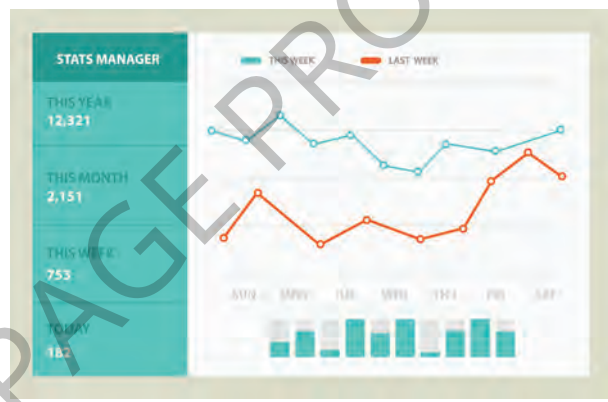
TOPIC 14
Construction and interpretation of graphs

14.1 Overview

14.1.1 Introduction



Graphs are a means of providing visual representations of within data. Large amounts of data are very unwieldly but once organised into suitable graphs can be analysed and interpreted. Every organisation, large and small, use graphs to assist decision making. Corporations and businesses rely on graphs to provide important information to all their stakeholders in weekly, monthly and yearly reports. Scientists, economists, sales and marketing personnel, government departments, educators, traffic engineers and the media, to name just a few, all rely on graphs to make future predictions and hence convince stake holders that change may be necessary or even essential.

Recent data has shown that from Christmas 2016 to Christmas 2017, online spending on significant retail days, such as, Click Frenzy, Black Friday, Cyber Monday and the Amazon AU launch has demonstrated a significant increase. This increase is clearly shown in the graph of the data. The visual impact of these spikes is hard to ignore and can help retailers both online and offline to consider what affect this may have on their business in subsequent years and make decisions accordingly.

It is important that everyone has the skills and knowledge to interpret the graphs that are providing information about all aspects of life, including health, study scores, mobile phone accounts, electricity and gas usage and billing, housing costs, consumer trends and environmental issues. This knowledge at both a personal and global level ensures informed consumers and citizens.

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LEARNING SEQUENCE

14.1 Overview
14.2 Constructing and interpreting straight-line graphs
14.3 Line segments and step functions
14.4 Simultaneous equations and break-even point
14.5 Interpreting non-linear graphs
14.6 Constructing non-linear relations and graphs
14.7 Review: exam practice

Fully worked solutions for this topic are available in the Resources section of your eBookPLUS at www.jacplus.com.au.

14.1.2 Kick off with CAS

Solving simultaneous equations with CAS

Simultaneous equations are a set of two (or more) equations that contain the same variables. By solving simultaneous equations we are finding the solution(s) which satisfy all of the equations within the set.

To solve simultaneous equations graphically we can sketch graphs of the equations on the same set of axes. The intersection of the two graphs is the solution to the set of simultaneous equations.

1. Using CAS, sketch the graph of $y = 4x - 3$.
2. Using CAS, sketch the graph of $y = 6 - 3x$ on the same set of axes as the graph in question 1.
3. Use CAS to find the point of intersection of both of the graphs you have sketched.
4. Confirm your solution to question 3 by using CAS to solve the pair of simultaneous equations ($y = 4x - 3$ and $y = 6 - 3x$) algebraically on a calculator page.
5. Use CAS to sketch the graphs of $y = 2x + 5$ and $y = 2x - 1$ on the same set of axes.
6. Is there a point of intersection between the two graphs you plotted in question 5? What is the relationship between the two equations?

on Resources

Please refer to the Resources tab in the Prelims section of your eBookPLUS for a comprehensive step-by-step guide on how to use your CAS technology.

14.2 Constructing and interpreting straight-line graphs

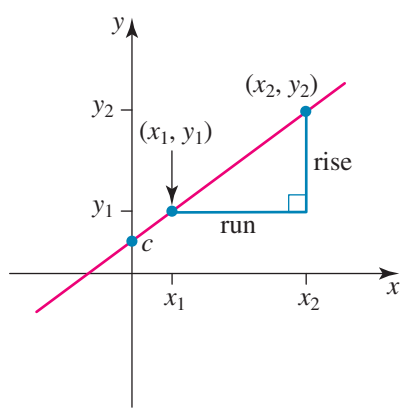
Equations and graphs are used to study the relationship between two variables such as distance and speed, tax payable and income, or radioactivity and time.

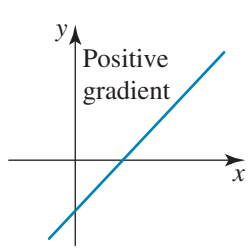
If a relationship exists between the variables, one can be said to be a **function** of the other. A function can be described by a table, a rule or a graph. A function whose graph is a straight line is called a **linear function**.

The general equation of a straight line is $y = mx + c$, where m is the gradient and c is the y-intercept.

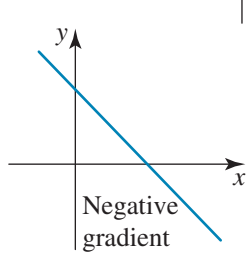
The ***y*-intercept** is the value of y where the graph cuts the y-axis.
The **gradient** (or slope) of a straight line is denoted by m where

$$m = \frac{\text{rise}}{\text{run}} \text{ or } m = \frac{y_2 - y_1}{x_2 - x_1}.$$





A graph that rises as x increases will have a positive gradient.



A graph that falls as x increases will have a negative gradient.

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WORKED EXAMPLE 1

Consider the equation $y = 3x + 4$.
a. What is the value of y when $x = 2$?
b. What is the value of x when $y = 13$?
c. Does the point (2, 4) lie on the graph of $y = 3x + 4$?
d. Does the point (5, 19) lie on the graph of $y = 3x + 4$?
e. State the value of the gradient and the y-intercept.

THINK	WRITE
a. Substitute $x = 2$ into the equation.	a. When $x = 2$, $y = 3 \times 2 + 4$ $= 10$
b. Substitute $y = 13$ into the equation and solve for x.	b. When $y = 13$, $13 = 3x + 4$ $3x = 9$ $x = 3$
c. Substitute 2 for x and 4 for y. If this makes the equation true then the point lies on the line; otherwise it does not lie on the line.	c. Does $4 = 3 \times 2 + 4$? 4 does not equal 10. The equation is not true, therefore the point (2, 4) does not lie on the line.
d. Substitute 5 for x and 19 for y. If this makes the equation true then the point lies on the line; otherwise it does not lie on the line.	d. Does $19 = 3 \times 5 + 4$? The equation is true, therefore the point (5, 19) does lie on the line.
e. Compare the given equation with that of the general equation of a straight line, $y = mx + c$. The gradient is the coefficient of the x-term, m, and the y-intercept is the constant term, c.	e. The gradient is 3 and the y-intercept is 4.

WORKED EXAMPLE 2

Two points A (1, 7) and B (3, 13) lie on the same line. Use the points A and B to calculate the gradient of the line.

THINK	WRITE
1. Write the gradient formula and match points A and B with (x_1, y_1) and (x_2, y_2).	$m = \frac{y_2 - y_1}{x_2 - x_1}$ where $(x_1, y_1) = (1, 7)$ and $(x_2, y_2) = (3, 13)$
2. Substitute the values into the formula and evaluate.	$m = \frac{13 - 7}{3 - 1}$ $= \frac{6}{2}$ $= 3$ The gradient is 3.

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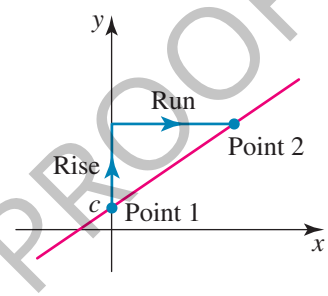
14.2.1 Sketching straight-line graphs

In previous years of study you have learned that only one straight line can be drawn through two distinct points. Therefore, the graph of a straight line can be obtained by plotting and joining together any two points on the line. Thus, to sketch a straight-line graph by hand, we first need to find the coordinates of any two points on the line. This can be done in a number of different ways. The three most common methods are outlined as follows.

Gradient–intercept method

This method is used if the equation is in $y = mx + c$ form.

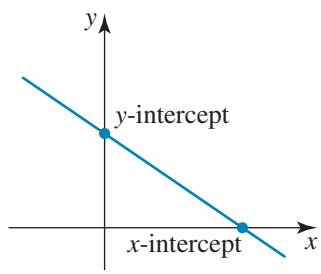
1. The first point plotted is the y-intercept, given by the value of c. Plot it on a set of axes.
2. The gradient is given by $m = \frac{\text{rise}}{\text{run}}$. Write the gradient as a fraction and identify the values of the rise and the run.
3. To obtain the second point, start from the y-intercept and move up (or down) and across, as suggested by the gradient.
4. Join the two points together with a straight line and label the graph.



x- and y-intercept method

This method is used if the equation is in $ax + by = c$ form, or if you are required to show both the x- and y-intercepts.

1. If a point is on the y-axis, its x-coordinate is 0. To find the y-intercept, substitute 0 for x and solve the resultant equation.
2. If a point is on the x-axis, its y-coordinate is 0. To find the x-intercept, substitute 0 for y and solve the resultant equation.
3. Plot the x-intercept and the y-intercept on a set of axes.
4. Join the two points together with a straight line and label the graph.

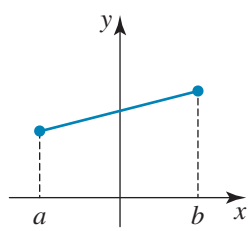


Sketching a line over a required interval

If a graph needs to be sketched between two given x-values, its end points must be shown. Since only two points are needed to sketch a line, we can obtain the coordinates of the end points and join them together. To construct a graph of a straight line between a and b, follow these steps:

1. Rearrange the equation to make y the subject.
2. Substitute each of the two given x-values (that is, a and b) into the equation and find corresponding values of y.
3. Plot the end points on a set of axes.
4. Join the two points together with a straight line and label the graph.

Worked example 3 illustrates the use of these three methods.



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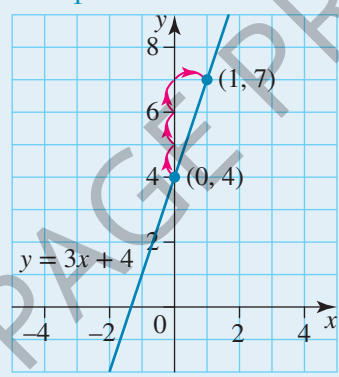
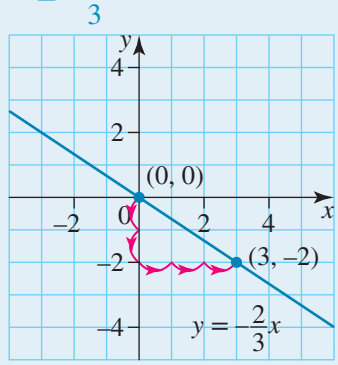
WORKED EXAMPLE 3

Sketch the graph of each of the following equations.

a. $y = 3x + 4$ **b.** $y = -\frac{2}{3}x$ **c.** $2x + 3y = 6$

d. $x - y = 3$ **between** $x = -2$ **and** $x = 6$.

THINK	WRITE/DRAW
a. 1. The equation is in $y = mx + c$ form, so use the gradient-intercept method. State the value of the y-intercept.	**a.** $y = 3x + 4$ $c = 4$, so the y-intercept is $(0,4)$.
2. Write the gradient as a fraction and identify the values of the rise and the run.	$m = \frac{\text{rise}}{\text{run}}$ $= \frac{3}{1}$
3. Draw a set of axes and plot the y-intercept. To obtain the second point, start from the y-intercept and move up and across as suggested by the positive gradient; that is, 3 units up and 1 unit right. The second point is $(1, 7)$.	
4. Join the two points with a straight line and label the graph.	
b. 1. The equation is in $y = mx + c$ form, so use the gradient-intercept method. State the value of the y-intercept. Since the value of c is 0, the line passes through the origin.	**b.** $y = -\frac{2}{3}x$ $c = 0$, so the y-intercept is $(0, 0)$.
2. Write the gradient as a fraction and identify the values of the rise and the run. Note that the negative sign always belongs to the rise.	$m = \frac{\text{rise}}{\text{run}}$ $= -\frac{2}{3}$
3. Draw a set of axes and plot they-intercept. To obtain the second point, start from the origin and move down and across as suggested by the negative gradient; that is, 2 units down and 3 units right. The second point is $(3, -2)$.	
4. Join the two points with a straight line and label the graph.	
c. 1. The equation is in $ax + by = c$ form, so find the x- and y-intercepts.	**c.** $2x + 3y = 6$
2. Find the y-intercept by substituting $x = 0$ into the equation and solving.	$2(0) + 3y = 6$ $3y = 6$ $y = 2$

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3. State the coordinates of the y-intercept.

The coordinates of the y-intercept are $(0, 2)$.

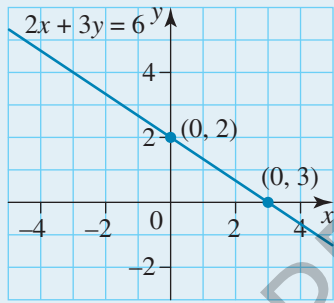
4. Find the x-intercept by substituting $y = 0$ into the equation and solving.

$$2x + 3y = 6$$
$$2x + 3(0) = 6$$
$$2x = 6$$
$$x = 3$$

5. State the coordinates of the x-intercept.

The coordinates of the x-intercept are $(3, 0)$.

6. Plot the intercepts on a set of axes and join them together with a straight line. Label the graph.



d. 1. The graph needs to be sketched between two x-values. Rearrange the equation to make y the subject.

d. $x - y = 3$
$$-y = -x + 3$$
$$y = x - 3$$

2. Substitute each of the two given x-values (that is, -2 and 6) into the equation and find corresponding values of y.

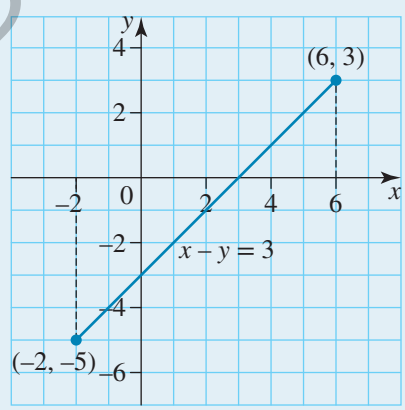
When $x = -2$:
$y = x - 3$
$= -2 - 3$
$= -5$

When $x = 6$:
$y = x - 3$
$= 6 - 3$
$= 3$

3. State the coordinates of the end points.

The end points are $(-2, -5)$ and $(6, 3)$.

4. Plot the end points on a set of axes and join them with a straight line. Label the graph. *Note*: Do not extend the line beyond the end points.



14.2.2 Applications of straight-line graphs

Many real-life situations involve variables whose relationship is linear and hence can be described by a linear rule and represented graphically by a straight line.

When modelling a linear relationship, remember that the y-intercept represents the value of the function when $x = 0$. In most situations, it represents the initial (or original) value of something, a fixed cost or flag fall.

The gradient represents the rate of change of the y-value with respect to x. It shows the change (increase or decrease) in y, as $x$increases by 1 unit. For example, let the equation $V = 1000 - 200t$ describe the volume

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(in litres) of water in a tub t minutes after a plug is removed. The y-intercept shows that the initial amount of water in the tub is 1000 litres. The gradient means that the volume of water decreases (since the gradient is negative) at a rate of 200 litres every minute.

WORKED EXAMPLE 4

Mikaela works as a car salesperson. She is paid a retainer of $150 a week and receives 2% commission on her sales.

a. How much does Mikaela earn in a week in which she sold cars worth $40 000 in total?

b. Construct a table to describe the relationship between earnings and sales using sales of $0, $10 000, $20 000 and $80 000.

c. Write an equation that relates earnings (E) to sales (S).

d. State the value of the gradient and the y-intercept and interpret their meaning.

e. Draw a graph of the relationship between earnings and sales. That is, draw a graph of E versus S.

THINK

a. Calculate earnings by finding the sum of the retainer and the commission.

$2\% = \frac{2}{100}$

$= 0.02$

b. For each of the four values of sales, repeat the working of **a** opposite.

c. Use the pattern seen in **a** and **b** to relate E and S. Writing the relationship in words may assist in finding the equation.

d. 1. Compare the equation relating earnings to sales with that of the general equation of a straight line, $y = mx + c$. The gradient is the coefficient of the x-term, m, and the y-intercept is the constant term, c.

2. Interpret the meaning of the gradient and the y-intercept.

WRITE/DRAW

a. Earnings = retainer + commission

= retainer + 2% of sales

$= 150 + 2\% \times 40\,000$

$= 150 + 0.02 \times 40\,000$

$= 150 + 800$

$= 950$

Mikaela earns $950.

b.

Sales	Earnings
0	150
10 000	350
20 000	550
80 000	1750

c. Earnings = retainer + 2% of sales

$E = 150 + 0.02 \times S$

or $E = 0.02S + 150$

d. The gradient is 0.02 and the y-intercept is 150.

The gradient represents the commission rate. It shows that for every $100 worth of sales, Mikaela earns $2 in commission. The y-intercept represents the value of the retainer; that is, the amount Mikaela is paid if no sales are made.

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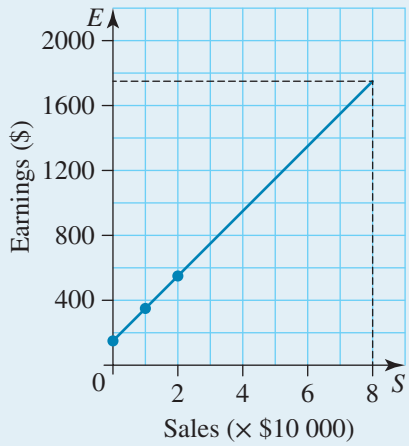
e. Earnings depends on sales. Thus sales is the *explanatory* variable and goes on the horizontal axis. Do not crowd the axes by writing the sales values in thousands — use a legend and write only the number of multiples of ten thousand.

e.



14.2.3 Extrapolation and interpolation

Two terms used widely in interpreting data are **extrapolation** and **interpolation**. Extrapolation means to examine the relationship between the variables by extending it beyond the data. Interpolation means to infer the relationship between distinct data points. Worked example 5 should make these meanings clear.

WORKED EXAMPLE 5

The tension (measured in Newtons) in a spring is linearly related to the extension of the spring (measured in centimetres). Some values relating tension (T) and extension (x) are given:

x	2	5	7
T	50	125	175

a. Plot these data.
b. By extrapolating from the data, predict the tension when the extension is 10 cm.
c. By interpolating, predict the tension when the extension is 4 cm.

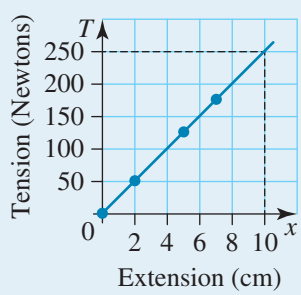
THINK

a. Plot the data on a set of axes that shows x-values up to 10. Draw a line through these points, extending it beyond the given values.

b. Extrapolation involves inference beyond the data range. Use the graph to find T when x is 10.

WRITE/DRAW

a.



b. When $x = 10$, $T = 250$
When the spring is stretched 10 cm, the tension is 250 newtons.

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c. Interpolation involves inference between the data points. Use the graph to find T when x is 4.

c. When $x = 4$, $T = 100$
When the spring is stretched 4 cm, the tension is 100 newtons.

Resources

- **Interactivity:** Equations of straight lines (int-6485)
- **Interactivity:** Linear graphs (int-6484)
- **Interactivity:** Constructing and interpreting straight-line graphs (int-6280)

studyon

Unit 4 › AOS M4 › Topic 1 › Concept 1

Straight-line graphs Summary screen and practice questions

Unit 4 › AOS M4 › Topic 1 › Concept 8

Interpolation and extrapolation Summary screen and practice questions

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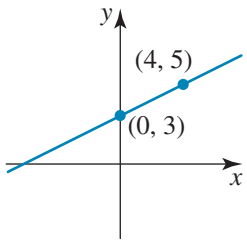
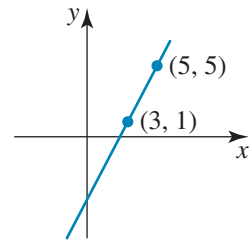
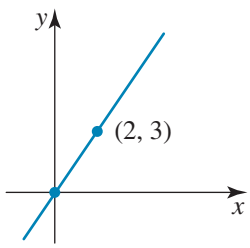
Exercise 14.2 Constructing and interpreting straight-line graphs

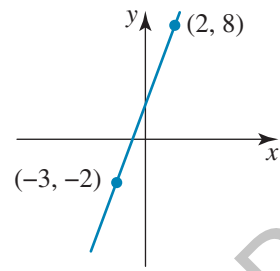
1. **WE 1** Consider the equation of the line $y = 2x + 5$.
   - **a.** What is the value of y when $x = 4$?
   - **b.** What is the value of x when $y = 17$?
   - **c.** Does the point $(3, 12)$ lie on the line?
   - **d.** State the value of the gradient and the y-intercept.
2. Consider the equation $5x - 2y = 10$.
   - **a.** Find the value of y when $x = 2$.
   - **b.** Find the value of x when $y = 5$.
   - **c.** Does the point $(-8, -25)$ lie on the line?
3. Consider the equation of the line $y = -2x + 4$.
   - **a.** What is the value of y when $x = 2$?
   - **b.** What is the value of x when $y = -4$?
   - **c.** Does the point $(0, 4)$ lie on the line?
   - **d.** State the value of the gradient and the y-intercept.
4. Consider the equation of the line $2y + 3x = 12$.
   - **a.** What is the value of y when $x = 2$?
   - **b.** What is the value of x when $y = 6$?
   - **c.** Does the point $(3, 2)$ lie on the line?
   - **d.** State the value of the gradient and the y-intercept.
5. **WE 2** Two points P$(2, 5)$ and Q$(4, 13)$ lie on the same line.
   Calculate the gradient of the line.
6. The points $(5, -3)$ and $(-2, 4)$ lie on a line. Calculate the gradient of this line.
7. Consider the equation of the line $2y + x = 8$.
   - **a.** What is the value of y when $x = 2$?
   - **b.** What is the value of x when $y = 1$?
   - **c.** Does the point $(3, 2)$ lie on the line?
   - **d.** Using the two points whose coordinates were found in **a** and **b**, calculate the gradient.

8. Calculate the gradient of the line shown in each of the following.

a.


b.


c.


d.


9. **WE 3** Sketch the graphs of the following functions.

a. $y = 3x - 4$

b. $y - 3x = 12$

c. $y = \frac{3}{4}x$

d. $4x - 3y = 12$ between $x = -2$ and $x = 4$

10. Sketch the graph of the following functions.

a. $5y + x = 10$

b. $y = -x + 3$

c. $y = 3x$

d. $y = 3x - 2$, between $x = -2$ and $x = 2$

11. Sketch the graphs of the following functions.

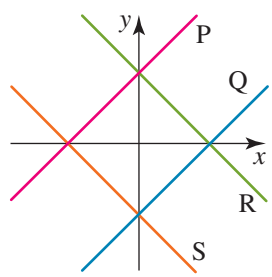
a. $3x + y = 12$

b. $y = -2x + 8$

c. $y = -2x$

d. $y = 2x + 1$ between $x = -2$ and $x = 2$

12. **MC** The graphs of the following lines are shown at right.



i. $y + x = 4$

ii. $y = x + 4$

iii. $y + x + 4 = 0$

iv. $y = x - 4$

The best match between graphs P to S and equations i to iv is:

A. P → iv Q → i R → iii S → ii

B. P → i Q → iii R → ii S → iv

C. P → iv Q → i R → ii S → iii

D. P → iv Q → iii R → i S → ii

E. P → ii Q → iv R → i S → iii

13. **WE 4** Greta sells boats and she is paid $275 per week plus 3% of her weekly sales.

a. How much does Greta receive if the value of her weekly sales is $12 000?

b. Construct a table to show Greta's weekly pay for the following values of weekly sales: $5 000, $10 000, $15 000 and $20 000.

c. Write an equation that relates her weekly pay (P) and the value of her weekly sales (S).

d. Draw a graph of the relationship between weekly pay and weekly sales. That is, draw a graph of P versus S. Place S on the horizontal axis.

e. State the value of the gradient and the y-intercept, and interpret their meaning.

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14. Sum, who works as a car salesperson, is paid a retainer of $200 a week and receives 3% commission on her sales.
    a. Copy and complete this table.

Sales	0	$20 000	$40 000	$60 000
Earnings				

    b. Write an equation that relates earnings (E) to sales (S).
    c. Draw the graph of earnings (E) versus sales (S) using the information in **a**.

15. **WE 5** For a farm water tank, the volume of water in the tank (V) is related linearly to the depth of water in the tank (D) for values of D greater than 450 mm.

Depth (mm)	**Volume (L)**
500	2500
700	4500
1000	7500

    a. Plot these data using suitable axes.
    b. By extrapolating from the data, predict the depth needed to give a volume of 13 000 litres.
    c. By interpolating from the data, predict the volume when the depth of the water is 600 mm.

16. The distance covered (in metres) of an object over a period of time (in seconds) is shown in the table.

Time (s)	2	4	5	7
Distance (m)	8	16	20	28

    a. Plot these data.
    b. By extrapolating from the data, predict the distance covered when 9 seconds have elapsed.
    c. By interpolating, predict the distance covered when the time elapsed is 6 seconds.

17. An object that falls freely due to gravity increases its speed (S) by 9.8 m/s each second. Assume that at the start the speed of the object was 0.
    a. How fast is the object travelling after 3 seconds?
    b. How long has the object been falling when its speed is 78.4 metres per second?
    c. If t stands for the number of seconds the object has been falling, write a formula that relates S to t.
    d. Draw a graph of S versus t.

18. Visa car rentals charge $75 per day plus $15 per hundred kilometres.
    a. How much would it cost to rent a car for one day if the car travelled 345 km?
    b. The bill for one day's rental came to $142.50. How many kilometres did the car travel?
    c. Sketch a graph of the cost of renting the car for one day (C) versus the number of kilometres travelled (d).

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19. A psychology experiment is testing the relationship between the number of errors (n) made by a child on a task and the time taken to complete the task in seconds (t). The results of five trials of the experiment are given in the table below.

If the number of errors made is linearly related to the time taken, which experiment does not fit the pattern?

Experiment	Time taken (seconds)	Number of errors
A	30	10
B	45	6
C	36	9
D	48	7
E	42	8

20. Janine sells cosmetics at a department store. She knows she is paid a retainer plus commission on sales but she is not sure of the exact rates. For three weeks she records her wages and the value of product she sold during that week.

Week	Sales	Wages
1	$1200	$494
2	$750	$440
3	$880	$455.60

a. In week 4 her sales totalled $1000. Predict her wage for that week.

b. Write a formula to calculate Janine's weekly wage in terms of her weekly sales.

21. In a one-day international cricket match, each team bats for 50 overs. After 20 overs Australia's score had reached 108 runs. If the target for Australia is 250 runs at the end of 50 overs:

a. how many runs per over does the team need to score for the remaining 30 overs?

b. Write an equation to relate the score (S) to the number of overs completed (n).

c. The graph of S versus n could be a straight line. What would be the gradient of that straight line?

22. Taxi hire charges are shown in the diagram.

a. Calculate the cost of travelling 15 km in a taxi.

b. If the taxi fare was $20.80, how far did the taxi travel?

c. Write an equation relating the fare (F) to the distance (d).

d. Draw a graph relating F to d.

e. What is the slope of this graph?

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14.3 Line segments and step functions

In this section we consider graphs which are not straight lines but are made from straight lines. We also consider graphs which are not straight lines but discrete sets of points.

WORKED EXAMPLE 6

When a real estate agent sells a property, he earns commission at the following rate:
1.5% on the first 20 000
0.9% on the remainder.

a. Calculate the commission earned on sales of:
i. $10 000 **ii. $20 000** **iii. $30 000** **iv. $40 000.**

b. Draw a line segment graph of commission (C) versus the value of the sales (S) up to a sales value of $40 000.

c. Give a reason for the difference of the gradients of the two segments.

THINK

a. i. For any amount up to and including $20 000, the commission is 1.5% of the amount.

ii. The commission is 1.5% of the amount.

iii. $30 000 is more than $20 000, so calculate 1.5% on $20 000 plus 0.9% on the remainder over $20 000; that is, $10 000.

iv. $40 000 is more than $20 000, so calculate 1.5% on $20 000 plus 0.9% on the remainder over $20 000; that is, $20 000.

b. 1. Since the commission rate changes when sales exceed $20 000, the graph will consist of two segments. One segment for all sales up to and including $20 000 and the other for sales of more than $20 000 and up to and including $40 000.

2. The starting point of the first segment is (0, 0) (since there is no commission when there are no sales). The end point of this segment is (20 000, 300) (as was found in part **a (ii)**). Plot these points and join them with a straight line.

WRITE/DRAW

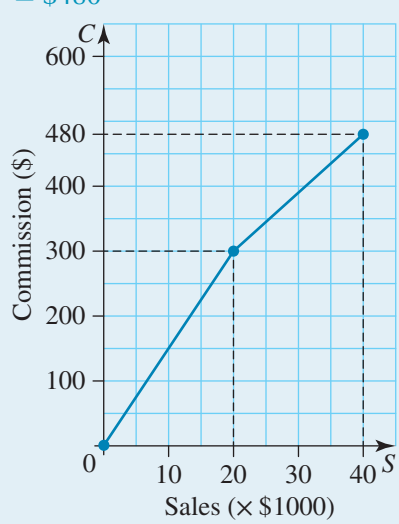
a. i. Commission on $10 000
$= 1.5\%$ of 10 000
$= 0.015 \times 10\,000$
$= \$150$

ii. Commission on $20 000
$= 1.5\%$ of 20 000
$= 0.015 \times 20\,000$
$= \$300$

iii. Commission on $30 000
$= 1.5\%$ of $20 000 + 0.9\%$ of $10 000
$= 0.015 \times 20\,000 + 0.009 \times 10\,000$
$= 300 + 90$
$= \$390$

iv. Commission on $40 000
$= 1.5\%$ of 20 000 $+ 0.9\%$ of 20 000
$= 0.015 \times 20\,000 + 0.009 \times 20\,000$
$= 300 + 180$
$= \$480$

b.



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3. The second segment starts where the first segment ends. The end point of the second segment is (40 000, 480) (as was found in part a (iv)). Plot this end point and join it to the first segment.

c. In this case, the gradient is the rate of commission. When the sales exceed $20 000, the commission rate decreases — this results in a line segment that is less steep.

c. The gradient of each line segment represents the commission rates. Since the rates on values lower than $20 000 differ from the rates on values greater than $20 000, the gradients also differ.

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Unit 4 > AOS M4 > Topic 1 > Concept 2 **Segment graphs** Summary screen and practice questions

WORKED EXAMPLE 7

An electrician charges the rates shown.

a. Calculate the charges for service calls of the following durations: 10 minutes, 25 minutes, 40 minutes, 55 minutes, 70 minutes and 85 minutes.

b. Draw a step graph of charges (C) versus the time of the service call in minutes (t) for calls of up to 120 minutes.

THINK

a. 'Part thereof' means that whether the repairs take 3 minutes, 10 minutes or 30 minutes, you are charged for 30 minutes. For a service call up to 30 minutes, the electrician charges $35 call-out fee plus $30. For a service call over 30 minutes long and up to 60 minutes, the charge is $35 call-out fee plus 2 lots of $30. Finally, for a service call over 60 minutes and up to 90 minutes, the charge is $3 call-out fee plus 3 lots of $30.

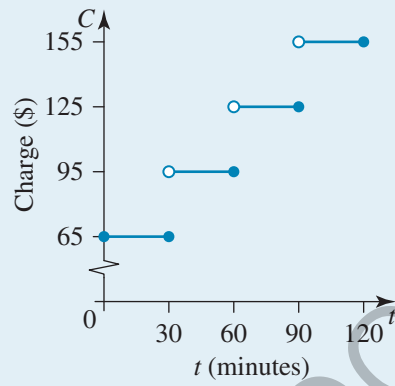
WRITE/DRAW

a.

Time	Charge
10 min	$35 + 30 = \$65$
25 min	$35 + 30 = \$65$
40 min	$35 + 2 \times 30 = \$95$
55 min	$35 + 2 \times 30 = \$95$
70 min	$35 + 3 \times 30 = \$125$
85 min	$35 + 3 \times 30 = \$125$

b. Draw the graph. Place t along the horizontal axis as it is the explanatory variable. The graph consists of four horizontal sections (steps). This reflects the fact that the fixed fee is charged for any amount of time up to and including 30 minutes; then a different fixed fee for any amount of time above 30 minutes and up to and including 60 minutes, and so on. Note that where the end point of the step is included in the step, it is shown as a full circle (closed circle), whereas if the end point is not included, it is shown as an empty circle (open circle).

b.



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Unit 4 > AOS M4 > Topic 1 > Concept 3 **Step graphs** Summary screen and practice questions

In the previous example, the explanatory variable, time (t), was a **continuous variable**; that is, t can take any values — 2, 15.5, 87 — and so on. In the next example we consider a situation where the explanatory variable can take whole number values only. A variable that is not continuous is called a **discrete variable**. If the data are discrete, the points on the graph are not joined together.

WORKED EXAMPLE 8

A bakery sells bread rolls for 50 cents each or $2.50 for 6.
a. Calculate the cost of 4, 5, 6, 7, 11, 12, 13, 17, 18 and 19 bread rolls.
b. Sketch this information on a graph for up to 24 rolls.

THINK

a. Calculate the cost for each number of bread rolls. If the number of rolls is above 5, it can be formed by combining pack(s) of 6 rolls and single rolls. For example, 7 rolls can be bought as a pack of 6 plus 1 single roll; 17 rolls as 2 packs of 6 plus 5 single rolls etc.

WRITE/ DRAW

a.

Number of rolls	Cost
4	$4 \times 50c = \$2.00$
5	$5 \times 50c = \$2.50$
6	$\$2.50$
7	$\$2.50 + 1 \times 50c = \$3.00$
11	$\$2.50 + 5 \times 50c = \$5.00$
12	$2 \times \$2.50 = \$5.00$
13	$2 \times \$2.50 + 1 \times 50c = \$5.50$
17	$2 \times \$2.50 + 5 \times 50c = \$7.50$
18	$3 \times \$2.50 = \$7.50$
19	$3 \times \$2.50 + 1 \times 50c = \$8.00$

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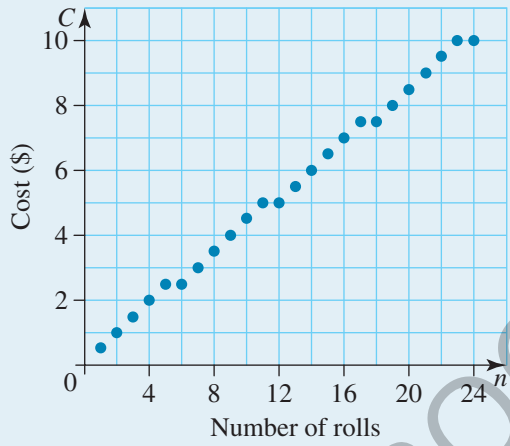
b. Sketch the information using the number of bread rolls as the explanatory variable. Note that the points on the graph are not joined together as the data are discrete.

b.



Resources

- **Interactivity:** Piecewise linear graphs (int-6486)
- **Interactivity:** Step functions (int-6281)

Exercise 14.3 Line segments and step functions

1. **WE 6** Suppose a real estate agent is paid commission at the following rates: 1.5% on the first $40 000 and 1% on the remainder.
   - **a.** Calculate the commission due on sales of $20 000, $30 000, $40 000, $50 000 and $60 000.
   - **b.** Draw a graph of commission (C) versus sales (S).
   - **c.** Give a reason for the difference of the slopes of the two segments.
2. Beena, who runs a real estate agency, sells a property and earns commission at the following rate:

   2% on the first $25 000
   0.8% on the remainder.

   Calculate the commission earned by Beena on sales of:
   - **a.** $10 000 **b.** $25 000 **c.** $35 000 **d.** $45 000.
   - **e.** Construct a graph of commission (C) versus the value of sales (S).
3. Copy and complete the following table by calculating the cost due for each of the consumptions.

Consumption (kWh)	Power bill ($)
200	
400	
600	
1000	
1500	

4. Using the data in the table, draw a graph of power bill versus consumption.

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5. **WE 7** A plumber charges these rates:

$55 call out fee
$40 per half hour or part thereof.

   a. Using this information, construct the graph of charges (C) versus time of the service call, in minutes (t), for calls of up to 120 minutes.
   b. Calculate the charges for these service calls:
   12 minutes, 23 minutes, 44 minutes, 56 minutes, 73 minutes, 87 minutes.

6. A telephone company charges users at a rate of 25 cents for each completed 30 seconds. This implies a call of less than 30 seconds is free.
   a. Copy and complete this table for the calls shown.
   b. Construct the graph of cost versus length of call for calls of up to 120 seconds.

Length of call (seconds)	Cost (cents)
15	
30	
45	
60	
75	
90	
105	
120	

7. An electrician charges at the following rates:

$45 call out fee plus
$35 per half hour or part thereof.

   a. Calculate the charges for the following service calls:
   20 minutes, 30 minutes, 45 minutes, 60 minutes, 80 minutes and 90 minutes.
   b. Draw a graph of charges (C) versus time of the service call in minutes (t) for calls of up to 90 minutes.

8. A mobile phone company charges users a rate of 15 cents for each completed 20 seconds of the call. This means a call of less than 20 seconds is free.
   a. Draw a table with the following headings and complete it for calls of 10, 20, 30, 40, 50, 60, 70, 80, and 90 seconds.

Length of call (seconds)	Cost (cents)

   b. Draw the graph of cost versus length of call for calls of up to 90 seconds.

9. **WE 8** A country bakery sells buns for 40 cents each or 6 for $2.00.
   a. Using this information, construct a graph for buying up to 20 buns using the number of buns as the independent variable.
   b. Calculate the cost of 2, 4, 6, 8 and 10 buns.

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10. The Explorindo Travel Company specialises in surfing tours of remote islands in Indonesia. They will take individuals but prefer to deal with groups of people. They have the following charges for a holiday package:

1 person	$900
2 people	$1650
Each extra person	$600

a. Draw a table with the following headings and complete it for costs for 1, 2, 3, 6, 8 and 10 people.

Number of people	Total cost ($)

b. Draw the graph of total cost versus number of people. (Include only the number of people discussed in part **a**.)

This information relates to questions **3** and **4**. The amount of electricity used around the home is measured in kilowatt hours (kWh). A light bulb left on for 10 hours will consume about 1 kWh of power. The local power supplier charges at the following rates.

Power	Cost per kWh ($)
First 400 kWh	0.20
Next 1000 kWh	0.15
Remaining kWh	0.10

11. MC Which of the following is clearly a discrete variable?

A. the time taken for a phone call
B. the number of CDs in a collection
C. the commission earned on sales
D. the power consumed by a hot-water jug
E. a person's weight

12. MC On a particular visit, a tradesperson charges the householder $100. It was noted he arrived at 11.30 am. Which of the following departure times could be correct?

A. 11.38 am **B.** 11.48 am **C.** 12.05 pm **D.** 12.20 pm **E.** 12.35 pm

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13. This is the 2011–12 tax table.

Taxable income	Tax on this income
0–$6000	Nil
$6001–$37 000	15c for each $1 over $6000
$37 001–$80 000	$4650 plus 30c for each $1 over $37 000
$80 001–$180 000	$17 550 plus 37c for each $1 over $80 000
$180 001 and over	$54 550 plus 45c for each $1 over $180 000

Use this tax table to calculate the amount of tax paid in 2012 by people with taxable incomes of:

a. $4000 **b.** $7000 **c.** $20 000 **d.** $35 000 **e.** $40 000
f. $60 000 **g.** $100 000 **h.** $200 000.

14. a. Using the values calculated in question **13** and any other values, draw the graph of income tax versus taxable income.

b. Use the graph to estimate the income tax payable on a taxable income of:

i. $24 000 **ii.** $95 000.

Confirm your answers with calculations.

15. This is the 2017–2018 tax table.

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$37 000	19c for each $1 over $18 200
$37 001–$87 000	$3572 plus 32.5c for each $1 over $37 000
$87 001–$180 000	$19 822 plus 37c for each $1 over $87 000
$180 001 and over	$54 232 plus 45c for each $1 over $180 000

Use this tax table to calculate the amount of tax paid in 2018 by people with taxable incomes of:

a. $4000 **b.** $7000 **c.** $20 000 **d.** $35 000 **e.** $40 000
f. $60 000 **g.** $100 000 **h.** $200 000.

16. a. Use the values calculated in question **15** to draw a graph of income tax versus taxable income.

b. Use the graph to estimate the income tax payable on a taxable income of:

i. $24 000
ii. $95 000.

Confirm your answers with calculations.

c. Compare your answers to questions **14b** and **16b**.

17. What situation could be described by the following graphs? In your response, clearly identify the variables used on both axes and explain how the graph represents that situation. For example, the first graph could measure the profit versus the number of units of production.

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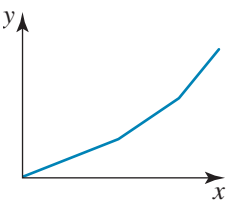
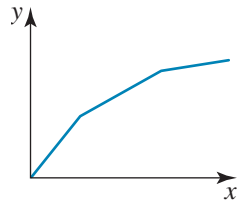
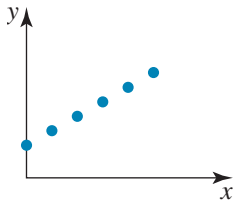
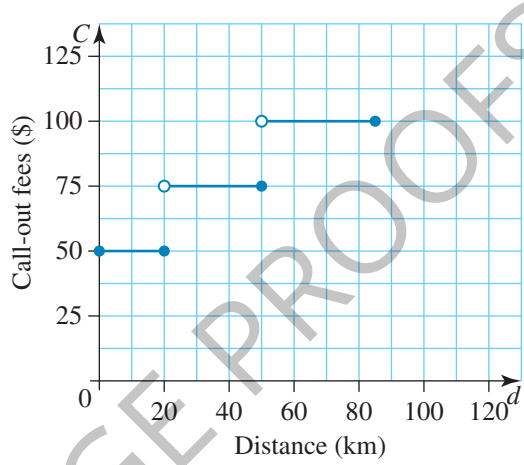
a. 

b. 

c. 

18. 'Fix-it-fast' is a photocopier repair service for schools in North-central Victoria. The call-out charge depends on the distance the repair person has to travel. The call-out fees for distances up to 85 km are shown on the following graph.



a. i. What is the call-out fee for a distance of 40 km?

ii. What is the maximum distance travelled for a call-out fee of $50?

b. Copy the shown graph and add this information to it. A call-out fee of $125 is charged for schools at a distance of more than 85 km but less than 120 km.

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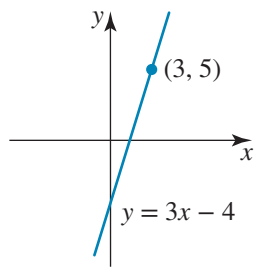
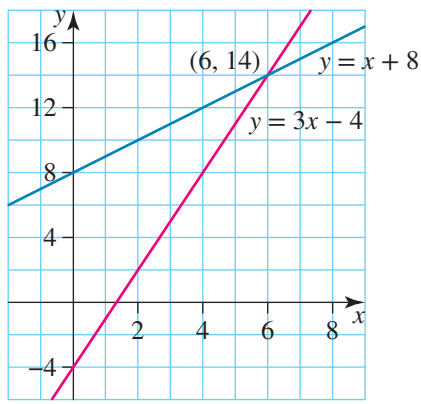
14.4 Simultaneous equations and break-even point

In this topic, we have seen that when an equation such as $y = 3x - 4$ is graphed it forms a straight line. Each point on the line will have an x- and a y-coordinate and these values of x and y satisfy the equation. The reverse is also true. Each pair, x and y, which satisfy the equation will be the coordinates of a point on the line.



14.4.1 Solving linear simultaneous equations

If we draw the graph of $y = 3x - 4$ and $y = x + 8$ on the same set of axes the result is shown at right.



The point where the lines intersect (6, 14) is called the **simultaneous solution** because $x = 6$ and $y = 14$ satisfies both equations.

In this example we found the simultaneous solution graphically; that is, from the graph. It is also possible to find the simultaneous solution algebraically, either by the substitution method or the elimination method.

WORKED EXAMPLE 9

Find the simultaneous solutions to these equations using algebraic methods.

a. $y = 3x - 4$ and $y = x + 8$ **b.** $2x + 3y = 13$ and $x - 4y = 1$

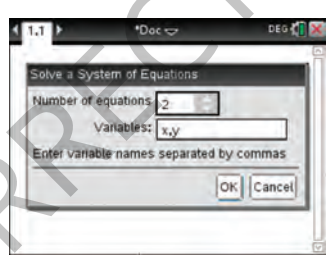
THINK	WRITE
a. 1. Since both equations start with $y =$, solve using the substitution method. Use equation [1] to substitute for y in equation [2].	**a.** $y = 3x - 4$ [1] $y = x + 8$ [2] Substitute [1] into [2].
2. Solve equation [3].	$3x - 4 = x + 8$ [3] $3x - x = 8 + 4$ $2x = 12$ $x = 6$
3. Find y by substituting $x = 6$ into one of the original equations.	Substitute $x = 6$ into [2]: $y = 6 + 8$ $= 14$
4. State the solution.	The solution is $(6, 14)$.
b. 1. If one of the equations was rearranged to a $y = mx + c$ format, the substitution method could be used. However, it will be more efficient to use the elimination method in this case.	**b.** $2x + 3y = 13$ [1] $x - 4y = 1$ [2]
2. Multiply equation [2] by 2 so that the x-coefficients of both equations are the same.	$[2] \times 2$: $2x - 8y = 2$ [3]
3. Eliminate x by subtracting [3] from [1].	[1] − [3]: $11y = 11$
4. Solve to find y.	$y = 1$
5. Find x by substituting $y = 1$ into one of the equations (into equation [2] is easiest).	**c.** Substitute $y = 1$ into [2]: $x - 4 \times 1 = 1$ $x - 4 = 1$ $x = 5$
6. State the solution.	The solution is $(5, 1)$.

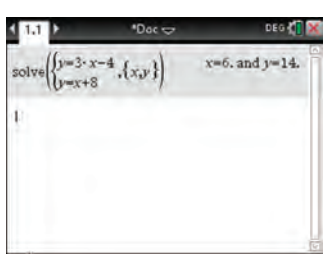
TI | THINK

a. 1. On a Calculator page, press MENU and then
3: Algebra
7: Solve System of Equations
1: Solve System of Equations
Complete the fields as shown.
Press ENTER.
Complete the simultaneous equations template as:
$y = 3x - 4$
$y = x + 8$
Press ENTER.

2. The answer appears on the screen.

WRITE





The solution is $(6, 14)$.

CASIO | THINK

a. 1. On the MAIN screen, open the keyboard and locate the simultaneous template in Math1.
Insert the first equation in the top box. Insert the second equation in the bottom box.
Insert x, y in the box on the right.
Press EXE.

2. The answer appears on the screen.

WRITE

The solution is $(6, 14)$.

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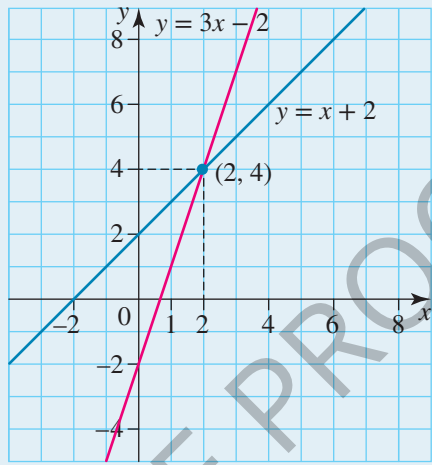
WORKED EXAMPLE 10

Find the simultaneous solution of $y = 3x - 2$ and $y = x + 2$ using a graphical method.

THINK

1. Draw both graphs on the one set of axes.

WRITE/DRAW



2. The simultaneous solution is the point where the graphs intersect.

The point of intersection is (2, 4) so the simultaneous solution is (2, 4) or $x = 2, y = 4$.

study on

Unit 4 > AOS M4 > Topic 1 > Concept 5

Algebraic solutions to simultaneous equations Summary screen and practice questions

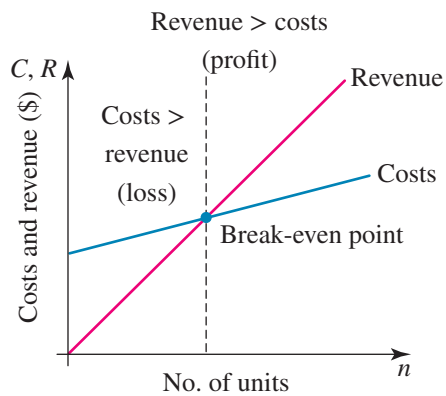
14.4.2 Break-even analysis

The aim of most businesses is to make a profit. The profit depends on the costs associated with the business (labour, raw materials and plant) and its revenue (the money it earns through sales). It represents the difference between the revenue and the costs.

Profit = revenue − costs

It is evident that a profit will occur if the revenue exceeds the costs. However, if the costs exceed the revenue, a loss will result. Finally, if the costs equal the revenue, there will be neither a profit nor a loss. This is referred to as a **break-even point**.

The diagram shows the graph of a cost function and a revenue function, drawn on the same set of axes. The point of intersection of the two lines represents the point at which costs and revenue are equal; that is, the break-even point. To the left of the break-even point, the cost line is above the revenue line. This means that the costs are higher than the revenue and will result in a loss. To the right of the break-even point, the cost line is below the revenue line. This means that costs are lower than the revenue and will result in a profit.



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WORKED EXAMPLE 11

The cost associated with publishing a particular maths book (C) is given by

$$C = 12n + 24\,000$$

where n represents the number of books. The revenue (R) made from selling n books is given by $R = 28n$. Both C and R are in dollars.

a. Copy and complete the following table.

Number of books (n)	Costs ($)	Revenue ($)
500		
1000		
1500		
2000		

b. Sketch the graph of the costs (C) versus number (n), and the graph of revenue (R) versus number (n) on the same set of axes.

c. How many books need to be published and sold so that the revenue equals the costs?

d. State the coordinates of the break-even point and interpret its meaning.

THINK

a. 1. Write the equations for the costs (C) and the revenue (R).

2. Calculate the value of C and R for each value of n given in the table.

3. Use the results from the calculations in step 2 to complete the table.

WRITE/DRAW

a. $C = 12n + 24\,000$, $R = 28n$

When $n = 500$,

$C = 12 \times 500 + 24\,000$
$= \$30\,000$

$R = 28 \times 500$
$= \$14\,000$

When $n = 1000$,

$C = 12 \times 1000 + 24\,000$
$= \$36\,000$

$R = 28 \times 1000$
$= \$28\,000$

When $n = 1500$,

$C = 12 \times 1500 + 24\,000$
$= \$42\,000$

$R = 28 \times 1500$
$= \$42\,000$

When $n = 2000$,

$C = 12 \times 2000 + 24\,000$
$= \$48\,000$

$R = 28 \times 2000$
$= \$56\,000$

Number of books (n)	Costs ($)	Revenue ($)
500	30 000	14 000
1000	36 000	28 000
1500	42 000	42 000
2000	48 000	56 000

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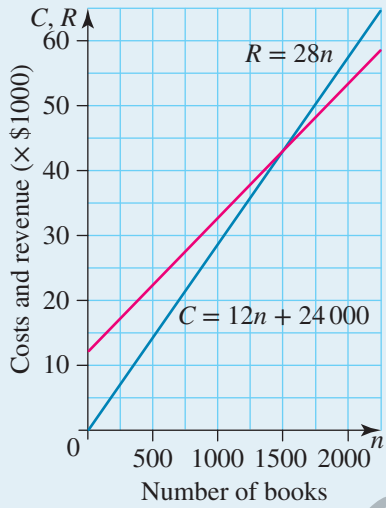
b. The number of books (n) is the explanatory variable, so place it on the horizontal axis. The graphs of the cost and revenue functions are straight lines, so they can be constructed by plotting any two points from the table then joining them with a straight line.

b.



c. From the table (or the graph), we can see that when $n = 1500$, the costs and the revenue are both the same.

c. 1500 books need to be published and sold so that the revenue equals the costs.

d. The break-even point is the point where the costs equal the revenue. On the graph, it is the point of intersection of the two lines.

d. The coordinates of the break-even point are (1500, 42 000). The cost associated with publishing and the revenue made from selling 1500 books is $42 000. If less than 1500 books are sold, the costs are higher than the revenue (loss). If more than 1500 books are sold, the revenue is higher than the costs (profit).

Note that the break-even point in the previous worked example could have been found using CAS or algebraically.

on Resources

- **Interactivity:** Solving simultaneous equations using substitution (int-6453)
- **Interactivity:** Break-even points (int-6454)

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Unit 4 › AOS M4 › Topic 1 › Concept 6

Break-even analysis Summary screen and practice questions

Unit 4 › AOS M4 › Topic 1 › Concept 4

Graphical solutions to simultaneous equations Summary screen and practice questions

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Exercise 14.4 Simultaneous equations and break-even point

1. **WE 9** Solve the following simultaneous equations.

$$y = 10x - 7$$
$$y = 2x + 1$$

2. Solve the following simultaneous equations.

$$6x - 11y = 2$$
$$5x - 9y = 1$$

3. Find the simultaneous solution, algebraically, to:
   a. $y = 4x - 4$, $y = 2x$
   b. $y = 3x - 4$, $y = 2x - 2$
   c. $2y + x = 5$, $3y - 2x = 4$
4. **WE 10** Find the simultaneous solution of $y = 2x - 3$ and $y = -x$ using a graphical method.
5. Find the simultaneous solution of $y = -2x - 3$ and $y = x + 3$ using a graphical method.
6. Find the simultaneous solution of each pair of linear equations in question 3 using a graphical method.
7. Find the simultaneous solution of $y = 3x - 10$ and $x = 2y + 5$ using the substitution method.
8. Find the simultaneous solution of $3x - y = -8$ and $2x + 2y = 0$ using the elimination method.
9. Find the simultaneous solution of $y = x$ and $y = -x$ using a graphical method.
10. Find the simultaneous solution of $-3x + 2y = 1$ and $-2x + 3y = 9$ using the elimination method.
11. **WE 11** The cost of manufacturing toys (C) is related to the number of toys produced (n), by the formula $C = 600 + 3n$. The revenue (R) made from selling n toys is $R = 7n$. Both C and R are in dollars.
    a. Copy and complete the following table.

Number of toys	Cost ($)	Revenue ($)
50		
100		
150		
200		

    b. Sketch the graph of cost (C) versus number (n) and the graph of revenue (R) versus number (n) on the same set of axes.
    c. How many toys need to be produced before revenue equals cost?
    d. State the coordinates of the break-even point and interpret its meaning.
12. The cost of manufacturing basketballs (C) is related to the number of basketballs produced (n) by the formula $C = 2800 + 4n$. The revenue (R) made from selling n basketballs is $R = 14n$.
    a. Copy and complete the table below.

Number of basketballs	Cost ($)	Revenue ($)
150		
200		
250		
300		
350		

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b. Using the information supplied, construct the graph of cost (C) versus number (n), and the graph of revenue (R) versus number (n), on the same set of axes.

c. Using the graph, write the number of basketballs produced before revenue equals cost to 'break even'.

13. Consider the phone connection plans shown at right.

Plan A: $15 monthly fee plus 60 cents a minute for calls.
Plan B: $25 monthly fee plus 30 cents a minute for calls.

a. Copy and complete the following table.

Call time (minutes)	Cost ($) — Plan A	Cost ($) — Plan B
10		
20		
30		
40		

b. Sketch the graph of *cost* versus *time* for each of the two plans on the same set of axes.

c. How many minutes of calls would you need to make for both plans to cost the same?

14. The cost of manufacturing electronic components (C) is related to the number of components produced (n), by the formula $C = 6000 + 2.5n$. The revenue (R) made from selling n components is $R = 4.5n - 8000$. Both C and R are in dollars.

a. Copy and complete the following table.

Number of components	Cost ($)	Revenue ($)
5000		
10 000		
15 000		
20 000		

b. Sketch the graph of cost (C) versus number (n) and the graph of revenue (R) versus number (n) on the same set of axes.

c. How many components need to be produced before revenue equals cost?

15. A new employee, whose job it is to sell a software package, is offered two different salary plans by Minitech:
Plan A: $400 per week plus $25 for each package sold
Plan B: $150 per week plus $45 for each package sold.

a. Copy and complete the following table.

Number of packages sold	Salary ($) Plan A	Salary ($) Plan B
5		
10		
15		
20		

b. Sketch the graph of salary versus number of packages sold for both Plan A and Plan B on the same set of axes.

c. How many packages need to be sold before Plan B is the better choice?

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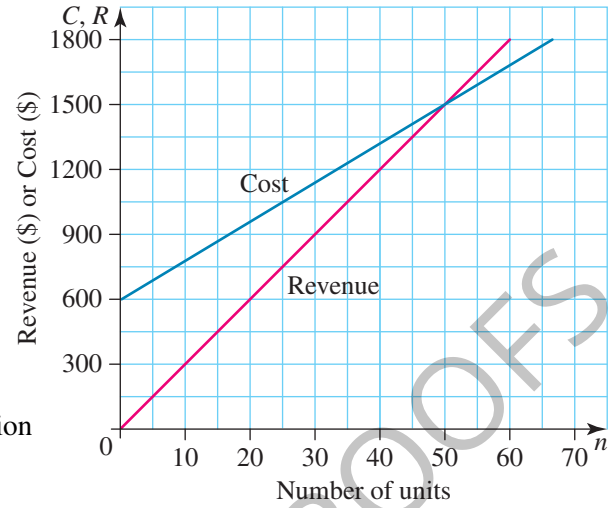
16. MC The statement that best matches the graph is:



- **A.** revenue exceeds costs when more than 50 units are sold
- **B.** revenue exceeds costs when less than 50 units are sold
- **C.** revenue exceeds costs when more than 1500 units are sold
- **D.** revenue exceeds costs when less than 1500 units are sold
- **E.** revenue exceeds costs when exactly 50 units are sold.

17. A factory producing mattresses finds that the equation linking cost in dollars (C) and the number of mattresses produced (m) is $C = 800m$.

- **a.** What is the cost of producing 10 mattresses?
- **b.** How many mattresses could be produced for $124 000?

18. From question **17**, if all mattresses are sold, the revenue in dollars (R) from the sale of m mattresses is $R = 1800m$.

- **a.** Draw graphs of C and R on the same set of axes for $0 \leq m \leq 200$.
- **b.** Determine the number of mattresses which would need to be sold for the factory to break even.
- **c.** From the two equations given, write a profit equation (P).
- **d.** Calculate the profit if 175 mattresses were produced and sold.

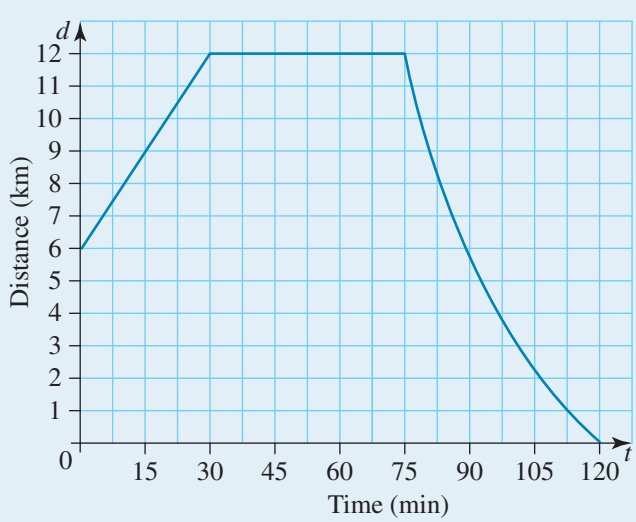
14.5 Interpreting non-linear graphs

Why are graphs used so widely in papers, magazines, journals, in education, in government, and in sales and marketing? Graphs are used because they have the capacity to convey a significant amount of information effectively.

To get the most from a graph, a user needs to learn to read and interpret information presented graphically. In this section, we will look at **non-linear graphs**. A non-linear graph is a graph which is not a straight line.

WORKED EXAMPLE 12

The graph shows the distance a cyclist is from her home over a period of 120 minutes.



- **a. How far from her home was the cyclist at the start of the time period?**
- **b. At what speed did the cyclist travel for the first 30 minutes?**
- **c. Describe the motion of the cyclist after 45 minutes.**
- **d. What was the cyclist's furthest distance from home?**
- **e. When did the cyclist turn for home?**
- **f. How long did the cyclist take to get home on the way back?**

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THINK

a. The start of the time period means $t = 0$ min. The graph shows a distance of 6 km when $t = 0$.

b. Use the relationship speed $= \frac{\text{distance}}{\text{time}}$.
The cyclist travels from 6 km to 12 km in 30 minutes or 0.5 hour.

c. Between $t = 30$ min and $t = 75$ min the distance from home remains at 12 km.

d. Observe the furthest distance from home from the graph.

e. From the graph it can be seen that the distance the cyclist is from home decreases when $t = 75$ min.

f. The cyclist began travelling home when $t = 75$ min. The cyclist reached home when $t = 120$ min.

WRITE

a. The cyclist was 6 km from home at the start of the time period.

b. Speed $= \frac{\text{distance}}{\text{time}}$
$= \frac{6\text{ km}}{0.5\text{ h}}$
$= 12$ km/h

c. At 45 minutes into the time period, the cyclist is not moving away from or closer to her home.

d. The furthest distance from home is 12 km.

e. The cyclist turns for home after 75 min.

f. Time taken to travel home $= (120 - 75)$ min
$= 45$ min

WORKED EXAMPLE 13

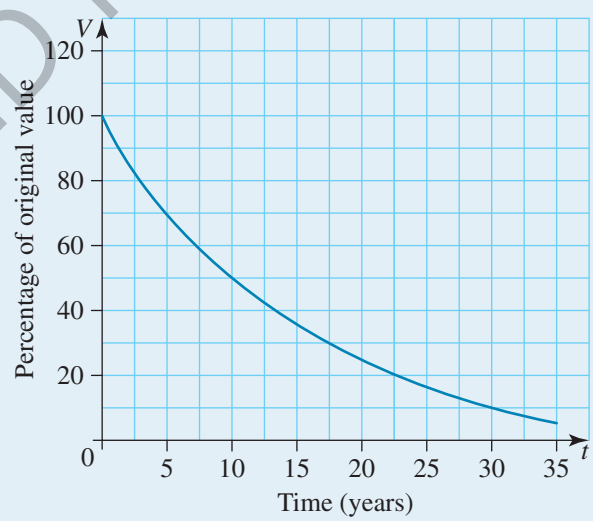
The value of a car originally worth $30 000 decreases over time. The graph describes the value of the car as a percentage of its starting price.

a. What is the value of the car, in percentage terms, after 5 years?

b. The 'half-life' is the time taken for the value to decrease by half. How long does it take for the car to lose half its original value?

c. How long does it take for the car to fall in value from 50% of its original value to 25% of its original value?

d. Estimate, in dollar terms, the value of the car after 30 years.



THINK

a. Read directly from the graph.

b. 'Half' means 50%. Reading from the graph, it takes 10 years for the value of the car to fall to 50%.

c. From the graph, the car is worth 50% of the original price at $t = 10$ and it is worth 25% at $t = 20$. Note that the half-life is also the time taken to fall from 50% to 25%.

WRITE

a. The value of the car after 5 years is approximately 70% of its original value.

b. The time taken to fall in value by 50% (half-life) is 10 years.

c. Time taken to fall from 50% to 25% of original value
$= 20 - 10$ years
$= 10$ years

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d. As the half-life is 10 years, use this to calculate the value after 30 years. That is, after 3 half-lives. Calculate 12.5% of $30 000 to obtain the value of the car in dollar terms.

d. Half-life = 10 years
Therefore, the value after:
10 years is 50%
20 years is 25%
30 years is 12.5%.
The value after 30 years
= 12.5% of $30 000
= 0.125 × $30 000
= $3750

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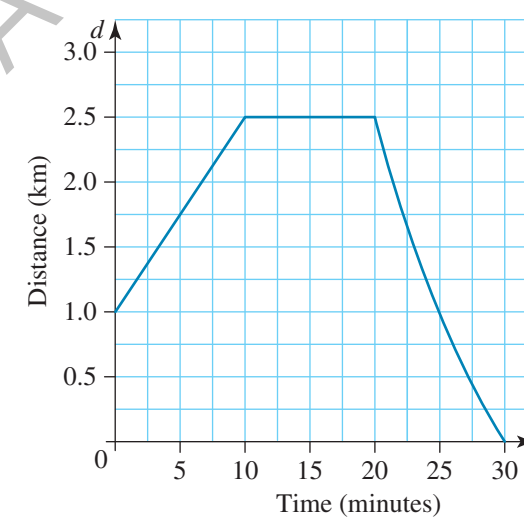
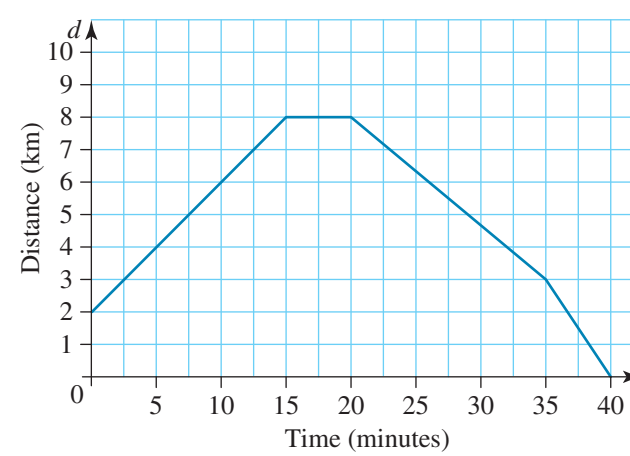
Unit 4 › AOS M4 › Topic 1 › Concept 1 **Non-linear graphs** Summary screen and practice questions

Exercise 14.5 Interpreting non-linear graphs

1. **WE 12** The graph shows the distance a skateboarder is from the skate park over a period of 30 minutes.
   - **a.** How far from the skate park was the boarder at the start of the time period?
   - **b.** At what speed did the boarder travel for the first 10 minutes?
   - **c.** Describe the motion of the boarder after 15 minutes.
   - **d.** What was the boarder's furthest distance from the skate park?
   - **e.** When did the boarder start travelling towards the skate park?
   - **f.** How long did it take the boarder to get back to the skate park on the way back?



2. The graph at right shows the distance a runner is from the finish line over a period of 40 minutes.
   - **a.** How far from the finish line was the runner at the start of the time period?
   - **b.** At what speed did the runner travel for the first 15 minutes?
   - **c.** Describe the motion of the runner after 25 minutes.
   - **d.** What was the runner's furthest distance from the finish line?
   - **e.** When did the runner start travelling towards the finish line?
   - **f.** How long did it take the runner to get back to the finish line on the way back?



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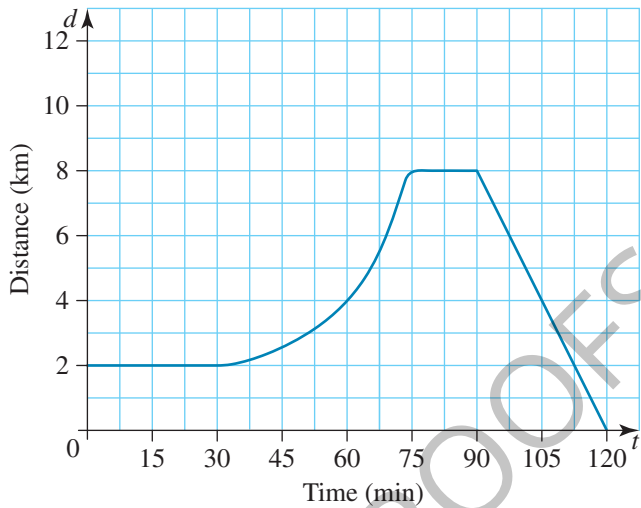
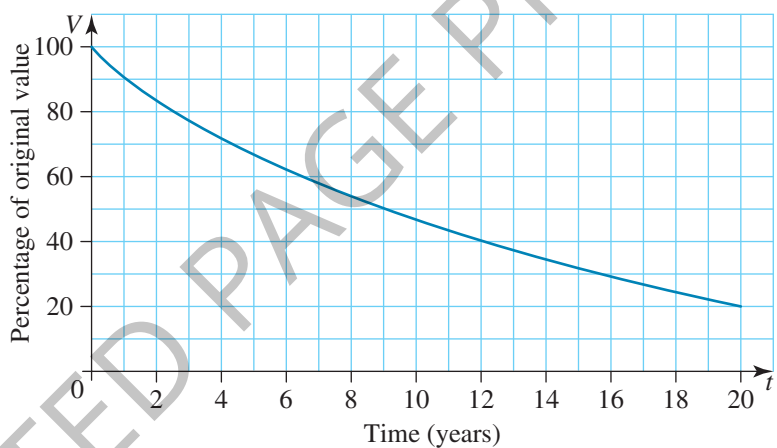
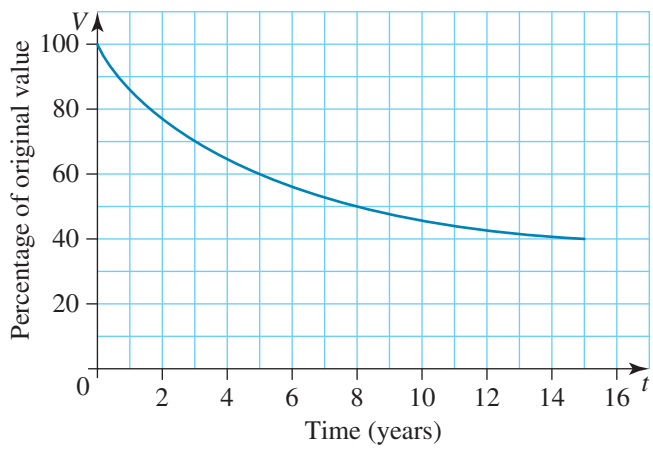
3. The graph shown gives the distance of a cyclist from her home over a period of 120 minutes.
   a. How far from her home was the cyclist at the start of the time period?
   b. At what speed did the cyclist travel for the first 30 minutes?
   c. Describe the motion of the cyclist after 7 minutes.
   d. What was the cyclist's furthest distance from home?
   e. When did the cyclist begin to travel rlaphome?
   f. How long did the cyclist take to get home?
   g. What was the average speed of the cyclist on the journey home?



4. **WE 13** The value of a ute originally worth $42 000 decreases over time. The graph describes the value of the car as a percentage of its starting price.
   a. What is the value of the ute, in percentage terms, after 5 years?
   b. The 'half-life' is the time taken for the value to decrease by half. How long does it take for the car to lose half its original value?
   c. How long does it take for the car to fall in value from 50% of its original value to 25% of its original value?
   d. Estimate, in dollar terms, the value of the car after 20 years.



5. The value of a boat originally worth $37 500 decreases over time. The graph drawn describes the value of the boat as a percentage of its starting price.
   a. What is the value of the boat, in percentage terms, after 3 years?
   b. The 'half-life' is the time taken for the value to decrease by half. How long does it take for the boat to lose half its original value?
   c. How long does it take for the boat to fall in value from 50% of its original value to 40% of its original value?
   d. Estimate, in dollar terms, the value of the boat after 15 years.



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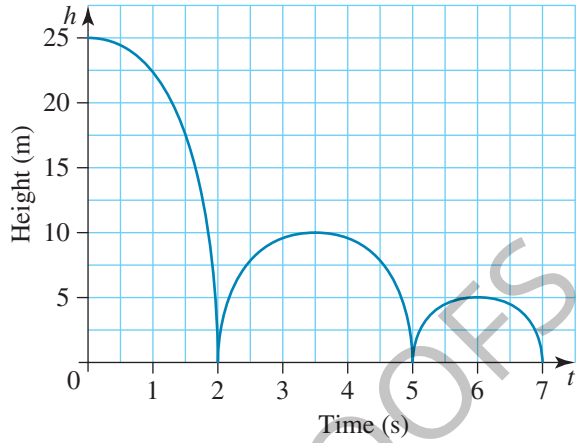
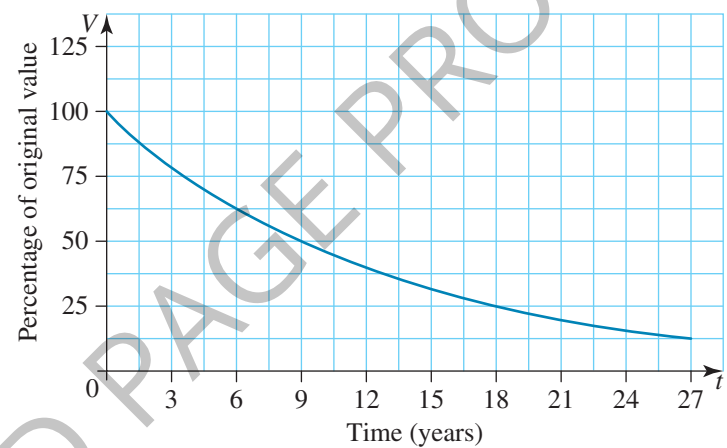
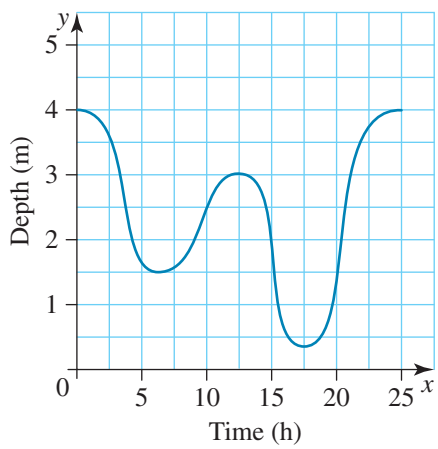
6. In an experiment, a ball is dropped from a height of 20 m. The height of the ball above ground level, as it bounces, is given in the graph.
   a. How long did it take the ball to reach the ground for the first time?
   b. How long was the ball in the air between the second and third bounce?
   c. What total distance is travelled by the ball in 7 seconds?
   d. Calculate the average speed of the ball for the first 2 seconds.



7. The value of a car, originally worth $32 000, decreases over time. The graph describes the value of the car as a percentage of its original price.
   a. What is the value of the car, as a percentage of its original price, after 6 years?
   b. The 'half-life' is the time taken for the value to decrease by half. How long does it take for the car to lose half its original value?
   c. How long does it take the car to fall in value from 50% of its original value to 25% of its original value?
   d. Estimate, in dollars, the value of the car after 27 years.



8. The depth of water across a sand bar varies according to the tide. This is shown in the graph.
   a. Approximately how long after high tide is the next low tide?
   b. How deep is the water at the first low tide?
   c. What is the lowest value in the depth of the water across the bar?
   d. What is the difference in the heights of the first and second high tide?
   e. If the first high tide is at 9.40 am Tuesday, at what time approximately will the first high tide occur on Wednesday?
   f. The water must be at least 1.5 m deep for Judy's boat to travel across the bar. Between what times, after the first high tide, can the boat *not* travel across the bar?



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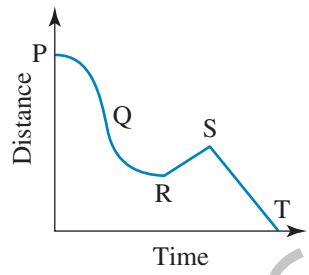
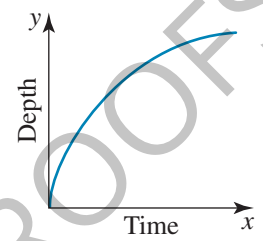
9. MC The graph shown describes the distance from home of an object as it travels.



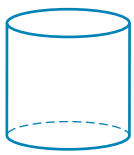
The point at which the object is moving with the greatest speed is:

A. P **B.** Q **C.** R **D.** S **E.** T

10. MC Water is poured into a container at a constant rate. The depth of water in the vessel is described in the graph. From the options following, the vessel that best matches the rate at which the vessel fills with water is:



A.



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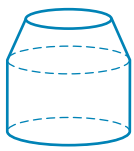

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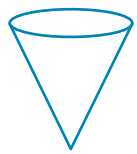
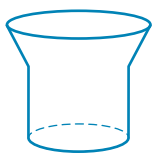


D.



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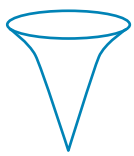

11. Carbon-14 is a radioactive element which breaks down over time. It has a half-life of about 6000 years; that is, every 6000 years, half of the material present breaks down.

a. Copy and complete the table for the amount of Carbon-14 (C-14) present.

Time (years)	0	6000	12 000	18 000
Amount of Carbon-14	800 units			

b. Plot the graph using these data.

c. Use the graph to estimate the time taken for the amount of C-14 to fall to 600 units.

d. Use extrapolation to estimate the amount of C-14 present after 24 000 years.

12. The following table displays the profit/loss a company makes from selling x units of toothpaste.

Units of toothpaste sold (× 1000)	0	1	2	3	4	5
Profit (× $1000)	−5	−6	−5	−2	1	9

a. Plot the graph using these data.

b. Use your graph to estimate how many units of toothpaste need to be sold in order to break even.

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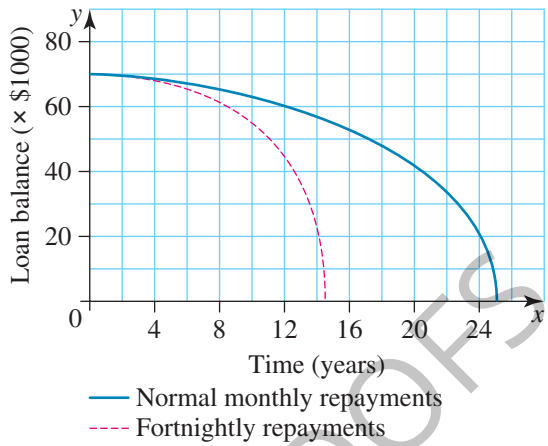
13. Yasmin borrows $70 000. She can pay a certain amount each month or half that value each fortnight. The graph shows the progress of the loan under both systems of payment.



a. How much time does it take to pay off the loan using monthly repayments?

b. How long does it take to pay off the loan using fortnightly repayments?

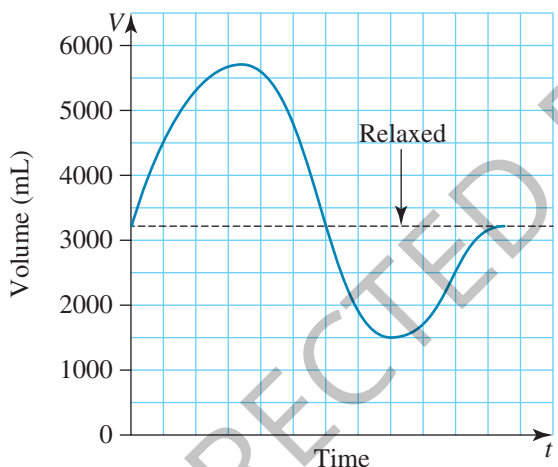
c. Using monthly repayments, after how many years has the balance fallen to 50% of its original value?

14. The graph shows the volume of air in a person's lungs during a cycle of breathing.

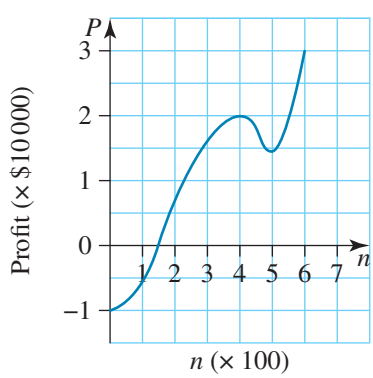
a. What is the maximum volume of the lungs?

b. When relaxed, how much air is contained in the lungs?

c. How much air is exhaled during the breathing cycle?



15. The graph at right shows the profits made by a manufacturing firm versus the number of units produced.



a. What profit is made when 400 units are produced?

b. What is the result if 0 units are produced?

16. a. Using the graph from question **15**, how many units need to be produced to break even?

b. Can you suggest an explanation for the dip in profits in the region $n = 500$?

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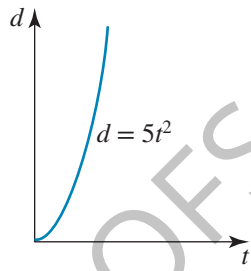
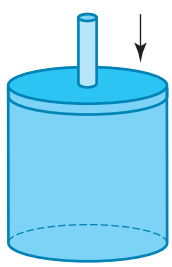
14.6 Constructing non-linear relations and graphs

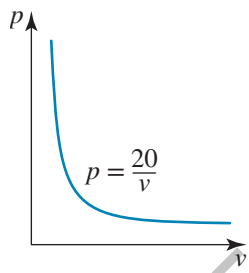
There are many important non-linear relationships where the graphs are not a straight line. Some examples follow.

A rivet is accidentally dropped from a tall pylon under construction. The relationship between the distance the rivet has fallen (d), and the time taken (t) is $d = 5t^2$.

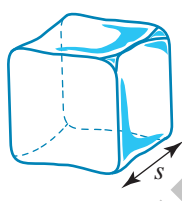


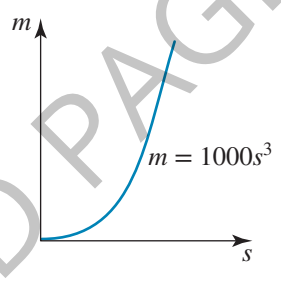
The gas in a piston is compressed. The relationship between the pressure (p) and the volume (v), is $p = 20v^{-1}$ or $p = \frac{20}{v}$.





The mass of a cubic block of ice (m) is related to the length of the side of the block (s) by the formula $m = 1000s^3$.

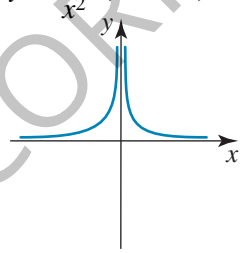




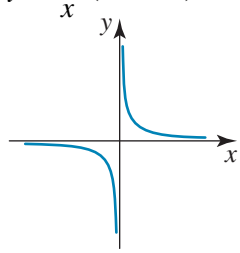
14.6.1 Graphical representation of relations of the form $y = kx^n$

Given a non-linear equation of the form $y = kx^n$, the easiest way to graph it is to use CAS. Alternatively, from the equation a table of values can be produced and this can be used to sketch a graph of the relationship. The general shape of the graphs of $y = kx^n$ for $n = -2, -1, 1, 2, 3$ are given here.

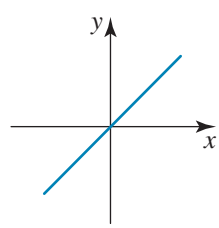
$y = \frac{k}{x^2}$ $(n = -2)$



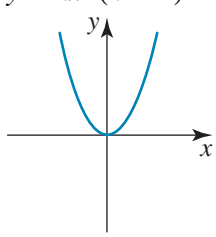
$y = \frac{k}{x}$ $(n = -1)$



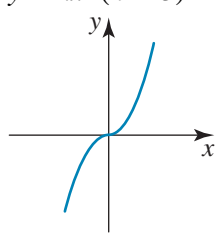
$y = kx$ $(n = 1)$



$y = kx^2$ $(n = 2)$



$y = kx^3$ $(n = 3)$



The value of k in these relationships is known as the **constant of proportionality**.

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WORKED EXAMPLE 14

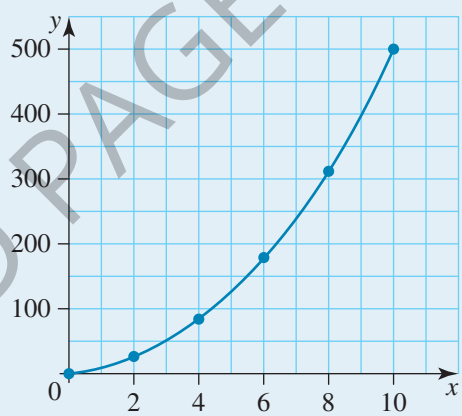
A research scientist has discovered that her data are related according to the equation $y = 5x^2$. Construct a table of values to draw the graph of the equation $y = 5x^2$ for x between 0 and 10.

THINK

1. Substitute values of x into $y = 5x^2$ to obtain corresponding y-values. For example, if $x = 4$ then
   $y = 5 \times 4^2$
   $= 5 \times 16$
   $= 80$.
2. Plot the points and draw a smooth curve through these points.

WRITE/DRAW

x	0	2	4	6	8	10
y	0	20	80	180	320	500



14.6.2 Finding a non-linear relationship using linear graphs

If you know the equation, it is reasonably straightforward to produce a table of values. The reverse process (that is, discovering a formula for a non-linear relationship from a table of values) is more difficult.

If the relationship between x and y is of the form $y = kx^n$, then plotting y against x^n will produce a straight line from the origin. The gradient of this line will equal the value of k, the constant of proportionality. The following algorithm can be used for finding the rule of the relationship, if the value of n is known.

1. Plot y against x^n. The result must be a straight line, coming from the origin.
2. Select any two points on the line to calculate the value of the gradient.
3. Since the gradient represents the value of k, substitute it into $y = kx^n$ to give the rule for the relationship.

Note that if the value of n is not known, plot y against x first. The shape of the graph will indicate the possible value(s) of n. Test this value by plotting y against x^n. If a straight line is produced, proceed as above; otherwise try a different value of n.

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WORKED EXAMPLE 15

By graphing y against x^2 find the equation for the relationship between x and y if the equation is of the form $y = kx^2$.

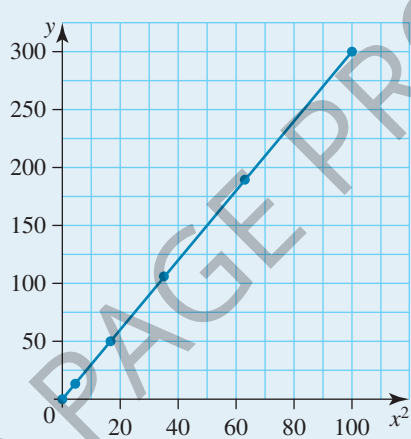
x	0	2	4	6	8	10
y	0	12	48	108	192	300

THINK

1. Since it is known that the relationship is of the form $y = kx^2$, draw up a table of values for x^2 and y.
2. Plot the values of x^2 on the horizontal axis and y on the vertical axis and join with a smooth line. (A straight line from the origin confirms that the relationship is of the form $y = kx^2$.)
3. Choose two points on the line, say (0, 0) and (100, 300), and calculate the gradient.
4. The gradient represents the value of k in $y = kx^2$. Replace k with 3 to state the equation for the relationship.

WRITE/DRAW

x^2	0	4	16	36	64	100
y	0	12	48	108	192	300



$$\text{Gradient} = \frac{300 - 0}{100 - 0}$$
$$= \frac{300}{100}$$
$$= 3$$

$y = kx^2$

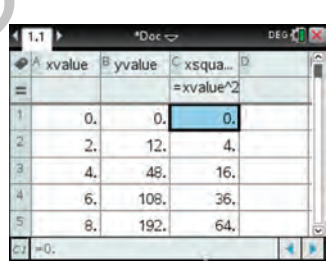
$k = 3$

Therefore, $y = 3x^2$.

TI | THINK

1. On a Lists & Spreadsheets page, enter the data into the Columns A and B and label the columns as *xvalue* and *yvalue*. Label Column C as *xsquared* and in the formula cell, complete the entry line as:
   $= xvalue^2$
   Press ENTER.

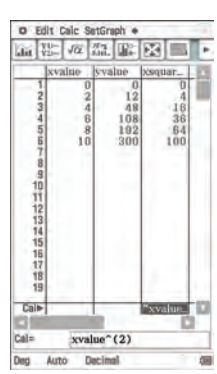
WRITE



CASIO | THINK

1. On the Statistics screen, enter the data into List1 and List2 and rename the lists as *xvalue* and *yvalue*. Label List3 as *xsquared* and in the formula cell, complete the entry line as:
   $xvalue^2$
   Press EXE.

WRITE



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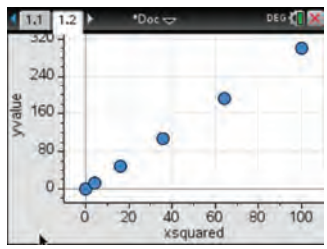
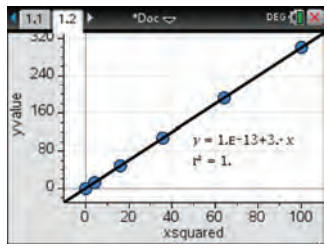
2. Press Ctrl + I and select:
5: Data & Statistics
Move the cursor to the horizontal axis and select *xsquared*.
Move the cursor to the vertical axis and select *yvalue*.
The points produce a straight line, so this confirms that the relationship is of the form $y = kx^2$.



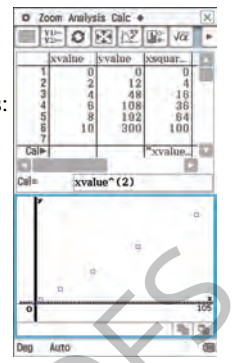
Press MENU
4: Analyze
6: Regression
2: Show Linear $(a + bx)$
The gradient b, represents the value of k in $y = kx^2$.



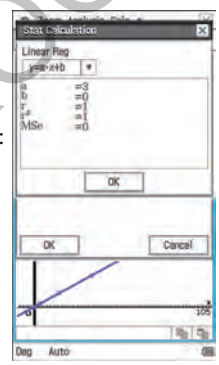
2. The answer appears on the screen.

$k = 3$
Therefore, $y = 3x^2$.

2. Tap:
SetGraph
Settings
Complete the Set StatGraphs fields as:
Draw: On
Type: Scatter
XList: main\xsquared
YList: main\yvalue
Freq: 1
Mark: square
Tap Set.
Tap the graph icon.



Tap:
Calc
Regression
Linear Reg
Complete the Set Calcuation fields as:
XList: main\squared
YList: main\yvalue
Freq: 1
Copy Formulae: y1
Copy Residua: Off
Tap OK.
The gradient a, represents the value of k in $y = kx^2$.



2. The answer appears on the screen.

$k = 3$
Therefore, $y = 3x^2$.

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studyon

Unit 4 › AOS M4 › Topic 1 › Concept 9
***y* = *kx*ⁿ** Summary screen and practice questions

Unit 4 › AOS M4 › Topic 1 › Concept 10
Constant of proportionality Summary screen and practice questions

Exercise 14.6 Constructing non-linear relations and graphs

1. **WE 14** A statistician found that his data are related according to the equation $y = 3x^2$. Construct a table of values and draw the graph of the equation $y = 3x^2$ for values of x between 0 and 10.
2. A researcher found that her data are related according to the equation $y = \frac{2}{x}$. Construct a table of values and draw the graph of the equation $y = \frac{2}{x}$ for values of x between 2 and 8.
3. Construct a table of values to draw the graphs of the following equations. (Use values of x from -5 to 5.)
   a. $y = x^2$
   b. $y = 2x^2$
   c. $y = 2x^3$
   d. $y = 0.5x^3$
   e. $y = \frac{0.5}{x}$
   f. $y = \frac{1}{x^2}$

   Use CAS to check your answers.

4. Construct a table of values to draw the graphs of the following equations. (Use values of x from -5 to 5.)

a. $y = 0.5x^2$ **b.** $y = x^3$ **c.** $y = \frac{1}{x}$ **d.** $y = \frac{2}{x}$

e. $y = \frac{2}{x^2}$ **f.** $y = \frac{0.5}{x^2}$

Use CAS to check your answers.

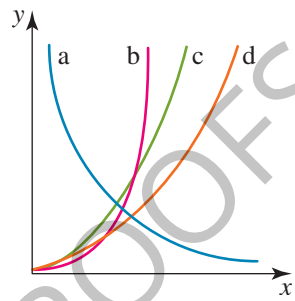
5. Write the letters **a** to **d** in your book. Against each letter that identifies a graph, write the equation which best matches that graph.

$y = x^2$

$y = 2x^2$

$y = x^3$

$y = \frac{3}{x}$



6. **WE 15** By graphing y against x^2 find the equation for the relationship between x and y if the equation is of the form $y = kx^2$.

x	0	2	4	6	8	10
y	0	8	32	72	128	200

7. By graphing y against x^2, find the equation for the relationship between x and y if the equation is of the form $y = kx^2$.

x	0	2	4	6	8	10
y	0	10	40	90	160	250

8. By graphing y against x^3, find the equation of the relationship between x and y, if the equation is of the form $y = kx^3$.

x	0	2	4	6	8
y	0	12	96	324	768

9. The Safety Council conducted research on the braking distance of vehicles and its relationship to the speed of the vehicle. The following data were obtained.

Speed (s) (km/h)	30	45	60	80	100
Braking distance (d) (metres)	7.5	16.9	30	53.3	83.3

a. Plot d versus s^2.

b. What is the equation relating d and s?

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10.

x	0.1	0.4	1	2	5
y	200	50	20	10	4

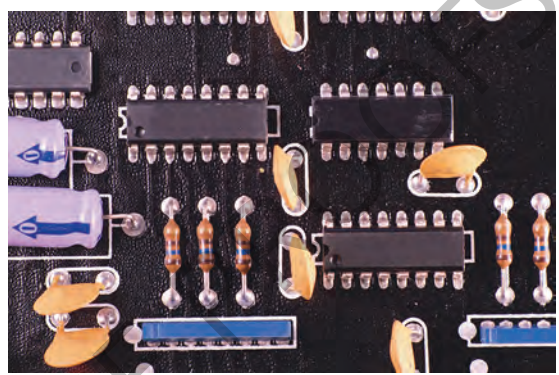
a. Plot these values on a set of axes.

b. Plot y versus $\frac{1}{x}$ and draw a straight line through the points.

c. What is the slope of the line?

d. Deduce the relationship between y and x.

11. In a physics experiment a student measured the current (I) flowing through a resistor for different values of the resistance (R) and obtained the following data.



Resistance (R) (ohms)	100	200	1000	1500
Current (I) (milliamps)	300	150	30	20

a. Plot values of I versus $\frac{1}{R}$.

b. Deduce a relationship between I and R.

12. Examine the following table of values.

x	0	2	4	6	8
y	0	1	8	27	64

a. Plot these values on a set of axes.

b. Plot y versus x^3 and draw a straight line through the points.

c. What is the slope of the line?

d. Deduce the relationship between y and x.

13. It is suspected that for male adults, mass (m) is related to height (h) by a formula like $m = kh^3$. Use the data in the table to find a relationship between m and h.

Mass (kg)	45	55	71	84
Height (cm)	150	160	175	185

14. The intensity of light drops off as you move away from the source. The relationship is of the form $I = \frac{k}{r^2}$. If the intensity (I) is 50 when $r = 20$, find:

a. the value of k

b. the distance at which the intensity of light falls below 35.

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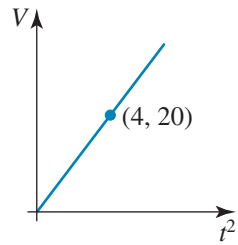
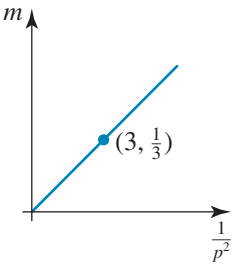
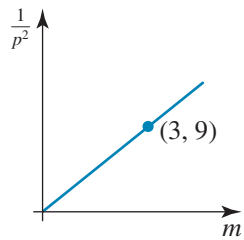
15. MC Which of the following equations describes the relationship shown on the graph at right?

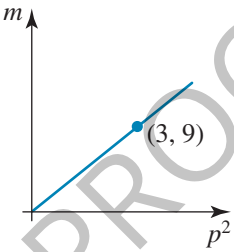


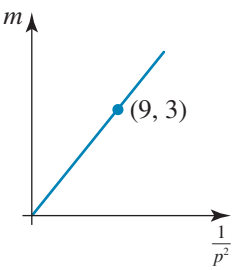
A. $V = 10t^2$
B. $V = 10t$
C. $V = 5t^2$
D. $V = 5t$
E. $V^2 = 5t$

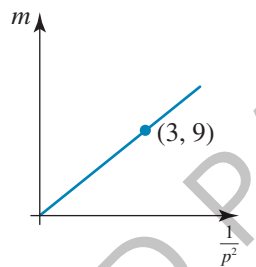
16. MC Which of the graphs shows the relationship $m = \frac{3}{p^2}$?

A.


B.


C.


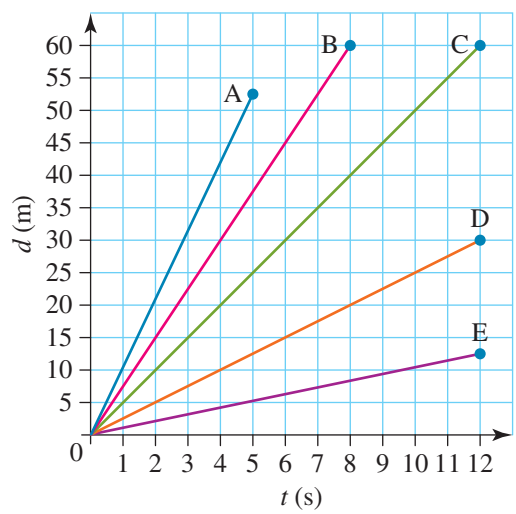
D.


E.


14.7 Review: exam practice

Multiple choice

Questions **1** to **3** relate to the following information. Karl and Lisa are travelling with a group of friends along the footpath and roadside at a constant speed. Lisa starts with Karl but, riding a bike at a constant speed, she moves faster than Karl. After 4 seconds, Lisa has moved 30 meters more than Karl.



1. The line that represents Karl is:
A. Line A **B.** Line B
C. Line C **D.** Line D
E. Line E

2. The line that represents the fastest person is:
A. Line A **B.** Line B
C. Line C **D.** Line D
E. Line E

3. The speed represented by line C is:
A. 60 m/s **B.** 12 m/s **C.** 5 m/s
D. 1 m/s **E.** 0.2 m/s

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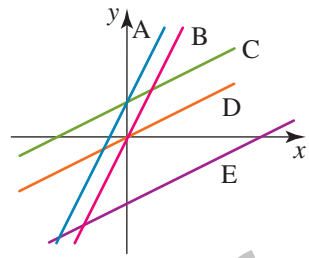
4. Match each of the following equations to the graph that fits best.

a. $y = 2x$

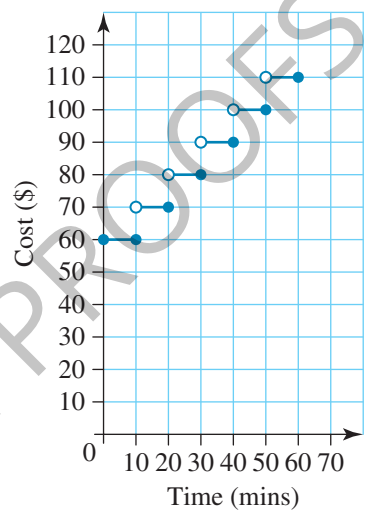
b. $y = 2x + 2$

c. $2y = x$

d. $2y = x + 4$



5. A plumber charges a call-out fee plus an amount for each 10 minutes or part thereof needed to complete a job. The graph below shows the cost charged for jobs up to 60 minutes in length.



a. The cost charged by the plumber for a job taking 20 minutes is:

A. \$60 **B.** \$70 **C.** \$80 **D.** \$90 **E.** \$100

b. The cost charged by the plumber for a job taking 35 minutes is:

A. \$80 **B.** \$85 **C.** \$90 **D.** \$95 **E.** \$100

c. The call-out fee is:

A. \$40 **B.** \$50 **C.** \$60 **D.** \$70 **E.** \$80

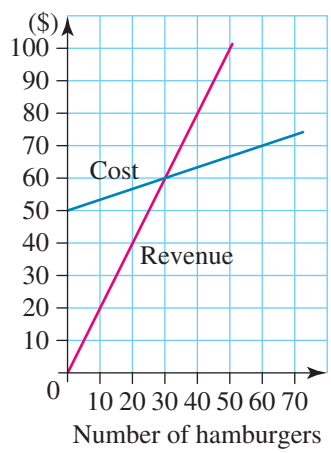
6. The solution to the simultaneous equations $y = 5x - 3$ and $y = 2x + 6$ is:

A. (1, 2) **B.** (2, 10) **C.** (2, 7) **D.** (3, 12) **E.** (3, 4)

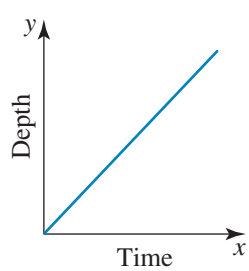
7. The graph shows the relationships for revenue and cost against the number of hamburgers sold at a school fete.

The number of hamburgers that need to be sold to break even is:

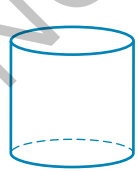
A. 0 **B.** 20 **C.** 30 **D.** 40 **E.** 50



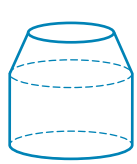
8. Water is poured into a container at a constant rate. The graph shows the depth of water in the container versus time. The container below that best matches this graph is:



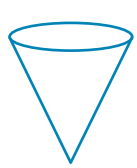
A.



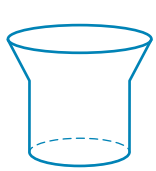
B.



C.



D.



E.

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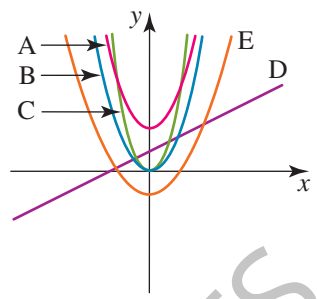
Questions 9 to 11 relate to the graph that shows the following (in random order):
$y = x^2$, $y = 2x^2$, $y = x^2 + 2$, $y = x^2 - 1$ and $y = x + 1$.



9. Which graph best matches the equation $y = x^2$?
10. Which graph best matches the equation $y = x^2 + 2$?
11. Which graph best matches the equation $y = 2x^2$?
12. A publisher produces a restaurant guide each year. To produce x copies, the cost is C dollars, where $C = 15\,000 + 15x$. If all of the x copies produced are sold, then the revenue gained is R dollars, where $R = 25x$.
    Which of the following statements is not true?
    - **A.** The cost and revenue equations are linear.
    - **B.** The selling price for each copy of the guide is $25.
    - **C.** It will cost $30 000 to produce 1000 copies of the guide.
    - **D.** The revenue from selling 1000 copies is $15 000.
    - **E.** The revenue is more than the cost if 1600 copies of the guide are sold.

Short answer

1. Bruno is an insurance salesman. He is paid $150 per week plus $120 for each insurance policy he sells.
   a. How much does Bruno receive if he sells 3 policies in a week?
   b. One week Bruno is paid $630. How many policies did he sell in that week?
   c. Write a rule that relates Bruno's weekly pay (P) and the number of policies he sells (n).
   d. Draw a graph of P versus n.
2. For the equation of the line $y = 3x - 3$:
   a. what is the value of y when $x = 4$?
   b. what is the value of x when $y = 18$?
   c. does the point $(0, -3)$ lie on the line?
   d. Using the two points whose coordinates were found in **a** and **b**, calculate the gradient.
3. Sketch the graphs of the following functions between $x = -5$ and $x = 5$.
   a. $y = 4x - 3$ b. $y + 2x = 6$ c. $y = 2x + 2$ d. $2y - x = 12$
4. A car salesperson is paid commission at the rate of 1.75% on the first $40 000 worth of cars that he or she sells in a month, and 1.2% on the remainder.

   a. Copy and complete the following table by calculating the commission due each month.

Sales	Commission
$20 000	
$30 000	
$40 000	
$50 000	
$60 000	

   b. Using the data in the table, draw a graph of commission versus sales.

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5. For each of the following pairs of simultaneous equations, find the solution using algebraic methods.

   a. $y = 4x - 2$
   $y = 3x$

   b. $2y + x = 4$
   $3y - 2x = 4$

6. The cost of manufacturing surfboards depends on labour, plant and raw materials. The monthly cost (C) is related to the number produced (n) by the equation $C = 4000 + 280n$.
   The revenue (R) made from selling n surfboards is $R = 450n$
   Both C and R are in dollars.

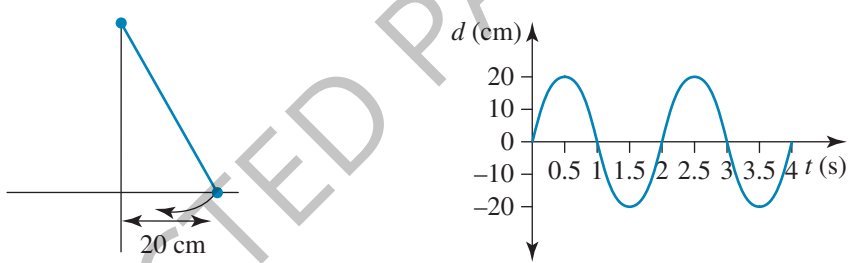
   a. Copy and complete the following table.

Number of surfboards made	Cost ($)	Revenue ($)
10		
20		
30		
40		

   b. Sketch the graph of cost (C) versus number (n) and the graph of revenue (R) versus number (n) on the same set of axes.

   c. How many surfboards need to be produced before the break-even point is reached?

7. A pendulum swings back and forth as shown in the diagram. The distance the bob of the pendulum is from its rest position is given as a function of time in the graph.



   a. How long does it take the pendulum to return to its starting position?

   b. Give the position of the pendulum after 1.5 seconds.

   c. Predict the position of the pendulum after 22.5 seconds.

8. It is known that y is related to x by the equation $y = kx^2$.

   a. Graph y versus x^2.

   b. Deduce the exact relationship of y versus x.

x	2	4	5
y	12	48	75

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Extended response

1. One day, Rachel and Nathan decided to travel to Mt Buffalo. For the first three hours they were driving at an average speed of 50 km/h, while for the last two hours they managed to increase their average speed to 60 km/h.
   a. Copy and complete the table below to show Rachel and Nathan's distance from home at various times of their journey.

Number of hours travelled (t)	Distance (km) from home (D)
1	
2	
3	
4	
5	

   b. Use the data from the table to represent the trip graphically.
   c. Name the type of graph formed in part b.
   d. Find the gradient and the y-intercept of the first section of the graph and hence, state the equation that relates time and distance travelled over the first 3 hours of the trip.
   e. Interpret the value of the gradient and the y-intercept in part d.
   f. Explain why the second section of the graph is steeper than the first.
   g. State the distance of Mt Buffalo from Rachel and Nathan's home.

2. Alex owns a small cake shop. To attract more customers, he decides to start making and selling coffee in addition to his cakes. The total cost of an espresso machine, cups and spoons adds up to $7500. In addition to this, Alex estimates that it will cost him an average of 20 cents to make each cup of coffee. He intends to sell the coffee at $2.70 per cup.
   a. Find a function that gives the total cost $C\,(\text{in }\$)$ of making n cups of coffee.
   b. Find a function that gives the revenue $R\,(\text{in }\$)$ from selling n cups of coffee.
   c. Draw the graphs of the cost and the revenue functions on the same set of axes.
   d. Find the coordinates of the point of intersection of the two graphs using algebraic methods.
   e. Interpret the meaning of the point of intersection.
   f. What is the smallest number of cups of coffee that Alex needs to make and sell to start making a profit?
   g. Write the function for the profit $P\,(\text{in }\$)$ in terms of n.
   h. How many cups of coffee does Alex have to make and sell in order to make a profit of $2000?

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3. In business, the rule of 72 is used to predict the growth of an investment. The rule states that if the growth rate of an investment is $r\%$ p.a., then the time taken for the investment to double is $72 \div r$ years.
It is estimated that a share portfolio, initially worth \$75 000, will grow at an annual rate of 9%.
   a. Use the rule of 72 to predict the value of the investment for the following years.

Time (years)	0	8	16
Value	\$75 000		

   b. Plot the data and draw a smooth curve through the points.
   c. Use this graph to estimate the value of the investment after 10 years.
   d. What do you predict the value of the investment to be in 40 years' time?

4. Deduce the relationship between y and x.

x	0	2	4	6	8
y	0	12	96	324	768

(*Hint*: It is of the form $y = kx^n$.)

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Unit 1 & 2 Sit topic test

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Answers

Topic 14 Construction and interpretation of graphs

Exercise 14.2 Constructing and interpreting straight-line graphs

1. **a.** 13 **b.** 6
c. No **d.** $m = 2, c = 5$

2. **a.** $y = 0$ **b.** $x = 4$ **c.** Yes

3. **a.** 0 **b.** 4
c. Yes **d.** $m = -2, c = 4$

4. **a.** 3 **b.** 0
c. No **d.** $m = \frac{-3}{2}, c = 6$

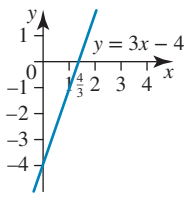
5. 4

6. -1

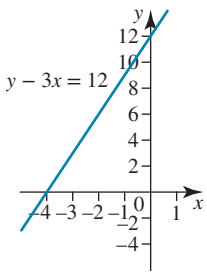
7. **a.** 3 **b.** 6 **c.** No **d.** -0.5

8. **a.** 0.5 **b.** 2 **c.** 1.5 **d.** 2

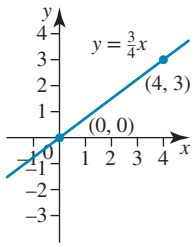
9. **a.**



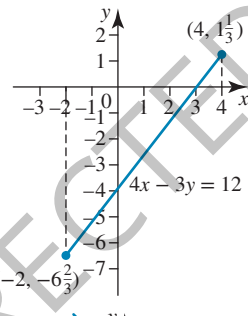
b.



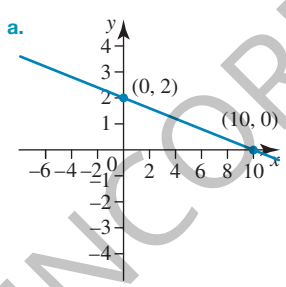
c.



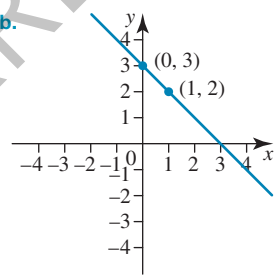
d.



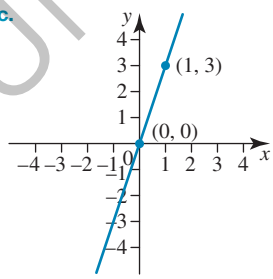
10. **a.**



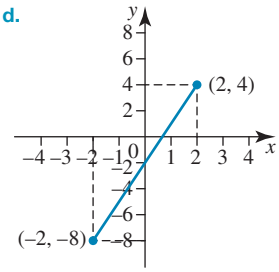
b.



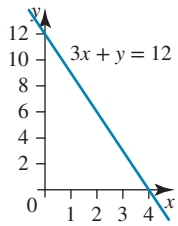
c.



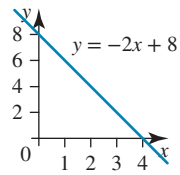
d.



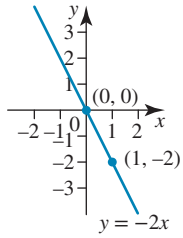
11. **a.**



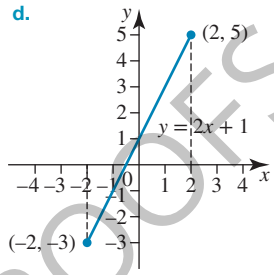
b.



c.



d.



12. E

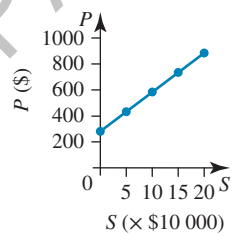
13. **a.** \$635

b.

Weekly sales	Weekly pay
5000	\$425
10 000	\$575
15 000	\$725
20 000	\$875

c. $P = 275 + 0.03S$

d.



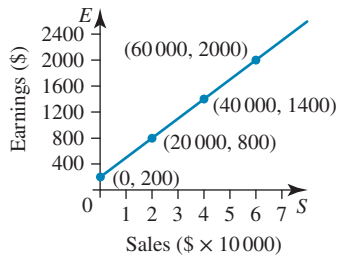
e. $m = 0.03$; y-intercept $= 275$; retainer amount

14. **a.**

Sales	0	\$20 000	\$40 000	\$60 000
Earnings	\$200	\$800	\$1400	\$2000

b. $E = 200 + 0.03S$

c.



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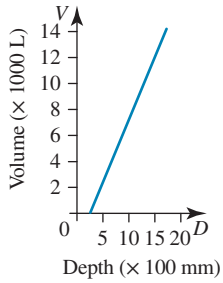
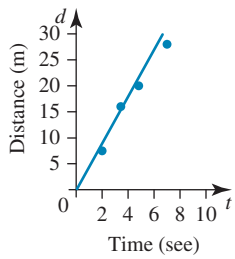
15. a.



b. 1550 mm

c. 3500 L

16. a.



b. $d = 36$ m

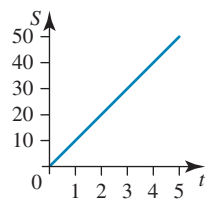
c. $d = 24$ m

17. a. 29.4 m/s

b. 8 s

c. $S = 9.8t$

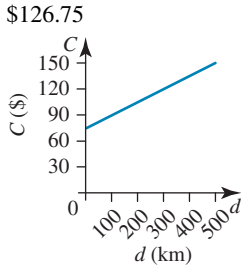
d.



18. a. $126.75

b. 450 km

c.



19. B

20. a. $470

b. $W = 350 + \frac{3S}{25}$

21. a. 4.73

b. $S = 108+(n-20)\times 4.73$

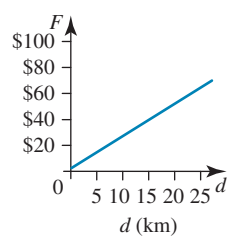
c. 4.73

22. a. $40.30

b. 7.2 km

c. $F = 2.8 + 2.5d$

d.



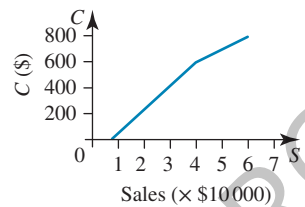
e. 2.5

Exercise 14.3 Line segments and step functions

1. a.

Sales ($)	Commission ($)
20 000	300
30 000	450
40 000	600
50 000	700
60 000	800

b.



c. Different commission rates mean different gradients.

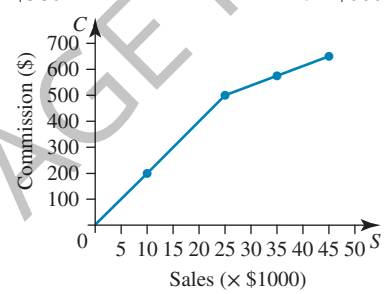
2. a. $200

b. $500

c. $580

d. $660

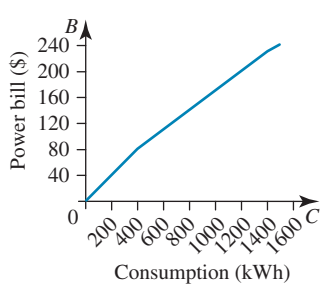
e.



3.

Consumption (kWh)	Power bill
200	$40
400	$80
600	$110
1000	$170
1500	$240

4.



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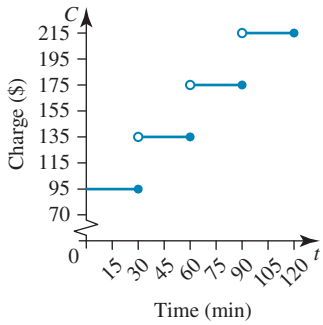
5. a.



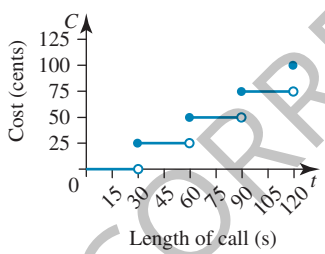
b.

Time	Charge
12 min	$55 + $40 = $95
23 min	$55 + $40 = $95
44 min	$55 + 2 × $40 = $135
56 min	$55 + 2 × $40 = $135
73 min	$55 + 3 × $40 = $175
87 min	$55 + 3 × $40 = $175

6. a.

Length of call (seconds)	Cost (cents)
15	0
30	25
45	25
60	50
75	50
90	75
105	75
120	100

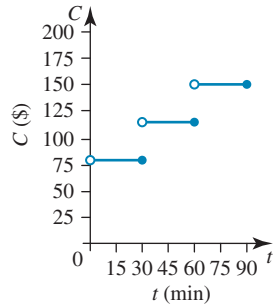
b.



7. a.

Time of service (minutes)	Cost
20	$80
30	$80
45	$115
60	$115
80	$150
90	$150

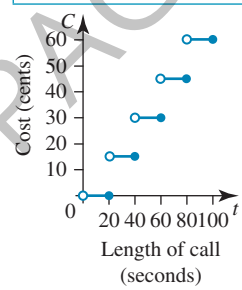
b.



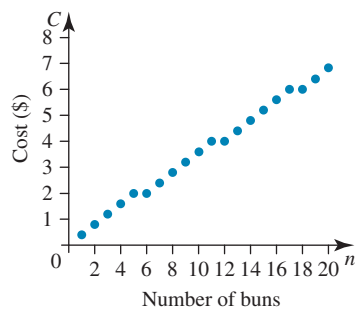
8. a.

Length of call (seconds)	Cost (cents)
10	0
20	15
30	15
40	30
50	30
60	45
70	45
80	60
90	60

b.



9. a.



b.

Number of buns	Cost
2	2 × 40c = $0.80
4	4 × 40c = $1.60
6	$2.00
8	$2.00 + 2 × 40c = $2.80
10	$2.00 + 4 × 40c = $3.60

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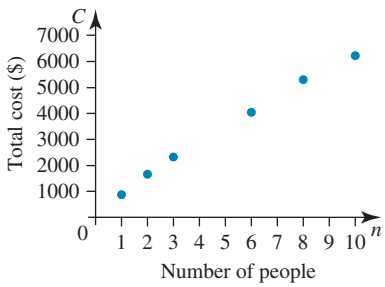
10. a.

Number of people	Total cost
1	$900
2	$1650
3	$2250
6	$4050
8	$5250
10	$6450

b.

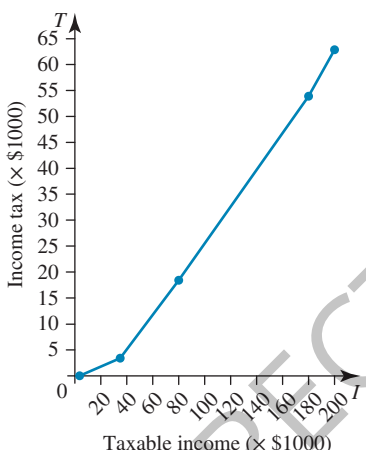


11. B

12. B

13. a. $0 **b.** $150 **c.** $2100
d. $4350 **e.** $5550 **f.** $11 550
g. $24 950 **h.** $63 550

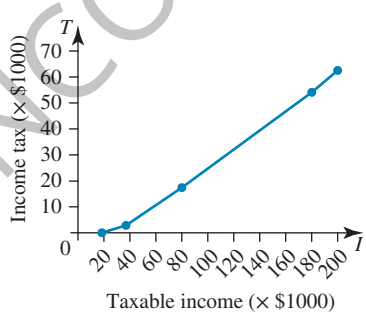
14. a.



b. i. $2700 **ii.** $23 100

15. a. $0 **b.** $0 **c.** $342
d. $3192 **e.** $4547 **f.** $11 047
g. $24 632 **h.** $63 232

16. a.

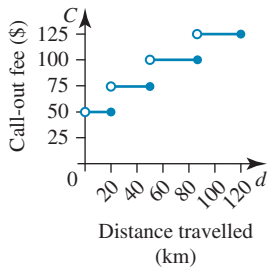


b. i. Approx $1100
ii. Approx $23 000
c. Answers will vary.

17. Answers will vary.

18. a. i. $75
ii. 20 km

b.



Exercise 14.4 Simultaneous equations and break-even point

1. (1, 3)

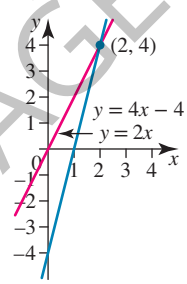
2. (−7, −4)

3. a. (2, 4) **b.** (2, 2) **c.** (1, 2)

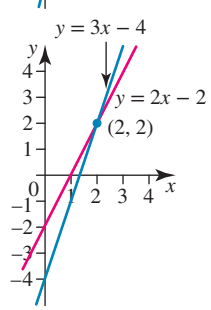
4. (1, −1)

5. (−2, 1)

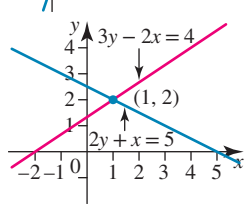
6. a.



b.



c.



7. (3, −1)

8. (−2, 2)

9. (0, 0)

10. (3, 5)

11. a.

Number of toys	Cost ($)	Revenue ($)
50	750	350
100	900	700
150	1050	1050
200	1200	1400

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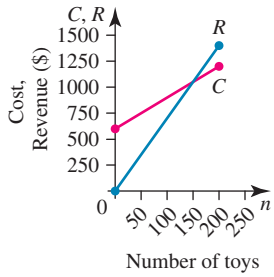
b.



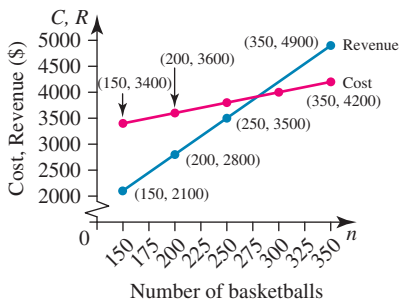
c. 150

d. (150, 1050). If 150 toys are produced, the cost and revenue is equal to $1050. No profit nor loss is made.

12. a.

Number of basketballs	Cost ($)	Revenue ($)
150	$3400	$2100
200	$3600	$2800
250	$3800	$3500
300	$4000	$4200
350	$4200	$4900

b.

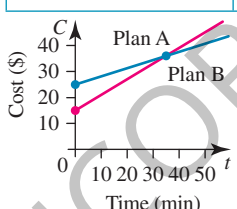


c. Number of basketballs (n) = 280 required to break even

13. a.

Call time (minutes)	Cost Plan A	Cost Plan B
10	$21	$28
20	$27	$31
30	$33	$34
40	$39	$37

b.

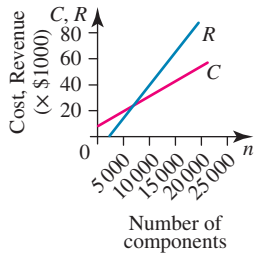


c. Approx.

14. a.

Number of components	Cost ($)	Revenue ($)
5000	18 500	14 500
10 000	31 000	37 000
15 000	43 500	59 500
20 000	56 000	82 000

b.

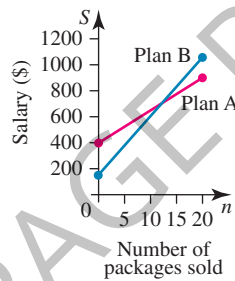


c. 7000

15. a.

Number of packages sold	Salary Plan A	Salary Plan B
5	$525	$375
10	$650	$600
15	$775	$825
20	$900	$1050

b.

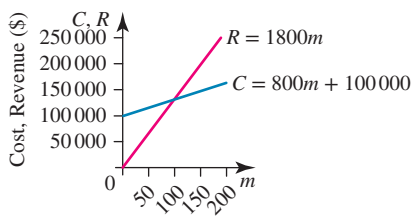


c. 13

16. A

17. a. $108 000 **b.** 30

18. a.



b. 100

c. $P = 1000m - 100\,000$

d. $75 000

Exercise 14.5 Interpreting non-linear graphs

1. a. 1.0 km

b. 0.15 km/min

c. The boarder is not moving.

d. 2.5 km

e. 20 mins

f. 10 mins

2. a. 2 km

b. 0.4 km/min

c. The runner is moving towards the finish line.

d. 8 km

e. 20 mins

f. 20 mins

3. a. 2 km

b. 0 km/h

c. Stationary (not moving)

d. 8 km

e. 90 min

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f. 30 min
g. 16 km/h

4. a. ≈ 70%
b. 9 years
c. 9 years
d. $8400

5. a. ≈ 70%
b. 8 years
c. 7 years
d. $15000

6. a. 2 s
b. 2 s
c. 50 m
d. 10 m/s

7. a. ≈ 62%
b. 9 years
c. 9 years
d. $4000

8. a. About 6 hours
b. 1.5 m
c. 0.5 m
d. 1 m
e. 10.40 am
f. Between about 15 hours and 21 hours after the first high tide

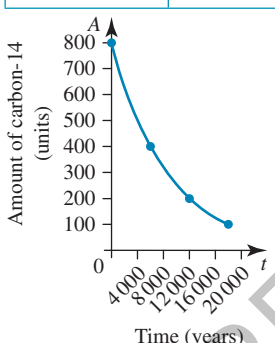
9. B

10. C

11. a.

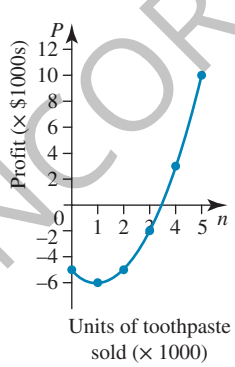
Time (number of years)	Amount of Carbon-14 (units)
0	800
6000	400
12 000	200
18 000	100

b.



c. 2500 years
d. 50 units

12. a.



b. 3450

13. a. 25 years
b. 14.5 years
c. Approximately 21 years

14. a. 5750 ml
b. 3100 ml
c. 4250 ml

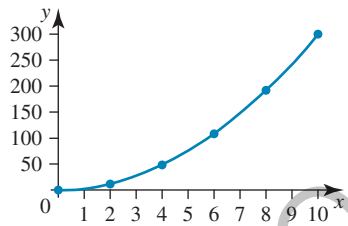
15. a. $20 000
b. Loss of $10 000

16. a. 150
b. May need to employ extra staff or install new plant.

Exercise 14.6 Constructing non-linear relations and graphs

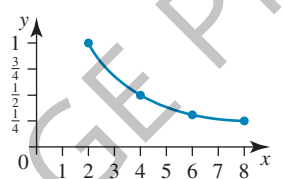
1. $y = 3x^2$

x	0	2	4	6	8	10
y	0	12	48	108	192	300

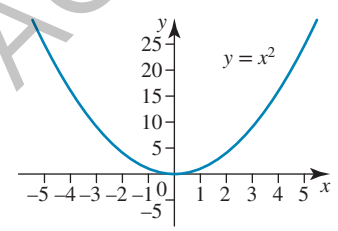


2.

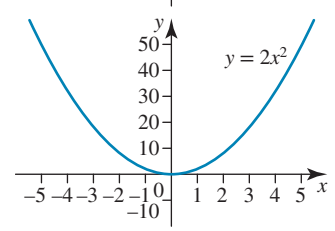
x	2	4	6	8
y	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$



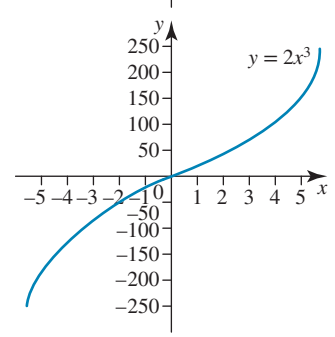
3. a.



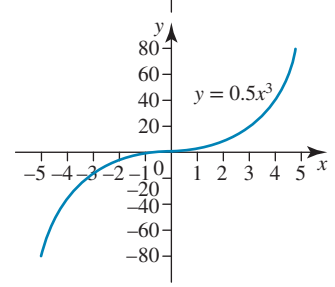
b.



c.



d.



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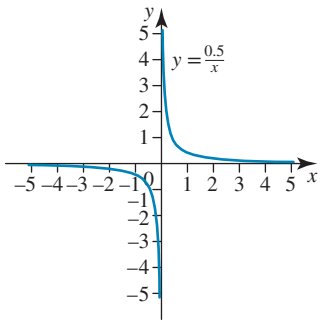
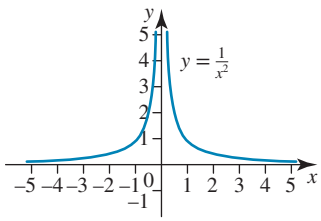
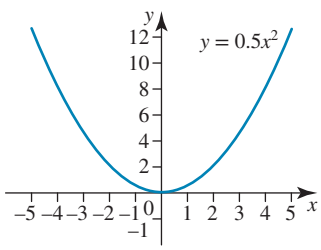
e.



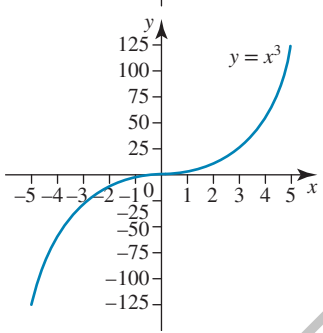
f.



4. a.



b.



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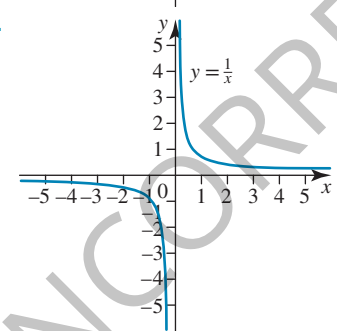
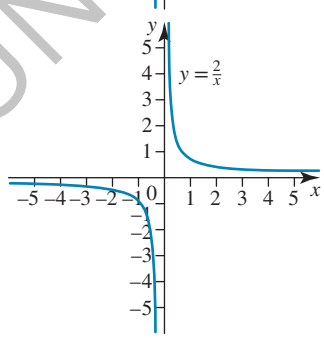
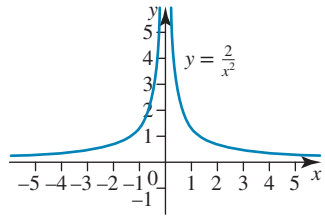


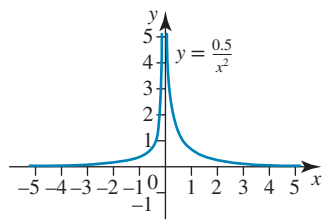
d.



e.



f.



5. a. $y = \frac{3}{x}$ **b.** $y = x^3$

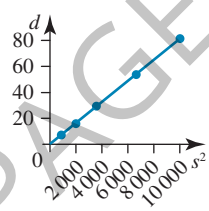
c. $y = 2x^2$ **d.** $y = x^2$

6. $y = 2x^2$

7. $y = 2.5x^2$

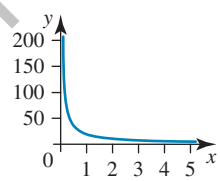
8. $y = 1.5x^3$

9. a.

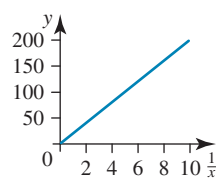


b. $d = 0.0083s^2$

10. a.



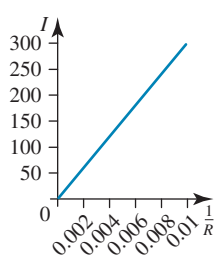
b.



c. 20

d. $y = \frac{20}{x}$

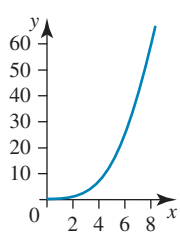
11. a.



b. $I = \frac{30\,000}{R}$

R	100	200	1000	1500
$\frac{1}{R}$	0.01	0.005	0.001	0.000 67
I	300	150	30	20

12. a.



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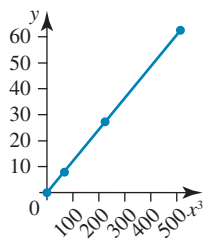
b.



x	0	2	4	6	8
x^3	0	8	64	216	512
y	0	1	8	27	64

c. 0.125

d. $y = 0.125x^3$

13. $m = \dfrac{h^3}{75\,000}$

14. a. $k = 20\,000$ **b.** 23.9

15. C

16. E

14.7 Review: exam practice

Multiple choice

1. D

2. A

3. C

4. a. B **b.** A **c.** D **d.** C

5. a. B **b.** C **c.** B

6. D

7. C

8. A

9. B

10. A

11. C

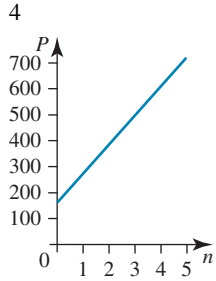
12. D

Short answer

1. a. \$510 **b.** 4

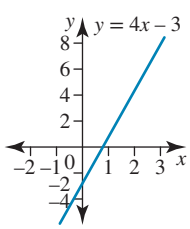
c. $P = 150 + 120n$

d.

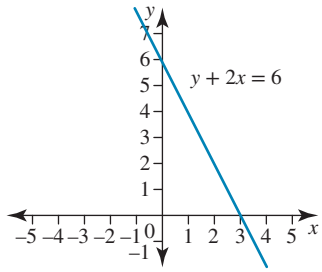


2. a. 9 **b.** 7 **c.** Yes **d.** 3

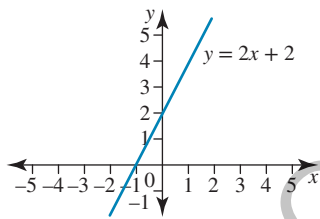
3. a.



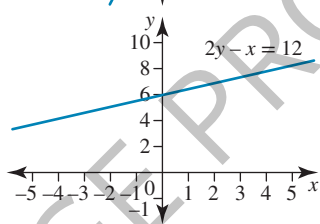
b.



c.



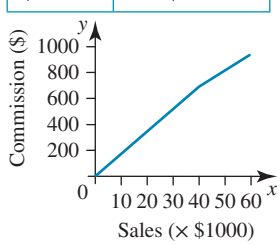
d.



4. a.

Sales	Commission
\$20 000	\$350
\$30 000	\$525
\$40 000	\$700
\$50 000	\$820
\$60 000	\$940

b.

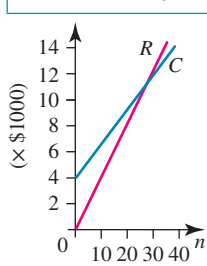


5. a. $x = 2, y = 6$ **b.** $x = \frac{4}{7}, y = \frac{12}{7}$

6. a.

Number of surfboards made	Cost (\$)	Revenue (\$)
10	6800	4500
20	9600	9000
30	12 400	13 500
40	15 200	18 000

b.



c. About 23 (That is, when 23 boards are produced, small loss occurs when 24 boards we produced; and sold, small profit occurs.)

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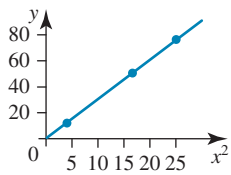
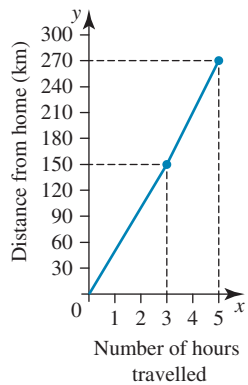
7. a. 1 second
   b. 20 cm to the left
   c. 20 cm to the right

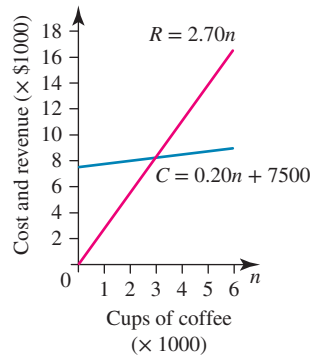
8. a. 

   b. $y = 3x^2$

Extended response

1. a. 50 km, 100 km, 150 km, 210 km, 270 km
   b. 

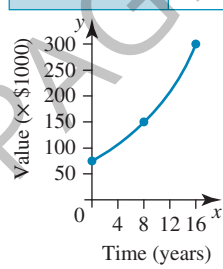
   c. Line segment graph
   d. $m = 50$; y-intercept $= 0$; $D = 50t$
   e. The gradient represents the average speed (50 km/h) during the first section of the trip. The y-intercept represents the initial distance from home; since it is 0, this means that they have started their trip from home.
   f. The steepness is determined by the gradient; in this case the gradient represents the average speed. The second section is steeper because the average speed increased from 50 km/h to 60 km/h.
   g. 270 km

2. a. $c = 0.2n + 7500$
   b. $R = 2.7n$
   c. 

   d. (3000, 8100)
   e. The point of intersection is the break-even point. It shows that by making and selling 3000 cups of coffee, neither a profit nor a loss will be made.
   f. 3001
   g. $P = 2.5n - 7500$
   h. 3800

3. a.

Time (years)	0	8	16
Value	$75 000	$150 000	$300 000

   b. 

   c. $170 000
   d. $2.4 million

4. $y = 1.5x^3$

UNCORRECTED PAGE PROOFS