TOPIC 1 Indices

1.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

1.1.1 Why learn this?

Don't you wish that your money could grow as quickly as a culture of bacteria? Perhaps it can — both financial investments and a culture of bacteria can grow exponentially, that is, according to the laws of indices. Indices are useful when a number is continually multiplied by itself, becoming very large, or perhaps very small.

1.1.2 What do you know?

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1. **THINK** List what you know about indices. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of indices.

LEARNING SEQUENCE

1.1 Overview
1.2 Review of index laws
1.3 Negative indices
1.4 Fractional indices
1.5 Combining index laws
1.6 Review

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Watch this eLesson: The story of mathematics: Leibniz
Searchlight ID: eles-1840

1.2 Review of index laws

- When a number or pronumeral is repeatedly multiplied by itself, it can be written in a shorter form called index form.
- A number written in index form has two parts, the **base** and the **index**, and is written as:

- Another name for an index is an exponent or a power.
- Performing operations on numbers or pronumerals written in index form requires the application of the index laws.

First Index Law: When terms with the same base are multiplied, the indices are added.

$$a^m \times a^n = a^{m+n}$$

Second Index Law: When terms with the same base are divided, the indices are subtracted.

$$a^m \div a^n = a^{m-n}$$

WORKED EXAMPLE 1

TI | CASIO

Simplify each of the following.

a $m^4n^3p \times m^2n^5p^3$ **b** $2a^2b^3 \times 3ab^4$ **c** $\dfrac{2x^5y^4}{10x^2y^3}$

THINK

a 1 Write the expression.

2 Multiply the terms with the same base by adding the indices. *Note:* $p = p^1$.

b 1 Write the expression.

2 Simplify by multiplying the coefficients, then multiply the terms with the same base by adding the indices.

c 1 Write the expression.

2 Simplify by dividing both of the coefficients by the same factor, then divide terms with the same base by subtracting the indices.

WRITE

a $m^4n^3p \times m^2n^5p^3$

$= m^{4+2}n^{3+5}p^{1+3}$

$= m^6n^8p^4$

b $2a^2b^3 \times 3ab^4$

$= 2 \times 3 \times a^{2+1} \times b^{3+4}$

$= 6a^3b^7$

c $\dfrac{2x^5y^4}{10x^2y^3}$

$= \dfrac{1x^{5-2}y^{4-3}}{5}$

$= \dfrac{x^3y}{5}$

Third Index Law: Any term (excluding 0) with an index of 0 is equal to 1.

$$a^0 = 1, \ a \neq 0$$

WORKED EXAMPLE 2

Simplify each of the following.

a $(2b^3)^0$ **b** $-4(a^2b^5)^0$

THINK	WRITE
a 1 Write the expression.	**a** $(2b^3)^0$
2 Apply the Third Index Law, which states that any term (excluding 0) with an index of 0 is equal to 1.	$= 1$
b 1 Write the expression.	**b** $-4(a^2b^5)^0$
2 The entire term inside the brackets has an index of 0, so the bracket is equal to 1.	$= -4 \times 1$
3 Simplify.	$= -4$

Fourth Index Law: When a power (a^m) is raised to a power, the indices are multiplied.

$$(a^m)^n = a^{mn}$$

Fifth Index Law: When the base is a product, raise every part of the product to the index outside the brackets.

$$(ab)^m = a^m b^m$$

Sixth Index Law: When the base is a fraction, multiply the indices of both the numerator and denominator by the index outside the brackets.

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

WORKED EXAMPLE 3

TI | CASIO

Simplify each of the following.

a $(2n^4)^3$ **b** $(3a^2b^7)^3$ **c** $\left(\frac{2x^3}{y^4}\right)^4$ **d** $(-4)^3$

THINK	WRITE
a 1 Write the term.	**a** $(2n^4)^3$
2 Apply the Fourth Index Law and simplify.	$= 2^{1\times3} \times n^{4\times3}$ $= 2^3 n^{12}$ $= 8n^{12}$
b 1 Write the expression.	**b** $(3a^2b^7)^3$
2 Apply the Fifth Index Law and simplify.	$= 3^{1\times3} \times a^{2\times3} \times b^{7\times3}$ $= 3^3 a^6 b^{21}$ $= 27a^6b^{21}$
c 1 Write the expression.	**c** $\left(\frac{2x^3}{y^4}\right)^4$
2 Apply the Sixth Index Law and simplify.	$= \frac{2^{1\times4} \times x^{3\times4}}{y^{4\times4}}$ $= \frac{16x^{12}}{y^{16}}$

d **1** Write the expression.
2 Write in expanded form.
3 Simplify, taking careful note of the negative sign.

d $(-4)^3$
$= -4 \times -4 \times -4$
$= -64$

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Complete this digital doc: SkillSHEET: Index form
Searchlight ID: doc-5168

Complete this digital doc: SkillSHEET: Using a calculator to evaluate numbers given in index form
Searchlight ID: doc-5169

Exercise 1.2 Review of index laws

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Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–f, 2a–f, 3a–f, 4a–f, 6, 7a–f, 9, 10	Questions: 1d–i, 2d–i, 3a–f, 4e–l, 6, 7a–f, 9–11, 15	Questions: 1d–l, 2d–l, 3, 4d–o, 5, 6, 7d–i, 8–16

Individual pathway interactivity: int-4562

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Fluency

1. WE1a, b Simplify each of the following.

a. $a^3 \times a^4$ **b.** $a^2 \times a^3 \times a$ **c.** $b \times b^5 \times b^2$

d. $ab^2 \times a^3b^5$ **e.** $m^2n^6 \times m^3n^7$ **f.** $a^2b^5c \times a^3b^2c^2$

g. $mnp \times m^5n^3p^4$ **h.** $2a \times 3ab$ **i.** $4a^2b^3 \times 5a^2b \times \frac{1}{2}b^5$

j. $3m^3 \times 2mn^2 \times 6m^4n^5$ **k.** $4x^2 \times \frac{1}{2}xy^3 \times 6x^3y^3$ **l.** $2x^3y^2 \times 4x \times \frac{1}{2}x^4y^4$

2. WE1c Simplify each of the following.

a. $a^4 \div a^3$ **b.** $a^7 \div a^2$ **c.** $b^6 \div b^3$

d. $\frac{4a^7}{3a^3}$ **e.** $\frac{21b^6}{7b^2}$ **f.** $\frac{48m^8}{12m^3}$

g. $\frac{m^7n^3}{m^4n^2}$ **h.** $\frac{2x^4y^3}{4x^4y}$ **i.** $6x^7y \div 8x^4$

j. $7ab^5c^4 \div ab^2c^4$ **k.** $\frac{20m^5n^3p^4}{16m^3n^3p^2}$ **l.** $\frac{14x^3y^4z^2}{28x^2y^2z^2}$

3. WE2 Simplify each of the following.

a. a^0 **b.** $(2b)^0$ **c.** $(3m^2)^0$

d. $3x^0$ **e.** $4b^0$ **f.** $-3 \times (2n)^0$

g. $4a^0 - \left(\frac{a}{4}\right)^0$ **h.** $5y^0 - 12$ **i.** $5x^0 - (5xy^2)^0$

4. WE3 Simplify each of the following.

a. $(a^2)^3$ **b.** $(2a^5)^4$ **c.** $\left(\frac{m^2}{3}\right)^4$

d. $\left(\frac{2n^4}{3}\right)^2$ **e.** $(a^2b)^3$ **f.** $(3a^3b^2)^2$

g. $(2m^3n^5)^4$ **h.** $\left(\frac{3m^2n}{4}\right)^3$ **i.** $\left(\frac{a^2}{b^3}\right)^2$

j. $\left(\frac{5m^3}{n^2}\right)^4$ **k.** $\left(\frac{7x}{2y^5}\right)^3$ **l.** $\left(\frac{3a}{5b^3}\right)^4$

m. $(-3)^5$ **n.** $(-7)^2$ **o.** $(-2)^5$

5. MC **a.** $2m^{10}n^5$ is the simplified form of:

A. $m^5n^3 \times 2m^4n^2$ **B.** $\frac{6m^{10}n^4}{3n}$ **C.** $(2m^5n^2)^2$ **D.** $2n(m^5)^2 \times n^4$ **E.** $\left(\frac{2m^5}{n^3}\right)^2$

b. The value of $4 - (5a)^0$ is:

A. -1 **B.** 9 **C.** 1 **D.** 3 **E.** 5

6. MC **a.** $4a^3b \times b^4 \times 5a^2b^3$ simplifies to:

A. $9a^5b^8$ **B.** $20a^5b^7$ **C.** $20a^5b^8$ **D.** $9a^5b^7$ **E.** $21a^5b^8$

b. $\frac{15x^9 \times 3x^6}{9x^{10} \times x^4}$ simplifies to:

A. $5x^9$ **B.** $9x$ **C.** $5x^{29}$ **D.** $9x^9$ **E.** $5x$

c. $\frac{3p^7 \times 8q^9}{12p^3 \times 4q^5}$ simplifies to:

A. $2q^4$ **B.** $\frac{p^4q^4}{2}$ **C.** $\frac{q^4}{2}$ **D.** $\frac{p^4q^4}{24}$ **E.** $\frac{q^4}{24}$

d. $\frac{7a^5b^3}{5a^6b^2} \div \frac{7b^3a^2}{5b^5a^4}$ simplifies to:

A. $\frac{49a^3b}{25}$ **B.** $\frac{25a^3b}{49}$ **C.** a^3b **D.** ab^3 **E.** $\frac{25ab^3}{49}$

Understanding

7. Evaluate each of the following.

a. $2^3 \times 2^2 \times 2$ **b.** $2 \times 3^2 \times 2^2$ **c.** $(5^2)^2$

d. $\frac{3^5 \times 4^6}{3^4 \times 4^4}$ **e.** $(2^3 \times 5)^2$ **f.** $\left(\frac{3}{5}\right)^3$

g. $\frac{4^4 \times 5^6}{4^3 \times 5^5}$ **h.** $(3^3 \times 2^4)^0$ **i.** $4(5^2 \times 3^5)^0$

8. Simplify each of the following.

a. $(x^y)^{3z}$ **b.** $a^b \times (p^q)^0$ **c.** $m^a \times n^b \times (mn)^0$

d. $\left(\frac{a^2}{b^3}\right)^x$ **e.** $\frac{n^3m^2}{n^pm^q}$ **f.** $(a^{m+n})^p$

Reasoning

9. Explain why $a^3 \times a^2 = a^5$ and not a^6.

10. Is $2x$ ever the same as x^2? Explain your reasoning using examples.

11. Explain the difference between $3x^0$ and $(3x)^0$.

12. a. In the following table, enter the values of $3a^2$ and $5a$ when $a = 0, 1, 2$ and 3.

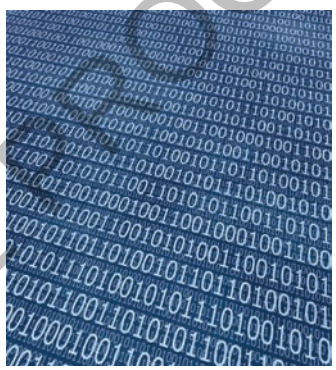
a	0	1	2	3
$3a^2$				
$5a$				
$3a^2 + 5a$				
$3a^2 \times 5a$				

b. Enter the values of $3a^2 + 5a$ and $3a^2 \times 5a$ in the table.

c. What do you think will happen as a becomes very large?

13. Find algebraically the exact value of x if $4^{x+4} = 2^{x^2}$. Justify your answer.

14. Binary numbers (base 2 numbers) are used in computer operations. As the name implies, binary uses only two types of numbers, 0 and 1, to express all numbers. A binary number such as 101 (read one, zero, one) means $(1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) = 4 + 0 + 1 = 5$ (in base 10, the base we are most familiar with).



The number 1010 (read one, zero, one, zero) means $(1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) = 8 + 0 + 2 + 0 = 10$.

If we read the binary number from right to left, the index of 2 increases by one each time, beginning with a power of zero. Using this information, write out the numbers 1 to 10 in binary (base 2) form.

Problem solving

15. Solve for x:

a. $\dfrac{7^x \times 7^{1+2x}}{(7^x)^2} = 16\,807$

b. $2^{2x} - 5(2^x) = -4$

16. For the following:

a. calculate the correct answer

b. identify the error in the solution.

$$\left(\frac{a^2b^3c}{a^2b^2}\right)^3 \times \left(\frac{a^3b^2c^2}{a^2b^3}\right)^2 = \left(\frac{b^3c}{b^2}\right)^3 \times \left(\frac{ab^2c^2}{b^3}\right)^2$$
$$= \left(\frac{bc}{1}\right)^3 \times \left(\frac{ac^2}{b}\right)^2$$
$$= \left(\frac{abc^3}{b}\right)^6$$
$$= \left(\frac{ac^3}{1}\right)^6$$
$$= a^6c^{18}$$

Reflection

Why are these laws called index laws?

CHALLENGE 1.1

It was estimated that there were 4×10^{10} locusts in the largest swarm ever seen. If each locust can consume 2 grams of grain in a day, how long would it take the swarm to consume 1 tonne of grain?

1.3 Negative indices

- Consider the expression $\frac{a^3}{a^5}$. This expression can be simplified in two different ways.

  1. Written in expanded form: $\frac{a^3}{a^5} = \frac{a \times a \times a}{a \times a \times a \times a \times a}$
  $= \frac{1}{a \times a}$
  $= \frac{1}{a^2}$

  2. Using the Second Index Law: $\frac{a^3}{a^5} = a^{3-5}$
  $= a^{-2}$
  So, $a^{-2} = \frac{1}{a^2}$.

- In general, $\frac{1}{a^n} = \frac{a^0}{a^n}$ $(1 = a^0)$
  $= a^{0-n}$ (using the Second Index Law)
  $= a^{-n}$

  Seventh Index Law: $a^{-n} = \frac{1}{a^n}$

- The convention is that an expression should be written using positive indices and with pronumerals given in alphabetical order.

WORKED EXAMPLE 4

Express each of the following with positive indices.

a x^{-3} **b** $2m^{-4}n^2$ **c** $\frac{4}{a^{-3}}$

THINK	WRITE
a 1 Write the expression.	**a** x^{-3}
2 Apply the Seventh Index Law.	$= \frac{1}{x^3}$
b 1 Write the expression.	**b** $2m^{-4}n^2$
2 Apply the Seventh Index Law to write the expression with positive indices.	$= \frac{2n^2}{m^4}$
c 1 Write the expression and rewrite the fraction, using a division sign.	**c** $\frac{4}{a^{-3}} = 4 \div a^{-3}$
2 Apply the Seventh Index Law to write the expression with positive indices.	$= 4 \div \frac{1}{a^3}$
3 To divide the fraction, change fraction division into multiplication.	$= 4 \times \frac{a^3}{1}$
	$= 4a^3$

- Part **c** from Worked Example 4 demonstrates the **converse** of the Seventh Index Law $\frac{1}{a^{-n}} = a^n$.

WORKED EXAMPLE 5

TI | CASIO

Simplify each of the following, expressing the answers with positive indices.

a $a^2b^{-3} \times a^{-5}b$ **b** $\dfrac{2x^4y^2}{3xy^5}$ **c** $\left(\dfrac{2m^3}{n^{-2}}\right)^{-2}$

THINK	WRITE
a 1 Write the expression.	**a** $a^2b^{-3} \times a^{-5}b$
2 Apply the First Index Law. Multiply terms with the same base by adding the indices.	$= a^{2+-5}b^{-3+1}$ $= a^{-3}b^{-2}$
3 Express the answer with positive indices.	$= \dfrac{1}{a^3b^2}$
b 1 Write the expression.	**b** $\dfrac{2x^4y^2}{3xy^5}$
2 Apply the Second Index Law. Divide terms with the same base by subtracting the indices.	$= \dfrac{2x^{4-1}y^{2-5}}{3}$ $= \dfrac{2x^3y^{-3}}{3}$
3 Express the answer with positive indices.	$= \dfrac{2x^3}{3y^3}$
c 1 Write the expression.	**c** $\left(\dfrac{2m^3}{n^{-2}}\right)^{-2}$
2 Apply the Sixth Index Law. Multiply the indices of both the numerator and denominator by the index outside the brackets.	$= \dfrac{2^{-2}m^{-6}}{n^4}$
3 Express all terms with positive indices.	$= \dfrac{1}{2^2m^6n^4}$
4 Simplify.	$= \dfrac{1}{4m^6n^4}$

- Numbers in index form can be easily evaluated if they are expressed with positive indices first. Consider the following example.

WORKED EXAMPLE 6

Evaluate 6×3^{-3} without using a calculator.

THINK	WRITE
1 Write the multiplication.	6×3^{-3}
2 Apply the Seventh Index Law to write 3^{-3} with a positive index.	$= 6 \times \dfrac{1}{3^3}$
3 Multiply the numerator of the fraction by the whole number.	$= \dfrac{6}{3^3}$

4 Evaluate the denominator. $= \frac{6}{27}$

5 Cancel by dividing both the numerator and denominator by the highest common factor (3). $= \frac{2}{9}$

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Try out this interactivity: Colour code breaker
Searchlight ID: int-2777

Exercise 1.3 Negative indices

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Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–i, 2a–i, 3a–f, 4, 5a–e, 6a–b, 8a–c, 9, 11a, 12	Questions: 1a–i, 2a–i, 3c–h, 4, 5a–g, 6, 7, 8b–e, 9, 11a–b, 12, 13, 15, 18	Questions: 1, 2c–o, 3c–l, 4, 5d–j, 6, 7, 8c–f, 9–18

Individual pathway interactivity: int-4563

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE4 Express each of the following with positive indices.

a. x^{-5} **b.** y^{-4} **c.** $2a^{-9}$

d. $\frac{4}{5}a^{-3}$ **e.** $3x^2y^{-3}$ **f.** $2^{-2}m^{-3}n^{-4}$

g. $6a^3b^{-1}c^{-5}$ **h.** $\frac{1}{a^{-6}}$ **i.** $\frac{2}{3a^{-4}}$

j. $\frac{6a}{3b^{-2}}$ **k.** $\frac{7a^{-4}}{2b^{-3}}$ **l.** $\frac{2m^3n^{-5}}{3a^{-2}b^4}$

2. WE5 Simplify each of the following, expressing the answers with positive indices.

a. $a^3b^{-2} \times a^{-5}b^{-1}$ **b.** $2x^{-2}y \times 3x^{-4}y^{-2}$ **c.** $3m^2n^{-5} \times m^{-2}n^{-3}$

d. $4a^3b^2 \div a^5b^7$ **e.** $2xy^6 \div 3x^2y^5$ **f.** $5x^{-2}y^3 \div 6xy^2$

g. $\frac{6m^4n}{2n^3m^6}$ **h.** $\frac{4x^2y^9}{x^7y^{-3}}$ **i.** $\frac{2m^2n^{-4}}{6m^5n^{-1}}$

j. $(2a^3m^4)^{-5}$ **k.** $4(p^7q^{-4})^{-2}$ **l.** $3(a^{-2}b^{-3})^4$

m. $\left(\frac{2p^2}{3q^3}\right)^{-3}$ **n.** $\left(\frac{a^{-4}}{2b^{-3}}\right)^2$ **o.** $\left(\frac{6a^2}{3b^{-2}}\right)^{-3}$

3. WE6 Evaluate each of the following without using a calculator.

a. 2^{-3} **b.** 6^{-2} **c.** 3^{-4}

d. $3^{-2} \times 2^3$ **e.** $4^{-3} \times 2^2$ **f.** 5×6^{-2}

g. $\dfrac{6}{2^{-3}}$ **h.** $\dfrac{4 \times 3^{-3}}{2^{-3}}$ **i.** $\frac{1}{3} \times 5^{-2} \times 3^4$

j. $\dfrac{16^0 \times 2^4}{8^2 \times 2^{-4}}$ **k.** $\dfrac{5^3 \times 25^0}{25^2 \times 5^{-4}}$ **l.** $\dfrac{3^4 \times 4^2}{12^3 \times 15^0}$

4. Write each of these numbers as a power of 2.

a. 8 **b.** $\frac{1}{8}$ **c.** 32 **d.** $\frac{1}{64}$

5. Complete each statement by writing the correct index.

a. $125 = 5^{\cdots}$ **b.** $\frac{1}{16} = 4^{\cdots}$ **c.** $\frac{1}{7} = 7^{\cdots}$ **d.** $216 = 6^{\cdots}$

e. $0.01 = 10^{\cdots}$ **f.** $1 = 8^{\cdots}$ **g.** $64 = 4^{\cdots}$ **h.** $\frac{1}{64} = 4^{\cdots}$

i. $\frac{1}{64} = 2^{\cdots}$ **j.** $\frac{1}{64} = 8^{\cdots}$

6. Evaluate the following expressions.

a. $\left(\frac{2}{3}\right)^{-1}$ **b.** $\left(\frac{5}{4}\right)^{-1}$ **c.** $\left(3\frac{1}{2}\right)^{-1}$ **d.** $\left(\frac{1}{5}\right)^{-1}$

7. Write the following expressions with positive indices.

a. $\left(\dfrac{a}{b}\right)^{-1}$ **b.** $\left(\dfrac{a^2}{b^3}\right)^{-1}$ **c.** $\left(\dfrac{a^{-2}}{b^{-3}}\right)^{-1}$ **d.** $\left(\dfrac{m^3}{n^{-2}}\right)^{-1}$

8. Evaluate each of the following, using a calculator.

a. 3^{-6} **b.** 12^{-4} **c.** 7^{-5}

d. $\left(\frac{1}{2}\right)^{-8}$ **e.** $\left(\frac{3}{4}\right)^{-7}$ **f.** $(0.04)^{-5}$

9. MC **a.** x^{-5} is the same as:

A. $-x^5$ **B.** $-5x$ **C.** $5x$ **D.** $\dfrac{1}{x^5}$ **E.** $\dfrac{1}{x^{-5}}$

b. $\dfrac{1}{a^{-4}}$ is the same as:

A. $4a$ **B.** $-4a$ **C.** a^4 **D.** $\dfrac{1}{a^4}$ **E.** $-a^4$

c. $\frac{1}{8}$ is the same as:

A. 2^3 **B.** 2^{-3} **C.** 3^2 **D.** 3^{-2} **E.** $\dfrac{1}{2^{-3}}$

10. MC **a.** Which of the following, when simplified, gives $\dfrac{3m^4}{4n^2}$?

A. $\dfrac{3m^{-4}n^{-2}}{4}$ **B.** $3 \times 2^{-2} \times m^4 \times n^{-2}$ **C.** $\dfrac{3n^{-2}}{2^{-2}m^{-4}}$

D. $\dfrac{2^2n^{-2}}{3^{-1}m^{-4}}$ **E.** $3m^4 \times 2^2n^{-2}$

b. When simplified, $3a^{-2}b^{-7} \div \frac{3}{4}a^{-4}b^6$ is equal to:

A. $\dfrac{4}{a^6b^{13}}$ **B.** $\dfrac{9b}{4a^6}$ **C.** $\dfrac{9a^2}{4b}$ **D.** $\dfrac{4a^2}{b^{13}}$ **E.** $\dfrac{4a^2}{b}$

c. When $(2x^6y^{-4})^{-3}$ is simplified, it is equal to:

A. $\dfrac{2x^{18}}{y^{12}}$ **B.** $\dfrac{x^{18}}{8y^{12}}$ **C.** $\dfrac{y^{12}}{8x^{18}}$ **D.** $\dfrac{8y^{12}}{x^{18}}$ **E.** $\dfrac{x^{18}}{6y^{12}}$

d. If $\left(\frac{2a^x}{b^y}\right)^3$ is equal to $\frac{8b^9}{a^6}$, then x and y (in that order) are:

A. -3 and -6 **B.** -6 and -3 **C.** -3 and 2
D. -3 and -2 **E.** -2 and -3

Understanding

11. Simplify, expressing your answer with positive indices.

a. $\frac{m^{-3}n^{-2}}{m^{-5}n^6}$ **b.** $\frac{(m^3n^{-2})^{-7}}{(m^{-5}n^3)^4}$ **c.** $\frac{5(a^3b^{-3})^2}{(ab^{-4})^{-1}} \div \frac{(5a^{-2}b)^{-1}}{(a^{-4}b)^3}$

12. Simplify, expanding any expressions in brackets.

a. $(r^3 + s^3)(r^3 - s^3)$ **b.** $(m^5 + n^5)^2$

c. $\frac{(x^{a+1})^b \times x^{a+b}}{x^{a(b+1)} \times x^{2b}}$ **d.** $\left(\frac{p^{x+1}}{p^{x-1}}\right)^{-4} \times \frac{p^{8(x+1)}}{(p^{2x})^4} \times \frac{p^2}{(p^{12x})^0}$

13. Write $\left(\frac{2^r \times 8^r}{2^{2r} \times 16}\right)$ in the form 2^{ar+b}.

14. Write $2^{-m} \times 3^{-m} \times 6^{2m} \times 3^{2m} \times 2^{2m}$ as a power of 6.

15. Solve for x if $4^x - 4^{x-1} = 48$.

Reasoning

16. Explain why each of these statements is false. Illustrate each answer by substituting a value for the pronumeral.

a. $5x^0 = 1$ **b.** $9x^5 \div 3x^5 = 3x$ **c.** $a^5 \div a^7 = a^2$ **d.** $2c^{-4} = \frac{1}{2c^4}$

Problem solving

17. Solve for x and y if $5^{x-y} = 625$ and $3^{2x} \times 3^y = 243$.

Hence, evaluate $\frac{35^x}{7^{-2y} \times 5^{-3y}}$.

18. Solve for n. Verify your answers.

a. $(2^n)^n \times (2^n)^3 \times 4 = 1$ **b.** $\frac{(3^n)^n \times (3^n)^{-1}}{81} = 1$

Reflection

Are there any index laws from Section 1.2 that do not apply to negative indices?

1.4 Fractional indices

- Terms with fractional indices can be written as surds, using the following laws:

1. $a^{\frac{1}{n}} = \sqrt[n]{a}$
2. $a^{\frac{m}{n}} = \sqrt[n]{a^m}$
   $= (\sqrt[n]{a})^m$

- To understand how these laws are formed, consider the following numerical examples.

We know $4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^1$

and that $\sqrt{4} \times \sqrt{4} = \sqrt{16}$
$= 4$

It follows, then, that $4^{\frac{1}{2}} = \sqrt{4}$.

Similarly, we know that $8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8^1$

and that $\sqrt[3]{8} \times \sqrt[3]{8} \times \sqrt[3]{8} = \sqrt[3]{512}$

$= 8$

It follows, then that $8^{\frac{1}{3}} = \sqrt[3]{8}$.

This observation can be generalised to $a^{\frac{1}{n}} = \sqrt[n]{a}$.

Now consider: $a^{\frac{m}{n}} = a^{m \times \frac{1}{n}}$ or $a^{\frac{m}{n}} = a^{\frac{1}{n} \times m}$

$= (a^m)^{\frac{1}{n}}$ $= (a^{\frac{1}{n}})^m$

$= \sqrt[n]{a^m}$ $= (\sqrt[n]{a})^m$

Eighth Index Law: $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

WORKED EXAMPLE 7

Evaluate each of the following without using a calculator.

a $9^{\frac{1}{2}}$ **b** $16^{\frac{3}{2}}$

THINK	WRITE
a 1 Rewrite the number using the Eighth Index Law.	**a** $9^{\frac{1}{2}} = \sqrt{9}$
2 Evaluate.	$= 3$
b 1 Rewrite the number using $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$.	**b** $16^{\frac{3}{2}} = (\sqrt{16})^3$
2 Simplify and evaluate the result.	$= 4^3$
	$= 64$

WORKED EXAMPLE 8

TI | CASIO

Simplify each of the following.

a $m^{\frac{1}{5}} \times m^{\frac{2}{5}}$ **b** $(a^2b^3)^{\frac{1}{6}}$ **c** $\left(\dfrac{x^{\frac{2}{3}}}{y^{\frac{3}{4}}}\right)^{\frac{1}{2}}$

THINK	WRITE
a 1 Write the expression.	**a** $m^{\frac{1}{5}} \times m^{\frac{2}{5}}$
2 Apply the First Index Law to multiply terms with the same base by adding the indices.	$= m^{\frac{3}{5}}$
b 1 Write the expression.	**b** $(a^2b^3)^{\frac{1}{6}}$
2 Use the Fourth Index Law to multiply each index inside the brackets by the index outside the brackets.	$= a^{\frac{2}{6}}b^{\frac{3}{6}}$
3 Simplify.	$= a^{\frac{1}{3}}b^{\frac{1}{2}}$

c 1 Write the expression.

c $\left(\dfrac{x^{\frac{2}{3}}}{y^{\frac{3}{4}}}\right)^{\frac{1}{2}}$

2 Use the Sixth Index Law to multiply the index in both the numerator and denominator by the index outside the brackets.

$= \dfrac{x^{\frac{1}{3}}}{y^{\frac{3}{8}}}$

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Exercise 1.4 Fractional indices

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Fluency

1. WE7 Evaluate each of the following without using a calculator.

a. $16^{\frac{1}{2}}$ **b.** $25^{\frac{1}{2}}$ **c.** $81^{\frac{1}{2}}$ **d.** $8^{\frac{1}{3}}$ **e.** $64^{\frac{1}{3}}$ **f.** $81^{\frac{1}{4}}$

2. Write the following in surd form.

a. $15^{\frac{1}{2}}$ **b.** $m^{\frac{1}{4}}$ **c.** $7^{\frac{2}{5}}$ **d.** $7^{\frac{5}{2}}$
e. $w^{\frac{3}{8}}$ **f.** $w^{1.25}$ **g.** $5^{3\frac{1}{3}}$ **h.** $a^{0.3}$

3. Write the following in index form.

a. $\sqrt{t}$ **b.** $\sqrt[4]{5^7}$ **c.** $\sqrt[6]{6^{11}}$ **d.** $\sqrt[7]{x^6}$
e. $\sqrt[6]{x^7}$ **f.** $\sqrt[5]{w^{10}}$ **g.** $\sqrt[10]{w^5}$ **h.** $\sqrt{11^n}$

4. Without using a calculator, find the exact value of each of the following.

a. $8^{\frac{2}{3}}$ **b.** $8^{\frac{4}{3}}$ **c.** $32^{\frac{3}{5}}$ **d.** $32^{\frac{4}{5}}$
e. $25^{\frac{3}{2}}$ **f.** $27^{\frac{2}{3}}$ **g.** $27^{\frac{-2}{3}}$ **h.** $81^{\frac{3}{4}}$
i. $10^{\frac{6}{2}}$ **j.** $36^{1\frac{1}{2}}$ **k.** $7^{\frac{1}{2}}$ **l.** $12^{\frac{1}{3}}$

5. Using a calculator, evaluate each of the following. Give the answer correct to 2 decimal places.

a. $3^{\frac{1}{3}}$ **b.** $5^{\frac{1}{2}}$ **c.** $7^{\frac{1}{5}}$

d. $8^{\frac{1}{9}}$ **e.** $12^{\frac{3}{8}}$ **f.** $(0.6)^{\frac{4}{5}}$

g. $\left(\frac{2}{3}\right)^{\frac{3}{2}}$ **h.** $\left(\frac{3}{4}\right)^{\frac{3}{4}}$ **i.** $\left(\frac{4}{5}\right)^{\frac{2}{3}}$

6. WE8a Simplify each of the following.

a. $4^{\frac{3}{5}} \times 4^{\frac{1}{5}}$ **b.** $2^{\frac{1}{8}} \times 2^{\frac{3}{8}}$ **c.** $a^{\frac{1}{2}} \times a^{\frac{1}{3}}$

d. $x^{\frac{3}{4}} \times x^{\frac{2}{5}}$ **e.** $5m^{\frac{1}{3}} \times 2m^{\frac{1}{5}}$ **f.** $\frac{1}{2}b^{\frac{3}{7}} \times 4b^{\frac{2}{7}}$

g. $-4y^2 \times y^{\frac{2}{9}}$ **h.** $\frac{2}{5}a^{\frac{3}{8}} \times 0.05a^{\frac{3}{4}}$ **i.** $5x^3 \times x^{\frac{1}{2}}$

7. Simplify each of the following.

a. $a^{\frac{2}{3}}b^{\frac{3}{4}} \times a^{\frac{1}{3}}b^{\frac{3}{4}}$ **b.** $x^{\frac{3}{5}}y^{\frac{2}{9}} \times x^{\frac{1}{5}}y^{\frac{1}{3}}$ **c.** $2ab^{\frac{1}{3}} \times 3a^{\frac{3}{5}}b^{\frac{4}{5}}$

d. $6m^{\frac{3}{7}} \times \frac{1}{3}m^{\frac{1}{4}}n^{\frac{2}{5}}$ **e.** $x^3y^{\frac{1}{2}}z^{\frac{1}{3}} \times x^{\frac{1}{6}}y^{\frac{1}{3}}z^{\frac{1}{2}}$ **f.** $2a^{\frac{2}{5}}b^{\frac{3}{8}}c^{\frac{1}{4}} \times 4b^{\frac{3}{4}}c^{\frac{3}{4}}$

8. Simplify each of the following.

a. $3^{\frac{1}{2}} \div 3^{\frac{1}{3}}$ **b.** $5^{\frac{2}{3}} \div 5^{\frac{1}{4}}$ **c.** $12^2 \div 12^{\frac{3}{2}}$

d. $a^{\frac{6}{7}} \div a^{\frac{3}{7}}$ **e.** $x^{\frac{3}{2}} \div x^{\frac{1}{4}}$ **f.** $\dfrac{m^{\frac{4}{5}}}{m^{\frac{5}{9}}}$

g. $\dfrac{2x^{\frac{3}{4}}}{4x^{\frac{3}{5}}}$ **h.** $\dfrac{7n^2}{21n^{\frac{4}{3}}}$ **i.** $\dfrac{25b^{\frac{3}{5}}}{20b^{\frac{1}{4}}}$

9. Simplify each of the following.

a. $x^3y^2 \div x^{\frac{4}{3}}y^{\frac{3}{5}}$ **b.** $a^{\frac{5}{9}}b^{\frac{2}{3}} \div a^{\frac{2}{5}}b^{\frac{2}{5}}$ **c.** $m^{\frac{3}{8}}n^{\frac{4}{7}} \div 3n^{\frac{3}{8}}$

d. $10x^{\frac{4}{5}}y \div 5x^{\frac{2}{3}}y^{\frac{1}{4}}$ **e.** $\dfrac{5a^{\frac{3}{4}}b^{\frac{3}{5}}}{20a^{\frac{1}{5}}b^{\frac{1}{4}}}$ **f.** $\dfrac{p^{\frac{7}{8}}q^{\frac{1}{4}}}{7p^{\frac{2}{3}}q^{\frac{1}{6}}}$

10. Simplify each of the following.

a. $(2^{\frac{3}{4}})^{\frac{3}{5}}$ **b.** $(5^{\frac{2}{3}})^{\frac{1}{4}}$ **c.** $(7^{\frac{1}{5}})^6$

d. $(a^3)^{\frac{1}{10}}$ **e.** $(m^{\frac{4}{9}})^{\frac{3}{8}}$ **f.** $(2b^{\frac{1}{2}})^{\frac{1}{3}}$

g. $4(p^{\frac{3}{7}})^{\frac{14}{15}}$ **h.** $(x^{\frac{m}{n}})^{\frac{n}{p}}$ **i.** $(3m^{\frac{a}{b}})^{\frac{b}{c}}$

11. WE8b, c Simplify each of the following.

a. $(a^{\frac{1}{2}}b^{\frac{1}{3}})^{\frac{1}{2}}$ **b.** $(a^4b)^{\frac{3}{4}}$ **c.** $(x^{\frac{3}{5}}y^{\frac{7}{8}})^2$

d. $(3a^{\frac{1}{3}}b^{\frac{3}{5}}c^{\frac{3}{4}})^{\frac{1}{3}}$ **e.** $5(x^{\frac{1}{2}}y^{\frac{2}{3}}z^{\frac{2}{5}})^{\frac{1}{2}}$ **f.** $\left(\dfrac{a^{\frac{3}{4}}}{b}\right)^{\frac{2}{3}}$

g. $\left(\dfrac{m^{\frac{4}{5}}}{n^{\frac{7}{8}}}\right)^2$ **h.** $\left(\dfrac{b^{\frac{3}{5}}}{c^{\frac{4}{9}}}\right)^{\frac{1}{2}}$ **i.** $\left(\dfrac{4x^7}{2y^{\frac{3}{4}}}\right)^{\frac{1}{2}}$