Equations for Inventory Management

Chapter 1 Stocks and inventories

Empirical observation for the amount of stock held in a number of locations:

\[ AS(N_2) = AS(N_1) \times \sqrt{\frac{N_2}{N_1}} \]

where:
- \( N_2 \) = number of planned future facilities
- \( N_1 \) = number of existing facilities
- \( AS(N_i) \) = aggregate stock with \( N_i \) facilities

Chapter 3 Economic order quantity

The variables used here, and throughout the book, are:

- \( Q \) = order quantity
- \( Q_o \) = optimal order quantity
- \( D \) = demand
- \( UC \) = unit cost
- \( RC \) = reorder cost
- \( HC \) = holding cost
- \( T \) = cycle length
- \( To \) = optimal cycle length
- \( VC \) = variable cost per unit time
- \( VCo \) = optimal variable cost per unit time
- \( TC \) = total cost per unit time
- \( TCo \) = optimal total cost per unit time
- \( ROL \) = reorder level
- \( LT \) = lead time

- Economic order quantity:
  \[ Q_o = \sqrt{\frac{2 \times RC \times D}{HC}} \]

- Optimal stock cycle length:
  \[ To = \frac{Q_o}{D} = \sqrt{\frac{2 \times RC}{D \times HC}} \]
Equations for Inventory Management

- Variable cost per unit time:
  \[ VC = \frac{RC \times D}{Q} + \frac{HC \times Q}{2} \]

- Optimal value of variable cost per unit time:
  \[ VCo = HC \times Qo = \frac{2 \times RC \times D}{Qo} = \sqrt{2 \times RC \times HC \times D} \]

- Total cost per unit time:
  \[ TC = UC \times D + VC \]

- Optimal cost per unit time:
  \[ TCo = UC \times D + VCo \]

- Change of variable cost moving away from the EOQ:
  \[ \frac{VC}{VCo} = \frac{1}{2} \times \left[ \frac{Qo}{Q} + \frac{Q}{Qo} \right] \]

- Reorder level:
  \[ ROL = LT \times D - n \times Qo \]

Chapter 4 Models for known demand

Model for finite replenishment rate, \( P \)

- Optimal order quantity:
  \[ Qo = \sqrt{\frac{2 \times RC \times D}{HC}} \times \sqrt{\frac{P}{P - D}} \]

- Optimal time cycle time:
  \[ To = \sqrt{\frac{2 \times RC}{HC \times D}} \times \sqrt{\frac{P}{P - D}} \]

- Optimal variable cost:
  \[ VCo = \sqrt{2 \times RC \times HC \times D} \times \sqrt{\frac{P - D}{P}} \]

- Optimal total cost:
  \[ TCo = UC \times D + VCo \]
Model for planned shortages and backorders

SC = shortage cost per unit per unit time

- Optimal order quantity:
  \[ Q_o = \sqrt{\frac{2 \times RC \times D \times (HC + SC)}{HC \times SC}} \]

- Optimal amount to be backordered:
  \[ S_o = \sqrt{\frac{2 \times RC \times HC \times D}{SC \times (HC + SC)}} \]

- Time during which demand is met:
  \[ T_1 = \frac{(Q_o - S_o)}{D} \]

- Time during which demand is backordered:
  \[ T_2 = \frac{S_o}{D} \]

- Cycle time;
  \[ T = T_1 + T_2 \]

Model for shortages with lost orders

R = revenue

Z = proportion of demand met

- Cost of each unit of lost sales including loss of profits:
  \[ LC = DC + SP - UC \]

- Optimal revenue:
  \[ Ro = Z \times [D \times LC - \sqrt{2 \times RC \times HC \times D}] \]

Model for constraints on space

AC = additional cost related to the storage area (or volume) used by each unit of the item.

Si = amount of space occupied by one unit of item i.

- The total holding cost per unit per unit time:
  \[ HC + AC \times S_i \]
Equations for Inventory Management

- Optimal order quantities:
  \[ Q_i = \sqrt{\frac{2 \times RC_i \times D_i}{HC_i + AC \times S_i}} \]

Model for constraint on investment

\[ UL = \text{upper limit on the total average investment} \]

- Optimal order quantities:
  \[ Q_i = Q_{oi} \times \frac{2 \times UL \times HC}{UC \times \sum_{i=1}^{N} VCo_i} \]

Model for discrete variable demand

- Test for the point where it is more expensive to order for \( N + 1 \) periods than to order for \( N \) periods:
  \[ N \times (N + 1) \times D_{N+1} > \frac{2 \times RC}{HC} \]

- Confirming that it is more expensive to order for \( N + 2 \) periods than to order for \( N \) periods:
  \[ N \times (N + 2) \times [D_{N+1} + D_{N+2}] > \frac{4 \times RC}{HC} \]

- Variable cost per period:
  \[ VC_N = \frac{RC}{N} + \frac{HC \times \sum_{i=1}^{N} D_i}{2} \]

Chapter 5 Models for uncertain demand

Model for the newsboy problem

- Selling price
- Scrap value

- Test for the optimal order size:
  \[ \text{Prob}(D \geq Qo) > \frac{UC - SV}{SP - SV} > \text{Prob}(D \geq Qo + 1) \]

- Expected profit with buying \( Q \) units:
  \[ EP(Q) = SP \times \left[ \sum_{D=0}^{Q} D \times \text{Prob}(D) + Q \times \sum_{D=Q+1}^{\infty} \text{Prob}(D) \right] - Q \times UC \]
Model for discrete demand with shortages

\[ A = \text{Actual stock level} \]

- Test for the optimal stock level:

\[ \text{Prob}(D \leq Ao) \geq \frac{SC}{HC + SC} \geq \text{Prob}(D \leq Ao - 1) \]

Approach to intermittent demand

- Service level = 1 – \text{Prob(shortage)}
  
  \[ = 1 - [\text{Prob(there is a demand)} \times \text{Prob(demand > A)}] \]

Joint calculation of order quantity and reorder level with shortages

- Calculation for order quantity:

\[ Q = \sqrt{\frac{2 \times D}{HC} \times \left[ RC + SC \times \sum_{D=ROL}^{\infty} (D - ROL) \times \text{Prob}(D) \right]} \]

- Calculation for reorder level:

\[ \frac{HC \times Q}{SC \times D} = \sum_{D=ROL}^{\infty} \text{Prob}(D) \]

Model for order quantity with shortages

- Order quantity:

\[ Q = \sqrt{\frac{2 \times D}{HC} \times \left[ RC + SC \times \sum_{D=ROL}^{\infty} (D - ROL) \times \text{Prob}(D) \right]} \]

Model for uncertain lead time demand

- Safety stock:

\[ SS = Z \times \text{standard deviation of lead time} = Z \times \sigma \times \sqrt{LT} \]

- Reorder level:

\[ ROL = \text{lead time demand} + \text{safety stock} = LT \times D + Z \times \sigma \times \sqrt{LT} \]

Model for service level with uncertain lead time

- Service level = \text{Prob} (LT \times D < ROL) = \text{Prob}(LT < ROL/D)
Model for periodic review method

- Target stock level:
  
  \[ TSL = D \times (T + LT) + Z \times \sigma \times \sqrt{T + LT} \]

Chapter 6 Sources of information

Accounting information

- Cost of products sold = opening stock + net purchases − closing stock
- Value of stock = number of units in stock × unit value
- Average cost = \( \frac{\text{Total cost of units}}{\text{Number of units bought}} \)
- Closing stock = opening stock + purchases − sales
- Gross profit = sales revenue − cost of units sold

Chapter 7 Forecasting demand

Value of demand in a time series

Actual demand = underlying pattern + random noise

Linear relationship

  dependent variable = a + b \times \text{independent variable}

  \[ y = a + bx \]
  
  x = value of the independent variable
  
  y = value of the dependent variable
  
  a = intercept, where the line crosses the y axis
  
  b = gradient of the line.

- Equations for linear regression:

  \[ b = \frac{n \times \sum (x \times y) - \sum x \times \sum y}{n \times \sum x^2 - (\sum x)^2} \]
  
  \[ a = \frac{\sum y}{n} - b \times \frac{\sum x}{n} \]

- Coefficient of determination = \( (\text{coefficient of correlation})^2 \)
Multiple regression

\[ y = a + b_1 \times \text{variable 1} + b_2 \times \text{variable 2} + b_3 \times \text{variable 3} + b_4 \times \text{variable 4} \ldots \]

Exponential smoothing

- New Forecast = \( \alpha \times \) latest demand + \((1 - \alpha) \times \) previous forecast
- \( \alpha \) is the smoothing constant (usually between 0.1 and 0.2)
- Tracking signal = \( \frac{\text{sum of forecast errors}}{\text{mean absolute deviation}} \)
- Seasonal index = \( \frac{\text{seasonal value}}{\text{deseasonalized value}} \)
- Demand = (underlying value + trend) \times \text{seasonal index} + \text{noise}

Chapter 8 Planning and stocks

Stock and planning

Stock at end of last + during – met during – from earlier + met in later period = period

Chapter 9 Material requirements planning

- Basic calculation

Gross requirements = number of units made \( \times \) amount of material for each unit

Net requirements = gross requirements – current stock – stock on order

- Batching rule to find \( N \)

\[ N \times (N + 1) \times D_{N+1} > \frac{2 \times RC}{HC} \]

Where:

- \( N \) = the period number in a cycle
- \( D_{N+1} \) = demand in period \( N + 1 \) of a cycle
Equations for Inventory Management

**Chapter 10 Just-in-time**

- Number of *kanbans* to maintain smooth operations

\[
K = \frac{D \times (TP + TD)}{C} \quad \text{Number of kanbans}
\]

Where:

- \(C\) = number of units held in each container
- \(TP\) = time container spends in production part of a cycle (waiting, being filled and moving to the store of work in progress)
- \(TD\) = time container spends in demand part of a cycle (waiting, being emptied and moving to the store of work in progress)

Total cycle length = \(TP + TD\)

- Number of *kanbans* with safety factor

\[
K < \frac{D \times (TP + TD) \times (1 + SF)}{C} \quad \text{SF = safety factor (generally less than 0.1)}
\]

- Maximum stock of work in progress

\[
\text{Maximum stock level} = K \times C = D \times (TP + TD) \times (1 + SF)
\]