text{ kV} \]

The input power is 800 MW and the power transmitted to the load is

\[ P_{\text{load}} = I^2 R_{\text{load}} \]
\[ = 734 \text{ MW} \]

Therefore, the power loss in the transmission line is

\[ P_{\text{line}} = P_{\text{in}} - P_{\text{load}} = I^2 R_{\text{line}} \]
\[ = 66 \text{ MW} \]

Since \( P = VI \), suppose now that the utility company supplied power at 200 kV and 4 kA. What effect would this have on our transmission network? Without making a single calculation, we know that because power is proportional to the square of the current, there would be a large increase in the power loss in the line and, therefore, the efficiency of the facility would decrease substantially. That is why, in general, we transmit power at high voltage and low current.

MULTIPLE SOURCE/RESISTOR NETWORKS  At this point we wish to extend our analysis to include a multiplicity of voltage sources and resistors. For example, consider the circuit shown in Fig. 2.18a. Here we have assumed that the current flows in a clockwise direction, and we have defined the variable \( i(t) \) accordingly. This may or may not be the case, depending on the value of the various voltage sources. Kirchhoff’s voltage law for this circuit is

\[ +v_{R_1} + v_2(t) - v_3(t) + v_4(t) + v_5(t) - v_1(t) = 0 \]

or, using Ohm’s law,

\[ (R_1 + R_2)i(t) = v_1(t) - v_2(t) + v_3(t) - v_4(t) - v_5(t) \]

which can be written as

\[ (R_1 + R_2)i(t) = v(t) \]

where

\[ v(t) = v_1(t) + v_3(t) - [v_2(t) + v_4(t) + v_5(t)] \]

so that under the preceding definitions, Fig. 2.18a is equivalent to Fig. 2.18b. In other words, the sum of several voltage sources in series can be replaced by one source whose value is the algebraic sum of the individual sources. This analysis can, of course, be generalized to a circuit with \( N \) series sources.

Figure 2.18
Equivalent circuits with multiple sources.
Now consider the circuit with \( N \) resistors in series, as shown in Fig. 2.19a. Applying Kirchhoff’s voltage law to this circuit yields
\[
v(t) = RSi(t)
\]
and therefore,
\[
v(t) = RSi(t)
\]
Thus, using Eq. (2.13), we can draw the circuit in Fig. 2.19b as an equivalent circuit for the one in Fig. 2.19a.

Equation (2.13) illustrates that the equivalent resistance of \( N \) resistors in series is simply the sum of the individual resistances. Thus, using Eq. (2.13), we can draw the circuit in Fig. 2.19b as an equivalent circuit for the one in Fig. 2.19a.

**Example 2.15**

Given the circuit in Fig. 2.20a, let us find \( I, V_{ab}, \) and the power absorbed by the 30-k\( \Omega \) resistor. Finally, let us use voltage division to find \( V_{bc}. \)

**Figure 2.19**
Equivalent circuits.

**Figure 2.20**
Circuit used in Example 2.15.
**SOLUTION**  
KVL for the network yields the equation
\[ 10kI + 20kI + 12 + 30kI - 6 = 0 \]
\[ 60kI = -6 \]
\[ I = -0.1 \text{ mA} \]

Therefore, the magnitude of the current is 0.1 mA, but its direction is opposite to that assumed.

The voltage can be calculated using either of the closed paths \( abdea \) or \( bcdb \). The equations for both cases are
\[ 10kI + V_{bd} + 30kI - 6 = 0 \]
and
\[ 20kI + 12 - V_{bd} = 0 \]

Using \( I = -0.1 \text{ mA} \) in either equation yields \( V_{bd} = 10 \text{ V} \). Finally, the power absorbed by the 30-kΩ resistor is
\[ P = I^2R = 0.3 \text{ mW} \]

Now from the standpoint of determining the voltage \( V_{bs} \), we can simply add the sources since they are in series, add the remaining resistors since they are in series, and reduce the network to that shown in Fig. 2.20b. Then
\[ V_{bs} = \frac{20}{20k + 40k} (-6) \]
\[ = -2 \text{ V} \]

---

**Example 2.16**

A dc transmission facility is modeled by the approximate circuit shown in Fig. 2.21. If the load voltage is known to be \( V_{load} = 458.3 \text{ kV} \), we wish to find the voltage at the sending end of the line and the power loss in the line.

![Figure 2.21](image-url)  
Circuit used in Example 2.16.

**SOLUTION**  
Knowing the load voltage and load resistance, we can obtain the line current using Ohm’s law:
\[ I_L = \frac{458.3k}{220} \]
\[ = 2.083 \text{ kA} \]

The voltage drop across the line is
\[ V_{line} = (I_L)(R_{line}) \]
\[ = 41.66 \text{ kV} \]

Now, using KVL,
\[ V_S = V_{line} + V_{load} \]
\[ = 500 \text{ kV} \]

Note that since the network is simply a voltage-divider circuit, we could obtain \( V_S \) immediately from our knowledge of \( R_{line}, R_{load} \) and \( V_{load} \). That is,
\[ V_{load} = \left( \frac{R_{load}}{R_{load} + R_{line}} \right) V_S \]

and \( V_S \) is the only unknown in this equation.
The power absorbed by the line is
\[ P_{\text{line}} = I_{\text{line}}^2 R_{\text{line}} = 86.79 \text{ MW} \]

**Learning Extensions**

**E2.8** Find \( I \) and \( V_{bd} \) in the circuit in Fig. E2.8.

**Answer:** \( I = -0.05 \) mA and \( V_{bd} = 10 \) V.

**E2.9** In the network in Fig. E2.9, if \( V_{ad} \) is 3 V, find \( V_s \).

**Answer:** \( V_s = 9 \) V.

### 2.4 Single-Node-Pair Circuits

**Current Division** An important circuit is the single-node-pair circuit. If we apply KVL to every loop in a single-node-pair circuit, we discover that all of the elements have the same voltage across them and, therefore, are said to be connected in parallel. We will, however, apply Kirchhoff’s current law and Ohm’s law to determine various unknown quantities in the circuit.

Following our approach with the single-loop circuit, we will begin with the simplest case and then generalize our analysis. Consider the circuit shown in Fig. 2.22. Here we have an independent current source in parallel with two resistors.
Since all of the circuit elements are in parallel, the voltage \( v(t) \) appears across each of them. Furthermore, an examination of the circuit indicates that the current \( i(t) \) is into the upper node of the circuit and the currents \( i_1(t) \) and \( i_2(t) \) are out of the node. Since KCL essentially states that what goes in must come out, the question we must answer is how \( i_1(t) \) and \( i_2(t) \) divide the input current \( i(t) \).

Applying Kirchhoff’s current law to the upper node, we obtain

\[
i(t) = i_1(t) + i_2(t)
\]

and, employing Ohm’s law, we have

\[
i(t) = \frac{v(t)}{R_1} + \frac{v(t)}{R_2} = \frac{1}{R_1} + \frac{1}{R_2} v(t) = \frac{v(t)}{R_p},
\]

where

\[
\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \quad 2.16
\]

\[
R_p = \frac{R_1 R_2}{R_1 + R_2} \quad 2.17
\]

Therefore, the equivalent resistance of two resistors connected in parallel is equal to the product of their resistances divided by their sum. Note also that this equivalent resistance \( R_p \) is always less than either \( R_1 \) or \( R_2 \). Hence, by connecting resistors in parallel we reduce the overall resistance. In the special case when \( R_1 = R_2 \), the equivalent resistance is equal to half of the value of the individual resistors.

The manner in which the current \( i(t) \) from the source divides between the two branches is called *current division* and can be found from the preceding expressions. For example,

\[
v(t) = R_p i(t) = \frac{R_1 R_2}{R_1 + R_2} i(t) \quad 2.18
\]

and

\[
i_1(t) = \frac{v(t)}{R_1} \quad 2.19
\]

and

\[
i_2(t) = \frac{v(t)}{R_2} = \frac{R_1}{R_1 + R_2} i(t) \quad 2.20
\]

Equations (2.19) and (2.20) are mathematical statements of the current-division rule.

**HINT**

The parallel resistance equation

**HINT**

The manner in which current divides between two parallel resistors

**Figure 2.22**

Simple parallel circuit.
Example 2.17
Given the network in Fig. 2.23a, let us find $I_1$, $I_2$, and $V_o$.

**SOLUTION** First, it is important to recognize that the current source feeds two parallel paths. To emphasize this point, the circuit is redrawn as shown in Fig. 2.23b. Applying current division, we obtain

$$I_1 = \left[ \frac{40k + 80k}{60k + (40k + 80k)} \right](0.9 \times 10^{-3}) = 0.6 \text{ mA}$$

and

$$I_2 = \left[ \frac{60k}{60k + (40k + 80k)} \right](0.9 \times 10^{-3}) = 0.3 \text{ mA}$$

Note that the larger current flows through the smaller resistor, and vice versa. In addition, note that if the resistances of the two paths are equal, the current will divide equally between them. KCL is satisfied since $I_1 + I_2 = 0.9 \text{ mA}$.

The voltage $V_o$ can be derived using Ohm’s law as

$$V_o = 80kI_2 = 24 \text{ V}$$

The problem can also be approached in the following manner. The total resistance seen by the current source is $40 \text{ k} \Omega$, that is, $60 \text{ k} \Omega$ in parallel with the series combination of $40 \text{ k} \Omega$ and $80 \text{ k} \Omega$ as shown in Fig. 2.23c. The voltage across the current source is then

$$V_1 = (0.9 \times 10^{-3})40k = 36 \text{ V}$$

Now that $V_1$ is known, we can apply voltage division to find $V_o$,

$$V_o = \left( \frac{80k}{80k + 40k} \right)V_1 = \left( \frac{80k}{120k} \right)36 = 24 \text{ V}$$

![Figure 2.23](image-url) Circuits used in Example 2.17.
Example 2.18

A typical car stereo consists of a 2-W audio amplifier and two speakers represented by the diagram shown in Fig. 2.24a. The output circuit of the audio amplifier is in essence a 430-mA current source, and each speaker has a resistance of 4 Ω. Let us determine the power absorbed by the speakers.

**Solution**  The audio system can be modeled as shown in Fig. 2.24b. Since the speakers are both 4-Ω devices, the current will split evenly between them, and the power absorbed by each speaker is

\[
P = I^2 R = (215 \times 10^{-3})^2(4) = 184.9 \text{ mW}
\]

---

**Learning Extension**

E2.10 Find the currents \( I_1 \) and \( I_2 \) and the power absorbed by the 40-kΩ resistor in the network in Fig. E2.10.

**Answer:** \( I_1 = 12 \text{ mA} \), \( I_2 = -4 \text{ mA} \), and \( P_{40 \text{kΩ}} = 5.76 \text{ W} \).

---

**Multiple Source/Resistor Networks**  Let us now extend our analysis to include a multiplicity of current sources and resistors in parallel. For example, consider the circuit shown in Fig. 2.25a. We have assumed that the upper node is \( v(t) \) volts positive with respect to the lower node. Applying Kirchhoff’s current law to the upper node yields

\[
i_1(t) - i_2(t) - i_3(t) + i_4(t) - i_6(t) = 0
\]

or

\[
i_1(t) - i_2(t) + i_4(t) - i_6(t) = i_3(t) + i_5(t)
\]

---

**Figure 2.24**

Circuits used in Example 2.18.

**Figure E2.10**

Circuits used in Example E2.10.

**Figure 2.25**

Equivalent circuits.
The terms on the left side of the equation all represent sources that can be combined algebraically into a single source; that is,
\[ i_o(t) = i_1(t) - i_3(t) + i_4(t) - i_6(t) \]
which effectively reduces the circuit in Fig. 2.25a to that in Fig. 2.25b. We could, of course, generalize this analysis to a circuit with \( N \) current sources. Using Ohm’s law, we can express the currents on the right side of the equation in terms of the voltage and individual resistances so that the KCL equation reduces to
\[ i_o(t) = \left( \frac{1}{R_1} + \frac{1}{R_2} \right) v(t) \]

Now consider the circuit with \( N \) resistors in parallel, as shown in Fig. 2.26a. Applying Kirchhoff’s current law to the upper node yields
\[ i_o(t) = i_1(t) + i_2(t) + \cdots + i_N(t) \]

or
\[ i_o(t) = \frac{v(t)}{R_p} \]

where
\[ \frac{1}{R_p} = \sum_{i=1}^{N} \frac{1}{R_i} \]

so that as far as the source is concerned, Fig. 2.26a can be reduced to an equivalent circuit, as shown in Fig. 2.26b.

The current division for any branch can be calculated using Ohm’s law and the preceding equations. For example, for the \( j \)th branch in the network of Fig. 2.26a,
\[ i_j(t) = \frac{v(t)}{R_j} \]

Using Eq. (2.22), we obtain
\[ i_j(t) = \frac{R_p}{R_j} i_o(t) \]

which defines the current-division rule for the general case.

---

**Example 2.19**

Given the circuit in Fig. 2.27a, we wish to find the current in the 12-k\( \Omega \) load resistor.

**SOLUTION**  To simplify the network in Fig. 2.27a, we add the current sources algebraically and combine the parallel resistors in the following manner:

\[ \frac{1}{R_p} = \frac{1}{18k} + \frac{1}{9k} + \frac{1}{12k} \]

\[ R_p = 4 \text{ k}\Omega \]
Using these values we can reduce the circuit in Fig. 2.27a to that in Fig. 2.27b. Now, applying current division, we obtain

\[ I_L = -\left[ \frac{4k}{4k + 12k} \right] (1 \times 10^{-3}) \]

\[ = -0.25 \text{ mA} \]

**Problem-Solving Strategy**

**Single-Node-Pair Circuits**

**Step 1.** Define a voltage \( v(t) \) between the two nodes in this circuit. We know from KVL that there is only one voltage for a single-node-pair circuit. A polarity is assigned to the voltage such that one of the nodes is assumed to be at a higher potential than the other node, which we will call the reference node.

**Step 2.** Using Ohm’s law, define a current flowing through each resistor in terms of the defined voltage.

**Step 3.** Apply KCL at one of the two nodes in the circuit.

**Step 4.** Solve the single KCL equation for \( v(t) \). If \( v(t) \) is positive, then the reference node is actually at a lower potential than the other node; if not, the reference node is actually at a higher potential than the other node.

**Learning Extension**

**E2.11** Find the power absorbed by the 6-kΩ resistor in the network in Fig. E2.11.  

**PSV**  

**ANSWER:**  

\[ P = 2.67 \text{ mW.} \]

**2.5 Series and Parallel Resistor Combinations**

We have shown in our earlier developments that the equivalent resistance of \( N \) resistors in series is

\[ R_s = R_1 + R_2 + \cdots + R_N \]

2.25

and the equivalent resistance of \( N \) resistors in parallel is found from

\[ \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \]

2.26

Let us now examine some combinations of these two cases.
Example 2.20
We wish to determine the resistance at terminals A-B in the network in Fig. 2.28a.

\[ R_{AB} = \frac{1}{\frac{2}{2} + \frac{2}{2} + \frac{2}{6} + \frac{6}{2}} \]

(b)

\[ 12 \Omega = 10 \Omega + (6 \Omega \text{ in parallel with } 3 \Omega) \]

(c)

\[ 6 \Omega = 2 \Omega + (6 \Omega \text{ in parallel with } 12 \Omega) \]

(d)

\[ 12 \Omega = 9 \Omega + (6 \Omega \text{ in parallel with } 6 \Omega) \]

(e)

\[ 3 \Omega = (4 \Omega \text{ in parallel with } 12 \Omega) \]

Figure 2.28 Simplification of a resistance network.

SOLUTION Starting at the opposite end of the network from the terminals and combining resistors as shown in the sequence of circuits in Fig. 2.28, we find that the equivalent resistance at the terminals is 5 kΩ.

Learning Extension

E2.12 Find the equivalent resistance at the terminals A-B in the network in Fig. E2.12. ANSWER: \( R_{AB} = 22 \text{kΩ} \).
PROBLEM-SOLVING STRATEGY

Simplifying Resistor Combinations

When trying to determine the equivalent resistance at a pair of terminals of a network composed of an interconnection of numerous resistors, it is recommended that the analysis begin at the end of the network opposite the terminals. Two or more resistors are combined to form a single resistor, thus simplifying the network by reducing the number of components as the analysis continues in a steady progression toward the terminals. The simplification involves the following:

**Step 1. Resistors in series.** Resistors $R_1$ and $R_2$ are in series if they are connected in tandem and carry exactly the same current. They can then be combined into a single resistor $R_S$, where $R_S = R_1 + R_2$.

**Step 2. Resistors in parallel.** Resistors $R_1$ and $R_2$ are in parallel if they are connected to the same two nodes and have exactly the same voltage across their terminals. They can then be combined into a single resistor $R_p$, where $R_p = R_1 R_2 / (R_1 + R_2)$.

These two combinations are used repeatedly, as needed, to reduce the network to a single resistor at the pair of terminals.

LEARNING EXTENSION

E2.13 Find the equivalent resistance at the terminals $A$-$B$ in the circuit in Fig. E2.13.  

**Figure E2.13**

<table>
<thead>
<tr>
<th>$R_{AB}$</th>
<th>$4 \text{k}\Omega$</th>
<th>$3 \text{k}\Omega$</th>
<th>$12 \text{k}\Omega$</th>
<th>$8 \text{k}\Omega$</th>
</tr>
</thead>
</table>

**Example 2.21**

A standard dc current-limiting power supply shown in Fig. 2.29a provides 0–18 V at 3 A to a load. The voltage drop, $V_R$, across a resistor, $R$, is used as a current-sensing device, fed back to the power supply and used to limit the current $I$. That is, if the load is adjusted so that the current tries to exceed 3 A, the power supply will act to limit the current to that value. The feedback voltage, $V_R$, should typically not exceed 600 mV.

**Figure 2.29**

Circuits used in Example 2.21.
CHAPTER 2 RESISTIVE CIRCUITS

RESISTOR SPECIFICATIONS

Some important parameters that are used to specify resistors are the resistor’s value, tolerance, and power rating. The tolerance specifications for resistors are typically 5% and 10%. A listing of standard resistor values with their specified tolerances is shown in Table 2.1.

The power rating for a resistor specifies the maximum power that can be dissipated by the resistor. Some typical power ratings for resistors are 1/4 W, 1/2 W, 1 W, 2 W, and so forth, up to very high values for high-power applications. Thus, in selecting a resistor for some particular application, one important selection criterion is the expected power dissipation.

If we have a box of standard 0.1-Ω, 5-W resistors, let us determine the configuration of these resistors that will provide \( V_R = 600 \text{ mV} \) when the current is 3 A.

**SOLUTION** Using Ohm’s law, the value of \( R \) should be

\[
R = \frac{V_R}{I}
\]

\[
= \frac{0.6}{3}
\]

\[
= 0.2 \Omega
\]

Therefore, two 0.1-Ω resistors connected in series, as shown in Fig. 2.29b, will provide the proper feedback voltage. Suppose, however, that the power supply current is to be limited to 9 A. The resistance required in this case to produce \( V_R = 600 \text{ mV} \) is

\[
R = \frac{0.6}{9}
\]

\[
= 0.0667 \Omega
\]

We must now determine how to interconnect the 0.1-Ω resistor to obtain \( R = 0.0667 \Omega \). Since the desired resistance is less than the components available (i.e., 0.1-Ω), we must connect the resistors in some type of parallel configuration. Since all the resistors are of equal value, note that three of them connected in parallel would provide a resistance of one-third their value, or 0.0333 Ω. Then two such combinations connected in series, as shown in Fig. 2.29c, would produce the proper resistance.

Finally, we must check to ensure that the configurations in Figs. 2.29b and c have not exceeded the power rating of the resistors. In the first case, the current \( I = 3 \text{ A} \) is present in each of the two series resistors. Therefore, the power absorbed in each resistor is

\[
P = I^2R
\]

\[
= (3)^2(0.1)
\]

\[
= 0.9 \text{ W}
\]

which is well within the 5-W rating of the resistors.

In the second case, the current \( I = 9 \text{ A} \). The resistor configuration for \( R \) in this case is a series combination of two sets of three parallel resistors of equal value. Using current division, we know that the current \( I \) will split equally among the three parallel paths and, hence, the current in each resistor will be 3 A. Therefore, once again, the power absorbed by each resistor is within its power rating.
Example 2.22

Given the network in Fig. 2.30, we wish to find the range for both the current and power dissipation in the resistor if \( R \) is a 2.7-k\( \Omega \) resistor with a tolerance of 10%.

**SOLUTION**

Using the equations \( I = \frac{V}{R} = \frac{10}{R} \) and \( P = \frac{V^2}{R} = \frac{100}{R} \), the minimum and maximum values for the resistor, current, and power are outlined next.

- **Minimum resistor value** = \( R(1 - 0.1) = 0.9 \) \( R = 2.43 \) k\( \Omega \)
- **Maximum resistor value** = \( R(1 + 0.1) = 1.1 \) \( R = 2.97 \) k\( \Omega \)
- **Minimum current value** = \( \frac{10}{2970} = 3.37 \) mA
- **Maximum current value** = \( \frac{10}{2430} = 4.12 \) mA
- **Minimum power value** = \( \frac{100}{2970} = 33.7 \) mW
- **Maximum power value** = \( \frac{100}{2430} = 41.2 \) mW

Thus, the ranges for the current and power are 3.37 mA to 4.12 mA and 33.7 mW to 41.2 mW, respectively.

---

**Example 2.23**

Given the network shown in Fig. 2.31: (a) find the required value for the resistor \( R \); (b) use Table 2.1 to select a standard 10% tolerance resistor for \( R \); (c) using the resistor selected in (b), determine the voltage across the 3.9-k\( \Omega \) resistor; (d) calculate the percent error in the voltage \( V_1 \), if the standard resistor selected in (b) is used; and (e) determine the power rating for this standard component.

---

**Table 2.1** Standard resistor values for 5% and 10% tolerances (10% values shown in boldface)

<table>
<thead>
<tr>
<th>Value</th>
<th>1.0</th>
<th>10</th>
<th>100</th>
<th>1.0k</th>
<th>10k</th>
<th>100k</th>
<th>1.0M</th>
<th>10M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>11</td>
<td>110</td>
<td>1.1k</td>
<td>11k</td>
<td>110k</td>
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46 CHAPTER 2 RESISTIVE CIRCUITS

SOLUTION

a. Using KVL, the voltage across \( R \) is 19 V. Then using Ohm’s law, the current in the loop is

\[
I = \frac{19}{3.9k} = 1.282 \text{ mA}
\]

The required value of \( R \) is then

\[
R = \frac{19}{0.001282} = 14.82 \text{ k}\Omega
\]

b. As shown in Table 2.1, the nearest standard 10% tolerance resistor is 15 k\( \Omega \).

c. Using the standard 15-k\( \Omega \) resistor, the actual current in the circuit is

\[
I = \frac{24}{18.9k} = 1.2698 \text{ mA}
\]

and the voltage across the 3.9-k\( \Omega \) resistor is

\[
V = IR = (0.0012698)(3.9k) = 4.952 \text{ V}
\]

d. The percent error involved in using the standard resistor is

\[
\% \text{ Error} = \frac{(4.952 - 5)}{5} \times 100 = -0.96\%
\]

e. The power absorbed by the resistor \( R \) is then

\[
P = IR = (0.0012698)^2(15k) = 24.2 \text{ mW}
\]

Therefore, even a quarter-watt resistor is adequate in this application.

2.6 Circuits with Series-Parallel Combinations of Resistors

At this point we have learned many techniques that are fundamental to circuit analysis. Now we wish to apply them and show how they can be used in concert to analyze circuits. We will illustrate their application through a number of examples that will be treated in some detail.

Example 2.24

We wish to find all the currents and voltages labeled in the ladder network shown in Fig. 2.32a.

![Diagram of a ladder network](image)

**SOLUTION** To begin our analysis of the network, we start at the right end of the circuit and combine the resistors to determine the total resistance seen by the 12-V source. This will allow us to calculate the current \( I_4 \). Then employing KVL, KCL, Ohm’s law, and/or voltage and current division, we will be able to calculate all currents and voltages in the network.
At the right end of the circuit, the 9-kΩ and 3-kΩ resistors are in series and, thus, can be combined into one equivalent 12-kΩ resistor. This resistor is in parallel with the 4-kΩ resistor, and their combination yields an equivalent 3-kΩ resistor, shown at the right edge of the circuit in Fig. 2.32b. In Fig. 2.32b the two 3-kΩ resistors are in series, and their combination is in parallel with the 6-kΩ resistor. Combining all three resistances yields the circuit shown in Fig. 2.32c.

Applying Kirchhoff’s voltage law to the circuit in Fig. 2.32c yields
\[ I_1(9k + 3k) = 12 \]
\[ I_1 = 1 \text{ mA} \]

\( V_a \) can be calculated from Ohm’s law as
\[ V_a = I_1(3k) \]
\[ V_a = 3 \text{ V} \]
or, using Kirchhoff’s voltage law,
\[ V_a = 12 - 9kI_1 \]
\[ = 12 - 9 \]
\[ = 3 \text{ V} \]

Knowing \( I_1 \) and \( V_a \), we can now determine all currents and voltages in Fig. 2.32b. Since \( V_a = 3 \text{ V} \), the current \( I_2 \) can be found using Ohm’s law as
\[ I_2 = \frac{3}{6k} \]
\[ I_2 = \frac{1}{2} \text{ mA} \]

Then, using Kirchhoff’s current law, we have
\[ I_1 = I_2 + I_3 \]
\[ 1 \times 10^{-3} = \frac{1}{2} \times 10^{-3} + I_3 \]
\[ I_3 = \frac{1}{2} \text{ mA} \]

Note that the \( I_3 \) could also be calculated using Ohm’s law:
\[ V_a = (3k + 3k)I_3 \]
\[ I_3 = \frac{3}{6k} \]
\[ I_3 = \frac{1}{2} \text{ mA} \]

Applying Kirchhoff’s voltage law to the right-hand loop in Fig. 2.32b yields
\[ V_a - V_b = 3kI_3 \]
\[ 3 - V_b = \frac{3}{2} \]
\[ V_b = \frac{3}{2} \text{ V} \]
or, since \( V_b \) is equal to the voltage drop across the 3-kΩ resistor, we could use Ohm’s law as
\[ V_b = 3kI_1 \]
\[ = \frac{3}{2} \text{ V} \]
We are now in a position to calculate the final unknown currents and voltages in Fig. 2.32a. Knowing \( V_b \), we can calculate \( I_4 \) using Ohm’s law as
\[
V_b = 4kI_4
\]
\[
I_4 = \frac{3}{7} \frac{3}{4k}
\]
\[
= \frac{3}{8} \text{mA}
\]

Then, from Kirchhoff’s current law, we have
\[
I_3 = I_4 + I_5
\]
\[
\frac{1}{2} \times 10^{-3} = \frac{3}{8} \times 10^{-3} + I_5
\]
\[
I_5 = \frac{1}{8} \text{mA}
\]

We could also have calculated \( I_4 \) using the current-division rule. For example,
\[
I_5 = \frac{4k}{4k + (9k + 3k)} I_3
\]
\[
= \frac{1}{8} \text{mA}
\]

Finally, \( V_c \) can be computed as
\[
V_c = I_4(3k)
\]
\[
= \frac{3}{8} \text{V}
\]

\( V_c \) can also be found using voltage division (i.e., the voltage \( V_c \) will be divided between the 9-k\( \Omega \) and 3-k\( \Omega \) resistors). Therefore,
\[
V_c = \left[ \frac{3k}{3k + 9k} \right] V_b
\]
\[
= \frac{3}{8} \text{V}
\]

Note that Kirchhoff’s current law is satisfied at every node and Kirchhoff’s voltage law is satisfied around every loop, as shown in Fig. 2.32d.

The following example is, in essence, the reverse of the previous example in that we are given the current in some branch in the network and are asked to find the value of the input source.

**Example 2.25**

Given the circuit in Fig. 2.33 and \( I_2 = 1/2 \text{ mA} \), let us find the source voltage \( V_o \).

**SOLUTION** If \( I_2 = 1/2 \text{ mA} \), then from Ohm’s law, \( V_b = 3 \text{ V} \). \( V_b \) can now be used to calculate \( I_3 = 1 \text{ mA} \). Kirchhoff’s current law applied at node \( y \) yields
\[
I_3 = I_4 + I_5
\]
\[
= 1.5 \text{ mA}
\]

Then, from Ohm’s law, we have
\[
V_o = (1.5 \times 10^{-3})(2k)
\]
\[
= 3 \text{ V}
\]
Since $V_o + V_b$ is now known, $I_5$ can be obtained:

$$I_5 = \frac{V_o + V_b}{3k + 1k} = 1.5 \text{ mA}$$

Applying Kirchhoff’s current law at node $x$ yields

$$I_1 = I_2 + I_5 = 3 \text{ mA}$$

Now KVL applied to any closed path containing $V_o$ will yield the value of this input source. For example, if the path is the outer loop, KVL yields

$$-V_o + 6kI_1 + 3kI_2 + 1kI_3 + 4kI_1 = 0$$

Since $I_1 = 3 \text{ mA}$ and $I_5 = 1.5 \text{ mA}$,

$$V_o = 36 \text{ V}$$

If we had selected the path containing the source and the points $x, y, z$, we would obtain

$$-V_o + 6kI_1 + V_a + V_b + 4kI_1 = 0$$

Once again, this equation yields

$$V_o = 36 \text{ V}.$$

**Problem-Solving Strategy**

**Analyzing Circuits Containing a Single Source and a Series-Parallel Interconnection of Resistors**

**Step 1.** Systematically reduce the resistive network so that the resistance seen by the source is represented by a single resistor.

**Step 2.** Determine the source current for a voltage source or the source voltage if a current source is present.

**Step 3.** Expand the network, retracing the simplification steps, and apply Ohm’s law, KVL, KCL, voltage division, and current division to determine all currents and voltages in the network.
To provide motivation for this topic, consider the circuit in Fig. 2.34. Note that this network has essentially the same number of elements as contained in our recent examples. However, when we attempt to reduce the circuit to an equivalent network containing the source and an equivalent resistor $R$, we find that nowhere is a resistor in series or parallel with another. Therefore, we cannot attack the problem directly using the techniques that we have learned thus far. We can, however, replace one portion of the network with an equivalent circuit, and this conversion will permit us, with ease, to reduce the combination of resistors to a single equivalent resistance. This conversion is called the *wye*-to-*delta* or *delta*-to-*wye* transformation.

Consider the networks shown in Fig. 2.35. Note that the resistors in Fig. 2.35a form a Δ (delta) and the resistors in Fig. 2.35b form a Y (wye). If both of these configurations are connected at only three terminals $a$, $b$, and $c$, it would be very advantageous if an equivalence could be established between them. It is, in fact, possible to relate the resistances of one network to those of the other such that their terminal characteristics are the same. This relationship between the two network configurations is called the *Y*-Δ transformation.

The transformation that relates the resistances $R_1$, $R_2$, and $R_3$ to the resistances $R_a$, $R_b$, and $R_c$ is derived as follows. For the two networks to be equivalent at each corresponding pair of terminals, it is necessary that the resistance at the corresponding terminals be equal (e.g., the resistance at terminals $a$ and $b$ with $c$ open-circuited must be the same for both networks).
Therefore, if we equate the resistances for each corresponding set of terminals, we obtain the following equations:

\[
R_{ab} = R_a + R_b = \frac{R_a(R_1 + R_3)}{R_2 + R_1 + R_3}
\]

\[
R_{bc} = R_b + R_c = \frac{R_b(R_1 + R_2)}{R_3 + R_1 + R_2}
\]

\[
R_{ca} = R_c + R_a = \frac{R_c(R_2 + R_3)}{R_1 + R_2 + R_3}
\]

Solving this set of equations for \(R_a\), \(R_b\), and \(R_c\) yields

\[
R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3}
\]

\[
R_b = \frac{R_2 R_3}{R_1 + R_2 + R_3}
\]

\[
R_c = \frac{R_1 R_3}{R_1 + R_2 + R_3}
\]

Similarly, if we solve Eq. (2.27) for \(R_1\), \(R_2\), and \(R_3\), we obtain

\[
R_1 = \frac{R_a R_b + R_c R_a + R_b R_c}{R_a}
\]

\[
R_2 = \frac{R_a R_b + R_c R_b + R_c R_a}{R_c}
\]

\[
R_3 = \frac{R_a R_b + R_c R_c + R_a R_b}{R_b}
\]

Equations (2.28) and (2.29) are general relationships and apply to any set of resistances connected in a Y or \(\Delta\). For the balanced case where \(R_a = R_b = R_c\) and \(R_1 = R_2 = R_3\), the equations above reduce to

\[
R_v = \frac{1}{3} R_\Delta
\]

\[
R_\Delta = 3R_v
\]

It is important to note that it is not necessary to memorize the formulas in Eqs. (2.28) and (2.29). Close inspection of these equations and Fig. 2.35 illustrates a definite pattern to the relationships between the two configurations. For example, the resistance connected to point \(a\) in the wye (i.e., \(R_a\)) is equal to the product of the two resistors in the \(\Delta\) that are connected to point \(a\) divided by the sum of all the resistances in the delta. \(R_b\) and \(R_c\) are determined in a similar manner. Similarly, there are geometrical patterns associated with the equations for calculating the resistors in the delta as a function of those in the wye.

Let us now examine the use of the delta \(\leftrightarrow\) wye transformation in the solution of a network problem.
Example 2.26
Given the network in Fig. 2.36a, let us find the source current $I_S$.

![Circuits used in Example 2.26.](image)

**SOLUTION** Note that none of the resistors in the circuit are in series or parallel. However, careful examination of the network indicates that the 12k-, 6k-, and 18k-ohm resistors, as well as the 4k-, 6k-, and 9k-ohm resistors each form a delta that can be converted to a wye. Furthermore, the 12k-, 6k-, and 4k-ohm resistors, as well as the 18k-, 6k-, and 9k-ohm resistors, each form a wye that can be converted to a delta. Any one of these conversions will lead to a solution. We will perform a delta-to-wye transformation on the 12k-, 6k-, and 18k-ohm resistors, which leads to the circuit in Fig. 2.36b. The 2k- and 4k-ohm resistors, like the 3k- and 9k-ohm resistors, are in series and their parallel combination yields a 4k-ohm resistor. Thus, the source current is

$$I_S = \frac{12}{(6k + 4k)} = 1.2 \text{ mA}.$$

**LEARNING EXTENSIONS**

**E2.17** Determine the total resistance $R_T$ in the circuit in Fig. E2.17.

**Answer:** $R_T = 34 \text{k}\Omega$.

**E2.18** Find $V_o$ in the network in Fig. E2.18.

**Answer:** $V_o = 24 \text{ V}$.
2.8 Circuits with Dependent Sources

In Chapter 1 we outlined the different kinds of dependent sources. These controlled sources are extremely important because they are used to model physical devices such as npn and pnp bipolar junction transistors (BJTs) and field-effect transistors (FETs) that are either metal-oxide-semiconductor field-effect transistors (MOSFETs) or insulated-gate field-effect transistors (IGFETs). These basic structures are, in turn, used to make analog and digital devices. A typical analog device is an operational amplifier (op-amp). This device is presented in Chapter 4. Typical digital devices are random access memories (RAMs), read-only memories (ROMs), and microprocessors. We will now show how to solve simple one-loop and one-node circuits that contain these dependent sources. Although the following examples are fairly simple, they will serve to illustrate the basic concepts.

**Problem-Solving Strategy**

**Circuits with Dependent Sources**

**Step 1.** When writing the KVL and/or KCL equations for the network, treat the dependent source as though it were an independent source.

**Step 2.** Write the equation that specifies the relationship of the dependent source to the controlling parameter.

**Step 3.** Solve the equations for the unknowns. Be sure that the number of linearly independent equations matches the number of unknowns.

The following four examples will each illustrate one of the four types of dependent sources: current-controlled voltage source, current-controlled current source, voltage-controlled voltage source, and voltage-controlled current source.

**Example 2.27**

Let us determine the voltage $V_o$ in the circuit in Fig. 2.37.

**SOLUTION** Applying KVL, we obtain

$$-12 + 3kI_1 - V_A + 5kI_1 = 0$$

where

$$V_A = 2000I_1$$

and the units of the multiplier, 2000, are ohms. Solving these equations yields

$$I_1 = 2 \text{ mA}$$

Then

$$V_o = (5 \text{ k})I_1 = 10 \text{ V}$$
Example 2.28

Given the circuit in Fig. 2.38 containing a current-controlled current source, let us find the voltage $V_o$.

**Figure 2.38** Circuit used in Example 2.28.

![Circuit](image)

**SOLUTION** Applying KCL at the top node, we obtain

$$10 \times 10^{-3} + \frac{V_s}{2k} + \frac{V_s}{4k} - 4I_o = 0$$

where

$$I_o = \frac{V_s}{3k}$$

Substituting this expression for the controlled source into the KCL equation yields

$$10^{-2} + \frac{V_s}{6k} + \frac{V_s}{3k} - 4\frac{V_s}{3k} = 0$$

Solving this equation for $V_s$, we obtain

$$V_s = 12 \text{ V}$$

The voltage $V_o$ can now be obtained using a simple voltage divider; that is,

$$V_o = \left(\frac{4k}{2k + 4k}\right)V_s$$

$$= 8 \text{ V}$$

Example 2.29

The network in Fig. 2.39 contains a voltage-controlled voltage source. We wish to find $V_o$ in this circuit.

**Figure 2.39** Circuit used in Example 2.29.

![Circuit](image)

**SOLUTION** Applying KVL to this network yields

$$-12 + 3kI + 2V_o + 1kI = 0$$

where

$$V_o = 1kI$$
Hence, the KVL equation can be written as
\[-12 + 3kI + 2kI + 1kI = 0\]
or
\[I = 2 \text{ mA}\]
Therefore,
\[V_o = 1kI\]
\[= 2 \text{ V}\]

**Example 2.30**

An equivalent circuit for a FET common-source amplifier or BJT common-emitter amplifier can be modeled by the circuit shown in Fig. 2.40a. We wish to determine an expression for the gain of the amplifier, which is the ratio of the output voltage to the input voltage.

**SOLUTION**

Note that although this circuit, which contains a voltage-controlled current source, appears to be somewhat complicated, we are actually in a position now to solve it with techniques we have studied up to this point. The loop on the left, or input to the amplifier, is essentially detached from the output portion of the amplifier on the right. The voltage across \[R_1\] is \([v_g(t)]\), which controls the dependent current source.

To simplify the analysis, let us replace the resistors \([R_3, R_4, \text{ and } R_5]\) with \([R_L]\) such that

\[\frac{1}{R_L} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\]

Then the circuit reduces to that shown in Fig. 2.40b. Applying Kirchhoff’s voltage law to the input portion of the amplifier yields

\[v_i(t) = i_i(t)(R_1 + R_2)\]

and

\[v_g(t) = i_i(t)R_2\]

Solving these equations for \([v_g(t)]\) yields

\[v_g(t) = \frac{R_2}{R_1 + R_2} v_i(t)\]

From the output circuit, note that the voltage \([v_o(t)]\) is given by the expression

\[v_o(t) = -g_m v_g(t)R_L\]
Combining this equation with the preceding one yields

\[ v_o(t) = \frac{-g_m R_2 R_3}{R_1 + R_2} v_i(t) \]

Therefore, the amplifier gain, which is the ratio of the output voltage to the input voltage, is given by

\[ \frac{v_o(t)}{v_i(t)} = -\frac{g_m R_2 R_3}{R_1 + R_2} \]

Reasonable values for the circuit parameters in Fig. 2.40a are \( R_1 = 100 \, \Omega \), \( R_2 = 1 \, k\Omega \), \( g_m = 0.04 \, S \), \( R_3 = 50 \, k\Omega \), and \( R_4 = R_5 = 10 \, k\Omega \). Hence, the gain of the amplifier under these conditions is

\[ \frac{v_o(t)}{v_i(t)} = \frac{-0.04 \times (4.545)(10^3)(1)(10^3)}{(1)(10^3)} = -165.29 \]

Thus, the magnitude of the gain is 165.29.

At this point it is perhaps helpful to point out again that when analyzing circuits with dependent sources, we first treat the dependent source as though it were an independent source when we write a Kirchhoff’s current or voltage law equation. Once the equation is written, we then write the controlling equation that specifies the relationship of the dependent source to the unknown variable. For instance, the first equation in Example 2.28 treats the dependent source like an independent source. The second equation in the example specifies the relationship of the dependent source to the voltage, which is the unknown in the first equation.

### Learning Extensions

#### E2.19
Find \( V_o \) in the circuit in Fig. E2.19.

**PSV**

**Answer:** \( V_o = 12 \, V \).

![Figure E2.19](image1)

#### E2.20
Find \( V_o \) in the network in Fig. E2.20.

**Answer:** \( V_o = 8 \, V \).

![Figure E2.20](image2)
2.9 Application Examples

Throughout this book we endeavor to present a wide variety of examples that demonstrate the usefulness of the material under discussion in a practical environment. To enhance our presentation of the practical aspects of circuit analysis and design, we have dedicated sections, such as this one, in most chapters for the specific purpose of presenting additional application-oriented examples.

Application Example 2.31

The eyes (heating elements) of an electric range are frequently made of resistive nichrome strips. Operation of the eye is quite simple. A current is passed through the heating element causing it to dissipate power in the form of heat. Also, a four-position selector switch, shown in Fig. 2.41, controls the power (heat) output. In this case the eye consists of two nichrome strips modeled by the resistors \( R_1 \) and \( R_2 \), where \( R_1 < R_2 \).

1. How should positions \( A \), \( B \), \( C \), and \( D \) be labeled with regard to high, medium, low, and off settings?
2. If we desire that high and medium correspond to 2000 W and 1200 W power dissipation, respectively, what are the values of \( R_1 \) and \( R_2 \)?
3. What is the power dissipation at the low setting?

Solution  Position \( A \) is the off setting since no current flows to the heater elements. In position \( B \), current flows through \( R_2 \) only, while in position \( C \) current flows through \( R_1 \) only. Since \( R_1 < R_2 \), more power will be dissipated when the switch is at position \( C \). Thus, position \( C \) is the medium setting, \( B \) is the low setting, and, by elimination, position \( D \) is the high setting.

When the switch is at the medium setting, only \( R_1 \) dissipates power, and we can write \( R_1 \) as

\[
R_1 = \frac{V_2^2}{P_1} = \frac{230^2}{1200}
\]

or

\[
R_1 = 44.08 \, \Omega
\]

On the high setting, 2000 W of total power is delivered to \( R_1 \) and \( R_2 \). Since \( R_1 \) dissipates 1200 W, \( R_2 \) must dissipate the remaining 800 W. Therefore, \( R_2 \) is

\[
R_2 = \frac{V_2^2}{P_2} = \frac{230^2}{800}
\]

or

\[
R_2 = 66.13 \, \Omega
\]

Finally, at the low setting, only \( R_2 \) is connected to the voltage source; thus, the power dissipation at this setting is 800 W.
Application Example 2.32

Have you ever cranked your car with the headlights on? While the starter kicked the engine, you probably saw the headlights dim then return to normal brightness once the engine was running on its own. Can we create a model to predict this phenomenon?

SOLUTION   Yes, we can. Consider the conceptual circuit in Fig. 2.42a, and the model circuit in Fig. 2.42b, which isolates just the battery, headlights, and starter. Note the resistor \( R_{\text{batt}} \). It is included to model several power loss mechanisms that can occur between the battery and the loads, that is, the headlights and starter. First, there are the chemical processes within the battery itself which are not 100\% efficient. Second, there are the electrical connections at both the battery posts and the loads. Third, the wiring itself has some resistance, although this is usually so small that it is negligible. The sum of these losses is modeled by \( R_{\text{batt}} \), and we expect the value of \( R_{\text{batt}} \) to be small. A reasonable value is 25 m\( \Omega \).

![Figure 2.42](IRWI02_017-081-hr.png)

A conceptual (a) model and (b) circuit for examining the effect of starter current on headlight intensity.

Next we address the starter. When energized, a typical automobile starter will draw between 90 and 120 A. We will use 100 A as a typical number. Finally, the headlights will draw much less current—perhaps only 1 A. Now we have values to use in our model circuit.

Assume first that the starter is off. By applying KCL at the node labeled \( V_L \), we find that the voltage applied to the headlights can be written as

\[ V_L = V_{\text{batt}} - I_{\text{HL}} R_{\text{batt}} \]

Substituting our model values into this equation yields \( V_L = 11.75 \text{ V} \)—very close to 12 V. Now we energize the starter and apply KCL again.

\[ V_L = V_{\text{batt}} - (I_{\text{HL}} + I_{\text{start}}) R_{\text{batt}} \]

Now the voltage across the headlights is only 9.25 V. No wonder the headlights dim! How would corrosion or loose connections on the battery posts change the situation? In this case, we would expect the quality of the connection from battery to load to deteriorate, increasing \( R_{\text{batt}} \) and compounding the headlight dimming issue.

Application Example 2.33

A Wheatstone Bridge circuit is an accurate device for measuring resistance. The circuit, shown in Fig. 2.43, is used to measure the unknown resistor \( R_x \). The center leg of the circuit contains a galvanometer, which is a very sensitive device that can be used to measure current in the microamp range. When the unknown resistor is connected to the bridge, \( R_x \) is adjusted until the current in the galvanometer is zero, at which point the bridge is balanced. In this balanced condition

\[ \frac{R_1}{R_2} = \frac{R_3}{R_4} \]

so that

\[ R_x = \left( \frac{R_3}{R_1} \right) R_3 \]
Engineers also use this bridge circuit to measure strain in solid material. For example, a system used to determine the weight of a truck is shown in Fig. 2.44a. The platform is supported by cylinders on which strain gauges are mounted. The strain gauges, which measure strain when the cylinder deflects under load, are connected to a Wheatstone bridge as shown in Fig. 2.44b. The strain gauge has a resistance of 120 Ω under no-load conditions and changes value under load. The variable resistor in the bridge is a calibrated precision device.

Weight is determined in the following manner. The ΔR required to balance the bridge represents the Δ strain, which when multiplied by the modulus of elasticity yields the Δ stress. The Δ stress multiplied by the cross-sectional area of the cylinder produces the Δ load, which is used to determine weight.

Let us determine the value of R3 under no load when the bridge is balanced and its value when the resistance of the strain gauge changes to 120.24 Ω under load.

**SOLUTION** Using the balance equation for the bridge, the value of R3 at no load is

\[
R_3 = \frac{R_1}{R_2} R_x
\]

\[
= \left(\frac{100}{110}\right) (120)
\]

\[= 109.0909 \, \Omega\]

Under load, the value of R3 is

\[
R_3 = \left(\frac{100}{110}\right) (120.24)
\]

\[= 109.3091 \, \Omega\]

Therefore, the ΔR is

\[
\Delta R_3 = 109.3091 - 109.0909
\]

\[= 0.2182 \, \Omega\]
2.10 Design Examples

Most of this text is concerned with circuit analysis; that is, given a circuit in which all the components are specified, analysis involves finding such things as the voltage across some element or the current through another. Furthermore, the solution of an analysis problem is generally unique. In contrast, design involves determining the circuit configuration that will meet certain specifications. In addition, the solution is generally not unique in that there may be many ways to satisfy the circuit/performance specifications. It is also possible that there is no solution that will meet the design criteria.

In addition to meeting certain technical specifications, designs normally must also meet other criteria, such as economic, environmental, and safety constraints. For example, if a circuit design that meets the technical specifications is either too expensive or unsafe, it is not viable regardless of its technical merit.

At this point, the number of elements that we can employ in circuit design is limited primarily to the linear resistor and the active elements we have presented. However, as we progress through the text we will introduce a number of other elements (for example, the op-amp, capacitor, and inductor), which will significantly enhance our design capability.

We begin our discussion of circuit design by considering a couple of simple examples that demonstrate the selection of specific components to meet certain circuit specifications.

---

**Design Example 2.34**

An electronics hobbyist who has built his own stereo amplifier wants to add a back-lit display panel to his creation for that professional look. His panel design requires seven light bulbs—two operate at and five at Luckily, his stereo design already has a quality 12-V dc supply; however, there is no 9-V supply. Rather than building a new dc power supply, let us use the inexpensive circuit shown in Fig. 2.45a to design a 12-V to 9-V converter with the restriction that the variation in be no more than In particular, we must determine the necessary values of and

**SOLUTION**

First, lamps and have no effect on Second, when lamps are on, they each have an equivalent resistance of

\[ R_{eq} = \frac{V_2}{I} = \frac{9}{0.005} = 1.8 \ \text{k}\Omega \]

As long as \( V_2 \) remains fairly constant, the lamp resistance will also be fairly constant. Thus, the requisite model circuit for our design is shown in Fig. 2.45b. The voltage \( V_2 \) will be at its maximum value of 9 + 5% = 9.45 V when \( L_3-L_7 \) are all off. In this case \( R_1 \) and \( R_2 \) are in series, and \( V_2 \) can be expressed by simple voltage division as

\[ V_2 = 9.45 = 12 \left( \frac{R_2}{R_1 + R_2} \right) \]

Rearranging the equation yields

\[ \frac{R_1}{R_2} = 0.27 \]

A second expression involving \( R_1 \) and \( R_2 \) can be developed by considering the case when \( L_3-L_7 \) are all on, which causes \( V_2 \) to reach its minimum value of 9—5%, or 8.55 V. Now, the effective resistance of the lamps is five 1.8-k\Omega resistors in parallel, or 360 \( \Omega \). The corresponding expression for \( V_2 \) is

\[ V_2 = 8.55 = 12 \left( \frac{R_2/360}{R_1 + (R_2/360)} \right) \]
Design Example 2.35

Let’s design a circuit that produces a 5-V output from a 12-V input. We will arbitrarily fix the power consumed by the circuit at 240 mW. Finally, we will choose the best possible standard resistor values from Table 2.1 and calculate the percent error in the output voltage that results from that choice.

which can be rewritten in the form

\[ \frac{360R_1 + 360 + R_1}{R_2} = \frac{12}{8.55} = 1.4 \]

Substituting the value determined for \( R_1/R_2 \) into the preceding equation yields

\[ R_1 = 360[1.4 - 1 - 0.27] \]

or

\[ R_1 = 48.1 \text{Ω} \]

and so for \( R_2 \)

\[ R_2 = 178.3 \text{Ω} \]
**SOLUTION**  The simple voltage divider, shown in Fig. 2.46, is ideally suited for this application. We know that $V_o$ is given by

$$V_o = V_i \left( \frac{R_2}{R_1 + R_2} \right)$$

which can be written as

$$R_1 = R_2 \left[ \frac{V_i}{V_o} - 1 \right]$$

**Figure 2.46**  
A simple voltage divider.

Since all of the circuit’s power is supplied by the 12-V source, the total power is given by

$$P = \frac{V_i^2}{R_1 + R_2} \leq 0.24$$

Using the second equation to eliminate $R_1$, we find that $R_2$ has a lower limit of

$$R_2 \geq \frac{V_i V_o}{P} = \frac{(5)(12)}{0.24} = 250 \Omega$$

Substituting these results into the second equation yields the lower limit of $R_1$, that is

$$R_1 = R_2 \left[ \frac{V_i}{V_o} - 1 \right] \geq 350 \Omega$$

Thus, we find that a significant portion of Table 2.1 is not applicable to this design. However, determining the best pair of resistor values is primarily a trial-and-error operation that can be enhanced by using an Excel spreadsheet. After a number of trials, we find that the best values that satisfy the constraint equations are

$$R_2 = 750 \Omega \quad \text{and} \quad R_1 = 1000 \Omega$$

and these values yield an output voltage of 5.14 V. The resulting error in the output voltage can be determined from the expression

$$\text{percent error} = \left[ \frac{5.14 - 5}{5} \right] 100\% = 2.8\%$$

Note that by selecting the best possible resistor values, we have an output error of 2.8% that is less than the 5% resistor tolerance.

It should be noted, however, that these resistor values are nominal, that is, typical values. To find the worst-case error, we must consider that each resistor as purchased may be as much as ±5% off from the nominal value. In this application, since $V_o$ is already greater than the target of 5 V, the worst-case scenario occurs when $V_i$ increases even further, that is, when $R_1$ is 5% too low (950 Ω) and $R_2$ is 5% too high (787.5 Ω). The resulting output voltage is 5.44 V, that is, an 8.8% error. Of course, most resistor values are closer to the nominal value than to the guaranteed maximum/minimum values. However, if we intend to build this circuit with a guaranteed tight output error, for example, 5%, we should purchase resistors with lower tolerances.

How much lower should the tolerances be? Our first equation can be altered to yield the worst-case output voltage by adding a tolerance, $\Delta$, to $R_2$ and subtracting the tolerance from $R_1$. Let us choose a worst-case output voltage of $V_{\text{max}} = 5.25$ V, that is, a 5% error.

$$V_{\text{max}} = 5.25 = V_i \left[ \frac{R_2(1 + \Delta)}{R_1(1 - \Delta) + R_2(1 + \Delta)} \right] = 12 \left[ \frac{750(1 + \Delta)}{750(1 + \Delta) + 1000(1 - \Delta)} \right]$$
The resulting value of $\Delta$ is 0.018, or 1.8%. Standard resistors are available in tolerances of 10%, 5%, 2%, and 1%. Tighter tolerances are available but very expensive. Thus, based on nominal values of 750 $\Omega$ and 1000 $\Omega$, we should choose 1% resistors to assure an output voltage error less than 5%.

**Design Example 2.36**

In factory instrumentation, process parameters such as pressure and flow rate are measured, converted to electrical signals, and sent some distance to an electronic controller. The controller then decides what actions should be taken. One of the main concerns in these systems is the physical distance between the sensor and the controller. An industry standard format for encoding the measurement value is called the 4–20 mA standard, where the parameter range is linearly distributed from 4 to 20 mA. For example, a 100 psi pressure sensor would output 4 mA if the pressure were 0 psi, 20 mA at 100 psi, and 12 mA at 50 psi. But most instrumentation is based on voltages between 0 and 5 V, not on currents.

Therefore, let us design a current-to-voltage converter that will output 5 V when the current signal is 20 mA.

**SOLUTION**

The circuit in Fig. 2.47a is a very accurate model of our situation. The wiring from the sensor unit to the controller has some resistance, $R_{\text{wire}}$. If the sensor output were a voltage proportional to pressure, the voltage drop in the line would cause measurement error even if the sensor output were an ideal source of voltage. But, since the data are contained in the current value, $R_{\text{wire}}$ does not affect the accuracy at the controller as long as the sensor acts as an ideal current source.

![Figure 2.47 The 4–20 mA control loop (a) block diagram, (b) with the current-to-voltage converter.](image)

As for the current-to-voltage converter, it is extremely simple—a resistor. For 5 V at 20 mA, we employ Ohm’s law to find

$$R = \frac{5}{0.02} = 250 \, \Omega$$

The resulting converter is added to the system in Fig. 2.47b, where we tacitly assume that the controller does not load the remaining portion of the circuit.

Note that the model indicates that the distance between the sensor and controller could be infinite. Intuitively, this situation would appear to be unreasonable, and it is. Losses that would take place over distance can be accounted for by using a more accurate model of the sensor, as shown in Fig. 2.48. The effect of this new sensor model can be seen from the equations that describe this new network. The model equations are

$$I_s = \frac{V_S}{R_S} + \frac{V_S}{R_{\text{wire}} + 250}$$

and

$$I_{\text{signal}} = \frac{V_S}{R_{\text{wire}} + 250}$$

Combining these equations yields

$$\frac{I_{\text{signal}}}{I_s} = \frac{1}{1 + \frac{R_{\text{wire}} + 250}{R_S}}$$
Thus, we see that it is the size of $R_S$ relative to $(R_{\text{wire}} + 250 \, \Omega)$ that determines the accuracy of the signal at the controller. Therefore, we want $R_S$ as large as possible. Both the maximum sensor output voltage and output resistance, $R_S$, are specified by the sensor manufacturer.

We will revisit this current-to-voltage converter in Chapter 4.

**Design Example 2.37**

The network in Fig. 2.49 is an equivalent circuit for a transistor amplifier used in a stereo preamplifier. The input circuitry, consisting of a 2-mV source in series with a 500-Ω resistor, models the output of a compact disk player. The dependent source, $R_{\text{in}}$, and $R_o$ model the transistor, which amplifies the signal and then sends it to the power amplifier. The 10-kΩ load resistor models the input to the power amplifier that actually drives the speakers. We must design a transistor amplifier as shown in Fig. 2.49 that will provide an overall gain of $-200$. In practice we do not actually vary the device parameters to achieve the desired gain; rather, we select a transistor from the manufacturer’s data books that will satisfy the required specification. The model parameters for three different transistors are listed as follows:

<table>
<thead>
<tr>
<th>Manufacturer’s transistor parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Part Number</strong></td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

Design the amplifier by choosing the transistor that produces the most accurate gain. What is the percent error of your choice?

**SOLUTION**

The output voltage can be written

$$V_o = -g_m V \left( \frac{R_{\text{in}}}{R_o} \right)$$

Using voltage division at the input to find $V$,

$$V = \frac{R_{\text{in}}}{R_{\text{in}} + R_S} V_S$$

Combining these two expressions, we can solve for the gain:

$$A_V = \frac{V_o}{V_S} = -g_m \left( \frac{R_{\text{in}}}{R_{\text{in}} + R_S} \right) \left( \frac{R_{\text{in}}}{R_o} \right)$$

Using the parameter values for the three transistors, we find that the best alternative is transistor number 2, which has a gain error of

$$\text{Percent error} = \left( \frac{211.8 - 200}{200} \right) \times 100\% = 5.9\%$$

![Figure 2.48](image) A more accurate model for the 4–20 mA control loop.
PROBLEMS

65

SUMMARY

- **Ohm’s law** $V = IR$
- **The passive sign convention with Ohm’s law** The current enters the resistor terminal with the positive voltage reference.
- **Kirchhoff’s current law (KCL)** The algebraic sum of the currents leaving (entering) a node is zero.
- **Kirchhoff’s voltage law (KVL)** The algebraic sum of the voltages around any closed path is zero.
- **Solving a single-loop circuit** Determine the loop current by applying KVL and Ohm’s law.
- **Solving a single-node-pair circuit** Determine the voltage between the pair of nodes by applying KCL and Ohm’s law.
- **The voltage-division rule** The voltage is divided between two series resistors in direct proportion to their resistance.
- **The current-division rule** The current is divided between two parallel resistors in reverse proportion to their resistance.
- **The equivalent resistance of a network of resistors** Combine resistors in series by adding their resistances. Combine resistors in parallel by adding their conductances. The wye-to-delta and delta-to-wye transformations are also an aid in reducing the complexity of a network.
- **Short circuit** Zero resistance, zero voltage; the current in the short is determined by the rest of the circuit.
- **Open circuit** Zero conductance, zero current; the voltage across the open terminals is determined by the rest of the circuit.

PROBLEMS

PSV CS both available on the web at: http://www.justask4u.com/irwin

SECTION 2.1

2.1 Find the current $I$ and the power supplied by the source in the network in Fig. P2.1.

2.2 In the network in Fig. P2.2, the power absorbed by $R_x$ is 20 mW. Find $R_x$.

2.3 Find the current $I$ and the power supplied by the source in the network in Fig. P2.3.

2.4 In the circuit in Fig. P2.4, find the voltage across the current source and the power absorbed by the resistor.
2.5 If the 5-kΩ resistor in the network in Fig. P2.5 absorbs 200 mW, find $V_S$.

![Figure P2.5](image)

2.6 In the network in Fig. P2.6, the power absorbed by $G_x$ is 20 mW. Find $G_x$.

![Figure P2.6](image)

2.7 A model for a standard two D-cell flashlight is shown in Fig. P2.7. Find the power dissipated in the lamp.

![Figure P2.7](image)

SECTION 2.2

2.10 Find $I_1$ in the network in Fig. P2.10.

![Figure P2.10](image)

2.11 Find $I_1$ and $I_2$ in the circuit in Fig. P2.11.

![Figure P2.11](image)

2.8 An automobile uses two halogen headlights connected as shown in Fig. P2.8. Determine the power supplied by the battery if each headlight draws 3 A of current.

![Figure P2.8](image)

2.9 Many years ago a string of Christmas tree lights was manufactured in the form shown in Fig. P2.9a. Today the lights are manufactured as shown in Fig. P2.9b. Is there a good reason for this change?

![Figure P2.9](image)
2.12 Find $I_1$ and $I_x$ in the circuit in Fig. P2.12.

![Figure P2.12](image1)

2.13 Find $I_x$ in the circuit in Fig. P2.13.

![Figure P2.13](image2)

2.14 Find $I_x$ in the circuit in Fig. P2.14.

![Figure P2.14](image3)

2.15 Find $I_x$ in the circuit in Fig. P2.15.

![Figure P2.15](image4)

2.16 Find $V_x$ in the circuit in Fig. P2.16.

![Figure P2.16](image5)

2.17 Find $V_{fb}$ and $V_{ce}$ in the circuit in Fig. P2.17.

![Figure P2.17](image6)

2.18 Find $V_m$ in the circuit in Fig. P2.18.

![Figure P2.18](image7)

2.19 Find $V_{da}$ and $V_{be}$ in the circuit in Fig. P2.19.

![Figure P2.19](image8)

2.20 The 10-V source absorbs 2.5 mW of power. Calculate $V_m$ and the power absorbed by the dependent voltage source in Fig. P2.20.
2.21 Find $V_o$ in the network in Fig. P2.21.

2.22 Find $V_o$ in the network in Fig. P2.22.

2.23 Find $V_x$ in the network in Fig. P2.23.

2.24 Find both $I$ and $V_{bd}$ in the circuit in Fig. P2.24.

2.25 Find $V_x$ in the circuit in Fig. P2.25.

2.26 Find $V_x$ in the network in Fig. P2.26.

2.27 Find the power absorbed by the 30-kΩ resistor in the circuit in Fig. P2.27.

2.28 In the network in Fig. P2.28, if $V_i = 12$ V, find $V_s$.

2.29 In the circuit in Fig. P2.29, $P_{kW} = 12$ mW. Find $V_s$. 

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### Figure P2.21

### Figure P2.22

### Figure P2.23

### Figure P2.24

### Figure P2.25

### Figure P2.26

### Figure P2.27

### Figure P2.28

### Figure P2.29
2.30 If $V_s = 4 \, \text{V}$ in the network in Fig. P2.30, find $V_x$.

![Figure P2.30](image)

**Figure P2.30**

2.31 If $V_s = 12 \, \text{V}$ in the circuit in Fig. P2.31, find $V_x$.

![Figure P2.31](image)

**Figure P2.31**

2.32 A commercial power supply is modeled by the network shown in Fig. P2.32.

(a) Plot $V_o$ versus $R_{\text{load}}$ for $1 \, \Omega \leq R_{\text{load}} \leq \infty$.

(b) What is the maximum value of $V_o$ in (a)?

(c) What is the minimum value of $V_o$ in (a)?

(d) If for some reason the output should become short circuited, that is, $R_{\text{load}} \to 0$, what current is drawn from the supply?

(e) What value of $R_{\text{load}}$ corresponds to maximum power consumed?

![Figure P2.32](image)

**Figure P2.32**

2.33 A commercial power supply is guaranteed by the manufacturer to deliver $5 \, \text{V} \pm 1\%$ across a load range of 0 to 10 A. Using the circuit in Fig. P2.33 to model the supply, determine the appropriate values of $R$ and $V$.

![Figure P2.33](image)

**Figure P2.33**

2.34 A power supply is specified to provide $48 \pm 2 \, \text{V}$ at 0–200 A and is modeled by the circuit in Fig. P2.34.

(a) What are the appropriate values for $V$ and $R$?

(b) What is the maximum power the supply can deliver? What values of $I_{\text{load}}$ and $V_o$ correspond to that level?

![Figure P2.34](image)

**Figure P2.34**

2.35 Although power supply loads are often modeled as either resistors or constant current sources, some loads are best modeled as constant power loads, as indicated in Fig. P2.35. Given the model shown in the figure,

(a) Write a $V$–$I$ expression for a constant power load that always draws $P_L$ watts.

(b) If $P_L = 40 \, \text{W}$, $V_{ps} = 9 \, \text{V}$ and $I_o = 5 \, \text{A}$, determine the values of $V_o$ and $R_{ps}$.

![Figure P2.35](image)

**Figure P2.35**

2.36 A student needs a 15-V voltage source for research. She has been able to locate two power supplies, a 10-V supply and a 5-V supply. The equivalent circuits for the two supplies are shown in Fig. P2.36.

(a) Draw an equivalent circuit for the effective 15-V supply.

(b) If she can tolerate a 0.5-V deviation from 15 V, what is the maximum current change the combined supply can satisfy?

![Figure P2.36](image)

**Figure P2.36**
2.37 Given the network in Fig. P2.37, we wish to obtain a voltage of $2 \leq V_o \leq 9$ V across the full range of the pot. Determine the values of $R_1$ and $R_2$.

![Figure P2.37]

**SECTION 2.4**

2.38 Determine $I_L$ in the circuit in Fig. P2.38.

![Figure P2.38]

2.39 Find $V_o$ in the circuit in Fig. P2.39.

![Figure P2.39]

2.40 Find $I_o$ in the network in Fig. P2.40.

![Figure P2.40]

2.41 Find $V_o$ in the network in Fig. P2.41.

![Figure P2.41]

2.42 In the network in Fig. P2.42, $P_{\text{ diss }} = 96$ mW. Find $I_L$.
2.43 In the circuit in Fig. P2.43, \( V_s = 12 \) V. Find \( V_x \).

![Figure P2.43](image)

2.44 In the circuit in Fig. P2.44, \( V_s = 6 \) V. Find \( I_s \).

![Figure P2.44](image)

2.45 Determine \( I_c \) in the circuit in Fig. P2.45.

![Figure P2.45](image)

2.46 Determine \( I_e \) in the circuit in Fig. P2.46.

![Figure P2.46](image)

2.47 Find \( R_{AB} \) in the circuit in Fig. P2.47.

![Figure P2.47](image)

2.48 Find \( R_{AB} \) in the network in Fig. P2.48.

![Figure P2.48](image)

2.49 Find \( R_{AB} \) in the circuit in Fig. P2.49.

![Figure P2.49](image)

2.50 Find \( R_{AB} \) in the circuit in Fig. P2.50.

![Figure P2.50](image)
2.51 Determine $R_{AB}$ in the circuit in Fig. P2.51.

![Figure P2.51](image1)

2.52 Find $R_{AB}$ in the network in Fig. P2.52.

![Figure P2.52](image2)

2.53 Find $R_{AB}$ in the network in Fig. P2.53.

![Figure P2.53](image3)

2.54 Find the equivalent resistance, $R_{eq}$, in the circuit in Fig. P2.54.

![Figure P2.54](image4)

2.55 Find the equivalent resistance, $R_{eq}$, in the network in Fig. P2.55.

![Figure P2.55](image5)

2.56 Find the range of resistance for the following resistors.
(a) 1 kΩ with a tolerance of 5%
(b) 470 Ω with a tolerance of 2%
(c) 22 kΩ with a tolerance of 10%

2.57 Given the network in Fig. P2.57, find the possible range of values for the current and power dissipated by the following resistors.
(a) 390 Ω with a tolerance of 1%
(b) 560 Ω with a tolerance of 2%

![Figure P2.57](image6)

2.58 Given the circuit in Fig. P2.58,
(a) find the required value of $R$.
(b) use Table 2.1 to select a standard 10% tolerance resistor for $R$.
(c) calculate the actual value of $I$.
(d) determine the percent error between the actual value of $I$ and that shown in the circuit.
(e) determine the power rating for the resistor $R$. 

![Figure P2.58](image7)
2.59 The resistors \( R_1 \) and \( R_2 \) shown in the circuit in Fig. P2.59 are 1 \( \Omega \) with a tolerance of 5\% and 2 \( \Omega \) with a tolerance of 10\%, respectively.

(a) What is the nominal value of the equivalent resistance?

(b) Determine the positive and negative tolerance for the equivalent resistance.

\[
\text{Figure P2.59}
\]

\[
\begin{align*}
R_{eq} & \quad R_1 \\
\text{Figure P2.59} & \quad R_2
\end{align*}
\]

SECTION 2.6

2.60 Find \( V_{ab} \) and \( V_{dc} \) in the circuit in Fig. P2.60.

\[
\text{Figure P2.60}
\]

2.61 Find \( I_1 \) and \( V_o \) in the circuit in Fig. P2.61.

\[
\text{Figure P2.61}
\]

2.62 Find \( I_1 \) and \( V_o \) in the circuit in Fig. P2.62.

\[
\text{Figure P2.62}
\]

2.63 Find \( I_o \) in the network in Fig. P2.63.

\[
\text{Figure P2.63}
\]

2.64 Find \( I_1 \) in the circuit in Fig. P2.64.

\[
\text{Figure P2.64}
\]

2.65 Determine \( V_o \) in the network in Fig. P2.65.

\[
\text{Figure P2.65}
\]

2.66 Determine \( I_o \) in the circuit in Fig. P2.66.

\[
\text{Figure P2.66}
\]
2.67 Determine $V_o$ in the network in Fig. P2.67.

![Figure P2.67](image)

2.68 Find $I_o$ in the circuit in Fig. P2.68.

![Figure P2.68](image)

2.69 Find the value of $V_i$ in the network in Fig. P2.69 such that the 5-A current source supplies 50 W.

![Figure P2.69](image)

2.70 Find the value of $V_i$ in the network in Fig. P2.70 such that $V_o = 0$.

![Figure P2.70](image)

2.71 Find the value of $V_i$ in the circuit in Fig. P2.71 such that the power supplied by the 5-A source is 60 W.

![Figure P2.71](image)

2.72 Find the value of $V_i$ in the network in Fig. P2.72 such that the power supplied by the current source is 0.

![Figure P2.72](image)

2.73 Find $V_o$ in the circuit in Fig. P2.73.

![Figure P2.73](image)

2.74 Find $I_o$ in the network in Fig. P2.74.

![Figure P2.74](image)
2.75 Find $I_o$ in the circuit in Fig. P2.75.

![Figure P2.75](image)

2.76 Determine $V_o$ in the circuit in Fig. P2.76.

![Figure P2.76](image)

2.77 Find $V_o$ in the circuit in Fig. P2.77.

![Figure P2.77](image)

2.78 Find $V_o$ in the circuit in Fig. P2.78.

![Figure P2.78](image)

2.79 Find $I_o$ in the circuit in Fig. P2.79.

![Figure P2.79](image)

2.80 Find $I_o$ in the circuit in Fig. P2.80.

![Figure P2.80](image)

2.81 Find $I_o$ in the circuit in Fig. P2.81.

![Figure P2.81](image)

2.82 Find $V_o$ in the circuit in Fig. P2.82.

![Figure P2.82](image)
2.83 Find $I_o$ in the circuit in Fig. P2.83.

2.84 Determine the value of $V_o$ in the circuit in Fig. P2.84.

2.85 Find $P_{1\Omega}$ in the network in Fig. P2.85.

2.86 Find $I_o$ in the network in Fig. P2.86.

2.87 In the network in Fig. P2.87, the power absorbed by the 4-$\Omega$ resistor is 100 W. Find $V_S$.

2.88 If $V_o = 2$ V in the circuit in Fig. P2.88, find $V_S$.

2.89 If $V_o = 6$ V in the circuit in Fig. P2.89, find $I_S$. 
2.90 If $I_s = 2 \text{ mA}$ in the circuit in Fig. P2.90, find $V_o$.

![Figure P2.90]

2.91 If $V_i = 5 \text{ V}$ in the circuit in Fig. P2.91, find $I_s$.

![Figure P2.91]

2.92 In the network in Fig. P2.92, $V_i = 12 \text{ V}$. Find $V_o$.

![Figure P2.92]

2.93 In the circuit in Fig. P2.93, $V_o = 2 \text{ V}$. Find $I_s$.

![Figure P2.93]

2.94 In the network in Fig. P2.94, $V_o = 6 \text{ V}$. Find $I_s$.

![Figure P2.94]

2.95 In $I_s = 4 \text{ mA}$ in the circuit in Fig. P2.95, find $I_s$.

![Figure P2.95]

2.96 If $V_o = 6 \text{ V}$ in the circuit in Fig. P2.96, find $I_s$.

![Figure P2.96]

2.97 Given that $V_o = 4 \text{ V}$ in the network in Fig. P2.97, find $V_s$.

![Figure P2.97]

2.98 Find $I_s$ in the circuit in Fig. P2.98.
2.99 Given $V_o$ in the network in Fig. P2.99, find $I_A$.

**Figure P2.99**

2.100 Given $I_o = 2$ mA in the circuit in Fig. P2.100, find $I_A$.

**Figure P2.100**

2.101 Given $I_o = 2$ mA in the network in Fig. P2.101, find $V_A$.

**Figure P2.101**

2.102 Find the power absorbed by the network in Fig. P2.102.

**Figure P2.102**

2.103 Find the value of $g$ in the network in Fig. P2.103 such that the power supplied by the 3-A source is 20 W.

**Figure P2.103**

2.104 Find the power supplied by the 24-V source in the circuit in Fig. P2.104.

**Figure P2.104**
2.105 Find $I_o$ in the circuit in Fig. P2.105.

![Figure P2.105](image)

2.106 Find $I_o$ in the circuit in Fig. P2.106.

![Figure P2.106](image)

2.107 Find $V_o$ in the network in Fig. P2.107.

![Figure P2.107](image)

2.108 Find $I_o$ in the circuit in Fig. P2.108.

![Figure P2.108](image)

2.109 Find $I_o$ in the circuit in Fig. P2.109.

![Figure P2.109](image)

2.110 Find $V_o$ in the circuit in Fig. P2.110.

![Figure P2.110](image)

2.111 Find $V_o$ in the network in Fig. P2.111.

![Figure P2.111](image)
2.112 Find $V_o$ in the network in Fig. P2.112.

![Figure P2.112](image1)

2.113 Find $I_o$ in the network in Fig. P2.113.

![Figure P2.113](image2)

2.114 Find the power absorbed by the 10-kΩ resistor in the circuit in Fig. P2.114.

![Figure P2.114](image3)

2.115 Find the value of $k$ in the network in Fig. P2.115 such that the power supplied by the 6-A source is 108 W.

![Figure P2.115](image4)

2.116 For the network in Fig. P2.116, choose the values of $R_m$ and $R_a$ such that $V_o$ maximized. What is the resulting ratio, $V_o/V_s$?

![Figure P2.116](image5)

2.117 A typical transistor amplifier is shown in Fig. P2.117. Find the amplifier gain $G$ (i.e., the ratio of the output voltage to the input voltage).

![Figure P2.117](image6)
2.118 In many amplifier applications we are concerned not only with voltage gain, but also with power gain.

\[
\text{Power gain} = \frac{A_p \text{ (power delivered to the load)}}{\text{power delivered to the input)}
\]

Find the power gain for the circuit in Fig. P2.118, where \( R_L = 60 \, \text{k}\Omega \).

![Figure P2.118](image_url)

**Typical Problems Found on the FE Exam**

**2FE-1** Find the power generated by the source in the network in Fig. 2PFE-1.

![Fig. 2PFE-1](image_url)

**2FE-2** Find the equivalent resistance of the circuit in Fig. 2PFE-2 at the terminals \( A-B \).

![Fig. 2PFE-2](image_url)

**2FE-3** Find the voltage \( V_o \) in the network in Fig. 2PFE-3.

![Fig. 2PFE-3](image_url)

**2FE-4** Find the current \( I_o \) in the circuit in Fig. 2PFE-4.

![Fig. 2PFE-4](image_url)