EXERCISE 8.1:
Design a positive pressure dilute-phase pneumatic transport system to carry 500 kg/hr of a powder of particle density 1800 kg/m³ and mean particle size 150 μm across a horizontal distance of 100 metres and a vertical distance of 20 metres using ambient air. Assume that the pipe is smooth, that four 90° bends are required and that the allowable pressure loss is 0.7 bar.

SOLUTION TO EXERCISE 8.1:
Design in this case means determine the pipe size and air flowrate which would give a total system pressure loss near to, but not exceeding, the allowable pressure loss. The design procedure requires trial and error calculations. Pipes are available in fixed sizes and so the procedure is to select a pipe size and determine the saltation velocity from Text-Equation 8.3. Saltation velocity is important since, in any system with horizontal and vertical pipelines, the saltation velocity is always greater than the choking velocity - so if we avoid saltation, we avoid choking. The system pressure loss is then calculated at a superficial gas velocity equal to 1.5 times the saltation velocity (this gives a reasonable safety margin bearing in mind the accuracy of the correlation in Text-Equation 8.3). The calculated system pressure loss is then compared with the allowable pressure loss. The pipe size selected may then be altered and the above procedure repeated until the calculated pressure loss matches that allowed.

Step 1 Selection of pipe size: Select 50 mm internal diameter pipe.

Step 2 Determine gas velocity
Use the Rizk correlation of Text-Equation (8.3) to estimate the saltation velocity, $U_{SALT}$. Text-Equation (8.3) rearranged becomes:

$$U_{SALT} = \left[\frac{4M_l 10^\alpha g \beta \left(\frac{\beta - 2}{2}\right)}{\pi \rho_f} \right]^{1/\beta}$$

where $\alpha = 1440x + 1.96$ and $\beta = 1100x + 2.5$. In the present case $\alpha = 2.176$, $\beta = 2.665$ and $U_{SALT} = 9.21$ m/s. Therefore, superficial gas velocity, $U = 1.5 \times 9.211$ m/s = 13.82 m/s.
Step 3 Pressure loss calculations

a) Horizontal Sections

Starting with Text-Equation 8.15 and expression for the total pressure loss in the horizontal sections of the transport line may be generated. We will assume that all the initial acceleration of the solids and the gas take place in the horizontal sections and so terms 1 and 2 are required. For term 3 the Fanning friction Equation is used assuming that the pressure loss due to gas/wall friction is independent of the presence of solids. For term 4 we employ the Hinkle correlation (Text-Equation 8.17). Terms 5 and 6 became zero as $\theta = 0$ for horizontal pipe. Thus, the pressure loss, $\Delta P_H$, in the horizontal sections of the transport line is given by:

$$\Delta P_H = \frac{\rho_f \varepsilon_H U_H^2}{2} + \frac{\rho_p (1 - \varepsilon_H) U^2}{2} + \frac{2 f_p \rho_f U^2 L_H}{D} + \frac{2 f_p \rho_p (1 - \varepsilon_H) U_{pH}^2 L_H}{D}$$

where the subscript H refers to the values specific to the horizontal sections.

To use this Equation we need to know $\varepsilon_H$, $U_{fH}$ and $U_{pH}$. Hinkle’s correlation gives us $U_{pH}$:

$$U_{pH} = U (1 - 0.0638 \times 0.3 \times 0.5) = 11.15 \text{ m/s}$$

From continuity, $G = \rho_p (1 - \varepsilon_H) U_{pH}$. Solids flux, $G = M_p / A = \frac{500}{3600} \times \frac{1}{\pi (0.05)^2} = 70.73 \text{ kg / m}^2 \cdot \text{s}$

thus $\varepsilon_H = 1 - \frac{G}{\rho_p U_{pH}} = 0.9965$

and $U_{fH} = \frac{U}{\varepsilon_H} = \frac{13.82}{0.9965} = 13.87 \text{ m / s}$

Friction factor $f_p$ is found from Text-Equation (8.19) with $C_D$ estimated at the relative velocity $(U_{fH} - U_{pH})$, using the approximate correlations given below, (or by using an appropriate $C_D$ versus Re chart [see Chapter 2])

- $\text{Re}_{p} < 1$ : $C_D = 24/\text{Re}_{p}$
- $1 < \text{Re}_{p} < 500$ : $C_D = 18.5 \text{ Re}_{p}^{-0.6}$
- $500 < \text{Re}_{p} < 2 \times 10^5$ : $C_D = 0.44$
Thus, for flow in the horizontal sections, $\text{Re}_p = \frac{\rho_f (U_{H} - U_{pH}) x}{\mu}$

For ambient air, $\rho_f = 1.2 \text{ kg/m}^3$ and $\mu = 18.4 \times 10^{-6} \text{ Pas}$, giving

$\text{Re}_p = \frac{150 \times 10^{-6} \times 1.2 \times (13.87 - 11.15)}{18.4 \times 10^{-6}} = 26.5$

and so, using the approximate correlations above, $C_D = 18.5 \text{ Re}^{-0.6} = 2.59$

Substituting $C_D = 2.59$ in Text-Equation 8.19 we have:

$$f_p = \frac{3}{8} \times \frac{1.2}{1800} \times 2.59 \times \frac{0.050}{150 \times 10^{-6}} \left\{ \frac{13.87 - 11.15}{11.15} \right\}^2 = 0.01277$$

To estimate the gas friction factor we use the Blasius correlation for smooth pipes, $f_g = 0.079 \times \text{Re}^{-0.25}$. The Reynolds number calculated based on the superficial gas velocity:

$$\text{Re} = \frac{0.05 \times 1.2 \times 13.82}{18.4 \times 10^{-6}} = 45065$$

which gives $f_g = 0.0054$.

Thus the components of the pressure loss in the horizontal pipe from Text-Equation 8.15 are:

Term 1 (gas acceleration): $= \frac{\rho_f \varepsilon_H U_{H}^2}{2} = \frac{1.2 \times 0.9965 \times 13.87^2}{2} = 114.9 \text{ Pa}$.

Term 2 (solids acceleration):

$$= \frac{\rho_p (1 - \varepsilon_H) U_{pH}^2}{2} = \frac{1800 \times (1 - 0.9965) \times 11.15^2}{2} = 394.4 \text{ Pa}.$$

Term 3 (gas friction):

$$= \frac{2 f_g \rho_f U_{H}^2 L_H}{D} = \frac{2 \times 0.0054 \times 1.2 \times 13.82^2 \times 100}{0.05} = 4968 \text{ Pa}.$$

Term 4 (solids friction):

$$= \frac{2 f_p \rho_p (1 - \varepsilon_p) U_{pH}^2}{D} = \frac{2 \times 0.01276 \times 1800 \times (1 - 0.9965) \times 11.15^2 \times 100}{0.05} = 40273 \text{ Pa}.$$

This gives $\Delta p_H = 45751 \text{ Pa}$. 

SOLUTIONS TO CHAPTER 8 EXERCISES: PNEUMATIC TRANSPORT
Starting again with Text-Equation 8.15, the general pressure loss Equation, an expression for the total pressure loss in the vertical section may be derived. Since the initial acceleration of solids and gas was assumed to take place in the horizontal sections, terms 1 and 2 become zero. The Fanning friction Equation is used to estimate the pressure loss due to gas-to-wall friction (term 3) assuming solids have negligible effect on this pressure loss. For term 4 the modified Konno and Saito correlation (Text-Equation 8.16) is used. For vertical transport \( \theta = 90^\circ \) in terms 5 and 6.

Thus, the pressure loss, \( \Delta p_v \), in the vertical sections of the transport line is given by:

\[
\Delta p_v = \frac{2f g \rho_p U^2 L_v}{D} + 0.057 g L_v \sqrt{\frac{g}{D} + \rho_g (1 - \varepsilon_v) g L_v + \rho_p \varepsilon_v g L_v}
\]

where subscript \( v \) refers to values specific to the vertical sections.

To use this Equation we need to calculate the voidage of the suspension in the vertical pipe line \( \varepsilon_v \):

Assuming particles behave as individuals, then slip velocity is equal to single particle terminal velocity, \( U_T \) (also noting that the superficial gas velocity in both horizontal and vertical section is the same and equal to \( U \))

\[
\text{i.e.} \quad U_{pv} = \frac{U}{\varepsilon_v} - U_T
\]

continuity gives particle mass flux, \( G = \rho_p (1 - \varepsilon_v) U_{pv} \)

Combining these Equations gives a quadratic in \( \varepsilon_v \) which has only one possible root.

\[
\varepsilon_v^2 U_T - \left[ U_T + U + \frac{G}{\rho_p} \right] \varepsilon_v + U = 0
\]

The single particle terminal velocity, \( U_T \) may be estimated as shown in Chapter 2, giving \( U_T = 0.715 \) m/s assuming the particles are spherical.

And so, solving the quadratic Equation, \( \varepsilon_v = 0.9970 \)

The components of the pressure loss in the vertical pipe are therefore:
Term 3 (gas friction): \[ \frac{2f_g \rho_f U^2 L_v}{D} = \frac{2 \times 0.0054 \times 1.2 \times 13.82^2 \times 20}{0.05} = 993.7 \text{ Pa.} \]

Term 4 (solids friction): \[ 0.057 \times GL_v \sqrt{\frac{g}{D}} = 0.057 \times 70.73 \times 20 \times \sqrt{\frac{9.81}{0.05}} = 1129.4 \text{ Pa} \]

Term 5 (solids gravitational head): \[ \rho_p (1 - \varepsilon_v) g L_v = 1800 \times (1 - 0.9970) \times 9.81 \times 20 = 1055.8 \text{ Pa.} \]

Term 6 (gas gravitational head): \[ \rho_f \varepsilon_f g L_v = 1.2 \times 0.9970 \times 9.81 \times 20 = 234.7 \text{ Pa.} \]

and thus total pressure loss across vertical sections, \( \Delta p_v = 3414 \text{ Pa} \)

c) Bends

The pressure loss across each 90 degree bend is taken to be equivalent to that across 7.5 m of vertical pipe.

Pressure loss per metre of vertical pipe = \( \frac{\Delta p_v}{L_v} = 170.7 \text{ Pa} / \text{m} \)

Therefore, pressure loss across four 90° bends

\[ = 4 \times 7.5 \times 170.7 \text{ Pa} \]
\[ = 5120.4 \text{ Pa} \]

And so,

\[ \begin{bmatrix} \text{total pressure loss} \\ \text{loss across vertical sections} \end{bmatrix} = \begin{bmatrix} \text{loss across horizontal sections} \end{bmatrix} + \begin{bmatrix} \text{loss across bends} \end{bmatrix} \]

\[ = 3413.6 + 45751.6 + 5120.4 \text{ Pa} \]

\[ = 0.543 \text{ bar} \]

Step 4 Compare calculated and allowable pressure losses

The allowable system pressure loss is 0.7 bar and so we may select a smaller pipe size and repeat the above calculation procedure. The table below gives the results for a range of pipe sizes.
<table>
<thead>
<tr>
<th>Pipe inside diameter (mm)</th>
<th>Total System Pressure Loss (bar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.543</td>
</tr>
<tr>
<td>40</td>
<td>0.857</td>
</tr>
</tbody>
</table>

In this case we would select 50 mm pipe work which gives a total system pressure loss of 0.543 bar. The design details for this selection are given below:

Pipe size: 50 mm inside diameter
Air flowrate = 0.027 m³/s
Air superficial velocity = 13.82 m/s
Saltation velocity = 9.21 m/s
Solids loading = 4.26 kg solid/kg air
Total system pressure loss = 0.543 bar

**EXERCISE 8.2:**

It is required to use an existing 50 mm inside diameter vertical smooth pipe as lift line to transfer 2000 kg/hr of sand of mean particle size 270 μm and particle density 2500 kg/m³ to a process 50 metres above the solids feed point. A blower is available which is capable of delivering 60 m³/hr of ambient air at a pressure of 0.3 bar. Will the system operate as required?

**SOLUTION TO EXERCISE 8.2:**

To test whether the system will operate, we will first check that the air volume flow rate is satisfactory:

The superficial gas velocity in the lift line must exceed the predicted choking velocity by a reasonable safety margin. The choking velocity is predicted using Text-Equation 8.1 and 8.2.

\[
\frac{U_{CH}}{e_{CH}} - U_T = \frac{G}{\rho_p (1 - e_{CH})}
\]  
(Text-Equation 8.1)

\[
\rho_t^{0.77} = \frac{2250D(e_{CH}^{-4.7} - 1)}{\left[U_{CH} - \frac{e_{CH}}{U_{CH}} - U_T\right]^2}
\]  
(Text-Equation 8.2)

The single particle terminal velocity, \(U_T\) may be estimated as shown in Chapter 2, giving \(U_T = 1.77\) m/s (assuming the particles are spherical).
Solids flux, \( G = \frac{M_p}{A} = \frac{2000}{3600} \times \frac{1}{\frac{\pi}{4} (0.05)^2} = 282.9 \text{ kg} / \text{m}^2.\text{s} \)

Substituting Text-Equation 8.1 into Text-Equation 8.2 gives:

\[
\rho_f^{0.77} = \frac{2250D \left(e_{CH}^{-4.7} - 1\right) \rho_p^2 (1 - \varepsilon_{CH})^2}{G^2}
\]

which can be solved by trial and error to give \( \varepsilon_{CH} = 0.9705 \).

Substituting back into Text-Equation 8.1 gives choking velocity \( U_{CH} = 5.446 \text{ m/s} \).

Actual maximum volume flow rate available at the maximum pressure is 60 m\(^3\)/h, which in a 50 mm diameter pipe gives a superficial gas velocity of 8.49 m/s. Operating at this superficial gas velocity would give us a 56\% safety margin over the predicted choking velocity (\( U = U_{CH} \times 1.56 \)), which is acceptable.

The next step is to calculate the lift line pressure loss at this gas flow rate and compare it with the available blower pressure at this flow rate.

Starting with Text-Equation 8.15, the general pressure loss Equation, an expression for the total pressure loss in the vertical lift line may be derived. Initial acceleration of solids and gas must be taken into account and so terms 1 and 2 are included. The Fanning friction Equation is used to estimate the pressure loss due to gas-to-wall friction (term 3) assuming solids have negligible effect on this pressure loss. For term 4 the modified Konno and Saito correlation (Text-Equation 8.16) is used. For vertical transport \( \theta \) is 90\(^\circ\) in terms 5 and 6.

Thus, the pressure loss, \( \Delta p_v \), in the vertical sections of the transport line is given by:

\[
\Delta p_v = \rho_f \varepsilon_v U_{pv}^2 \frac{2}{2} + \rho_p (1 - \varepsilon_v) U_{pv}^2 \frac{2}{2} + \frac{2g \rho_f U^2 L_v}{D} + 0.057gL_v \sqrt{\frac{g}{D}} + \rho_p (1 - \varepsilon_v) gL_v + \rho_f \varepsilon_v gL_v
\]

To use this Equation we need to calculate the voidage of the suspension in the vertical pipe line \( \varepsilon_v \):

Assuming particles behave as individuals, then slip velocity is equal to single particle terminal velocity, \( U_T \).

\[
i.e. \quad U_{pv} = \frac{U}{\varepsilon_v} - U_T
\]

continuity gives particle mass flux, \( G = \rho_p (1 - \varepsilon_v) U_{pv} \)
Combining these Equations gives a quadratic in $\varepsilon_v$ which has only one possible root.

$$\varepsilon_v^2 U_T - \left[ U_T + U + \frac{G}{\rho_p} \right] \varepsilon_v + U = 0$$

The single particle terminal velocity, $U_T$ was found above to be 1.77 m/s. And so, solving the quadratic Equation, $\varepsilon_v = 0.9835$ and actual gas velocity,

$$U_{tv} = \frac{U}{\varepsilon_v} = \frac{8.49}{0.9835} = 8.63 \text{ m/s}$$

Then actual solids velocity, $U_{pv} = U_{fv} - U_T = 8.63 - 1.77 = 6.86 \text{ m/s}$

The components of the pressure loss in the vertical pipe are therefore:

**Term 1 (gas acceleration):**

$$= \frac{\rho_f \varepsilon_v U_{fv}^2}{2} = \frac{1.2 \times 0.9835 \times 8.63^2}{2} = 43.9 \text{ Pa.}$$

**Term 2 (solids acceleration):**

$$= \frac{\rho_p (1 - \varepsilon_v) U_{pv}^2}{2} = \frac{2500 \times (1 - 0.9835) \times 6.86^2}{2} = 970.5 \text{ Pa.}$$

**Term 3 (gas friction):**

Estimate the gas friction factor using the Blasius correlation for smooth pipes, $f_g = 0.079 \times \text{Re}^{-0.25}$. The Reynolds number calculated based on the superficial gas velocity:

$$\text{Re} = \frac{0.05 \times 1.2 \times 8.49}{18.4 \times 10^{-6}} = 27679$$, which gives $f_g = 0.0061$.

Then, term 3

$$= 2 f_g \rho f U_{fv}^2 L_v = 2 \times 0.0061 \times 1.2 \times 8.49^2 \times 50 \times 0.05 = 1059.1 \text{ Pa.}$$

**Term 4 (solids friction):**

$$= 0.057 \times G L_v \sqrt{\frac{g}{D}} = 0.057 \times 282.9 \times 50 \times \sqrt{\frac{9.81}{0.05}} = 11293.6 \text{ Pa}$$

**Term 5 (solids gravitational head):**

$$= \rho_p (1 - \varepsilon_v) g L_v = 2500 \times (1 - 0.9835) \times 9.81 \times 50 = 20226 \text{ Pa.}$$
Term 6 (gas gravitational head): \[ = \rho_f \varepsilon_f g L_v = 1.2 \times 0.9835 \times 9.81 \times 50 = 579 \text{ Pa}. \]

and thus, total pressure loss across vertical sections, \( \Delta p_v = 33160 \text{ Pa (0.332 bar)} \)

The available blower pressure at this maximum flow rate is 0.3 bar and so the lift line will not operate as required. Reducing the gas velocity safety margin will not help, since this will cause the line pressure loss to increase.

**EXERCISE 8.3:**
Design a negative pressure dilute-phase pneumatic transport system to carry 700 kg/hr of plastic spheres of particle density 1000 kg/m\(^3\) and mean particle size 1 mm between two points in a factory separated by a vertical distance of 15 metres and a horizontal distance of 80 metres using ambient air. Assume that the pipe is smooth, that five 90 degree bends are required and that the allowable pressure loss is 0.4 bar.

**SOLUTION TO EXERCISE 8.3**
Design in this case means determine the pipe size and air flowrate which would give a total system pressure loss near to the allowable pressure loss.

The design procedure requires trial and error calculations. Pipes are available in fixed sized and so the procedure is to select a pipe size and determine the saltation velocity from Text-Equation 8.1. The system pressure loss is then calculated at a superficial gas velocity equal to 1.5 times the saltation velocity (this gives a reasonable safety margin bearing in mind the accuracy of the correlation in Text-Equation 8.1). The calculated system pressure loss is then compared with the allowable pressure loss. The pipe size selected may then be altered and the above procedure repeated until the calculated pressure loss matches that allowed.

**Step 1 Selection of pipe size:** Select 40 mm internal diameter pipe.

**Step 2 Determine gas velocity**
Use the Rizk correlation of Text-Equation (8.3) to estimate the saltation velocity, \( U_{SALT} \). Text-Equation (8.3) rearranged becomes:

\[
U_{SALT} = \left[ \frac{4M_e 10^\alpha g^2 D \left( \frac{\beta}{2} \right)^\beta}{\pi \rho_f} \right]^{1/b+1}
\]

where \( \alpha = 1440x + 1.96 \) and \( \beta = 1100x + 2.5 \).
In the present case $\alpha = 3.4$, $\beta = 3.6$ and $U_{SALT} = 10.94$ m/s.
Therefore, superficial gas velocity, $U = 1.5 \times 10.94$ m/s = 16.41 m/s.

**Step 3 Pressure loss calculations**

1) **Horizontal Sections**

Starting with Text-Equation 8.15 and expression for the total pressure loss in the horizontal sections of the transport line may be generated. We will assume that all the initial acceleration of the solids and the gas take place in the horizontal sections and so terms 1 and 2 are required. For term 3 the Fanning friction Equation is used assuming that the pressure loss due to gas/wall friction is independent of the presence of solids. For term 4 we employ the Hinkle correlation (Text-Equation 8.17). Terms 5 and 6 became zero as $\theta = 0$ for horizontal pipe. Thus, the pressure loss, $\Delta P_H$, in the horizontal sections of the transport line is given by:

$$\Delta P_H = \frac{\rho_f \varepsilon H U_f H^2}{2} + \frac{\rho_p (1 - \varepsilon H) U_p H^2}{2} + \frac{2 f_p \rho_f U_H^2 L_H}{D} + \frac{2 f_p \rho_p (1 - \varepsilon H) U_p H^2 L_H}{D}$$

where the subscript $H$ refers to the values specific to the horizontal sections.

To use this Equation we need to know $\varepsilon_H$, $U_f H$ and $U_p H$. Hinkle’s correlation gives us $U_p H$:

$$U_p H = U (1 - 0.0638 x 0.3 \rho_p^{0.5}) = 12.24$$ m/s

From continuity, $G = \rho_p (1 - \varepsilon_H) U_p H$.

Solids flux, $G = M_p / A = \frac{700}{3600} \times \frac{1}{\pi (0.04)^2} = 154.7$ kg / m$^2$.s

thus $\varepsilon_H = 1 - \frac{G}{\rho_p U_p H} = 0.9874$

and $U_f H = \frac{U}{\varepsilon_H} = \frac{16.41}{0.9874} = 16.62$ m / s

Friction factor $f_p$ is found from Text-Equation (8.19) with $C_D$ estimated at the relative velocity $(U_f H - U_p H)$, using the approximate correlations given below, (or by using an appropriate $C_D$ versus $Re$ chart [see Chapter 2]).
\[ \text{Re}_p < 1 \quad : \quad C_D = \frac{24}{\text{Re}_p} \]
\[ 1 < \text{Re}_p < 500 \quad : \quad C_D = 18.5 \text{Re}_p^{-0.6} \]
\[ 500 < \text{Re}_p < 2 \times 10^5 \quad : \quad C_D = 0.44 \]

Thus, for flow in the horizontal sections, \( \text{Re}_p = \frac{\rho_f (U_{BH} - U_{PH}) k}{\mu} \)

for ambient air \( \rho_f = 1.2 \text{ kg/m}^3 \) and \( \mu = 18.4 \times 10^{-6} \text{ Pas} \), giving
\[
\text{Re}_p = \frac{1.2 \times (16.62 - 12.24) \times 1 \times 10^{-3}}{18.4 \times 10^{-6}} = 285.5
\]

and so, using the approximate correlations above, \( C_D = 18.5 \text{ Re}^{-0.6} = 0.622 \)

Substituting \( C_D = 0.622 \) in Text-Equation 8.19 we have:
\[
f_f = \frac{3}{8} \times \frac{1.2}{1000} \times 0.622 \times \frac{0.040 \times (16.62 - 12.24)^2}{1 \times 10^{-3} \times 12.24} = 0.00143
\]

To estimate the gas friction factor we use the Blasius correlation for smooth pipes, \( f_g = 0.079 \times \text{Re}^{-0.25} \). The Reynolds number calculated based on the superficial gas velocity:
\[
\text{Re} = \frac{0.04 \times 1.2 \times 16.41}{18.4 \times 10^{-6}} = 42800, \text{ which gives } f_g = 0.0055.
\]

Thus the components of the pressure loss in the horizontal pipe from Text-Equation 8.15 are:

Term 1 (gas acceleration):
\[
\frac{\rho_f \epsilon_H U_{BH}^2}{2} = \frac{1.2 \times 0.9874 \times 16.62^2}{2} = 163.6 \text{ Pa.}
\]

Term 2 (solids acceleration):
\[
\frac{\rho_p (1 - \epsilon_H) U_{pH}^2}{2} = \frac{1000 \times (1 - 0.9874) \times 12.24^2}{2} = 946.8 \text{ Pa.}
\]

Term 3 (gas friction):
\[
\frac{2f_g \rho_f U_{BH}^2 L_H}{D} = \frac{2 \times 0.0055 \times 1.2 \times 16.41^2 \times 80}{0.04} = 7096 \text{ Pa.}
\]
Term 4 (solids friction):

\[ \frac{2f p_p (1 - \varepsilon_p)}{D} \left( \frac{U_p}{p_H} \right)^2 = \frac{2 \times 0.001432 \times 1000 \times (1 - 0.9874) \times 12.24^2 \times 80}{0.04} = 10847 \text{ Pa.} \]

This gives \( \Delta p_H = 19054 \text{ Pa.} \)

\[ \varepsilon \]

b) Vertical Sections

Starting again with Text-Equation 8.15, the general pressure loss Equation, an expression for the total pressure loss in the vertical section may be derived. Since the initial acceleration of solids and gas was assumed to take place in the horizontal sections, terms 1 and 2 become zero. The Fanning friction Equation is used to estimate the pressure loss due to gas-to-wall friction (term 3) assuming solids have negligible effect on this pressure loss. For term 4 the modified Konno and Saito correlation (Text-Equation 8.16) is used. For vertical transport \( \theta \) is 90° in terms 5 and 6.

Thus, the pressure loss, \( \Delta p_v \), in the vertical sections of the transport line is given by:

\[ \Delta p_v = \frac{2f_g \rho_1 U^2 L_v}{D} + 0.057 G L_v \sqrt{\frac{g}{D}} + \rho_g (1 - \varepsilon_v) g L_v + \rho_f \varepsilon_v g L_v \]

where subscript \( v \) refers to values specific to the vertical sections.

To use this Equation we need to calculate the voidage of the suspension in the vertical pipe line \( \varepsilon_v \):

Assuming particles behave as individuals, then slip velocity is equal to single particle terminal velocity, \( U_T \) (also noting that the superficial gas velocity in both horizontal and vertical section is the same and equal to \( U \))

\[ \text{i.e. } U_{pv} = \frac{U}{\varepsilon_v} - U_T \]

continuity gives particle mass flux, \( G = \rho_p (1 - \varepsilon_v) U_{pv} \)

Combining these Equations gives a quadratic in \( \varepsilon_v \) which has only one possible root.

\[ \varepsilon_v^2 U_T - \left[ U_T + U + \frac{G}{\rho_p} \right] \varepsilon_v + U = 0 \]
The single particle terminal velocity, \( U_T \) may be estimated as shown in Chapter 2, giving \( U_T = 4.1 \text{ m/s} \) assuming the particles are spherical (Reynolds number at \( U_T \) is 267).

And so, solving the quadratic Equation, \( \varepsilon_v = 0.9876 \)

The components of the pressure loss in the vertical pipe are therefore:

Term 1 (gas friction):

\[
\frac{2f_g \rho_f U_T^2 L_v}{D} = \frac{2 \times 0.0055 \times 1.2 \times 16.41^2 \times 15}{0.04} = 1330.6 \text{ Pa.}
\]

Term 2 (solids friction):

\[
0.057 \times G \sqrt{\frac{g}{D}} = 0.057 \times 154.7 \times 15 \times \sqrt{\frac{9.81}{0.04}} = 2071.6 \text{ Pa}
\]

Term 3 (solids gravitational head):

\[
\rho_p (1 - \varepsilon_v) g L_v = 1000 \times (1 - 0.9876) \times 9.81 \times 15 = 1819.6 \text{ Pa.}
\]

Term 4 (gas gravitational head):

\[
\rho_f \varepsilon_g g L_v = 1.2 \times 0.9876 \times 9.81 \times 15 = 174.4 \text{ Pa}.
\]

and thus, total pressure loss across vertical sections, \( \Delta p_v = 5396.1 \text{ Pa} \)

c) Bends

The pressure loss across each 90° bend is taken to be equivalent to that across 7.5 m of vertical pipe.

Pressure loss per metre of vertical pipe = \( \frac{\Delta p_v}{L_v} = 359.7 \text{ Pa} / \text{m} \)

Therefore, pressure loss across five 90° bends

\[
= 5 \times 7.5 \times 359.7 \text{ Pa} \\
= 13490.3 \text{ Pa}
\]

And so,

\[
\begin{bmatrix}
\text{total pressure loss} \\
\text{loss across vertical sections}
\end{bmatrix}
\begin{bmatrix}
\text{loss across horizontal sections} \\
\text{loss across bends}
\end{bmatrix}
= 5396.1 + 19054 + 13490.3 \text{ Pa}
\]

\[
= 0.3794 \text{ bar}
\]

Step 4 Compare calculated and allowable pressure losses

The allowable system pressure loss is 0.4 bar and so the 40 mm pipe looks OK. The table below gives the results for a range of pipe sizes.
Pipe inside diameter (mm) | Total System Pressure Loss (bar)
---|---
63 | 0.167
50 | 0.252
40 | 0.379

In this case we would select 40 mm pipe work which gives a total system pressure loss of 0.38 bar. The design details for this selection are given below:

Pipe size: 40 mm inside diameter  
Air flowrate = 0.0247 m$^3$/s  
Air superficial velocity = 16.41 m/s  
Saltation velocity = 10.94 m/s  
Solids loading = 7.87 kg solid/kg air  
Total system pressure loss = 0.38 bar

**EXERCISE 8.4**

A 25 m long standpipe carrying Group A solids at a rate of 75 kg/s is to be aerated in order to maintain fluidized flow with a voidage in the range 0.50 - 0.55. Solids enter the top of the standpipe at a voidage of 0.55. The pressure and gas density at the top of the standpipe are 1.4 bar (abs) and 1.1 kg/m$^3$ respectively. The particle density of the solids is 1050 kg/m$^3$. Determine the aeration positions and rates.

**SOLUTION TO EXERCISE 8.4:**
The objective is to add gas to the standpipe to prevent the voidage falling below the lowest acceptable. Text-Equation 8.29 gives us the maximum pressure ratio between an upper level, operating at an acceptable voidage, and a lower level at which the voidage has reached the limiting value:

\[
\frac{p_2}{p_1} = \frac{(1 - \varepsilon_2)}{\varepsilon_2} \cdot \frac{\varepsilon_1}{(1 - \varepsilon_1)}
\]

where $\varepsilon_1$ is voidage at the upper level and $\varepsilon_2$ is the lowest voidage acceptable (lower level). $\varepsilon_1 = 0.55$ and $\varepsilon_2 = 0.5$.

Therefore, pressure ratio, \[ \frac{p_2}{p_1} = \frac{(1 - 0.50)}{0.50} \cdot \frac{0.55}{(1 - 0.55)} = 1.222 \]
Therefore, \( p_2 = p_1 \times 1.222 = 1.711 \) bar

Pressure difference, \( p_2 - p_1 = (1.711 - 1.4) \times 10^5 = 0.311 \times 10^5 \) Pa.

Hence, from Text-Equation 8.30:

\[
(p_2 - p_1) = (\rho_p - \rho_f) [1 - \varepsilon_a] Hg
\]

(with \( \varepsilon_a = [0.5 + 0.55]/2 = 0.525 \)),

length to first aeration point, \( H = \frac{0.311 \times 10^5}{(1050 - 1.1) \times (1 - 0.525) \times 9.81} = 6.358 \) m

Assuming ideal gas behaviour, density at level 2,

\[
\rho_{f2} = \rho_{f1} \left( \frac{p_2}{p_1} \right) = 1.1 \times 1.222 = 1.344 \text{ kg} / \text{m}^3
\]

Applying Equation 8.34, \( M_{f2} = \frac{\varepsilon_1}{(1 - \varepsilon_1)} \frac{M_2}{\rho_p} (\rho_{f2} - \rho_n) \)

aeration gas mass flow at first aeration point,

\( M_{f2} = \frac{0.55}{(1 - 0.55)} \frac{75}{1050} (1.344 - 1.1) = 0.0213 \text{ kg} / \text{s} \)

The above calculation is repeated in order to determine the position and rates of subsequent aeration points. The results are summarised below:

<table>
<thead>
<tr>
<th>Distance from top of standpipe (m)</th>
<th>First point</th>
<th>Second point</th>
<th>Third point</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6.36</td>
<td>14.13</td>
<td>23.62</td>
</tr>
<tr>
<td>Aeration rate (kg/s)</td>
<td>0.0213</td>
<td>0.0261</td>
<td>0.0319</td>
</tr>
<tr>
<td>Pressure at aeration point (bar)</td>
<td>1.71</td>
<td>2.09</td>
<td>2.56</td>
</tr>
</tbody>
</table>

EXERCISE 8.5:

A 15 m long standpipe carrying Group A solids at a rate of 120 kg/s is to be aerated in order to maintain fluidized flow with a voidage in the range 0.50 - 0.54. Solids enter the top of the standpipe at a voidage of 0.54. The pressure and gas density at the top of the standpipe are 1.2 bar (abs) and 0.9 kg/m³ respectively. The particle density of the solids is 1100 kg/m³. Determine the aeration positions and rates. What is the pressure at the lowest aeration point?

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SOLUTION TO EXERCISE 8.5:
The objective is to add gas to the standpipe to prevent the voidage falling below the lowest acceptable. Text-Equation 8.29 gives us the maximum pressure ratio between an upper level, operating at an acceptable voidage, and a lower level at which the voidage has reached the limiting value:

\[
\frac{p_2}{p_1} = \frac{(1 - \varepsilon_2)}{\varepsilon_2} \frac{\varepsilon_1}{(1 - \varepsilon_1)}
\]

where \(\varepsilon_1\) is voidage at the upper level and \(\varepsilon_2\) is the lowest voidage acceptable (lower level). \(\varepsilon_1 = 0.54\) and \(\varepsilon_2 = 0.5\).

Therefore, pressure ratio, \(\frac{p_2}{p_1} = \frac{(1 - 0.50)}{0.50} \frac{0.54}{(1 - 0.54)} = 1.174\)

Therefore, \(p_2 = p_1 \times 1.174 = 1.4087\) bar

Pressure difference, \(p_2 - p_1 = (1.4087 - 1.2) \times 10^5 = 0.2087 \times 10^5\) Pa. Pa.

Hence, from Text-Equation 8.30:

\[
(p_2 - p_1) = (\rho_p - \rho_f) \left(1 - \varepsilon_a\right) \text{Hg}
\]

(with \(\varepsilon_a = [0.5 + 0.54]/2 = 0.52\),

length to first aeration point, \(H = \frac{0.2087 \times 10^5}{(1100 - 0.9) \times (1 - 0.52) \times 9.81} = 4.029\) m

Assuming ideal gas behaviour, density at level 2,

\[
\rho_{f2} = \rho_{f1} \left(\frac{p_2}{p_1}\right) = 0.9 \times 1.174 = 1.057\ \text{kg/m}^3
\]

Applying Equation 8.34, \(M_{f2} = \frac{\varepsilon_1}{(1 - \varepsilon_1)} \frac{\rho_p}{\rho_f} \left(p_{f2} - \rho_{f1}\right)\)

aeration gas mass flow at first aeration point,

\[
M_{f2} = \frac{0.54}{(1 - 0.54)} \frac{120}{1100} (1.057 - 0.9) = 0.0201\ \text{kg/s}
\]

The above calculation is repeated in order to determine the position and rates of subsequent aeration points. The results are summarised below:
EXERCISE 8.6:
A 5 m long vertical standpipe of inside diameter 0.3 m transports solids at flux of 500 kg/m².s from an upper vessel which is held at a pressure 1.25 bar to a lower vessel held at 1.6 bar. The particle density of the solids is 1800 kg/m³ and the surface-volume mean particle size is 200 mm. Assuming that the voidage is 0.48 and is constant along the standpipe, and that the effect of pressure change may be ignored, determine the direction and flow rate of gas passing between the vessels. (Properties of gas in the system: density, 1.5 kg/m³; viscosity 1.9 x 10⁻⁵ Pas).

SOLUTION TO EXERCISE 8.6:
First check that the solids are moving in packed bed flow. We do this by comparing the actual pressure gradient with the pressure gradient for fluidization.

Assuming that in fluidized flow the apparent weight of the solids will be supported by the gas flow, Text-Equation 8.26 gives the pressure gradient for fluidized bed flow:

\[
\frac{(-\Delta p)}{H} = (1 - 0.48) \times (1800 - 1.5) \times 9.81 = 9174.5 \text{ Pa/m}
\]

Actual pressure gradient = \(\frac{(1.6 - 1.25) \times 10^5}{5}\) = 7000 Pa/m

Since the actual pressure gradient is well below that for fluidized flow, the standpipe is operating in packed bed flow.

The pressure gradient in packed bed flow is generated by the upward flow of gas through the solids in the standpipe. The Ergun Equation (Text-Equation 8.25) provides the relationship between gas flow and pressure gradient in a packed bed.

Knowing the required pressure gradient, the packed bed voidage and the particle and gas properties, Text-Equation 8.26 can be solved for \(|U_{rel}|\), the magnitude of the relative gas velocity:
\[
\frac{(-\Delta p)}{H} = \left[150 \frac{\mu}{x_{sv}^2} \left(1-\frac{\varepsilon}{1-\varepsilon}\right)\right] U_{rel} + \left[1.75 \frac{\rho_f}{x_{sv}} \left(1-\frac{\varepsilon}{1-\varepsilon}\right)\right] U_{rel}^2
\]

\[
7000 = \left[150 \frac{1.9 \times 10^{-5} \times (1-0.48)^2}{(200 \times 10^{-6})^2} \right] U_{rel} + \left[1.75 \frac{1.5}{200 \times 10^{-6}} \times (1-0.48)\right] U_{rel}^2
\]

\[0 = 14219 U_{rel}^2 + 83620 |U_{rel}| - 7000\]

Ignoring the negative root of the quadratic, \(|U_{rel}| = 0.08255 \text{ m/s}\)

We now adopt a sign convention for velocities. For standpipes it is convenient to take downward velocities as positive. In order to create the pressure gradient in the required direction, the gas must flow upwards relative to the solids. Hence, \(U_{rel}\) is negative:

\[U_{rel} = -0.08255 \text{ m/s}\]

From the continuity for the solids (Text-Equation 8.11),

\[\text{solids flux, } \frac{M_p}{A} = U_p \left(1 - \varepsilon\right) \rho_p\]

The solids flux is given as 500 kg/m²s and so:

\[U_p = \frac{500}{(1-0.48) \times 1800} = 0.5342 \text{ m/s}\]

Solids flow is downwards, so \(U_p = +0.5342 \text{ m/s}\)

The relative velocity, \(U_{rel} = U_f - U_p\)

hence, actual gas velocity, \(U_f = -0.08255 + 0.5342 = 0.4516 \text{ m/s} \) (downwards)

Therefore the gas flows downwards at a velocity of 0.4516 m/s relative to the standpipe walls. The superficial gas velocity is therefore:

\[U = \varepsilon U_f = 0.48 \times 0.4516 = 0.217 \text{ m/s}\]

From the continuity for the gas (Text-Equation 8.12) mass flow rate of gas,

\[M_f = \varepsilon U_f \rho_f A\]

\[= 0.0230 \text{ kg/s}\]
So for the standpipe to operate as required, 0.0230 kg/s of gas must flow from upper vessel to lower vessel.

EXERCISE 8.7:
A vertical standpipe of inside diameter 0.3 m transports solids at flux of 300 kg/m².s from an upper vessel which is held at a pressure 2.0 bar to a lower vessel held at 2.72 bar. The particle density of the solids is 2000 kg/m³ and the surface-volume mean particle size is 220 μm. The density and viscosity of the gas in the system are 2.0 kg/m³ and 2 x 10⁻⁵ Pas respectively.

Assuming that the voidage is 0.47 and constant along the standpipe, and that the effect of pressure change may be ignored,

(a) Determine the minimum standpipe length required to avoid fluidized flow.
(b) If the actual standpipe is 8 m long, determine the direction and flow rate of gas passing between the vessels.

SOLUTION TO EXERCISE 8.7:
Assuming that in fluidized flow the apparent weight of the solids will be supported by the gas flow Text-Equation 8.26 gives the pressure gradient for fluidized bed flow:

\[
\frac{(-\Delta p)}{H} = (1 - 0.47) \times (2000 - 2) \times 9.81 = 10388 \text{ Pa} / \text{m}
\]

The minimum length to avoid fluidization is given by:
limiting pressure gradient = \(\frac{(2.72 - 2.0) \times 10^5}{H_{\text{min}}} = 10388 \text{ Pa.}\)

which gives \(H_{\text{min}} = 6.93 \text{ m.}\)

Since the actual standpipe is 8 m long the actual pressure gradient is well below that for fluidized flow, the standpipe is operating in packed bed flow.
Actual pressure gradient = \(\frac{(2.72 - 2.0) \times 10^5}{8} = 9000 \text{ Pa.}\)

The pressure gradient in packed bed flow is generated by the upward flow of gas through the solids in the standpipe. The Ergun Equation (Text-Equation 8.25) provides the relationship between gas flow and pressure gradient in a packed bed.
Knowing the required pressure gradient, the packed bed voidage and the particle and gas properties, Text-Equation 8.26 can be solved for $|U_{rel}|$, the magnitude of the relative gas velocity:

$$\frac{(-\Delta p)}{H} = \left[ 150 \frac{\mu}{X_{sv}^2} \left( \frac{1 - \varepsilon}{\varepsilon} \right)^2 \right] |U_{rel}| + \left[ 1.75 \frac{\rho_f}{X_{sv}^2} \left( \frac{1 - \varepsilon}{\varepsilon} \right) \right] |U_{rel}|^2$$

$$9000 = \left[ 150 \frac{2 \times 10^{-6} \times (1 - 0.47)^2}{(220 \times 10^{-6})^2} \right] |U_{rel}| + \left[ 1.75 \frac{2.0}{220 \times 10^{-6}} \times (1 - 0.47) \right] |U_{rel}|^2$$

$$0 = 17940|U_{rel}|^2 + 78819|U_{rel}| - 9000$$

Ignoring the negative root of the quadratic, $|U_{rel}| = 0.1114 \text{ m/s}$

We now adopt a sign convention for velocities. For standpipes it is convenient to take downward velocities as positive. In order to create the pressure gradient in the required direction, the gas must flow upwards relative to the solids. Hence, $U_{rel}$ is negative: $U_{rel} = -0.1114 \text{ m/s}$

From the continuity for the solids (Text-Equation 8.11), solids flux, $= \frac{M_p}{A} = U_p (1 - \varepsilon) \rho_p$

The solids flux is given as 300 kg/m²s and so:

$$U_p = \frac{300}{(1 - 0.47) \times 2000} = 0.283 \text{ m/s}$$

Solids flow is downwards, so $U_p = +0.283\text{m/s}$

The relative velocity, $U_{rel} = U_f - U_p$

Hence, actual gas velocity, $U_f = -0.1114 + 0.283 = 0.1716 \text{ m/s}$ (downwards)

Therefore the gas flows downwards at a velocity of 0.1716 m/s relative to the standpipe walls.

The superficial gas velocity is therefore:

$$U = \varepsilon U_f = 0.47 \times 0.1716 = 0.0807 \text{ m/s}$$
From the continuity for the gas (Text-Equation 8.12) mass flow rate of gas,
\[ M_f = \varepsilon U_f \rho_f A \]
\[ = 0.0114 \text{ kg/s}. \]

So for the standpipe to operate as required, 0.0114 kg/s of gas must flow from upper vessel to lower vessel.