Solutions Manual
Internal Combustion Engines: Applied Thermosciences

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Chapter 1

Introduction to Internal Combustion Engines

1.1) Compute the mean piston speed, bmep (bar), torque (Nm), and the power per piston area for the engines listed in Table 1.2.

Engine	Bore (mm)	Stroke (mm)	Cylinders	Speed (rpm)	Power (kW)
Marine	136	127	12	2600	1118
Truck	108	95	8	6400	447
Airplane	86	57	8	10500	522

Table 1.2 Engine Data for Homework Problems

a)

$$\begin{aligned}
\overline{U}_p &= 2Ns \\
\overline{U}_p &= 2{\cdot}2600\frac{\text{rev}}{\text{min}} \cdot \frac{\text{min}}{60\text{s}} \cdot 0.127 = 11\,{}^{\text{m}}\!/\!_{\text{s}} \\
&= 2{\cdot}6400 \cdot \frac{1}{60} \cdot 0.095 = 20\,{}^{\text{m}}\!/\!_{\text{s}} \\
&= 2{\cdot}10{,}500 \cdot \frac{1}{60} \cdot 0.057 = 19.95\,{}^{\text{m}}\!/\!_{\text{s}}
\end{aligned}$$

b)

$$\begin{aligned}
\text{bmep} = \frac{2\dot{W}}{V_d N} &= \frac{2\dot{W}}{n_c\left(\frac{\pi}{4}\right)(b)^2(s)(N)} \\
&= \frac{2\cdot 1118}{12\left(\frac{\pi}{4}\right)(0.136)^2(0.127)\left(\frac{2600}{60}\right)} = 2.33\times 10^3\,\text{kPa} = 23.3\,\text{bar} \\
&= \frac{2\cdot 447}{8\left(\frac{\pi}{4}\right)(0.108)^2(0.095)\left(\frac{6400}{60}\right)} = 1.20\times 10^3\,\text{kPa} = 12.0\,\text{bar} \\
&= \frac{2\cdot 522}{8\left(\frac{\pi}{4}\right)(0.086)^2(0.057)\left(\frac{10{,}500}{60}\right)} = 2.25\times 10^3\,\text{kPa} = 22.5\,\text{bar}
\end{aligned}$$

c)

$$
\begin{aligned}
\tau &= \frac{\dot{W}}{2\pi N} \\
&= \frac{(1118)\,(1000)}{(2\pi)\left(\frac{2600}{60}\right)} = 4106.0\,\mathrm{Nm} \\
&= \frac{(447)\,(1000)}{(2\pi)\left(\frac{6400}{60}\right)} = 667.0\mathrm{Nm} \\
&= \frac{(522)\,(1000)}{(2\pi)\left(\frac{10500}{60}\right)} = 474.7\,\mathrm{Nm}
\end{aligned}
$$

d)

$$
\begin{aligned}
\frac{\text{Power}}{\text{Piston Area}} &= \frac{\dot{W}}{n_c\left(\frac{\pi}{4}\right)(b)^2} \\
&= \frac{(1118)}{12\left(\frac{\pi}{4}\right)(0.136)^2} = 6413\,{}^{\mathrm{kN}}\!/\!{}_{\mathrm{m}^2} \\
&= \frac{(447)}{8\left(\frac{\pi}{4}\right)(0.108)^2} = 6099\,{}^{\mathrm{kN}}\!/\!{}_{\mathrm{m}^2} \\
&= \frac{(522)}{8\left(\frac{\pi}{4}\right)(0.086)^2} = 11{,}233\,{}^{\mathrm{kN}}\!/\!{}_{\mathrm{m}^2}
\end{aligned}
$$

1.2) A six-cylinder two-stroke engine with a compression ratio $r = 9$ produces a torque of 1100 Nm at a speed of 2100 rpm. It has a bore b of 123 mm and a stroke s of 127 mm. a.) What is the displacement volume and the clearance volume of a cylinder? b.) What is the engine bmep, brake work, and mean piston speed?

a)

$$V_d = \left(\frac{\pi}{4}\right)(b)^2(s) = \left(\frac{\pi}{4}\right)(0.123)^2(0.127)$$
$$V_d = 1.51 \times 10^{-3}\,\mathrm{m}^3$$

But we know that

$$r = \frac{V_d + V_c}{V_c} \longrightarrow V_c = \frac{V_d}{r-1}$$

Solving for the clearance volume we get

$$V_c = \frac{V_d}{r-1} = \frac{1.51 \times 10^3}{9-1}$$
$$V_c = 1.89 \times 10^{-4}\,\mathrm{m}^3$$

b)

$$\mathrm{bmep} = \frac{2\pi\tau}{n_c V_d} = \frac{(2\pi)(1100)}{(6)\left(1.51 \times 10^{-3}\right)} = 7.63 \times 10^5\,\mathrm{Pa} = 763\,\mathrm{kPa}$$
$$\dot{W} = 2\pi\tau N = (2\pi)(1100)\left(\frac{2100}{60}\right) = 2.42 \times 10^5\,\mathrm{W} = 242\,\mathrm{kW}$$
$$\overline{U}_p = 2Ns = 2\left(\frac{2100}{60}\right)(0.129) = 8.89\,\mathrm{m/s}$$

1.3) A 4 cylinder 2.5 L spark ignited engine is mounted on a dyno and operated at a speed of $N =$ 3000 rpm. The engine has a compression ratio of 10:1 and mass air-fuel ratio of 15:1. The inlet air manifold conditions are 80 kPa and 313 K. The engine produces a torque of 160 Nm and has a volumetric efficiency of 0.82. a.) What is the brake power $\dot{W}_b$ (kW)? b.) What is the brake specific fuel consumption bsfc (g/kWh)?

a)

$$\dot{W}_b = 2\pi\tau N = (2\pi)(160)\left(\frac{3000}{60}\right) = 5.03e4\,\mathrm{Nm/s}$$
$$\dot{W}_b = 50.3\,\mathrm{kw}$$

b)

$$\dot{m}_f = \frac{\dot{m}_a}{AF} = \frac{\frac{1}{2}(e_v)(\rho)(V_d)(N)}{AF} = \frac{\frac{1}{2}(e_v)\left(\frac{P}{R_i T_i}\right)(V_d)(N)}{AF}$$
$$= \frac{\frac{1}{2}(0.82)\left(\frac{80}{(0.287)(313)}\right)\left(2.5\times10^{-3}\right)\left(\frac{3000}{60}\right)}{AF}$$
$$\dot{m}_f = 3.04\times10^{-5}\,\mathrm{kg/s}$$

$$\mathrm{bsfc} = \frac{\dot{m}_f}{\dot{W}_b} = \frac{3.04\times10^{-5}}{50.3} = 6.05\times10^{-5}\,\mathrm{kg/kJ}$$
$$\mathrm{bsfc} = 217\,\mathrm{g/kWh}$$

1.4) The volumetric efficiency of the fuel injected marine engine in Table 1.2 is 0.80 and the inlet manifold density is 50% greater than the standard atmospheric density of $\rho_{amb} = 1.17$ kg/m^3. If the engine speed is 2600 rpm, what is the air mass flow rate (kg/s)?

a)

$$e_v = \frac{2\left(\dot{m}_a + \dot{m}_f\right)}{\rho V_d N} = \frac{2\left(\dot{m}_a + \dot{m}_f\right)}{\rho\left[n_c\left(\frac{\pi}{4}\right)b^2 s\right]N}$$

We can rearrange the equation to solve for the mass flow rate of air

$$\dot{m}_a = \frac{1}{2}e_v\rho_i V_d N = \frac{1}{2}e_v\rho_i\left[n_c\left(\frac{\pi}{4}\right)b^2 s\right]N$$
$$\dot{m}_a = \frac{1}{2}0.8(1.5)\left(1.17\right)\left[(12)\left(\frac{\pi}{4}\right)(0.136)^2\,(0.127)\right]\left(\frac{2600}{60}\right)$$
$$\dot{m}_a = 0.672\,{}^{\mathrm{kg}}/_{\mathrm{s}}$$

1.5) A 380 cc single-cylinder two-stroke motorcycle engine is operating at 5500 rpm. The engine has a bore of 82 mm and a stroke of 72 mm. Performance testing gives a bmep = 6.81 bar, bsfc = 0.49 kg/kW hr, and delivery ratio of 0.748. (a) What is the fuel to air ratio, FA? (b) What is the air mass flow rate (kg/s)?

a)

$$\rho_{amb} = 1.17\ \mathrm{kg/m^3} \longrightarrow \text{Assumed value for density}$$

$$FA = \frac{\dot{m}_f}{\dot{m}_a}$$

$$\text{bmep} = \frac{\dot{W}_b}{V_d N}$$

$$\text{bsfc} = \frac{\dot{m}_f}{\dot{W}_b} \longrightarrow \dot{W}_b = \frac{\dot{m}_f}{\text{bsfc}} \longrightarrow \text{bmep} = \frac{\dot{m}_f}{(\text{bsfc})\,(V_d)\,(N)}$$

$$D_r = \frac{\dot{m}_a}{\rho V_d N}$$

Solving for the fuel mass flow rate:

$$\dot{m}_f = (\text{bmep})\,(\text{bsfc})\,(V_d)\,(N) = \left(6.81 \times 10^5\right)\left(\frac{0.49}{3600}\right)\left[\left(\frac{\pi}{4}\right)(0.082)^2\,(0.072)\right]\left(\frac{5500}{60}\right) = 3.23\ \mathrm{g/s}$$

$$\dot{m}_f = 3.23 \times 10^{-3}\ \mathrm{kg/s}$$

Solving for the air mass flow rate:

$$\dot{m}_a = (D_r)\,(\rho_a)\,(V_d)\,(N) = (0.748)\,(1.17)\left[\left(\frac{\pi}{4}\right)(0.082)^2\,(0.072)\right]\left(\frac{5500}{60}\right) = 30.48\ \mathrm{g/s}$$

$$\dot{m}_a = 3.048 \times 10^{-2}\ \mathrm{kg/s}$$

Solving for the fuel to air ratio:

$$FA = \frac{\dot{m}_f}{\dot{m}_a} = \frac{3.23}{30.48}$$

$$FA = 0.106$$

b) See the calculation above

$$\dot{m}_a = 3.048 \times 10^{-2}\ \mathrm{kg/s}$$

1.6) A 3.8 L four-stroke 4 four cylinder fuel-injected automobile engine has a power output of 88 kW at 4000 rpm and volumetric efficiency of 0.85. The bsfc is 0.35 kg/kW hr. If the fuel has a heat of combustion of 42 MJ/kg, what are the bmep, thermal efficiency, and air to fuel ratio? Assume atmospheric conditions of 298 K and 1 bar.

a)

$$\text{bmep} = \frac{2\dot{W}}{V_d N} = \frac{2\,(88)}{\left(3.8\times 10^{-3}\right)\left(\frac{4000}{60}\right)}$$
$$\text{bmep} = 694\,\text{kPa}$$

b)

$$\eta = \frac{1}{\text{bsfc}\,(q_0)} = \frac{1}{0.35\left(\frac{42\times 10^3}{3600}\right)}$$
$$\eta = 0.24$$

c)

$$\dot{m}_f = \dot{W}\,(\text{bsfc}) = (88)\left(\frac{0.35}{3600}\right) = 8.55\times 10^{-3}\,{}^{\text{kg}}/_{\text{s}}$$
$$\dot{m}_a = \frac{e_v\,(\rho_a)\,(V_d)\,(N)}{2} = \frac{(0.85)\,(1.17)\left(3.8\times 10^{-3}\right)\left(\frac{4000}{60}\right)}{2} = 0.125\,{}^{\text{kg}}/_{\text{s}}$$
$$A = \frac{\dot{m}_a}{\dot{m}_f} = \frac{0.125}{8.55\times 10^{-3}}$$
$$A = 14.7$$

1.7) A 4.0 L six cylinder engine is operating at 3000 rpm. The engine has a compression ratio of 10:1, and volumetric efficiency of 0.85. If the bore and stroke are equal, what is the stroke, the mean piston speed, cylinder clearance volume, and air mass flow rate into the engine? Assume standard inlet conditions.

a)

$$V_d = n_c \frac{\pi}{4} b^3 \longrightarrow b = s = \left[\frac{4V_d}{\pi n_c}\right]^{1/3} = \left[\frac{(4)\left(4.0 \times 10^{-3}\right)}{(\pi)(6)}\right]^{1/3}$$

$$s = 0.0947\,\mathrm{m}$$

b)

$$\overline{U}_p = 2Ns = 2\left(\frac{3000}{60}\right)(0.0947)$$

$$\overline{U}_p = 9.47\,\mathrm{m/s}$$

c)

$$V_c = \frac{V_d}{r-1} = \frac{4.0 \times 10^{-3}}{10-1}$$

$$V_c = 4.44 \times 10^{-4}\,\mathrm{m}^3$$

d)

$$\dot{m}_a = \frac{1}{2} e_v \rho_i V_d N = \frac{1}{2}(e_v)\left(\frac{P}{RT}\right)(V_d)(N) = \frac{1}{2}(0.85)\left(\frac{100}{(0.287)(298)}\right)\left(4.0 \times 10^{-3}\right)\left(\frac{3000}{60}\right)$$

$$\dot{m}_a = 9.94 \times 10^{-2}\,\mathrm{kg/s}$$

1.8) Chose an automotive, marine, or aviation engine of interest, and compute the engine's mean piston speed, bmep, power/volume, mass/volume, and power/mass. Compare your calculated values with those presented in Table 1.1.

1.9) Compare the approximate, Equation (1.29), and exact, Equation (1.26), dimensionless cylinder volume versus crank angle profiles for $r = 8, s = 100$ mm, and $l = 150$ mm. What is the maximum error and at what crank angle does it occur ?

Using the program `Volume.m`, the exact and approximate volume profiles are computed and plotted below. The program output indicates the maximum error is 16.3 % at crank angles of ± 62 degrees.

```
function [ ]=Volume( )
% this program computes and plots the exact
% and approximate cylinder volume
clear();
r = 8; % compression ratio
s = 100;   % stroke (mm)
len= 150; %connecting rod length (mm)
ep=s/(2*len);
theta=-180:1:180; %crankangle theta vector
ys1=(1-cosd(theta))/2; %approx y/s
ys2= ys1+ (1-(1- ep^2*sind(theta).^2).^(1/2))/(2*ep); %exact y/s
vol1 = 1+(r-1)*ys1; %approx volume
vol2= 1+(r-1)*ys2;  % exact volume
diff = abs(vol1-vol2)./vol1 * 100;
[diffmax, id_max] = max(diff);
thmax=theta(id_max);
%plot results
fprintf('Max Diff (percent) = %5.2f  at theta (deg) = %5.2f \n', diffmax,thmax);
figure;
plot(theta,vol1,'--',theta,vol2,'-','linewidth',2);
set(gca,'Xlim',[-180 180],'Ylim',[0 r],'fontsize',18,'linewidth',2);
xlabel('Crank Angle (deg)','fontsize', 18);
ylabel('Dim. Cylinder Volume','fontsize', 18);
legend('Approx. Volume', 'Exact Volume','Location', 'North');
figure;
plot(theta,diff,'linewidth',2);
set(gca,'Xlim',[-180 180],'fontsize',18,'linewidth',2);
xlabel('Crank Angle (deg)','fontsize', 18);
ylabel('Dim. Error (%)','fontsize', 18);
end
```

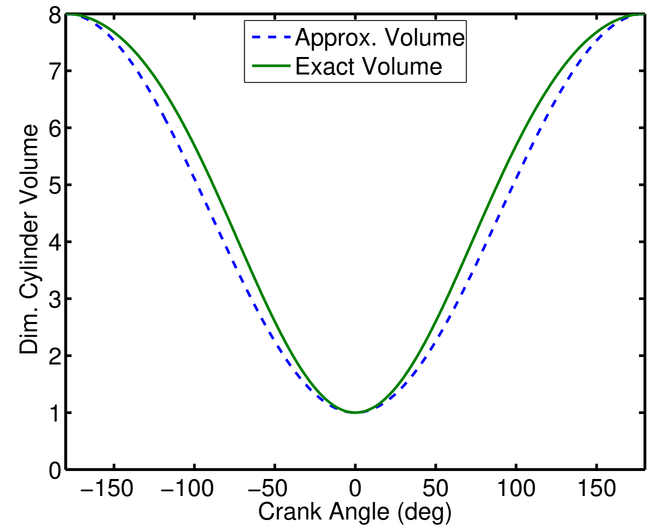

Figure 1.1: Problem 1-9 volume plot

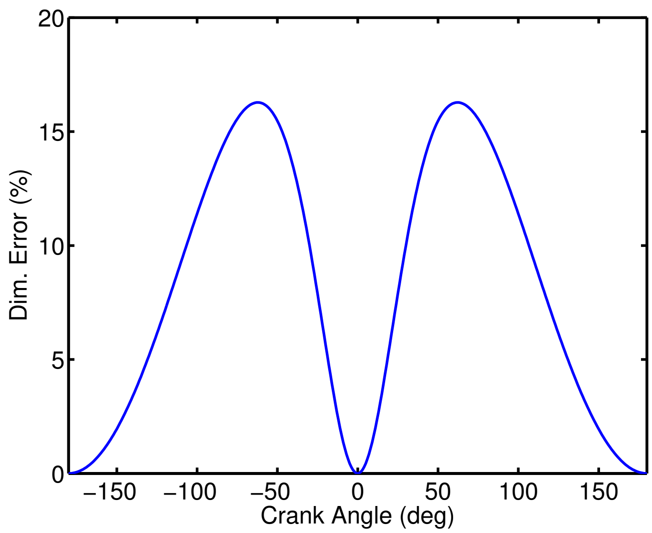

Figure 1.2: Problem 1-9 volume error plot

1.10) Plot the dimensionless piston velocity for an engine with a stroke $s = 100$ mm and connecting rod length $l = 150$ mm.

Using the program `Velocity.m`, the dimensional and dimensionless velocity profiles are computed and plotted below. The program output indicates the maximum velocity is 11.04 m/s at an angle of 73 degrees.

```
function [ ]=Velocity( )
% this program computes and plots the
% dimensional and dimensionless piston velocity
clear();
N=2000; %rev/min
s = 0.10;   % stroke (m)
len= 0.150; %connecting rod length (m)
ep=s/(2*len);
theta=0:1:180; %crankangle theta vector
term1=pi/2*sind(theta);
term2= (1+(ep*cosd(theta))./(1 - ep^2*sind(theta).^2).^(1/2)); %exact y/s
Upbar=term1.*term2; %dimensionless velocity
Up=Upbar*2*N*s/60; % m/s
[umax, id_max] = max(Up); %get max velocity
thmax=theta(id_max); % and crank angle
%output results
fprintf( 'U max (m/s) = %5.2f  at theta (deg) = %5.2f \n', umax,thmax);
figure;
plot(theta,Up,'linewidth',2);
set(gca,'Xlim',[0 180],'fontsize',18,'linewidth',2);
xlabel('Crank Angle (deg)','fontsize', 18);
ylabel(' Piston Velocity (m/s)','fontsize', 18);
figure;
plot(theta,Upbar,'linewidth',2);
set(gca,'Xlim',[0 180],'Ylim',[0 1.8],'fontsize',18,'linewidth',2);
xlabel('Crank Angle (deg)','fontsize', 18);
ylabel('Dimensionless Piston Velocity','fontsize', 18);
end
```

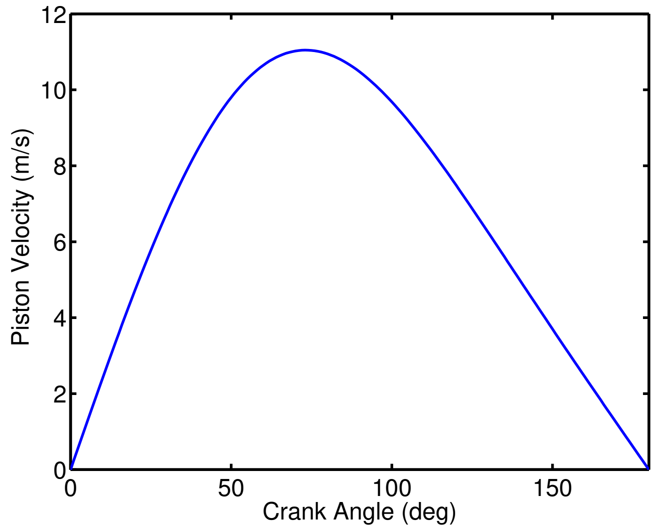

Figure 1.3: Problem 1-10 velocity plot

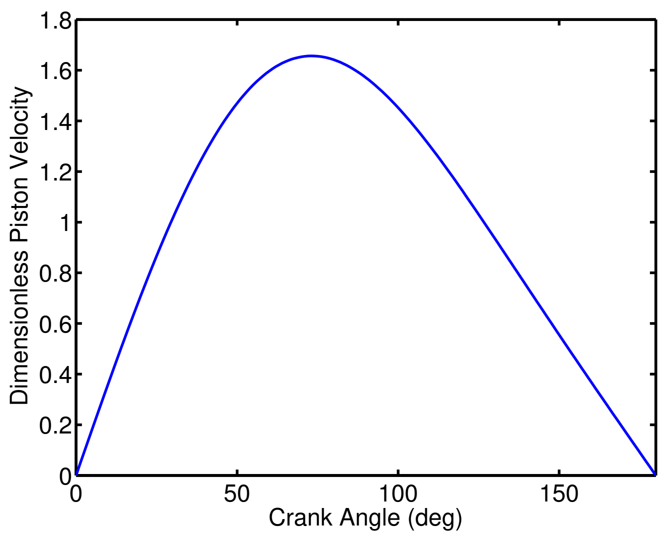

Figure 1.4: Problem 1-10 dimensionless velocity plot

1.11) Assuming that the mean effective pressure, mean piston speed, power per unit piston area, and mass per unit displacement volume are all size independent, how will the power per unit weight of an engine depend upon the number of cylinders if the total displacement is constant? To make the analysis easier, assume that the bore and stroke are equal.

a) The power/weight can be expanded in terms of the power/area and mass/displacement volume

$$\frac{\dot{W}}{m} = \frac{\frac{\dot{W}}{A}\frac{\pi}{4}b^2 n_c}{\frac{m}{V_d}\frac{\pi}{4}b^2 s n_c}$$

If $b = s$, then

$$V_d = \frac{\pi}{4}b^3 n_c \longrightarrow b = s = \left(\frac{4V_d}{\pi n_c}\right)^{1/3}$$

Substituting for s

$$\frac{\dot{W}}{m} = \frac{\frac{\dot{W}}{A}}{\frac{m}{V_d}}\left(\frac{\pi n_c}{4V_d}\right)^{1/3} \approx {n_c}^{1/3}$$

The power per unit weight scales as the number of cylinders to the 1/3 power.