using brain science to improve math instruction
+ practical classroom activities for grades 4 and 5

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Give babies a set of blocks and they will build and order them, fascinated by the ways the edges line up. Children will look up at the sky and be delighted by the V formations in which birds fly. We want to see patterns in the world and to understand the rhythms of the universe.

But the joy and fascination young children experience with mathematics are quickly replaced by dread and dislike when they start school mathematics and are introduced to a dry set of methods they think they just have to accept and remember. The inquisitiveness of our children’s early years fades away and is replaced by a strong belief that math is all about following instructions and rules. There is a clear gap between what research has shown to work in teaching math and what happens in schools and at home.

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Understanding How the Brain Processes Mathematics Learning

Mathematics is a conceptual subject and it is important for students to be thinking slowly, deeply, and conceptually about mathematical ideas, not racing through methods that they try to memorize. One reason that students need to think conceptually has to do with the ways the brain processes mathematics. When we learn new mathematical ideas, they take up a large space in our brain as the brain works out where they fit and what they connect with. But with time, as we move on with our understanding, the knowledge becomes compressed in the brain, taking up a very small space. For first graders, the idea of addition takes up a large space in their brains as they think about how it works and what it means, but for adults the idea of addition is compressed and it takes up a small space.

The best and most important start we can give our students is to encourage them to play with numbers and shapes, thinking about what patterns and ideas they can see.

You will probably agree with me that not many students think of mathematics as a “real joy,” and part of the reason is that they are not compressing mathematical ideas in their brain. This is because the brain only compresses concepts, not methods. So, if students are thinking that mathematics is a set of methods to memorize, they are on the wrong pathway, and it is critical that we change that. It is very important that students think deeply and conceptually about ideas.

MINDSET MATTERS

Research has shown definitively the importance of a growth mindset—the belief that intelligence grows and the more you learn, the smarter you get. We need students to have growth beliefs about themselves and accompany them with growth beliefs about the nature of mathematics and their role in relation to it. Children need to see math as a conceptual, growth subject that they should think about and make sense of. When students see math as a series of short questions, they cannot see the role for their own inner growth and learning. They think that math is a fixed set of methods that either they get or they don’t. When students see math as a broad landscape of unexplored puzzles in which they can wander around, asking questions and thinking about relationships, they understand that their role is thinking, sense making, and growing. When students see mathematics as a set of ideas and relationships and their role as one of thinking about the ideas, and making sense of them, they have a mathematical mindset.
Building Brain Connections: Introducing Big Ideas

Mathematics is a set of connected ideas that need to be understood. When students understand the “big ideas” in mathematics, the methods and rules fall into place. One of the reasons any set of curriculum standards is flawed is that standards take the beautiful subject of mathematics and its many connections and divide it into small pieces that make the connections disappear. Instead of starting with the small pieces, start with the big ideas and important connections intrinsically linked.

The figure below shows some of the connections between big ideas in fifth-grade math, and you may be able to see others. It is very important to share with students that mathematics is a subject of connections and to highlight the connections as students work.

SEE THE BIG IDEAS To see maps of big ideas for all grades K through 8, read our paper “What Is Mathematical Beauty?” available on youcubed.org.

![Big Ideas and Connections (Grade 5)](image)
Visualize, Play, Investigate

Visualizing, playing, and investigating pave the way for open student thinking, for powerful brain connections, for engagement, and for deep understanding.

VISUALIZE

Brain scientists now know that when we work on mathematics, even when we perform a bare number calculation, five areas of the brain are involved. Two of the five brain pathways—the dorsal and ventral pathways—are visual. Now brain scientists know that our brains “see” fingers when we calculate, and knowing fingers well—what they call finger perception—is critical for the development of an understanding of numbers.

Our brain wants to think visually about mathematics, yet few curriculum materials engage students in visual thinking. Some mathematics books show pictures, but they rarely ever invite students to do their own visualizing and drawing.

In addition to the brain development that occurs when students think visually, we have found that visual activities are really engaging for students. Connecting visual and numerical representations encourages important brain connections as well as deep student engagement.

PLAY

Opening mathematics involves inviting students to see ideas differently, explore with ideas, and ask their own questions. Students can gain access to the same mathematical ideas and methods through creativity and exploration that they can by being taught methods that they practice. Albert Einstein famously once said that “play is the highest form of research.” This is because play is an opportunity for ideas to be used and developed in the service of something enjoyable. Play activities can invite students to work with each other, building understanding together.

INVESTIGATE

A crucial finding from neuroscience is the importance of students struggling and making mistakes—these are the times when brains grow the most. In one of my meetings with a leading neuroscientist, he stated it very clearly: if students are not struggling, they are not learning. We want to put students into situations where they feel that work is hard, but within their reach.

Do not worry if students ask questions that you don’t know the answer to; that is a good thing. It is good to say to your students, “That is a great question that we can all think about” or “I have never thought about that idea; let’s investigate it together.” It is even good to make mistakes in front of students, as it shows them that mistakes are an important part of mathematical work.
Classroom Activities

The activities that follow invite students to engage in the mathematical acts that are listed in the imperative Common Core practice standards. An essential role of the teacher is to give students time to think deeply and conceptually. Each activity is designed to take at least a class period, but it could go longer, especially if students ask deep questions or start an investigation into a cool idea.

About the Authors

**DR. JO BOALER** is a professor of mathematics education at Stanford University and co-founder and faculty director of youcubed. She serves as an advisor to several Silicon Valley companies and is a White House presenter on girls and STEM (Science, Technology, Engineering, and Math). The author of seven books, including *Mathematical Mindsets*, and numerous research articles, she is a regular contributor to news and radio in the United States and England.

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Learn more about science-based math instruction and how it can engage students and improve learning with these additional resources from Dr. Jo Boaler.

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Seeing Patterns inside Numbers

Numbers make up our world, and they are used throughout our lives, whatever our age, job, or level of interest. But many people develop a narrow relationship with numbers, seeing them as something to use in calculations, rather than as a fascinating set of ideas that can enrich their world. Our first big idea invites students to become captivated by numbers and to get to know numbers deeply. What is enchanting about numbers is that they are all made up of different arrangements, have different factors, can be seen differently, and have their own intricate system to be explored.

When we first came across Brent Vorgey’s number visual (Figure 1.1), we were enthralled, as we immediately saw the creativity, beauty, and insights that the visual representations revealed.

In our Visualize activity, we invite students to explore this depiction of numbers and to see what patterns are uncovered by the visual representations. We invite them to see what primes look like and to see the different factors inside numbers. We invite them to investigate patterns among the numbers, seeing what their positioning on the diagram reveals. Also, we invite students to see numbers visually and to develop a realization that numbers contain all sorts of information that make them different from each other, special, and interesting.
In the Play activity, we extend students’ time with the number visuals in a more playful setting. Students play a game with the number visual page as a game board, and move between visual and numerical representations. This, as with the other two tasks in this big idea, encourages important connections between different areas of the brain.

In the Investigate activity, we invite students to think carefully about number flexibility. One of the ways that numbers are different from one another is the number of factors they have and the degree of flexibility they give us when using them. For example, 24 is a very flexible number, as it can be broken up in all sorts of different ways. This makes it a useful number for packaging, for designing, and for measuring time. In this activity, we invite students to give value to different numbers according to their flexibility, helping them develop an appreciation for these numbers. The activity also invites students to make equal groups and gives teachers an opportunity to discuss whether students are thinking additively or multiplicatively and what those differences mean.

All three activities give students an opportunity to develop new insights into the numbers that they will use for the rest of their lives.
Brain science tells us that when students are engaging with numbers as symbols, such as the numeral 4, and with numbers as visuals, as shown in Figure 1.2, they are connecting between different areas of the brain, and such connections are critical for mathematics learning and achievement. The activities in this big idea will invite a lot of brain connecting, with students developing pathways that will help them as they go forward in their mathematical careers.

Jo Boaler

Figure 1.2
Visualizing Numbers

Snapshot

In this activity, students work with the number visual page to explore the patterns that they can see inside of numbers. In this activity, we open the door to understanding factors, multiples, and primes, as well as other number patterns.

Connection to CCSS

4.OA.4

Agenda

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<th>Activity</th>
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<tbody>
<tr>
<td>Launch</td>
<td>5 min</td>
<td>Generate multiple ways that numbers can be represented and introduce the number visual page.</td>
<td>Number visual page reproduced for students and one to display</td>
</tr>
<tr>
<td>Explore</td>
<td>20+min</td>
<td>Students look for patterns inside the number visual page and color-code them.</td>
<td>Colors for students (colored pencils, markers, or pens)</td>
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<tr>
<td>Discuss</td>
<td>10 min</td>
<td>Discuss the different patterns students found and how their color coding makes the patterns visible.</td>
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| Explore    | 20+min | Student look for patterns shared across the different numbers. Student cut their papers so that they can group or arrange them to show shared patterns. | • Number visual page, one per group  
• Colors  
• Scissors  
• Optional: posters or large paper |
| Discuss    | 15 min | Students share the different ways they have grouped the numbers and discuss the shared patterns they have found. |                                                                            |
To the Teacher

The length of this lesson depends largely on how long students would like to explore patterns. We’ve found that some students want to explore patterns in depth, and they should be given time to do that. Follow your students’ lead and interest. This activity can easily be spread across multiple days.
Activity

Launch
Numbers can be represented in lots of different ways. For example, 6 can be written as a numeral, but 6 can also be shown in other ways, as in Figure 1.3.

![Figure 1.3](image)

When you launch this activity, you may want to share some of these ways with students or have them generate ways that numbers are represented in their world. Give each student a copy of the number visual page and ask them to notice what numbers are shown. Have them record the actual number value by each visual. There are patterns all over this page. Ask students, what patterns do you notice?

Explore
Ask students to investigate the patterns in the number visual page.

- What patterns do you see?

  Provide students with colors (colored pencils, pens, or markers).

- How can you use color to show the patterns within these numbers?

  Students might notice equal-size groups within some numbers. For instance, 4, 8, 12, 16, 20, 24, 28, and 32 all have square clusters of 4 inside them. Students might notice that some numbers have no groups inside them; numbers like 11, 13, 17, and 19 are circles. Students might notice how some numbers grow outward from a central pattern. For instance, 6 has a group of 3 in the center, and each corner has been added onto with one dot. Students might also notice multiple numbers inside one
number. For instance, 18 has three groups of 6, but also has 9 pairs. Some of these patterns are shown in Figure 1.4 as an example of how students might use color to highlight different patterns they see.

Discuss

Ask students to share the different ways they have color-coded their numbers to reveal patterns. What do different ways of coloring show? You may want to focus discussion on a single number to compare the different patterns inside. For instance, you could look at the different patterns inside of 12 that different ways of coloring make clear.

What do different numbers have in common? If you focused on a particular number, you might ask, What other numbers are like this number? How are they alike? If students notice the clusters within each number, give them the term factor to describe these clusters. For instance, if students see the three clusters of 4 inside 12, you can say that 4 is a factor of 12, or that 12 has 4 as a factor.
**Explore**

Now ask students to return to their color-coded number visuals and look for patterns that different numbers share. Provide students with a new number visual page and scissors to cut this new page apart so that they can group, sort, or web numbers by common features and color-code those features. Students should work with a partner or in a small group to find patterns.

- What patterns do different numbers share?
- How can you group or arrange the numbers to show what numbers have in common?

You might have students glue or tape their arrangements onto a poster to make sharing easier. This way, they could label the groups or the relationships between the numbers.

**Discuss**

Ask students to share the patterns they notice between numbers. You may want to have students hang their posters and do a gallery walk, or ask each group to share what they have found. In either case, discuss as a class the following questions:

- What patterns do different numbers share?
- What are you wondering now about these numbers?
- What are you wondering about the numbers we haven’t looked at yet?

If students notice the clusters that different number share, be sure to tell them that we say that they share a factor. If students notice the circles and the lack of clusters within, be sure to probe what this means. You can name these numbers, where no equal groups are possible, as *prime*.

**Look-Fors**

- **Do students notice that numbers are inside of other numbers?** For instance, does anyone see three clusters of 4 inside of 12? One goal of this activity is for students to see the building blocks of numbers.
- **Do students notice that some numbers are made only of individual dots?** This is the beginning of noticing primes.
- **Are students thinking multiplicatively or additively?** Although numbers can be broken apart through addition, we want to push students to notice patterns of equal groups. This is an interesting point of discussion.

- **Are students noticing that some numbers have similar building blocks?** This is the beginning of noticing common factors.

**Reflect**

What do you think is the most interesting number on this page? Why?
BIG IDEA 1: SEEING PATTERNS INSIDE NUMBERS

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Thinking in Cubes

The new brain science shows that five different pathways are involved when people think about mathematical ideas, and two of these are visual. When we make mathematics visual for students, we help them learn and hold ideas in powerful ways in their brains, as the introduction to this book explains. Similarly, we now know that movement really helps with mathematical ideas and is important for brain development. When students move with mathematics, it means that the mathematical ideas are held in the sensory-motor portions of the brain, which helps students understand the ideas powerfully. We see evidence of people holding mathematical ideas in these parts of their brains when they gesture to illustrate an idea; when people talk about circles, for example, they often draw a circle in the air. This big idea gives students opportunities to touch and feel mathematical ideas, and that is meaningful to students of any age.

In our Visualize activity, students will build with cubes and develop connections between two- and three-dimensional representations of solids. They will be asked to think about the outside and inside of cubes, which is important geometric thinking. As they physically model and also draw, they will build significant brain connections.

In the Play activity, students will construct cities of cubes that match views that we give them, again using brain pathways that will develop mathematical thinking. Students also will build their own cities, which will be engaging and exciting for them, enhancing the learning potential of the activities. As students think visually and also bring in numerical thinking, their brains will develop pathways between the areas that are used for these different types of thinking.
In our Investigate activity, students will again have the opportunity to feel cubes and consider their size physically and with numbers, encouraging brain connections. They also get to work with some constraints that will guide their thinking and learning. Students will be asked to investigate the volume of rectangular solids by packing little boxes into larger boxes of their own design. Any time that students are asked to bring their own ideas into mathematics, such as when they make their own designs, they are working with agency, which will help them enjoy mathematics and also see it as an active subject that they should think deeply about. When students work with agency, their work is closer to that of a mathematician, and inviting students to combine their own ideas with formal mathematical ideas is a really worthwhile goal. The activities that make up this big idea provide plenty of opportunities for students to combine their own thinking with major mathematical ideas and principles.

Jo Boaler
**Solids, Inside and Out**

**Snapshot**

Students build connections between two- and three-dimensional representations of solids by using views of a rectangular solid to construct a model with cubes. Students investigate what the inside looks like and compare results.

**Connection to CCSS**

5.MD.3
5.MD.4

**Agenda**

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<tbody>
<tr>
<td><strong>Launch</strong></td>
<td>5 min</td>
<td>Show students the two-dimensional views of a rectangular solid constructed out of 60 cubes. Challenge students to build this solid.</td>
<td>Rectangular Solids Sheet, to display for the class</td>
</tr>
<tr>
<td><strong>Explore</strong></td>
<td>30 min</td>
<td>Partnerships try to build a rectangular solid from 60 cubes so that it matches the views provided. Students then consider what the inside looks like and figure out how to construct and draw a model of the cubes that cannot be seen.</td>
<td>Rectangular Solids Sheets, one per partnership</td>
</tr>
</tbody>
</table>
| **Discuss** | 20 min | Students compare their results and discuss how they used the views to construct the solid. Students discuss the differences between their models of the inside. | }
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| Extend   | 30–60 min | Partnerships construct their own rectangular solid puzzles and swap with other groups to solve. | • Drawing Solids Sheet, at least one per partnership  
• Cubes  
• Colors  
• Baskets, trays, or bags for students’ puzzles |

**To the Teacher**

Students often struggle when moving between two-dimensional representations of solids objects, like the ones shown in Figure 1.1, and three-dimensional representations. Two-dimensional drawings of solids force us to imagine the parts we cannot see, and students need experiences with mentally rotating and imagining these invisible parts. Similarly, we often encounter multiple views of three-dimensional objects aimed at helping us see what one view does not show. These offer a different challenge: constructing the three-dimensional whole from parts. In this visual activity, students are asked to move repeatedly between two-dimensional and three-dimensional representations to build connections and a deeper sense of what it means to be solid.
The extension activity pushes these connections even further by asking students to create and draw their own solids and then try to build solids designed by others. This extension could easily take another full day or more, depending on how excited students are. They may want to construct multiple puzzles and solve many as well. We encourage you to follow students’ interests and allow as much time as students want for constructing puzzles. The repeated engagement with building, drawing, and mentally manipulating the solids will help develop important ways of thinking about solids and volume for future work.
**Activity**

**Launch**

Launch this lesson by telling students that we are going to be exploring ways to fill space. In the past, they have likely spent a lot of time thinking about how to cover two-dimensional shapes with squares to find the area, but now we are going to think about how to fill three-dimensional solids with cubes. Show students the views provided on the Rectangular Solid Sheet shown in Figure 1.1. Tell students that all of these images are of the same rectangular solid, and each image is called a *view*. Tell students that the solid is made of 60 blocks: 15 are red, 15 are green, 15 are yellow, and 15 are blue. Their challenge today is to build this solid. Partners will each get a copy of the views and enough blocks to construct the solid. If you are not able to copy the Rectangular Solid Sheet in color for students, we suggest you display the images in color using a projector and provide students with the blank version, which they can color in themselves. Coloring in the individual cubes may also help students attend their arrangement on each face and support students with building the solid.

**Explore**

Have students work in partnerships. Provide each partnership with 15 red, 15 green, 15 yellow, and 15 blue snap or multilink cubes, and a copy of the Rectangular Solid Sheet. Partnerships use the views to try to construct the rectangular solid so that it matches the views.

After students have built the solid and both partners agree that it matches the views and uses all 60 cubes, then challenge students to figure out what cubes are on the inside. Ask, What cubes are we not able see? Can you make a model of what is inside your solid? Provide partners with additional cubes and a copy of the Drawing Solids Sheet. Ask them to draw an image of what is inside their solid once they have built their model.

**Discuss**

Bring students together with their models of the rectangular solid, models of the inside, and drawings of the inside. Pair partnerships up to share their two models and drawings, as in a turn and talk. Then bring all students together to discuss the following questions:
- How did you figure out how to build your solid? What did you have to think about?
- How did you use the images to help you?
- What was hard about building the rectangular solid so that it matched the views you were given?
- What did you notice when you compared your models to those others made?

Then discuss the models of the inside of the solid:

- What was inside your solid? How did you figure it out?
- How are the insides of our solids similar? Different? Why?

**Extend**

Invite students to create their own rectangular solid puzzle for another partnership to solve. Provide students with additional copies of the Drawing Solids Sheet, colors, and a container for their puzzle. Students should first use any number of their cubes to build a rectangular solid. They then use their solid to construct multiple views of it on a copy of the Drawing Solids Sheet. Push students to think carefully about which views they will show so that it is challenging but possible to build the solid with the views they provide. Once they are confident that their views are accurate, ask students to deconstruct their solid and place their views and cubes into a container (basket, tray, or bag). You may want to ask students to name their puzzle or put their names on it. Then students can trade puzzles with another group and try to build their solid. After students have spent some time exploring one another’s puzzles, you may want to bring them back together to discuss questions like these:

- What makes a good puzzle?
- What made solving a rectangular solid puzzle easier or harder?
- What strategies for solving these puzzles have you and your partner come up with?

**Look-Fors**

- **Are students creating a solid?** Students may be tempted to construct only what they see in the images and presume that there is nothing inside the solid at all. Challenge students who have leftover cubes by asking: Is your
solid constructed with 60 cubes? Why not? Does it match the views you were given? How can you make your solid meet all of the constraints of the problem?

- **How are students mentally rotating the images?** Students may struggle to mentally manipulate the images provided to figure out how the pieces fit together. Often students don’t realize that rotating the paper can help them see the images differently. Some students may be overwhelmed by the images and not know how to get started. Consider prompting students by asking: Is there one image that makes sense to you? How could you get started with that one image and then see how it fits with the others? What could you do with your paper to help you see how the pieces fit together?

- **How are students interpreting the images?** Drawing a three-dimensional object on two-dimensional paper is a challenge. If students have not yet had experience with isometric dot paper (see appendix), you may want to ask them to try drawing a single cube first, just to explore how the dot paper works. You might also ask them to think about the views they were provided and how they could use those as models.

**Reflect**

What surprised you about building rectangular solids from cubes and views?
Rectangular Solids Sheet

Use 15 yellow, 15 red, 15 blue, and 15 green cubes to build the rectangular solid.