Applications of trigonometry

9.1 Kick off with CAS
9.2 Trigonometric ratios
9.3 Applications of trigonometric ratios
9.4 The sine rule
9.5 The cosine rule
9.6 Area of triangles
9.7 Review
Please refer to the Resources tab in the Prelims section of your eBookPLUS for a comprehensive step-by-step guide on how to use your CAS technology.
9.2 Trigonometric ratios

A ratio of the lengths of two sides of a right-angled triangle is called a trigonometric ratio. The three most common trigonometric ratios are sine, cosine and tangent. They are abbreviated as sin, cos and tan respectively. Trigonometric ratios are used to find the unknown length or acute angle size in right-angled triangles.

It is important to identify and label the features given in a right-angled triangle. The labelling convention of a right-angled triangle is as follows:

The longest side of a right-angled triangle is always called the hypotenuse and is opposite the right-angle. The other two sides are named in relation to the reference angle, $\theta$. The opposite side is opposite the reference angle, and the adjacent side is next to the reference angle.

Angles

Angles are usually measured in degrees ($^\circ$). To be more precise, we can measure and write angles either in degrees and minutes form or in decimal form.

60 minutes is equal to 1 degree: $60^\prime = 1^\circ$. So a minute is a fraction of a degree and can be written as $1^\prime = \frac{1}{60}^\circ$.

Degrees and minutes form, for example $30^\circ20^\prime$, can be converted to decimal form by keeping the degrees component as is and dividing the minutes component by 60. Both components can then be added together.

$$30^\circ20^\prime = 30 + \frac{20}{60}^\circ = 30.33^\circ$$

Decimal form, for example $24.7^\circ$, can be converted to degrees and minutes form by first separating the whole and decimal portions, multiplying the decimal portion by 60, and then adding the portions together.

$$24.7^\circ = 24^\circ + (0.7 \times 60)^\prime = 24^\circ + 42^\prime = 24^\circ42^\prime$$

When using CAS for trigonometric calculations, make sure that you are in degree mode.

The sine ratio

The sine ratio is used when we want to find an unknown value given two out of the three combinations: opposite, hypotenuse and reference angle.
The sine ratio of $\theta$ is written as $\sin(\theta)$ and is defined as follows:

$$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

or

$$\sin(\theta) = \frac{O}{H}$$

The inverse sine function is used to find the value of the unknown reference angle given the lengths of the hypotenuse and opposite.

$$\theta = \sin^{-1}\left(\frac{O}{H}\right)$$

**WORKED EXAMPLE 1**

Calculate the length of $x$ correct to 2 decimal places.

1. Label all the given information on the triangle.

2. Since we have been given the combination of opposite, hypotenuse and the reference angle $\theta$, we need to use the sine ratio. Substitute the given values into the ratio equation.

3. Rearrange the equation to make the unknown the subject and solve.

   Make sure your calculator is in degree mode.

   $$x = 10 \sin(59^\circ) = 8.57$$

   The opposite side length is 8.57 cm

**WORKED EXAMPLE 2**

Find the value of the unknown angle, $\theta$, expressed in decimal form.
The cosine ratio

The cosine ratio is used when we want to find an unknown value given two out of the three combinations: adjacent, hypotenuse and reference angle.

The cosine ratio of \( \theta \) is written as \( \cos(\theta) \) and is defined as follows:

\[
\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}} \quad \text{or} \quad \cos(\theta) = \frac{A}{H}
\]

The inverse cosine function is used to find the value of the unknown reference angle when given lengths of the hypotenuse and adjacent side.

\[
\theta = \cos^{-1}\left(\frac{A}{H}\right)
\]

WORKED EXAMPLE 3

Calculate the length of \( y \) correct to 2 decimal places.

THINK

1. Label all the given information on the triangle.

WRITE

\[
\theta = \cos^{-1}\left(\frac{A}{H}\right)
\]

\[
\theta = \cos^{-1}\left(\frac{5.3}{8.8}\right)
\]

\[
\theta = 37.03^\circ
\]

**THINK**

1. Label all the given information on the triangle.

WRITE

\[
\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}} \quad \text{or} \quad \cos(\theta) = \frac{A}{H}
\]

\[
\sin(\theta) = \frac{O}{H}
\]

\[
\theta = \sin^{-1}\left(\frac{5.3}{8.8}\right)
\]

\[
\theta = 37.03^\circ
\]
2 Convert the angle from degrees and minutes form to decimal form.

\[ \theta = 63^\circ + \left(\frac{15}{60}\right)' \]
\[ = 63.25^\circ \]

3 Since we have been given the combination of adjacent, hypotenuse and the reference angle \( \theta \), we need to use the cosine ratio. Substitute the given values into the ratio equation.

\[ \cos(\theta) = \frac{A}{H} \]
\[ \cos(63.25^\circ) = \frac{4.71}{y} \]

4 Rearrange the equation to make the unknown the subject and solve.

Make sure your calculator is in degree mode.

\[ y = \frac{4.71}{\cos(63.25^\circ)} \]
\[ = 10.46 \]

The length of the hypotenuse is 10.46 mm.

WORKED EXAMPLE 4 Find the value of the unknown angle, \( \theta \), expressed in degrees and minutes.

THINK

1 Label all the given information on the triangle.

WRITE

2 Since we have been given the combination of adjacent, hypotenuse and the reference angle \( \theta \), we need to use the cosine ratio. Substitute the given values into the ratio equation.

\[ \cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{A}{H} \]
\[ \cos(\theta) = \frac{3.3}{4.2} \]

3 To find angle \( \theta \), we need to use the inverse cosine function.

\[ \theta = \cos^{-1}\left(\frac{3.3}{4.2}\right) \]
\[ = 38.21^\circ \]

4 Convert the angle decimal form to degrees and minutes form.

\[ = 38^\circ + (0.21 \times 6000)' \]
\[ = 38^\circ 13' \]

The tangent ratio

The tangent ratio is used when we want to find an unknown value given two out of the three combinations: opposite, adjacent and reference angle.
The tangent ratio of \( \theta \) is written as \( \tan(\theta) \) and is defined as follows:

\[
\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}} \quad \text{or} \quad \tan(\theta) = \frac{O}{A}
\]

The inverse tangent function is used to find the value of the unknown reference angle given the lengths of the adjacent and opposite sides.

\[
\theta = \tan^{-1}\left(\frac{O}{A}\right)
\]

WORKED EXAMPLE 5

Calculate the length of \( x \) correct to 2 decimal places.

THINK

1. Label all the given information on the triangle.

2. Since we have been given the combination of opposite, adjacent and the reference angle \( \theta \), we need to use the tangent ratio. Substitute the given values into the ratio equation.

3. Rearrange the equation to make the unknown the subject and solve.
   Make sure your calculator is in degree mode.

WRITE

\[
\tan(58^\circ) = \frac{x}{9.4}
\]

\[
x = 9.4 \tan(58^\circ)
\]

\[
x = 15.04
\]

The adjacent side length is 15.04 cm.

WORKED EXAMPLE 6

Find the value of the unknown angle, \( \theta \), expressed in decimal form.
**THINK**

1. Label all the given information on the triangle.

2. Since we have been given the combination of opposite, adjacent and the reference angle $\theta$, we need to use the tangent ratio. Substitute the given values into the ratio equation.

3. To find the angle $\theta$, we need to use the inverse tangent function. Make sure your calculator is in degree mode.

**WRITE**

**The unit circle**

If we draw a circle of radius 1 in the Cartesian plane with its centre located at the origin, then we can locate the coordinates of any point on the circumference of the circle by using right-angled triangles.

In this diagram, the length of the hypotenuse is 1 and the coordinates of $A$ can be found using the trigonometric ratios.

$$\cos(\theta) = \frac{A}{H} \quad \sin(\theta) = \frac{O}{H}$$

$$= \frac{a}{1} \quad \text{and} \quad = \frac{b}{1}$$

$$= a \quad \text{and} \quad = b$$
Therefore the base length of the triangle, $a$, is equal to $\cos(\theta)$, and the height of the triangle, $b$, is equal to $\sin(\theta)$. This gives the coordinates of $A$ as $(\cos(\theta), \sin(\theta))$. For example, if we have a right-angled triangle with a reference angle of $30^\circ$ and a hypotenuse of length 1, then the base length of the triangle will be 0.87 and the height of the triangle will be 0.5, as shown in the following triangle.

Similarly, if we calculate the value of $\cos(30^\circ)$ and $\sin(30^\circ)$, we get 0.87 and 0.5 respectively.

We can actually extend this definition to any point $B$ on the unit circle as having the coordinates $(\cos(\theta), \sin(\theta))$, where $\theta$ is the angle measured from $OX$ to the hypotenuse of the triangle in an anticlockwise direction.
Extending sine and cosine to 180°

We can place any right-angled triangle with a hypotenuse of 1 in the unit circle so that one side of the triangle lies on the positive x-axis. The following diagram shows a triangle with base length 0.64, height 0.8 and reference angle 50°.

The coordinates of point \( B \) in this triangle are \( (0.64, 0.77) \) or \( (\cos(50°), \sin(50°)) \).

Now reflect the circle in the y-axis as shown in the following diagram.

We can see that the coordinates of point \( B' \) are \( (-0.64, 0.77) \) or \( (-\cos(50°), \sin(50°)) \).
We have previously determined the coordinates of any point $B$ on the circumference of the unit circle as $(\cos(\theta), \sin(\theta))$, where $\theta$ is the angle measured from $OX$ to the hypotenuse in an anticlockwise direction. In this instance the value of

$$\theta = 180^\circ - 50^\circ = 130^\circ.$$

Therefore, the coordinates of point $B'$ are $(\cos(130^\circ), \sin(130^\circ))$.

This discovery can be extended when we place any right-angled triangle with a hypotenuse of 1 inside the unit circle.

As previously determined, the point $B$ has coordinates $(\cos(\theta), \sin(\theta))$.

Reflecting this triangle in the $y$-axis gives:
So the coordinates of point \( B' \) are \((-\cos(\theta), \sin(\theta))\). We also know that the coordinates of \( B' \) are \((\cos(180 - \theta), \sin(180 - \theta))\) from the general rule about the coordinates of any point on the unit circle.

Equating the two coordinates for \( B' \) gives us the following equations:

\[-\cos(\theta) = \cos(180 - \theta)\]
\[\sin(\theta) = \sin(180 - \theta)\]

So, to calculate the values of the sine and cosine ratios for angles up to 180°, we can use:

\[\cos(\theta) = -\cos(180 - \theta)\]
\[\sin(\theta) = \sin(180 - \theta)\]

Remember that if two angles sum to 180°, then they are supplements of each other. So if we are calculating the sine or cosine of an angle between 90° and 180°, then start by finding the supplement of the given angle.

### WORKED EXAMPLE 7

Find the values of:

\[\text{a} \quad \sin 140^\circ\]
\[\text{b} \quad \cos 160^\circ\]

giving your answers correct to 2 decimal places.

**THINK**

**WRITE**

\[\text{a} \quad 180^\circ - 140^\circ = 40^\circ\]
\[\sin(40^\circ) = 0.642787\ldots\]
\[= 0.64 \text{ (2 decimal places)}\]

\[\text{b} \quad 180^\circ - 160^\circ = 20^\circ\]
\[\cos(20^\circ) = 0.939692\ldots\]
\[= 0.94 \text{ (2 d.p.)}\]

\[\sin(140^\circ) = \sin(40^\circ)\]
\[= 0.64 \text{ (2 d.p.)}\]

\[\cos(160^\circ) = -\cos(20^\circ)\]
\[= -0.94 \text{ (2 d.p.)}\]

### SOH–CAH–TOA

Trigonometric ratios provide relationships between the sides and angles of a right-angled triangle.

In solving trigonometric ratio problems for sine, cosine and tangent, we need to:

1. determine which ratio to use
2. write the relevant equation
3. substitute values from given information
4. make sure the calculator is in degree mode
5. solve the equation for the unknown lengths, or use the inverse trigonometric functions to find unknown angles.

To assist in remembering the trigonometric ratios, the mnemonic **SOH–CAH–TOA** has been developed.
This mnemonic stands for:
- Sine is Opposite over Hypotenuse
- Cosine is Adjacent over Hypotenuse
- Tangent is Opposite over Adjacent

**Exercise 9.2**

**Trigonometric ratios**

1. **WE1** Find the value of \( x \) correct to 2 decimal places.

   \[
   \sin \theta = \frac{2.7}{x}
   \]

   \( x = 2.7 \sin 53° \)

2. **WE2** Find the value of \( x \) correct to 2 decimal places.

   \[
   \cos \theta = \frac{5.3}{x}
   \]

   \( x = \frac{5.3}{\cos 30°} \)

3. **WE2** Find the value of the unknown angle, \( \theta \), expressed in decimal form.

   \[
   \tan \theta = \frac{6.3}{4.6}
   \]

   \( \theta = \tan^{-1} \left( \frac{6.3}{4.6} \right) \)

4. **WE3** Find the value of the unknown angle, \( \theta \), expressed in decimal form.

   \[
   \tan \theta = \frac{8.2}{11.5}
   \]

   \( \theta = \tan^{-1} \left( \frac{8.2}{11.5} \right) \)

5. **WE3** Find the value of \( y \) correct to 2 decimal places.

   \[
   \sin \theta = \frac{y}{1.8}
   \]

   \( y = 1.8 \sin 25°12' \)

6. **WE3** Find the value of \( y \) correct to 2 decimal places.

   \[
   \tan \theta = \frac{y}{3.19}
   \]

   \( y = 3.19 \tan 34°30' \)
7. Find the value of the unknown angle, $\theta$, expressed in degrees and minutes.

8. Find the value of the unknown angle, $\theta$, expressed in degrees and minutes.

9. Find the value of $x$ correct to 2 decimal places.

10. Find the value of $y$ correct to 2 decimal places.

11. Find the value of the unknown angle, $\theta$, expressed in decimal form.

12. Find the value of the unknown angle, $\theta$, expressed in degrees and minutes.

13. Find the values of:
   a. $\sin 125^\circ$
   b. $\cos 152^\circ$
   giving your answers correct to 2 decimal places.
14 Find the values of:
   a \( \sin 99.2^\circ \)  
   b \( \cos 146.7^\circ \)
giving your answers correct to 2 decimal places.

15 Find the value of \( x \) correct to 2 decimal places.

16 Find the value of the unknown angle, \( \theta \), expressed in degrees and minutes.

17 A kitesurfer has a kite of length 2.5 m and strings of length 7 m as shown. 
Find the value of the angles \( \theta \) and \( \alpha \), expressed in degrees and minutes.

18 A daredevil is to be catapulted into the air in a capsule at an angle of 64.3° to the ground. Assuming the stuntman travels in a straight path, what horizontal distance will he have covered, correct to 2 decimal places, if he reached a height of 20 metres?

19 A yacht race follows a triangular course as shown below. Calculate, correct to 1 decimal place:
   a the distance of the final leg, \( y \)  
   b the total distance of the course.

20 A railway line rises for 300 metres at a uniform slope of 6° with the horizontal. 
What is the distance travelled by the train, correct to the nearest metre?
21 A truss is used to build a section of a roof. If the vertical height of the truss is 1.5 metres and the span (horizontal distance between the walls) is 8 metres wide, calculate the pitch of the roof, expressed in decimal form.

22 If 3.5 metres of Christmas lights are attached directly from the tip of the bottom branch to the top of a Christmas tree at an angle of 35.2° from the ground, how high is the Christmas tree, correct to 2 decimal places?

23 A 2.5 m ladder is placed against a wall. The base of the ladder is 1.7 m from the wall.
   a Calculate the angle, expressed in degrees and minutes, the ladder makes with the ground.
   b Find how far the ladder reaches up the wall, correct to 2 decimal places.

24 A school is building a wheelchair ramp of length 4.2 m to be inclined at an angle of 10.5°.
   a Find the horizontal length of the ramp, correct to 1 decimal place.
   b The ramp is too steep at 10.5°. Instead, the vertical height of the ramp needs to be 0.5 m and the ramp inclined at an angle of 5.7°. Calculate the new length of the wheelchair ramp, correct to 2 decimal places.

25 A play gym for monkeys is constructed at the zoo. A rope is tied from a tree branch 1.6 m above the ground to another tree branch 2.5 m above the ground. The monkey swings along the rope, which makes an angle of 11°52′ to the vertical. How far apart are the trees, correct to 2 decimal places?

26 A dog training obstacle course ABCDEA is shown in the diagram below with point B vertically above point D. Find the total length of the obstacle course in metres, giving your answer correct to 2 decimal places.

9.3 Applications of trigonometric ratios

Angles of elevation and depression

An angle of elevation is the angle between a horizontal line from the observer to an object that is above the horizontal line.
An angle of depression is the angle between a horizontal line from the observer to an object that is below the horizontal line.

We use angles of elevation and depression to locate the positions of objects above or below the horizontal (reference) line. Angles of elevation and angles of depression are equal as they are alternate angles.

Think Write

1. Draw a diagram to represent the information.

2. Label all the given information on the triangle.

3. Since we have been given the combination of opposite, adjacent and the reference angle $\theta$, we need to use the tangent ratio. Substitute the given values into the ratio equation.

The angle of depression from a scuba diver at the water’s surface to a hammerhead shark on the sea floor of the Great Barrier Reef is $40^\circ$. The depth of the water is 35 m. Calculate the horizontal distance from the scuba diver to the shark.
Bearings

Bearings are used to locate the positions of objects or the direction of a journey on a two-dimensional plane.

The four main directions or standard bearings of a directional compass are known as cardinal points. They are North (N), South (S), East (E) and West (W).

There are two types of bearings: conventional (compass) bearings and true bearings.

<table>
<thead>
<tr>
<th>Conventional (compass) bearings</th>
<th>True bearings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional bearings are written by first identifying whether the point is north or south, and then identifying whether the point is east or west.</td>
<td>True bearings are measured in a clockwise direction from the north–south line. They are written with all three digits of the angle stated.</td>
</tr>
<tr>
<td>The conventional bearing of a point is stated as the number of degrees east or west of the north–south line.</td>
<td>If the angle measured is less than 100°, place a zero in front of the angle. For example, if the angle measured is 20° from the north–south line the bearing is 020°T</td>
</tr>
</tbody>
</table>

Rearrange the equation to make the unknown the subject and solve.

Make sure your calculator is in degree mode.

\[
x = \frac{35}{\tan(40°)}
\]

\[
x = 41.71
\]

The horizontal distance from the scuba diver to the shark is 41.71 m.
True bearings and conventional bearings are interchangeable, for example $030^\circ T = N30^\circ E$.

For the remainder of this topic, we will only use true bearings.

**Bearings from A to B**

The bearing from A to B is **not** the same as the bearing from B to A.

When determining a bearing from a point to another point, it is important to follow the instructions and draw a diagram. Always draw the centre of the compass at the starting point of the direction requested.

When a problem asks to find the bearing of A from B, start at B, mark in north, and join a directional line to A to work out the bearing. To return to where you came from is a change in bearing of $180^\circ$. 
Find the true bearing from:

a. Town A to Town B
b. Town B to Town A.

**THINK**

a. 1. To find the bearing from Town A to Town B, make sure the centre of the compass is marked at town A. The angle is measured clockwise from north to the bearing line at Town B.

2. A true bearing is written with all three digits of the angle followed by the letter T.

b. 1. To find the bearing from Town B to Town A, make sure the centre of the compass is marked at Town B. The angle is measured clockwise from north to the bearing line at Town A.

2. A true bearing is written with all three digits of the angle followed by the letter T.

**WRITE**

a. The angle measure from north is 60°.

The true bearing from Town A to Town B is 060°T.

b. The angle measure from north is 60° + 180° = 240°.

The true bearing from Town B to Town A is 240°T.

---

**Using trigonometry in bearings problems**

As the four cardinal points (N, E, S, W) are at right angles to each other, we can use trigonometry to solve problems involving bearings.

When solving a bearings problem with trigonometry, always start by drawing a diagram to represent the problem. This will help you to identify what information you already have, and determine which trigonometric ratio to use.

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**WORKED EXAMPLE 10**

A boat travels for 25 km in a direction of 310°T.

a. How far north does the boat travel, correct to 2 decimal places?

b. How far east does the boat travel, correct to 2 decimal places?
Applications of trigonometric ratios

1. The angle of depression from a scuba diver at the water’s surface to a hammerhead shark on the sea floor of the Great Barrier Reef is 41°. The depth of the water is 32 m. Calculate the horizontal distance from the scuba diver to the shark.

2. The angle of elevation from a hammerhead shark on the sea floor of the Great Barrier Reef to a scuba diver at the water’s surface is 35°. The depth of the water is 33 m. Calculate the horizontal distance from the shark to the scuba diver.

3. In the figure below, find the true bearing from:
   a. Town A to Town B
   b. Town B to Town A.

EXERCISE 9.3 Applications of trigonometric ratios

1. The angle of depression from a scuba diver at the water’s surface to a hammerhead shark on the sea floor of the Great Barrier Reef is 41°. The depth of the water is 32 m. Calculate the horizontal distance from the scuba diver to the shark.

2. The angle of elevation from a hammerhead shark on the sea floor of the Great Barrier Reef to a scuba diver at the water’s surface is 35°. The depth of the water is 33 m. Calculate the horizontal distance from the shark to the scuba diver.

3. In the figure below, find the true bearing from:
   a. Town A to Town B
   b. Town B to Town A.
4 In the figure below, find the true bearing from:
   a Town A to Town B
   b Town B to Town A.

5 A boat travels for 36 km in a direction of 155°T.
   a How far south does the boat travel, correct to 2 decimal places?
   b How far west does the boat travel, correct to 2 decimal places?

6 A boat travels north for 6 km, west for 3 km, then south for 2 km.
   What is the boat’s true bearing from its starting point? Give your answer in decimal form to 1 decimal place.

7 Find the angle of elevation of the kite from the ground, expressed in decimal form.

8 Find the angle of depression from the boat to the treasure at the bottom of the sea, expressed in degrees and minutes.

9 A crocodile is fed on a 'Jumping crocodile tour' on the Adelaide River. The tour guide dangles a piece of meat on a stick at an angle of elevation of 60° from the boat, horizontal to the water. If the stick is 2 m long and held 1 m above the water, find the vertical distance the crocodile has to jump out of the water to get the meat, correct to 2 decimal places.

10 A ski chair lift operates from the Mt Buller village and has an angle of elevation of 45° to the top of the Federation ski run. If the vertical height is 707 m, calculate the ski chair lift length, correct to 2 decimal places.

11 From the figure below, find the true bearing of
   a B from A
   b C from B
   c A from C.
12 A student uses an inclinometer to measure an angle of elevation of 50° from the ground to the top of Uluru (Ayers Rock). If the student is standing 724 m from the base of Uluru, determine the height of Uluru.

13 A tourist looks down from the Eureka Tower’s Edge on the Skydeck to see people below on the footpath. If the angle of depression is 88° and the people are 11 m from the base of the tower, how high up is the tourist standing in the glass cube?

14 A student uses an inclinometer to measure the height of his house. The angle of elevation is 54°. He is 1.5 m tall and stands 7 m from the base of the house. Calculate the height of the house correct to 1 decimal place.

15 A tourist 1.72 m tall is standing 50 m away from the base of the Sydney Opera House. The Opera House is 65 m tall. Calculate the angle of elevation, to the nearest degree, from the tourist to the top of the Opera House.

16 A parachutist falls from a height of 5000 m to the ground while travelling over a horizontal distance of 150 m. What was the angle of depression of the descent?

17 Air traffic controllers in two control towers, which are both 87 m high, spot a plane at an altitude of 500 metres. The angle of elevation from tower A to the plane is 5° and from tower B to the plane is 7°. Find the distance between the two control towers.
18 A footballer takes a set shot at goal, with the following graph showing the path that the ball took as it travelled towards the goal.

If the footballer’s eye level is at 1.6 metres, calculate the angle of elevation from his eyesight to the ball after:

a 1 second  

b 2 seconds  

c 3 seconds  

d 4 seconds  

e 5 seconds.

9.4 The sine rule

The sine rule can be used to find the side length or angle in non-right-angled triangles.

To help us solve non-right-angled triangle problems, the labelling convention of a non-right-angled triangle, ABC, is as follows:

Angle A is opposite side length a.

Angle B is opposite side length b.

Angle C is opposite side length c.

The largest angle will always be opposite the longest side length, and the smallest angle will always be opposite to the smallest side length.

Formulating the sine rule

We can divide an acute non-right-angled triangle into two right-angled triangles as shown in the following diagrams.

If we apply trigonometric ratios to the two right-angled triangles we get:

\[
\frac{h}{c} = \sin(A) \quad \frac{h}{a} = \sin(C)
\]

\[
h = c \sin(A) \quad h = a \sin(C)
\]

Equating the two expressions for \( h \) gives:

\[
c \sin(A) = a \sin(C)
\]

\[
\frac{a}{\sin(A)} = \frac{c}{\sin(C)}
\]

In a similar way, we can split the triangle into two using side a as the base, giving us:

\[
\frac{b}{\sin(B)} = \frac{c}{\sin(C)}
\]
This gives us the sine rule:

\[
\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}
\]

We can apply the sine rule to determine all of the angles and side lengths of a triangle if we are given either:

- 2 side lengths and 1 corresponding angle
- 1 side length and 2 angles.

**WORKED EXAMPLE 11**

Find the value of the unknown length \(x\), correct to 2 decimal places.

**THINK**

1. Label the triangle with the given information, using the conventions for labelling.
   - Angle \(A\) is opposite to side \(a\).
   - Angle \(B\) is opposite to side \(b\).

2. Substitute the known values into the sine rule.

3. Rearrange the equation to make \(x\) the subject and solve.
   - Make sure your calculator is in degree mode.

4. Write the answer.

**WRITE**

\[
\frac{x}{\sin(35^\circ)} = \frac{10}{\sin(40^\circ)}
\]

\[
x = \frac{10 \sin(35^\circ)}{\sin(40^\circ)}
\]

\[
x = 8.92
\]

The unknown side length \(x\) is 8.92 cm.

**WORKED EXAMPLE 12**

A non-right-angled triangle has values of side \(b = 12.5\), angle \(A = 25.3^\circ\) and side \(a = 7.4\). Calculate the value of angle \(B\).

**THINK**

1. Draw a non-right-angled triangle, labelling with the given information.
   - Angle \(A\) is opposite to side \(a\).
   - Angle \(B\) is opposite to side \(b\).

**WRITE**

\[
\frac{a}{\sin(A)} = \frac{b}{\sin(B)}
\]

\[
\frac{7.4}{\sin(25.3^\circ)} = \frac{12.5}{\sin(B)}
\]

\[
B = \arcsin\left(\frac{12.5 \sin(25.3^\circ)}{7.4}\right)
\]

\[
B = 50.1^\circ
\]
2 Substitute the known values into the sine rule.

\[
\frac{a}{\sin(A)} = \frac{b}{\sin(B)}
\]

\[
\frac{7.4}{\sin(25.3^\circ)} = \frac{12.5}{\sin(B)}
\]

3 Rearrange the equation to make \(\sin(B)\) the subject and solve.

Make sure your calculator is in degree mode.

\[
\sin(B) = \frac{7.4 \sin(25.3^\circ)}{12.5}
\]

\[
B = \sin^{-1}\left(\frac{12.5 \sin(25.3^\circ)}{7.4}\right)
\]

\[
= 46.21^\circ
\]

4 Write the answer.

Angle \(B\) is 46.21°.

The ambiguous case of the sine rule

When we are given two sides lengths of a triangle and an acute angle opposite one of these side lengths, there are two different triangles we can draw. So far we have only dealt with triangles that have all acute angles; however, it is also possible to draw triangles with an obtuse angle. This is known as the ambiguous case of the sine rule.

For example, take the triangle ABC, where \(a = 12\), \(c = 8\) and \(C = 35^\circ\).

When we solve this for angle \(A\) we get an acute angle as shown:

\[
\frac{8}{\sin(35)} = \frac{12}{\sin(A)}
\]

\[
8 \sin(A) = 12 \sin(35)
\]

\[
\sin(A) = \frac{12 \sin(35)}{8}
\]

\[
A = \sin^{-1}\left(\frac{12 \sin(35)}{8}\right)
\]

\[
= 59.36^\circ
\]

However, there is also an obtuse-angled triangle that can be drawn from this given information.

In this case, the size of the obtuse angle is the supplement of the acute angle calculated previously.

\[
A' = 180^\circ - A
\]

\[
= 180^\circ - 59.36^\circ
\]

\[
= 120.64^\circ
\]

Determining when we can use the ambiguous case

The ambiguous case of the sine rule does not work for every example. This is due to the way the ratios are set in the development of the sine rule, since side length \(a\) must be longer than \(h\), where \(h\) is the length of the altitude from angle \(B\) to the base line \(b\).
For the ambiguous case to be applicable, the following conditions must be met:
• The given angle must be acute.
• The adjacent side must be bigger than the opposite side.
• The opposite side must be bigger than the adjacent side multiplied by the sine of the given angle.

When using the sine rule to calculate a missing angle, it is useful to first identify whether the ambiguous case is applicable to the problem or not.

**WORKED EXAMPLE 13**

Find the two possible values of angle \( A \) for triangle ABC, given \( a = 15 \), \( c = 8 \) and \( C = 25^\circ \).

**THINK**
1. Draw a non-right-angled triangle, labelling with the given information.
   - Angle \( A \) is opposite to side \( a \).
   - Angle \( C \) is opposite to side \( b \).
   - Note that two triangles can be drawn, with angle \( A \) being either acute or obtuse.
2. Substitute the known values into the sine rule.
3. Rearrange the equation to make \( \sin(A) \) the subject and solve.
   - Make sure your calculator is in degree mode.
   - The calculator will only give the acute angle value.
4. Solve for the obtuse angle \( A' \).
5. Write the answer.

**WRITE**

\[
\frac{a}{\sin(A)} = \frac{c}{\sin(C)}
\]

\[
\frac{15}{\sin(A)} = \frac{8}{\sin(25^\circ)}
\]

\[
15 \sin(25^\circ) = 8 \sin(A)
\]

\[
\sin(A) = \frac{15 \sin(25^\circ)}{8}
\]

\[
A = \sin^{-1} \left( \frac{15 \sin(25^\circ)}{8} \right)
\]

\[
= 52.41^\circ
\]

\[
A' = 180^\circ - A
\]

\[
= 180^\circ - 52.41^\circ
\]

\[
= 127.59^\circ
\]

The two possible value for \( A \) are 52.41° and 127.59°.

---

**EXERCISE 9.4**

**PRACTISE**

**The sine rule**

1. **WE11** Find the value of the unknown length \( x \), correct to 2 decimal places.

![Diagram with angles 45° and 55°, side 12 cm and unknown side x cm]
2 Find the value of the unknown length $x$, correct to 2 decimal places.

3 **WE12** A non-right-angled triangle has values of side $b = 10.5$, angle $A = 22.3^\circ$ and side $a = 8.4$. Calculate the value of angle $B$.

4 A non-right-angled triangle has values of side $b = 7.63$, angle $A = 15.8^\circ$ and side $a = 4.56$. Calculate the value of angle $B$.

5 **WE13** Find the two possible values of angle $A$ for triangle $ABC$, given $a = 8$, $c = 6$ and $C = 43^\circ$.

6 Find the two possible values of angle $A$ for triangle $ABC$, given $a = 7.5$, $c = 5$ and $C = 32^\circ$.

7 Find the value of the unknown length $x$, correct to 2 decimal places.

8 Find the value of the unknown length $x$, correct to the nearest cm.

9 If triangle $ABC$ has values $b = 19.5$, $A = 25.3^\circ$ and $a = 11.4$, find both possible angle values of $B$.

10 For triangle $ABC$ shown below, find the acute value of $\theta$, correct to 1 decimal place.

11 Find all the side lengths, correct to 2 decimal places, and angle values for the triangle $ABC$, given $a = 10.5$, $B = 60^\circ$ and $C = 72^\circ$. 
12 Find the acute and obtuse angles that have a sine value of approximately 0.573 58. Give your answer to the nearest degree.

13 Part of a roller-coaster track is in the shape of an isosceles triangle, ABC, as shown in the following triangle. Calculate the track length AB, correct to 2 decimal places.

14 The shape and length of a water slide follows the path of PY and YZ in the following diagram.

Calculate, correct to 2 decimal places:

a the total length of the water slide

b the height of the water slide, PX.

15 Find the two unknown angles shown in the diagram below.

16 In the triangle ABC, \( a = 11.5 \, \text{m}, \, c = 6.5 \, \text{m} \) and \( C = 25^\circ \).

a Draw the two possible triangles with this information.

b Find the two possible values of angle \( A \), and hence the two possible values of angle \( B \).

17 At a theme park, the pirate ship swings back and forth on a pendulum. The centre of the pirate ship is secured by a large metal rod that is 5.6 metres in length. If one of the swings covers an angle of 122°, determine the distance between the point where the rod meets the ship at both extremes of the swing.
Andariel went for a ride on her dune buggy in the desert. She rode east for 6 km, then turned 125° to the left for the next stage of her ride. After 5 minutes riding in the same direction, she turned to the left again, and from there travelled the 5.5 km straight back to her starting position.

How far did Andariel travel in the second section of her ride?

9.5

The cosine rule

Formulating the cosine rule

The cosine rule, like the sine rule, is used to find the length or angle in a non-right-angled triangle. We use the same labelling conventions for non-right-angled triangles as when using the sine rule.

As with the sine rule, the cosine rule is derived from a non-right-angled triangle being divided into two right-angled triangles, where the base side lengths are equal to \((b - x)\) and \(x\).

Using Pythagoras’ theorem we get:

\[
\begin{align*}
  c^2 &= x^2 + h^2 \\
  h^2 &= c^2 - x^2
\end{align*}
\]

and

\[
\begin{align*}
  a^2 &= (b - x)^2 + h^2 \\
  h^2 &= a^2 - (b - x)^2
\end{align*}
\]

Equating the two expressions for \(h^2\) gives:

\[
\begin{align*}
  c^2 - x^2 &= a^2 - (b - x)^2 \\
  a^2 &= (b - x)^2 + c^2 - x^2 \\
  a^2 &= b^2 - 2bx + c^2
\end{align*}
\]

Substituting the trigonometric ratio \(x = c \cos(A)\) from the right-angled triangle into the expression, we get:

\[
\begin{align*}
  a^2 &= b^2 - 2b (c \cos(A)) + c^2 \\
  &= b^2 + c^2 - 2bc \cos(A)
\end{align*}
\]

This is known as the cosine rule, and we can interchange the pronumerals to get:

\[
\begin{align*}
  a^2 &= b^2 + c^2 - 2bc \cos(A) \\
  b^2 &= a^2 + c^2 - 2ac \cos(B) \\
  c^2 &= a^2 + b^2 - 2ab \cos(C)
\end{align*}
\]

We can apply the cosine rule to determine all of the angles and side lengths of a triangle if we are given either:

* 3 side lengths
* 2 side lengths and the included angle.

The cosine rule can also be transposed to give:

\[
\begin{align*}
  \cos(A) &= \frac{b^2 + c^2 - a^2}{2bc} \\
  \cos(B) &= \frac{a^2 + c^2 - b^2}{2ac} \\
  \cos(A) &= \frac{b^2 + c^2 - a^2}{2bc}
\end{align*}
\]
WORKED EXAMPLE 14  
Find the value of the unknown length \( x \), correct to 2 decimal places.

**THINK**

1. Draw the non-right-angled triangle, labelling with the given information.
   - Angle \( A \) is opposite to side \( a \).
   - If three sides lengths and one angle are given, always label the angle as \( A \) and the opposite side as \( a \).

2. Substitute the known values into the cosine rule.

3. Solve for \( x \).
   - Make sure your calculator is in degree mode.

4. Write the answer.

**WRITE**

10 cm

\[ 35^\circ \]

12.5 cm

\[ x \text{ cm} \]

\[ a^2 = b^2 + c^2 - 2bc \cos(A) \]

\[ x^2 = 10^2 + 12.5^2 - 2 \times 10 \times 12.5 \cos(35^\circ) \]

\[ x^2 = 51.462 \]

\[ x = \sqrt{51.462} \]

\[ x = 7.17 \]

The unknown length \( x \) is 7.17 cm.

---

WORKED EXAMPLE 15  
A non-right-angled triangle \( \triangle ABC \) has values \( a = 7 \), \( b = 12 \) and \( c = 16 \). Find the magnitude of angle \( A \).

**THINK**

1. Draw the non-right-angled triangle, labelling with the given information.

2. Substitute the known values into the cosine rule.

3. Rearrange the equation to make \( \cos(A) \) the subject and solve.
   - Make sure your calculator is in degree mode.

4. Write the answer.

**WRITE**

\[ 7 \]

\[ 12 \]

\[ 16 \]

\[ a^2 = b^2 + c^2 - 2bc \cos(A) \]

\[ 7^2 = 12^2 + 16^2 - 2 \times 12 \times 16 \cos(A) \]

\[ \cos(A) = \frac{12^2 + 16^2 - 7^2}{2 \times 12 \times 16} \]

\[ A = \cos^{-1}\left(\frac{12^2 + 16^2 - 7^2}{2 \times 12 \times 16}\right) \]

\[ A = 23.93^\circ \]

The magnitude of angle \( A \) is 23.93°.

*Note:* In the above example it would have been quicker to substitute the known values directly into the transposed cosine rule for \( \cos(A) \).
Sets of sufficient information to determine a triangle

Knowing which rule to use for different problems will save time and help to reduce the chance for errors to appear in your working. The following table should help you determine which rule to use.

<table>
<thead>
<tr>
<th>Type of triangle</th>
<th>What you want</th>
<th>What you know</th>
<th>What to use</th>
<th>Rule</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side length</td>
<td>Two other sides</td>
<td>Pythagoras’ theorem</td>
<td></td>
<td>$a^2 + b^2 = c^2$</td>
<td><img src="1" alt="Example" /></td>
</tr>
</tbody>
</table>
| Side length      | A side length and an angle | Trigonometric ratios |                             | $\sin(\theta) = \frac{O}{H}$  
$\cos(\theta) = \frac{A}{H}$  
$\tan(\theta) = \frac{O}{A}$ | ![Example](2) |
| Angle            | Two side lengths    | Trigonometric ratios       |                             | $\sin(\theta) = \frac{O}{H}$  
$\cos(\theta) = \frac{A}{H}$  
$\tan(\theta) = \frac{O}{A}$ | ![Example](3) |
| Side length      | Angle opposite unknown side and another side/angle pair | Sine rule               |                             | $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$ | ![Example](4) |
| Angle            | Side length opposite unknown side and another side/angle pair | Sine rule               |                             | $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$ | ![Example](5) |
| Side length      | Two sides and the angle between them | Cosine rule           |                             | $a^2 = b^2 + c^2 - 2bc \cos(A)$ | ![Example](6) |
| Angle            | Three sides         | Cosine rule               |                             | $\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$ | ![Example](7) |
EXERCISE 9.5

The cosine rule

1. Find the value of the unknown length $x$, correct to 2 decimal places.

2. Find the value of the unknown length $x$, correct to 2 decimal places.

3. A non-right-angled triangle ABC has values $a = 8$, $b = 13$ and $c = 17$. Find the magnitude of angle $A$.

4. A non-right-angled triangle ABC has values $a = 11$, $b = 9$ and $c = 5$. Find the magnitude of angle $A$.

5. Find the value of the unknown length $x$, correct to 1 decimal place.

6. Find the value of the unknown length $x$, correct to 2 decimal places.

7. For triangle ABC, find the magnitude of angle $A$, expressed in decimal form, given $a = 5$, $b = 7$ and $c = 4$.

8. For triangle ABC with $a = 12$, $B = 57^\circ$ and $c = 8$, find the side length $b$ correct to 2 decimal places.

9. Find the largest angle, expressed in decimal form, between any two legs of the following sailing course.
10 A triangular paddock has sides of length 40 m, 50 m and 60 m. Find the magnitude of the largest angle between the sides, expressed in decimal form.

11 A triangle has side lengths of 5 cm, 7 cm and 9 cm. Find the size of the smallest angle, correct to 2 decimal places.

12 ABCD is a parallelogram. Find the length of the diagonal AC, correct to 2 decimal places.

13 An orienteering course is shown in the following diagram. Find the total distance of the course, correct to 2 decimal places.

14 Two air traffic control towers are 180 km apart. At the same time, they both detect a plane, P. The plane is at a distance 100 km from Tower A at the bearing shown in the diagram below. Find the distance of the plane from Tower B, correct to 2 decimal places.

15 Britney is mapping out a new running path around her local park. She is going to run west for 2.1 km, before turning 105° to the right and running another 3.3 km. From there, she will run in a straight line back to her starting position. How far will Britney run in total? Give your answer correct to the nearest metre.

16 A cruise boat is travelling to two destinations. To get to the first destination it travels for 4.5 hours at a speed of 48 km/h. From there, it takes a 98° turn to the left and travels for 6 hours at a speed of 54 km/h to reach the second destination. The boat then travels directly back to the start of its journey. How long will this leg of the journey take if the boat is travelling at 50 km/h? Give your answer to the nearest minute.
9.6 Area of triangles

You should be familiar with calculating the area of a triangle using the rule:
\[ \text{area} = \frac{1}{2}bh, \]
where \( b \) is the base length and \( h \) is the perpendicular height of the triangle. However, for many triangles we are not given the perpendicular height, so this rule cannot be directly used.

Take the triangle ABC as shown below.

![Diagram of triangle ABC]

If \( h \) is the perpendicular height of this triangle, then we can calculate the value of \( h \) by using the sine ratio:

\[ \sin(A) = \frac{h}{c} \]

Transposing this equation gives \( h = c \sin(A) \), which we can substitute into the rule for the area of the triangle to give:

\[ \text{area} = \frac{1}{2}bc \sin(A) \]

Note: We can label any sides of the triangle \( a, b \) and \( c \), and this formula can be used as long as we have the length of two sides of a triangle and know the value of the included angle.

WORKED EXAMPLE 16

Find the area of the following triangles. Give both answers correct to 2 decimal places.

a A triangle with sides of length 5 cm and 7 cm, and an included angle of 63°.

b A triangle with sides of length 8 cm and 7 cm, and an included angle of 55°.

THINK

a 1 Label the vertices of the triangle.

WRITE

a
2 Write down the known information.

\[ b = 5 \text{ cm} \]
\[ c = 7 \text{ cm} \]
\[ A = 63^\circ \]

3 Substitute the known values into the formula to calculate the area of the triangle.

\[ \text{Area} = \frac{1}{2}bc \sin(A) \]
\[ = \frac{1}{2} \times 5 \times 7 \times \sin(63^\circ) \]
\[ = 15.592\ldots \]
\[ = 15.59 \text{ (2 d.p.)} \]

4 Write the answer, remembering to include the units.

The area of the triangle is 15.59 cm\(^2\), correct to 2 decimal places.

---

b 1 Draw a diagram to represent the triangle.

---

2 Write down the known information.

\[ b = 8 \text{ cm} \]
\[ c = 7 \text{ cm} \]
\[ A = 55^\circ \]

3 Substitute the known values into the formula to calculate the area of the triangle.

\[ \text{Area} = \frac{1}{2}bc \sin(A) \]
\[ = \frac{1}{2} \times 8 \times 7 \times \sin(55^\circ) \]
\[ = 22.936\ldots \]
\[ = 22.94 \text{ (2 d.p.)} \]

4 Write the answer, remembering to include the units.

The area of the triangle is 22.94 cm\(^2\), correct to 2 decimal places.

---

**Heron's formula**

As shown in topic 7, we can also use Heron's formula to calculate the area of a triangle.

To use Heron’s formula, we need to know the length of all three sides of the triangle.

\[ \text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{a+b+c}{2} \]

---

**WORKED EXAMPLE 17**

Find the area of a triangle with sides of 4 cm, 7 cm and 9 cm, giving your answer correct to 2 decimal places.

**THINK**

1 Write down the known information.

\[ a = 4 \text{ cm} \]
\[ b = 7 \text{ cm} \]
\[ c = 9 \text{ cm} \]

**WRITE**
2 Calculate the value of $s$ (the semi-perimeter).

$$s = \frac{a + b + c}{2}$$

$$= \frac{4 + 7 + 9}{2}$$

$$= \frac{20}{2}$$

$$= 10$$

3 Substitute the values into Heron’s formula to calculate the area.

$$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)}$$

$$= \sqrt{10(10 - 4)(10 - 7)(10 - 9)}$$

$$= \sqrt{10 \times 6 \times 3 \times 1}$$

$$= \sqrt{180}$$

$$= 13.416\ldots$$

$$= 13.42 \text{ (to 2 d.p.)}$$

4 Write the answer, remembering to include the units.

The area of the triangle is $13.42 \text{ cm}^2$ correct to 2 decimal places.

Determining which formula to use

In some situations you may have to perform some calculations to determine either a side length or angle size before calculating the area. This may involve using the sine or cosine rule. The following table should help if you are unsure what to do.

<table>
<thead>
<tr>
<th>Known information</th>
<th>What to do</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>The base length and perpendicular height</td>
<td>Use area $= \frac{1}{2}bh$.</td>
<td>[Diagram of a triangle with base 8 cm and perpendicular height 13 cm]</td>
</tr>
<tr>
<td>Two side lengths and the included angle</td>
<td>Use area $= \frac{1}{2}bc \sin(A)$.</td>
<td>[Diagram of a triangle with sides 5 cm and 9 cm, included angle 104°]</td>
</tr>
</tbody>
</table>
| Three side lengths | Use Heron’s formula:

$$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)},$$

where $s = \frac{a + b + c}{2}$

| | | |
| Two angles and one side length | Use the sine rule to determine a second side length, and then use area $= \frac{1}{2}bc \sin(A)$. Note: The third angle may have to be calculated. | [Diagram of a triangle with angles 75° and 29°, and side 9 cm] |
### Known information

<table>
<thead>
<tr>
<th>What to do</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use the sine rule to calculate the other angle opposite one of these lengths, then determine the final angle before using area = ( \frac{1}{2}bc \sin(A) ). <strong>Note:</strong> Check if the ambiguous case is applicable.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
</tbody>
</table>

### Area of triangles

**1.** Find the area of the following triangle, correct to 2 decimal places.

\[
\begin{align*}
\text{1. Area of a triangle with sides of length 14.3 mm and 6.5 mm, and an inclusive angle of 32°.}
\end{align*}
\]

**2.** Find the area of a triangle with sides of length 11 cm, 12 cm and 13 cm, giving your answer correct to 2 decimal places.

**3.** Find the area of a triangle with sides of 22.2 mm, 13.5 mm and 10.1 mm, giving your answer correct to 2 decimal places.

**4.** Find the area of the following triangles.

**5.** Find the area of the following triangles.

**6.** Find the area of the following triangles.

- **a** Triangle ABC, given \( a = 12 \text{ cm}, b = 15 \text{ cm}, c = 20 \text{ cm} \)
- **b** Triangle ABC, given \( a = 10.5 \text{ mm}, b = 11.2 \text{ mm} \) and \( C = 40° \)
- **c** Triangle DEF, given \( d = 19.8 \text{ cm}, e = 25.6 \text{ cm} \) and \( D = 33° \)
- **d** Triangle PQR, given \( p = 45.9 \text{ cm}, Q = 45.5° \) and \( R = 67.2° \)
7 A triangular field is defined by three trees, each of which sits in one of the corners of the field, as shown in the following diagram.

![Diagram of a triangular field with sides 1.9 km, 2.3 km, and 2.5 km, and angles at 93°, 35°, and unknown.]

Calculate the area of the field correct to the nearest metre.

8 A triangle has one side length of 8 cm and an adjacent angle of 45.5°. If the area of the triangle is 18.54 cm², calculate the length of the other side that encloses the 45.5° angle.

9 A triangle ABC has values \( a = 11 \) cm, \( b = 14 \) cm and \( A = 31.3° \).
   a Calculate the size of the other two angles of the triangle.
   b Calculate the other side length of the triangle.
   c Calculate the area of the triangle.

10 The smallest two sides of a triangle are 10.2 cm and 16.2 cm respectively, and the largest angle of the same triangle is 104.5°. Calculate the area of the triangle.

11 A triangle has side lengths of \( 3x \), \( 4x \) and \( 5x \). If the area of the triangle is 121.5 cm², use any appropriate method to determine the value of \( x \).

12 A triangular-shaped piece of jewellery has two side lengths of 8 cm and an area of 31.98 cm². Use trial and error to find the length of the third side correct to 1 decimal place.

13 A triangle has two sides of length 9.5 cm and 13.5 cm, and one angle of 40.2°. Calculate all three possible areas of the triangle.

14 A BMX racing track encloses two triangular sections, as shown in the following diagram.

![Diagram of a BMX race track with sides 330 m, 445 m, 425 m, 550 m, and angles at 93°, 35°, and unknown.]

Calculate the total area that the race track encloses to the nearest m².
15 Find the area of the following shape.

16 A dry field is in the shape of a quadrilateral, as shown in the following diagram.

How much grass seed is needed to cover the field in 1 mm of grass seed?
The Maths Quest Review is available in a customisable format for you to demonstrate your knowledge of this topic.

The Review contains:
- **Short-answer** questions — providing you with the opportunity to demonstrate the skills you have developed to efficiently answer questions using the most appropriate methods
- **Multiple-choice** questions — providing you with the opportunity to practise answering questions using CAS technology
- **Extended-response** questions — providing you with the opportunity to practise exam-style questions.

A summary of the key points covered in this topic is also available as a digital document.

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Interactivities

A comprehensive set of relevant interactivities to bring difficult mathematical concepts to life can be found in the Resources section of your eBookPLUS.
9 Answers

EXERCISE 9.2

1. $x = 2.16 \text{ cm}$
2. $x = 10.6 \text{ cm}$
3. $\theta = 46.9^\circ$
4. $\theta = 45.5^\circ$
5. $y = 1.99 \text{ cm}$
6. $y = 2.63 \text{ cm}$
7. $\theta = 73^\circ58'$
8. $\theta = 35^\circ35'$
9. $x = 13.15 \text{ cm}$
10. $y = 0.75 \text{ cm}$
11. $\theta = 53.20^\circ$
12. $\theta = 36^\circ23'$
13. $a = 0.82$
14. $a = 0.99$
15. $x = 5.16$
16. $\theta = 49^\circ19'$
17. $\theta = 10^\circ17', \alpha = 79^\circ43'$
18. $\theta = 9.63 \text{ m}$
19. $\theta = 27.2 \text{ km}$
20. $2870 \text{ m}$
21. $\theta = 20.6^\circ$
22. $2.02 \text{ m}$
23. $a = 47^9$
24. $a = 4.1 \text{ m}$
25. $4.28 \text{ m}$

EXERCISE 9.3

1. $36.81 \text{ m}$
2. $47.13 \text{ m}$
3. $a = 110^\circ T$
4. $a = 237^\circ T$
5. $a = 31.72 \text{ km}$
6. $323.1^\circ T$
7. $36.25^\circ$
8. $999.85 \text{ m}$
9. $999.85 \text{ m}$
10. $050^\circ T$
11. $b = 127^\circ T$
12. $862.83 \text{ m}$
13. $315 \text{ m}$
14. $11.13 \text{ m}$
15. $52^\circ$
16. $1.72^\circ$
17. $1357 \text{ m}$
18. $a = 83.59^\circ$
19. $d = 74.97^\circ$
20. $b = 82.35^\circ$
21. $c = 79.93^\circ$
22. $e = 60.67^\circ$

EXERCISE 9.4

1. $x = 10.36 \text{ cm}$
2. $x = 10.38 \text{ cm}$
3. $B = 28.3^\circ$
4. $B = 27.1^\circ$
5. $A = 65.41^\circ \text{ or } 114.59^\circ$
6. $A = 52.64^\circ \text{ or } 127.36^\circ$
7. $x = 9.05 \text{ cm}$
8. $x = 18 \text{ cm}$
9. $B = 46.97^\circ \text{ or } 133.03^\circ$
10. $\theta = 47.2^\circ$
11. $b = 12.24, c = 13.43$
12. $35^\circ \text{ and } 145^\circ$
13. $35.85 \text{ m}$
14. $a = 23.18 \text{ m}$
15. $x = 142.4^\circ, y = 37.6^\circ$

EXERCISE 9.5

1. $x = 2.74 \text{ km}$
2. $x = 10.49 \text{ m}$
3. $A = 26.95^\circ$
4. $A = 99.59^\circ$
5. $x = 8.5 \text{ km}$
6. $x = 4.48 \text{ m}$
7. $A = 44.42^\circ$
8. $b = 10.17$
9. $79.66^\circ$
10. $82.82^\circ$
11. $33.56^\circ$
12. $13.29 \text{ cm}$
13. $31.68 \text{ km}$
14. $98.86 \text{ km}$
15. $8822 \text{ m}$
16. $7 \text{ hours, } 16 \text{ minutes}$

EXERCISE 9.6

1. $52.75 \text{ mm}^2$
2. $24.63 \text{ mm}^2$
3. $61.48 \text{ cm}^2$
4. $43.92 \text{ mm}^2$
5. $a = 113.49 \text{ cm}^2$
6. $b = 43.93 \text{ mm}^2$
7. $c = 216.10 \text{ cm}^2$
8. $d = 122.48 \text{ cm}^2$
9. $a = 89.67 \text{ cm}^2$
10. $b = 37.80 \text{ mm}^2$
11. $c = 247.68 \text{ cm}^2$
12. $d = 750.79 \text{ cm}^2$
13. $2082 \text{ km}^2$
14. $6.50 \text{ cm}$
15. $B = 41.39^\circ, C = 107.31^\circ$
16. $b = 20.21 \text{ cm}$
17. $c = 73.51 \text{ cm}^2$
18. $10.99 \text{ cm}^2$
19. $11 \times 4.5 \text{ cm}$
20. $11.1 \text{ cm}$
21. $41.39 \text{ cm}^2, 61.41 \text{ cm}^2 \text{ and } 59.12 \text{ cm}^2$
22. $167 \text{ 330 m}^2$
23. $15 \text{ 205.39 cm}^2$
24. $7.77 \text{ m}^3$