TOPIC 2
Algebra and equations

2.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

2.1.1 Why learn this?

Do you speak mathematics? Algebra is the language of mathematics; it holds the key to understanding the rules, formulas and relationships that summarise much of our understanding of the universe. Every student of mathematics needs a mathematical tool chest, a set of algebraic skills to manipulate and process mathematical information.

2.1.2 What do you know?

assessON

1. **THINK** List what you know about linear equations. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of linear equations.

LEARNING SEQUENCE

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2.2 Substitution

When the numerical values of pronumerals are known, they can be substituted into an algebraic expression and the expression can then be evaluated. It can be useful to place any substituted values in brackets when evaluating an expression.

WORKED EXAMPLE 1

If $a = 4$, $b = 2$ and $c = -7$, evaluate the following expressions.

a $a - b$

b $a^3 + 9b - c$

THINK	WRITE
a 1 Write the expression.	**a** $a - b$
2 Substitute $a = 4$ and $b = 2$ into the expression.	$= 4 - 2$
3 Simplify.	$= 2$
b 1 Write the expression.	**b** $a^3 + 9b - c$
2 Substitute $a = 4$, $b = 2$ and $c = -7$ into the expression.	$= (4)^3 + 9(2) - (-7)$
3 Simplify.	$= 64 + 18 + 7$ $= 89$

WORKED EXAMPLE 2 TI | CASIO

If $c = \sqrt{a^2 + b^2}$, calculate c if $a = 12$ and $b = -5$.

THINK	WRITE
1 Write the expression.	$c = \sqrt{a^2 + b^2}$
2 Substitute $a = 12$ and $b = -5$ into the expression.	$= \sqrt{(12)^2 + (-5)^2}$
3 Simplify.	$= \sqrt{144 + 25}$ $= \sqrt{169}$ $= 13$

2.2.1 Number laws

- Recall from previous studies that when dealing with numbers and pronumerals, particular rules must be obeyed. Before progressing further, let us briefly review the Commutative, Associative, Identity and Inverse Laws.
- Consider any three pronumerals x, y and z, where x, y and z are elements of the set of Real numbers.

Commutative Law

1. $x + y = y + x$ (example: $3 + 2 = 5$ and $2 + 3 = 5$)
2. $x - y \neq y - x$ (example: $3 - 2 = 1$ but $2 - 3 = -1$)
3. $x \times y = y \times x$ (example: $3 \times 2 = 6$ and $2 \times 3 = 6$)
4. $x \div y \neq y \div x$ (example: $3 \div 2 = \frac{3}{2}$, but $2 \div 3 = \frac{2}{3}$)

Therefore, the **Commutative Law** holds true for addition and multiplication, since the order in which two numbers or pronumerals are added or multiplied does not affect the result. However, the Commutative Law does not hold true for subtraction or division.

Associative Law

1. $x + (y + z) = (x + y) + z$ [example: $2 + (3 + 4) = 2 + 7 = 9$ and $(2 + 3) + 4 = 5 + 4 = 9$]

2. $x - (y - z) \neq (x - y) - z$ [example: $2 - (3 - 4) = 2 - -1 = 3$ and $(2 - 3) - 4 = -1 - 4 = -5$]

3. $x \times (y \times z) = (x \times y) \times z$ [example: $2 \times (3 \times 4) = 2 \times 12 = 24$ and $(2 \times 3) \times 4 = 6 \times 4 = 24$]

4. $x \div (y \div z) \neq (x \div y) \div z$ [example: $2 \div (3 \div 4) = 2 \div \frac{3}{4} = 2 \times \frac{4}{3} = \frac{8}{3}$ but $(2 \div 3) \div 4 = \frac{2}{3} \div 4 = \frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$]

The **Associative Law** holds true for addition and multiplication since grouping two or more numbers or pronumerals and calculating them in a different order does not affect the result. However, the Associative Law does not hold true for subtraction or division.

Identity Law

The **Identity Law** states that in general:

$$x + 0 = 0 + x = x$$
$$x \times 1 = 1 \times x = x$$

In both of the examples above, x has not been changed (that is, it has kept its identity) when zero is added to it or it is multiplied by 1.

Inverse Law

The **Inverse Law** states that in general:

$$x + -x = -x + x = 0$$
$$x \times \frac{1}{x} = \frac{1}{x} \times x = 1$$

That is, when the additive inverse of a number or pronumeral is added to itself, it equals 0. When the multiplicative inverse of a number or pronumeral is multiplied by itself, it equals 1.

Closure Law

A law that you may not yet have encountered is the Closure Law. The **Closure Law** states that, when an operation is performed on an element (or elements) of a set, the result produced must also be an element of that set. For example, addition is closed on natural numbers (that is, positive integers: 1, 2, 3, …) since adding a pair of natural numbers produces a natural number. Subtraction is not closed on natural numbers. For example, 5 and 7 are natural numbers and the result of adding them is 12, a natural number. However, the result of subtracting 7 from 5 is −2, which is not a natural number.

WORKED EXAMPLE 3

Find the value of the following expressions, given the integer values $x = 4$ and $y = -12$. Comment on whether the Closure Law for integers holds for each of the expressions when these values are substituted.

a $x + y$ **b** $x - y$ **c** $x \times y$ **d** $x \div y$

THINK	WRITE
a **1** Substitute each pronumeral into the expression.	**a** $x + y = 4 + -12$
2 Evaluate and write the answer.	$= -8$
3 Determine whether the Closure Law holds; that is, is the result an integer?	The Closure Law holds for these substituted values.

b Repeat steps **1–3** of part **a**.	**b** $x - y = 4 - -12$ $= 16$ The Closure Law holds for these substituted values.
c Repeat steps **1–3** of part **a**.	**c** $x \times y = 4 \times -12$ $= -48$ The Closure Law holds for these substituted values.
d Repeat steps **1–3** of part **a**.	**d** $x \div y = 4 \div -12$ $= \frac{4}{-12}$ $= -\frac{1}{3}$ The Closure Law does not hold for these substituted values since the answer obtained is a fraction, not an integer.

- It is important to note that, although a particular set of numbers may be closed under a given operation, for example multiplication, another set of numbers may not be closed under that same operation. For example, in part **c** of Worked example 3, integers were closed under multiplication. However, in some cases, the set of *irrational numbers* is not closed under multiplication, since $\sqrt{3} \times \sqrt{3} = \sqrt{9} = 3$. In this example, two irrational numbers produced a rational number under multiplication.

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Searchlight ID: doc-5185

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Exercise 2.2 Substitution

assesson

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–f, 2a–f, 3c–d, 4a–c, 5a–c, 6a–e, 7, 8, 10, 14	Questions: 1c–i, 2a–f, 3c–d, 4, 5a–c, 6d–j, 9, 10, 14	Questions: 1e–l, 2c–i, 3, 4, 5, 6d–j, 9–15

Individual pathway interactivity: int-4566

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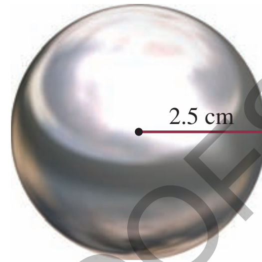
To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. **WE1** If $a = 2$, $b = 3$ and $c = 5$, evaluate the following expressions.
   a. $a + b$
   b. $c - b$
   c. $c - a - b$
   d. $c - (a - b)$
   e. $7a + 8b - 11c$
   f. $\frac{a}{2} + \frac{b}{3} + \frac{c}{5}$
   g. abc
   h. $ab(c - b)$
   i. $a^2 + b^2 - c^2$
   j. $c^2 + a$
   k. $-a \times b \times -c$
   l. $2.3a - 3.2b$
2. If $d = -6$ and $k = -5$, evaluate the following.
   a. $d + k$
   b. $d - k$
   c. $k - d$
   d. kd
   e. $-d(k + 1)$
   f. d^2
   g. k^3
   h. $\frac{k - 1}{d}$
   i. $3k - 5d$
3. If $x = \frac{1}{3}$ and $y = \frac{1}{4}$, evaluate the following.
   a. $x + y$
   b. $y - x$
   c. xy
   d. $\frac{x}{y}$
   e. x^2y^3
   f. $\frac{9x}{y^2}$
4. If $x = 3$, find the value of the following.
   a. x^2
   b. $-x^2$
   c. $(-x)^2$
   d. $2x^2$
   e. $-2x^2$
   f. $(-2x)^2$
5. If $x = -3$, find the value of the following.
   a. x^2
   b. $-x^2$
   c. $(-x)^2$
   d. $2x^2$
   e. $-2x^2$
   f. $(-2x)^2$
6. **WE2** Calculate the unknown variable in the following real-life mathematical formulas.
   a. If $c = \sqrt{a^2 + b^2}$, calculate c if $a = 8$ and $b = 15$.
   b. If $A = \frac{1}{2}bh$, determine the value of A if $b = 12$ and $h = 5$.
   c. The perimeter, P, of a rectangle is given by $P = 2L + 2W$. Calculate the perimeter, P, of a rectangle, given $L = 1.6$ and $W = 2.4$.
   d. If $T = \frac{C}{L}$, determine the value of T if $C = 20.4$ and $L = 5.1$.
   e. If $K = \frac{n + 1}{n - 1}$, determine the value of K if $n = 5$.
   f. Given $F = \frac{9C}{5} + 32$, calculate F if $C = 20$.
   g. If $v = u + at$, evaluate v if $u = 16$, $a = 5$, $t = 6$.
   h. The area, A, of a circle is given by the formula $A = \pi r^2$. Calculate the area of a circle, correct to 1 decimal place, if $r = 6$.
   i. If $E = \frac{1}{2}mv^2$, calculate m if $E = 40$, $v = 4$.
   j. Given $r = \sqrt{\frac{A}{\pi}}$, evaluate A to 1 decimal place if $r = 14.1$.
7. **MC** a. If $p = -5$ and $q = 4$, then pq is equal to:
   A. 20 B. 1 C. -1 D. -20 E. $-\frac{5}{4}$
   b. If $c^2 = a^2 + b^2$, and $a = 6$ and $b = 8$, then c is equal to:
   A. 28 B. 100 C. 10 D. 14 E. 44
   c. Given $h = 6$ and $k = 7$, then kh^2 is equal to:
   A. 294 B. 252 C. 1764 D. 5776 E. 85

Understanding

8. Knowing the length of two sides of a right-angled triangle, the third side can be calculated using Pythagoras' theorem. If the two shorter sides have lengths of 1.5 cm and 3.6 cm, calculate the length of the hypotenuse.
9. The volume of a sphere can be calculated using the formula $\frac{4}{3}\pi r^3$. What is the volume of a sphere with a radius of 2.5 cm? Give your answer correct to 2 decimal places.



10. A rectangular park is 200 m by 300 m. If Blake runs along the diagonal of the park, how far will he run? Give your answer to the nearest metre.

Reasoning

11. **WE3** Determine the value of the following expressions, given the integer values $x = 1$, $y = -2$ and $z = -1$. Comment on whether the Closure Law for integers holds true for each of the expressions when these values are substituted.
    - **a.** $x + y$
    - **b.** $y - z$
    - **c.** $y \times z$
    - **d.** $x \div z$
    - **e.** $z - x$
    - **f.** $x \div y$
12. Find the value of the following expressions, given the natural number values $x = 8$, $y = 2$ and $z = 6$. Comment on whether the Closure Law for natural numbers holds true for each of the expressions.
    - **a.** $x + y$
    - **b.** $y - z$
    - **c.** $y \times z$
    - **d.** $x \div z$
    - **e.** $z - x$
    - **f.** $x \div y$
13. For each of the following, complete the relationship to illustrate the stated law. Justify your reasoning.
    - **a.** $(a + 2b) + 4c =$ ____________ Associative Law
    - **b.** $(x \times 3y) \times 5c =$ ____________ Associative Law
    - **c.** $2p \div q \neq$ ________________ Commutative Law
    - **d.** $5d + q =$ ________________ Commutative Law
    - **e.** $3z + 0 =$ ________________ Identity Law
    - **f.** $2x \times$ __________ $=$ __________ Inverse Law
    - **g.** $(4x \div 3y) \div 5z \neq$ ____________ Associative Law
    - **h.** $3d - 4y \neq$ ________________ Commutative Law

Problem solving

14. $s = ut + \frac{1}{2}at^2$
    where t is the time in seconds, s is the displacement in metres, u is the initial velocity and a is the acceleration due to gravity.
    - **a.** Calculate s when $u = 16.5$ m/s, $t = 2.5$ seconds and $a = 9.8$ m/s^2.
    - **b.** A body has an initial velocity of 14.7 m/s and after t seconds has a displacement of 137.2 metres. Find the value of t if $a = 9.8$ m/s^2.

15. Find the value of m if $n = p\sqrt{1 + \frac{1}{m}}$, when $n = 6$ and $p = 4$.

Reflection

Why is understanding of the Commutative Law useful?

CHALLENGE 2.1

The lowest common multiple of four terms is $24a^3bc^2d$. Three of the terms are $12a^2bc$, $8ab$ and $4a^2cd$. The fourth term contains only two pronumerals, and its coefficient is an odd prime number. What is the fourth term?

2.3 Adding and subtracting algebraic fractions

- In an algebraic fraction, the denominator, the numerator or both are algebraic expressions. For example, $\frac{x}{2}, \frac{3x+1}{2x-5}$ and $\frac{1}{x^2+5}$ are all **algebraic fractions**.
- As with all fractions, algebraic fractions must have a common denominator if they are to be added or subtracted, so an important step is to find the lowest common denominator (LCD).

WORKED EXAMPLE 4

TI | CASIO

Simplify the following expressions.

a $\frac{2x}{3} - \frac{x}{2}$

b $\frac{x+1}{6} + \frac{x+4}{4}$

THINK / **WRITE**

a 1 Write the expression.

a $\frac{2x}{3} - \frac{x}{2}$

2 Rewrite each fraction as an equivalent fraction using the LCD of 3 and 2, which is 6.

$= \frac{2x}{3} \times \frac{2}{2} - \frac{x}{2} \times \frac{3}{3}$

$= \frac{4x}{6} - \frac{3x}{6}$

3 Express as a single fraction.

$= \frac{4x - 3x}{6}$

4 Simplify the numerator.

$= \frac{x}{6}$

b 1 Write the expression.

b $\frac{x+1}{6} + \frac{x+4}{4}$

2 Rewrite each fraction as an equivalent fraction using the LCD of 6 and 4, which is 12.

$= \frac{x+1}{6} \times \frac{2}{2} + \frac{x+4}{4} \times \frac{3}{3}$

$= \frac{2(x+1)}{12} + \frac{3(x+4)}{12}$

3 Express as a single fraction.

$= \frac{2(x+1) \quad 3(x+4)}{12}$

4 Simplify the numerator by expanding brackets and collecting like terms.

$= \frac{2x + 2 + 3x + 12}{12}$

$= \frac{5x + 14}{12}$

Pronumerals in the denominator

- If pronumerals appear in the denominator, the process involved in adding and subtracting the fractions is to find a lowest common denominator as usual.
- When there is an algebraic expression in the denominator of each fraction, we can obtain a common denominator by writing the product of the denominators. For example, if $x+3$ and $2x-5$ are in the denominator of each fraction, then a common denominator of the two fractions will be $(x+3)(2x-5)$.

WORKED EXAMPLE 5

Simplify $\frac{2}{3x}-\frac{1}{4x}$.

THINK	WRITE
1 Write the expression.	$\frac{2}{3x}-\frac{1}{4x}$
2 Rewrite each fraction as an equivalent fraction using the LCD of $3x$ and $4x$, which is $12x$. *Note:* $12x^2$ is not the lowest LCD.	$=\frac{2}{3x}\times\frac{4}{4}-\frac{1}{4x}\times\frac{3}{3}$ $=\frac{8}{12x}-\frac{3}{12x}$
3 Express as a single fraction.	$=\frac{8-3}{12x}$
4 Simplify the numerator.	$=\frac{5}{12x}$

WORKED EXAMPLE 6

Simplify $\frac{x+1}{x+3}+\frac{2x-1}{x+2}$ by writing it first as a single fraction.

THINK	WRITE
1 Write the expression.	$\frac{x+1}{x+3}+\frac{2x-1}{x+2}$
2 Rewrite each fraction as an equivalent fraction using the LCD of $x+3$ and $x+2$, which is the product $(x+3)(x+2)$.	$=\frac{(x+1)}{(x+3)}\times\frac{(x+2)}{(x+2)}+\frac{(2x-1)}{(x+2)}\times\frac{(x+3)}{(x+3)}$ $=\frac{(x+1)(x+2)}{(x+3)(x+2)}+\frac{(2x-1)(x+3)}{(x+3)(x+2)}$
3 Express as a single fraction.	$=\frac{(x+1)(x+2)+(2x-1)(x+3)}{(x+3)(x+2)}$
4 Simplify the numerator by expanding brackets and collecting like terms. *Note:* The denominator is generally kept in factorised form. That is, it is not expanded.	$=\frac{(x^2+2x+x+2)+(2x^2+6x-x-3)}{(x+3)(x+2)}$ $=\frac{(x^2+3x+2+2x^2+5x-3)}{(x+3)(x+2)}$ $=\frac{3x^2+8x-1}{(x+3)(x+2)}$

WORKED EXAMPLE 7

TI | CASIO

Simplify $\frac{x+2}{x-3}+\frac{x-1}{(x-3)^2}$ by writing it first as a single fraction.

THINK	WRITE
1 Write the expression.	$\frac{x+2}{x-3}+\frac{x-1}{(x-3)^2}$
2 Rewrite each fraction as an equivalent fraction using the LCD of $x-3$ and $(x-3)^2$, which is $(x-3)^2$.	$=\frac{x+2}{x-3}\times\frac{x-3}{x-3}+\frac{x-1}{(x-3)^2}$ $=\frac{(x+2)(x-3)}{(x-3)^2}+\frac{x-1}{(x-3)^2}$ $=\frac{x^2-x-6}{(x-3)^2}+\frac{x-1}{(x-3)^2}$
3 Express as a single fraction.	$=\frac{x^2-x-6+x-1}{(x-3)^2}$
4 Simplify the numerator.	$=\frac{x^2-7}{(x-3)^2}$

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Complete this digital doc: SkillSHEET: Writing equivalent algebraic fractions with the lowest common denominator
Searchlight ID: doc-5190

Exercise 2.3 Adding and subtracting algebraic fractions

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–f, 2a–f, 3a–f, 4–6	Questions: 1d–i, 2a–f, 3a–i, 4–7, 9	Questions: 1g–i, 2e–j, 3d–i, 4–10

Individual pathway interactivity: int-4567

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Fluency

1. Simplify each of the following.

a. $\frac{4}{7}+\frac{2}{3}$ **b.** $\frac{1}{8}+\frac{5}{9}$ **c.** $\frac{3}{5}+\frac{6}{15}$

d. $\frac{4}{9}-\frac{3}{11}$ **e.** $\frac{3}{7}-\frac{2}{5}$ **f.** $\frac{1}{5}-\frac{x}{6}$

g. $\frac{5x}{9}-\frac{4}{27}$ **h.** $\frac{3}{8}-\frac{2x}{5}$ **i.** $\frac{5}{x}-\frac{2}{3}$

2. **WE4** Simplify the following expressions.

a. $\frac{2y}{3} - \frac{y}{4}$
b. $\frac{y}{8} - \frac{y}{5}$
c. $\frac{4x}{3} - \frac{x}{4}$
d. $\frac{8x}{9} + \frac{2x}{3}$
e. $\frac{2w}{14} - \frac{w}{28}$
f. $\frac{y}{20} - \frac{y}{4}$
g. $\frac{12y}{5} + \frac{y}{7}$
h. $\frac{10x}{5} + \frac{2x}{15}$
i. $\frac{x+1}{5} + \frac{x+3}{2}$
j. $\frac{x+2}{4} + \frac{x+6}{3}$
k. $\frac{2x-1}{5} - \frac{2x+1}{6}$
l. $\frac{3x+1}{2} + \frac{5x+2}{3}$

3. **WE5** Simplify the following.

a. $\frac{2}{4x} + \frac{1}{8x}$
b. $\frac{3}{4x} - \frac{1}{3x}$
c. $\frac{5}{3x} + \frac{1}{7x}$
d. $\frac{12}{5x} + \frac{4}{15x}$
e. $\frac{1}{6x} + \frac{1}{8x}$
f. $\frac{9}{4x} - \frac{9}{5x}$
g. $\frac{2}{100x} + \frac{7}{20x}$
h. $\frac{1}{10x} + \frac{5}{x}$
i. $\frac{4}{3x} - \frac{3}{2x}$

4. **WE6, 7** Simplify the following by writing as single fractions.

a. $\frac{2}{x+4} + \frac{3x}{x-2}$
b. $\frac{2x}{x+5} + \frac{5}{x-1}$
c. $\frac{5}{2x+1} + \frac{x}{x-2}$
d. $\frac{2x}{x+1} - \frac{3}{2x-7}$
e. $\frac{4x}{x+7} + \frac{3x}{x-5}$
f. $\frac{x+2}{x+1} + \frac{x-1}{x+4}$
g. $\frac{x+8}{x+1} - \frac{2x+1}{x+2}$
h. $\frac{x+5}{x+3} - \frac{x-1}{x-2}$
i. $\frac{x+1}{x+2} - \frac{2x-5}{3x-1}$
j. $\frac{2}{x-1} - \frac{3}{1-x}$
k. $\frac{4}{(x+1)^2} + \frac{3}{x+1}$
l. $\frac{3}{x-1} - \frac{1}{(x-1)^2}$

Understanding

5. A classmate attempted to complete an algebraic fraction subtraction problem.

$$\frac{x}{x-1} - \frac{3}{x-2} = \frac{x}{x-1} \times \frac{(x-2)}{(x-2)} - \frac{3}{x-2} \times \frac{(x-1)}{(x-1)}$$
$$= \frac{x(x-2) - 3(x-1)}{(x-1)(x+2)}$$
$$= \frac{x^2 - 2x - 3x - 1}{(x-1)(x+2)}$$
$$= \frac{x^2 - 5x - 1}{(x-1)(x+2)}$$

a. What mistake did she make?
b. What is the correct answer?

Reasoning

Adding and subtracting algebraic fractions can become more complicated if you add a third fraction into the expression.

6. Simplify the following.

a. $\frac{1}{x+2} + \frac{2}{x+1} + \frac{1}{x+3}$
b. $\frac{1}{x-1} + \frac{4}{x+2} + \frac{2}{x-4}$
c. $\frac{3}{x+1} + \frac{2}{x+3} - \frac{1}{x+2}$
d. $\frac{2}{x-4} - \frac{3}{x-1} + \frac{5}{x+3}$

7. Why is the process that involves finding the lowest common denominator important in question **6**?

8. The reverse process of adding or subtracting algebraic fractions is quite complex. Use trial and error, or technology, to determine the value of a if $\dfrac{7x-4}{(x-8)(x+5)} = \dfrac{a}{x-8} + \dfrac{3}{x+5}$.

Problem solving

9. Simplify $\dfrac{3}{x^2+7x+12} - \dfrac{1}{x^2+x-6} + \dfrac{2}{x^2+2x-8}$.

10. Simplify $\dfrac{x^2+3x-18}{x^2-x-42} - \dfrac{x^2-3x+2}{x^2-5x+4}$.

Reflection

Why can't we just add the numerators and the denominators of fractions; for example, $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{a+c}{b+d}$?

2.4 Multiplying and dividing algebraic fractions

2.4.1 Simplifying algebraic fractions

- Algebraic fractions can be simplified using the index laws and by cancelling factors common to the numerator and denominator.
- A fraction can only be simplified if:
  - there is a common factor in the numerator and the denominator
  - the numerator and denominator are both written in factorised form, that is, as the *product* of two or more factors.

$$\frac{3ab}{12a} = \frac{{}^{1}\cancel{3} \times {}^{1}\cancel{a} \times b}{{}^{4}\cancel{12} \times {}^{1}\cancel{a}} \begin{matrix} \leftarrow \text{product of factors} \\ \leftarrow \text{product of factors} \end{matrix}$$

$$= \frac{b}{4}$$

$$\frac{3a+b}{12a} = \frac{3 \times a + b}{12 \times a} \begin{matrix} \leftarrow \text{not a product of factors} \\ \leftarrow \text{product of factors} \end{matrix}$$

Cannot be simplified

2.4.2 Multiplying algebraic fractions

- Multiplication of algebraic fractions follows the same rules as multiplication of numerical fractions: multiply the numerators, then multiply the denominators.

WORKED EXAMPLE 8

Simplify each of the following.

a $\dfrac{5y}{3x} \times \dfrac{6z}{7y}$

b $\dfrac{2x}{(x+1)(2x-3)} \times \dfrac{x+1}{x}$

THINK	WRITE
a 1 Write the expression.	**a** $\dfrac{5y}{3x} \times \dfrac{6z}{7y}$
2 Cancel common factors in the numerator and denominator. The y can be cancelled in the denominator and the numerator. Also, the 3 in the denominator can divide into the 6 in the numerator.	$= \dfrac{5\cancel{y}^{1}}{{}_{1}\cancel{3}x} \times \dfrac{\cancel{6}^{2}z}{7\cancel{y}^{1}}$ $= \dfrac{5}{x} \times \dfrac{2z}{7}$

3 Multiply the numerators, then multiply the denominators. $= \frac{10z}{7x}$

b 1 Write the expression. **b** $\frac{2x}{(x+1)(2x-3)} \times \frac{x+1}{x}$

2 Cancel common factors in the numerator and the denominator. $(x + 1)$ and the x are both common in the numerator and the denominator and can therefore be cancelled.

$= \frac{2\cancel{x}^1}{{}^1\cancel{(x+1)}(2x-3)} \times \frac{\cancel{x+1}^1}{\cancel{x}^1}$

$= \frac{2}{2x-3} \times \frac{1}{1}$

3 Multiply the numerators, then multiply the denominators. $= \frac{2}{2x-3}$

2.4.3 Dividing algebraic fractions

- When dividing algebraic fractions, follow the same rules as for division of numerical fractions: write the division as a multiplication and invert the second fraction. This process is sometimes known as multiplying by the **reciprocal**.

WORKED EXAMPLE 9 TI | CASIO

Simplify the following expressions.

a $\frac{3xy}{2} \div \frac{4x}{9y}$ **b** $\frac{4}{(x+1)(3x-5)} \div \frac{x-7}{x+1}$

THINK	WRITE
a 1 Write the expression.	**a** $\frac{3xy}{2} \div \frac{4x}{9y}$
2 Change the division sign to a multiplication sign and write the second fraction as its reciprocal.	$= \frac{3xy}{2} \times \frac{9y}{4x}$
3 Cancel common factors in the numerator and denominator and cancel. The pronumeral x is common to both the numerator and denominator and can therefore be cancelled.	$= \frac{3y}{2} \times \frac{9y}{4}$
4 Multiply the numerators, then multiply the denominators.	$= \frac{27y^2}{8}$
b 1 Write the expression.	**b** $\frac{4}{(x+1)(3x-5)} \div \frac{x-7}{x+1}$
2 Change the division sign to a multiplication sign and write the second fraction as its reciprocal.	$= \frac{4}{(x+1)(3x-5)} \times \frac{x+1}{x-7}$
3 Cancel common factors in the numerator and denominator and cancel. $(x + 1)$ is common to both the numerator and denominator and can therefore be cancelled.	$= \frac{4}{3x-5} \times \frac{1}{x-7}$
4 Multiply the numerators, then multiply the denominators.	$= \frac{4}{(3x-5)(x-7)}$

learnON RESOURCES — ONLINE ONLY

- **Complete this digital doc:** SkillSHEET: Multiplication of fractions
  Searchlight ID: doc-5187
- **Complete this digital doc:** SkillSHEET: Division of fractions
  Searchlight ID: doc-5188
- **Complete this digital doc:** SkillSHEET: Simplification of algebraic fractions
  Searchlight ID: doc-5191
- **Complete this digital doc:** WorkSHEET 2.1
  Searchlight ID: doc-13847

Exercise 2.4 Multiplying and dividing algebraic fractions

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–f, 2a–f, 3a–i, 4a–b	Questions: 1d–i, 2a–f, 3a–i, 4, 5, 7	Questions: 1g–l, 2e–j, 3d–l, 4–8

Individual pathway interactivity: int-4568 — **learnON** ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE8a Simplify each of the following.

a. $\frac{x}{5}\times\frac{20}{y}$ **b.** $\frac{x}{4}\times\frac{12}{y}$ **c.** $\frac{y}{4}\times\frac{16}{x}$

d. $\frac{x}{2}\times\frac{9}{2y}$ **e.** $\frac{x}{10}\times\frac{-25}{2y}$ **f.** $\frac{3w}{-14}\times\frac{-7}{x}$

g. $\frac{3y}{4x}\times\frac{8z}{7y}$ **h.** $\frac{-y}{3x}\times\frac{6z}{-7y}$ **i.** $\frac{x}{3z}\times\frac{-9z}{2y}$

j. $\frac{5y}{3x}\times\frac{x}{8y}$ **k.** $\frac{-20y}{7x}\times\frac{-21z}{5y}$ **l.** $\frac{y}{-3w}\times\frac{x}{2y}$

2. WE8b Simplify the following expressions.

a. $\frac{2x}{(x-1)(3x-2)}\times\frac{x-1}{x}$ **b.** $\frac{5x}{(x-3)(4x+7)}\times\frac{4x+7}{x}$

c. $\frac{9x}{(5x+1)(x-6)}\times\frac{5x+1}{2x}$ **d.** $\frac{(x+4)}{(x+1)(x+3)}\times\frac{x+1}{x+4}$

e. $\frac{2x}{x+1}\times\frac{x-1}{(x+1)(x-1)}$ **f.** $\frac{2}{x(2x-3)}\times\frac{x(x+1)}{4}$

g. $\frac{2x}{4(a+3)}\times\frac{3a}{15x}$ **h.** $\frac{15c}{12(d-3)}\times\frac{21d}{6c}$

i. $\frac{6x^2}{20(x-2)^2}\times\frac{15(x-2)}{16x^4}$ **j.** $\frac{7x^2(x-3)}{5x(x+1)}\times\frac{3(x-3)(x+1)}{14(x-3)^2(x-1)}$

3. WE9a Simplify the following expressions.

a. $\frac{3}{x}\div\frac{5}{x}$ **b.** $\frac{2}{x}\div\frac{9}{x}$ **c.** $\frac{4}{x}\div\frac{12}{x}$

d. $\frac{20}{y}\div\frac{20}{3y}$ **e.** $\frac{1}{5w}\div\frac{5}{w}$ **f.** $\frac{7}{2x}\div\frac{3}{5x}$

g. $\frac{3xy}{7} \div \frac{3x}{4y}$

h. $\frac{2xy}{5} \div \frac{5x}{y}$

i. $\frac{6y}{9} \div \frac{3x}{4xy}$

j. $\frac{8wx}{5} \div \frac{3w}{4y}$

k. $\frac{2xy}{5} \div \frac{3xy}{5}$

l. $\frac{10xy}{7} \div \frac{20x}{14y}$

4. WE9b Simplify the following expressions.

a. $\frac{9}{(x-1)(3x-7)} \div \frac{x+3}{x-1}$

b. $\frac{1}{(x+2)(2x-5)} \div \frac{x-9}{2x-5}$

c. $\frac{12(x-3)^2}{(x+5)(x-9)} \div \frac{4(x-3)}{7(x-9)}$

d. $\frac{13}{6(x-4)^2(x-1)} \div \frac{3(x+1)}{2(x-4)(x-1)}$

Reasoning

5. Is $\frac{3}{x+2}$ the same as $\frac{1}{x+2} + \frac{1}{x+2} + \frac{1}{x+2}$? Explain your reasoning.

6. a. Simplify $\frac{(x-4)(x+3)}{4x-x^2} \times \frac{x^2-x}{(x+3)(x-1)}$.

b. Find and describe the error in the following reasoning.

$$\frac{(x-4)(x+3)}{4x-x^2} \times \frac{x^2-x}{(x+3)(x-1)}$$
$$= \frac{(x-4)(x+3)}{x(4-x)} \times \frac{x(x-1)}{(x+3)(x-1)}$$
$$= 1$$

Problem solving

7. Simplify $\frac{x^2-2x-3}{x^4-1} \times \frac{x^2+4x-5}{x^2-5x+6} \div \frac{x^2+7x+10}{x^4-3x^2-4}$.

8. Simplify $\frac{x+1}{x-\frac{x}{\frac{x}{a}}}$ where $a = \frac{x-1}{x+1}$.

Reflection

How are multiplying and dividing algebraic fractions different to adding and subtracting them?

CHALLENGE 2.2

Simplify the expression: $\left(1 + \frac{\frac{1}{x}}{1+\frac{1}{x}}\right) \times \left(1 + \frac{\frac{1}{x}}{1-\frac{1}{x}}\right)$.

2.5 Solving simple equations

- **Equations** show the equivalence of two expressions.
- Equations can be solved using inverse operations.
- When solving equations, the last operation performed on the pronumeral when building the equation is the first operation undone by applying inverse operations to both sides of the equation. For example, the equation $2x + 3 = 5$ is built from x by multiplying x by 2 and then adding 3 to give the result of 5. To solve the equation, undo the adding 3 by subtracting 3, then undo the multiplying by 2 by dividing by 2.

$+$ and $-$ are inverse operations
$\times$ and $\div$ are inverse operations
^{2} and $\sqrt{\ }$ are inverse operations

One-step equations

- Equations that require one step to solve are called one-step equations.

WORKED EXAMPLE 10

Solve the following equations.

a $a + 27 = 71$ **b** $\frac{d}{16} = 3\frac{1}{4}$ **c** $\sqrt{e} = 0.87$ **d** $f^2 = \frac{4}{25}$

THINK	WRITE
a **1** Write the equation.	**a** $a + 27 = 71$
2 27 has been added to a resulting in 71. The addition of 27 has to be reversed by subtracting 27 from both sides of the equation to obtain the solution.	$a + 27 - 27 = 71 - 27$ $a = 44$
b **1** Write the equation.	**b** $\frac{d}{16} = 3\frac{1}{4}$
2 Express $3\frac{1}{4}$ as an improper fraction.	$\frac{d}{16} = \frac{13}{4}$
3 The pronumeral d has been divided by 16 resulting in $\frac{13}{4}$. Therefore the division has to be reversed by multiplying both sides of the equation by 16 to obtain d.	$\frac{d}{16} \times 16 = \frac{13}{4} \times 16$ $d = 52$
c **1** Write the equation.	**c** $\sqrt{e} = 0.87$
2 The square root of e has been taken to result in 0.87. Therefore, the square root has to be reversed by squaring both sides of the equation to obtain e.	$\left(\sqrt{e}\right)^2 = 0.87^2$ $e = 0.7569$
d **1** Write the equation.	**d** $f^2 = \frac{4}{25}$
2 The pronumeral f has been squared, resulting in $\frac{4}{25}$. Therefore the squaring has to be reversed by taking the square root of both sides of the equation to obtain f. Note that there are two possible solutions, one positive and one negative, since two negative numbers can also be multiplied together to produce a positive result.	$f = \pm\sqrt{\frac{4}{25}}$ $f = \pm\frac{2}{5}$

2.5.1 Two-step equations

- Two-step equations involve the inverse of two operations in their solutions

WORKED EXAMPLE 11 TI | CASIO

Solve the following equations.

a $5y - 6 = 7$ **b** $\frac{4x}{9} = 5$

THINK	WRITE
a **1** Write the equation.	**a** $5y - 6 = 79$
2 Step 1: Add 6 to both sides of the equation.	$5y - 6 + 6 = 79 + 6$ $5y = 85$
3 Step 2: Divide both sides of the equation by 5 to obtain *y*.	$\frac{5y}{5} = \frac{85}{5}$ $y = 17$
b **1** Write the equation.	**b** $\frac{4x}{9} = 5$
2 Step 1: Multiply both sides of the equation by 9.	$\frac{4x}{9} \times 9 = 5 \times 9$ $4x = 45$
3 Step 2: Divide both sides of the equation by 4 to obtain *x*.	$\frac{4x}{4} = \frac{45}{4}$ $x = \frac{45}{4}$
4 Express the improper fraction as a mixed number.	$x = 11\frac{1}{4}$

2.5.2 Equations where the pronumeral appears on both sides

- In solving equations where the pronumeral appears on both sides, subtract the smaller pronumeral term so that it is eliminated from both sides of the equation.

WORKED EXAMPLE 12

Solve the following equations.

a $5h + 13 = 2h - 2$ **b** $14 - 4d = 27 - d$ **c** $2(x - 3) = 5(2x + 4)$

THINK	WRITE
a **1** Write the equation.	**a** $5h + 13 = 2h - 2$
2 Eliminate the pronumeral from the right-hand side by subtracting 2*h* from both sides of the equation.	$3h + 13 = -2$
3 Subtract 13 from both sides of the equation.	$3h = -15$
4 Divide both sides of the equation by 3 and write your answer.	$h = -5$
b **1** Write the equation.	**b** $14 - 4d = 27 - d$
2 Add 4*d* to both sides of the equation.	$14 = 27 + 3d$
3 Subtract 27 from both sides of the equation.	$-13 = 3d$
4 Divide both sides of the equation by 3.	$-\frac{13}{3} = d$

5 Express the improper fraction as a mixed number. $-4\frac{1}{3} = d$

6 Write your answer so that d is on the left-hand side. $d = -4\frac{1}{3}$

c 1 Write the equation. **c** $2(x - 3) = 5(2x + 4)$

2 Expand the brackets on both sides of the equation. $2x - 6 = 10x + 20$

3 Subtract $2x$ from both sides of the equation. $2x - 2x - 6 = 10x - 2x + 20$

4 Subtract 20 from both sides of the equation. $-6 - 20 = 8x + 20 - 20$

$-26 = 8x$

5 Divide both sides of the equation by 8. $-\frac{26}{8} = x$

6 Simplify and write your answer with the pronumeral on the left-hand side. $x = -\frac{13}{4}$

Exercise 2.5 Solving simple equations

assess on

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–f, 2a–e, 3a–f, 4a–b, 5a–b, 6a–b, 7a–f, 8a–d, 9a–b, 10a–b, 11a–c, 12a–b, 13a–c, 15a–d, 16a–d, 17a–d, 19–21, 25	Questions: 1d–i, 2d–i, 3a–f, 4, 5a–b, 6a–b, 7d–i, 8c–f, 9c–g, 10a–d, 11c–f, 12, 13d–i, 15c–f, 16c–f, 17c–f, 19–21, 23, 25, 26	Questions: 3–6, 7d–i, 8c–f, 9e–i, 10d–f, 11d–f, 12, 14, 15g–i, 16g–i, 17g–i, 18–26

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Fluency

1. WE10a Solve the following equations.

a. $a + 61 = 85$ **b.** $k - 75 = 46$ **c.** $g + 9.3 = 12.2$
d. $r - 2.3 = 0.7$ **e.** $h + 0.84 = 1.1$ **f.** $i + 5 = 3$
g. $t - 12 = -7$ **h.** $q + \frac{1}{3} = \frac{1}{2}$ **i.** $x - 2 = -2$

2. WE10b Solve the following equations.

a. $\frac{f}{4} = 3$ **b.** $\frac{i}{10} = -6$ **c.** $6z = -42$
d. $9v = 63$ **e.** $6w = -32$ **f.** $\frac{k}{12} = \frac{5}{6}$
g. $4a = 1.7$ **h.** $\frac{m}{19} = \frac{7}{8}$ **i.** $\frac{y}{4} = 5\frac{3}{8}$

3. WE10c, d Solve the following equations.

a. $\sqrt{t} = 10$ **b.** $y^2 = 289$ **c.** $\sqrt{q} = 2.5$
d. $f^2 = 1.44$ **e.** $\sqrt{h} = \frac{4}{7}$ **f.** $p^2 = \frac{9}{64}$
g. $\sqrt{g} = \frac{15}{22}$ **h.** $j^2 = \frac{196}{961}$ **i.** $a^2 = 2\frac{7}{9}$

4. Solve the following equations.

a. $\sqrt{t} - 3 = 2$ **b.** $5x^2 = 180$ **c.** $3\sqrt{m} = 12$
d. $-2t^2 = -18$ **e.** $t^2 + 11 = 111$ **f.** $\sqrt{m} + 5 = 0$

5. Solve the following equations.

a. $\sqrt[3]{x} = 2$
b. $x^3 = -27$
c. $\sqrt[3]{m} = \frac{1}{2}$
d. $x^3 = \frac{27}{64}$
e. $\sqrt[3]{m} = 0.2$
f. $w^3 = 15\frac{5}{8}$

6. Solve the following equations.

a. $x^3 + 1 = 0$
b. $3x^3 = -24$
c. $\sqrt[3]{m} + 5 = 6$
d. $-2 \times \sqrt[3]{w} = 16$
e. $\sqrt[3]{t} - 13 = -8$
f. $2x^3 - 14 = 2$

7. **WE11a** Solve the following.

a. $5a + 6 = 26$
b. $6b + 8 = 44$
c. $8i - 9 = 15$
d. $7f - 18 = 45$
e. $8q + 17 = 26$
f. $10r - 21 = 33$
g. $6s + 46 = 75$
h. $5t - 28 = 21$
i. $8a + 88 = 28$

8. Solve the following.

a. $\frac{f}{4} + 6 = 16$
b. $\frac{g}{6} + 4 = 9$
c. $\frac{r}{10} + 6 = 5$
d. $\frac{m}{9} - 12 = -10$
e. $\frac{n}{8} + 5 = 8.5$
f. $\frac{p}{12} - 1.8 = 3.4$

9. Solve the following.

a. $6(x + 8) = 56$
b. $7(y - 4) = 35$
c. $5(m - 3) = 7$
d. $3(2k + 5) = 24$
e. $5(3n - 1) = 80$
f. $6(2c + 7) = 58$
g. $2(x - 5) + 3(x - 7) = 19$
h. $3(x + 5) - 5(x - 1) = 12$
i. $3(2x - 7) - (x + 3) = -60$

10. **WE11b** Solve the following.

a. $\frac{3k}{5} = 15$
b. $\frac{9m}{8} = 18$
c. $\frac{7p}{10} = -8$
d. $\frac{8u}{11} = -3$
e. $\frac{11x}{4} = 2$
f. $\frac{4v}{15} = 0.8$

11. Solve the following.

a. $\frac{x - 5}{3} = 7$
b. $\frac{2m + 1}{3} = -3$
c. $\frac{3w - 1}{4} = 6$
d. $\frac{t - 5}{2} = 0$
e. $\frac{6 - x}{3} = -1$
f. $\frac{3n - 5}{4} = -6$

12. **MC** a. The solution to the equation $\frac{p}{5} + 2 = 7$ is:

A. $p = 5$
B. $p = 25$
C. $p = 45$
D. $p = 10$
E. $p = 1$

b. If $5h + 8 = 53$, then h is equal to:

A. $\frac{1}{5}$
B. 12.2
C. 225
D. 10
E. 9

c. The exact solution to the equation $14x = 75$ is:

A. $x = 5.357\,142\,857$
B. $x = 5.357$ (to 3 decimal places)
C. $x = 5\frac{5}{14}$
D. $x = 5.4$
E. $x = 5.5$

13. Solve the following equations.

a. $-x = 5$
b. $2 - d = 3$
c. $5 - p = -2$
d. $-7 - x = 4$
e. $-5h = 10$
f. $-6t = -30$
g. $-\frac{v}{5} = 4$
h. $-\frac{r}{12} = \frac{1}{4}$
i. $-4g = 3.2$