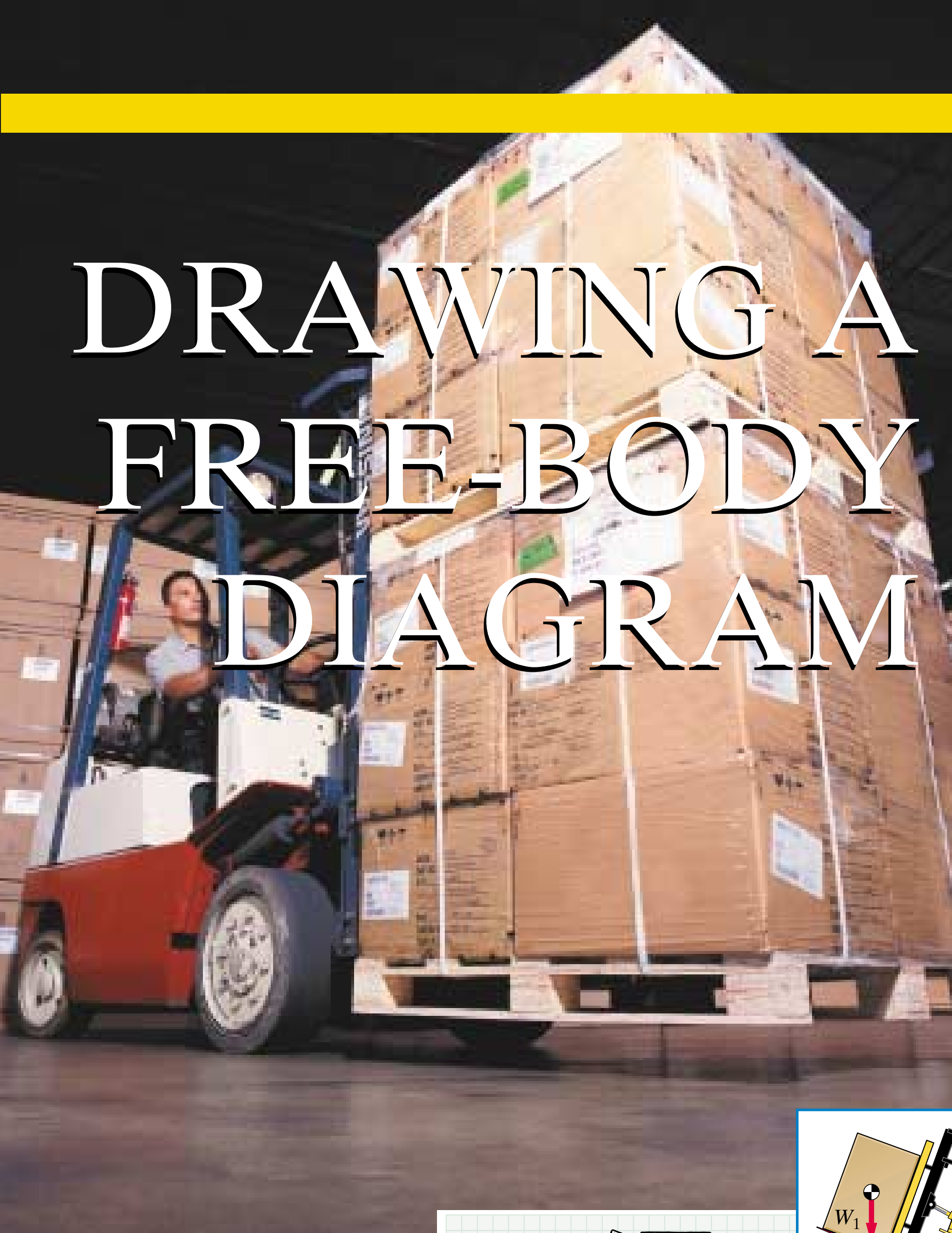
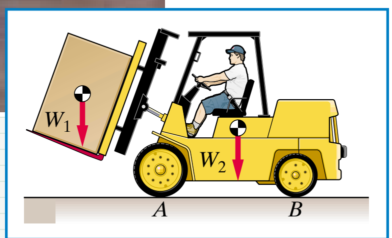
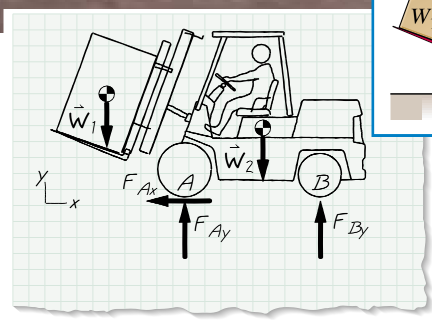


# DRAWING A FREE-BODY DIAGRAM

The **free-body diagram** is the most important tool in this book. It is a drawing of a system and the loads acting on it. Creating a free-body diagram involves mentally separating the system (the portion of the world you're interested in) from its surroundings (the rest of the world), and then drawing a simplified representation of the system. Next you identify all the loads (forces and moments) acting on the system and add them to the drawing.



## OBJECTIVES

**On completion of this chapter, you will be able to:**

- **Isolate a system from its surroundings and identify the supports**
- **Define the loads associated with supports and represent these loads in terms of vectors**
- **Inspect a system and determine whether it can be modeled as a planar system**
- **Represent the system and the external loads acting on it in a diagram called a free-body diagram**

As an example of creating a free-body diagram, consider the two friends leaning against each other in **Figure 6.1*a***. The free-body diagram of Friend 1 is shown in **Figure 6.1*c***. In going from **Figure 6.1*a*** to **6.1*c***, we zoomed in on and drew a boundary around the system (Friend 1) to isolate him from his surroundings, as shown in **Figure 6.1*b***. This boundary is an imaginary surface and the system is (by definition) the "stuff" inside this imaginary surface. The system's surroundings are everything else. You can think of the boundary as a shrink wrap around the system.

After drawing the boundary, we identified the external loads acting on the system either at or across this boundary and drew them at their points of application. These loads represent how the surroundings push, pull, and twist the system. In a free-body diagram we draw the system somewhat realistically and replace the surroundings with the loads they apply to the system. It is important to recognize that we are not ignoring the surroundings—we simply replace them with the loads the system experiences because of them. For example, in **Figure 6.1*c*** we replace the back of Friend 2 with the normal force he applies to the back of Friend 1 ($\boldsymbol{F}_{\text{normal, back 2 on back 1}}$).

This chapter is devoted exclusively to creating free-body diagrams. We build on the work in prior chapters on forces and moments and on the engineering analysis procedure presented in Chapter 1. Creating a free-body diagram is part of the DRAW step in the analysis procedure.

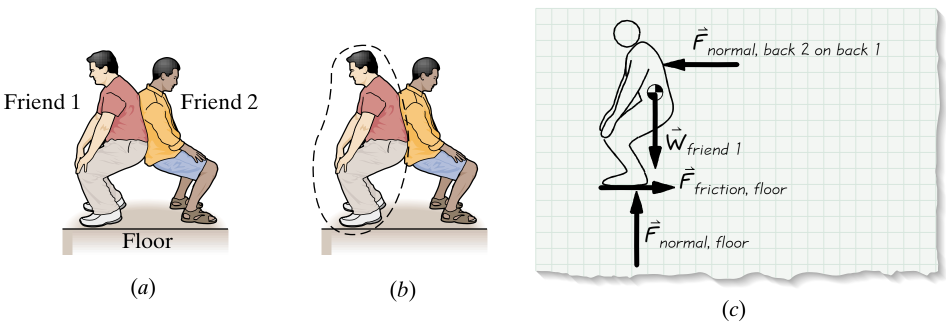



**Figure 6.1** (*a*) Two friends leaning against one another; (*b*) isolate Friend 1 by drawing a boundary. Friend 1 is the system; (*c*) a free-body diagram of Friend 1

## 6.1 TYPES OF EXTERNAL LOADS ACTING ON SYSTEMS

Some of the external loads acting on a system act *across* the system boundary; the principal example of this type of load is **gravity** (which manifests itself as weight). Another example is the magnetic force, which results from electromagnetic field interaction. The magnetic force is what turns a motor.

Other external loads act *directly on* the boundary. Consider where:

- The boundary passes through a connection between the system and its surroundings, commonly referred to as a **boundary support** (or **support** for short). A support may be, for example, a bolt, cable or a weld, or simply where the system rests against its surroundings. We replace each support with the loads it applies to the system. These loads consist of the contact forces discussed in Chapter 4 (normal contact, friction, tension, compression, and shear). Synonyms for the term *supports* are *reactions* and *boundary connections*.
- The boundary separates the system from fluid surroundings. We refer to this as a **fluid boundary**. We replace the fluid with the loads the fluid applies to the system. This load consists of the pressure (force per unit area) of the fluid pressing on and/or moving along the boundary.

In practice, a system may be acted on by a combination of cross-boundary loads (usually gravity), loads at supports, and loads at fluid boundaries, as illustrated in **Figure 6.2**.

Notice that at some boundary locations no loads act. At other locations there are what are called **known loads**—for example, in **Figure 6.2**, the 40-kN gravity force is a known load.

Depending on the nature of an external load acting on a system, when that load is drawn on a free-body diagram it is represented either by a vector acting at a point of application or as a distributed load acting on an area. The load is given a unique variable label, and its magnitude (if it is a known load) is written next to the vector.

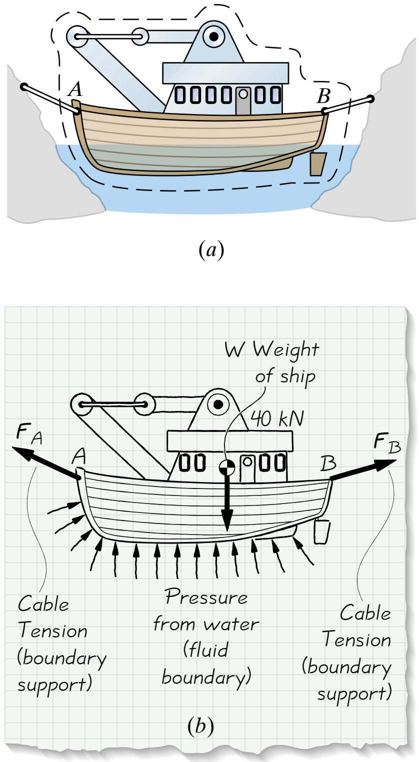



**Figure 6.2** (*a*) Isolate the ship by drawing a boundary. The ship is the system; (*b*) a free-body diagram of the ship

## EXERCISES 6.1

**6.1.1.** The system to be considered is a coat rack with some items hanging off of it as shown in **E6.1.1**. In your mind draw a boundary around the system to isolate it from its surroundings.

**a.** Make a sketch of the coat rack and the external loads acting on it. Show the loads as vectors and label them with variables, and where possible give word descriptions of the loads.

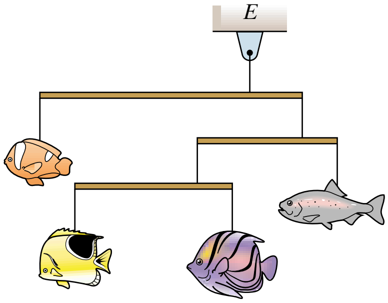
**b.** List any uncertainties you have about the free-body diagram you have created.

**E6.1.1**

**6.1.2.** The system to be considered is defined as a mobile as shown in **E6.1.2**. In your mind draw a boundary around the system to isolate it from its surroundings at point E.

**a.** Make a sketch of the mobile and the external loads acting on it. Show the loads as vectors and label them with variables, and where possible give word descriptions of the loads.

**b.** List any uncertainties you have about the free-body diagram you have created.




**E6.1.2**

**6.1.3.** The system to be considered is a person and a ladder, as shown in **E6.1.3**. In your mind draw a boundary around the system to isolate it from its surroundings.

**a.** Make a sketch of the system and the external loads acting on it. Show the loads as vectors and label them with variables, and where possible give word descriptions of the loads.

**b.** List any uncertainties you have about the free-body diagram you have created.

**E6.1.3**

**6.1.4.** Visit a weight room, and take a look at one of the exercise stations—preferably one that is in use!

**a.** Consider where the person is standing, hanging, laying, and/or pushing on it. In your mind, draw a boundary around *the person* to define him or her as the system. Make a sketch of the system and the external loads acting on it, showing the loads as vectors with variable labels. Where possible give word descriptions of the loads. List any uncertainties you have about the free-body diagram you have created.

**b.** Consider where the person is standing, hanging, laying, and/or pushing on it. In your mind, draw a boundary around the *exercise machine* to define it as the system. Make a sketch of the system and the external loads acting on it, showing the loads as vectors with variable labels. Where possible give word descriptions of the loads. List any uncertainties you have about the free-body diagram you have created.

**6.1.5.** Visit a local playground near campus, and take a look at a jungle gym—preferably one that is in use! Consider where the children (or adults!) are standing/hanging. In your mind, draw a boundary around the *jungle gym* to define it as your system. Make a sketch of the system and the external loads acting on it, showing the loads as vectors with variable labels. List any uncertainties you have about the free-body diagram you have created.

**6.1.6.** Visit a local pet store or zoo and look at the fish tanks.

**a.** In your mind, draw a boundary around the *fish tank including the water* to define it as your system. Make a sketch of the system and the external loads acting on it, showing the loads as vectors with variable labels. List any uncertainties you have about the free-body diagram you have created.

**b.** In your mind, draw a boundary around the *fish tank excluding the water* to define it as your system. Make a sketch of the system and the external loads acting on it, showing the loads as vectors with variable labels. List any uncertainties you have about the free-body diagram you have created.

## 6.2 PLANAR SYSTEM SUPPORTS

We now consider how to identify supports and represent the loads associated with them when working with **planar systems**. A system is planar if all the forces acting on it can be represented in a single plane and all moments are about an axis perpendicular to that plane. If a system is not planar it is a **nonplanar system**. In Section 6.4 we will lay out guidelines for identifying planar and nonplanar systems. For now, we assume that all of the systems we are dealing with in this section are planar.

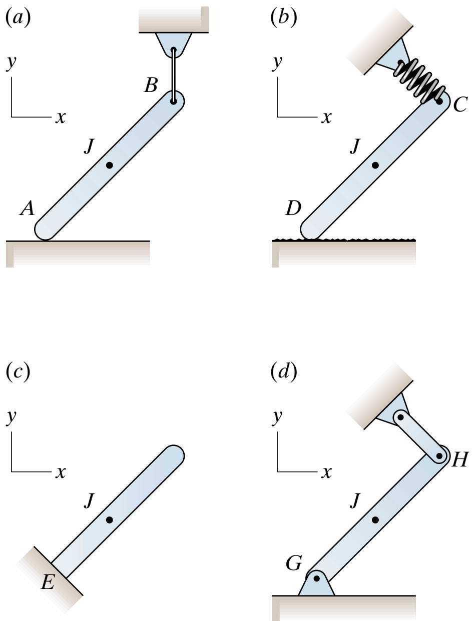



**Figure 6.3** An object connected to its surroundings by various supports. The object can be modeled as a planar system

Consider the systems in **Figure 6.3** for which we want to draw fee-body diagrams. Each system consists of a uniform bar of weight $W$, oriented so that gravity acts in the negative $y$ direction. However, each has different supports that connect it to its surroundings. For example:

In **Figure 6.3*a***, the supports consist of
- normal contact without friction at $A$, and
- cable attached to the system at $B$.

In **Figure 6.3*b***, the supports consist of
- a spring attached to the system at $C$, and
- normal contact with friction at $D$.

In **Figure 6.3*c***, the support consists of
- a system fixed to its surroundings at $E$.

In **Figure 6.3*d***, the supports consist of
- a system pinned to its surroundings at $G$, and
- a link attached to the system at $H$.

At each support we consider whether the surroundings act on the system with a force and/or a moment. As a general rule, *if a support prevents the translation of the system in a given direction, then a force acts on the system at the location of the support in the opposing direction. Likewise, if rotation is prevented, a moment opposing the rotation acts on the system at the location of the support.*

**At Point *A* (Normal Contact Without Friction).** At this support the system rests against a smooth, frictionless surface. A normal force prevents the system from moving into the surface and is oriented so

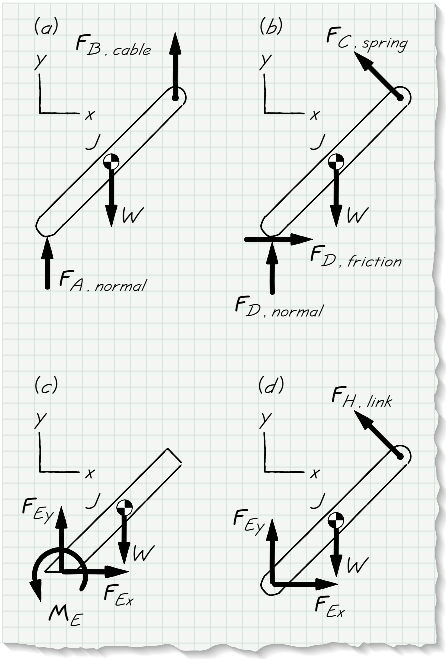



**Figure 6.4** Free-body diagrams of the planar systems shown in Figure 6.3

as to push on the system. Because the supporting surface at $A$ is smooth, friction between the system and its surroundings is nonexistent. Therefore there is no force component parallel to the surface. In **Figure 6.4*a*** (the free-body diagram of **Figure 6.3*a***) the force resulting from normal contact at $A$ is represented by $\boldsymbol{F}_{A,\text{normal}}$; we know its direction is normal to the surface so as to *push* on the system.

**At Point *B* (Cable).** At this support a force acts on the system; the line of action of the force is along the cable. The force represents the cable pulling on the system because the cable can only act in tension. In **Figure 6.4*a*** the force from the cable at $B$ is represented by $\boldsymbol{F}_{B,\text{cable}}$; we know its direction is along the cable axis in the direction that allows the cable to *pull* on the system.

**At Point *C* (Spring).** At this support there is a force that either pushes or pulls on the system; the line of action of the force is along the axis of the spring. If the spring is extended by an amount $\Delta$, the spring is in tension and the force is oriented so as to pull on the system. If the spring is compressed by an amount $\Delta$, the force is oriented so as to push on the system. The magnitude of the force is proportional to the amount of spring extension or compression, and the proportionality constant is the spring constant, $k$. In other words, the size of the force is equal to the product of $k$ and the spring extension or compression:

$$F_C = k(\Delta) \tag{6.1}$$

where the dimensions of $k$ are force/length (e.g., N/mm). The value of $F_C$ in (6.1) will be positive when the spring is in tension (since $\Delta$ will be positive) and negative when the spring is compressed (since $\Delta$ will be negative).

In **Figure 6.4*b*** the spring force at $C$ is represented by $\boldsymbol{F}_{C,\text{spring}}$; we know its direction is along the spring axis. If the spring is in tension, the force acts so as to *pull* on the system. If the spring is in compression, the force acts so as to push on the system. We have drawn the direction of $\boldsymbol{F}_{C,\text{spring}}$ to indicate that the spring is in tension. We could equally well have chosen the direction of $\boldsymbol{F}_{C,\text{spring}}$ to indicate that the spring is in compression, but as we will see in the next chapter, drawing the spring in tension will make interpreting numerical answers easier.

**At Point *D* (Normal Contact with Friction).** At this support there are two forces acting on the system. One is a normal force (just like normal contact without friction). The second is due to friction and is parallel to the surface against which the system rests—therefore it is perpendicular to the normal force.

The force due to friction ($\boldsymbol{F}_{\text{friction}}$) is related to and limited by normal contact force ($\boldsymbol{F}_{\text{normal}}$) and the characteristics of the contact. Often the relationship between $\boldsymbol{F}_{\text{friction}}$ and $\boldsymbol{F}_{\text{normal}}$ is represented in terms of the Coulomb Friction Model. This model states that if $\|\boldsymbol{F}_{\text{friction}}\| < \mu_{\text{static}} \|\boldsymbol{F}_{\text{normal}}\|$ the system will not slide relative to its surroundings, where

$\mu_{\text{static}}$ is the coefficient of static friction and typically ranges from 0.01 to 0.70, depending on the characteristics of the contact. If $\|\boldsymbol{F}_{\text{friction}}\| = \mu_{\text{static}} \|\boldsymbol{F}_{\text{normal}}\|$, the condition is that of impending motion.

In **Figure 6.4*b*** the normal force at $D$ is represented by $\boldsymbol{F}_{D,\text{normal}}$; we know its direction is normal to the surface so as to push on the system. The friction force is represented by $\boldsymbol{F}_{D,\text{friction}}$ and is parallel to the surface and perpendicular to the normal force. We could have drawn $\boldsymbol{F}_{D,\text{friction}}$ to point to the right or to the left; we arbitrarily chose to draw it upward to the right.

**At Point *E* (System Fixed to Surroundings, Referred to as a Fixed Support).** At this support a force and a moment act on the system. To get a feeling for a fixed boundary, consider the setup shown in **Figure 6.5** (better yet, reproduce it yourself). The ruler is the system, and your hands are the surroundings. Grip one end of the ruler firmly with your left hand and apply a force with a finger of your right hand, as shown in **Figure 6.5*a***. Notice that your left hand automatically applies a load (consisting of a force and a moment) to the ruler in order to keep the gripped end from translating and rotating; this load "fixes" the gripped end relative to your left hand. The load of your left hand acting on the ruler (a fixed support) can be represented as a force of $\boldsymbol{F}_{\text{LH}} = F_{\text{LH},x}\boldsymbol{i} + F_{\text{LH},y}\boldsymbol{j}$ and a moment of $\boldsymbol{M}_{\text{LH}} = M_{\text{LH},z}\boldsymbol{k}$ that are the net effect of your left hand gripping the ruler (see **Figure 6.5*b***).

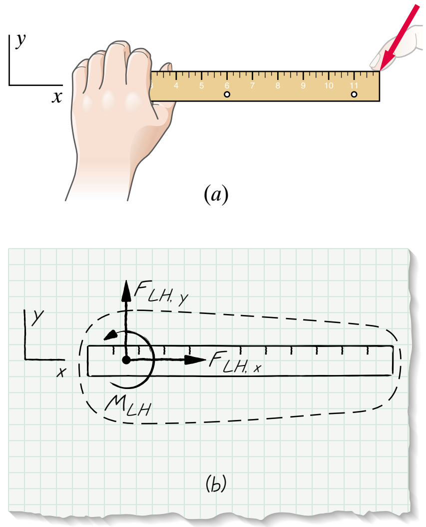



**Figure 6.5** Illustrating a planar fixed boundary connection: (*a*) applying loads to a ruler; (*b*) the resulting free-body diagram if the ruler is defined as the system. Note how the loads the left-hand applies to the ruler are depicted.

Returning now to the system depicted in **Figure 6.3*c***, we can describe the loads acting at the fixed support at $E$ as $\boldsymbol{F}_E = F_{Ex}\boldsymbol{i} + F_{Ey}\boldsymbol{j}$ and $\boldsymbol{M}_E = M_{Ez}\boldsymbol{k}$. These loads are shown in **Figure 6.4*c***.

**At Point *G* (System Pinned to Its Surroundings, Referred to as a Pin Connection).** A pin connection consists of a pin that is loosely fitted in a hole. At this support a force acts on the system. To get a feeling for the force at a pin connection consider the physical setup in **Figure 6.6*a***. The ruler (which is the system) is lying on a flat surface in position 1. A pencil, which is acting like a pin, is placed in the hole in the ruler and is gripped firmly with your left hand. The pencil and your hands constitute the surroundings. Now load the system with your right hand as shown in **Figure 6.6*a***; notice how your left hand reacts with a force to counter the right-hand force. If you next orient the ruler and right hand load as shown in **Figure 6.6*b***, again your left hand counters with a force. Finally, load the ruler as shown in **Figure 6.6*c***, and notice that the ruler rotates because your left hand is unable to counter with an opposing moment. This exercise tells you that there is a force ($\boldsymbol{F}_{\text{LH}}$) acting on the system at the pin connection but no moment. The force $\boldsymbol{F}_{\text{LH}}$ lies in the plane perpendicular to the pencil's length. For the situation in **Figure 6.6**, this means that $\boldsymbol{F}_{\text{LH}}$ can be written $\boldsymbol{F}_{\text{LH}} = F_{\text{LH},x}\boldsymbol{i} + F_{\text{LH},y}\boldsymbol{j}$ (**Figure 6.6*d***).

For the system in **Figure 6.3*d***, we can describe the load acting at the pin connection at $G$ as $\boldsymbol{F}_G = F_{Gx}\boldsymbol{i} + F_{Gy}\boldsymbol{j}$, as shown in **Figure 6.4*d***. We have arbitrarily chosen to draw both components in their respective positive direction.

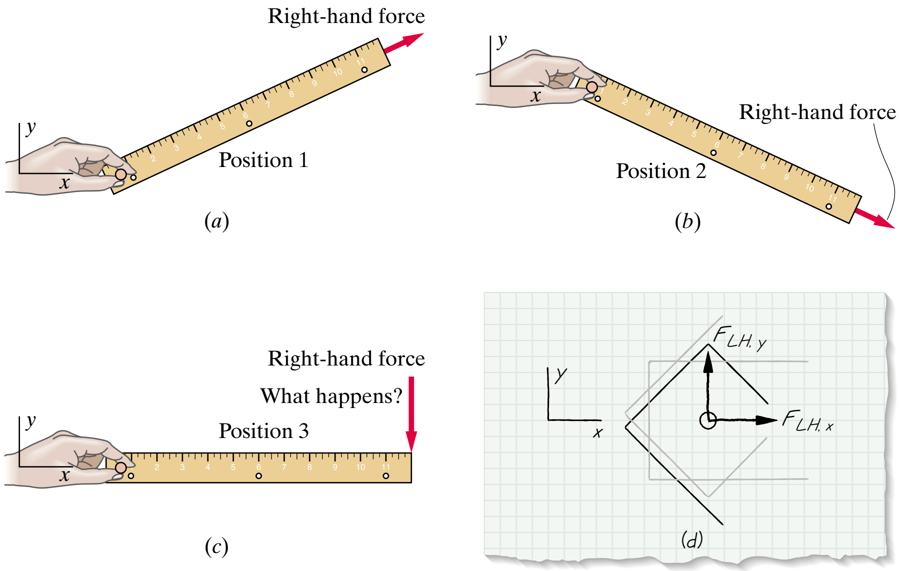



**Figure 6.6** Illustrating a planar pin connection: (*a*) applying loads to a ruler (Position 1); (*b*) applying loads to a ruler (Position 2); (*c*) applying loads to a ruler (Position 3); (*d*) loads acting on the ruler at the pin connection

**At Point *H* (Link).** At this support there is a force that either pushes or pulls on the system; the line of action of the force is along the axis of the link. We shall have a lot more to say about links in the next chapter—for now we simply say that a link is a member with a pin connection at each end and no other loads acting on it.

In **Figure 6.4*d*** the force at $H$ is represented by $\boldsymbol{F}_{H,\text{link}}$ acting along the long axis of the link. A link may either *push or pull* on the system, and here we have chosen to assume pulling. We could equally well have chosen the direction of $\boldsymbol{F}_{H,\text{link}}$ to indicate that the link is pushing, but as we will see in the next chapter, drawing the link as pulling will make interpreting numerical answers easier.

The free-body diagrams of the planar systems in **Figure 6.3** are presented in **Figure 6.4**. These diagrams include loads due to supports, as well as the load due to gravity acting at $J$. Each load is represented as a vector and is given a variable label. If the magnitude of a load is known, this value is included on the diagram.

## Summary

**Table 6.1** summarizes the loads associated with the planar supports discussed, along with some other commonly found planar supports. Don't feel that you need to memorize all the supports in this table—it is presented merely as a ready reference. On the other hand, you should be familiar with the loads associated with these standard planar supports.

**Table 6.1 Standard Supports for Planar Systems**

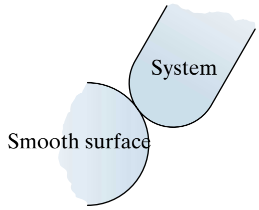
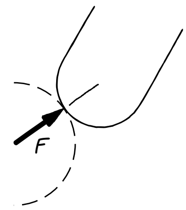
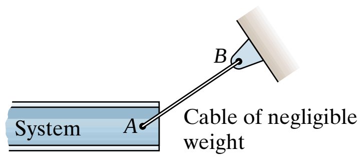
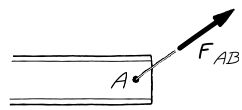
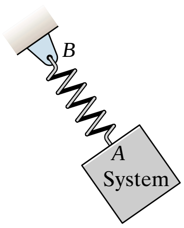
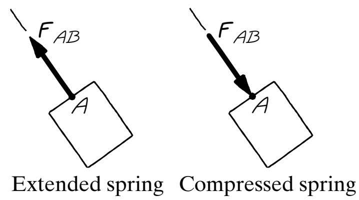
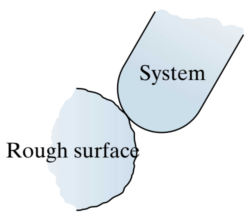
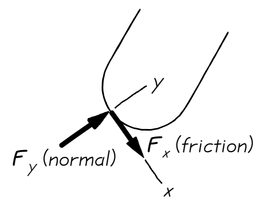
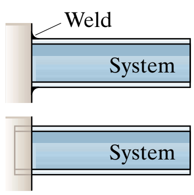
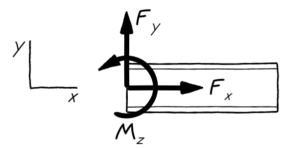
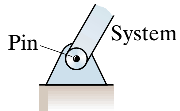
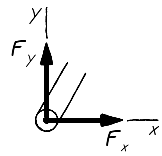
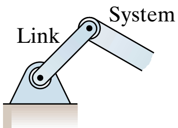
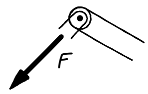
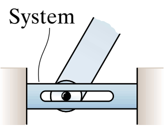
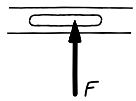
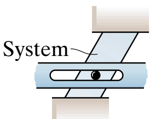
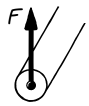
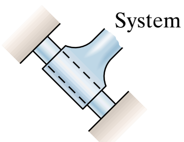
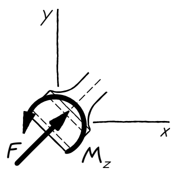
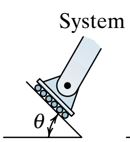
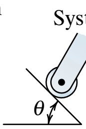
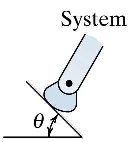
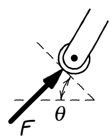
| (A)<br>Supports | Description of Loads | (B)<br>Loads to Be Shown on<br>Free-Body Diagram |
|---|---|---|
| 1. **Normal contact without friction**<br> | **Force** ($\boldsymbol{F}$) oriented normal to surface on which system rests. Direction is such that force pushes on system. | $\boldsymbol{F}$<br> |
| 2. **Cable, rope, wire**<br> | **Force** ($\boldsymbol{F}$) oriented along cable. Direction is such that cable pulls on the system. | $\boldsymbol{F}$<br> |
| 3. **Spring**<br> | **Force** ($\boldsymbol{F}$) oriented along long axis of spring. Direction is such that spring pulls on system if spring is in tension, and pushes if spring is in compression. | $\boldsymbol{F}$<br> |
| 4. **Normal contact with friction**<br> | **Two forces**, one ($\boldsymbol{F}_y$) oriented normal to surface on which the system rests so as to push on system, other force ($\boldsymbol{F}_x$) is tangent to surface. | $\boldsymbol{F}_y$<br>$\boldsymbol{F}_x$<br> |
| 5. **Fixed support**<br> | **Force** in $xy$ plane represented in terms of components $\boldsymbol{F}_x$ and $\boldsymbol{F}_y$.<br>**Moment** about $z$ axis ($\boldsymbol{M}_z$). | $\boldsymbol{F}_x + \boldsymbol{F}_y$<br>$\boldsymbol{M}_z$<br> |

**Table 6.1** **(Cont.)**

| (A)<br>Supports | Description of Loads | (B)<br>Loads to Be Shown on Free-Body Diagram |
|---|---|---|
| 6. **Pin connection**<br>(pin or hole is part of system)<br> | **Force** perpendicular to pin represented in terms of components $\boldsymbol{F}_x$ and $\boldsymbol{F}_y$. Point of application is at center of pin. | $\boldsymbol{F}_x + \boldsymbol{F}_y$<br> |
| 7. **Link**<br> | **Force** ($\boldsymbol{F}$) oriented along link length; force can push or pull on the system. | $\boldsymbol{F}$<br> |
| 8. **Slot-on-pin**<br>(slotted member is part of system)<br> | **Force** ($\boldsymbol{F}$) oriented normal to long axis of slot. Direction is such that force can pull or push on system. | $\boldsymbol{F}$<br> |
| 9. **Pin-in-slot**<br>(pin is part of system)<br> | **Force** ($\boldsymbol{F}$) oriented normal to long axis of slot. Direction is such that force can pull or push on system. | $\boldsymbol{F}$<br> |
| 10. **Smooth collar on smooth shaft**<br> | **Force** ($\boldsymbol{F}$) oriented perpendicular to long axis of shaft. Direction is such that force can pull or push on system.<br>**Moment** ($\boldsymbol{M}_z$) about $z$ axis. | $\boldsymbol{F}$<br>$\boldsymbol{M}_z$<br> |
| 11. **Roller or rocker**<br>Roller<br>Roller<br>Rocker | **Force** ($\boldsymbol{F}$) oriented normal to surface on which system rests. Direction is such that force pushes on system. | $\boldsymbol{F}$<br> |

## EXAMPLE 6.1 COMPLETE FREE-BODY DIAGRAMS

In **Figure 6.7** and **Figure 6.8**, a block is supported at several points and the system is defined as the block. Gravity acts downward in the $-y$ direction at the indicated center of gravity (CG). In **Figure 6.7**, the surface at $B$ is rough. As shown in **Figure 6.8**, the magnitudes of $\boldsymbol{F}_C$ and $\boldsymbol{M}_C$ are 10 lb and 40 in.-lb, respectively.

(a) Explain why these figures are not free-body diagrams.

(b) Create a free-body diagram of each system.

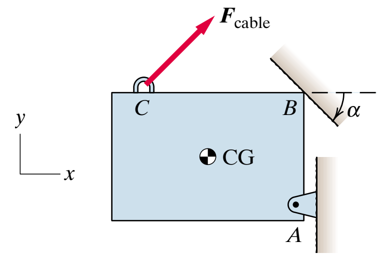


Figure 6.7

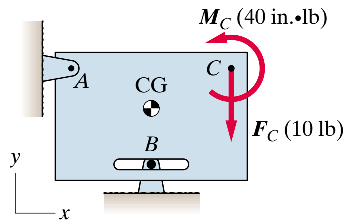


Figure 6.8

**Goal** Explain why the two figures are not free-body diagrams and create complete correct free-body diagrams.

**Given** We are given two systems with specified loads and supports.

**Assume** We assume that the system in each figure is planar because the known loads and the loads applied by supports all lie in a single plane. We also assume the slot-pin connection at $B$ in **Figure 6.8** is smooth (frictionless).

**Draw** For each system, isolate it by drawing a boundary around the block as is done in **Figure 6.9** and **Figure 6.10**. Then replace each support by its associated loads.

**Solution** **(a)** Neither figure is a free-body diagram because the system (the block) has not been isolated from its surroundings—at $A$ and $B$ each block is still shown connected to its surroundings.

**(b) Figure 6.7**: We isolate the block using the boundary shown in **Figure 6.9**, establish the $xy$ coordinate system for the entire system and the $x^*y^*$ coordinate system to simplify the representation of the support at $B$. Then we note the following:

*At A* a pin connection attaches the system to its surroundings. According to **Table 6.1**, a pin connection applies a force to the system. As we do not know the direction or magnitude of this force, we represent it as two components, $\boldsymbol{F}_{Ax}$ and $\boldsymbol{F}_{Ay}$, which we arbitrarily draw in the positive $x$ and $y$ directions (**Figure 6.11**).

*At B* the system rests against a surface inclined at angle $\alpha$ relative to the horizontal. Since we know that the surface is rough, we must consider the friction between surface and system. According to **Table 6.1**, there will be a normal force $\boldsymbol{F}_{B,\text{normal}}$ acting on the system, oriented perpendicular to the surface so as to push on the system, as shown in **Figure 6.11**. We do not know its magnitude. There is also the friction force $\boldsymbol{F}_{B,\text{friction}}$, perpendicular to $\boldsymbol{F}_{B,\text{normal}}$. We do not know the magnitude of $\boldsymbol{F}_{B,\text{friction}}$ or whether it acts in the $+x^*$ or $-x^*$ direction, and so we arbitrarily draw it in the $+x^*$ direction.

*At C* a cable pulls on the system, which we represent as a force of unknown magnitude but known direction ($\boldsymbol{F}_C$).

*At CG (the center of gravity)* a force $\boldsymbol{W}$ acts in the $-y$ direction.

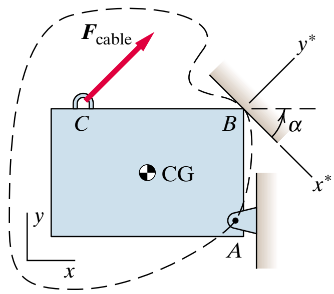


Figure 6.9

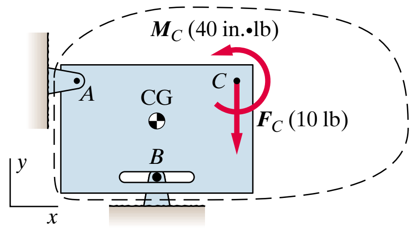


Figure 6.10

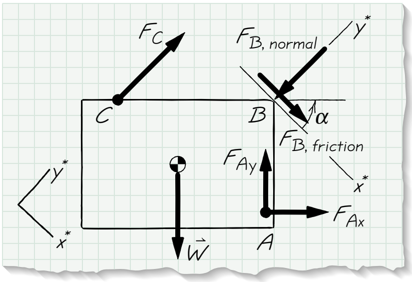



**Figure 6.11**

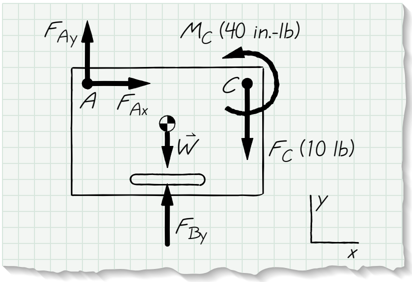



**Figure 6.12**

The block presented in **Figure 6.11**, with forces $\boldsymbol{F}_{Ax}$, $\boldsymbol{F}_{Ay}$, $\boldsymbol{F}_{B,\text{normal}}$, $\boldsymbol{F}_{B,\text{friction}}$, $\boldsymbol{F}_C$, and $\boldsymbol{W}$ each drawn at its point of application is a free-body diagram of the system in **Figure 6.7**.

**Figure 6.8**: We isolate the block using the boundary shown in **Figure 6.10**, establish an $xy$ coordinate system as shown, and then we note the following:

*At A* a pin connection attaches the system to its surroundings. According to **Table 6.1**, a pin connection applies a force to the system. As we do not know the direction or magnitude of this force, we represent it as two components, $\boldsymbol{F}_{Ax}$ and $\boldsymbol{F}_{Ay}$, which we arbitrarily draw in the positive $x$ and $y$ directions (**Figure 6.12**).

*At B* a slot in the block is attached to a slider that allows the block to move in the $x$ direction but prevents movement in the $y$ direction. Therefore, we include a force $\boldsymbol{F}_{By}$, which we have arbitrarily drawn in the positive direction.

*At C* there is a known force ($\boldsymbol{F}_C$) and a moment ($\boldsymbol{M}_C$). Known values are written next to the vectors.

*At CG (the center of gravity)* a force $\boldsymbol{W}$ acts in the $-y$ direction.

The block presented in **Figure 6.12**, with forces $\boldsymbol{F}_{Ax}$, $\boldsymbol{F}_{Ay}$, $\boldsymbol{F}_{By}$, and $\boldsymbol{F}_{C,\text{known}}$, $\boldsymbol{W}$, and moment $\boldsymbol{M}_{C,\text{known}}$ each drawn at its point of application constitutes a free-body diagram of the system. Notice that this diagram includes the known magnitudes of $\boldsymbol{F}_C$ and $\boldsymbol{M}_C$.

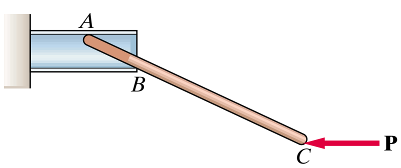
***Answer***

(a) **Figures 6.7** and **6.8** are not free-body diagrams of systems because each system (the block) has not been fully separated from all supports.

(b) The free-body diagrams of the systems in **Figures 6.7** and **6.8** are shown in **Figures 6.11** and **6.12**, respectively.

## EXAMPLE 6.2 EVALUATING THE CORRECTNESS OF FREE-BODY DIAGRAMS

Consider the description of each planar system in **Figures 6.13–6.21** and determine whether the associated free-body diagram is correct. Unless otherwise stated, assume that the weight of the system is negligible and therefore can be ignored.




(*a*)

**Figure 6.13**

**(a)** A thin rod is supported by a smooth tube that is fixed to the wall (**Figure 6.13*a***). The rod is touching the tube interior at $A$ and leaning on the tube end at $B$. A known force $P$ is pushing on the rod at $C$. Assume no friction on the surface of the tube.

**Answer** *Tube:* The proposed free-body diagram of the tube in **Figure 6.13*b*** is *correct*. It accounts for the fixed end at the left and the normal contact between the tube and the rod.

**Answer** *Rod:* The proposed free-body diagram of the tube in **Figure 6.13*b*** is *not correct*. The force $\boldsymbol{F}_{B,\text{tube on rod}}$ in this diagram, representing the normal force of the tube pushing on the rod, should be in the other direction (Newton's third law). As shown, the tube is pulling on the rod which is not physically possible.

**(b)** A beam, pinned at $A$ and resting against a roller at $B$, is loaded by a 2-kN force and a 2.4-kN · m moment (**Figure 6.14*a***).

**Answer** The proposed free-body diagram of the beam in **Figure 6.14*b*** is *not correct*. The normal force, $\boldsymbol{F}_{B,\text{normal}}$, acting on the beam at $B$ should be oriented perpendicular to the inclined surface.

**(c)** A uniform beam weighing 200 lb is fixed at $A$. A 400-lb and 1400-lb load are applied at $B$ and $C$ as shown (**Figure 6.15*a***).

**Answer** The proposed free-body diagram in **Figure 6.15*b*** is *correct*. The weight of the beam is included at the $x = 5$ ft position since the beam is uniform.

**(d)** A door that weighs $W$ hangs from hinges at $A$ and $B$. Hinge $A$ acts like a pin connection, and hinge $B$ acts like a vertical slot support (**Figure 6.16*a***).

**Answer** The proposed free-body diagram in **Figure 6.16*b*** is *not correct*. The hinge at $B$ does not apply a vertical force to the door, so this force should not be on the free-body diagram. The vertical force component at $A$ ($\boldsymbol{F}_{By}$) is a thrust force.

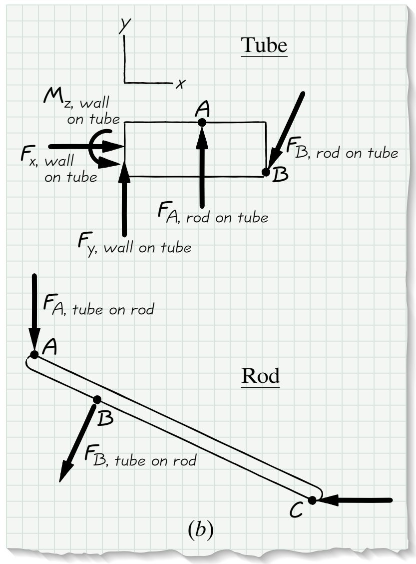



**Figure 6.13 (Cont.)** Proposed free-body diagram

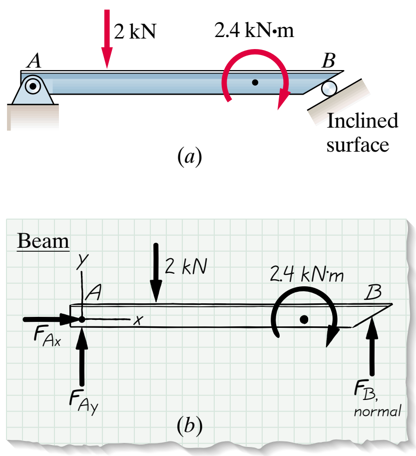



**Figure 6.14**

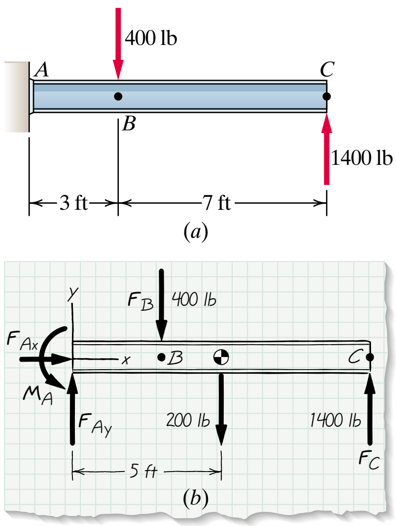



**Figure 6.15**

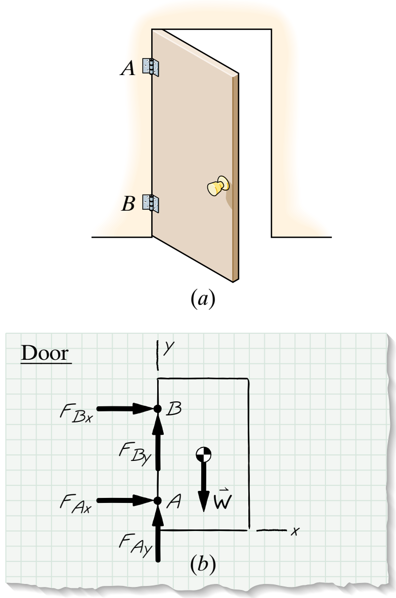



**Figure 6.16**

**(e)** A man weighing 800 N sits at the picnic table halfway between its two ends. His center of gravity ($CG_{man}$) is noted. The table weighs 200 N, with a center of gravity at $CG_{table}$ (**Figure 6.17*a***). Assume that any friction between the legs and the ground can be neglected.

***Answer*** The proposed free-body diagram in **Figure 6.17*b*** is *not correct*. The label for the normal force acting on the table at $B$ should be $2\boldsymbol{F}_B$, (not $\boldsymbol{F}_B$), because the force vector at $B$ represents the normal force for two legs.

**(f)** A frame used to lift a hatch is pinned to the hatch at $A$ and $B$. A force $\boldsymbol{F}$ is applied to the frame at $C$ (**Figure 6.18*a***).

***Answer*** This free-body diagram in **Figure 6.18*b*** is *not correct*. At each pin connection force components in the $x$ and $y$ directions should be shown.

**(g)** A frame consisting of members $AB$ and $CD$ supports the pulleys, cable, and block $L$ (**Figure 6.19*a***).

***Answer*** *Whole frame:* The proposed free-body diagram in **Figure 6.19*b*** is *correct*. It includes the forces at pins $A$ and $C$, the cable tension $\boldsymbol{F}_{cable}$ pulling on the frame, and the gravity force from block $L$ ($\boldsymbol{W}_L$).
*Member CD:* The proposed free-body diagram in **Figure 6.19*b*** is *incorrect* because the forces at the pin connection at $D$ have not been included
*Member AB:* The proposed free-body diagram in **Figure 6.19*b*** is *correct*.

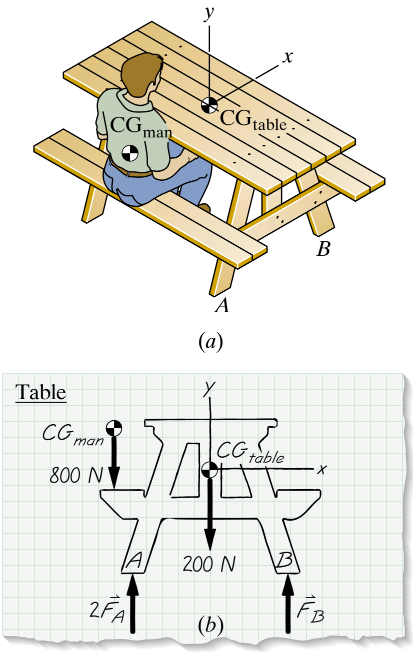



**Figure 6.17**

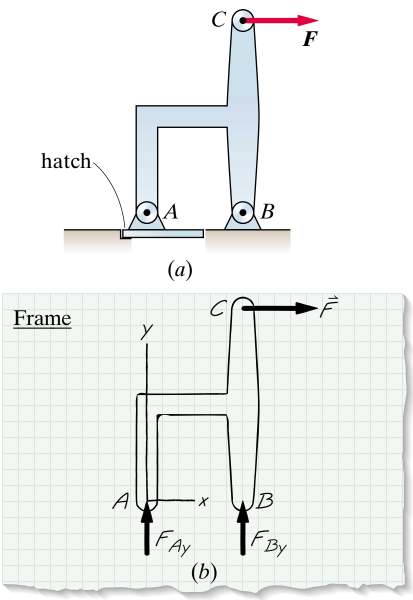



**Figure 6.18**

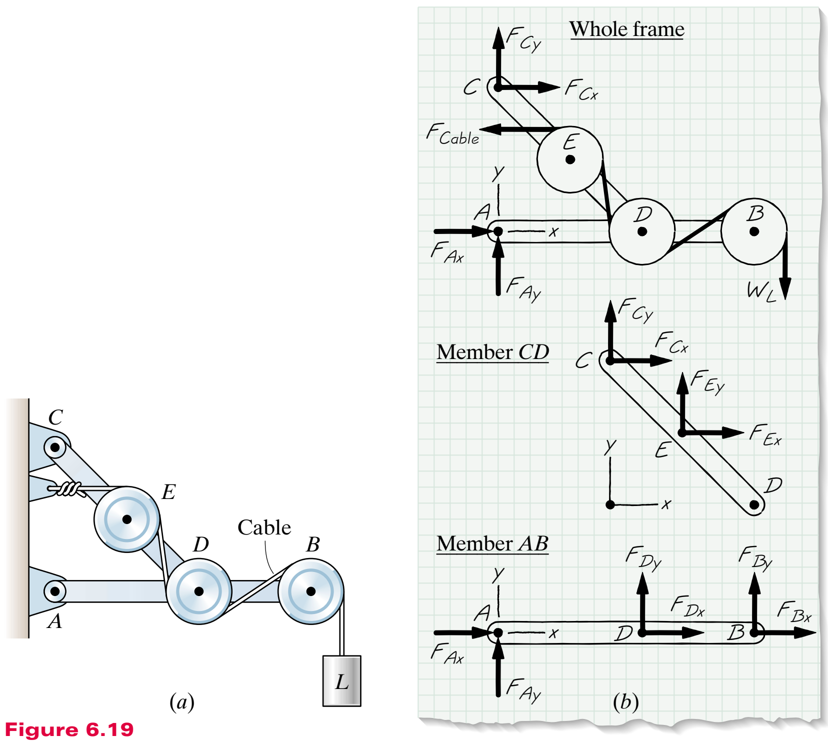



**Figure 6.19**

**(h)** A 50-kg roller is pulled up a 20° incline with a force $P$. As it is pulled over a smooth step, all of its weight rests against the step (**Figure 6.20*a***).

***Answer*** The proposed free-body diagram in **Figure 6.20*b*** is *not correct* because the gravity force should be drawn in the negative $y^*$ direction. The mass of the roller has been properly converted to weight on earth. The force $\boldsymbol{F_B}$ is drawn correctly.

**(i)** A 1200-lb object is held up by a force $T$ on a rope threaded through a system of frictionless pulleys (**Figure 6.21*a***).

***Answer*** *Pulley A:* The proposed free-body diagram in **Figure 6.21*b*** is *correct*.
Since the pulleys are frictionless, the force throughout the rope is constant (we will prove this in Chapter 7). The tension on the rope is $T$ wherever one cuts the rope.
*Pulley B:* The proposed free-body diagram in **Figure 6.21*b*** is *correct*.

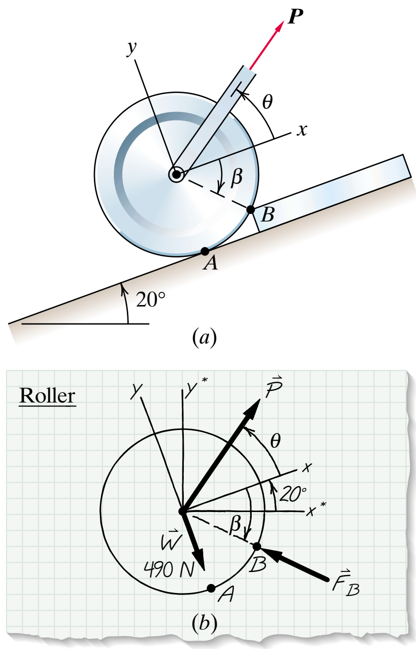


Figure 6.20

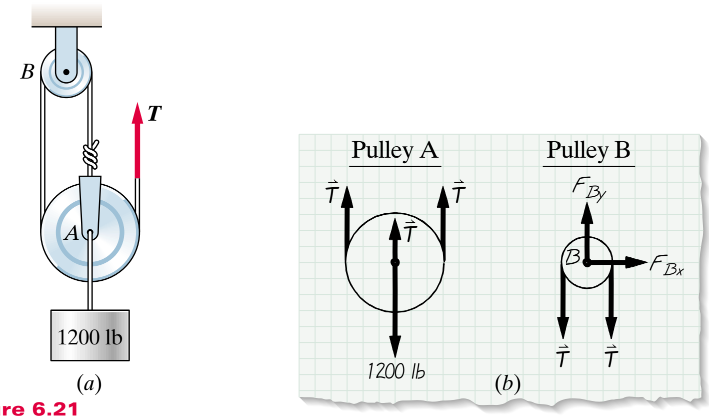


Figure 6.21

## EXERCISES 6.2

**6.2.1.** Consider the description of each planar system and determine whether the proposed free-body diagram is correct.

**a.** A curved beam of weight $W$ is supported at $A$ by a pin connection and at $B$ by a rocker, as shown in **E6.2.1*a***.

**b.** A beam is pinned at $B$ and rests against a smooth incline at $A$ as shown in **E6.2.1*b***. The total weight of the beam is $W$.

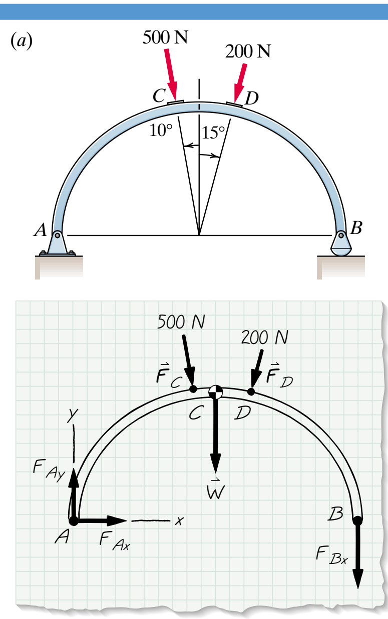
**c.** A forklift is lifting a crate of weight $W_1$ as shown in **E6.2.1*c***. The weight of the forklift is $W_2$. The front wheels are free to turn and the rear wheels are locked.

**d.** A mobile hangs from the ceiling from a cord as shown in **E6.2.1*d***.

**e.** A force acts on a brake pedal, as shown in **E6.2.1*e***.

**f.** A child balances on the beam as shown in **E6.2.1*f***. Planes $A$ and $B$ are smooth. The weight of the beam is negligible.

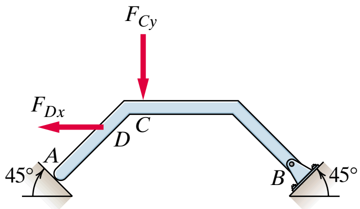
**g.** A beam is bolted to a wall at $B$ as shown in **E6.2.1*g***. The weight of the beam is negligible.

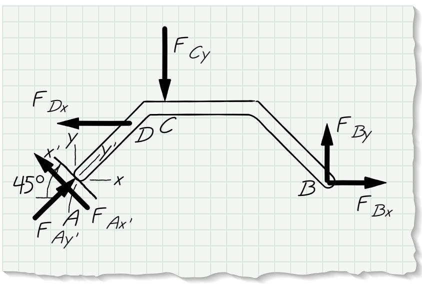


E6.2.1

Proposed free-body diagram.

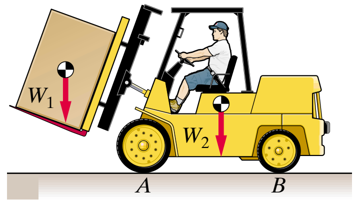
(*b*)

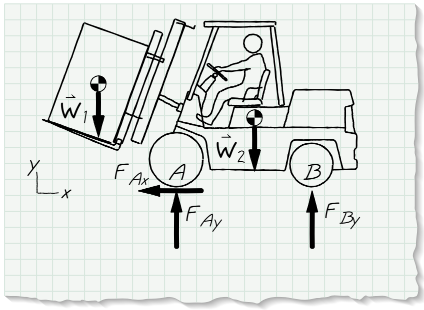



Proposed free-body diagram.

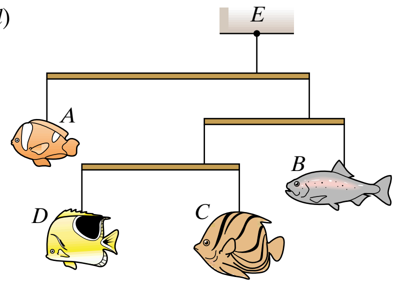
(*c*)

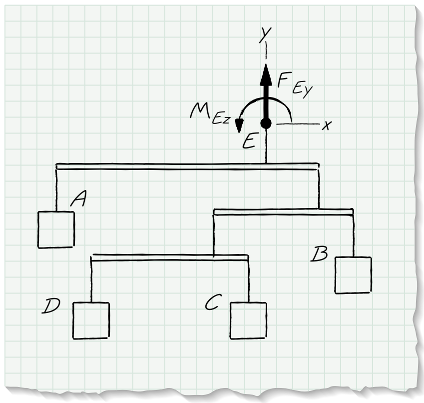



Proposed free-body diagram.

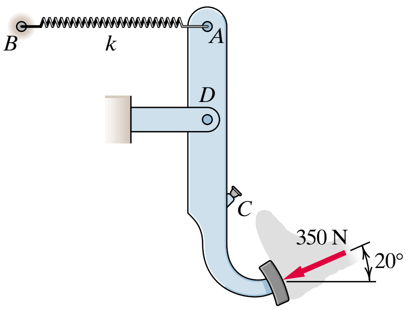
(*d*)

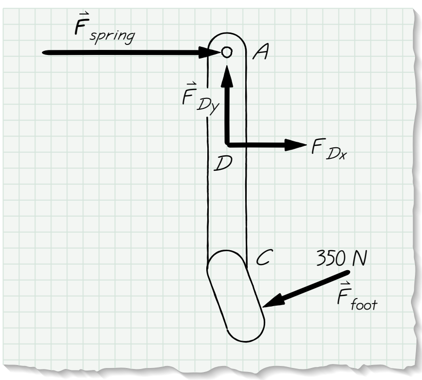



Proposed free-body diagram.

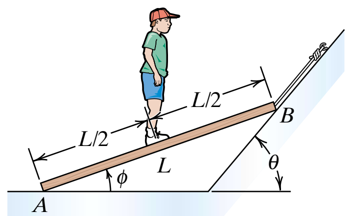
(*e*)

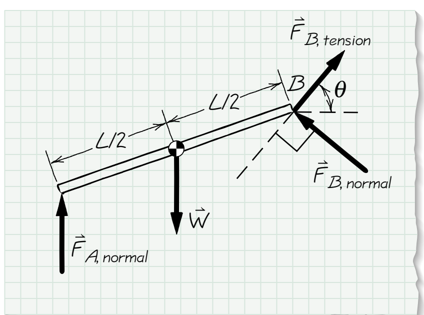



Proposed free-body diagram.

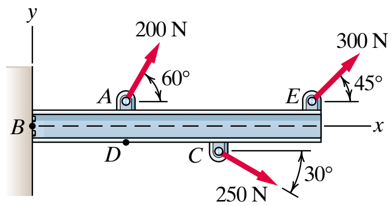
(*f*)

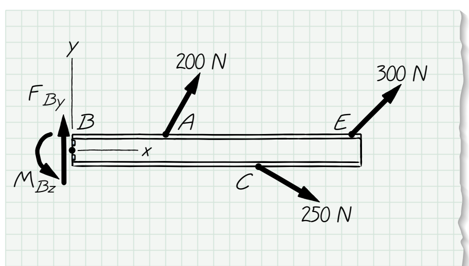



Proposed free-body diagram.

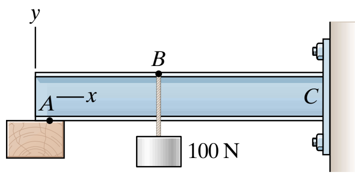
(*g*)



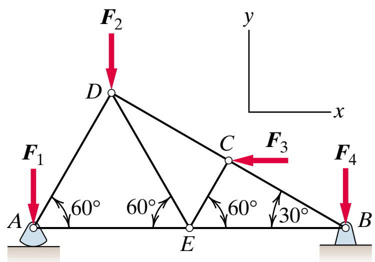

Proposed free-body diagram. **E6.2.1 (Cont.)**

**6.2.2.** The beam of uniform weight is fixed at $C$ and rests against a smooth block at $A$ (**E6.2.2**). In addition, a 100-N weight hangs from point $B$. Based on information in **Table 6.1**, what loads do you expect to act on the beam at $C$ due to the fixed condition? What loads do you expect to act on the beam at $A$ where it rests on the smooth block? Present your answer in terms of a sketch of the beam that shows the loads acting on it at $A$, $B$, and $C$. Also comment on whether the sketch you created is or is not a free-body diagram.



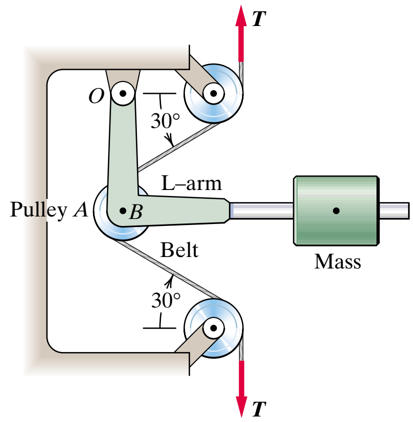
**E6.2.2**

**6.2.3.** The truss is attached to the ground at $A$ with a rocker and at $B$ with a pin connection (**E6.2.3**). Additional loads acting on the beam are as shown. Based on information in **Table 6.1**, what loads do you expect to act on the truss at $A$ due to the rocker connection? What loads do you expect to act on the truss at $B$ due to the pin connection? Present your answer in terms of a sketch of the truss that shows the loads acting on it at $A$ and $B$. Also comment on whether the sketch you created is or is not a free-body diagram.



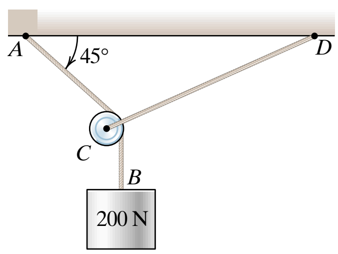
**E6.2.3**

**6.2.4.** The belt-tensioning device is as shown in **E6.2.4**. Pulley $A$ is pinned to the L-arm at $B$. Based on information in **Table 6.1**, what loads do you expect to act on the pulley at $B$? What loads do you expect to act on the pulley due to the belt tension? Present your answer in terms of a sketch of the pulley that shows loads acting on it at $B$ and due to the belt tension. Also comment on whether the sketch you created is or is not a free-body diagram.



**E6.2.4**

**6.2.5.** Cable $AB$ passes over the small frictionless pulley $C$ without a change in tension and holds up the metal cylinder (**E6.2.5**). Based on information in **Table 6.1**, what loads do you expect to act on the cylinder? Present your answer in terms of a sketch of the cylinder that shows all the loads acting on it. Also comment on whether the sketch you created is or is not a free-body diagram.



**E6.2.5**

## 6.3 NONPLANAR SYSTEM SUPPORTS

Now we consider how to identify nonplanar system supports and represent the loads associated with them. A system is nonplanar if all the forces acting on it *cannot* be represented in a single plane or all moments acting on it *are not* about an axis perpendicular to that plane. You will see similarities to our discussion in the prior section on planar systems *and* some important differences. As with planar systems, *if a boundary location prevents the translation of a nonplanar system in a given direction, then a force acts on the system at the location of the support in the opposite direction. Likewise, if rotation is prevented,* *a moment opposite the rotation acts on the system at the location of the support.*

Consider the systems in **Figure 6.22**, for which we want to draw free-body diagrams. The three nonplanar supports in **Figure 6.22*a*** (normal contact without friction, cable, spring) are identical to their planar counterparts. Associated with each support is a force acting along a known line of action, as depicted in **Figure 6.23*a***.

The two nonplanar supports in **Figure 6.22*b*** are similar to their planar counterparts. **Normal contact with friction** involves normal and friction forces, where the friction force is in the plane perpendicular to the normal force; these are represented in **Figure 6.22*b*** as $\boldsymbol{F}_{4z}$, $\boldsymbol{F}_{4x}$, and $\boldsymbol{F}_{4y}$, respectively. The **fixed support** of a nonplanar system is able to prevent the system from translating along and rotating about any axis—therefore it involves a force with three components ($\boldsymbol{F}_5 = F_{5x}\boldsymbol{i} + F_{5y}\boldsymbol{j} + F_{5y}\boldsymbol{k}$) and a moment with three components ($\boldsymbol{M}_5 = M_{5x}\boldsymbol{i} + M_{5y}\boldsymbol{j} + M_{5z}\boldsymbol{k}$). The free-body diagram of the system in **Figure 6.22*b*** is depicted in **Figure 6.23*b***.

Another commonly found nonplanar support is a **single hinge** (**Figure 6.24*a***). It does not restrict rotation of the system about the hinge

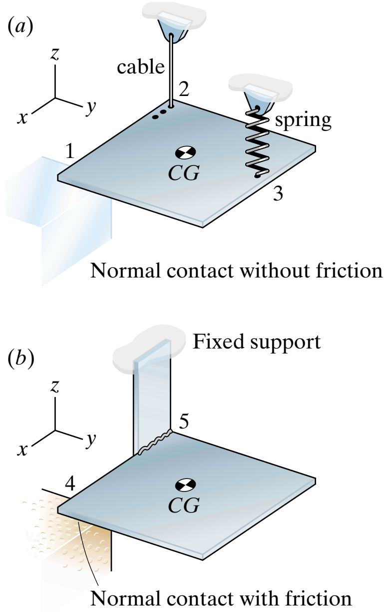



**Figure 6.22** A plate connected to its surroundings by various supports. The plate must be modeled as a nonplanar system.

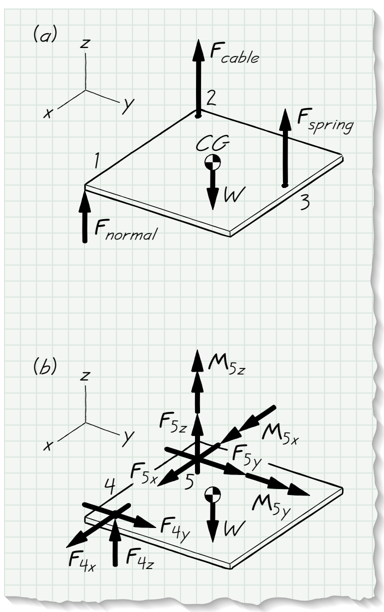
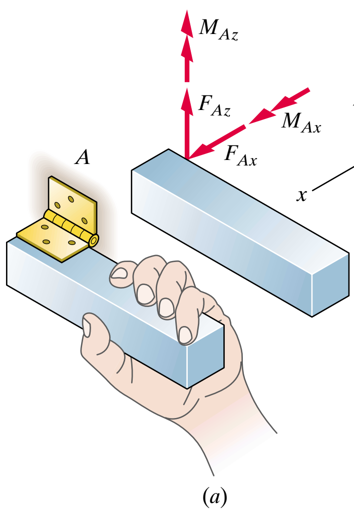
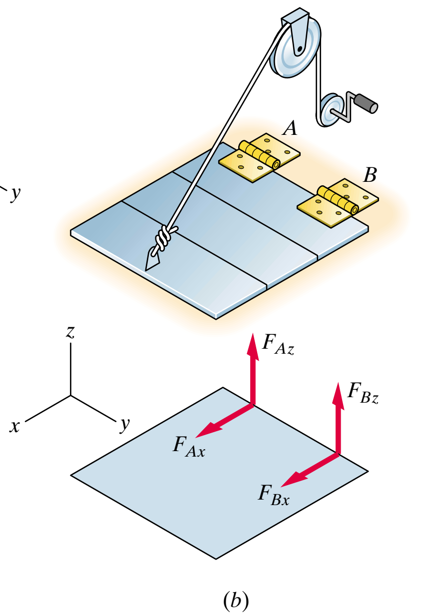



**Figure 6.23** Free-body diagram of the nonplanar systems shown in Figure 6.22



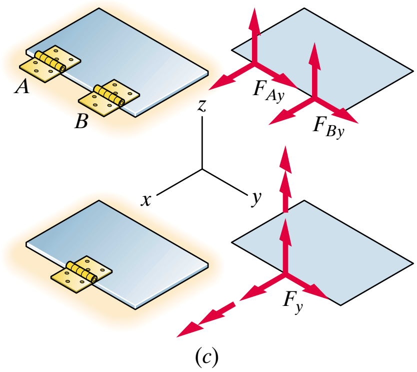




**Figure 6.24** (*a*) The forces acting on the system (*A* block) at a single-hinge connection; (*b*) the forces acting on the system (a trap door) at multiple hinges; (*c*) hinges designed to prevent motion along the hinge axis. The force component $F_z$ is commonly referred to as a thrust force.

pin. A single hinge applies a force (with two components) and a moment (with two components) perpendicular to the axis of the hinge. For the example in **Figure 6.24*a***, we represent the loads acting at the single hinge support as $\boldsymbol{F}_A = F_{Ax}\boldsymbol{i} + F_{Ay}\boldsymbol{j}$ and moment $\boldsymbol{M}_A = M_A\boldsymbol{i} + M_{Ay}\boldsymbol{j}$.

If a hinge is one of several properly aligned hinges attached to a system, each hinge applies a force perpendicular to the hinge axis (see **Figure 6.24*b***) and no moment. Depending on the design of a hinge (and regardless of whether it is a single hinge or one of several), it may also apply a force along the axis of the pin ($\boldsymbol{F}_z$ in **Figure 6.24*c***). The experiments outlined in Example 6.3 are intended to illustrate the difference in the loads involved with single versus multiple hinges.

## Summary

**Table 6.2** summarizes the loads associated with supports for nonplanar systems. Other supports commonly found with nonplanar systems are also included in the table. For example, the **ball-and-socket support** restricts all translations of the system by applying a force to the system, but it does not restrict rotation of the system about any axis. An example of a ball-and-socket support familiar to everyone is the human hip joint (**Figure 6.25**). As another example, a **journal bearing** does not restrict system rotation about one axis, while restricting translation in a plane perpendicular to the axis (**Figure 6.25*d***). Take a few minutes to study **Table 6.2** and notice the similarities and differences between hinges, journal bearings, and thrust bearings.

**Table 6.2** is not an exhaustive list of nonplanar supports. It contains commonly found and representative examples. If you find yourself considering a support that is not neatly classified as one of these, remember that you can always return to the basic characteristics associated with any support: *If a support prevents the translation of the system in a given direction, then a force acts on the system in the opposing direction. If rotation is prevented, a moment opposing the rotation is exerted on the system.*

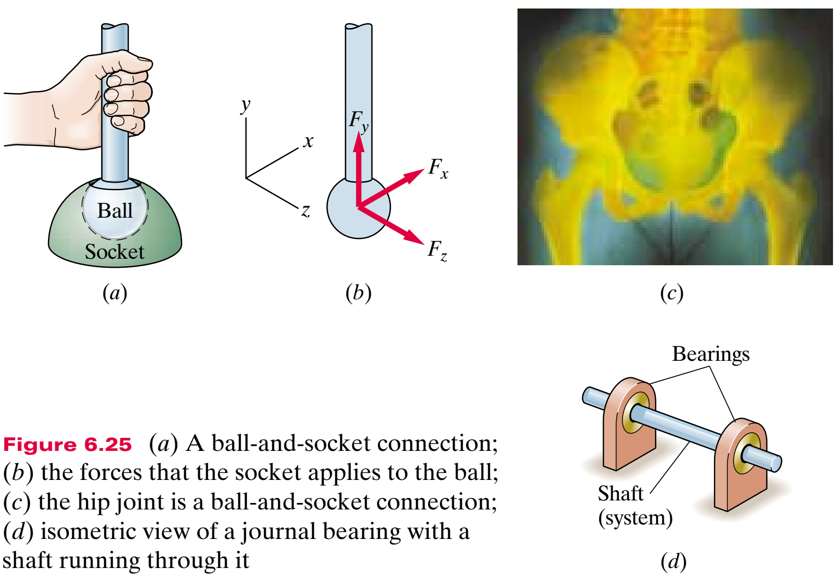
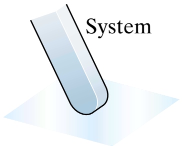
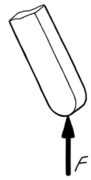
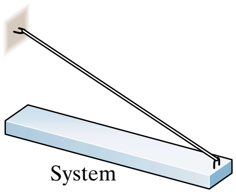
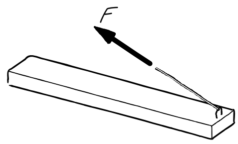
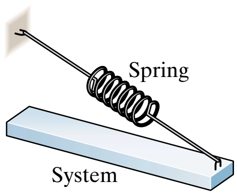
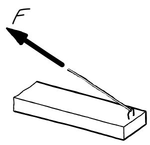
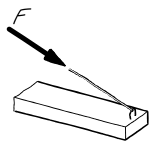
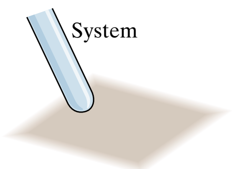
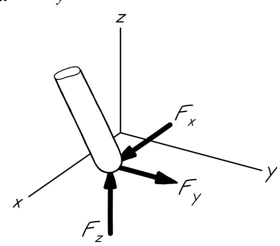
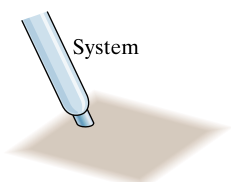
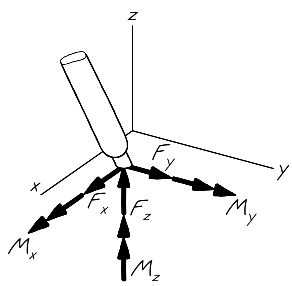
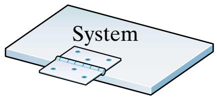
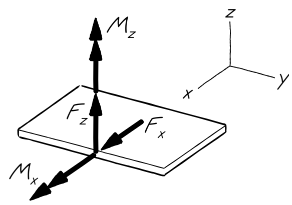
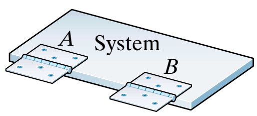
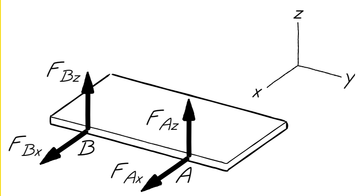
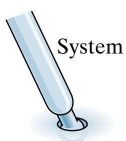
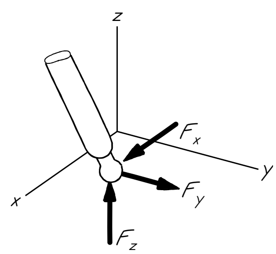
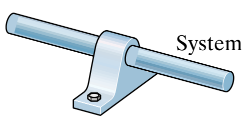
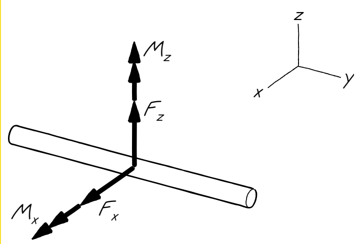



**Figure 6.25** (*a*) A ball-and-socket connection; (*b*) the forces that the socket applies to the ball; (*c*) the hip joint is a ball-and-socket connection; (*d*) isometric view of a journal bearing with a shaft running through it

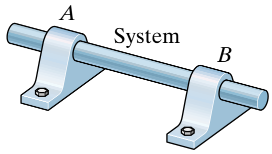
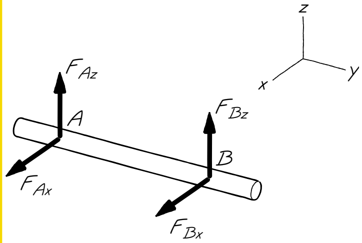
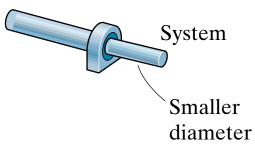
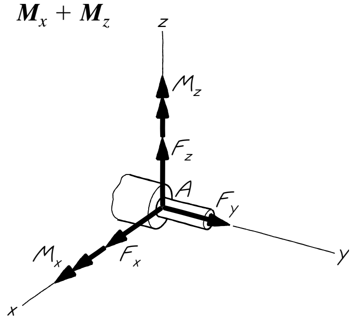
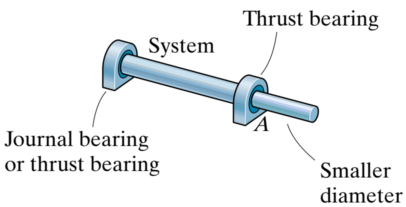
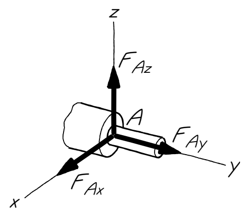
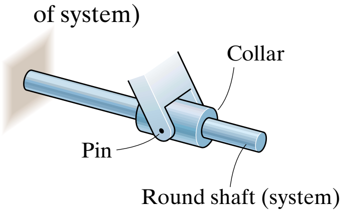
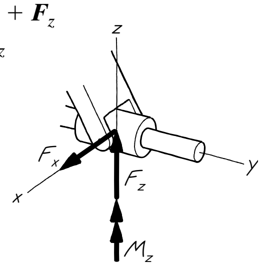
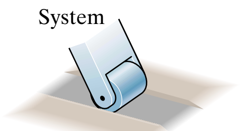
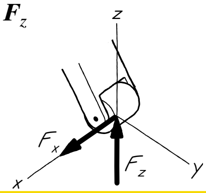
**Table 6.2 Standard Supports for Nonplanar Systems**

| (A)<br>Support | Description of Boundary Loads | (B)<br>Loads to Be Shown in Free-Body Diagram |
|---|---|---|
| 1. **Normal contact without friction**<br> | **Force** (***F***) oriented normal to surface on which system rests. Direction is such that force pushes on system. | ***F***<br> |
| 2. **Cable, rope, wire**<br> | **Force** (***F***) oriented along cable. Direction is such that force pulls on system. | ***F***<br> |
| 3. **Spring**<br> | **Force** (***F***) oriented along long axis of spring. Direction is such that force pulls on system if spring is in tension and pushes if spring is in compression. | ***F***<br><br>Spring in tension<br>$+\Delta$<br><br>Spring in compression<br>$-\Delta$ |
| 4. **Normal contact with friction**<br> | **Two forces**, one ($\boldsymbol{F}_z$) oriented normal to surface so as to push on system, other force is tangent to surface on which the system rests and is represented in terms of its components ($\boldsymbol{F}_x + \boldsymbol{F}_y$). | $\boldsymbol{F}_z$<br>$\boldsymbol{F}_x + \boldsymbol{F}_y$<br> |
| 5. **Fixed support**<br> | **Force** represented in terms of components ($\boldsymbol{F}_x + \boldsymbol{F}_y + \boldsymbol{F}_z$).<br>**Moment** represented in terms of components ($\boldsymbol{M}_x + \boldsymbol{M}_y + \boldsymbol{M}_z$). | $\boldsymbol{F}_x + \boldsymbol{F}_y + \boldsymbol{F}_z$<br>$\boldsymbol{M}_x + \boldsymbol{M}_y + \boldsymbol{M}_z$<br> |

<table>
<tr><th>(A)<br>Support</th><th>Description of Boundary Loads</th><th>(B)<br>Loads to Be Shown in Free-Body Diagram</th></tr>
<tr><td>6A. Single hinge<br>(shaft and articulated collar)<br></td><td>Force in plane perpendicular to shaft axis; represented as x and y components ($\boldsymbol{F}_x + \boldsymbol{F}_z$).<br>Moment with components about axes perpendicular to shaft axis ($\boldsymbol{M}_x + \boldsymbol{M}_z$).<br>Depending on the hinge design, may also apply force along axis of shaft, ($\boldsymbol{F}_y$).</td><td>$\boldsymbol{F}_x + \boldsymbol{F}_z$<br>$\boldsymbol{M}_x + \boldsymbol{M}_z$<br><br>OR<br>$\boldsymbol{F}_x + \boldsymbol{F}_y + \boldsymbol{F}_z$<br>$\boldsymbol{M}_x + \boldsymbol{M}_z$<br></td></tr>
<tr><td>6B. Multiple Hinges<br>(one of two or more properly aligned hinges)<br></td><td>Force in plane normal to shaft axis represented in terms of components ($\boldsymbol{F}_x + \boldsymbol{F}_z$). Point of application at center of shaft.<br>Depending on design, may also apply force along axis of shaft ($\boldsymbol{F}_y$).</td><td>At hinge A: $\boldsymbol{F}_{Ax} + \boldsymbol{F}_{Az}$<br>At hinge B: $\boldsymbol{F}_{Bx} + \boldsymbol{F}_{Bz}$<br><br>OR<br>$\boldsymbol{F}_{Ax} + \boldsymbol{F}_{Ay} + \boldsymbol{F}_{Az}$<br>$\boldsymbol{F}_{Bx} + \boldsymbol{F}_{By} + \boldsymbol{F}_{Bz}$<br></td></tr>
<tr><td>7. Ball and socket support<br>(ball or socket as part of system)<br></td><td>Force represented as three components.</td><td>$\boldsymbol{F}_x + \boldsymbol{F}_y + \boldsymbol{F}_z$<br></td></tr>
<tr><td>8A. Single journal bearing<br>(frictionless collar that holds a shaft)<br></td><td>Force in plane perpendicular to shaft axis; represented as x and y components ($\boldsymbol{F}_x + \boldsymbol{F}_z$).<br>Moment with components about axes perpendicular to shaft axis ($\boldsymbol{M}_x + \boldsymbol{M}_z$).</td><td>$\boldsymbol{F}_x + \boldsymbol{F}_z$<br>$\boldsymbol{M}_x + \boldsymbol{M}_z$<br></td></tr>
</table>

(*Continued*)

**Table 6.2 (Cont.)**

| (A) Support | Description of Boundary Loads | (B) Loads to Be Shown in Free-Body Diagram |
|---|---|---|
| 8B. **Multiple journal bearings** (two or more properly aligned journal bearings holding a shaft)  | **Force** in plane perpendicular to shaft axis represented in terms of components ($\boldsymbol{F}_{Ax} + \boldsymbol{F}_{Az}$). Point of application at center of shaft. | At journal bearing $A$:<br>$\boldsymbol{F}_{Ax} + \boldsymbol{F}_{Az}$<br>At journal bearing $B$:<br>$\boldsymbol{F}_{Bx} + \boldsymbol{F}_{Bz}$  |
| 9A. **Single thrust bearing** (journal bearing that also restricts motion along axis of shaft)  | **Force** represented in terms of three components ($\boldsymbol{F}_x + \boldsymbol{F}_z + \boldsymbol{F}_z$). Component in direction of shaft axis ($\boldsymbol{F}_y$) is sometimes referred to as the "thrust force." Point of application is at center of shaft.<br>**Moment** with components perpendicular to shaft axis ($\boldsymbol{M}_x + \boldsymbol{M}_z$). | $\boldsymbol{F}_x + \boldsymbol{F}_z + \boldsymbol{F}_z$<br>$\boldsymbol{M}_x + \boldsymbol{M}_z$  |
| 9B. **Multiple thrust bearings** (one of two or more properly aligned thrust bearings)  | **Force** represented in terms of three components ($\boldsymbol{F}_x + \boldsymbol{F}_z + \boldsymbol{F}_z$). Component in direction of shaft axis ($\boldsymbol{F}_y$) is sometimes referred to as the "thrust force." Point of application is at center of shaft. | At thrust bearing $A$:<br>$\boldsymbol{F}_{Ax} + \boldsymbol{F}_{Ay} + \boldsymbol{F}_{Az}$  |
| 10. **Clevis: Collar on shaft with pin** (collar and shaft are part of system)  | **Force** with components perpendicular to shaft axis ($\boldsymbol{F}_y + \boldsymbol{F}_z$).<br>**Moment** with components perpendicular to shaft axis ($\boldsymbol{M}_z$). | $\boldsymbol{F}_y + \boldsymbol{F}_z$<br>$\boldsymbol{M}_z$  |
| 11. **Smooth roller in guide**  | **Force** represented as two components. One component ($\boldsymbol{F}_z$) normal to surface on which system rests; the other is perpendicular to rolling direction ($\boldsymbol{F}_x$). | $\boldsymbol{F}_x + \boldsymbol{F}_z$  |

## EXAMPLE 6.3 EXPLORING SINGLE AND DOUBLE BEARINGS AND HINGES

For each situation described below, draw a free-body diagram and describe the loads involved. To create the situations yourself, you will need a yardstick, a rubber band, and a candy bar (to serve as a weight). Your hands will serve as models of bearings and hinges. When considering the system, ignore the weight of the yardstick.

**Situation 1:** Hold the yardstick level as shown in **Figure 6.26*a***. The left hand is at $x = 0$ in. and the right hand is at $x = 18$ in. The candy bar is hanging at the far right.

**Description** The left-hand fingers push down on the top of the yardstick. Notice that you can move your left thumb away from the stick because there is no load on it.

The right thumb pushes up on the bottom of the yardstick. Notice that you can move your right-hand fingers away from the stick because there is no load on them.

**Free-Body Diagram for Situation 1** Consider how the hands apply loads to the yardstick. Defining the yardstick as the system, draw these loads on the yardstick to create the free-body diagram (**Figure 6.26*b***).

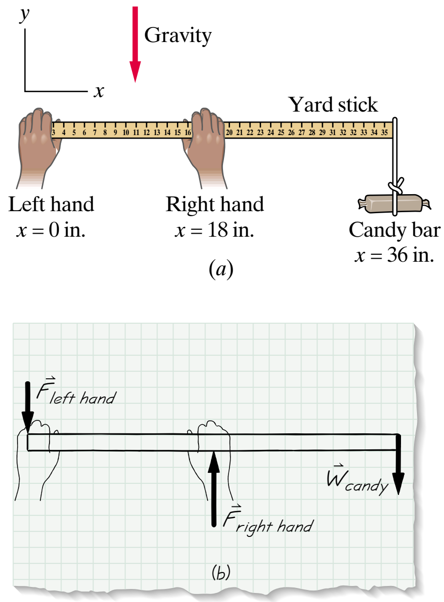



**Figure 6.26** (*a*) Situation 1; (*b*) free-body diagram of Situation 1

**Situation 2:** Hold the yardstick level as shown in **Figure 6.27*a***. The left hand is at $x = 9$ in. and the right hand is at $x = 18$ in. The candy bar is hanging at the far right. Each hand acts like a bearing or hinge.

**Description** The description for (1) still holds. The difference is, that in order to keep the yardstick level, the forces involved in pushing down with the left fingers and up with the right thumb are larger in magnitude.

**Free-Body Diagram for Situation 2** Consider how the hands apply loads to the yardstick. Defining the yardstick as the system, draw these loads on the yardstick to create the free-body diagram (**Figure 6.27*b***).

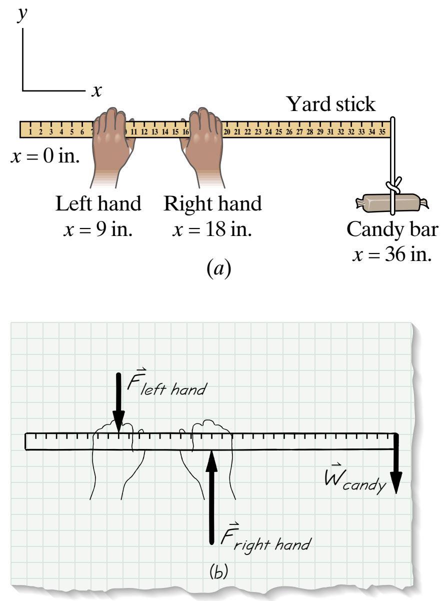



**Figure 6.27** (*a*) Situation 2; (*b*) free-body diagram of Situation 2

**Situation 3:** Hold the yardstick level as shown in **Figure 6.28*a***. The right hand is at $x = 18$ in., and the candy bar is hanging at the far right. The right hand acts like a single bearing or hinge.

**Description** The thumb pushes up on the bottom of the yardstick and in conjunction with the right-hand fingers works to prevent the stick from rotating; in doing this, the right hand applies a moment and pushes upward with a force.

**Free-Body Diagram for Situation 3** Consider how the hand applies loads to the yardstick. Defining the yardstick as the system, draw these loads on the yardstick to create the free-body diagram (**Figure 6.28*b***).

**Summary** Situations 1 and 2 are analogous to systems with two properly aligned bearings or hinges, with the hands playing the role of bearings/hinges. Although each hand applies only a force, each does create an equivalent moment at a specified moment center (as introduced in Chapter 5). For example, if we call $x = 18$ in. the moment center (this is the point of

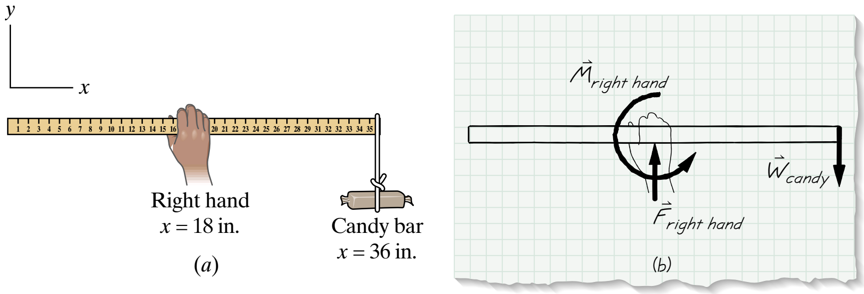



**Figure 6.28** (*a*) Situation 3; (*b*) free-body diagram of Situation 3

application of $\boldsymbol{F}_{\text{right hand}}$) in Situation 1, $\boldsymbol{F}_{\text{left hand}}$ creates a counterclockwise equivalent moment of (18 in. $\|\boldsymbol{F}_{\text{right hand}}$) that counters the clockwise equivalent moment created by the dangling candy of (18 in. $\|\boldsymbol{W}_{\text{candy}}$).

In situation 2 the candy stays at the same position, creating the same clockwise equivalent moment of (18 in. $\|\boldsymbol{W}_{\text{candy}}\|$) about $x = 18$ in. Since the left hand is placed at $x = 9$ in., it must exert a larger force to maintain the same counterclockwise moment.

In situation 3 the right hand acts like a single bearing or hinge, and must apply a force and a moment to counter the clockwise moment created by the dangling candy that is 18 inches from the hand. Notice that in **Figure 6.28*b*** the right hand applied both a force and a moment to the yard stick.

## EXAMPLE 6.4 EVALUATING THE CORRECTNESS OF FREE-BODY DIAGRAMS

Consider the description of each nonplanar system in **Figures 6.29–6.34** and determine whether the associated free-body diagram is correct or not correct. Unless stated otherwise, assume that gravity forces can be ignored.

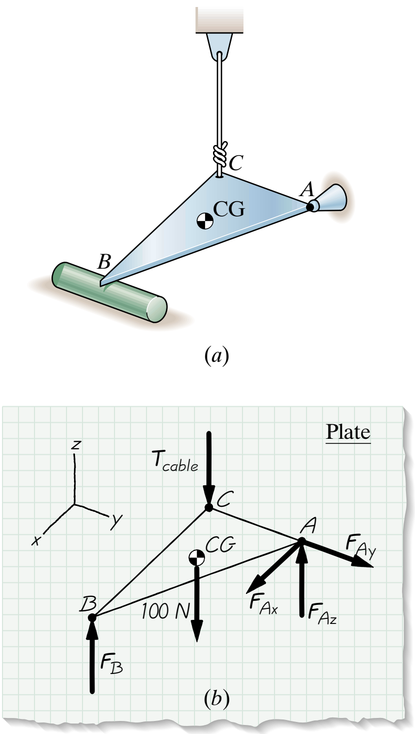



**Figure 6.29**

**Situation A:** The triangular plate *ABC* in **Figure 6.29*a*** is supported by a ball and socket at *A*, a roller at *B*, and a cable at *C*. The plate weighs 100 N.

***Answer*** The proposed free-body diagram in **Figure 6.29*b*** is *not correct*. Because the cable is in tension, it will pull on the plate in the $+y$ direction (not push on it in the $-y$ direction, as shown).

**Situation B:** A bar is supported by three well-aligned journal bearings at *A*, *B*, and *C* and supports a 200-N load (**Figure 6.30*a***).

***Answer*** The proposed free-body diagram in **Figure 6.30*b*** is *correct*. As summarized in **Table 6.2**, because there is more than one journal bearing supporting the system, each bearing applies a force (and no moment) to the system.

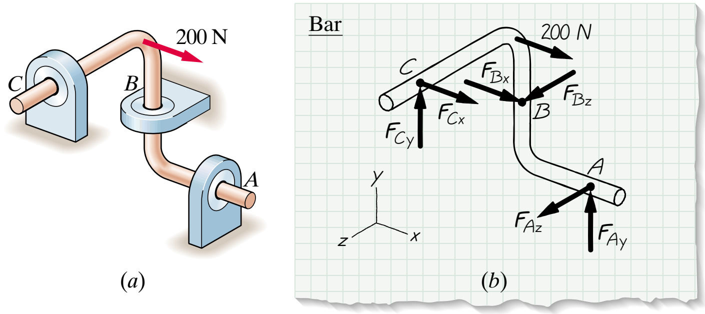



**Figure 6.30**

**Situation C:** A rod is supported by a thrust bearing at $A$ and a cable that extends from $B$ to $C$. A known force $\boldsymbol{F}$ is applied as shown in **Figure 6.31*a***.

***Answer*** The proposed free-body diagram in **Figure 6.31*b*** is *not correct*. Because there is a single journal bearing supporting the system, the bearing also applies moments about the $y$ and $z$ axes at $A$.

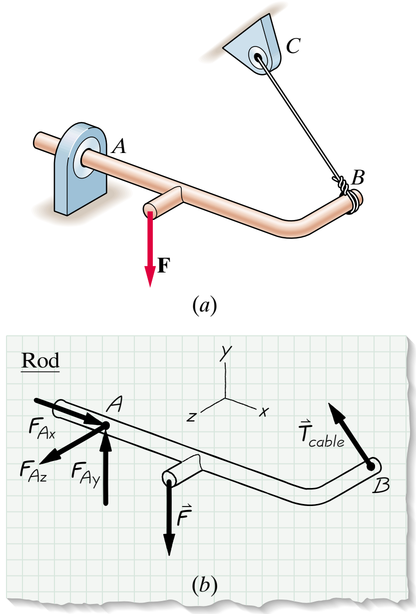



**Figure 6.31**

**Situation D:** A bar $ABC$ has built-in support at $A$ and loads applied at $B$ and $C$ as shown in **Figure 6.32*a***.

***Answer*** The proposed free-body diagram in **Figure 6.32*b*** is *correct*.

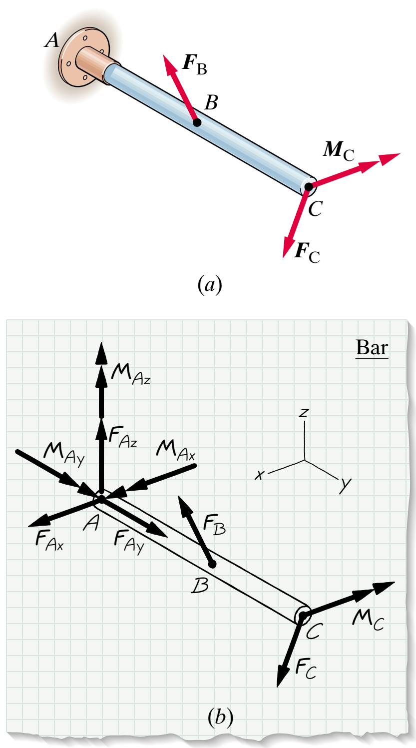



**Figure 6.32**

**Situation E:** The L-bar is supported at $B$ by a cable and at $A$ by a smooth square rod that just fits through the square hole of the collar. A known vertical load $F$ is applied as shown in **Figure 6.33*a***.

***Answer*** The proposed free-body diagram in **Figure 6.33*b*** is *not correct*. Since the rod is square and therefore prevents rotation of the collar, the connection at $A$ also applies a moment about the $y$ axis. Since the square rod is smooth, it cannot apply a force in the $y$ direction at $A$.

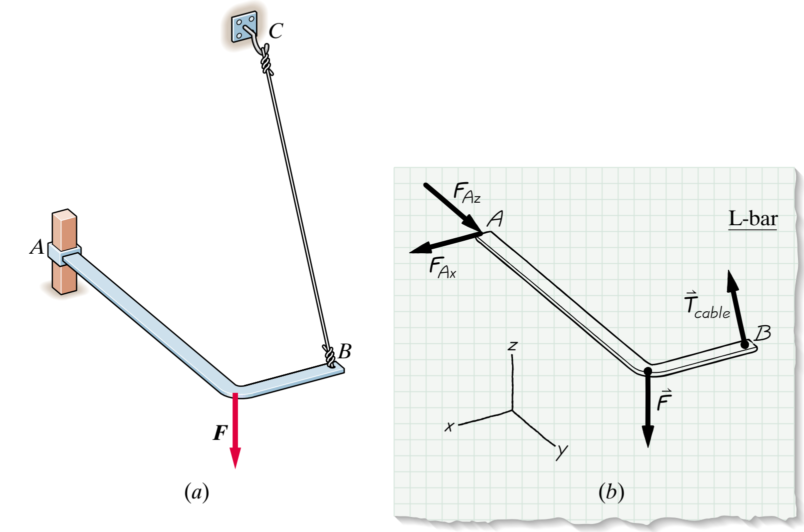



**Figure 6.33**

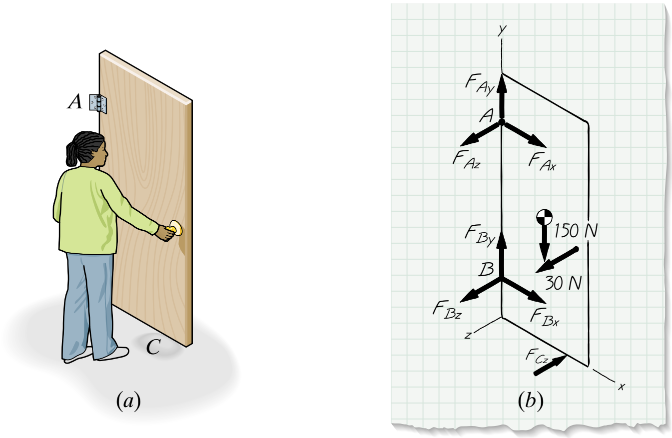



**Figure 6.34**

**Situation F:** The 150-N door is supported at $A$ and $B$ by hinges. Someone attempts to open the door by applying a force of 30 N to the handle, but because of a high spot in the floor at $C$, the door won't open. Both hinges are able to apply forces along their pin axis (**Figure 6.34*a***).

***Answer*** The proposed free-body diagram in **Figure 6.34*b*** is *correct*. Notice that unlike Example 6.3(D), here the door must be treated as a nonplanar system, and therefore the hinge forces in the $z$ direction must be considered.

## EXERCISES 6.3

**6.3.1.** Consider the description of each nonplanar system (part (*a*) of each figure) and determine whether the proposed free-body diagram (part (*b*) of each figure) is correct.

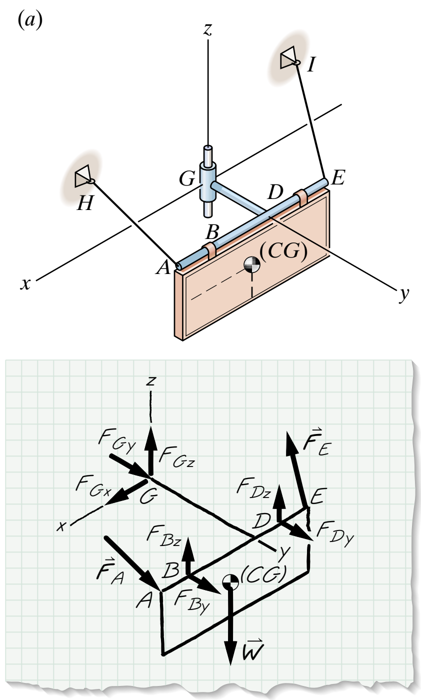
**a.** A sign of weight $W$ (500 N) with center of gravity as shown is supported by cables and a collar joint (**E6.1.1*a***).

**b.** A crankshaft is supported by a journal bearing at $B$ and a thrust bearing at $D$. Ignore the weight of the crank (**E6.1.1*b***).

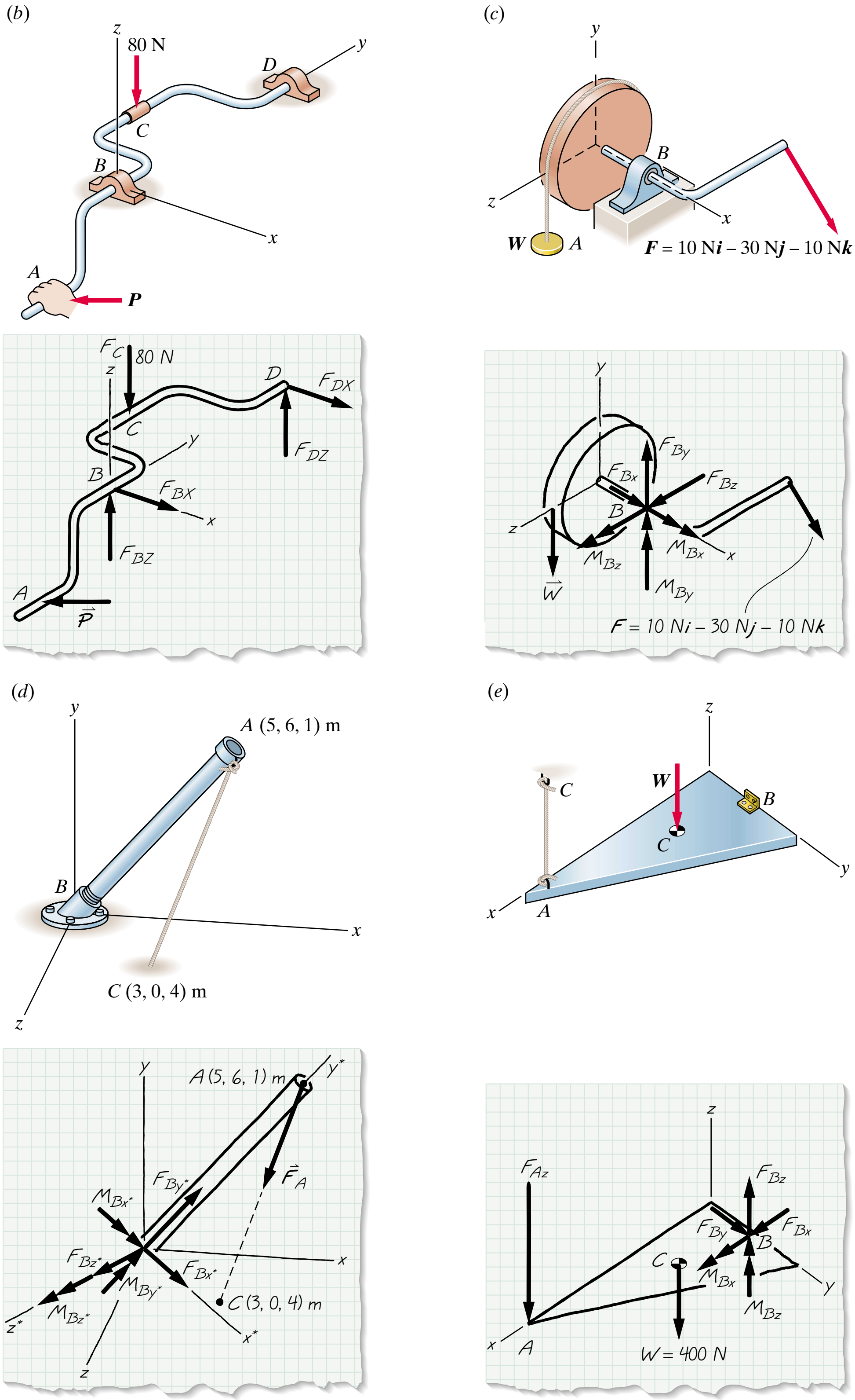
**c.** A pulley is used to lift a weight $W$. The shaft of the pulley is supported by a journal bearing, as shown. Ignore the weights of the pulley and the shaft (**E6.1.1*c***).

**d.** A pole is fixed at $B$ and tethered by a rope, as shown. Ignore the weight of the pole (**E6.1.1*d***).

**e.** A triangular plate is supported by a rope at $A$ and a hinge at $B$. Its weight of 400 N acts at the plate's center of gravity at $C$, as shown (**E6.1.1*e***).

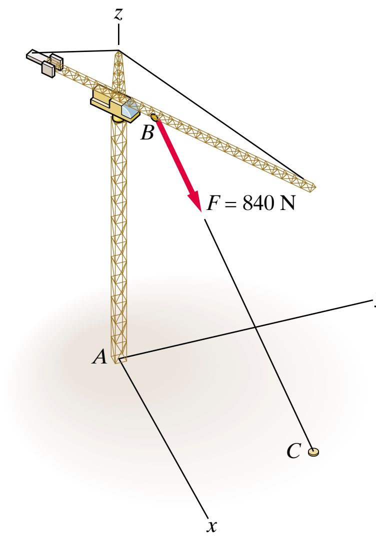



**E6.3.1**


(b)
80 N
A
B
C
D
P
$F_C$
$F_{DX}$
$F_{DZ}$
$F_{BX}$
$F_{BZ}$
(c)
W
A
B
F = 10 N$i$ – 30 N$j$ – 10 N$k$
$F_{By}$
$F_{Bx}$
$F_{Bz}$
$M_{Bx}$
$M_{By}$
$M_{Bz}$
(d)
A (5, 6, 1) m
C (3, 0, 4) m
B
$F_{By^*}$
$M_{Bx^*}$
$F_{Bz^*}$
$F_{Bx^*}$
$M_{By^*}$
$M_{Bz^*}$
$\vec{F}_A$
(e)
W
A
B
C
$F_{Az}$
$F_{Bz}$
$F_{By}$
$F_{Bx}$
$M_{Bx}$
$M_{Bz}$
W = 400 N

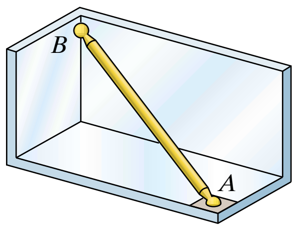
**E6.3.1 (Cont.)**

**6.3.2.** A tower crane is fixed to the ground at $A$ as shown in **E6.3.2**. Based on information in **Table 6.2**, what loads do you expect to act on the tower at $A$? Present your answer in terms of a sketch of the tower that shows the loads acting on it at $A$. Also comment on whether the sketch you created is or is not a free-body diagram.



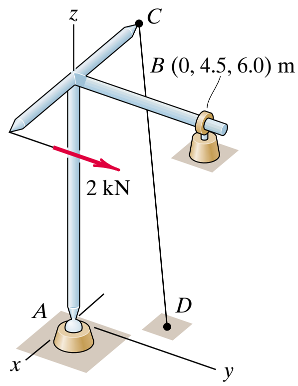

**E6.3.2**

**6.3.3.** The uniform 7-m steel shaft in **E.6.3.3** is supported by a ball-and-socket connection at $A$ in the horizontal floor. The ball end $B$ rests against the smooth vertical walls, as shown. Based on information in **Table 6.2**, what loads do you expect to act on the shaft at $A$? What loads do you expect to act on the shaft at $B$? Present your answer in terms of a sketch of the shaft that shows the loads acting on it at $A$ and $B$. Also comment on whether the sketch you created is or is not a free-body diagram.



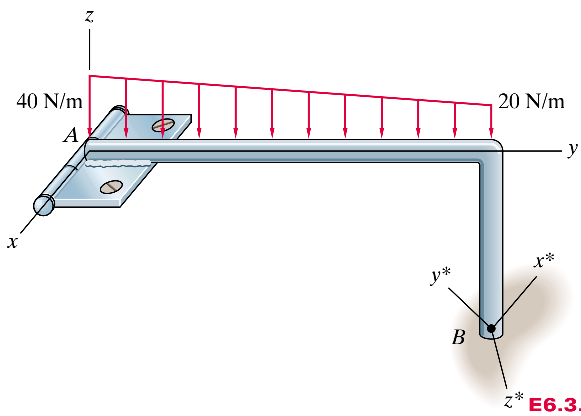

**E6.3.3**

**6.3.4.** The welded tubular frame in **E6.3.4** is secured to the horizontal $xy$ plane by a ball-and-socket connection at $A$ and receives support from a loose-fitting ring at $B$. Based on information in **Table 6.2**, what loads do you expect to act on the frame at $A$? What loads do you expect to act on the frame at $B$? Present your answer in terms of a sketch of the frame that shows the loads acting on it at $A$ and $B$. Also comment on whether the sketch you created is or is not a free-body diagram.




**E6.3.4**

**6.3.5.** A bar is supported at $A$ by a hinge, and at $B$ it rests against a rough surface. The surface is defined as the $x^*z^*$ plane. Based on information in **Table 6.2**, what loads do you expect to act on the bar at $A$? What loads do you expect to act on the bar at $B$? Present your answer in terms of a sketch that shows the loads acting on the bar at $A$ and $B$. Also comment on whether the sketch you created is or is not a free-body diagram.




**E6.3.5**

## 6.4 PLANAR AND NONPLANAR SYSTEMS

Defining the loads at a system's boundary is simplified if we can classify the system as a planar system, which is one in which all the forces acting on the system lie in the same plane and all moments are about an axis perpendicular to that plane. In this case, cross-boundary loads (e.g., gravity), known loads, fluid boundary loads, and supports are all in a single plane. **Figure 6.35*a*** shows an example of a system that can be classified as a planar system—planar because the gravity force and supports *A* and *B* are all in a single plane. Planar systems are referred to as **two-dimensional systems**. As we saw in Section 6.2, the free-body diagram associated with a planar system typically requires only a single view of the system.

A system in which the loads do not all lie in a single plane can be treated as a planar system for the purpose of static analysis if the system has a plane of symmetry *with regard to its geometry and the loads acting on it*. A **plane of symmetry** is one that divides the system into two sections that are mirror images of each other. None of the forces acting on the system has a component perpendicular to the plane of symmetry, and all moments acting on the system are about an axis perpendicular to the plane. **Figure 6.35*b*** illustrates a system that has a plane of symmetry and therefore can be treated as a planar system. Other examples in which a plane of symmetry was used to classify a system as planar are the Golden Gate Bridge in Chapter 2, and the ladder–person example in Chapter 4 (**Figures 4.19**, **4.20**, and **4.22**, but not **Figure 4.21**). **Figure 6.35*c*** illustrates a system with geometric symmetry, but because the cable forces acting on it have a component perpendicular to the *xy* plane, we are not able to classify the system as planar.

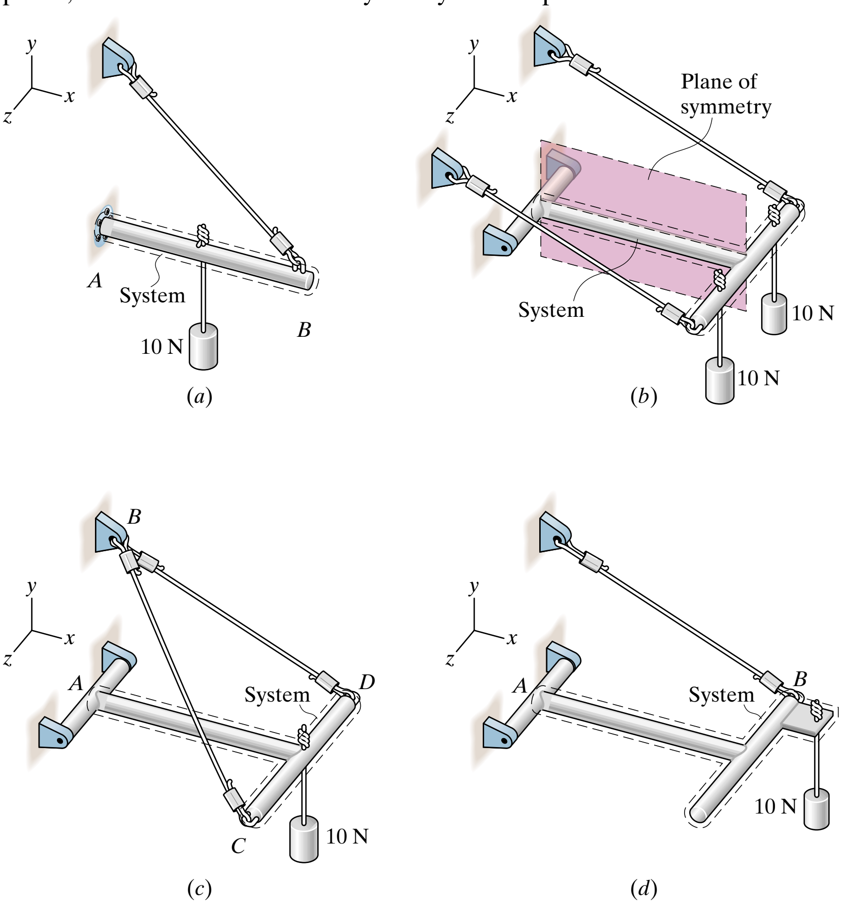


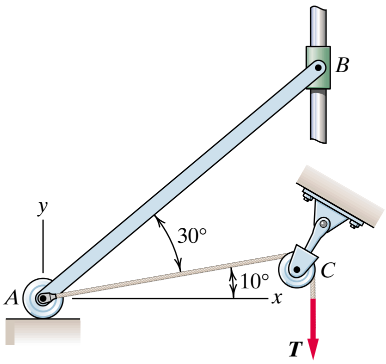

**Figure 6.35** (*a*) System that can be modeled as planar; (*b*) system with plane of symmetry can be modeled as planar; (*c*) and (*d*) system that cannot be modeled as planar

If it is not possible to define a single plane in which all forces and moments lie, or there is no plane of symmetry, the system is classified as nonplanar. In the system of **Figure 6.35*d***, for instance, it is not possible to define a single plane that contains the gravity force and supports $A$ and $B$, and there is no plane of symmetry. Nonplanar systems are referred to as **three-dimensional systems**. The free-body diagram associated with a nonplanar system typically requires an isometric drawing or multiple views.

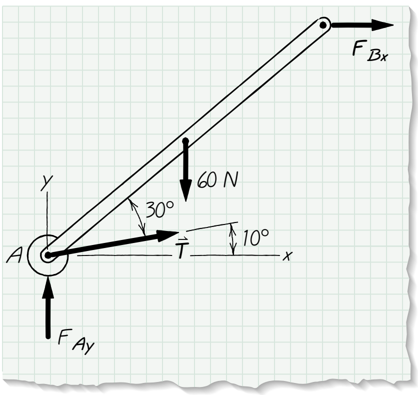
In Section 6.2 we dealt exclusively with planar systems and in Section 6.3 with nonplanar systems. Drawing the free-body diagram for a planar system is generally more straightforward because only external forces in the plane and external moments about an axis perpendicular to the plane must be considered. In performing analysis in engineering practice you will not be told whether a physical situation can be modeled as a planar system or must be modeled as a nonplanar system—the choice will be up to you. The discussion in this section is intended to give you some guidelines with which to make such a judgment.

## EXAMPLE 6.5 IDENTIFYING PLANAR AND NONPLANAR SYSTEMS

Consider the description of each system and determine whether the system can be classified as planar or nonplanar. (No system is really planar, because we live in a three-dimensional world. Even something as thin as a sheet of paper has a third dimension; BUT under certain conditions we can model it as planar for the purpose of static analysis).




(*a*)




(*b*)

**Figure 6.36**

**Goal** We are asked to determine, for a number of different cases, whether a system can be classified as planar or nonplanar.

**Given** We are given a specified system and the loads that act on it.

**Assume** Unless specified otherwise, assume that gravity is considered and acts in the negative $y$ direction.

**Draw** An additional drawing is not required to determine the classification of each system; however, you may want to draw a free-body diagram to help you better understand the geometry of the system and the external loads.

**Situation A:** The uniform bar $AB$ in **Figure 6.36*a*** weighs 60 N and is pulled on by a rope at $A$. The system is the arm and the wheel at $A$.

***Answer*** **Planar.** The gravity force of 60 N is in the $xy$ plane. Furthermore, the forces at the supports $A$ (normal force and tension in rope $AC$) and $B$ (collar guide) are also in the $xy$ plane. Because it is possible to define a single plane that contains all known forces and moments, gravity force, and forces applied at supports, this system can be treated as planar. The free-body diagram of $AB$ is shown in **Figure 6.36*b***.

*Question:* If gravity acted in the $z$ direction, would we reach the same conclusion? If the pulley at $C$ is not in the $xy$ plane, would we reach the same conclusion?

**Situation B:** The space truss in **Figure 6.37*a*** is of negligible weight and is supported by rollers at *B, C,* and *D*. It supports a vertical 800-N force at *A*. The system is the space truss.

***Answer*** **Nonplanar.** It is not possible to define a single plane that contains the points of application of supports (*B, C, D*) and the 800-N force. Therefore, this system must be treated as a nonplanar system.
Our answer would be unchanged if we had included gravity forces acting on the space truss. The free-body diagram of the space truss is shown in **Figure 6.37*b***.

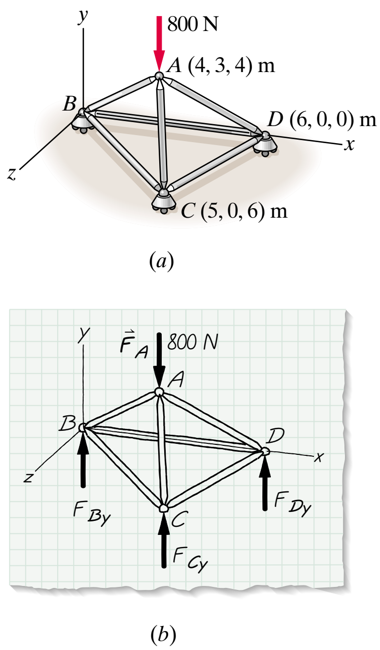



Figure 6.37

**Situation C:** The beam *AC* in **Figure 6.38*a*** is pinned to its surroundings at *A* and rests against a rocker at *B*. Ignore gravity. The system is the beam.

***Answer*** **Planar.** The 800-N · m moment at *C* and the 500-N force are in the *xy* plane, as are the points of application of supports at *A* and *B*.
The free-body diagram of the beam is shown in **Figure 6.38*b***.

*Question:* If gravity is considered and acts in the negative *y* direction, would we reach the same conclusion?

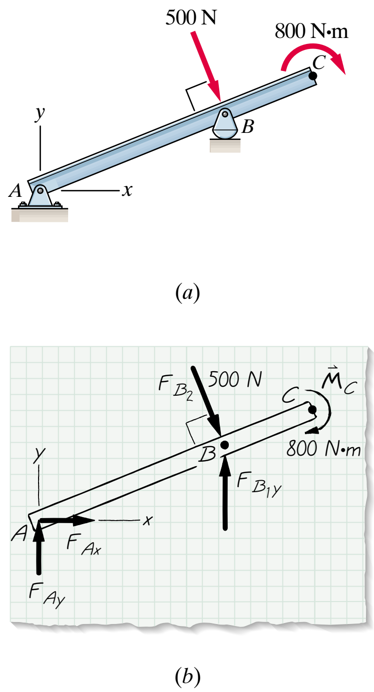



Figure 6.38

**Situation D:** The beam *AC* in **Figure 6.39*a*** rests on a block at *A*. In addition, there is a pin connection at *A* and a rocker at *B*. The system is the beam.

***Answer*** **Nonplanar.** It is not possible to define a single plane that contains the gravity force and the normal contact force at *B* and the 500-N force (both in the *xz* plane).
The free-body diagram of beam *AC* is shown in **Figure 6.39*b***.

*Question:* If gravity acted in the positive *z* direction, would we reach the same conclusion? How would our conclusion change if we were to ignore gravity?

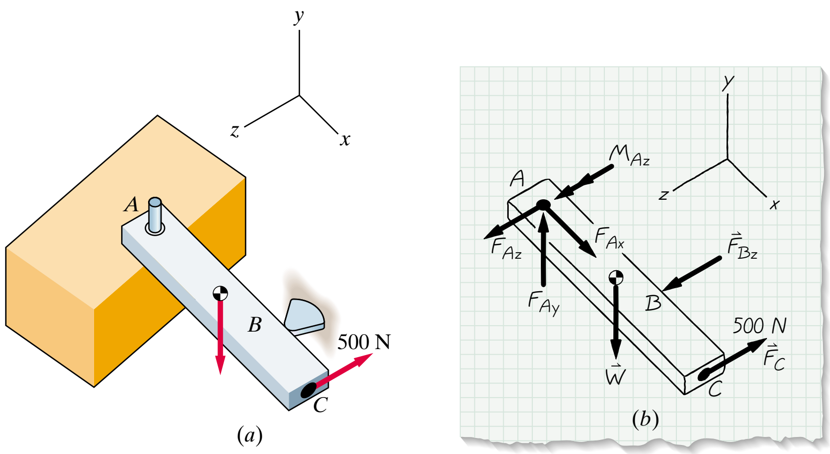



Figure 6.39

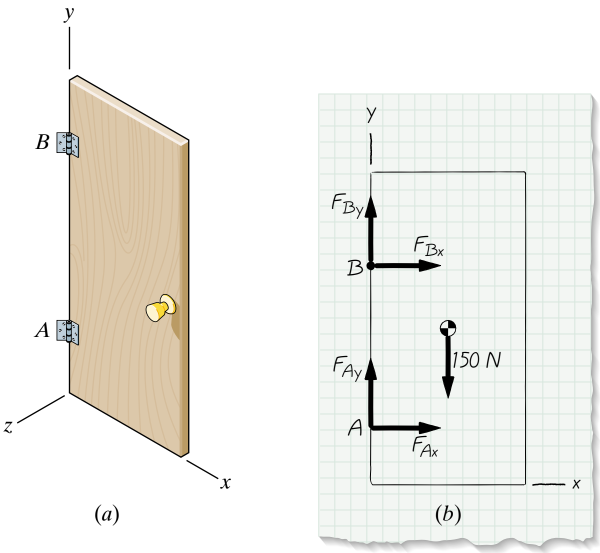



Figure 6.40

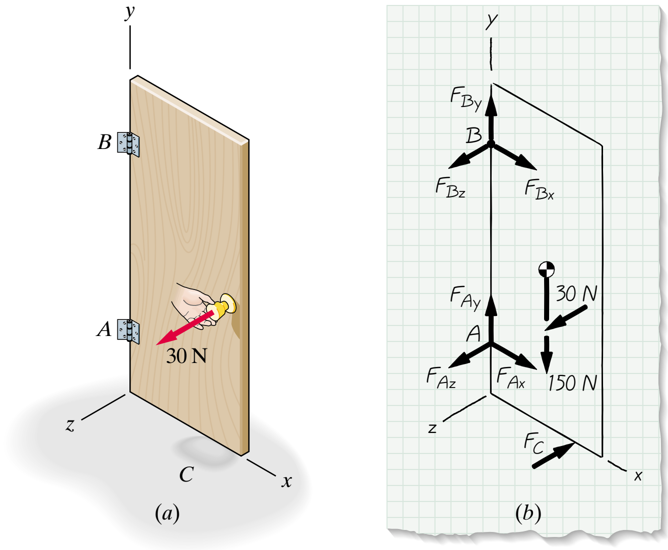



Figure 6.41

**Situation E:** The 150-N door in **Figure 6.40*a*** is supported at *A* and *B* by hinges. The system is the door.

***Answer*** **Planar.** If we assume that the door is of uniform density, we place the 150-N gravity force at the door's center. Furthermore, because the door is thin relative to its other dimensions, this gravity force and the loads due to the supports at *A* and *B* can be assumed to lie in the *xy* plane. The free-body diagram of the door is shown in **Figure 6.40*b***.

**Situation F:** The 150-N door in **Figure 6.41*a*** is supported at *A* and *B* by hinges. Someone attempts to open the door by applying a force of 30 N to the handle, but because of a high spot in the floor at *C*, the door won't open. The system is the door.

***Answer*** **Nonplanar.** It is not possible to define a single plane that contains all of the forces. The free-body diagram of the door is shown in **Figure 6.41*b***.

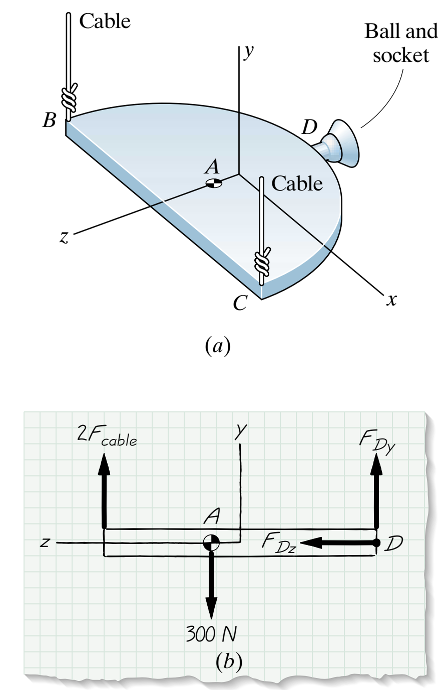



Figure 6.42

**Situation G:** A semicircular plate in **Figure 6.42*a*** weighs 300 N, which is represented by a point force at the center of gravity. Vertical cables support the plate at *B* and *C*, and a ball-and-socket joint supports the plate at *D*. The system is the plate.

***Answer*** **Planar.** The *yz* plane is a plane of symmetry for this system—the portion of the system at $+x$ (a quarter circle and cable force) is the mirror image of the portion of the system at $-x$ (a quarter circle and cable force). Consequently, it is possible to represent all of the forces as projections onto the *yz* plane. The free-body diagram of the plate is shown in **Figure 6.42*b***.

*Question:* If the 300 N-force acted in the positive $x$ direction, would we reach the same conclusion? If it acted in the positive $z$ direction, what conclusion should be drawn?

**Situation H:** The same plate as in **G** is supported by diagonal cables at $B$ and $C$, and a ball-and-socket joint at $D$, as shown in **Figure 6.43*a***. The system is the plate.

***Answer*** **Nonplanar.** As in **G**, the $yz$ plane is a plane of symmetry for this system. However, since the cable forces have components perpendicular to the plane of symmetry, we cannot represent this as a planar system.
The free-body diagram of the plate is shown in **Figure 6.43*b***.

**Situation I:** A man weighing 800 N sits at the picnic table halfway between its two ends (**Figure 6.44*a***). His center of gravity ($CG_{man}$) is noted. The table weighs 200 N, with a center of gravity at $CG_{table}$. Assume that any friction between the legs and the ground can be neglected. The system is the table.

***Answer*** **Planar.** The $xy$ plane is a plane of symmetry for this system—the portion of the system at $+z$ (half of the picnic table and supports at $A$ and $B$) is a mirror image of the portion of the system at $-z$ (the other half of the picnic table).
The free-body diagram of the table is shown in **Figure 6.44*b***.

**Situation J:** A child sits down next to the man at the picnic table in **I** (**Figure 6.45*a***). The system is again the table.

***Answer*** **Nonplanar.** With the child sitting next to the man, there is no longer any plane of symmetry.
The free-body diagram of the table is shown in **Figure 6.45*b***.

*Question:* If the child sits directly across from the man, could the system be modeled as planar?

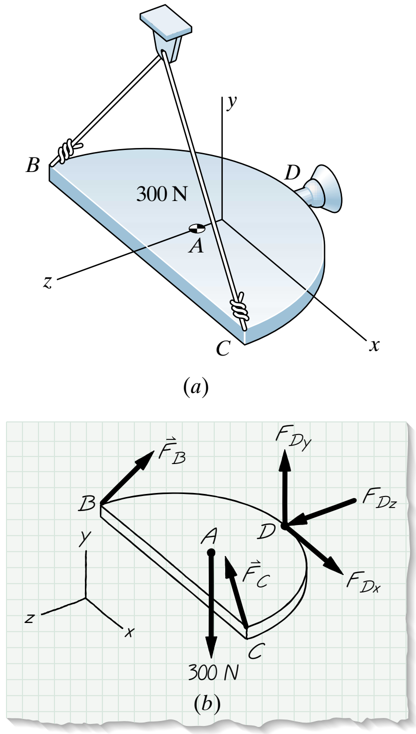



**Figure 6.43**

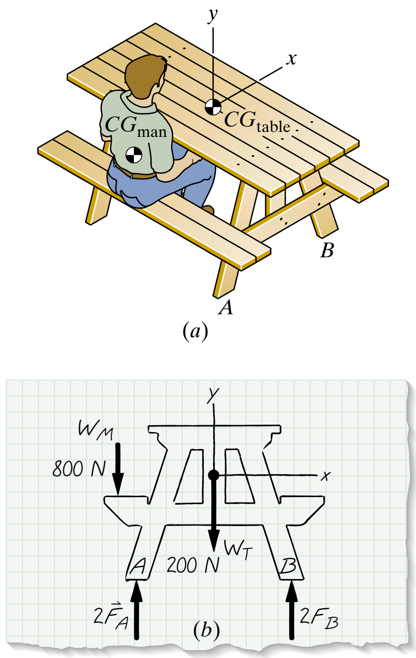



**Figure 6.44**

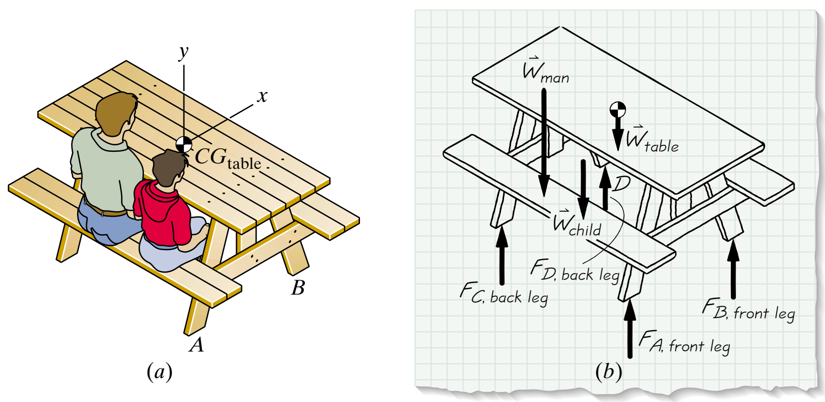



**Figure 6.45**

## EXAMPLE 6.6 USING QUESTIONS TO DETERMINE LOADS AT SUPPORTS

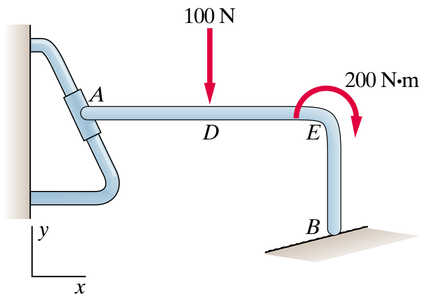



Figure 6.46

A bar is supported at $A$ by a frictionless collar guide. At $B$ it rests against a rough surface. Known forces act at $D$ and $E$, as shown in **Figure 6.46**.

(a) What loads act at $A$ and $B$? Use the general rule about the surroundings preventing translation and/or rotation at each support to answer this question.

(b) Draw a free-body diagram of the bar.

**Solution** **(a)** The bar (defined as the system) can be classified as planar. This means that in considering motion, we need consider only translations in the $xy$ plane and rotations about the $z$ axis.

**At A:**
Define the $x'y'$ coordinate system at $A$ (see **Figure 6.47*a***).

| Possible motion | Answer | Implication |
|---|---|---|
| Is $x'$ translation at $A$ possible? | Yes | There is no force acting on the bar in the $x'$ direction, since it is frictionless. |
| Is $y'$ translation at $A$ possible? | No | There is a force $\boldsymbol{F}_{Ay'}$. |
| Is $z$ rotation at $A$ possible? | No | There is a moment about the $z$ axis, $\boldsymbol{M}_{Az}$. |

**At B:**
Define the $x^*y^*$ coordinate system at $B$ (see **Figure 6.47*a***).

| Possible motion | Answer | Implication |
|---|---|---|
| Is $x^*$ translation at $B$ possible? | No, unless the force applied in the $x^*$ direction exceeds the maximum friction force that can be applied by the rough surface | There is a force $\boldsymbol{F}_{Bx^*}$. |
| Is $y^*$ translation at $B$ possible? | No, it is not possible in the negative $y$ direction. It is possible in the positive $y^*$ direction | There is a force $\boldsymbol{F}_{By^*}$ in the positive $y^*$ direction. |
| Is $z$ rotation at $B$ possible? | Yes | There is no moment about the $z$ axis. |

***Answer*** *At A:* $\boldsymbol{F}_{Ay'}$, $\boldsymbol{M}_{Az}$
*At B:* $\boldsymbol{F}_{Bx^*}$, $\boldsymbol{F}_{By^*}$ (in the positive $y^*$ direction).

**(b)** The free-body diagram of the bar is as shown.

***Answer*** See **Figure 6.47*b***.

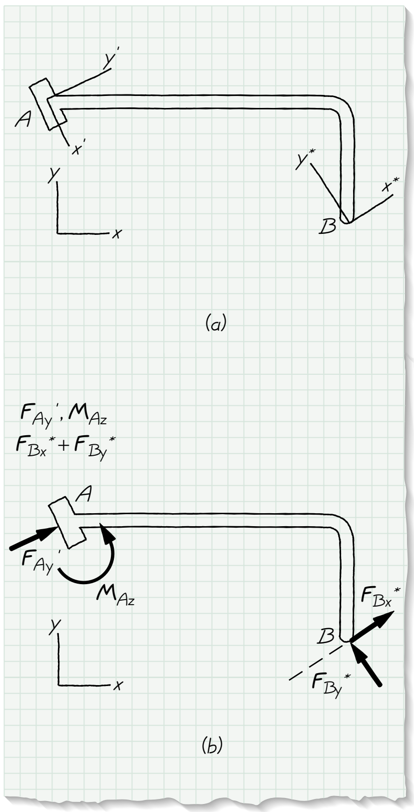



Figure 6.47

## EXAMPLE 6.7 USING QUESTIONS TO DETERMINE LOADS AT SOLID SUPPORTS

A crank is supported at $B$ by a pin connection, as shown in **Figure 6.48**. A cable (in the $xy$ plane) is attached to the bracket at $A$.

(a) What loads act at $A$ and $B$? Use the general rule about the surroundings preventing translation and/or rotation at each support to answer this question.

(b) Draw a free-body diagram of the crank.

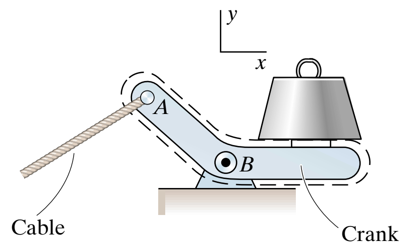



**Figure 6.48**

**Solution** **(a)** The crank (defined as the system) can be classified as planar. This means that in considering motion, we need consider only translations in the $xy$ plane and rotations about the $z$ axis.

**At *B* there is a pin connection:**

| Possible motion | Answer | Implication |
|---|---|---|
| Is $x$ translation at $B$ possible? | No | There is a force $\boldsymbol{F}_{Bx}$. |
| Is $y$ translation at $B$ possible? | No | There is a force $\boldsymbol{F}_{By}$. |
| Is $z$ rotation at $B$ possible? | Yes | There is no moment about the $z$ axis. |

**Check** The answers can be confirmed with **Table 6.1** for a pin connection.

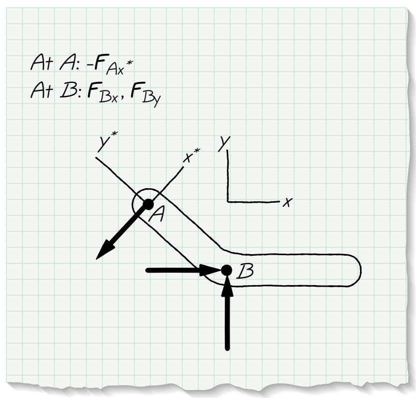



**Figure 6.49**

**At *A* a cable is attached to the system:**
Define the $x^*y^*$ coordinate system at $A$ (see **Figure 6.49**).

| Possible motion | Answer | Implication |
|---|---|---|
| Is $x^*$ translation at $A$ possible? | No, it is not possible in the positive $x^*$ direction. It is possible in the negative $x^*$ direction. | There is a force $\boldsymbol{F}_{Ax^*}$ in the negative $x^*$ direction. |
| Is $y^*$ translation at $A$ possible? | Yes | There is no force in the positive $y^*$ direction. |
| Is $z$ rotation at $A$ possible? | Yes | There is no moment about the $z$ axis. |

***Answer*** *At B:* $\boldsymbol{F}_{Bx}$, $\boldsymbol{F}_{By}$
*At A:* $\boldsymbol{F}_{Ax^*}$ in the negative $x^*$ direction See **Figure 6.49**.

**(b)** The free-body diagram of the crank is as shown.

***Answer*** See **Figure 6.50**.

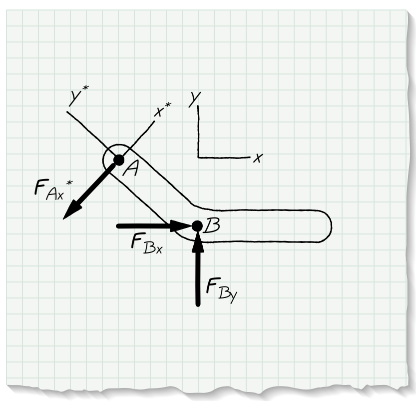



**Figure 6.50**

## EXAMPLE 6.8 USING QUESTIONS TO DETERMINE LOADS AT SUPPORTS

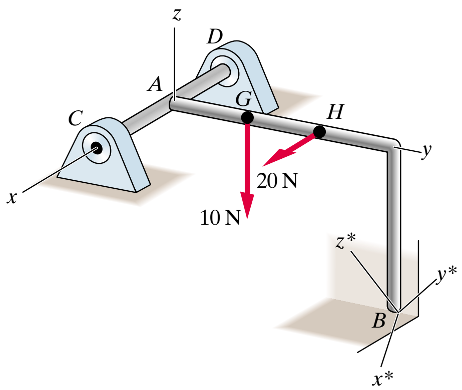



Figure 6.51

An L-shaped bar is supported at $A$ by a hinge and rests against a rough surface at $B$. Known loads act as shown in **Figure 6.51**. Ignore the weight of the bar.

(a) What loads act at $A$ and $B$? Use the general rule about the surroundings preventing translation and/or rotation at each support to answer this question.

(b) Draw a free-body diagram of the bar.

**Solution** **(a)** The bar (defined as the system) is nonplanar. This means that in considering motion, we must consider translations and rotation in all three directions.

**At A:**
Define the $xyz$ coordinate system at $A$ (**Figure 6.51**).

| Possible motion | Answer | Implication |
|---|---|---|
| Is $x$ translation at $A$ possible? | No, if we can assume that the connection at $C$ and/or $D$ prevents motion in the $x$ direction. | There is force in the $x$ direction, $\boldsymbol{F}_{Ax}$. |
| Is $y$ translation at $A$ possible? | No | There is force in the $y$ direction, $\boldsymbol{F}_{Ay}$. |
| Is $z$ translation at $A$ possible? | No | There is force in the $z$ direction, $\boldsymbol{F}_{Az}$. |
| Is rotation about the $x$ axis possible at $A$? | Yes | There is no moment about $x$ axis. |
| Is rotation about the $y$ axis possible at $A$? | No | There is moment about the $y$ axis, $\boldsymbol{M}_{Ay}$. |
| Is rotation about the $z$ axis possible at $A$? | No | There is moment about the $z$ axis, $\boldsymbol{M}_{Az}$. |

**At B:**
Define the $x^*y^*z^*$ coordinate system at $B$ (**Figure 6.51**).

| Possible motion | Answer | Implication |
|---|---|---|
| Is $x^*$ translation at $B$ possible? | No, unless the force applied exceeds the maximum friction force that can be applied by the rough surface. | There is a force $\boldsymbol{F}_{Bx^*}$. |
| Is $y^*$ translation at $B$ possible? | No, unless the force applied exceeds the maximum friction force that can be applied by the rough surface. | There is a force $\boldsymbol{F}_{By^*}$. |

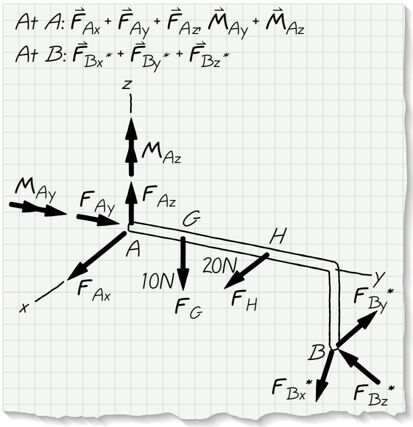



Figure 6.52

| Possible motion | Answer | Implication |
|---|---|---|
| Is $z^*$ translation at $B$ possible? | No, it is not possible in the negative $z$ direction. It is possible in the positive $z$ direction. | There is a force $\boldsymbol{F}_{Bz^*}$ in the positive $z^*$ direction. |
| Is rotation about the $x^*$ axis possible at $B$? | Yes | There is no moment about the $x^*$ axis. |
| Is rotation about the $y^*$ axis possible at $B$? | Yes | There is no moment about the $y^*$ axis. |
| Is rotation about the $z^*$ axis possible at $B$? | Yes | There is no moment about the $z^*$ axis. |

***Answer*** At A: $\boldsymbol{F}_{Ax}, \boldsymbol{F}_{Ay}, \boldsymbol{F}_{Az}, \boldsymbol{M}_{Ay}, \boldsymbol{M}_{Az}$
At B: $\boldsymbol{F}_{Bx^*}, \boldsymbol{F}_{By^*}, \boldsymbol{F}_{Bz^*}$ (in the positive $z^*$ direction)

**(b)** The free-body diagram of the bar is shown in **Figure 6.52**.

***Answer*** See **Figure 6.52**.

## EXERCISES 6.4

**6.4.1.** Consider the description of each system in **E6.4.1** and determine whether it can be classified as planar or nonplanar. Describe your reasoning. Unless otherwise stated, ignore the effect of gravity.

**a.** The uniform L-bar is pinned to its surroundings at $A$, and slides along a wall at $B$. A vertical force $\boldsymbol{F}$ acts at $C$. Gravity acts in the negative $y$ direction. The system is taken as the L-bar (**E6.4.1*a***).

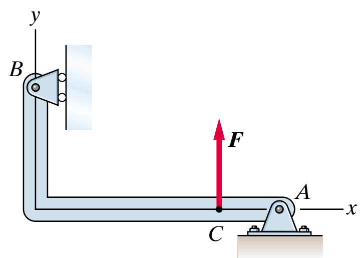
**b.** The wheelbarrow is loaded, with a center of gravity as shown in **E6.4.1*b***. The system is taken as the wheelbarrow.

**c.** The wheelbarrow is loaded as shown in **E6.4.1*c***. The system is taken as the wheelbarrow.

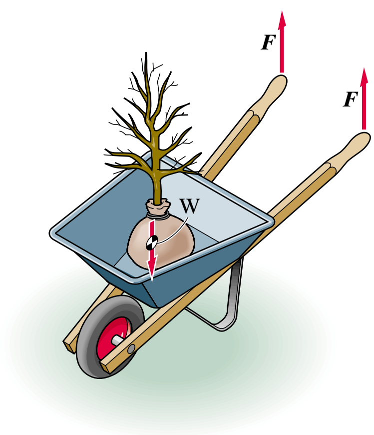
**d.** A tower 70 m tall is tethered by three cables, as shown in **E6.4.1*d***. The system is taken as the tower.

**e.** A uniform glass rod having a length $L$ is placed in a smooth hemispherical bowl having a radius $r$. The system is taken as the rod (**E6.4.1*e***).

**f.** For the situation shown in **E6.4.1*e***, define the bowl as the system.

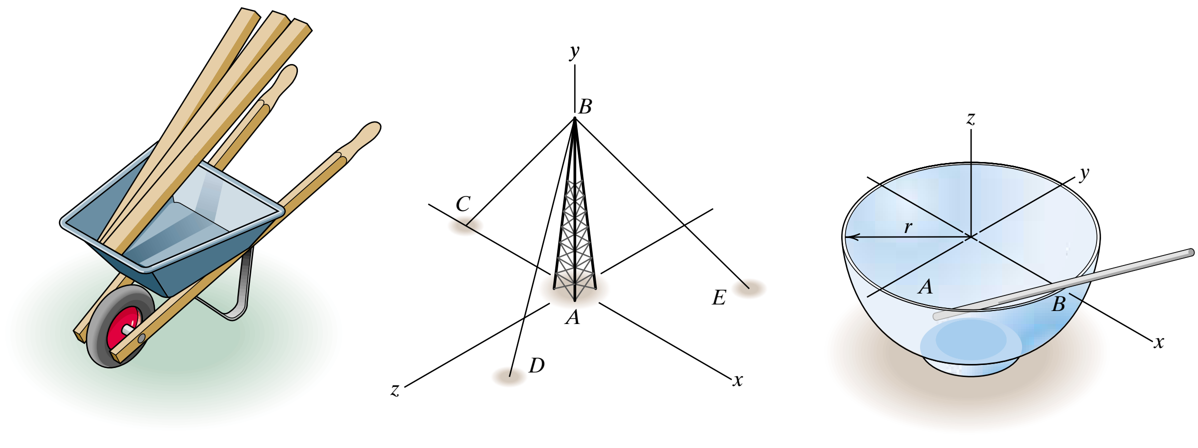



(*a*)




(*b*)

E6.4.1




(c) (d) (e)

E6.4.1 (Cont.)

**6.4.2.** Consider the description of each system in E6.4.2 and determine whether it can be classified as planar or nonplanar. Describe your reasoning. Unless otherwise stated, ignore the effect of gravity.

**a.** A bar *AB* is fixed to a wall at end *B*. At end *A*, a force acts, as shown in **E6.4.2*a***. The system is taken as the bar.

**b.** A bar *AB* is fixed to a wall at end *B*. At end *A* and at *C*, forces act, as shown in **E6.4.2*b***. The system is taken as the bar.

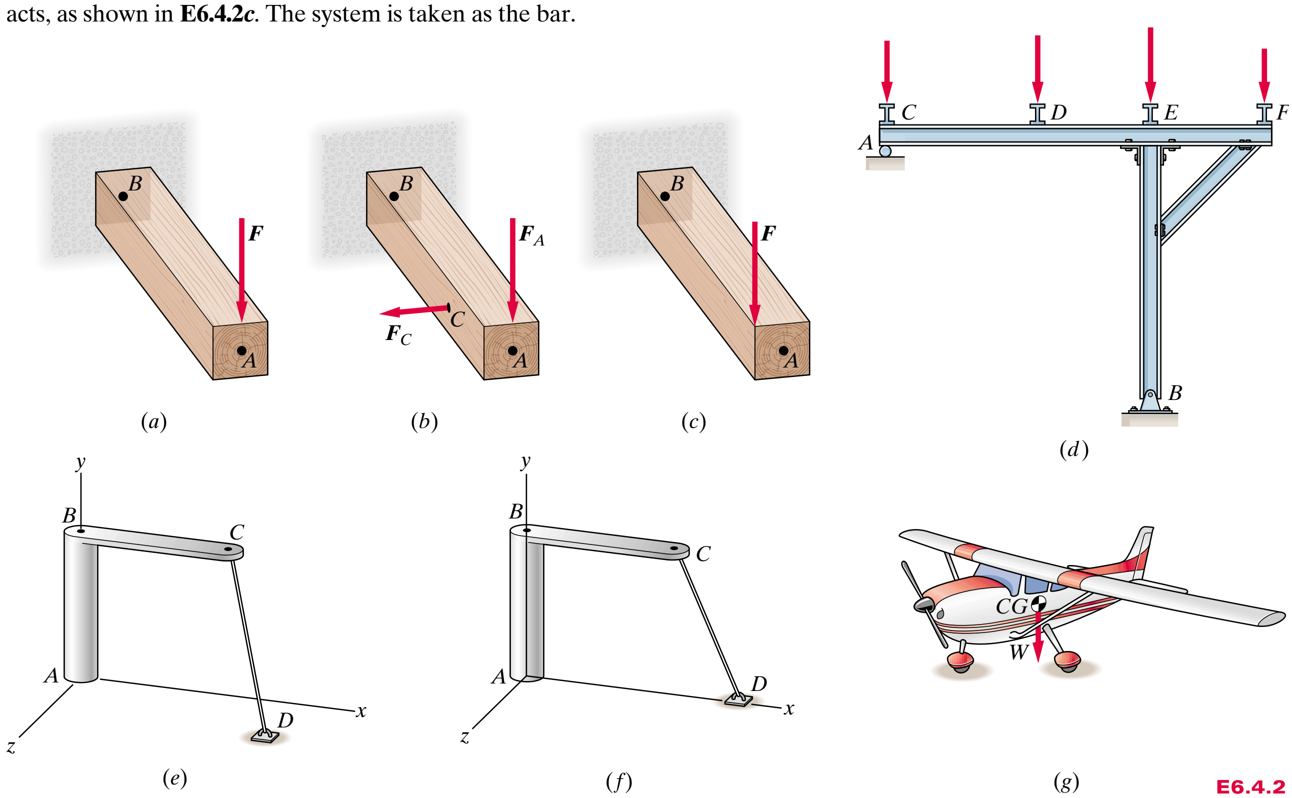
**c.** A bar *AB* is fixed to a wall at end *B*. At end *A* a force acts, as shown in **E6.4.2*c***. The system is taken as the bar.

**d.** The frame is supported at *A* and *B*. Loads act at *C*, *D*, *E*, and *F*, as shown in **E6.4.2*d***. The system is taken as the frame.

**e.** The bracket *ABC* in **E6.4.2*e*** is tethered as shown with cable *CD*. The bracket is taken as the system.

**f.** The bracket *ABC* in **E6.4.2*f*** is tethered as shown with cable *CD*. The bracket is taken as the system.

**g.** The airplane in **E6.4.2*g*** weighing 8000 N sits on the tarmac. It has one front wheel and two rear wheels. Its center of gravity is as shown.

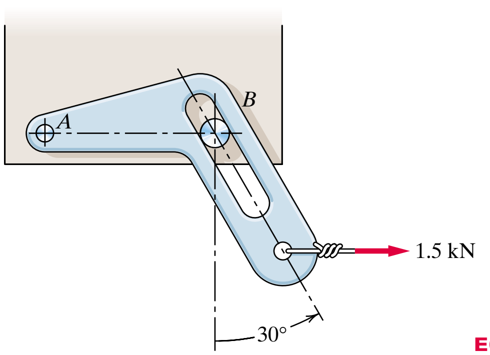



(a) (b) (c) (d)

(e) (f) (g)

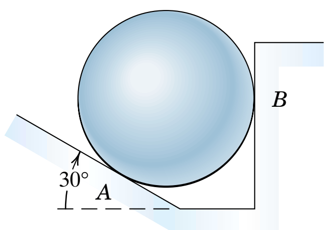
E6.4.2

**6.4.3.** A cable pulls on the bracket in **E6.4.3** with a force of 1.5 kN. At $A$ the bracket is attached to the wall with a pin connection, and at $B$ there is a pin-in-slot connection. If the system is defined as the bracket and is considered to be planar, what loads act on the bracket at $A$ and $B$? Use the general rule about "prevention of motion" to answer this question (*Strategy:* Review Examples 6.6–6.8.) Confirm that your answers are consistent with the information on loads in **Table 6.1** or **Table 6.2** (whichever is appropriate). Present your answer in words and as a sketch of the bracket that shows these loads acting on it at $A$ and $B$. Also comment on whether the sketch you created is or is not a free-body diagram.



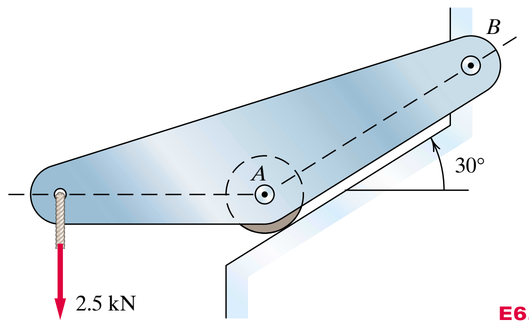
**E6.4.3**

**6.4.4.** A steel sphere sits in the groove, as shown in **E6.4.4**. Surfaces $A$ and $B$ are smooth. If the system is defined as the sphere and is considered to be planar, what loads act on the sphere at $A$ and $B$? Use the general rule about "prevention of motion" to answer this question. (*Strategy:* Review Examples 6.6–6.8.) Confirm that your answers are consistent with the information on loads in **Table 6.1** or **Table 6.2** (whichever is appropriate). Present your answer in words and as a sketch of the sphere that shows the loads acting on it at $A$ and $B$. Also comment on whether the sketch you created is or is not a free-body diagram.



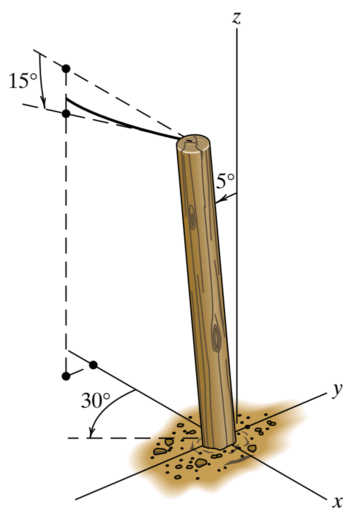
**E6.4.4**

**6.4.5.** A cable pulls on the bracket with a force of 2.5 kN (**E6.4.5**). At $A$ the bracket rests against a smooth surface, and at $B$ it is pinned to the wall. If the system is defined as the bracket and is considered to be planar, what loads act on the bracket at $A$ and $B$? Use the general rule about "prevention of motion" to answer this question. (*Strategy:* Review Examples 6.6–6.8.) Confirm that your answers are consistent with the information on loads in **Table 6.1** or **Table 6.2** (whichever is appropriate). Present your answer in words and as a sketch of the bracket that shows the loads acting on it at $A$ and $B$. Also comment on whether the sketch you created is or is not a free-body diagram.



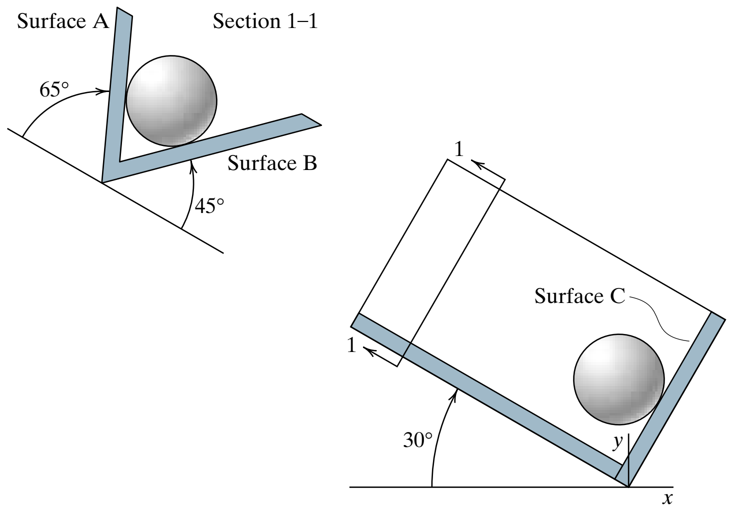
**E6.4.5**

**6.4.6.** Because of a combination of soil conditions and the tension in the single power cable, the utility pole in **E6.4.6** has developed the indicated 5° lean. The 9-m uniform pole has a mass per unit length of 25 kg/m, and the tension in the power cable is 900 N. If the system is defined as the power pole, what loads act on the pole at its base where it is fixed into the ground? Use the general rule about "prevention of motion" to answer this question. (*Strategy:* Review Examples 6.6–6.8.) Confirm that your answer is consistent with the information on loads in **Table 6.1** or **Table 6.2** (whichever is appropriate). Present your answer in words and as one or more sketches of the pole that show the loads acting on its base. Also comment on whether the sketch you created is or is not a free-body diagram.



**E6.4.6**

**6.4.7.** A steel sphere sits in the grooved trough, as shown in **E6.4.7**. Surfaces *A*, *B*, and *C* are smooth. Gravity acts in the negative *y* direction. If the system is defined as the sphere, what loads act on the sphere at *A*, *B*, and *C*? Use the general rule about "prevention of motion" to answer this question. (*Strategy:* Review Examples 6.6–6.8.) Confirm that your answer is consistent with the information on loads in **Table 6.1** or **Table 6.2** (whichever is appropriate). Present your answer in words and as one or more sketches of the sphere that show the loads acting on it at *A*, *B*, and *C*.




**E6.4.7**

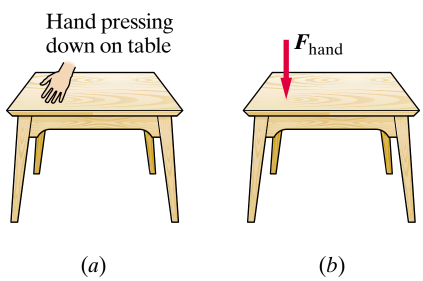



**Figure 6.53** Hand pressing down on a table modeled as a force

## 6.5 DISTRIBUTED FORCES

Up to this point we have modeled supports as loads acting at a single location on the system boundary. In actuality, all supports consist of forces distributed over a finite surface area. For example, if you press down on a table with your hand, the force you apply to the table is distributed over a finite area (**Figure 6.53*a***). For many practical applications, we can "condense" this distributed force into a single point force (**Figure 6.53*b***).

There are, however, some situations for which we explicitly consider the loads to be distributed; **Figure 6.54** shows some examples. The key idea we want to get across is that these distributed forces must be included in the system's free-body diagram. They can be represented in the diagram either as distributed forces (**Figure 6.55*b***) or as an equivalent point force (**Figure 6.55*c***). This equivalent point force is the total force represented by the distributed force and is located so as to create the same moment as the distributed force. For the uniformly distributed

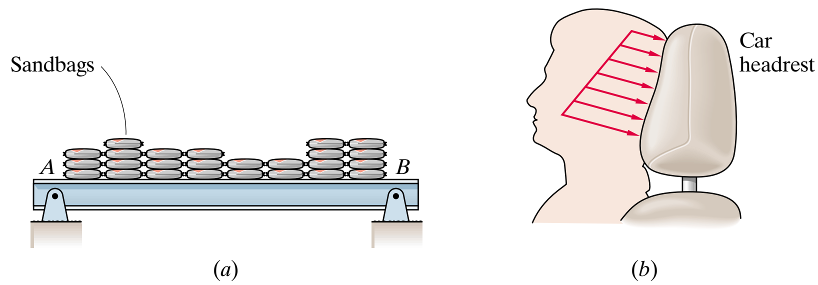



**Figure 6.54** Distributed loads: (*a*) 60-lb bags of concrete stacked on beam *AB*; (*b*) a head presses back on a head-rest

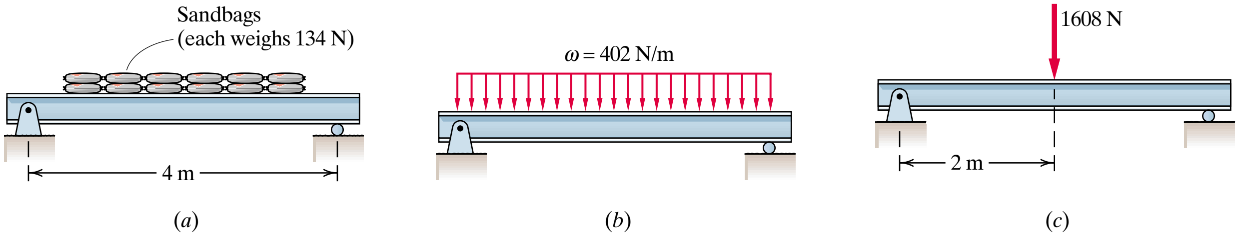



**Figure 6.55** (*a*) Sandbags sitting on a beam; (*b*) weight of sandbags represented as distributed load; (*c*) weight of sandbags represented as an equivalent force

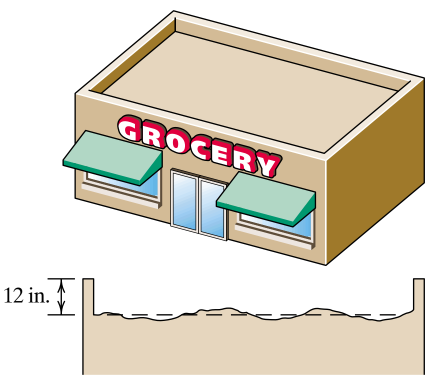
force in **Figure 6.55*a*** we are able to find this location by inspection. In Chapter 8 we shall show how to find the location for nonuniformly distributed forces. The important point to remember for your current work is that these distributed forces must be included in the free-body diagram.

By their very nature, fluids acting on a system boundary are distributed. Like distributed loads associated with supports, the loads at fluid boundaries are included in a free-body diagram, either as distributed loads or as an equivalent force. In Chapter 8 we discuss in greater detail distributed loads due to fluids acting on the system.

## EXERCISES 6.5

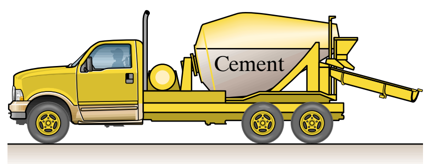
**6.5.1** Consider the coat rack in Exercise 6.1.1. Redraw the sketch showing the distributed load between the base of the coat rack and the floor.

**6.5.2.** Consider a building with a nominally flat roof. The actual roof surface is slightly irregular and can be represented by the profile shown in **E6.5.2**. After a night of heavy rain, an average rainfall total of 2 in. was recorded. Make a sketch of the distributed force that rain water applies to the nominally flat roof. Indicate the magnitude of the forces with the length of the vectors.




**E6.5.2**

**6.5.3.** Consider a cement truck with a tank that is half full in **E6.5.3**. Draw the distributed load applied to the inside of the tank. Indicate the magnitude of the loads with the length of the vectors.




**E6.5.3**

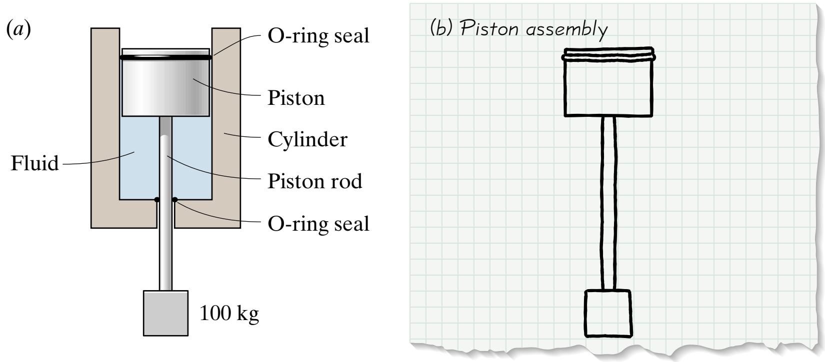
**6.5.4.** Identify three different systems on which distributed forces act. Make a sketch of each system and show what you think the distributed forces look like; indicate the magnitude of the forces with the length of vectors.

**6.5.5.** Consider a person wanting to cross a frozen pond. She has a choice of going on foot, wearing snow shoes, or using skis.

**a.** Make three sketches showing the distribution of her body weight on the frozen pond given the three types of footwear.

**b.** Which type of footwear would you choose? Why?

**6.5.6.** A hydraulic cylinder works by pumping fluid in and out of a piston assembly as shown in **E6.5.6*a***. Draw the loads acting on piston assembly shown in **E6.5.6*b***.




E6.5.6

## 6.6 FREE-BODY DIAGRAM DETAILS

We now outline a process for drawing a free-body diagram of a system; this is the DRAW step in our engineering analysis procedure.

1. Before diving into drawing, take time to **study the physical situation**. Consider what loads are present at boundaries and ask yourself whether you have ever seen a similar support. Study actual hardware (if available); pick it up or walk around it to really get a sense of how the loads act on the system. This inspection helps in making modeling assumptions. **Classify the system as planar or nonplanar**. If the system can be classified as planar, drawing the free-body diagram and writing and solving the conditions of equilibrium (covered in the next chapter) all become easier. If you are unsure, consider the system to be nonplanar. Also, consider asking for advice and opinions from others.

2. Define (either by imagining or actually drawing) a boundary that isolates the system from the rest of the world, then **draw the system** that is within the boundary. The drawing should contain enough detail so that distances and locations of loads acting on the system can be shown accurately. Sometimes multiple views of the system will be needed, especially if the system is nonplanar. **Establish a coordinate system**. **State any assumptions** you make.

3. Identify **cross-boundary forces** acting on the system and draw them at appropriate centers of gravity.[1] Include a variable label and the force magnitude (if known). Continue to state any assumptions you make.

4. Identify all **known loads** acting at the boundary and add these to the drawing, placing each known load at its point (or surface area) of application; identify each load on the drawing with a variable label and magnitude.

5. Identify the loads associated with each **support**, including those loads that act at discrete points and those that consist of distributed forces. If possible, classify each support as one of the standard supports (**Table 6.1** for planar systems and **Table 6.2** for nonplanar systems) to

[1]In Chapter 8 we will show how to find the center of gravity of a system.

help in identifying the loads. If this is not possible, consider how the surroundings restrict motion at a particular support (either translation and/or rotation) in order to identify the loads acting on the system that restrict this motion. Add all these loads to the drawing. Identify each load on the drawing with a variable label.

6. Identify **fluid boundaries**. Add loads associated with these boundaries to the drawing, showing them either as distributed or discrete point loads. Add variable labels.

You now have a free-body diagram of a system, as well as a list of the assumptions made in creating it. The diagram consists of a depiction of the system and the external loads acting on the system. The loads are represented in the diagram as vectors and with variable labels, and magnitudes (if known) are indicated.

A free-body diagram is an idealized model of a real system. By making assumptions about the behavior of supports, dimensions, and the material, you are able to simplify the complexity of the real system into a model that you can analyze. You might want the model to describe the real situation exactly, but this is generally not an achievable goal, due to limitations such as information, time, and money. What you do want, however, is a model that you can trust and that gives results that closely approximate the real situation.

In creating a model, an engineer must decide which loads acting on the system are significant. For example, a hinge on a door is often modeled as having no friction about its axis. Yet for most hinges, grease, dust, and dirt have built up, and there is actually some friction—some resistance to rotation. If friction is large enough the engineer should include it in the model. However, if the friction is small enough that the door can still swing freely, the engineer may conclude that it is not significant for the problem at hand, and model the hinge loads as shown in **Figure 6.56**.

Often the significance of loads is judged by their relative magnitude or location. For example, the weight of a sack of groceries is insignificant relative to the weight of an automobile carrying them but very significant if the vehicle is a bicycle. Whenever you are in doubt about the significance of a load, consider it significant. In many of the examples in this book, we will set the stage by making some of the assumptions regarding significance. In others, though, it will be up to you to judge the significance of a load based either on your own experience or on the advice of other engineers. Any loads considered insignificant are not included in the free-body diagram and should be noted in the assumption list.

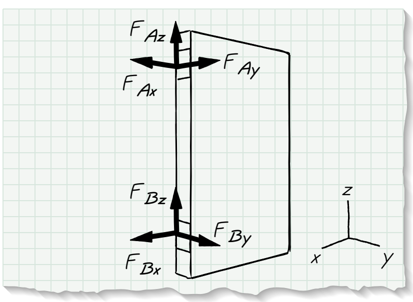



**Figure 6.56** Forces acting at hinges *A* and *B* when hinges are frictionless

## EXAMPLE 6.9 CREATING A FREE-BODY DIAGRAM OF A PLANAR SYSTEM

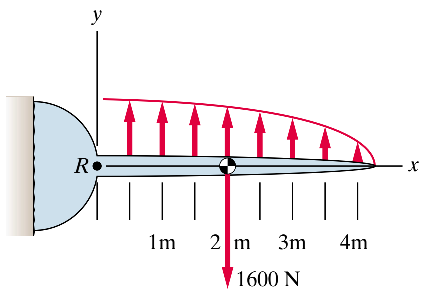



Figure 6.57

The lift force on an airplane wing (which is actually a distributed load) can be modeled by eight forces as shown in **Figure 6.57**. The magnitude of each force is given in terms of its position $x$ on the wing by

$$F_i = 300\sqrt{1-\left(\frac{x_i}{17}\right)^2}\text{ N},\quad \text{with } i = 1,2,\ldots,8 \tag{1}$$

The location $x = 0$ is at the root of the wing (location $R$); this is the point where the wing connects with the fuselage.

The weight of the wing $W = 1600$ N can be located at midpoint of the wing's length. Create a free-body diagram of the wing.

**Goal** Draw a free-body diagram of the wing.

**Given** We are given a planar view of the wing and some dimensions. In addition, we are told that we can model the lift as eight forces acting upward at locations $x_i = 0.5$ m, 1.0 m, 1.5 m, . . . , 4.0 m. We are also told that the weight of the wing is 1600 N, with a point of application at $x = 2.0$ m.

**Assume** Ignoring any slight differences in the leading and trailing edges of the wing, we assume there is a plane of symmetry (the $xy$ plane in **Figure 6.57**); therefore we can treat the wing as a planar system. This means we are ignoring any twist on the wings that would occur from asymmetry.

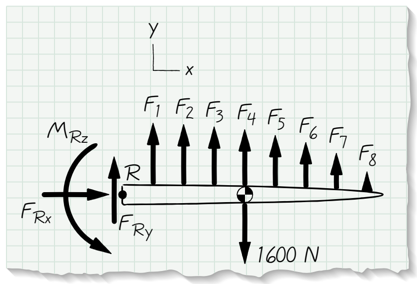



Figure 6.58

**Draw** Based on the information given in the problem and our assumptions, we isolate the wing at its root. The boundary condition at $R$ can be modeled as a fixed boundary condition; therefore, from **Table 6.1** we find that there will be a force (represented as $x$ and $y$ components) and a moment present. We also determine the values of the lift force at each of the eight locations using function (1), obtaining the values shown in **Table 6.3**. The free-body diagram is given in **Figure 6.58**.

**Table 6.3** Point Loads Representing Lift on a Wing

| $x$(m) | | $F$(N) |
|---|---|---|
| 0.5 | $F_1$ | 300 |
| 1 | $F_2$ | 299 |
| 1.5 | $F_3$ | 299 |
| 2 | $F_4$ | 298 |
| 2.5 | $F_5$ | 297 |
| 3 | $F_6$ | 295 |
| 3.5 | $F_7$ | 294 |
| 4 | $F_8$ | 292 |

## EXAMPLE 6.10 CREATING A FREE-BODY DIAGRAM OF A PLANAR SYSTEM

The ladder in **Figure 6.59** rests against the wall of a building at $A$ and on the roof of an adjacent building at $B$. If the ladder has a weight of 100 N and length 3 m, and the surfaces at $A$ and $B$ are assumed smooth, create a free-body diagram of the ladder.

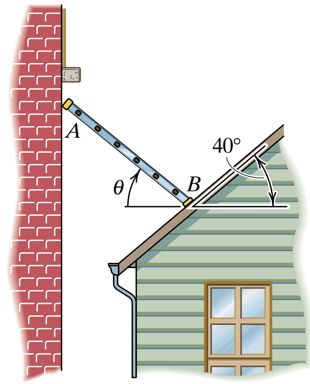



Figure 6.59

**Goal** Draw a free-body diagram of the ladder.

**Given** We are given a planar view of the ladder and some spatial information (length of ladder and angle of orientation with respect to the roof and wall). In addition we are told that the roof is at 40° with respect to the horizontal and that the surfaces at $A$ and $B$ are smooth.

**Assume** First, we assume that the ladder is uniform. By this we mean that the rungs (cross-pieces) are identical to one another, as are the two stringers (sides of ladder which the rungs are attached). Next we assume that the weight of the ladder is significant and that its center of mass can be located at its midpoint (since it is uniform). We also assume that gravity works downward in the vertical direction. Finally, we assume that there is an $xy$ plane of symmetry; therefore we can treat the ladder as a planar system.

**Draw** Based on the information given and our assumptions, we isolate the ladder from the wall (at $A$) and from the roof (at $B$). At each surface there is a normal force present that pushes on the ladder. There is no frictional force because both surfaces are smooth. The resulting free-body diagram is shown in **Figure 6.60**. Notice that there is a factor of two associated with both normal forces; this factor reflects that there are two stringers.

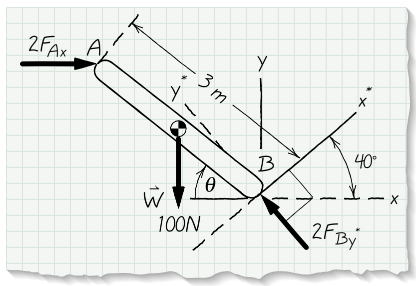



Figure 6.60

## EXAMPLE 6.11 CREATING A FREE-BODY DIAGRAM OF A PLANAR SYSTEM

A 500-N crane boom is supported by a pin at $A$ and a hydraulic cylinder at $B$. At $C$, the boom supports a stack of lumber weighing $W$. Create a free-body diagram of the boom (**Figure 6.61**).

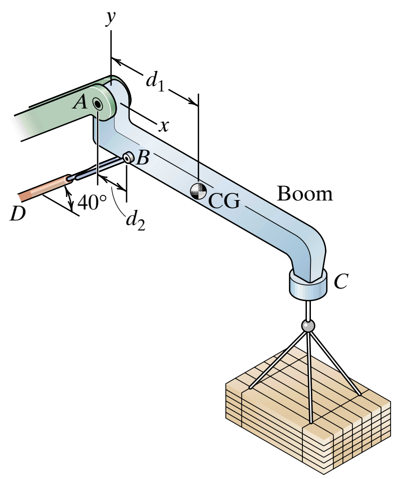



Figure 6.61

**Goal** Draw a free-body diagram of the boom.

**Given** We are given an isometric view of the boom. The joint at $A$ is a pin connection, and member $DB$ is a hydraulic cylinder. The hydraulic cylinder acts like a link; therefore there will be a force acting on the boom at $B$ that runs along $DB$. The boom is being used to lift a load $W$.

**Assume** We assume that the weight of the boom can be modeled as a force acting at the point marked $CG$ in **Figure 6.61**. We also assume that gravity works in the negative $y$ direction. Finally, we assume that there is an $xy$ plane of symmetry; therefore we can treat the boom as a planar system.

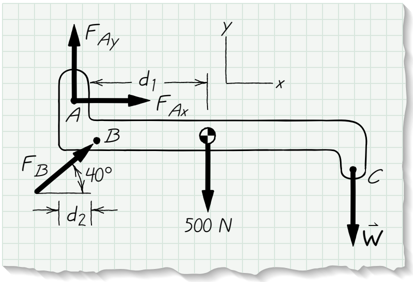



Figure 6.62

**Draw** Based on the information given and our assumptions, we isolate the boom. At $A$ there is a pin connection, and according to **Table 6.1**, this means that there is a force present (which we represent in terms of its $x$ and $y$ components). The resulting free-body diagram is shown in **Figure 6.62**.

## EXAMPLE 6.12 CREATING A FREE-BODY DIAGRAM OF A NONPLANAR SYSTEM

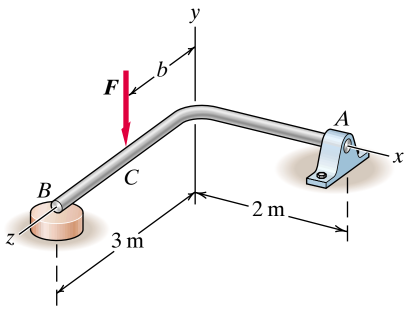



Figure 6.63

The L-shaped bar is supported by a bearing at $A$ and rests on a smooth horizontal surface at $B$, with $\| \boldsymbol{F} \| = 800$ N and $b = 1.5$ m (**Figure 6.63**). Ignore gravity. Create a free-body diagram of the bar.

**Goal** Draw a free-body diagram of the bar.

**Given** We are given the dimensions of the bar and that surface $B$ is smooth. Joint $A$ looks like a journal or thrust bearing. We are told to ignore the weight of the bar.

**Assume** We will assume that the bearing at $A$ is a journal bearing. Since the loads involved at $A$, $B$, and $C$ do not lie in a single plane, we must treat the bar as a nonplanar system.

**Draw** Based on the information given and our assumptions, we isolate the bar. At $A$ there is a single thrust bearing; according to **Table 6.2**, this means there are forces and moments present. At the smooth surface at $B$ there is a normal force that acts to push up on the bar. The resulting free-body diagram is shown in **Figure 6.64**.

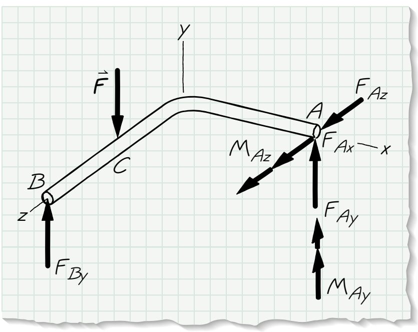



Figure 6.64

## EXAMPLE 6.13 CREATING A FREE-BODY DIAGRAM OF A NONPLANAR SYSTEM

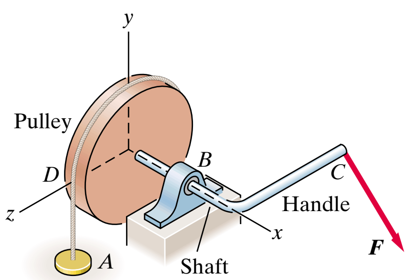



Figure 6.65

The cable in **Figure 6.65** is attached at $A$ to a stationary surface and is wrapped around the pulley. At $B$ is a single thrust bearing, and at $C$, force $\boldsymbol{F} = (10 \text{ N } \boldsymbol{i} - 30 \text{ N } \boldsymbol{j} - 10 \text{ N } \boldsymbol{k})$ acts. Ignore gravity. Create a free-body diagram of the shaft–handle–pulley system.

**Goal** We are to draw a free-body diagram of the shaft–handle–pulley system.

**Given** We are given dimensions of the shaft, pulley, and handle. In addition, we are told the magnitude and direction of the applied force $\boldsymbol{F}$, and that the bearing at $B$ is a thrust bearing.

**Assume** Since the loads acting on the shaft–handle–pulley system do not lie in a single plane, we must treat the shaft–handle–pulley system as nonplanar. Finally, we assume that the weights of the shaft, handle, and pulley are negligible (because we are not told anything about their weights).

**Draw** Based on the information given and our assumptions, we isolate the shaft–handle–pulley system. At $D$ we show the pull of the cable as $\boldsymbol{F}_{DA}$. At the single bearing at $B$ we include a force and moment (**Table 6.2**). Finally, at $C$ we show the force $\boldsymbol{F}$. Figure 6.66 shows the completed free-body diagram.

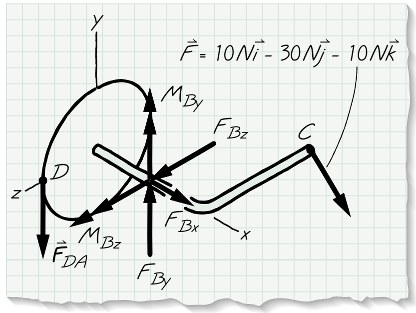



Figure 6.66

## EXAMPLE 6.14 CREATING A FREE-BODY DIAGRAM OF A NONPLANAR SYSTEM

Figure 6.67 shows an improved design relative to Example 6.13. The bearing at $B$ is a thrust bearing, and the bearing at $E$ is a journal bearing. At $C$, force $\boldsymbol{F} = (10\text{ N}\,\boldsymbol{i} - 30\text{ N}\,\boldsymbol{j} - 10\text{ N}\,\boldsymbol{k})$ acts. Ignore gravity. Create a free-body diagram of the shaft–handle–pulley system. Why do you think this design is improved relative to Example 6.13?

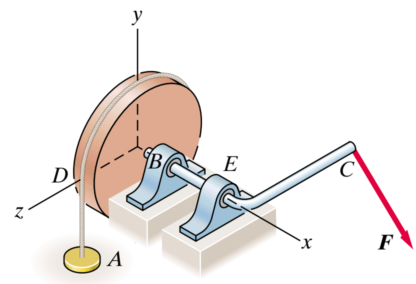



Figure 6.67

**Goal** Draw a free-body diagram of the shaft–handle–pulley system.

**Given** We are given the dimensions of the shaft, pulley, and handle. In addition, we are told the magnitude and direction of the applied force $\boldsymbol{F}$, and to ignore gravity.

**Assume** We assume that the bearings at $B$ and $E$ are properly aligned. This means (according to **Table 6.2**) that forces are applied to the bar at the location of each bearing. In addition, we are told that bearing $B$ is a thrust bearing; therefore it will apply an axial force to the shaft. Since the loads involved do not lie in a single plane, we must treat the shaft–handle–pulley system as nonplanar.

**Draw** Based on the information given and our assumptions, we isolate the shaft–handle–pulley system. At $D$ we include the pull of the cable. At the thrust bearing at $B$ we include radial ($F_{By}$, $F_{Bz}$) forces and an axial force ($F_{Bx}$). At the journal bearing at $E$ we include radial ($F_{Ey}$, $F_{Ez}$) forces. Finally, at $C$ we show the force $\boldsymbol{F}$ (Figure 6.68).

This design is superior to the design in Example 6.13 because the bearings apply only forces to the shaft; this results in better wear of both the shaft and the bearings. In addition, there will be less "radial play" of the system, generally a desirable characteristic of rotating systems. It is generally good practice to design systems with two properly aligned bearings.

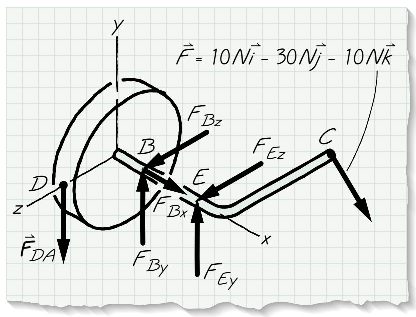
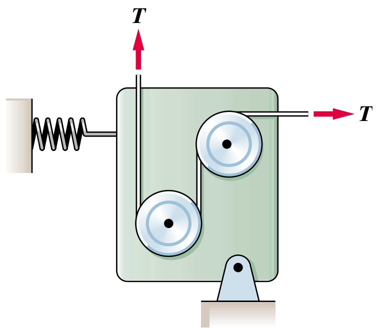



Figure 6.68

## EXERCISES 6.6

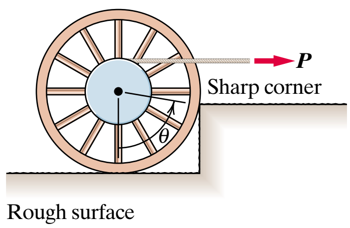
*For each of the exercises in this section, follow the steps outlined in Section 6.6 for presenting your work.*

**6.6.1.** A tape guide assembly is subjected to the loading as shown in **E6.6.1**. Draw the free-body diagram of the tape guide assembly.




E6.6.1

**6.6.2.** A wheel and pulley assembly is subjected to the loading as shown in **E6.6.2**. Draw the free-body diagram of the assembly.




E6.6.2

**6.6.3.** Reconsider the cable–pulley–cylinder assembly described in **E6.2.5**. Define

**a.** the cylinder as the system and draw its free-body diagram

**b.** the pulley as the system and draw its free-body diagram

**6.6.4.** Reconsider the belt-tensioning device described in **E6.2.4**. Define

**a.** the pulley $A$ as the system and draw its free-body diagram

**b.** the mass as the system and draw its free-body diagram

**c.** the pulley, L-arm, and mass as the system and draw its free-body diagram

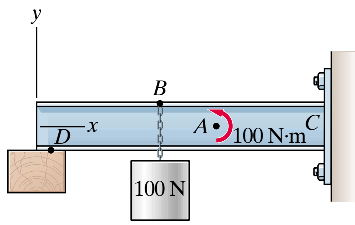
**6.6.5.** Reconsider the simple truss described in **E6.2.3**. Define the simple truss as the system and draw its free-body diagram.

**6.6.6.** Reconsider the beam described in **E6.2.2**. Define

**a.** the beam as the system and draw its free-body diagram

**b.** the beam and the 100-N cylinder as the system and draw its free-body diagram

**6.6.7.** A beam is fixed to the wall at $C$ and rests against a block at $D$. Additional loads act on the beam, as shown in **E6.6.7**. Define the beam as the system. Draw its free-body diagram.



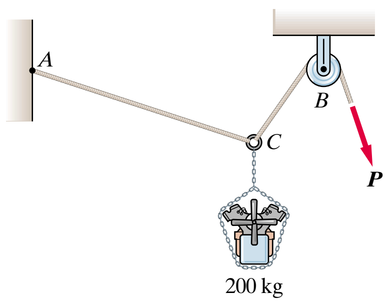

E6.6.7

**6.6.8.** An engine is lifted with the pulley system shown in **E6.6.8**. Define

**a.** the pulley at $B$ as the system and draw its free-body diagram

**b.** the engine and chain as the system and draw its free-body diagram

**c.** the ring at $C$ as the system and draw its free-body diagram



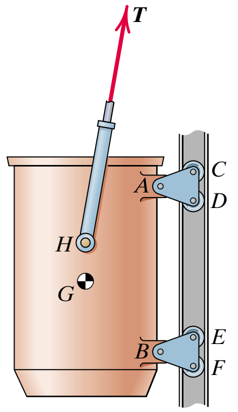

E6.6.8

**6.6.9.** A concrete hopper and its contents have a combined mass of 400 kg, with mass center at $G$. The hopper is being elevated at constant velocity along its vertical guide by cable tension $\boldsymbol{T}$. The design calls for two sets of guide rollers at $A$, one on each side of the hopper, and two sets at $B$. Define

**a.** the hopper and the triangular guides (including the wheels) at $A$ and $B$ as the system and draw its free-body diagram

**b.** the triangular guide at $A$ (including the two wheels) as the system and draw its free-body diagram

**c.** the hopper minus the triangular guides at $A$ and $B$ as the system and draw its free-body diagram

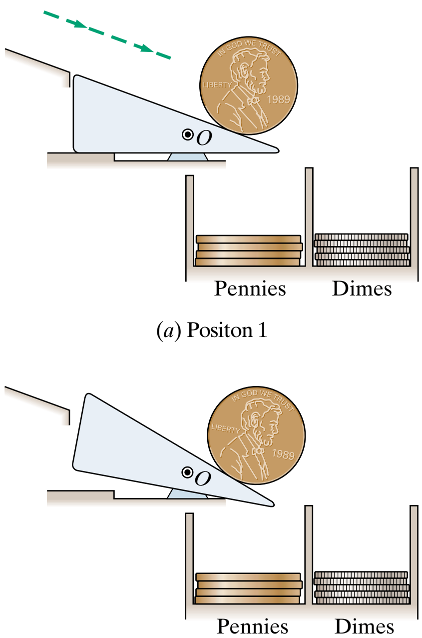



**E6.6.9**

**6.6.10.** A portion of a mechanical coin sorter is shown in **E6.6.10*a***. Pennies and dimes roll down the 20° incline, the triangular portion of which pivots freely about a horizontal axis through $O$. Dimes are light enough (2.28 grams mass each) so that the triangular portion remains stationary, and the dimes roll into the right collection column. Pennies, on the other hand, are heavy enough (3.06 grams mass each) so that the triangular portion pivots clockwise, and the pennies roll into the left collection column. Define

**a.** the triangular portion in Position 1 (**E6.6.10*a***) as the system and draw its free-body diagram

**b.** the triangular portion in Position 2 as the system (**E6.6.10*b***) (the triangle has just started to pivot) and draw its free-body diagram

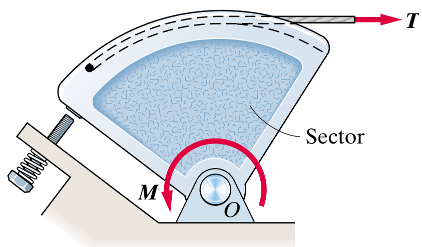



($a$) Positon 1

($b$) Position 2

**E6.6.10**

**6.6.11.** The throttle-control sector pivots freely at $O$. An internal torsional spring at $O$ exerts a return moment of magnitude $\|\boldsymbol{M}\| = 2\ \text{N} \cdot \text{m}$ on the sector when in the position shown. Define the sector as the system. Draw its free-body diagram.




**E6.6.11**

**6.6.12.** The following three cases involve a $2 \times 4$ wooden board and a wrench. In each case a force is applied, and hands are used to react to this force, as shown in **E6.6.12**. Imagine what forces the hands would need to apply to the $2 \times 4$, then:

**a.** *Consider Case 1.* Define the system as the $2 \times 4$, bolt and wrench. Draw its free-body diagram.

**b.** *Consider Case 2.* Define the system as the $2 \times 4$, bolt and wrench. Draw its free-body diagram.

**c.** *Consider Case 3.* Define the system as the $2 \times 4$, bolt and wrench. Draw its free-body diagram.

**d.** Repeat **a**, **b**, and **c** if the system is defined as just being the $2 \times 4$ and the bolt.

**6.6.13.** Consider the mechanism used to weigh mail in **E6.6.13**. A package placed at $A$ causes the weight pointer to rotate through an angle $\alpha$. Neglect the weights of the members except for the counterweight at $B$, which has a mass of 4 kg. For a particular package, $\alpha = 20°$. Define

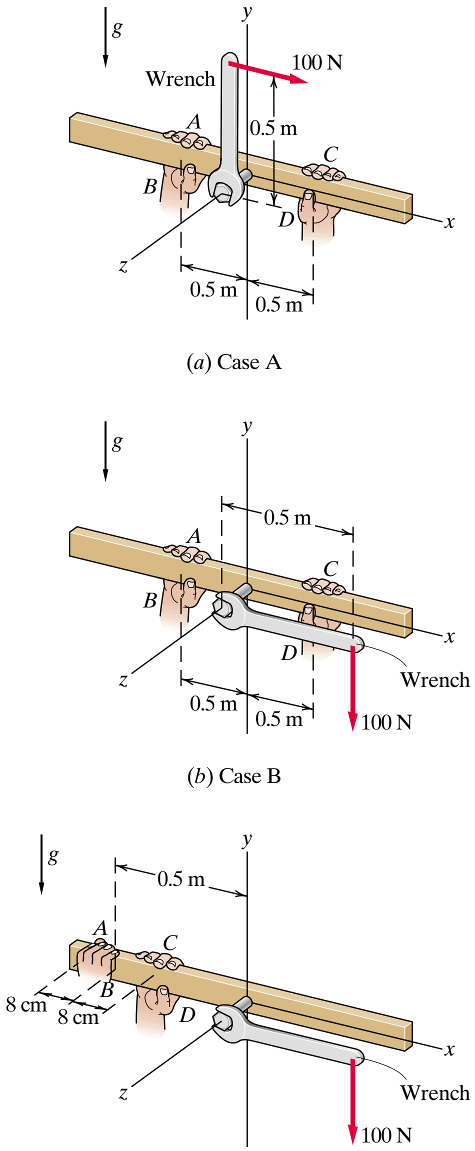
**a.** the system as shown in **E6.6.13*a*** and draw its free-body diagram

**b.** the system as shown in **E6.6.13*b*** and draw its free-body diagram

**c.** the system as shown in **E6.6.13*c*** and draw its free-body diagram

**d.** the system as shown in **E6.6.13*d*** and draw its free-body diagram

**e.** the system as shown in **E6.6.13*e*** and draw its free-body diagram

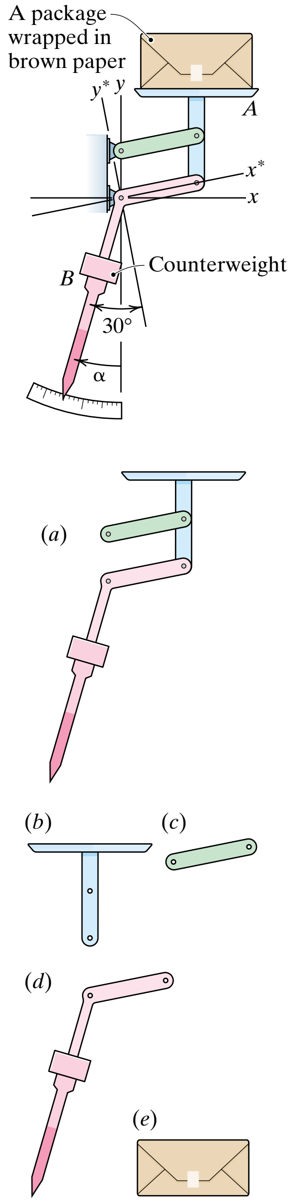



(a) Case A

(b) Case B

(c) Case C

E6.6.12




E6.6.13

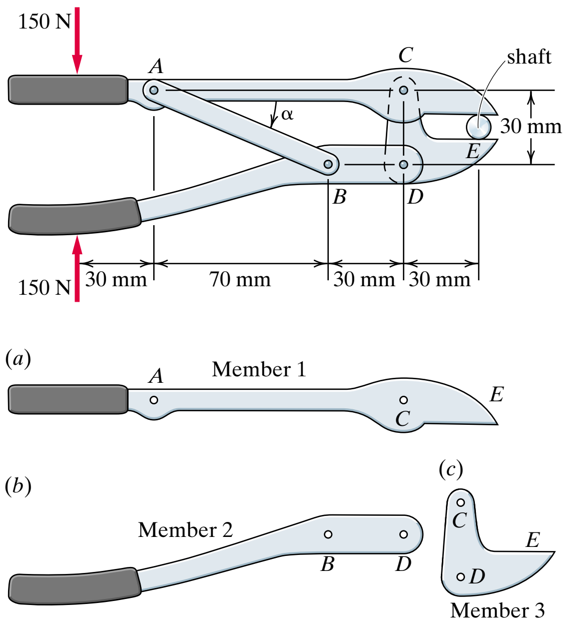
**6.6.14.** Consider the pair of pliers in **E6.6.14**. Define

**a.** Member 1 (**E6.6.14*a***) as the system and draw its free-body diagram

**b.** Member 2 (**E6.6.14*b***) as the system and draw its free-body diagram

**c.** Member 3 (**E6.6.14*c***) as the system and draw its free-body diagram

**d.** the entire pair of pliers as the system (minus the shaft it is clamping) and draw its free-body diagram



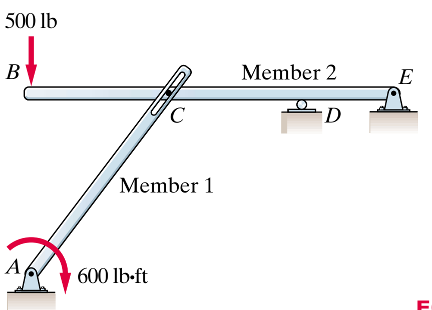

**E6.6.14**

**6.6.15.** Consider the frame in **E6.6.15**. Define

**a.** the entire frame as the system and draw its free-body diagram

**b.** Member 1 as the system and draw its free-body diagram

**c.** Member 2 as the system and draw its free-body diagram



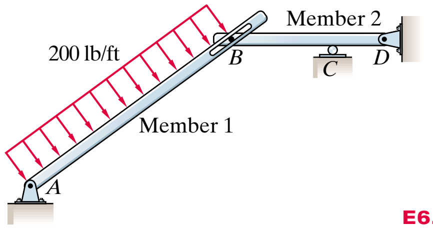

**E6.6.15**

**6.6.16.** Consider the frame in **E6.6.16**. Define

**a.** the entire frame as the system and draw its free-body diagram

**b.** Member 1 as the system and draw its free-body diagram

**c.** Member 2 as the system and draw its free-body diagram



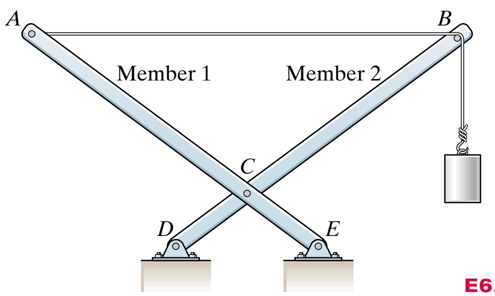

**E6.6.16**

**6.6.17.** Consider the frame in **E6.6.17**. Define

**a.** the entire frame as the system and draw its free-body diagram

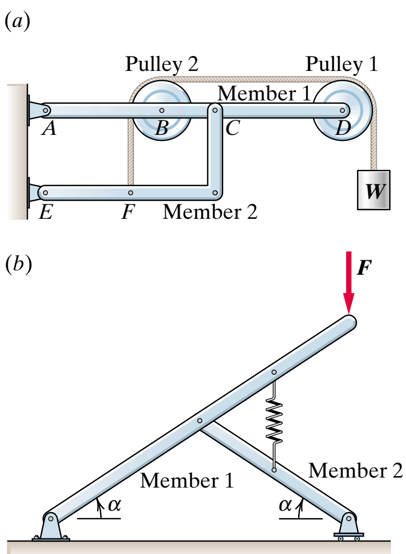
**b.** Member 1 as the system and draw its free-body diagram

**c.** Member 2 as the system and draw its free-body diagram




**E6.6.17**

**6.6.18.** Consider the frames in **E6.6.18**. Define

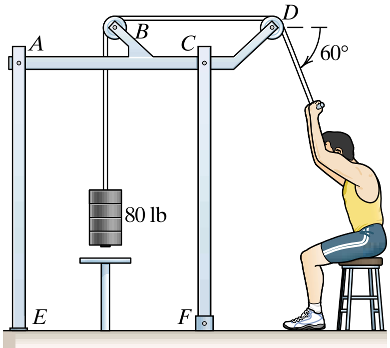



**E6.6.18**

**a.** Draw the free-body diagrams for the entire frame (**E6.6.18*a***), Member 1, Member 2, Pulley 1, and Pulley 2.

**b.** Draw the free-body diagrams for the entire frame (**E6.6.18*b***), Member 1, and Member 2.

**6.6.19.** Consider the exercise frame in E6.6.19. Define the entire frame as the system and draw its free-body diagram.

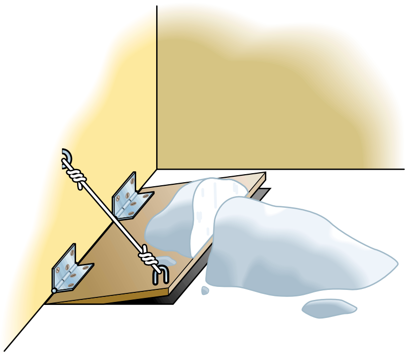



E6.6.19

**6.6.20.** A roof access cover is buried under a pile of snow as shown in E6.6.20.

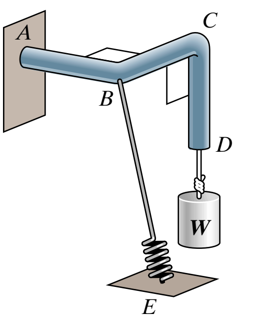
**a.** Draw the free-body diagram of the cover.

**b.** Describe in words how you showed the snow load and why you chose to show it in this manner.



E6.6.20

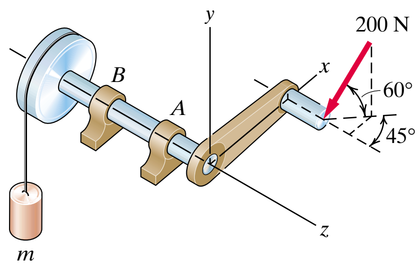
**6.6.21.** The bent bar is loaded and attached as shown in E6.6.21. Draw the free-body diagram of the bar.




E6.6.21

**6.6.22.** Reconsider the utility pole described in **E6.4.6** and draw the free-body diagram of the pole.

**6.6.23.** A 200-N force is applied to the handle of the hoist in the direction shown in E6.6.23. There is a thrust bearing at $A$ and a journal bearing at $B$. Draw a free-body diagram of the handle–shaft–pulley assembly. Make sure to record any assumptions.



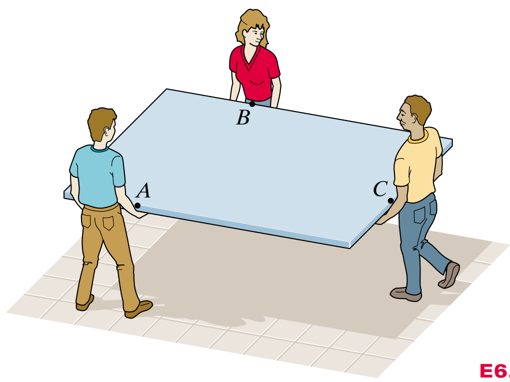

E6.6.23

**6.6.24.** Reconsider the welded tubular frame described in **E6.3.4** and draw the free-body diagram of the frame.

**6.6.25.** Reconsider the tower crane described in **E6.3.2** and draw the free-body diagram of the crane.

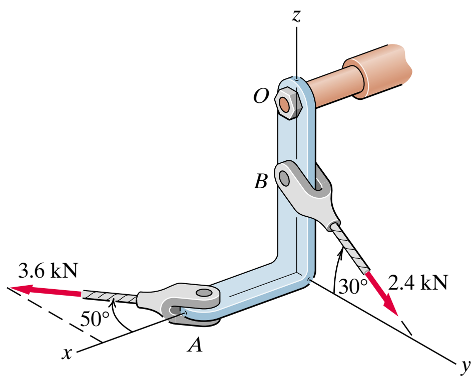
**6.6.26.** Reconsider the steel shaft described in **E6.3.3** and draw the free-body diagram of the shaft.

**6.6.27.** Three workers are carrying a 4-ft by 8-ft panel in the horizontal position shown in E6.6.27. The panel weight is 100 lb. Define the panel as the system and draw a free-body diagram of the panel.



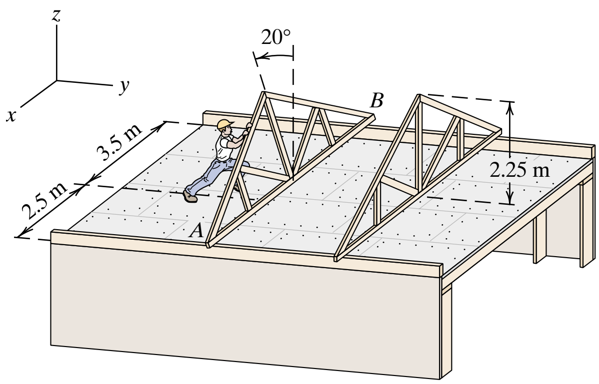

E6.6.27

**6.6.28.** A bracket is bolted to the shaft at $O$. Cables load the bracket, as shown in E6.6.28. Define the bracket as the system and draw a free-body diagram.




E6.6.28

**6.6.29.** A carpenter is slowly pushing the 90-kg roof truss into place. In the current position shown in **E6.6.29** it is oriented 20° from the vertical. The mass center of the symmetric triangular truss is located up one-third of its 2.25-m dimension from its base. Define the truss of the system and draw its free-body diagram.



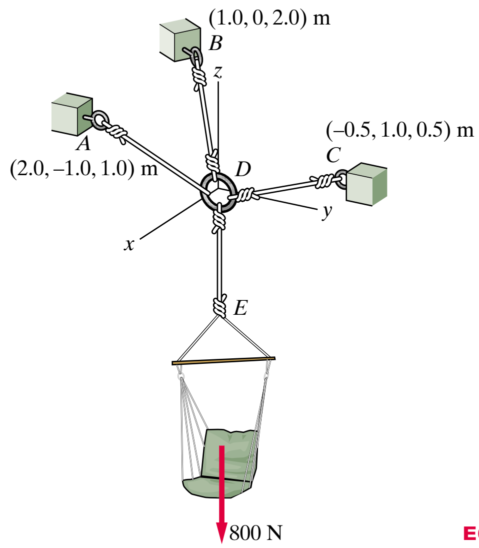

E6.6.29

**6.6.30.** A hanging chair is suspended, as shown in **E6.6.30**. A person weighing 800 N is sitting in the chair (but is not shown). Define

**a.** the ring at $D$ as the system and draw its free-body diagram

**b.** the chair (including the knot at $E$) as the system and draw its free-body diagram

**c.** the eyelet fastener at $A$ as the system and draw its free-body diagram




E6.6.30

## 6.7 JUST THE FACTS

In this chapter we looked at free-body diagrams—what they are and how to create them. To create a free-body diagram:

1. Study the physical situation. Classify the system as planar or nonplanar. A **planar system** is one in which all the forces acting on the system lie in the same plane and all moments are about an axis perpendicular to that plane or there is a **plane of symmetry**. Planar systems are also referred to as **two-dimensional systems**. If a system is not planar, it is **nonplanar** and is also referred to as **three-dimensional**.

2. Define (either by imagining or actually drawing) a boundary that isolates the system from its surroundings, then **draw the system** that is within the boundary. **Establish a coordinate system**. Planar systems typically require a single view drawing, whereas nonplanar systems may require multiple views or an isometric drawing. **State any assumptions** you make.

3. Identify cross-boundary forces (e.g., **gravity**) acting on the system and draw them at appropriate centers of gravity. Include a variable label and the force magnitude (if known). Continue to state any assumptions you make.

4. Identify all **known loads** acting at the boundary and add these to the drawing, placing each known load at its point (or surface area) or application; identify each load on the drawing with a variable label and magnitude.

5. Identify the loads associated with each **support**, both those loads that act at discrete points and those that consist of distributed forces. The loads associated with various planar supports (e.g., normal contact, links, cable) are presented in **Table 6.1**, and those with various nonplanar supports (e.g., hinges, journal bearings) are presented in **Table 6.2**.

Irrespective of whether a system is planar or nonplanar, if a particular support is not described in **Table 6.1** (planar systems) or **Table 6.2** (nonplanar systems), the general rule that describes a boundary's restriction of motion can be used to identify the loads at the support. This rule states:

*If a support prevents the translation of the system in a given direction, then a force acts on the system at the location of the support in the opposite direction. Furthermore, if rotation is prevented, a moment opposite the rotation acts on the system at the location of the support.*

6. Identify **fluid boundaries**. Add the loads at these boundaries to the drawing, showing them either as distributed or discrete point loads. Add variable labels.

You now have a **free-body diagram** of a system, as well as a list of the assumptions made in creating it. The diagram consists of a depiction of the system and the external loads acting on the system. The loads are represented in the diagram as vectors and with variable labels, and magnitudes (if known) are indicated. In the next chapter we consider how to use a free-body diagram in conjunction with Newton's first law to consider equilibrium of the system.

# SYSTEM ANALYSIS (SA) EXERCISES

## SA6.1 Check on the Design of a Chair

To increase the seating capacity during basketball games, collapsible and portable floors on rollers have been installed, as shown in **Figure SA6.1.1**. The chairs that go with this portable flooring system are placed onto the floors but are not connected to the floor. **Figure SA6.1.2** shows the dimensions of one of these chairs.

*Situation:* The basketball game of Wolfpack against UNC Chapel Hill is underway. As Sierra, an engineering student taking Statics, is cheering her team, she notices a woman sitting on the front edge of one of the chairs described above. Because of the small size of the hinge that holds the chair's seat, Sierra becomes concerned for the safety of the woman.

**Figure SA6.1.1** Mobile floor and chairs in the Reynolds Coliseum

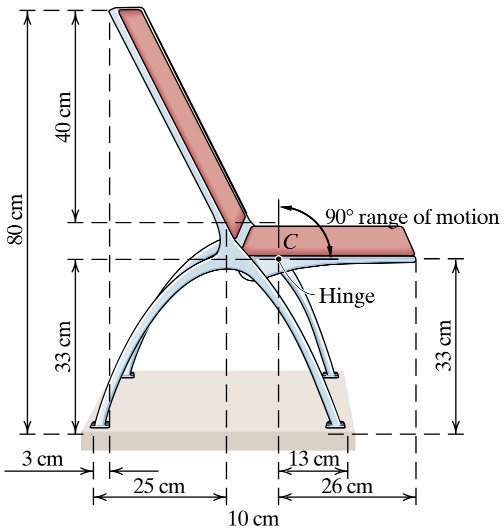



**Figure SA6.1.2** Dimensions of the chair

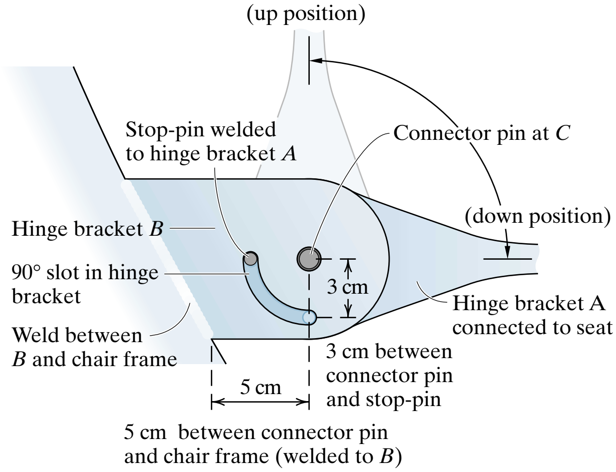



**Figure SA6.1.3** Basic design of hinge for seat

Imagine that you are in Sierra's place and re-create what goes through her mind as she has a sudden flashback to moments and the concept of free-body diagrams covered in her Statics class.

(a) Assuming that the mass of the woman is 61 kg, what is the maximum moment that the slotted hinge has to bear? **Figure SA6.1.3** may be helpful.

(b) As indicated in **Figure SA6.1.3**, the entire moment created by the woman has to be held by the connector pins and stop-pins. Consider the chair with the dimensions in **Figures SA6.1.2**. Draw free-body diagrams of the: (1) seat with hinge bracket *A* and (2) hinge bracket *B* with 90° slot. The dimensions in **Figure SA6.1.4** will be useful. Remember that the 90° slot is milled into bracket *B* while the stop-pin is welded to bracket *A* at a distance of 3 cm from *C*, the center of rotation at the connector pin.

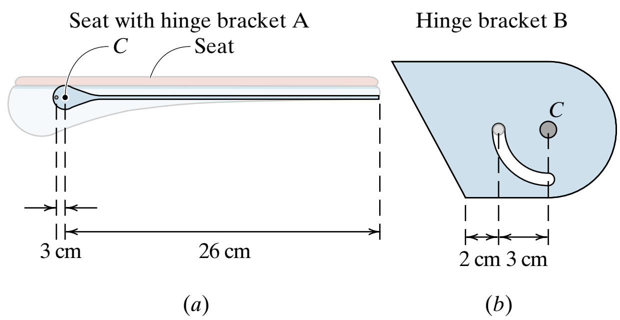



**Figure SA6.1.4** Dimensions associated with (*a*) seat with hinge bracket *A*; (*b*) hinge bracket *B* with 90° slot

(c) Based on the free-body diagram of the seat with hinge bracket *A* created in (b), write expressions for the equivalent load (consisting of an expression for the equivalent force and an expression for the equivalent moment) acting at a moment center at *C*. If the magnitude of the expression for equivalent moment is zero, what force must the stop-pin apply to the seat plate?

(d) **Figure SA6.1.5** provides a plot that shows the maximum allowable force that the stop-pin can safely hold. What size pin is needed to ensure that the woman sitting on the chair is safe? (*Remember:* Each chair has two hinges and stop-pins.)

(e) Assume that the maximum mass of a person for which each chair should be designed is 100 kg. What pin size would you recommend?

(f) Finally, it is very likely that in the process of sitting down, a person might actually "fall" into the chair. Since you did not cover the dynamic effect yet, let's assume you should triple the static force to account for the person's deceleration. What would your final recommendation be to the seat manufacturer regarding the pin size? (*Hint:* You may have to extend the plot by identifying the function that underlies the curve, which depends on the cross-sectional area of the pin and a fixed force per cm$^2$.)

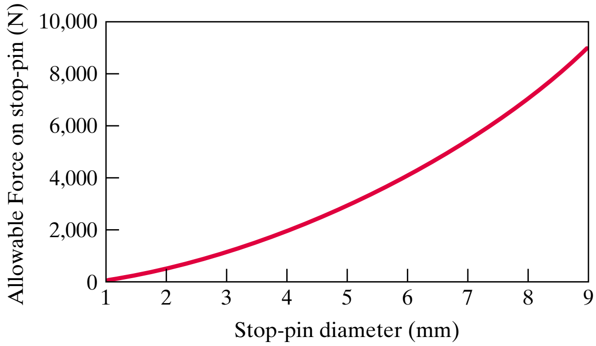



**Figure SA6.1.5** Shear strength versus pin diameter

## SA6.2 Follow the Path of the Gravitational Force

As you are leaving the coliseum after the game, you recognize still another mechanical "beauty" in a corner—a large trash cart ready for action. Here is a picture of it and its dimensions (**Figure SA6.2.1**).

We can safely assume that the cart would be able to carry a total of 150 kg.

### *Part I: Lifting a load*

(a) *Situation 1:* Draw a free-body diagram for the situation in which the janitor is just starting to dump a full cart by moving the handle on the back of the cart upward. The center of gravity for the full load distributed is shown in **Figure SA6.2.2**. If the equivalent moment about a moment center at *C* (the contact point between the front wheels and the ground) is zero, what is the magnitude of the force that the janitor must apply to the bin when the rear wheels just lift off the ground? State any assumptions you make.

**Figure SA6.2.1** Trash cart

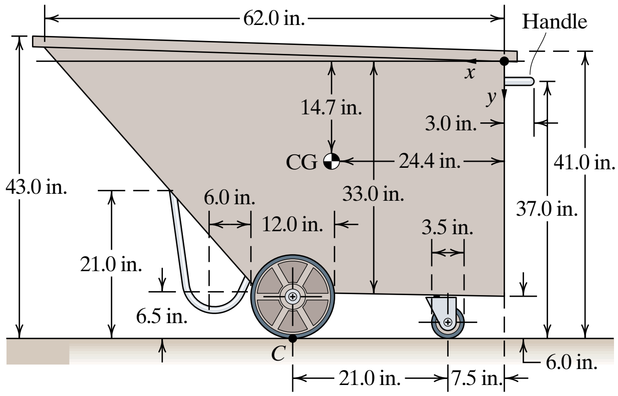



**Figure SA6.2.2** Dimensions of trash cart load distributions

(b) *Situation 2:* Instead of the load shown in **Figure SA6.2.1**, the cart is loaded with several heavy concrete pieces with a total mass of 150 kg that are placed close to the handle at position *A*; see **Figure SA6.2.3*a***. Draw a free-body diagram for the situation in which the janitor is just starting to dump the cart by moving the handle on the back of the cart upward. If the equivalent moment about a moment center at *C* is zero, what is the magnitude of the force that the janitor must apply to the bin when the rear wheels just lift off the ground? State any assumptions you make.

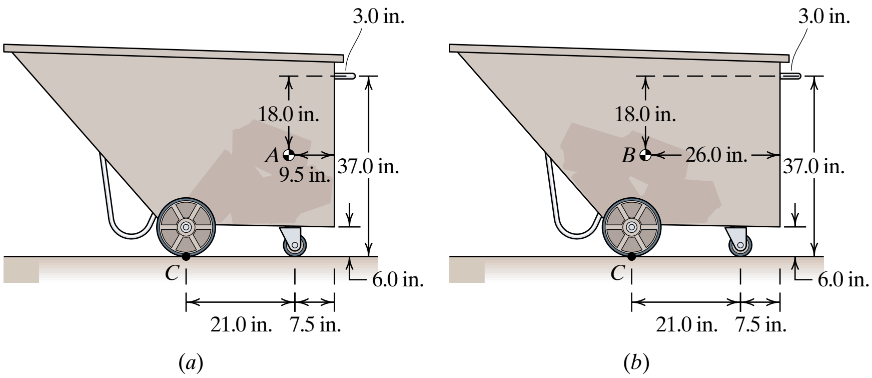



**Figure SA6.2.3** The cart is carrying 150 kg mass at (*a*) Position *A*; (*b*) Position *B*

(c) *Situation 3:* If the heavy concrete pieces with a total mass of 150 kg are placed at *B* (**Figure SA6.2.3*b***), what force must the janitor apply to just lift the rear wheels off the ground? Should the janitor worry more about hurting his or her back in Situation 1, 2, or 3, and why?

(d) Will the magnitude of the force that the janitor is required to apply to the cart to move it from Position 1 to Position 2 (see **Figure SA6.2.4**) decrease, increase, or remain the same as the cart goes from Position 1 to Position 2? Include the rationale for your answer (no calculations are required).

***Part II: How the load is transferred to the ground***

You notice that the cart has large front wheels and a sturdy main axle that connects the wheels to one another. The main axle is attached to a trash bin as shown in **Figure SA6.2.5**. Because the axle is attached to each of the front wheels with a bearing, the wheels rotate and the axle does not. Let's consider how the weight of the trash contained in the bin is transferred to the ground. To do this we will create a series of free-body diagrams.

Consider the following cross-sectional view of the trash bin in **Figure SA6.2.6** with various components labeled.

(a) Draw a free-body diagram of the system defined by Boundary 1 (the cart and the trash it is holding).

(b) Draw a free-body diagram of the system defined by Boundary 2 (the trash).

(c) Draw a free-body diagram of the main axle and wheel assembly.

(d) Now we are ready to separate the main axle from the wheels by pushing the shaft out of the bearing wheel hub. **Figure SA6.2.7** shows details of the axle–wheel

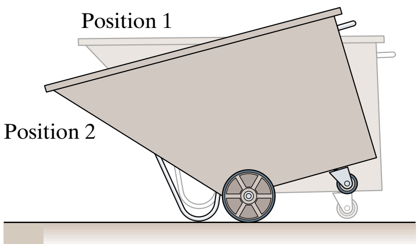



**Figure SA6.2.4** From Position 1 to Position 2

**Figure SA6.2.5** Cart suspension system

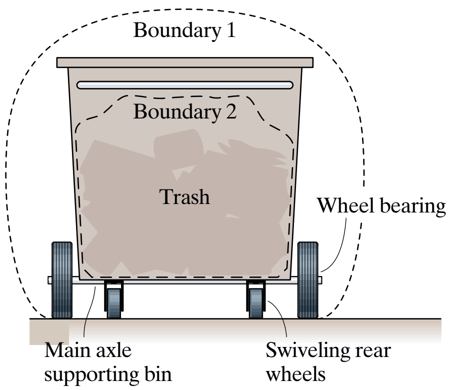



**Figure SA6.2.6** Cross section through cart

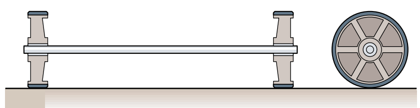


**Figure SA6.2.7** Axle and wheels

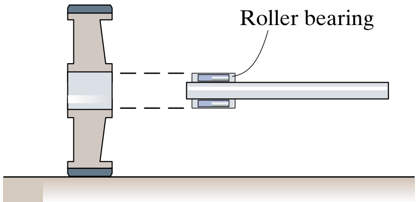



**Figure SA6.2.8** Details of axle–wheel connection

connection and the bearings that connect the axle and the wheels. Draw free-body diagrams for the wheels and the axle after they are separated from one another. **Figure SA6.2.8** may be useful in visualizing this.

(e) Based on the free-body diagrams you created in (a)–(d), which of the schematics shown in **Figure SA6.2.9** most accurately depicts how the weight of the trash is transferred to the ground?

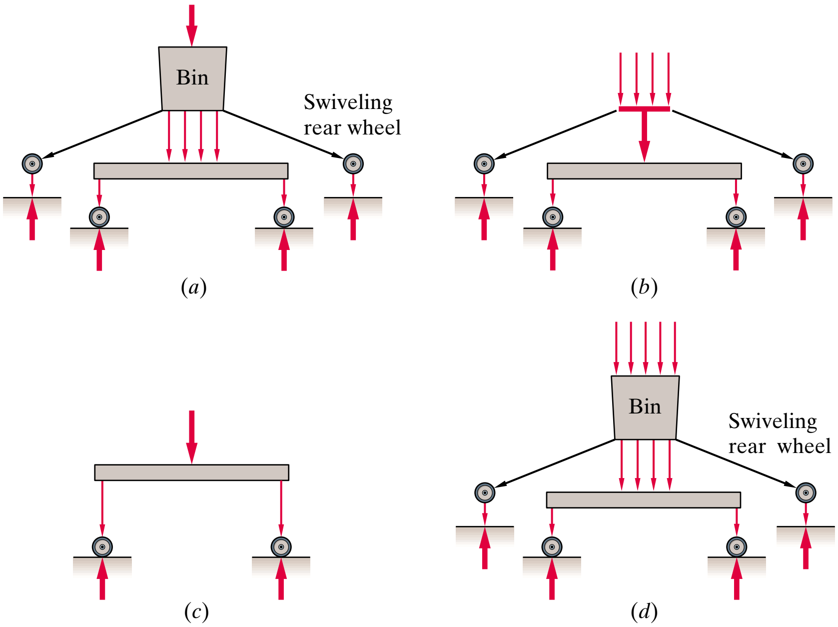



**Figure SA6.2.9** Load path schematics

## SA6.3 Perform the Experiment Described Below, then Follow the Steps to Create a Free-Body Diagram of the Situation.

*Materials needed:* One wire clothes hanger, rubber band, paper clip, a weight (a candy bar is suggested).

*Experiment:* Configure the hanger, rubber band, paper clip, and weight as shown in **Figure SA6.3.1**. Using both hands, keep the hanger level. Notice how your hands push or pull on the wire.

(a) Consider the hanger to be your system. **Draw** it. Add a coordinate system such that the $y$ axis is aligned with gravity.

(b) **List** (in words) the forces acting on the hanger when in the configuration shown.

(c) **Draw** each of the forces listed in (b) as a vector on the drawing created in (a). Clearly mark the points of application of each force and add variable labels. If the magnitudes of any of the forces are known, include this information on the drawing. You have now created a free-body diagram of the hanger. Make sure to **list any assumptions** you made and any uncertainties you have.

(d) Study your free-body diagram and **identify any couples** (describe in words).

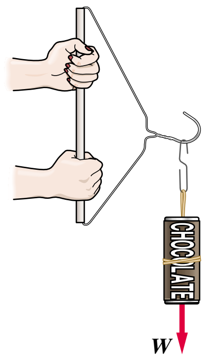



**Figure SA6.3.1** Configure hanger and candy bar as shown

## SA6.4 Perform the Experiment Described Below, then Follow the Steps to Create a Free-Body Diagram of the Situation.

*Materials needed:* Two wire clothes hangers, a friend.

*Experiment:* Hook the two clothes hangers together, as shown in Figure SA6.4.1. Have your friend push and pull on her hanger, as shown. At the same time, use your hands (in the positions shown) to keep the hanger level. Notice how your hands and those of your friend push and/or pull on the wire.

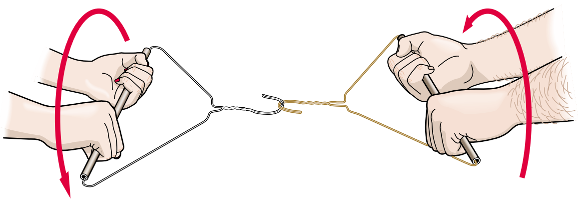


**Figure SA6.4.1** Configure two hangers as shown

(a) Consider the two hangers to be your system. **Draw** the structure. Add a coordinate system such that the $y$ axis is aligned with gravity.

(b) **List** (in words) the forces acting on the system.

(c) **Draw** each of the forces listed in (b) as a vector on the drawing created in (a). Clearly mark the points of application of each force and add variable labels. If the magnitudes of any of the forces are known, include this information on the drawing. You have now created a free-body diagram of the system. Make sure to **list any assumptions** you made and any uncertainties you have.

(d) Study your free-body diagram and **identify any couples** (describe in words).

(e) Repeat (a)–(d) if the system is defined as one of the hangers.

## SA6.5 The Bicycle Revisited

Consider the bottom bracket assembly of a bicycle, as shown in Figure SA6.5.1. The assembly consists of an axle, chainring, left and right cranks, and left and right pedals. The axle is held in the frame by two sets of ball bearings. For the position shown, draw a free-body diagram of the bottom bracket assembly.

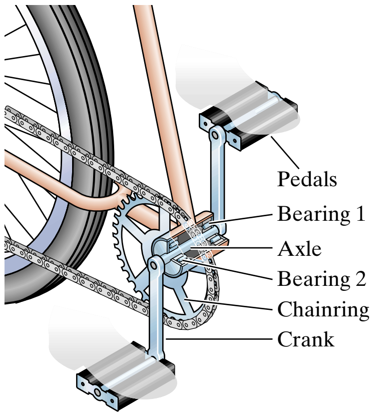



**Figure SA6.5.1** Bottom bracket assembly