13

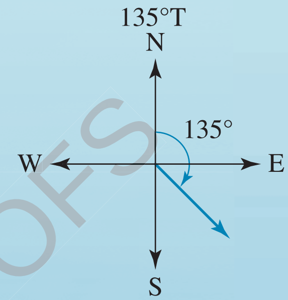
Applications of geometry and trigonometry

13.1 Kick off with CAS

Exploring bearings with CAS

True bearings allow us to state the direction from one object to another object, with respect to the north direction.

All true bearings should be stated as 3-digit figures, with the angle measure taken from north, as in the diagram at right.



True bearings can be used to solve navigation and triangulation problems, allowing us to accurately determine the location of objects if given limited information.

A cruise ship set off for sail on a bearing of 135°T and travelled in this direction for 6 kilometres.

1 Determine how far east the ship is from its starting position by completing the following steps:
   a Copy the above diagram and mark the blue direction line as having a length of 6 km.
   b Determine the angle between the 135° line and the east direction on the compass.
   c Use the east direction (adjacent side), the 135° line (hypotenuse) and the angle calculated in part b to draw a right-angled triangle to represent the situation.
   d Use CAS and the cosine ratio to calculate the length of the adjacent side length, which will tell you how far east the ship is from its starting position.

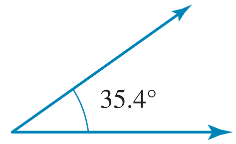
2 Use CAS and the sine ratio to calculate how far south the ship is from its starting position.

Please refer to the Resources tab in the Prelims section of your **eBookPLUS** for a comprehensive step-by-step guide on how to use your CAS technology.

13.2 Angles

An angle represents the space between two intersecting lines or surfaces at the point where they meet.

Angles are measured in degrees (°). In navigation, accuracy can be critical, so fractions of a degree are also used. For example, a cruise ship travelling 1000 kilometres on a course that is out by half a degree would miss its destination by almost 9 kilometres.



Adding and subtracting angles

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Interactivity
Adding and subtracting angles
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Angles can be added and subtracted using basic arithmetic. When using CAS or other technology to solve problems involving angles, make sure you are in degrees mode.

WORKED EXAMPLE 1

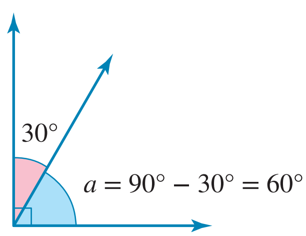
a **Add 46.37° and 65.49°.**

b **Subtract 16.55° from 40.21°.**

THINK	WRITE
a 1 Add the angles together, remembering to include the decimal points.	**a** $\begin{array}{r} 46.37° \\ +\ 65.49° \\ \hline 111.86° \end{array}$
2 Write the answer.	111.86°
b 1 Subtract 16.55° from 40.21°, remembering to include the decimal points.	**b** $\begin{array}{r} 40.21° \\ -\ 16.55° \\ \hline 23.66° \end{array}$
2 Write the answer.	23.66°

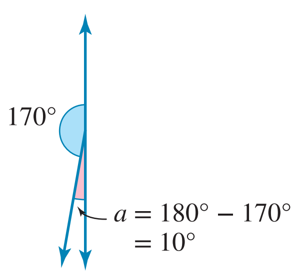
Some geometry (angle) laws

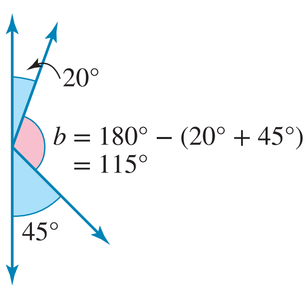
The following angle laws will be valuable when finding unknown values in the applications to be examined in this topic. These laws were dealt with at the start of topic 11, but we will review them here. Often we will need the laws to convert given directional bearings into an angle in a triangle.



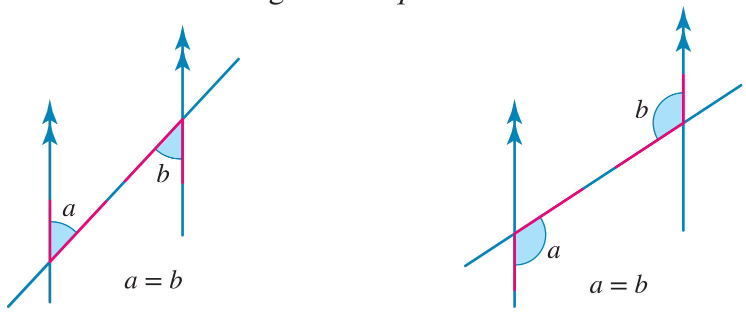
Two or more angles are **complementary** if they add up to 90°.

Two or more angles are **supplementary** if they add up to 180°. An angle of 180° is also called a *straight angle*.

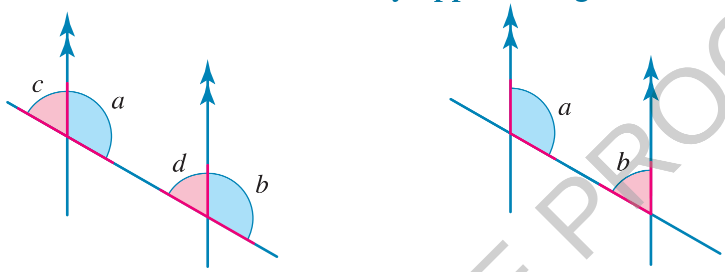




For **alternate angles** to exist we need a minimum of one pair of parallel lines and one transverse line. Alternate angles are *equal*.

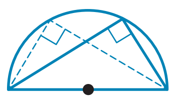


Other types of angles to be considered are **corresponding angles**, **co-interior angles**, *triangles in a semicircle* and **vertically opposite angles**.

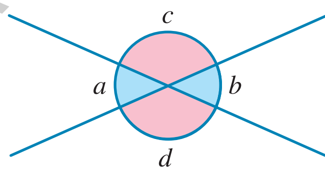


Corresponding angles are equal:
$a = b$
$c = d$

Co-interior angles are supplementary:
$a + b = 180°$



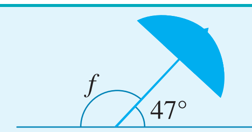
A triangle in a semicircle always gives a right-angled triangle.

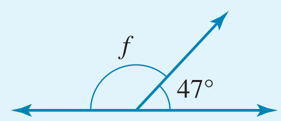


Vertically opposite angles are equal:
$a = b$
$c = d$

WORKED EXAMPLE 2

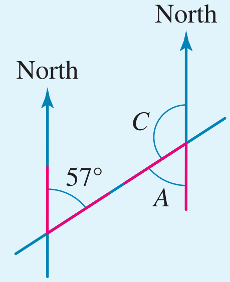
Find the value of the pronumeral, *f*, the angle a beach umbrella makes with the ground.



THINK	WRITE/DRAW
1 Recognise that the horizontal line is a straight angle, or 180°.	
2 To find the unknown angle, use the supplementary or straight angle law.	$180° = 47° + f$ $f = 180° - 47°$ $= 133°$

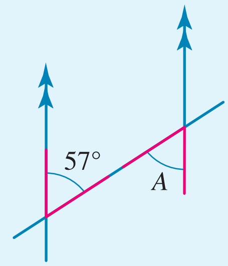
WORKED EXAMPLE 3

Find the value of the pronumerals *A* and *C* in the directions shown.



THINK | **WRITE/DRAW**

1 Recognise that the two vertical lines are parallel lines.



2 To find the unknown angle A, use the alternate angle law.

$A = 57°$

3 To find the unknown angle C, use the straight angle law. Alternatively, the co-interior angle law could be used with the same solution.

$$180° = 57° + C$$
$$C = 180° - 57°$$
$$= 123°$$

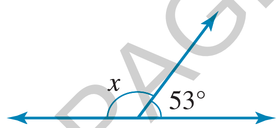
EXERCISE 13.2 Angles

PRACTISE

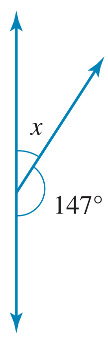
1 WE1 a Add 28.29° and 42.12°. b Subtract 21.38° from 74.51°.

2 a Add 51.33° and 18.74°. b Subtract 35.17° from 80.13°.

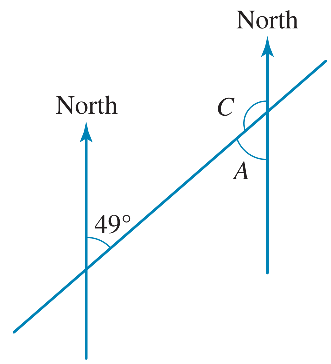
3 WE2 Find the value of the pronumeral.



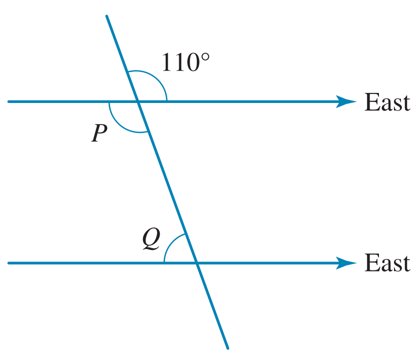
4 Find the value of the pronumeral.



5 WE3 Find the values of the pronumerals A and C in the diagram shown.



6 Find the values of the pronumerals P and Q in the diagram shown.



CONSOLIDATE

7 Which of the following angle calculations gives the greatest result?

A 27.16° + 33.83° **B** 13.84° + 42.69° **C** 83.09° − 24.55°
D 9.12° + 50.84° **E** 90.11° − 29.25°

8 a What is the supplementary angle of 45.56°?

b What is the complementary angle of 72.18°?

9 Use your calculator to find the values of the following trigonometric ratios correct to 3 decimal places.

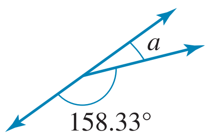
a sin(40.25°) **b** cos(122.33°) **c** tan(82.10°)
d cos(16.82°) **e** sin(147.50°) **f** tan(27.46°)

10 Add and then subtract the pairs of angles.

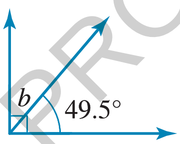
a 40.25°, 28.08° **b** 122.21°, 79.35° **c** 82.06°, 100.83°
d 247.52°, 140.58° **e** 346.37°, 176.84° **f** 212.33°, 6.64°

For questions **11**, **12** and **13**, find the values of the pronumerals.

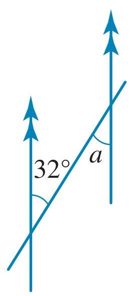
11 a



b



12 a



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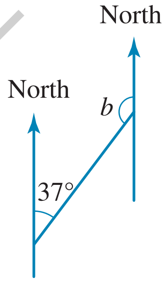
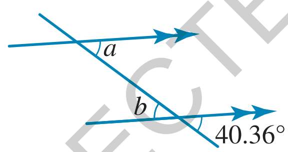


13 a



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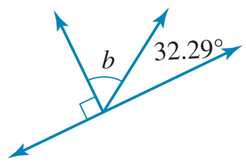
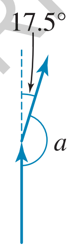
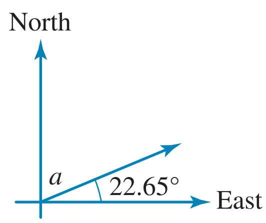


For questions **14**, **15** and **16**, find the values of the pronumerals.

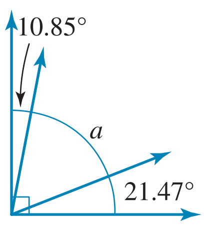
14 a



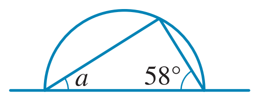
b



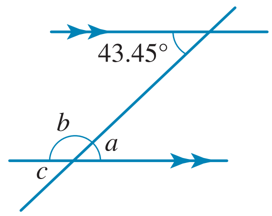
15 a



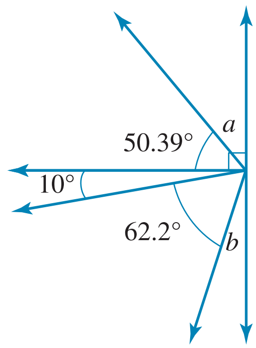
b



16 a



b



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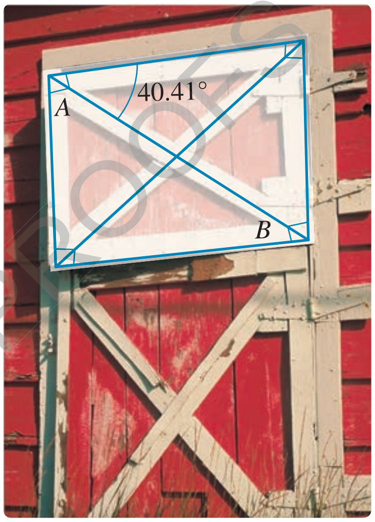
17 A barn door is shown here.



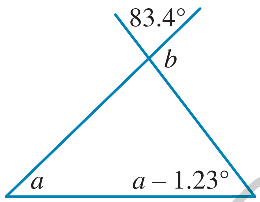
a The value of angle A is:

A 40.41° **B** 149.59° **C** 49.41°
D 50 **E** 90°

b The value of angle B is:

A 40.41° **B** 49.59° **C** 49.41°
D 50° **E** 139.59°

18 Find the values of the pronumerals in the figure.

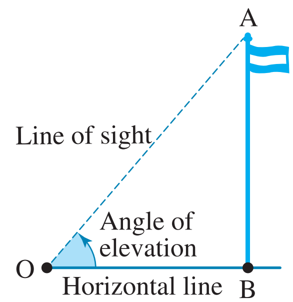


13.3 Angles of elevation and depression

One method for locating an object in the real world is by its position above or below a horizontal plane or reference line.

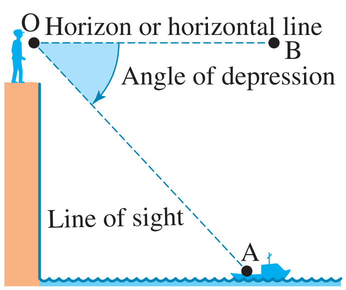
The **angle of elevation** is the angle *above* the horizon or horizontal line.

Looking up at the top of the flagpole from position O, the angle of elevation, ∠AOB, is the angle between the horizontal line OB and the line of sight OA.



The **angle of depression** is the angle *below* the horizon or horizontal line.

Looking down at the boat from position O, the angle of depression, ∠AOB, is the angle between the horizontal line, OB, and the line of sight, OA.

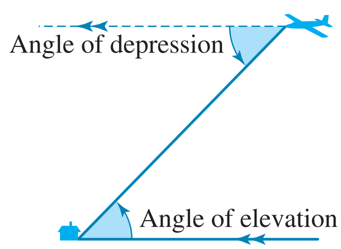


studyon

Unit 4
AOS M3
Topic 1
Concept 8

Angles in the horizontal and vertical planes
Concept summary
Practice questions

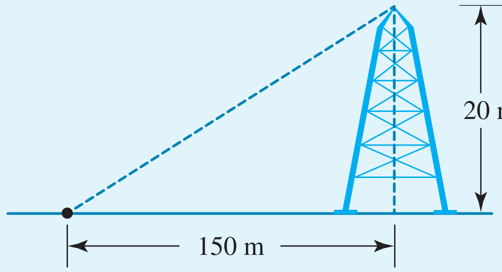
> **Angles of elevation and depression are in a *vertical plane.***



We can see from the diagram the angle of depression given from one location can give us the angle of elevation from the other position using the alternate angle law.

WORKED EXAMPLE 4

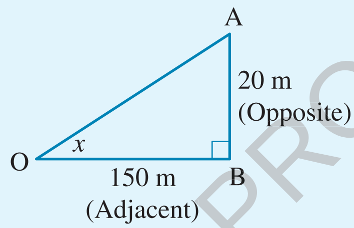
Find the angle of elevation (correct 2 decimal places) of the tower measured from the road as given in the diagram.



THINK

1 The angle of elevation is $\angle AOB$. Use ΔAOB and trigonometry to solve the problem.

WRITE/DRAW



2 The problem requires the tangent ratio. Substitute the values and simplify.

$$\tan(\theta) = \frac{\text{length of opposite side}}{\text{length of adjacent side}} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan(x) = \frac{20}{150} = 0.133\,33$$

3 Evaluate x and round correct to 2 decimal places.

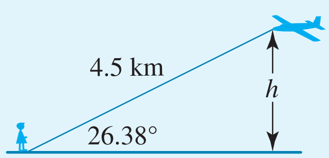
$$x = \tan^{-1}(0.133\,33) = 7.5946° \approx 7.59°$$

4 Write the answer.

From the road the angle of elevation to the tower is 7.59°, correct to 2 decimal places.

WORKED EXAMPLE 5

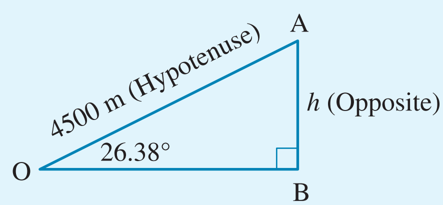
Find the altitude of a plane (correct to the nearest metre) if the plane is sighted 4.5 km directly away from an observer who measures its angle of elevation as 26.38°.



THINK

1 Draw a suitable diagram. Change distance to metres.

WRITE/DRAW



2 Use the sine ratio and simplify.

$$\sin(\theta) = \frac{\text{length of opposite side}}{\text{length of hypotenuse side}} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin(26.38°) = \frac{h}{4500}$$

$$h = 4500 \times \sin(26.38°)$$

3 Evaluate.

$$= 1999.45$$

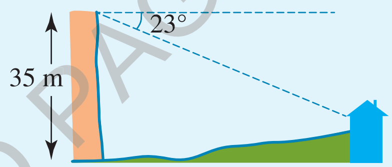
4 Write the answer in correct units.

The plane is flying at an altitude of 2000 m, correct to the nearest metre.

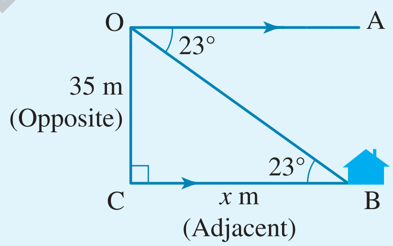
WORKED EXAMPLE 6 **The angle of depression from the top of a 35-metre cliff to a house at the bottom is 23°. How far from the base of the cliff is the house (correct to the nearest metre)?**

THINK | **WRITE/DRAW**

1 Draw a suitable diagram.



2 Angle of depression is ∠AOB. Use the alternate angle law to give the angle of elevation ∠CBO.



3 Use the tangent ratio. Substitute into the formula and evaluate.

$$\tan(\theta) = \frac{\text{length of opposite side}}{\text{length of adjacent side}} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan(23°) = \frac{35}{x}$$

$$\frac{1}{\tan(23°)} = \frac{x}{35}$$

$$x = \frac{35}{\tan(23°)} = 82.4548...$$

4 Write the answer in correct units.

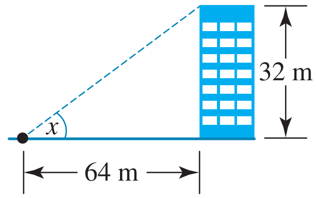
The distance from the house to the base of the cliff is 82 metres, correct to the nearest metre.

EXERCISE 13.3 Angles of elevation and depression

PRACTISE

For questions 1–6, find the values of the pronumerals.

1 WE4



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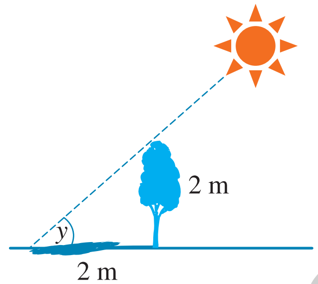
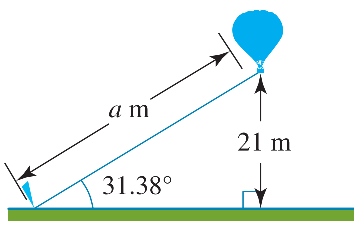


3 WE5



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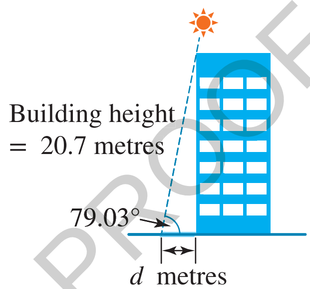
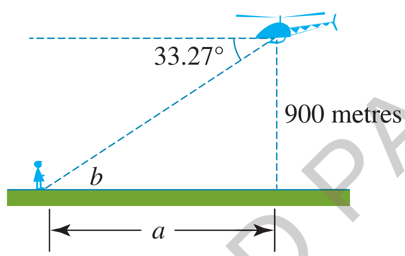


5 WE6



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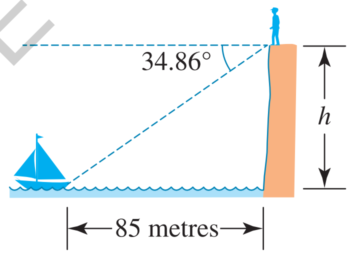
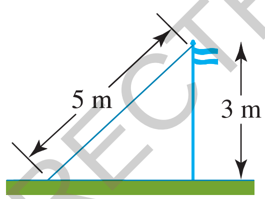


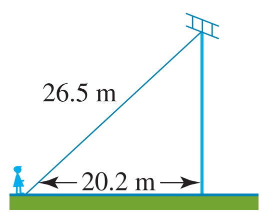
CONSOLIDATE

7 Find the angle of elevation (correct to 2 decimal places) in the following situations.

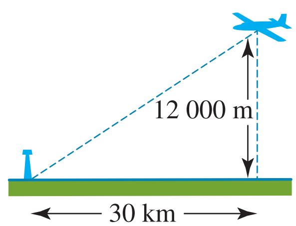
a



b

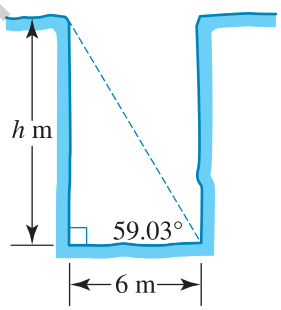


c

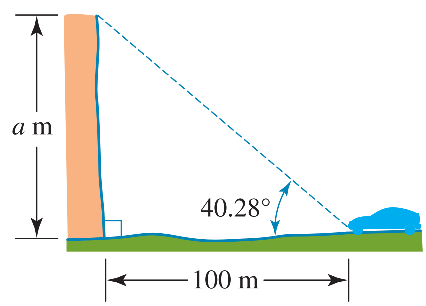


8 Find the values of the pronumerals (correct to the nearest metre).

a



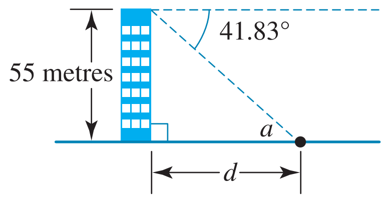
b



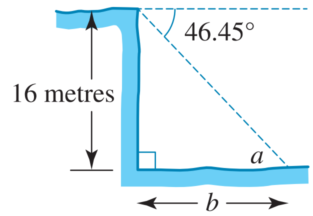
9 The angle of elevation of the sun at a particular time of the day was 49°. What was the length of a shadow cast by a 30-metre tall tower at that time?

10 Find the values of these pronumerals (correct to 2 decimal places).

a

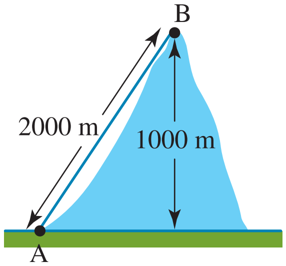


b

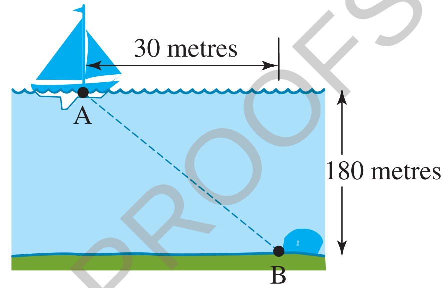


11 Find the angle of elevation or depression from an observer positioned at point A to the object at point B in each situation shown below, correct to two decimal places. State clearly whether it is an angle of depression or elevation.

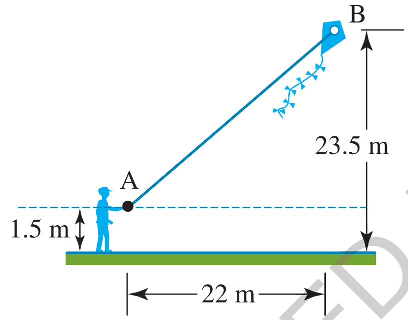
a



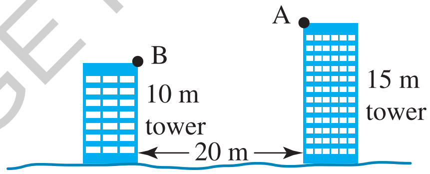
b



c



d



12 A hole has a diameter of 4 metres and is 3.5 metres deep. What is the angle of depression from the top of the hole to the bottom opposite side of the hole?

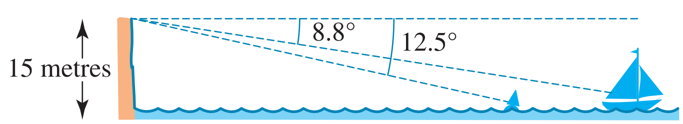
13 The angle of elevation of the top of a tree from a point 15.2 m from the base of the tree is 52.2°. The height of the tree is closest to:

A 12 m **B** 15 m **C** 19 m **D** 20 m **E** 25 m

14 A supporting wire for a 16-m high radio tower is 23.3 m long and is attached at ground level and to the top of the tower. The angle of depression of the wire from the top of the tower is:

A 34.48° **B** 43.37° **C** 46.63° **D** 55.52° **E** 45.16°

15 The angle of depression to a buoy from the top of a 15-metre cliff is 12.5°. A boat is observed to be directly behind but with an angle of depression of 8.8°.



Find (correct to the nearest metre):

a the distance to the buoy from the base of the cliff

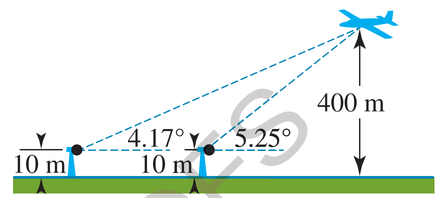
b the distance between the boat and the buoy.

16 Two buildings are 50 metres apart. Building A is 110 metres high. Building B is 40 metres high.

a Find the angle of elevation from the bottom of building A to the top of building B.
b Find the angle of depression from the top of building A to the bottom of building B.
c Find the angle of depression from the top of building B to the bottom of building A.

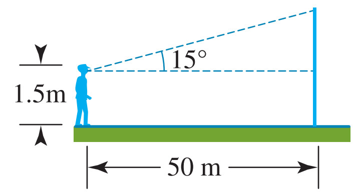
MASTER

17 Watchers in two 10-metre observation towers each spot an aircraft at an altitude of 400 metres. The angles of elevation from the two towers are shown in the diagram.
(Assume all three objects are in a direct line.)



a What is the horizontal distance between the nearest tower and the aircraft (correct to the nearest 10 metres)?
b How far apart are the two towers from each other (correct to the nearest 100 metres)?

18 A boy standing 1.5 metres tall measures the angle of elevation to the top of the goalpost using an inclinometer.



a If the angle was 15° when measured 50 metres from the base of the goalpost, how tall is the goalpost?
b If the angle of elevation to the top of the goalpost is now 55.5°, how far is the boy from the base of the goalpost?
c The angle of elevation is measured at ground level and is found to be 45°. Find the distance from the base of the goalpost to where the measurement was made.
d The result in part c is the same as the height of the goalpost. Explain why.

13.4 Bearings

Bearings are used to locate the position of objects or the direction of a journey on a two-dimensional horizontal plane. Bearings or directions are straight lines from one point to another. To find bearings, a compass rose (a diagram, as shown below, showing N, S, E and W) should be drawn centred on the point from where the bearing measurement is taken.

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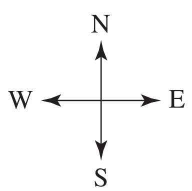
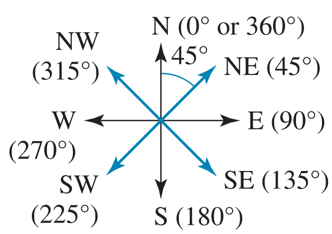
There are three main ways of specifying bearings or direction:

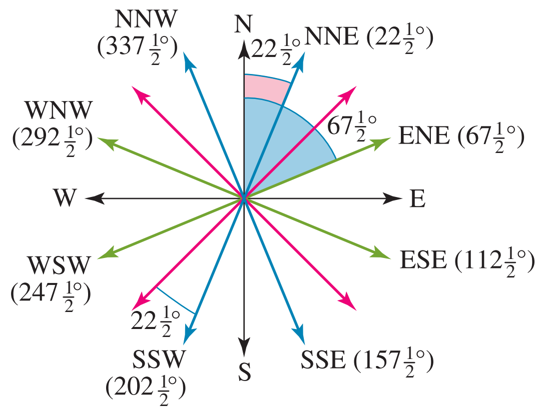
1. standard compass bearings (for example, N, SW, NE)
2. other compass bearings (for example, N10°W, S30°E, N45.37°E)
3. **true bearings** (for example, 100°T, 297°T, 045°T, 056°T)

Standard compass bearings

There are 16 main standard bearings as shown in the diagrams. The N, S, E and W standard bearings are called **cardinal points**.

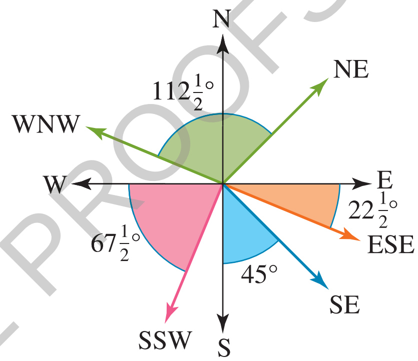






It is important to consider the angles between any two bearings. For example, the angles from north (N) to all 16 bearings are shown in brackets in the diagrams above and at the end of the opposite page.

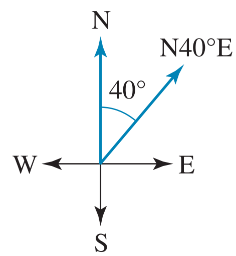
It can be seen that the angle between two adjacent bearings is $22\frac{1}{2}°$. Some other angles that will need to be considered are shown in the diagram.



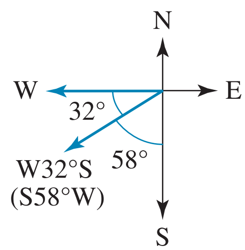
Other compass bearings

Often the direction required is not one of the 16 standard bearings. To specify other bearings the following approach is taken.

1. Start from north (N) or south (S).
2. Turn through the angle specified towards east (E) or west (W).

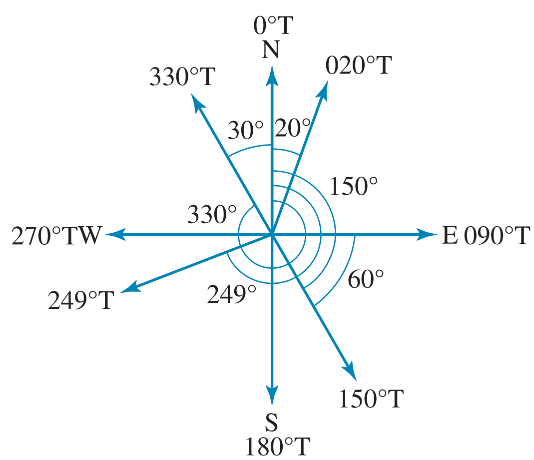


Sometimes the direction may be specified unconventionally, for example, starting from east or west as given by the example W32°S. This bearing is equivalent to S58°W.



True bearings

True bearings is another method for specifying directions and is commonly used in navigation.



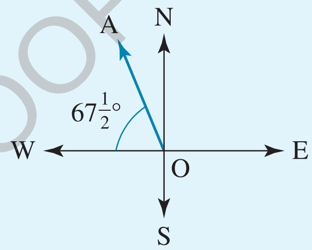
To specify true bearings, first consider the following:

1. the angle is measured from north
2. the angle is measured in a clockwise direction to the bearing line
3. the angle of rotation may take any value from 0° to 360°
4. the symbol T is used to indicate it is a true bearing, for example, 125°T, 270°T
5. for bearings less than 100°T, use three digits with the first digit being a zero to indicate it is a bearing, for example, 045°T, 078°T.

WORKED EXAMPLE 7

Specify the direction in the figure as:

a a standard compass bearing
b a compass bearing
c a true bearing.



THINK

a 1 Find the angle between the bearing line and north, that is, ∠AON.

WRITE

a $\angle AON = 90° - 67\frac{1}{2}°$
$= 22\frac{1}{2}°$

2 Since the angle is $22\frac{1}{2}°$, the bearing is a standard bearing. Refer to the standard bearing diagram.

The standard bearing is NNW.

b The bearing lies between the north and the west. The angle between north and the bearing line is $22\frac{1}{2}°$.

b The compass bearing is N$22\frac{1}{2}°$W.

c Find the angle from north to the bearing line in a clockwise direction. The bearing of west is 270°T.

c Angle required $= 270° + 67\frac{1}{2}°$
$= 337\frac{1}{2}°$
The true bearing is $337\frac{1}{2}°$T.

WORKED EXAMPLE 8

Draw a suitable diagram to represent the following directions.

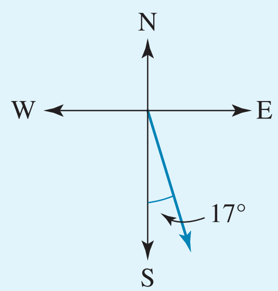
a S17°E
b 252°T

THINK

a Draw the 4 main standard bearings. A compass bearing of S17°E means start from south; turn 17° towards east. Draw a bearing line at 17°. Mark in an angle of 17°.

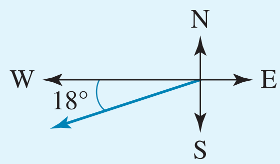
WRITE/DRAW

a



b A true bearing of 252°T is more than 180° and less than 270°, so the direction lies between south and west. Find the difference between the bearing and west (or south). Draw the 4 main standard bearings and add the bearing line. Add the angle from west (or south).

b Difference from west $= 270° - 252°$
$= 18°$



WORKED EXAMPLE 9

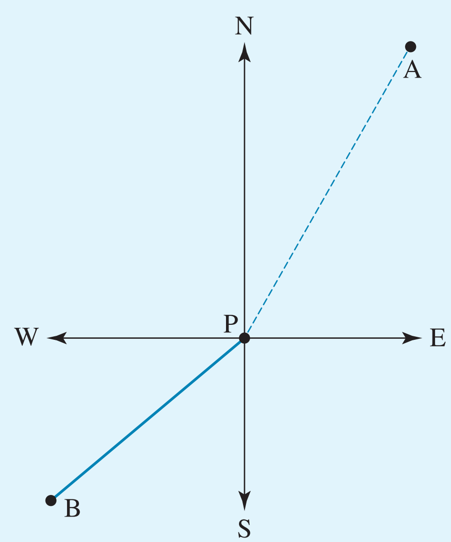
Convert:

a **the true bearing, 137°T, to a compass bearing**
b **the compass bearing, N25°W, to a true bearing.**

THINK	WRITE
a **1** The true bearing 137°T means the direction is between south and east. Find the angle from south to the bearing line.	**a** Angle required $= 180° - 137°$ $= 43°$
2 Write the compass bearing.	Compass bearing is S43°E.
b **1** State the angle between the bearing line and north.	**b** Angle from north $= 25°$
2 Find the angle from north to the bearing line in a clockwise direction.	Angle required $= 360° - 25°$ $= 335°$
3 Write the true bearing.	True bearing is 335°T.

WORKED EXAMPLE 10

Use your protractor to find the bearing of points A and B from location P. State the directions as true bearings and as compass bearings.



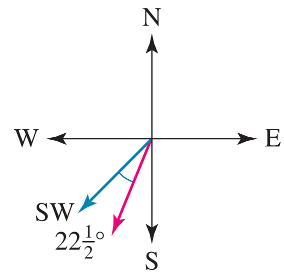
THINK	WRITE
1 Find ∠NPA and write as a true bearing and as a compass bearing.	$\angle NPA = 30°$ True bearing of A from P is 030°T. Compass bearing is N30°E.
2 Repeat for location B, this time with reference to south.	$\angle SPB = 50°$ True bearing of B from P is $180° + 50° = 230°$T. Compass bearing is S50°W.

EXERCISE 13.4 Bearings

PRACTISE

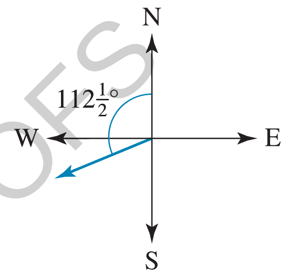
1 WE7 Specify the direction of the pink arrow as:

a a standard compass bearing
b a compass bearing
c a true bearing.



2 Specify the direction of the figure shown as:

a a standard compass bearing
b a compass bearing
c a true bearing.



3 WE8 Draw a suitable diagram to represent 080°T direction.

4 Draw a suitable diagram to represent N70°W direction.

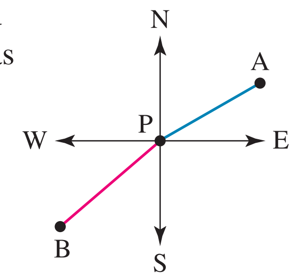
5 WE9 Convert:

a the true bearing, 152°T, to a compass bearing
b the compass bearing, N37°W, to a true bearing.

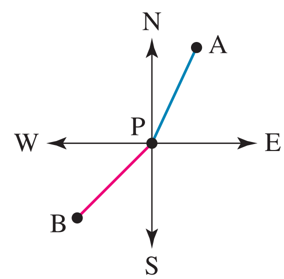
6 Convert:

a the true bearing, 239°T, to a compass bearing
b the compass bearing, S69°E, to a true bearing.

7 WE10 Use your protractor to find the bearing of points A and B from location P. State the directions as true bearings and as compass bearings.



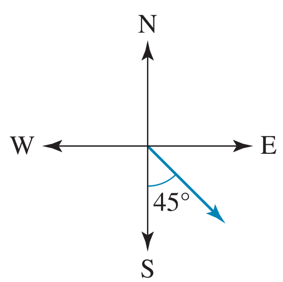
8 Use your protractor to find the bearing of points A and B from location P. State the directions as true bearings and as compass bearings.



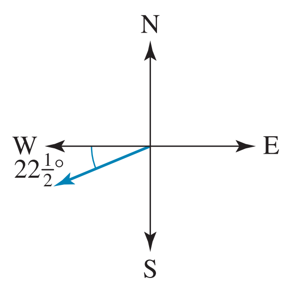
CONSOLIDATE

9 Specify the following directions as standard compass bearings.

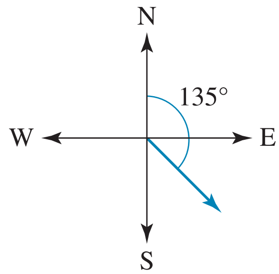
a



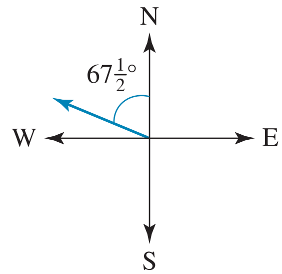
b



c

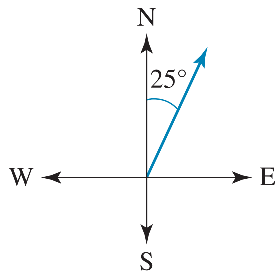


d



10 Specify the following directions as true bearings.

a



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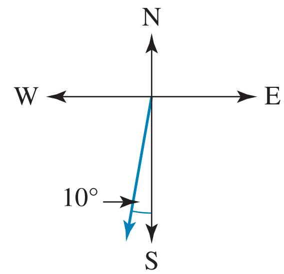
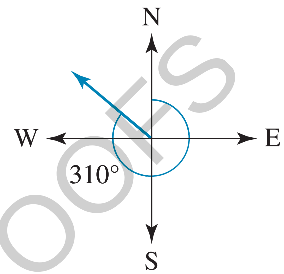


c



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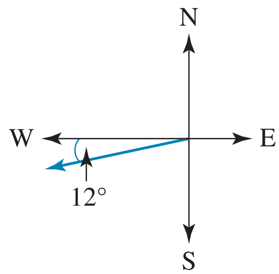
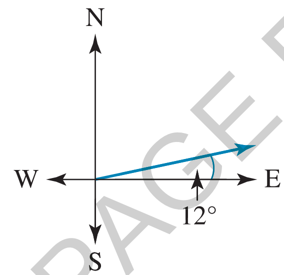
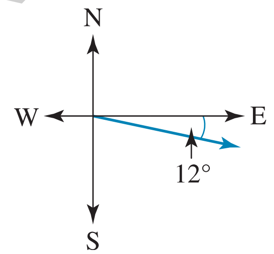


e



f



11 Draw suitable diagrams to represent the following directions.

a 045°T b 200°T c 028°T d 106°T e 270°T f 120°T

12 Convert the following true bearings to compass bearings.

a 040°T b $022\frac{1}{2}$°T c 180°T d 350°T

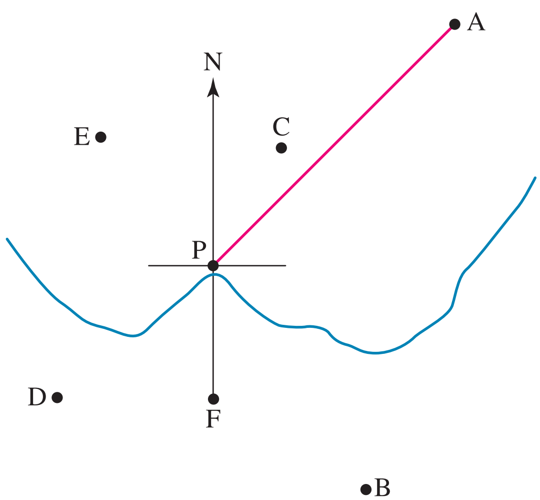
e 147°T f 67°30′T g 120°T h 135°T

13 Convert the following compass bearings to true bearings.

a N45°W b S$40\frac{1}{2}$°W c S d S35°E

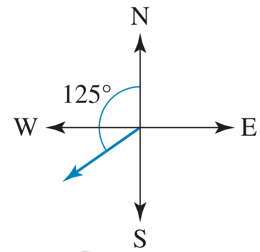
e N47°E f S67°30′W g NNW h S5°E

14 Use your protractor to find the bearing of each of the points from location P. State the directions as true bearings.



15 Now find the bearing of each of the points in the diagram from question **14** from location B (as true bearings). Also include the bearing from B to P and compare it to the direction from P to B.

16 The direction shown in the diagram is:



A 225°T **B** 305°T **C** 145°T
D 235°T **E** 125°T

17 An unknown direction — given that a second direction, 335°T, makes a straight angle with it — is:

A 165°T **B** 25°T **C** 155°T **D** 235°T **E** 135°T

18 The direction of a boat trip from Sydney directly to Auckland was 160°T. The direction of the return trip would be:

A 200°T **B** 250°T **C** 020°T **D** 235°T **E** 340°T

MASTER

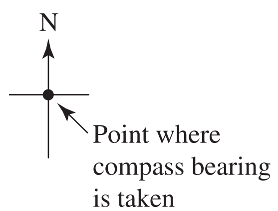
19 The direction of the first leg of a hiking trip was 220°T. For the second leg, the hikers turn 40° right. The new direction for the second leg of the hike is:

A 270°T **B** 180°T **C** 260°T **D** 040°T **E** 280°T

20 A hiker heads out on the direction 018°T. After walking 5 km they adjust their bearing 12° to their right and on the last leg of their hike they adjust their bearing 5° to their left. Calculate the bearing of their last leg.

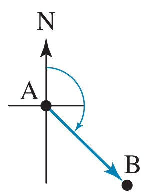
13.5 Navigation and specification of locations

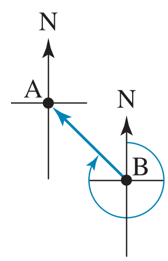
In most cases when you are asked to solve problems, a carefully drawn sketch of the situation will be given. When a problem is described in words only, very careful sketches of the situation are required. Furthermore, these sketches of the situation need to be converted to triangles with angles and lengths of sides included. This is so that Pythagoras' theorem, trigonometric ratios, areas of triangles, similarity and sine or cosine rules may be used.



Hints

1. Carefully follow given instructions.
2. Always draw the compass rose at the starting point of the direction requested.
3. Key words are *from* and *to*. For example:
   The bearing *from* A *to* B (see diagram left) is very different from the bearing *from* B *to* A (see diagram right).

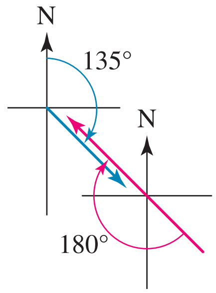


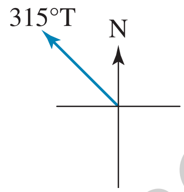


4. When you are asked to determine the direction to return directly back to an initial starting point, it is a 180° rotation or difference. For example, to return directly back after heading north, we need to change the direction to head south.
Other examples are:

Returning directly back after heading 135°T

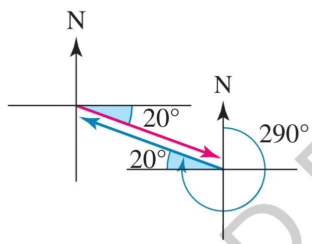
$$\begin{aligned}\text{New bearing} &= 135° + 180°\\ &= 315°\text{T}\end{aligned}$$

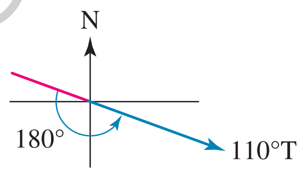




Returning directly back after heading 290°T

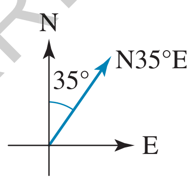
$$\begin{aligned}\text{New bearing} &= 290° - 180°\\ &= 110°\text{T}\end{aligned}$$

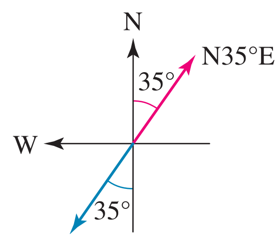




Returning directly back after heading N35°E

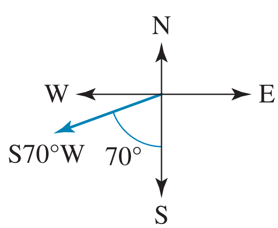
$$\begin{aligned}\text{New bearing} &= \text{N}35°\text{E} + 180°\\ &= \text{S}35°\text{W}\end{aligned}$$

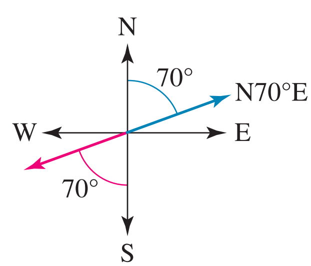




Returning directly back after heading S70°W

$$\text{New bearing} = \text{N}70°\text{E}$$





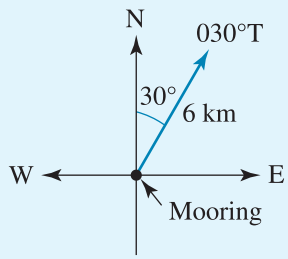
or simply use the opposite compass direction. North becomes south and east becomes west and vice versa.

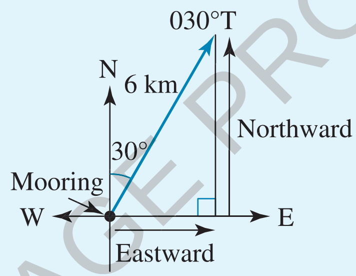
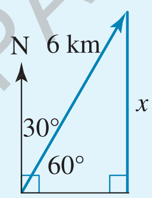
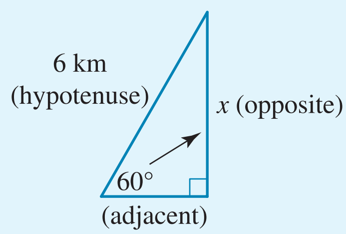
WORKED EXAMPLE 11

A ship leaves port, heading 030°T for 6 kilometres as shown.

a **How far north or south is the ship from its starting point (correct to 1 decimal place)?**

b **How far east or west is the ship from its starting point (correct to 1 decimal place)?**



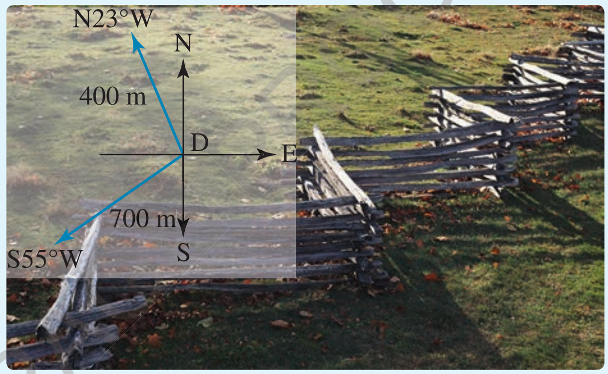
THINK	WRITE/DRAW
a **1** Draw a diagram of the journey and indicate or superimpose a suitable triangle.	**a** 
2 Identify the side of the triangle to be found. Redraw a simple triangle with the most important information provided. Use the bearing given to establish the angle in the triangle, that is, use the complementary angle law.	 $90° - 30° = 60°$
3 As the triangle is right-angled, the sine ratio can be used to find the distance north.	
4 Substitute and evaluate.	$\sin(\theta) = \dfrac{\text{length of opposite side}}{\text{length of hypotenuse side}}$ $= \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\sin(60°) = \dfrac{y}{6}$ $x = 6 \times \sin(60°)$ $= 6 \times 0.8660$ $= 5.196$
5 State the answer to the required number of decimal places.	The ship is 5.2 km north of its starting point, correct to 1 decimal place.

b **1** Use the same approach as in part **a**. This time the trigonometric ratio is cosine to find the distance east, using the same angle.	**b** $\cos(\theta) = \dfrac{\text{length of adjacent side}}{\text{length of hypotenuse side}} = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\cos(60°) = \dfrac{y}{6}$ $y = 6 \times \cos(60°) = 6 \times 0.5 = 3.0$
2 Answer in correct units and to the required level of accuracy.	The ship is 3.0 km east of its starting point, correct to 1 decimal place.

WORKED EXAMPLE 12

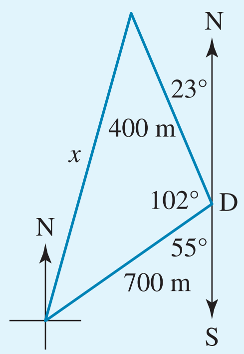
A triangular paddock has two complete fences. From location D, one fence line is on a bearing of 337°T for 400 metres. The other fence line is 235°T for 700 metres.

Find the length of fencing (correct to the nearest metre) required to complete the enclosure of the triangular paddock.



THINK | **WRITE/DRAW**

1 Redraw a simple triangle with the most important information provided. Identify the side of the triangle to be found.

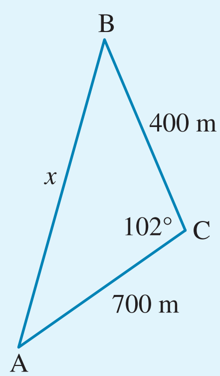


2 Use the bearings given to establish the angle in the triangle; that is, use the supplementary angle law.

In the triangle,
$\angle D = 180° - 23° - 55°$
$= 102°$

3 The cosine rule can be used, as two sides and the included angle are given.

State the values of the pronumerals and write the formula for finding the unknown side length using cosine rule.



$a = 400$ m, $b = 700$ m, $C = 102°$, $c = x$ m

THINK	WRITE
4 Substitute the values of the pronumerals into the formula and evaluate.	$c^2 = a^2 + b^2 - 2ab \times \cos(C)$ $x^2 = 400^2 + 700^2 - 2 \times 400 \times 700 \times \cos(102°)$ $= 650\,000 - 560\,000 \times -0.207\,91$ $= 766\,430.55$ $x = \sqrt{766\,430.55}$ $= 875.46$
5 Answer in correct units and to the required level of accuracy.	The new fence section is to be 875 metres long, correct to the nearest metre.

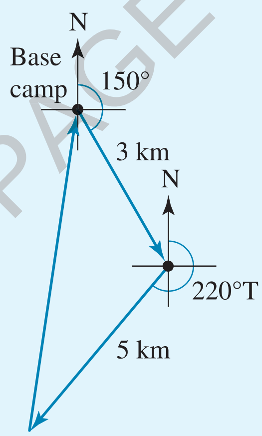
WORKED EXAMPLE 13

Soldiers on a reconnaissance set off on a return journey from their base camp. The journey consists of three legs. The first leg is on a bearing of 150°T for 3 km; the second is on a bearing of 220°T for 5 km.

Find the direction (correct to the nearest degree) and distance (correct to 2 decimal places) of the third leg by which the group returns to its base camp.

THINK | **WRITE/DRAW**

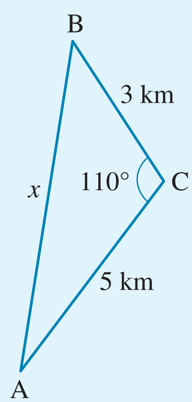
1 Draw a diagram of the journey and indicate or superimpose a suitable triangle.

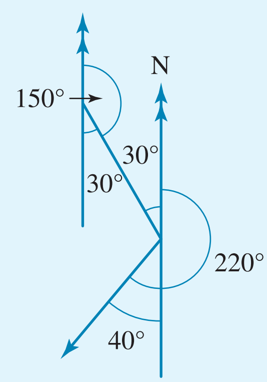


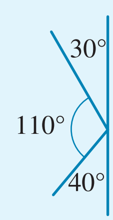
2 Identify the side of the triangle to be found. Redraw a simple triangle with the most important information provided.

(∠BCA = 180° − 30° − 40°.

∴ ∠BCA = 110°)







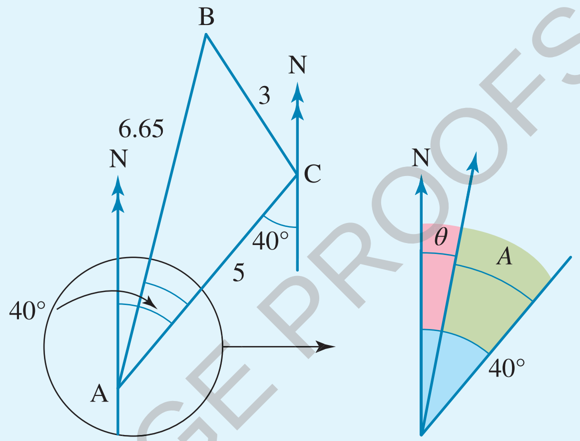
3 The cosine rule can be used to find the length of the side AB, as we are given two sides and the angle in between.

$a = 3$ km, $\quad b = 5$ km, $\quad C = 110°$, $\quad c = x$ km

4 Substitute the known values into the cosine rule and evaluate.

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab \times \cos(C) \\ x^2 &= 3^2 + 5^2 - 2 \times 3 \times 5 \times \cos(110°) \\ &= 44.260\,604 \\ x &= \sqrt{44.260\,604} \\ &= 6.65 \end{aligned}$$

5 For direction, we need to find the angle between the direction of the second and third legs first, that is, ∠BAC. Once the size of ∠BAC (or simply, ∠A) is established, it can be subtracted from 40° to find angle θ. This will give the bearing for the third leg of the journey. Since all 3 side lengths in ΔABC are now known, use the cosine rule to find ∠A.



$a = 3$, $\quad b = 5$, $\quad c = 6.65$ or $\sqrt{44.260\,604}$

6 Substitute the known values into the rearranged cosine rule.

Note: Use the most accurate form of the length of side c.

$$\begin{aligned} \cos(A) &= \frac{b^2 + c^2 - a^2}{2 \times b \times c} \\ &= \frac{5^2 + 44.260\,604 - 3^2}{2 \times 5 \times \sqrt{44.260\,604}} \\ &= 0.9058 \\ A &= 25.07° \\ &= 25° \text{(correct to the nearest degree)} \end{aligned}$$

7 Calculate the angle of the turn from the north bearing.

$$\begin{aligned} \theta &= 40° - 25° \\ &= 15° \end{aligned}$$

Bearing is 015°T.

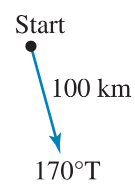
8 Write the answer in correct units and to the required level of accuracy.

The distance covered in the final leg is 6.65 km, correct to 2 decimal places, on a bearing of 015°T, correct to the nearest degree.

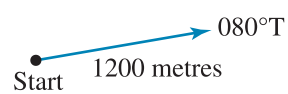
EXERCISE 13.5 Navigation and specification of locations

PRACTISE

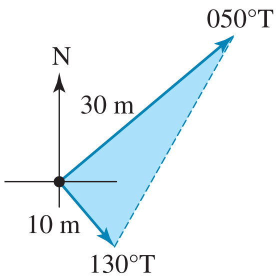
1 WE11 For the diagram at right, find how far north or south and east or west the end point is from the starting point (correct to 1 decimal place).



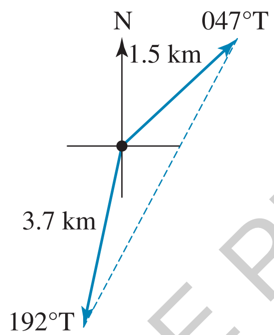
2 For the diagram at right, find how far north or south and east or west the end point is from the starting point (correct to 1 decimal place).



3 WE12 Find the length of the unknown side for the triangle (correct to the nearest unit).



4 Find the length of the unknown side for the triangle (correct to the nearest unit).

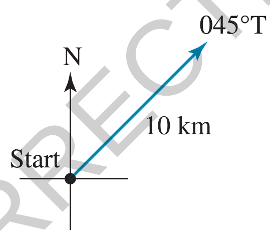


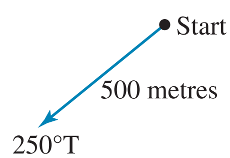
5 WE13 A scout troop travels on a hike of 2 km with a bearing of 048°T. They then turn and travel on a bearing of 125°T for 5 km, then camp for lunch. How far is the camp from the starting point? Give the answer correct to 2 decimal places.

6 Find the bearing required in question 5 to get the scout troop back to their starting point. Give your answer correct to the nearest degree.

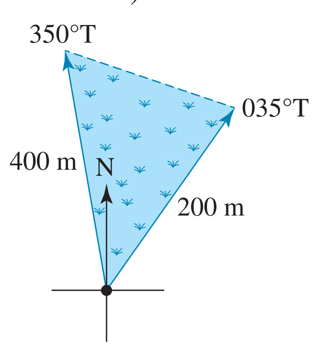
CONSOLIDATE

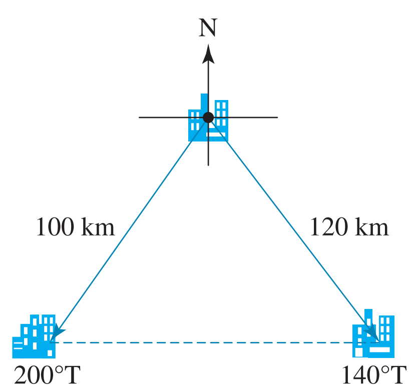
7 For the following, find how far north or south and east or west the end point is from the starting point (correct to 1 decimal place).

a


b


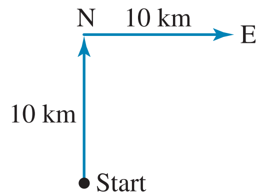
8 Find the length of the unknown side for each of the triangles (correct to the nearest unit).

a


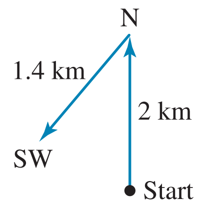
b


9 In each of the following diagrams, the first two legs of a journey are shown. Find the direction and distance of the third leg of the journey which returns to the start.

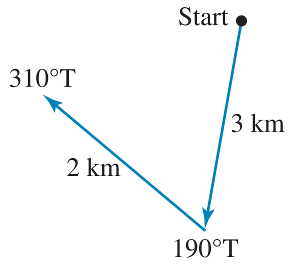
a



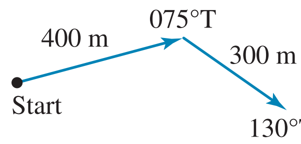
b



c



d



10 Draw a diagram to represent each of the directions specified below and give the direction required *to return* to the starting point:

a from A to B on a bearing of 320°T
b from C to E on a bearing of 157°T
c to F from G on a bearing of 215°T
d from B to A on a bearing of 237°T.

11 A boat sails from port A for 15 km on a bearing of N15°E before turning and sailing for 21 km in a direction of S75°E to port B.

a The distance between ports A and B is closest to:
A 15 km **B** 18 km **C** 21 km **D** 26 km **E** 36 km

b The bearing of port B from port A is closest to:
A N69.5°E **B** N54.5°E **C** N20.5°E **D** S20.5°E **E** 54.5°T

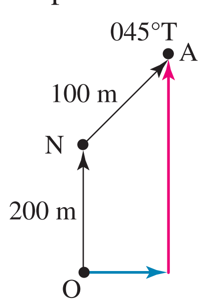
12 In a pigeon race, the birds start from the same place. In one race, pigeon A flew 35 km on a bearing of 295°T to get home, while pigeon B flew 26 km on a bearing of 174°T.

a The distance between the two pigeons' homes is closest to:
A 13 km **B** 18 km **C** 44 km **D** 50 km **E** 53 km

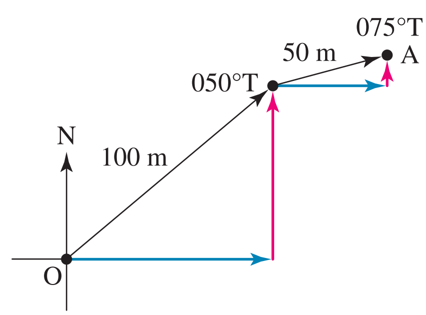
b The bearing of pigeon A's home from pigeon B's home is closest to:
A 332°T **B** 326°T **C** 320°T **D** 208°T **E** 220°T

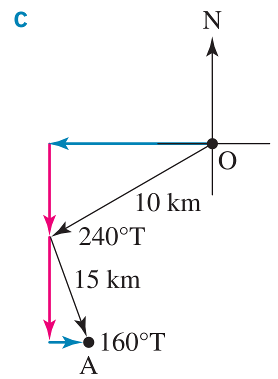
13 For each of the following, find how far north/south and east/west position A is from position O.

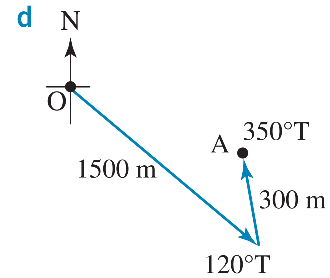
a



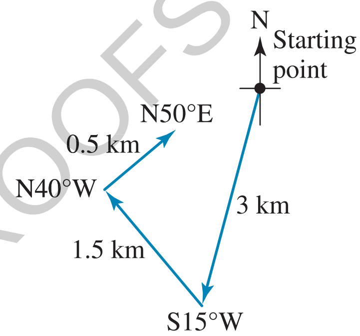
b







14 For the hiking trip shown in the diagram, find (correct to the nearest metre):

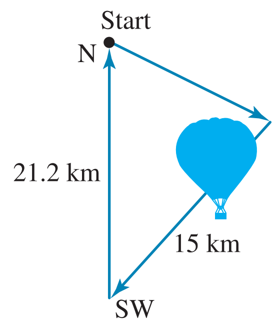


a how far south the hiker is from the starting point
b how far west the hiker is from the starting point
c the distance from the starting point
d the direction of the final leg to return to the starting point.

15 Captain Cook sailed from Cook Island on a bearing of 010°T for 100 km. He then changed direction and sailed for a further 50 km on a bearing of SE to reach a deserted island.

a How far from Cook Island is Captain Cook's ship (correct to the nearest kilometre)?
b Which direction would have been the most direct route from Cook Island to the deserted island (correct to 2 decimal places)?
c How much shorter would the trip have been using the direct route?

16 A journey by a hot-air balloon is as shown. The balloonist did not initially record the first leg of the journey. Find the direction and distance for the first leg of the balloonist's journey.



MASTER

17 A golfer is teeing off on the 1st hole. The distance and direction to the green is 450 metres on a bearing of 190°T. If the tee shot of the player was 210 metres on a bearing of 220°T, how far away from the green is the ball and in what direction should she aim to land the ball on the green with her second shot? (Give the distance correct to the nearest metre and the direction correct to the nearest degree.)

18 A boat begins a journey on a bearing of 063.28°T and travels for 20 km.

a How far east of its starting point is it?
It then changes to a bearing of 172.43°T and travels for a further 35 km.

b Through what angle did the boat turn?

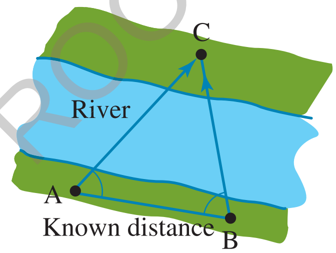
c How far is it now from its starting point?

d What is the bearing of its end point from the starting point?

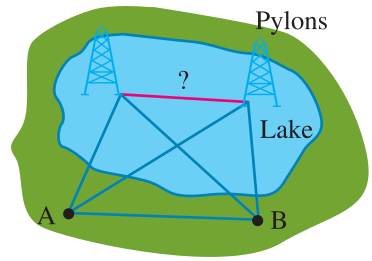
13.6 Triangulation — cosine and sine rules

In many situations, certain geographical or topographical features are not accessible to a survey. To find important locations or features, triangulation is used. This technique requires the coordination of bearings from two known locations, for example, fire spotting towers, to a third inaccessible location, the fire (see Worked example 14).

1. Triangulation should be used when:
   a. the distance between two locations is given and
   b. the direction from each of these two locations to the third inaccessible location is known.

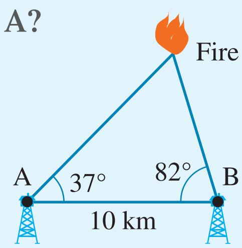


2. For triangulation:
   a. the sine rule is used to find distances from the known locations to the inaccessible one
   b. the cosine rule may be used occasionally for locating a fourth inaccessible location.



WORKED EXAMPLE 14

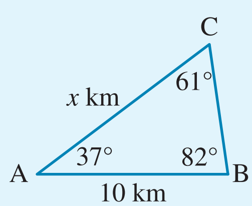
How far (correct to 1 decimal place) is the fire from Tower A?



THINK

1 Draw a triangle and identify it as a non-right-angled triangle with a given length and two known angles. Determine the value of the third angle and label appropriately for the sine rule.

WRITE/DRAW



$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

where

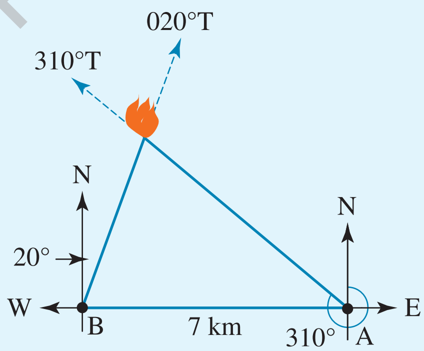
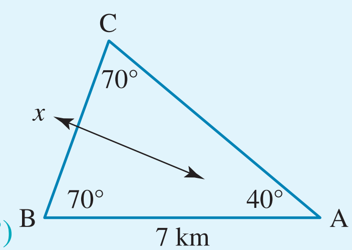
$c = 10\text{km}$ $\quad C = 180° - (37° + 82°)$
$\quad\quad\quad\quad\quad\quad = 61°$

$b = x$ $\quad B = 82°$

2 Substitute into the formula and evaluate.	$\dfrac{x}{\sin(82)°} = \dfrac{10}{\sin(61)°}$ $x = \dfrac{10 \times \sin(82)°}{\sin(61)°}$ $= 11.322$
3 Write the answer using correct units and to the required level of accuracy.	The fire is 11.3 km from Tower A, correct to 1 decimal place.

WORKED EXAMPLE 15

Two fire-spotting towers are 7 kilometres apart on an east–west line. From Tower A a fire is seen on a bearing of 310°T. From Tower B the same fire is spotted on a bearing of 020°T. Which tower is closest to the fire and how far is that tower from the fire (correct to 1 decimal place)?

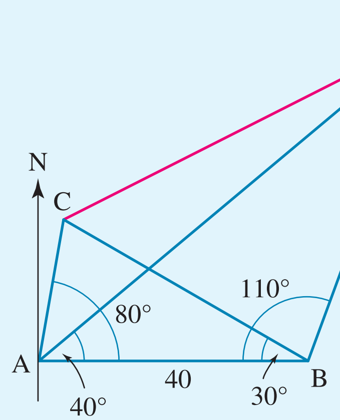
THINK	WRITE/DRAW
1 Draw a suitable sketch of the situation described. It is necessary to determine whether Tower A is east or west of Tower B.	
2 Identify the known values of the triangle and label appropriately for the sine rule. The *shortest side* of a triangle is *opposite* the *smallest angle*. Therefore, Tower B is closest to the fire. Use the sine rule to find the distance of Tower B from the fire.	$\dfrac{a}{\sin(A)} = \dfrac{c}{\sin(C)}$ where $a = x$ $c = 7$ km $A = 40°$ $C = 180° - (70° + 40°)$ $= 70°$ 
3 Substitute into the formula and evaluate. *Note:* ΔABC is an isosceles triangle, so Tower A is 7 km from the fire.	$\dfrac{x}{\sin(40)°} = \dfrac{7}{\sin(70)°}$ $x = \dfrac{7 \times \sin(40)°}{\sin(70)°}$ $= 4.788\,282$ km
4 Write the answer in the correct units.	Tower B is closest to the fire at a distance of 4.8 km, correct to 1 decimal place.

WORKED EXAMPLE 16

From the diagram find:

a **the length of CD (correct to 1 decimal place)**

b **the bearing from C to D (correct to 1 decimal place).**

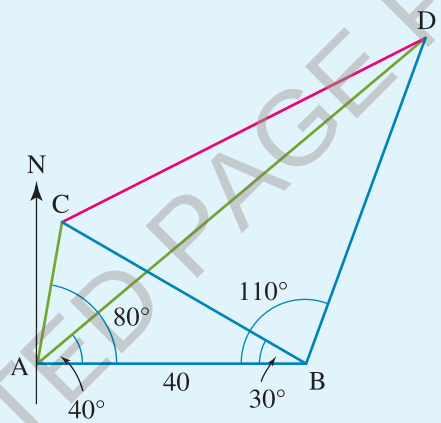


THINK

WRITE/DRAW

a **1** To evaluate the length of CD, we need to first determine the lengths of AC and AD. (Alternatively, we can find the lengths of BC and BD.)

a



2 Label ΔABC for the sine rule and evaluate the length of AC. ($\angle C = 180° - 30° - 80°$ $= 70°$)

$$\frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

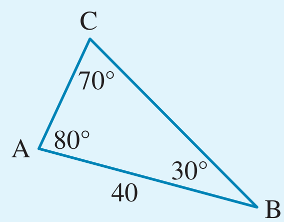
$b = \overline{AC}$, $B = 30°$

$c = 40$, $C = 70°$

$$\frac{\overline{AC}}{\sin(30)°} = \frac{40}{\sin(70)°}$$

$$\overline{AC} = \frac{40 \times \sin(30)°}{\sin(70)°}$$

$$= 21.283\,555$$



3 Label ΔABD for the sine rule and evaluate the length of AD. ($\angle D = 180° - 40° - 110°$ $= 30°$)

$$\frac{b}{\sin(B)} = \frac{d}{\sin(D)}$$

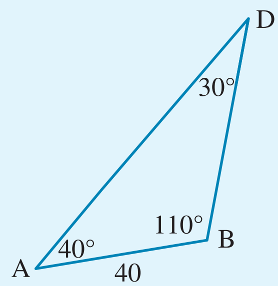
$b = \overline{AD}$, $B = 110°$

$d = 40$, $D = 30°$

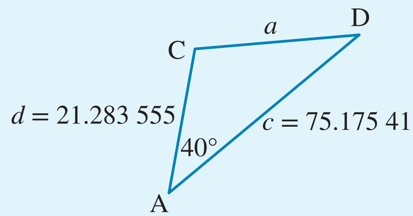
$$\frac{\overline{AD}}{\sin(110)°} = \frac{40}{\sin(30)°}$$

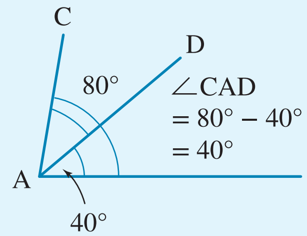
$$\overline{AD} = \frac{40 \times \sin(110)°}{\sin(30)°}$$

$$= 75.175\,41$$



4 Draw ΔACD, which is needed to find the length of CD. Use the two given angles to find ∠CAD. Now label it appropriately for the cosine rule.





$a = \overline{CD}$
$d = 21.283\,55$
$A = 40°$
$c = 75.175\,41$

5 Substitute into the formula and evaluate.

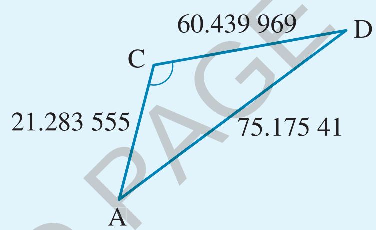
$$a^2 = c^2 + d^2 - 2cd \times \cos(A)$$
$$\overline{CD}^2 = 75.175\,41^2 + 21.283\,555^2 - 2 \times 75.175\,41 \times 21.283\,555 \times \cos(40°)$$
$$\overline{CD} = \sqrt{3652.9898}$$
$$= 60.439\,969$$

6 Write the answer in the correct units.

The length of $\overline{CD}$ is 60.4 units, correct to 1 decimal place.

b 1 Redraw ΔACD and label it with the known information.

b



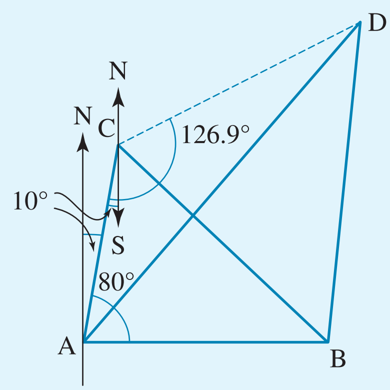
2 Bearing required is taken *from* C, so find ∠ACD by using the cosine rule first.

$a = 60.439\,969$
$c = 75.175\,41$
$d = 21.283\,555$

3 Substitute into the rearranged cosine rule and evaluate *C*.

$$\cos(C) = \frac{a^2 + d^2 - c^2}{2 \times a \times d}$$
$$= \frac{(60.439\,969)^2 + (21.283\,555)^2 - (75.175\,41)^2}{2 \times 60.439\,969 \times 21.283\,555}$$
$$= -0.6007$$
$$C = 126.9°$$

4 Redraw the initial diagram (from the question) with known angles at point C in order to find the actual bearing angle.



5 Determine the angle from south to the line CD.

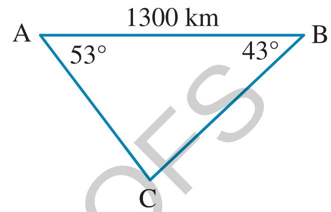
$$\angle SCD = 126.9° - 10°$$
$$= 116.9°$$

6	Determine the bearing angle.	$\angle NCD = 180° - 116.9°$ $= 63.1°$
7	Write the bearing of D from C.	The bearing of D from C is N63.1°E, correct to 1 decimal place.

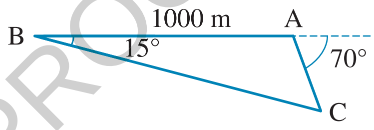
EXERCISE 13.6 Triangulation — cosine and sine rules

PRACTISE

1 WE14 Find the distance from A to C (correct to 1 decimal place).

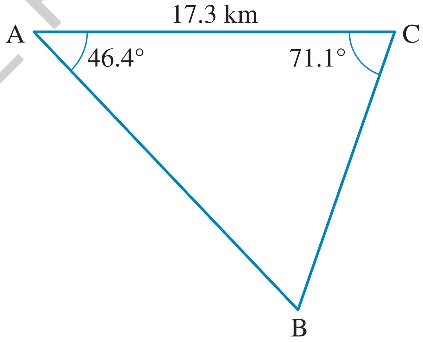


2 Find the distance from A to C (correct to 1 decimal place).



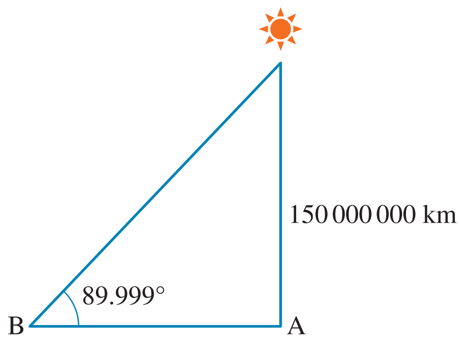
3 WE15 A boat (B) is spotted by 2 lighthouses (A and C), which are 17.3 km apart. The angles measured by each lighthouse are shown in the figure at right.

Determine the distance from each lighthouse to the boat.

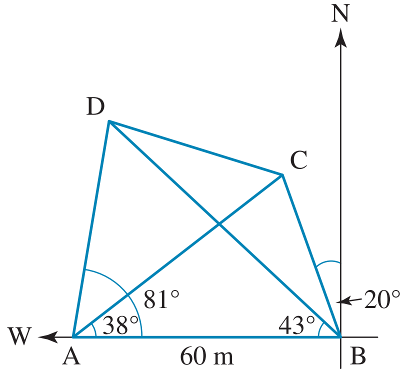


4 Two ancient astronomers lived in different towns (A and B) and wished to know the distance between the towns. At the same time they measured the angle of the sun, as shown in the diagram at right. Assuming the distance of the astronomer from town A to the sun was 150 000 000 km, find the distance between the towns (i.e. the distance AB).

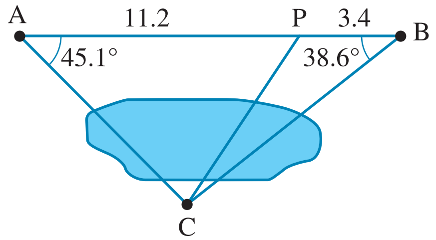
(Note that due to the nature of the angles, this is not an accurate way to measure this distance.)



5 WE16 Given the information in the diagram below, find the length of CD.



6 Three towns, A, B and C, are situated as shown in the diagram below.



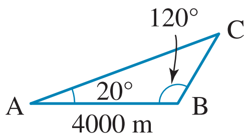
A mountain range separates town C from the other two, but a road is to be built, starting at P and tunnelling through the mountain in a straight line to C. From geographic information, the angles (45.1°, 38.6°) and distances (11.2 km, 3.4 km) between points are determined as shown in the figure.

Find the angle the road PC makes with the line AB (∠APC).

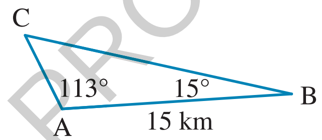
CONSOLIDATE

7 Find the distance from A to C in each case (correct to 1 decimal place).

a



b



8 a Two fire-spotting towers are 17 kilometres apart on an east–west line. From Tower A, a fire is seen on a bearing of 130°T. From Tower B, the same fire is spotted on a bearing of 200°T. Which tower is closest to the fire and how far is that tower from the fire?

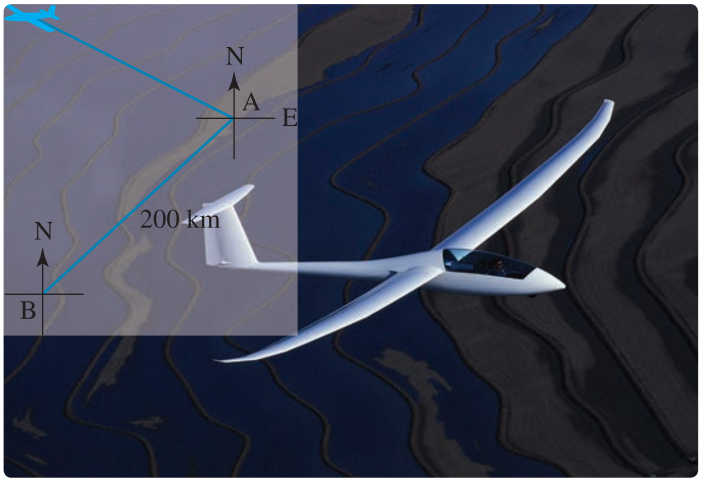
b Two fire-spotting towers are 25 kilometres apart on a north–south line. From Tower A, a fire is reported on a bearing of 082°T. Spotters in Tower B see the same fire on a bearing of 165°T. Which tower is closest to the fire and how far is that tower from the fire?

9 Two lighthouses are 20 km apart on a north–south line. The northern lighthouse spots a ship on a bearing of 100°T. The southern lighthouse spots the same ship on a bearing of 040°T.

a Find the distance from the northern lighthouse to the ship.

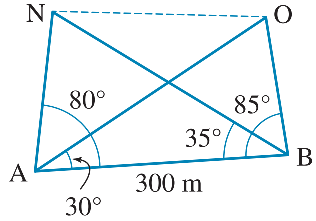
b Find the distance from the southern lighthouse to the ship.

10 Two air traffic control towers detect a glider that has strayed into a major air corridor. Tower A has the glider on a bearing of 315°T. Tower B has the glider on a bearing of north. The two towers are 200 kilometres apart on a NE line as shown. To which tower is the glider closer? What is the distance?

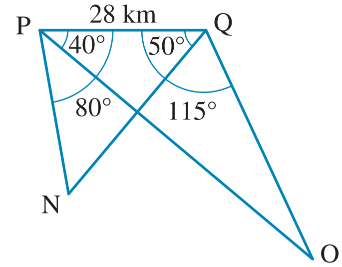


11 Find the value of line segment NO in each case (correct to 1 decimal place).

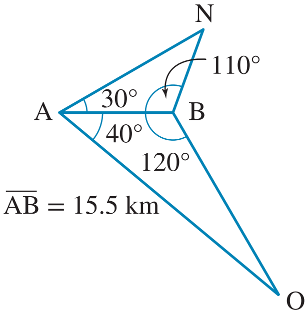
a



b

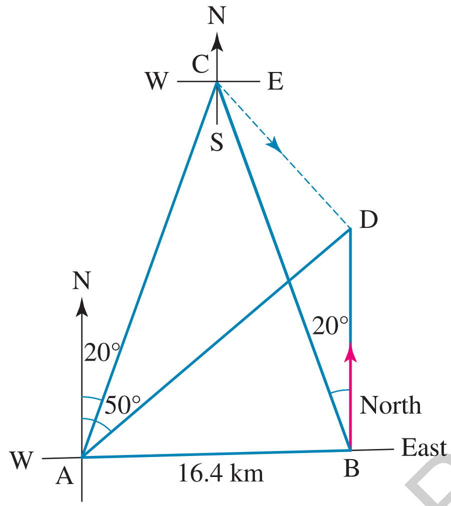


c

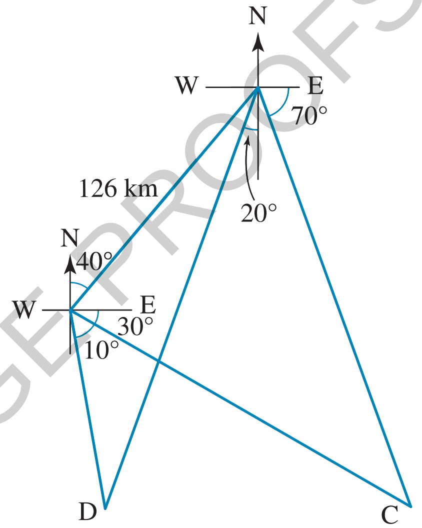


12 Find the distance and bearing from C to D (both correct to 1 decimal place).

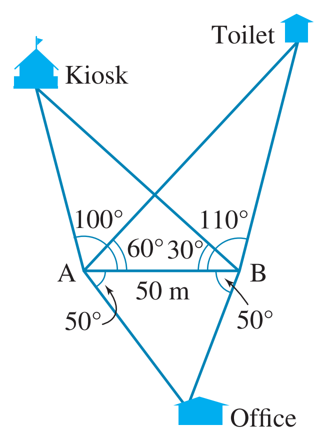
a



b



13 A student surveys her school grounds and makes the necessary measurements to 3 key locations as shown in the diagram.



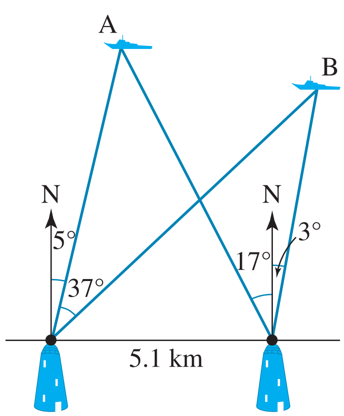
a Find the distance to the kiosk from:
 i location A
 ii location B.

b Find the distance to the toilets from:
 i location A
 ii location B.

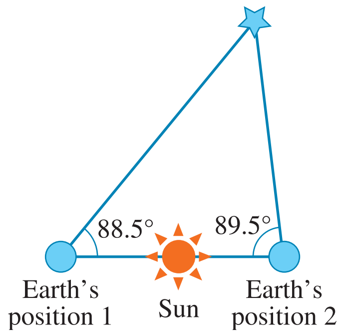
c Find the distance from the toilet to the kiosk.

d Find the distance from the office to location A.

14 From the diagram below, find the distance between the two ships and the bearing from ship A to ship B.



15 An astronomer uses direction measurements to a distant star taken 6 months apart, as seen in the diagram below (which is not drawn to scale). The known diameter of the Earth's orbit around the Sun is 300 million kilometres. Find the closest distance from Earth to the star (correct to the nearest million kilometres).



16 Two girls walk 100 metres from a landmark. One girl heads off on a bearing of 136°T, while the other is on a bearing of 032°T. After their walk, the distance between the two girls, correct to the nearest metre, is closest to:

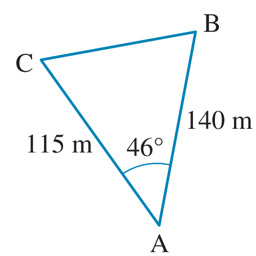
A 123 m **B** 158 m **C** 126 m **D** 185 m **E** 200 m

MASTER

17 Two ships leave the same port and sail the same distance, one ship on a bearing of NW and the other on a bearing of SSE. If they are 200 kilometres apart, what was the distance sailed by each ship?

A 100 km **B** 101 km **C** 102 km **D** 202 km **E** 204 km

18 In the swimming component of a triathlon, competitors have to swim around the buoys A, B and C, as marked in the figure, to end at point A.



a What is the shortest leg of the swimming component?
b What is the angle at buoy B?
c What is the total distance travelled?

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13.7 Review

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The Maths Quest Review is available in a customisable format for you to demonstrate your knowledge of this topic.

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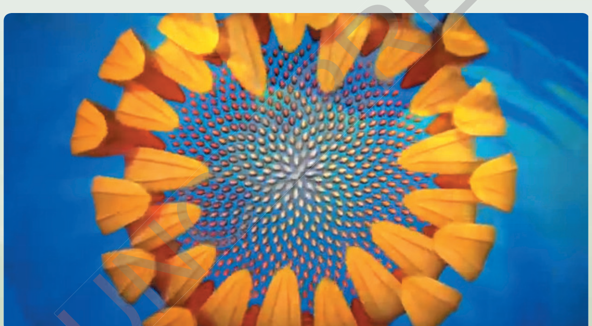

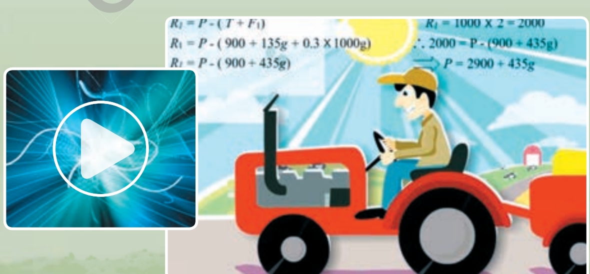

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13 ANSWERS

EXERCISE 13.2

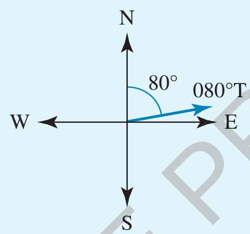
1 a 70.41° b 53.13°
2 a 70.07° b 44.96°
3 $x = 127°$
4 $x = 33°$
5 $A = 49°$, $C = 131°$
6 $P = 110°$, $Q = 70°$
7 A
8 a 134.44° b 17.82°
9 a 0.646 b −0.535
c 7.207 d 0.957
e 0.537 f 0.520
10 a 68.33°, 12.17° b 201.56°, 42.86°
c 182.89°, 18.77° d 388.1°, 106.94°
e 523.21°, 169.53° f 218.97°, 205.69°
11 a 21.67° b 40.5°
12 a 32° b 143°
13 a $a = b = 40.36°$ b 57.71°
14 a 162.5° b 67.35°
15 a 57.68° b 32°
16 a $a = c = 43.45°$, $b = 136.55°$ b $a = 39.61°$, $b = 17.8°$
17 a B b A
18 $a = 48.915°$, $b = 96.6°$

EXERCISE 13.3

1 26.57°
2 45°
3 40.33 m
4 4.01 m
5 $b = 33.27°$, $a = 1371.68$ m
6 59.21 m
7 a 36.87° b 40.34° c 21.80°
8 a 10 m b 85 m
9 26 metres
10 a $a = 41.83°$ $d = 61.45$ m
b $a = 46.45°$ $b = 15.21$ m
11 a Elevation 30° b Depression 80.54°
c Elevation 45° d Depression 14.04°
12 41.19°
13 D
14 B
15 a 68 m b 29 m
16 a 38.66° b 65.56° c 38.66°
17 a 4240 m b 1100 m
18 a 14.9 m b 9.2 m
c 14.9 m d A right-angled isosceles triangle

EXERCISE 13.4

1 a SSW b S $22\frac{1}{2}°$ W c $202\frac{1}{2}°$T
2 a WSW b S $67\frac{1}{2}°$ W c $247\frac{1}{2}°$T

3


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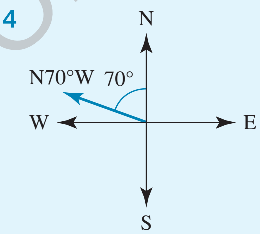
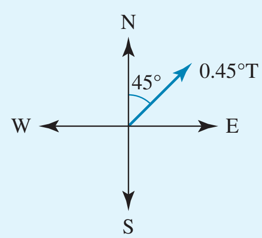
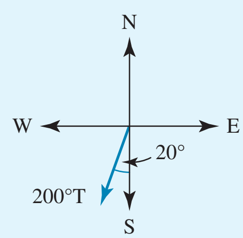
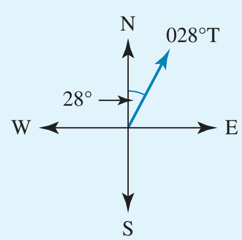


5 a S28°E b 323°T
6 a S59°W b 111°T
7 Point A: 060°T, N60°E Point B: 230°T, S50°W
8 Point A: 025°T, N25°E Point B: 225°T, S45°W
9 a SE b WSW
c SE d WNW
10 a 025°T b 190°T
c 310°T d 258°T
e 078°T f 102°T

11 a


b


c


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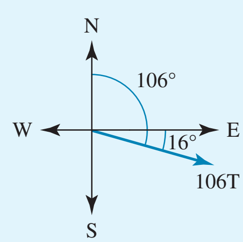
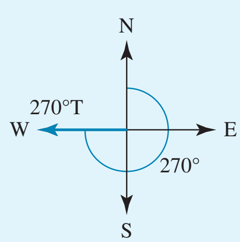
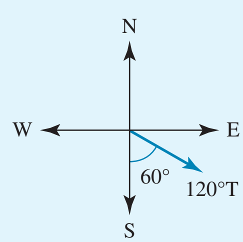


e


f


12 a N40°E b N22.5°E
c S d N10°W
e S33°E f N67°30′E
g S60°E h SE

13 a 315°T b 220.5°T
c 180°T d 145°T
e 047°T f 247°30′T
g $337\frac{1}{2}$°T h 175°T

14 A 045°T B 150°T
C 030°T D 230°T
E 318°T F 180°T

15 A 010°T C 347°T D 293°T
E 326°T F 310°T
From B to P: 330°T; From P to B: 150°T

16 D

17 C

18 E

19 C

20 025°T

EXERCISE 13.5

1 98.5 km south, 17.4 km east

2 208.4 m north, 1181.8 m east

3 30 m

4 5 km

5 5.79 km

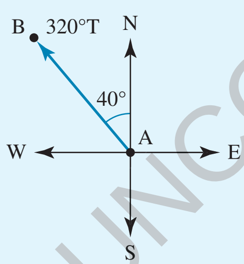
6 286°T

7 a 7.1 km north, 7.1 km east
b 171.0 m south, 469.8 m west

8 a 295 m b 111 km

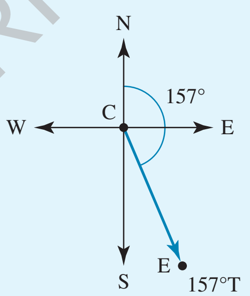
9 a 14.1 km SW or 225°T b 1.4 km SE or 135°T
c 2.65 km, 050.9°T d 622.6 m, 278.25°T

10 a



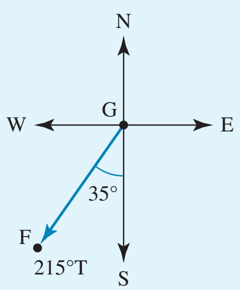
Return = 140°T

b



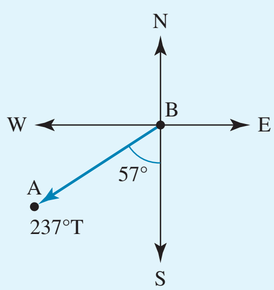
Return = 337°T

c



Return = 035°T

d



Return = 057°T

11 a D b A

12 a E b C

13 a 271 m north, 71 m east
b 77.2 m north, 124.9 m east
c 19.1 km south, 3.5 km west
d 454.6 m south, 1246.9 m east

14 a 1427 m b 1358 m
c 1970 m d N43°34′E

15 a 82 km b 039.87°T c 68 km

16 15 km, SE

17 288 m, 169°T

18 a 17.86 km b 109.15°
c 34.14 km d 119.02°T

EXERCISE 13.6

1 891.5 km

2 316.0 m

3 CB = 14.12 km, AB = 18.45 km

4 2618 km

5 40.87 m

6 53.9°

7 a 5389.2 m b 4.9 km

8 a Tower B, 11.6 km b Tower A, 6.5 km

9 a 14.8 km b 22.7 km

10 Tower A (200 km)

11 a 253.6 m b 42.6 km c 38.0 km

12 a 12.0 km, 136.1°T b 801.8 km, 222.6°T

13 a i 33 m
ii 64 m
b i 271 m
ii 249 m
c 247 m
d 39 m

14 8.1 km, 148.53°T

15 8593 million km

16 B

17 C

18 a BC
b 54°
c 357.26 m