Halliday ♦ Resnick ♦Walker

FUNDAMENTALS OF PHYSICS
SIXTH EDITION

Selected Solutions

Chapter 43

43.25
43.35
43.41
43.61
43.75

25. If a nucleus contains Z protons and N neutrons, its binding energy is $\Delta E_{\rm be} = (Zm_H + Nm_n - m)c^2$, where m_H is the mass of a hydrogen atom, m_n is the mass of a neutron, and m is the mass of the atom containing the nucleus of interest. If the masses are given in atomic mass units, then mass excesses are defined by $\Delta_H = (m_H - 1)c^2$, $\Delta_n = (m_n - 1)c^2$, and $\Delta = (m - A)c^2$. This means $m_H c^2 = \Delta_H + c^2$, $m_n c^2 = \Delta_n + c^2$, and $mc^2 = \Delta + Ac^2$. Thus $E = (Z\Delta_H + N\Delta_n - \Delta) + (Z + N - A)c^2 = Z\Delta_H + N\Delta_n - \Delta$, where $A = Z + N$ is used. For ${}^{197}_{79}\text{Au}$, $Z = 79$ and $N = 197 - 79 = 118$. Hence,

$$\Delta E_{\rm be} = (79)(7.29\,\text{MeV}) + (118)(8.07\,\text{MeV}) - (-31.2\,\text{MeV}) = 1560\,\text{MeV}\ .$$

This means the binding energy per nucleon is $\Delta E_{\rm ben} = (1560\,\text{MeV})/197 = 7.92\,\text{MeV}$.

35. (a) We assume that the chlorine in the sample had the naturally occurring isotopic mixture, so the average mass number was 35.453, as given in Appendix F. Then, the mass of ^{226}Ra was

$$m = \frac{226}{226 + 2(35.453)}(0.10\,\text{g}) = 76.1 \times 10^{-3}\ \text{g}\ .$$

The mass of a ^{226}Ra nucleus is $(226\,\text{u})(1.661 \times 10^{-24}\,\text{g/u}) = 3.75 \times 10^{-22}\,\text{g}$, so the number of ^{226}Ra nuclei present was $N = (76.1 \times 10^{-3}\,\text{g})/(3.75 \times 10^{-22}\,\text{g}) = 2.03 \times 10^{20}$.

(b) The decay rate is given by $R = N\lambda = (N \ln 2)/T_{1/2}$, where λ is the disintegration constant, $T_{1/2}$ is the half-life, and N is the number of nuclei. The relationship $\lambda = (\ln 2)/T_{1/2}$ is used. Thus,

$$R = \frac{(2.03 \times 10^{20}) \ln 2}{(1600\,\text{y})(3.156 \times 10^{7}\,\text{s/y})} = 2.79 \times 10^{9}\ \text{s}^{-1}\ .$$

41. If N is the number of undecayed nuclei present at time t, then

$$\frac{dN}{dt} = R - \lambda N$$

where R is the rate of production by the cyclotron and λ is the disintegration constant. The second term gives the rate of decay. Rearrange the equation slightly and integrate:

$$\int_{N_0}^{N} \frac{dN}{R - \lambda N} = \int_0^t dt$$

where N_0 is the number of undecayed nuclei present at time $t = 0$. This yields

$$-\frac{1}{\lambda} \ln \frac{R - \lambda N}{R - \lambda N_0} = t \, .$$

We solve for N:

$$N = \frac{R}{\lambda} + \left(N_0 - \frac{R}{\lambda} \right) e^{-\lambda t} \, .$$

After many half-lives, the exponential is small and the second term can be neglected. Then, $N = R/\lambda$, regardless of the initial value N_0. At times that are long compared to the half-life, the rate of production equals the rate of decay and N is a constant.

61. Since the electron has the maximum possible kinetic energy, no neutrino is emitted. Since momentum is conserved, the momentum of the electron and the momentum of the residual sulfur nucleus are equal in magnitude and opposite in direction. If p_e is the momentum of the electron and p_S is the momentum of the sulfur nucleus, then $p_S = -p_e$. The kinetic energy K_S of the sulfur nucleus is $K_S = p_S^2/2M_S = p_e^2/2M_S$, where M_S is the mass of the sulfur nucleus. Now, the electron's kinetic energy K_e is related to its momentum by the relativistic equation $(p_e c)^2 = K_e^2 + 2K_e mc^2$, where m is the mass of an electron. See Eq. 38-51. Thus,

$$\begin{aligned} K_S &= \frac{(p_e c)^2}{2M_S c^2} = \frac{K_e^2 + 2K_e mc^2}{2M_S c^2} = \frac{(1.71\,\text{MeV})^2 + 2(1.71\,\text{MeV})(0.511\,\text{MeV})}{2(32\,\text{u})(931.5\,\text{MeV/u})} \\ &= 7.83 \times 10^{-5}\,\text{MeV} = 78.3\text{ eV} \end{aligned}$$

where $mc^2 = 0.511\,\text{MeV}$ is used (see Table 38-3).

75. A generalized formation reaction can be written X + x → Y, where X is the target nucleus, x is the incident light particle, and Y is the excited compound nucleus (^{20}Ne). We assume X is initially at rest. Then, conservation of energy yields

$$m_X c^2 + m_x c^2 + K_x = m_Y c^2 + K_Y + E_Y$$

where m_X, m_x, and m_Y are masses, K_x and K_Y are kinetic energies, and E_Y is the excitation energy of Y. Conservation of momentum yields

$$p_x = p_Y \ .$$

Now, $K_Y = p_Y^2/2m_Y = p_x^2/2m_Y = (m_x/m_Y)K_x$, so

$$m_X c^2 + m_x c^2 + K_x = m_Y c^2 + (m_x/m_Y)K_x + E_Y$$

and

$$K_x = \frac{m_Y}{m_Y - m_x}\left[(m_Y - m_X - m_x)c^2 + E_Y\right] \ .$$

(a) Let x represent the alpha particle and X represent the ^{16}O nucleus. Then, $(m_Y - m_X - m_x)c^2 = (19.99244\,\text{u} - 15.99491\,\text{u} - 4.00260\,\text{u})(931.5\,\text{MeV/u}) = -4.722\,\text{MeV}$ and

$$K_\alpha = \frac{19.99244\,\text{u}}{19.99244\,\text{u} - 4.00260\,\text{u}}(-4.722\,\text{MeV} + 25.0\,\text{MeV}) = 25.35\ \text{MeV} \ .$$

(b) Let x represent the proton and X represent the ^{19}F nucleus. Then, $(m_Y - m_X - m_x)c^2 = (19.99244\,\text{u} - 18.99841\,\text{u} - 1.00783\,\text{u})(931.5\,\text{MeV/u}) = -12.85\,\text{MeV}$ and

$$K_\alpha = \frac{19.99244\,\text{u}}{19.99244\,\text{u} - 1.00783\,\text{u}}(-12.85\,\text{MeV} + 25.0\,\text{MeV}) = 12.80\ \text{MeV} \ .$$

(c) Let x represent the photon and X represent the ^{20}Ne nucleus. Since the mass of the photon is zero, we must rewrite the conservation of energy equation: if E_γ is the energy of the photon, then $E_\gamma + m_X c^2 = m_Y c^2 + K_Y + E_Y$. Since $m_X = m_Y$, this equation becomes $E_\gamma = K_Y + E_Y$. Since the momentum and energy of a photon are related by $p_\gamma = E_\gamma/c$, the conservation of momentum equation becomes $E_\gamma/c = p_Y$. The kinetic energy of the compound nucleus is $K_Y = p_Y^2/2m_Y = E_\gamma^2/2m_Y c^2$. We substitute this result into the conservation of energy equation to obtain

$$E_\gamma = \frac{E_\gamma^2}{2m_Y c^2} + E_Y \ .$$

This quadratic equation has the solutions

$$E_\gamma = m_Y c^2 \pm \sqrt{(m_Y c^2)^2 - 2m_Y c^2 E_Y} \ .$$

If the problem is solved using the relativistic relationship between the energy and momentum of the compound nucleus, only one solution would be obtained, the one corresponding to the negative sign above. Since $m_Y c^2 = (19.99244\,\text{u})(931.5\,\text{MeV/u}) = 1.862 \times 10^4\,\text{MeV}$,

$$\begin{aligned} E_\gamma &= (1.862 \times 10^4\,\text{MeV}) - \sqrt{(1.862 \times 10^4\,\text{MeV})^2 - 2(1.862 \times 10^4\,\text{MeV})(25.0\,\text{MeV})} \\ &= 25.0\ \text{MeV} \ . \end{aligned}$$

The kinetic energy of the compound nucleus is very small; essentially all of the photon energy goes to excite the nucleus.