

TOPIC 10

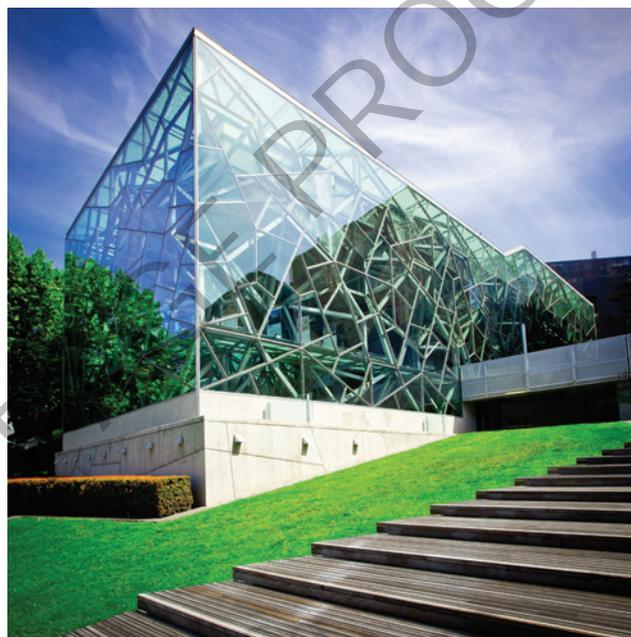
Deductive geometry

10.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

10.1.1 Why learn this?

Learning about geometry includes being able to reason deductively and to prove logically that certain mathematical statements are true. It is important to be able to prove theories meticulously and step by step in order to show that the conclusions reached are soundly based. Mathematicians spend most of their time trying to prove new theories, and they rely heavily on all the proofs that have gone before. Reasoning skills, and hence the ability to prove theories, can be developed and learned through practice and application.



10.1.2 What do you know?

assess on

- 1. THINK** List what you know about geometry. Use a thinking tool such as a concept map to show your list.
- 2. PAIR** Share what you know with a partner and then with a small group.
- 3. SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of geometry.

LEARNING SEQUENCE

- 10.1 Overview
- 10.2 Angles, triangles and congruence
- 10.3 Similar triangles
- 10.4 Quadrilaterals
- 10.5 Polygons
- 10.6 Review

learn on RESOURCES — ONLINE ONLY

Watch this video: The story of mathematics: Euclid
Searchlight ID: eles-1849

10.2 Angles, triangles and congruence

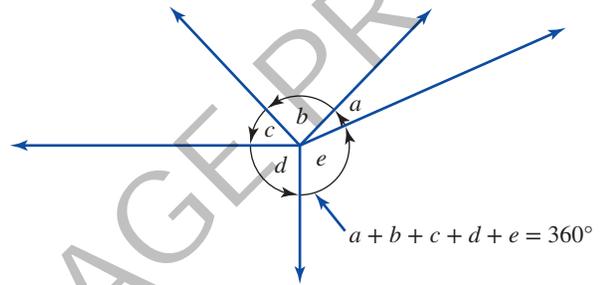
- Euclid (c. 300 BC) was the mathematician who developed a systematic approach to geometry, now referred to as Euclidean geometry, that relied on mathematical proofs.
- A **proof** is an argument that shows why a statement is true.
- A **theorem** is a statement that can be demonstrated to be true. To demonstrate that a statement is proven, formal language needs to be used. It is conventional to use the following structure when setting out a theorem.
 - **Given:** a summary of the information given
 - **To prove:** a statement that needs to be proven
 - **Construction:** a description of any additions to the diagram given
 - **Proof:** a sequence of steps that can be justified and form part of a formal mathematical proof.

10.2.1 Angles

Angles at a point

- The sum of the angles at a point is 360° .

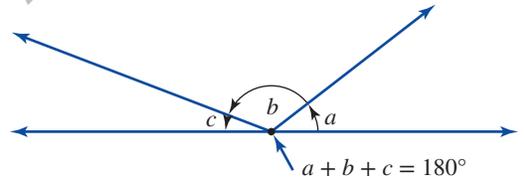
$$a + b + c + d + e = 360^\circ$$



Supplementary angles

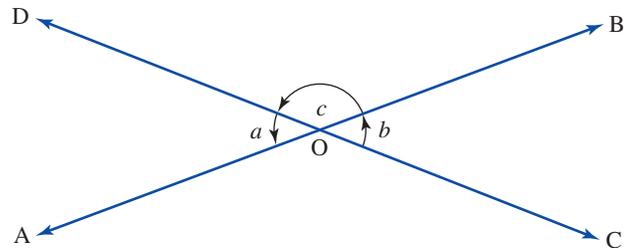
- The sum of the angles on a straight line is 180° .
- Angles a , b and c are **supplementary angles**.

$$a + b + c = 180^\circ$$



Vertically opposite angles

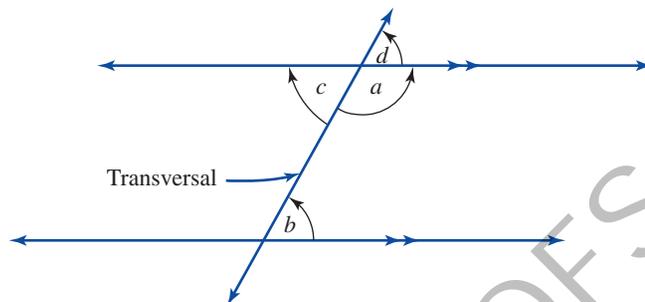
- **Theorem 1: Vertically opposite angles** are equal.



Given: Straight lines AB and CD intersect at O.
To prove: $\angle AOD = \angle BOC$ and $\angle BOD = \angle AOC$
Construction: Label $\angle AOD$ as a , $\angle BOC$ as b and $\angle BOD$ as c .
Proof: Let $\angle AOD = a^\circ$, $\angle BOC = b^\circ$ and $\angle BOD = c^\circ$.
 $a + c = 180^\circ$ (supplementary angles)
 $b + c = 180^\circ$ (supplementary angles)
 $\therefore a + c = b + c$
 $\therefore a = b$
 So, $\angle AOD = \angle BOC$.
 Similarly, $\angle BOD = \angle AOC$.

Parallel lines

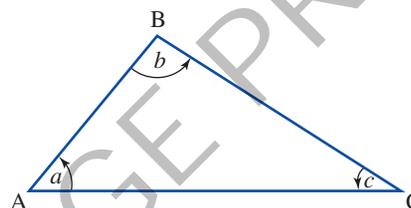
- If two lines are parallel and cut by a **transversal**, then:
 - co-interior angles are supplementary. For example, $a + b = 180^\circ$.
 - corresponding angles are equal. For example, $b = d$.
 - alternate angles are equal. For example, $b = c$.
 - opposite angles are equal. For example, $c = d$.



10.2.2 Angle properties of triangles

Theorem 2

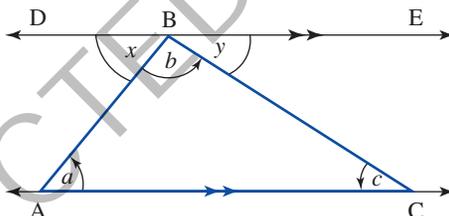
- **Theorem 2:** The sum of the interior angles of a triangle is 180° .



Given: $\triangle ABC$ with interior angles a , b and c

To prove: $a + b + c = 180^\circ$

Construction: Draw a line parallel to AC , passing through B and label it DE as shown. Label $\angle ABD$ as x and $\angle CBE$ as y .



Proof: $a = x$ (alternate angles)
 $c = y$ (alternate angles)
 $x + b + y = 180^\circ$ (supplementary angles)
 $\therefore a + b + c = 180^\circ$

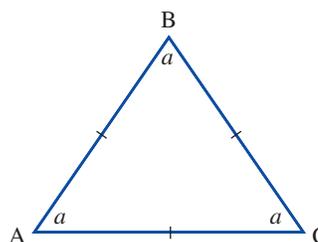
Equilateral triangles

- It follows from Theorem 2 that each interior angle of an **equilateral triangle** is 60° , and, conversely, if the three angles of a triangle are equal, then the triangle is equilateral.

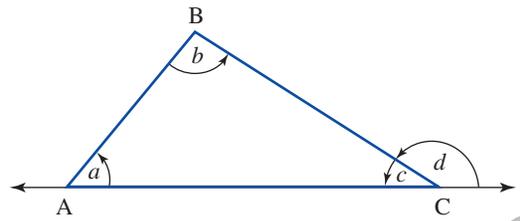
$$a + a + a = 180^\circ \quad (\text{sum of interior angles in a triangle is } 180^\circ)$$

$$3a = 180^\circ$$

$$a = 60^\circ$$



- **Theorem 3:** The exterior angle of a triangle is equal to the sum of the opposite interior angles.



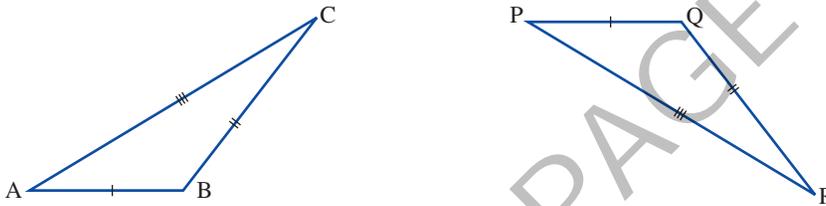
Given: $\triangle ABC$ with the exterior angle labelled d

To prove: $d = a + b$

Proof: $c + d = 180^\circ$ (supplementary angles)
 $a + b + c = 180^\circ$ (sum of interior angles in a triangle is 180°)
 $\therefore d = a + b$

10.2.3 Congruent triangles

- **Congruent triangles** have the same size and the same shape; that is, they are identical in all respects.
- The symbol used for congruency is \cong .
- For example, $\triangle ABC$ in the diagram below is congruent to $\triangle PQR$. This is written as $\triangle ABC \cong \triangle PQR$.



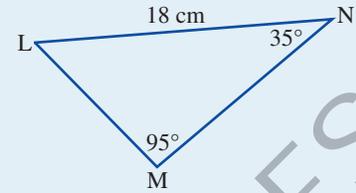
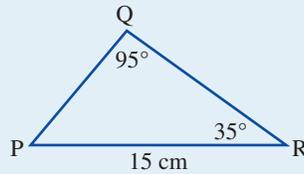
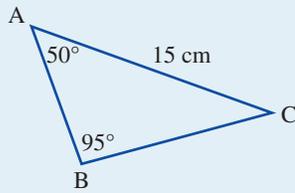
- Note that the vertices of the two triangles are written in corresponding order.
- There are five tests designed to check whether triangles are congruent. The tests are summarised in the table below.

Test	Diagram	Abbreviation
All three sides in one triangle are equal in length to the corresponding sides in the other triangle.		SSS
Two corresponding sides and the included angle are the same in both triangles.		SAS
Two corresponding angles and a pair of corresponding sides are the same in both triangles.		ASA
A pair of corresponding angles and a non-included side are equal in both triangles.		AAS
The hypotenuse and one pair of the other corresponding sides in two right-angled triangles are the same in two right-angled triangles.		RHS

- In each of the tests we need to show three equal measurements about a pair of triangles in order to show they are congruent.

WORKED EXAMPLE 1

Select a pair of congruent triangles from the diagrams below, giving a reason for your answer.



THINK

- In each triangle the length of the side opposite the 95° angle is given. If triangles are to be congruent, the sides opposite the angles of equal size must be equal in length. Draw your conclusion.
- To test whether $\triangle ABC$ is congruent to $\triangle PQR$, first find the angle C .
- Apply a test for congruence. Triangles ABC and PQR have a pair of corresponding sides equal in length and 2 pairs of angles the same, so draw your conclusion.

WRITE

All three triangles have equal angles, but the sides opposite the angle 95° are not equal.
 $AC = PR = 15$ and $LN = 18$ cm

$$\begin{aligned} \triangle ABC: \angle A &= 50^\circ, \angle B = 95^\circ, \\ \angle C &= 180^\circ - 50^\circ - 95^\circ \\ &= 35^\circ \end{aligned}$$

A pair of corresponding angles ($\angle B = \angle Q$ and $\angle C = \angle R$) and a non-included side ($AC = PR$) are equal.

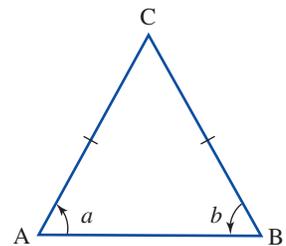
$$\triangle ABC \cong \triangle PQR \text{ (AAS)}$$

10.2.4 Isosceles triangles

- A triangle is isosceles if the lengths of two sides are equal but the third side is not equal.

Theorem 4

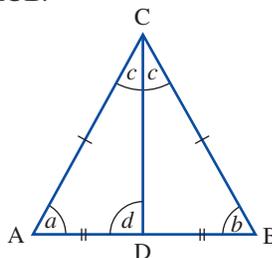
- Theorem 4:** The angles at the base of an **isosceles triangle** are equal.



Given: $AC = CB$

To prove: $\angle BAC = \angle CBA$

Construction: Draw a line from the vertex C to the midpoint of the base AB and label the midpoint D . CD is the bisector of $\angle ACB$.



Proof: In $\triangle ACD$ and $\triangle BCD$,
 $CD = CD$ (common side)
 $AD = DB$ (construction, D is the midpoint of AB)
 $AC = CB$ (given)
 $\Rightarrow \triangle ACD \cong \triangle BCD$ (SSS)
 $\therefore \angle BAC = \angle CBA$

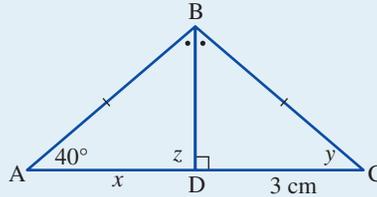
(common side)
 (construction, D is the midpoint of AB)
 (given)
 (SSS)

- Conversely, if two angles of a triangle are equal, then the sides opposite those angles are equal.
- It also follows that $\angle ADC = \angle BDC = d$

and that
$$\begin{aligned} 2d &= 180^\circ \\ \Rightarrow d &= 90^\circ \quad (\text{supplementary}) \end{aligned}$$

WORKED EXAMPLE 2

Given that $\triangle ABD \cong \triangle CBD$, find the values of the pronumerals in the figure below.



THINK

- 1 In congruent triangles corresponding sides are equal in length. Side AD (marked x) corresponds to side DC, so state the value of x .
- 2 Since the triangles are congruent, corresponding angles are equal. State the angles corresponding to y and z and hence find the values of these pronumerals.

WRITE

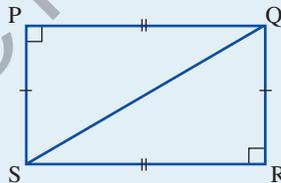
$\triangle ABD \cong \triangle CBD$
 $AD = CD$, $AD = x$, $CD = 3$
 So $x = 3$ cm.

$\angle BAD = \angle BCD$
 $\angle BAD = 40^\circ$, $\angle BCD = y$
 So $y = 40^\circ$

$\angle BDA = \angle BDC$
 $\angle BDA = z$, $\angle BDC = 90^\circ$
 So $z = 90^\circ$.

WORKED EXAMPLE 3

Prove that $\triangle PQS$ is congruent to $\triangle RSQ$.



THINK

- 1 Write the information given.
- 2 Write what needs to be proved.
- 3 Select the appropriate congruency test for proof. (In this case it is RHS because the triangles have an equal side, a right angle and a common hypotenuse.)

WRITE

Given: Rectangle PQRS with diagonal QS.

To prove: that $\triangle PQS$ is congruent to $\triangle RSQ$.

$QP = SR$ (given)
 $\angle SPQ = \angle SRQ = 90^\circ$ (given)
 QS is common.

So $\triangle PQS \cong \triangle RSQ$ (RHS)



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Complete this digital doc: SkillSHEET: Corresponding sides and angles of congruent triangles
Searchlight ID: doc-5277



Complete this digital doc: SkillSHEET: Angles and parallel lines
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Exercise 10.2 Angles, triangles and congruence

assesson

Individual pathways

PRACTISE

Questions:
1–5, 7, 9, 11

CONSOLIDATE

Questions:
1–5, 6, 8–10, 12, 13

MASTER

Questions:
1–14

Individual pathway interactivity: int-4612

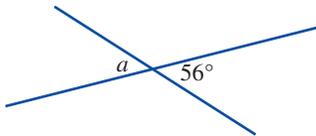
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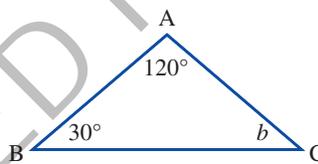
Fluency

1. Determine the values of the unknown in each of the following.

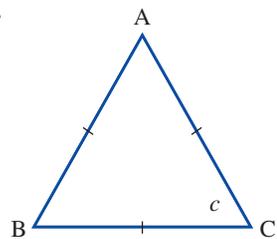
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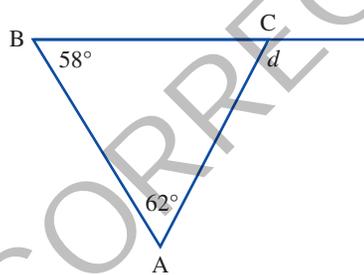
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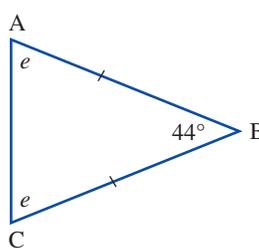
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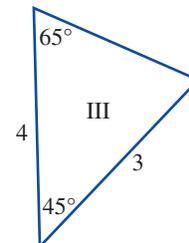
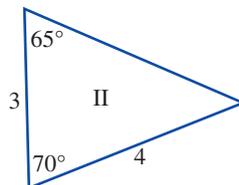
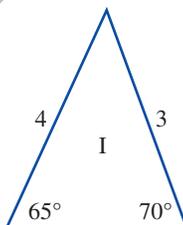


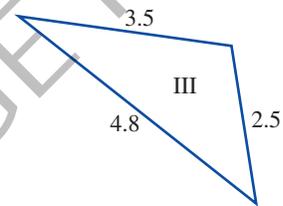
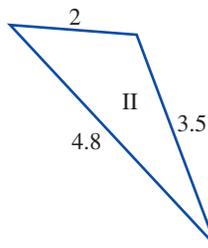
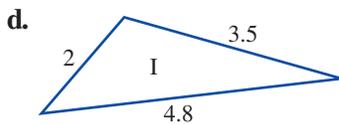
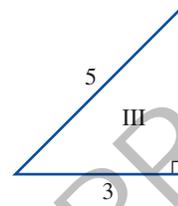
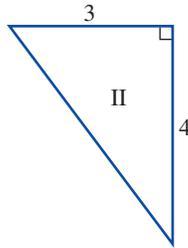
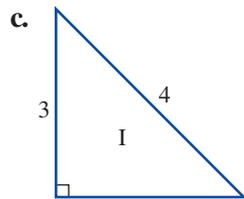
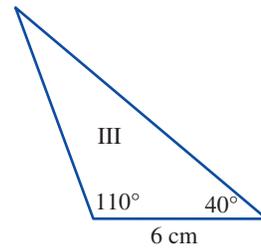
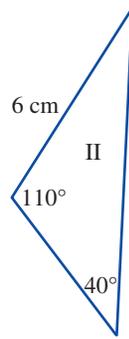
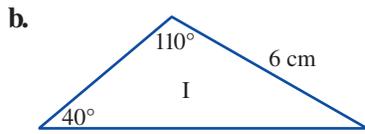
e.



2. **WE1** Select a pair of congruent triangles in each of the following, giving a reason for your answer. All side lengths are in cm.

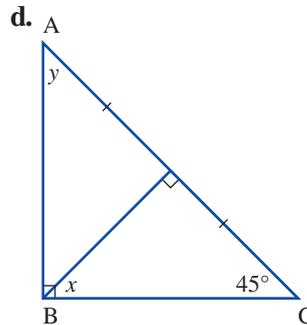
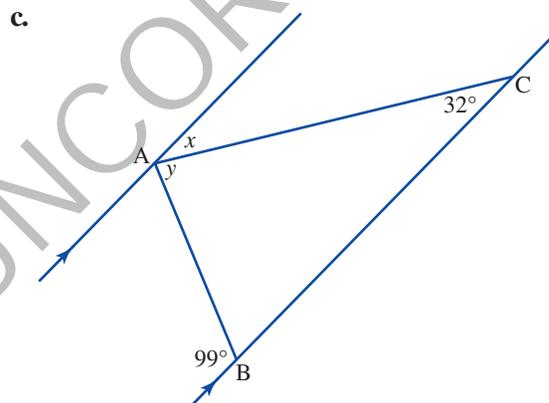
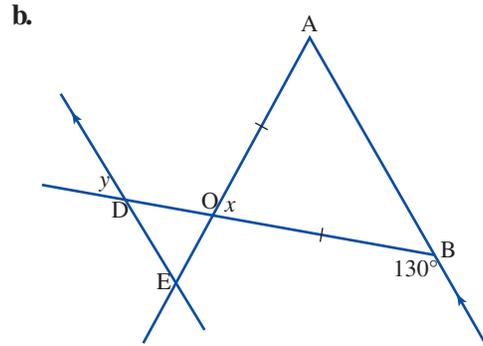
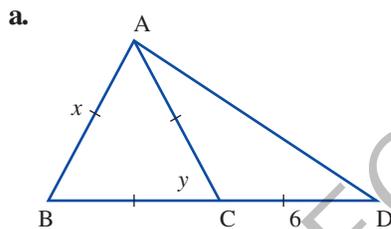
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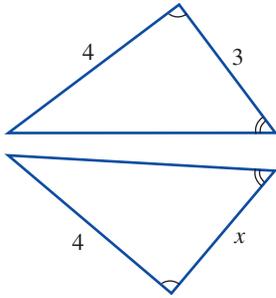
Understanding

3. Find the missing values of x and y in each of the following diagrams. Give reasons for your answers.

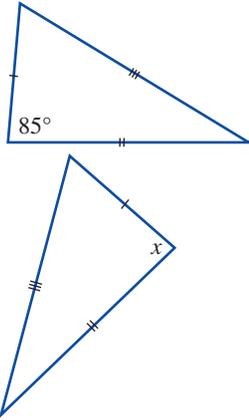


4. **WE2** Find the value of the pronumeral in each of the following pairs of congruent triangles. All side lengths are in cm.

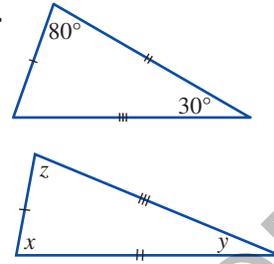
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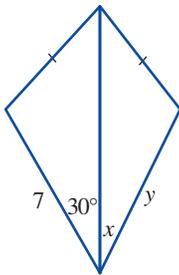
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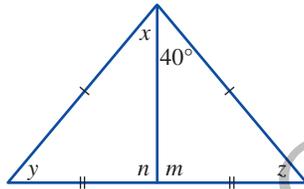
c.



d.



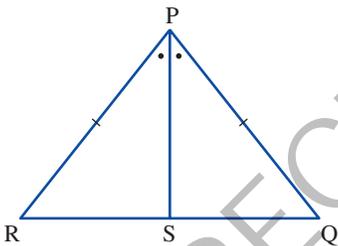
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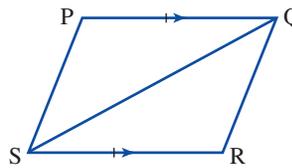
Reasoning

5. **WE3** Prove that each of the following pairs of triangles are congruent.

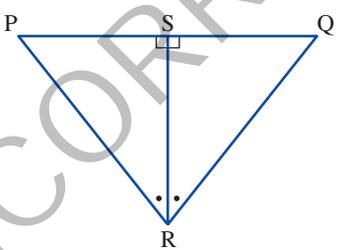
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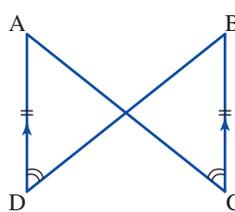
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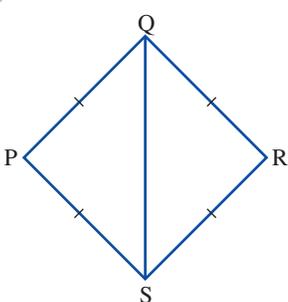
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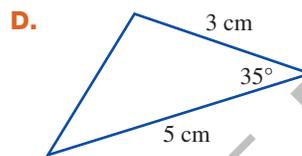
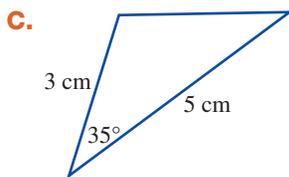
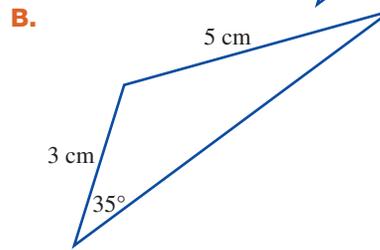
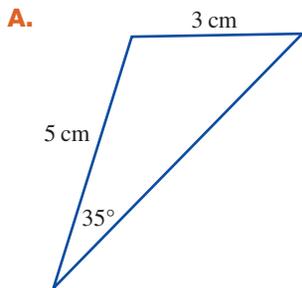
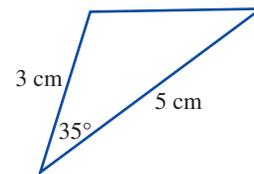


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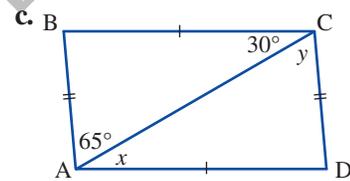
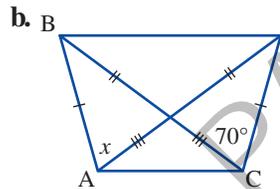
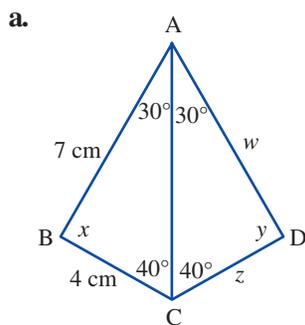


6. **MC** *Note:* There may be more than one correct answer.

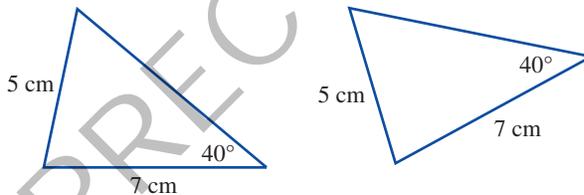
Which of the following is congruent to the triangle shown at right?



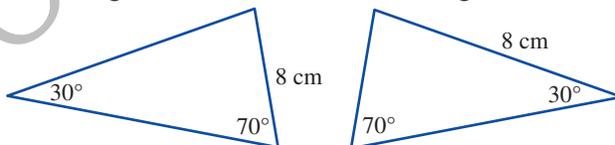
7. Prove that $\triangle ABC \cong \triangle ADC$ and hence find the values of the pronumerals in each of the following.



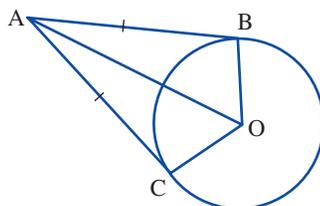
8. Explain why the triangles shown below are not necessarily congruent.



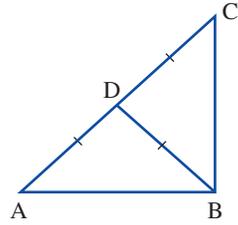
9. Explain why the triangles shown below are not congruent.



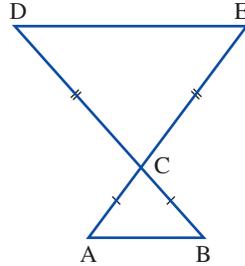
10. Show that $\triangle ABO \cong \triangle ACO$, if O is the centre of the circle.



11. If $DA = DB = DC$, prove that $\angle ABC$ is a right angle.

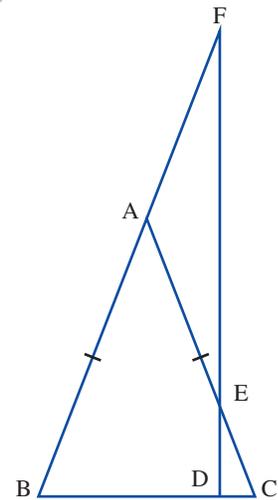


12. If $AC = CB$ and $DC = CE$ in the diagram shown, prove that $AB \parallel DE$.

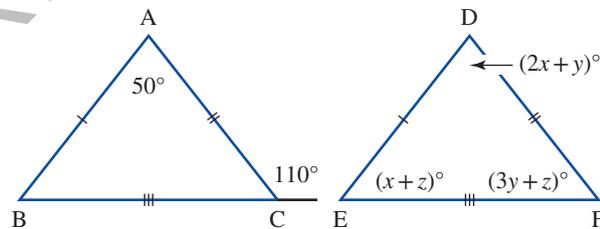


Problem solving

13. ABC is an isosceles triangle in which AB and AC are equal in length. BDF is a right-angled triangle. Show that triangle AEF is an isosceles triangle.



14. Triangles ABC and DEF are congruent.



Find the values of x , y and z .

Reflection

How can you be certain that two figures are congruent?

10.3 Similar triangles

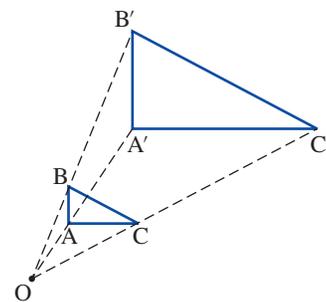
10.3.1 Similar figures

- Two geometric shapes are **similar** when one is an **enlargement** or reduction of the other shape.



– An enlargement increases the length of each side of a figure in all directions by the same factor. For example, in the diagram shown, triangle $A'B'C'$ is an enlargement of triangle ABC by a factor of 3 from its **centre of enlargement** at O .

- The symbol for similarity is \sim and is read as ‘is similar to’.
- The **image** of the original object is the enlarged or reduced shape.
- To create a similar shape, use a **scale factor** to enlarge or reduce the original shape.
- The scale factor can be found using the formula below and the lengths of a pair of corresponding sides.



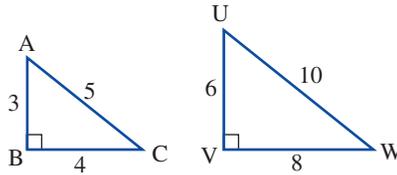
$$\text{Scale factor} = \frac{\text{image side length}}{\text{object side length}}$$

- If the scale factor is less than 1, the image is a reduced version of the original shape. If the scale factor is greater than 1, the image is an enlarged version of the original shape.

10.3.2 Similar triangles

- Two triangles are similar if:
 - the angles are equal, or
 - the corresponding sides are proportional.

- Consider the pair of **similar triangles** below.



- The following statements are true for these triangles.
 - Triangle UVW is similar to triangle ABC or, using symbols, $\Delta UVW \sim \Delta ABC$.
 - The corresponding angles of the two triangles are equal in size:
 $\angle CAB = \angle WUV$, $\angle ABC = \angle UVW$ and $\angle ACB = \angle UWV$.
 - The corresponding sides of the two triangles are in the same ratio. $\frac{UV}{AB} = \frac{VW}{BC} = \frac{UW}{AC} = 2$; that is, ΔUVW has each of its sides twice as long as the corresponding sides in ΔABC .
 - The scale factor is 2.

10.3.3 Testing triangles for similarity

- Triangles can be checked for similarity using one of the tests described in the table below.

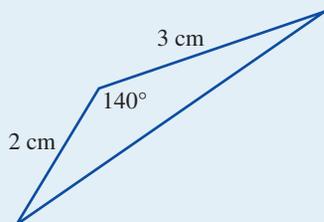
Test	Diagram	Abbreviation
Two angles of a triangle are equal to two angles of another triangle. This implies that the third angles are equal, as the sum of angles in a triangle is 180° .		AAA
The three sides of a triangle are proportional to the three sides of another triangle.		SSS
Two sides of a triangle are proportional to two sides of another triangle, and the included angles are equal.		SAS
The hypotenuse and a second side of a right-angled triangle are proportional to the hypotenuse and a second side of another right-angled triangle.		RHS

- Note:* When using the equiangular test, only two corresponding angles have to be checked. Since the sum of the interior angles in any triangle is a constant number (180°), the third pair of corresponding angles will automatically be equal, provided that the first two pairs match exactly.

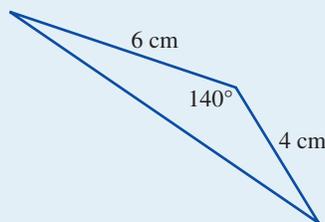
WORKED EXAMPLE 4

Find a pair of similar triangles among those shown. Give a reason for your answer.

a



b



c



THINK

- In each triangle the lengths of two sides and the included angle are known, so the SAS test can be applied. Since all included angles are equal (140°), we need to find the ratios of corresponding sides, taking two triangles at a time.
- Only triangles **a** and **b** have corresponding sides in the same ratio (and included angles are equal). State your conclusion, specifying the similarity test that has been used.

WRITE

For triangles **a** and **b**: $\frac{6}{3} = \frac{4}{2} = 2$

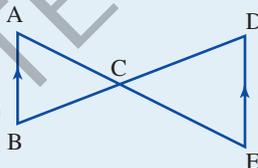
For triangles **a** and **c**: $\frac{5}{3} = 1.6, \frac{3}{2} = 1.5$

For triangles **b** and **c**: $\frac{5}{6} = 0.83, \frac{3}{4} = 0.75$

Triangle **a** ~ triangle **b** (SAS)

WORKED EXAMPLE 5

Prove that $\triangle ABC$ is similar to $\triangle EDC$.



THINK

- Write the information given. AB is parallel to DE . Transversal BD forms two alternate angles: $\angle ABC$ and $\angle EDC$.
- Write what is to be proved.
- Write the proof.

WRITE

- Given:**
 $\triangle ABC$ and $\triangle DCE$
 $AB \parallel DE$
 C is common.
- To prove:** $\triangle ABC \sim \triangle EDC$
- Proof:**
 $\angle ABC = \angle EDC$ (alternate angles)
 $\angle BAC = \angle DEC$ (alternate angles)
 $\angle BCA = \angle DCE$ (vertically opposite angles)
 $\therefore \triangle ABC \sim \triangle EDC$ (equiangular, AAA)



Complete this digital doc: SkillSHEET: Writing similarity statements
Searchlight ID: doc-5278



Complete this digital doc: SkillSHEET: Calculating unknown side lengths in a pair of similar triangles
Searchlight ID: doc-5281



Complete this digital doc: WorkSHEET 10.1
Searchlight ID: doc-5282

Exercise 10.3 Similar triangles

assesson

Individual pathways

PRACTISE

Questions:
1–10

CONSOLIDATE

Questions:
1–11, 15

MASTER

Questions:
1–15

Individual pathway interactivity: int-4613

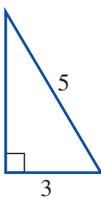
learnon ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. **WE4** Find a pair of similar triangles among those shown in each part. Give a reason for your answer.

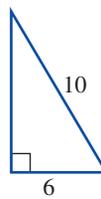
a. i.



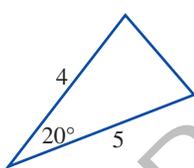
ii.



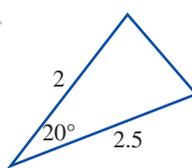
iii.



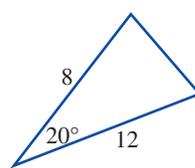
b. i.



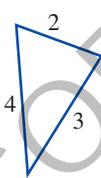
ii.



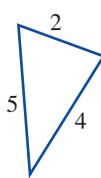
iii.



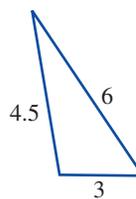
c. i.



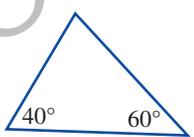
ii.



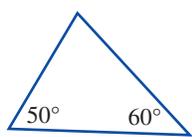
iii.



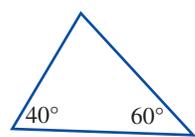
d. i.



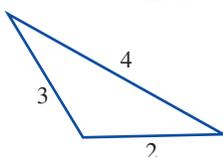
ii.



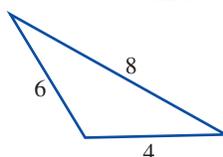
iii.



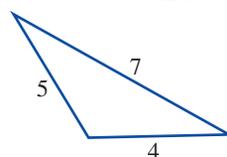
e. i.



ii.

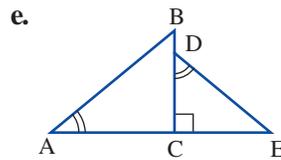
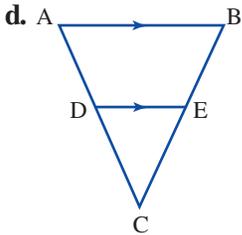
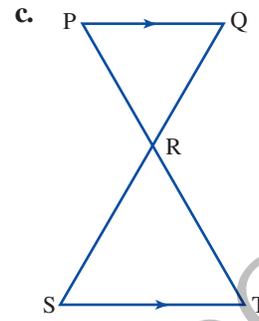
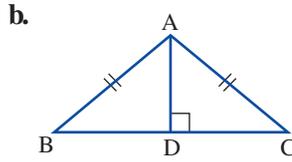
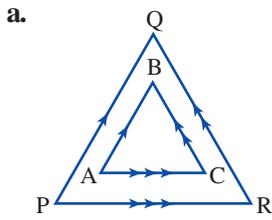


iii.



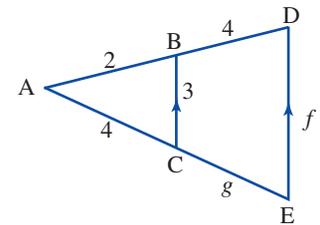
Understanding

2. Name two similar triangles in each of the following figures.

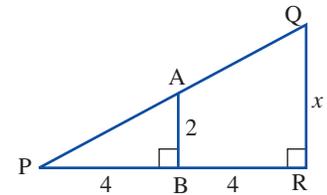


3. a. Complete this statement: $\frac{AB}{AD} = \frac{BC}{AE} = \frac{\quad}{\quad}$.

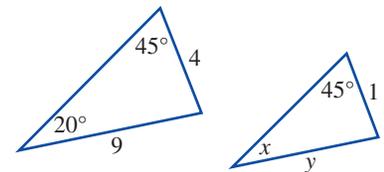
b. Find the value of the pronumerals.



4. Find the value of the pronumerals in the diagram at right.

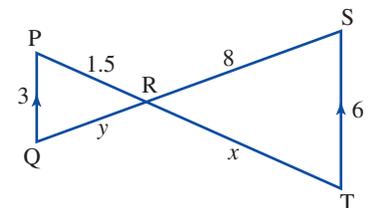


5. The triangles shown at right are similar. Find the value of x and y .

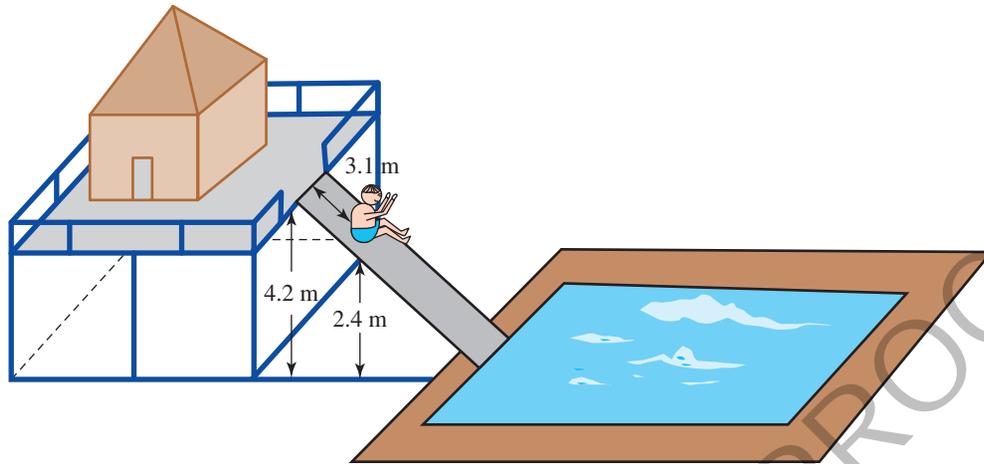


6. a. State why these two triangles shown at right are similar.

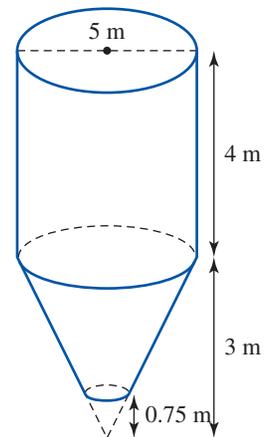
b. Find the values of x and y in the diagram.



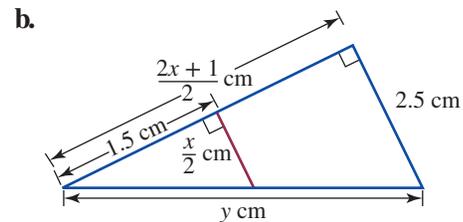
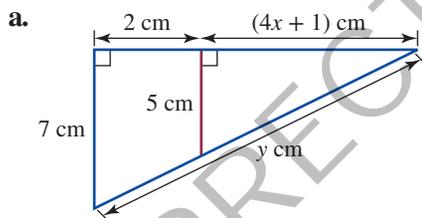
7. A waterslide is 4.2 m high and has a support 2.4 m tall. If a student reaches this support when she is 3.1 m down the slide, how long is the slide?



8. A storage tank as shown in the diagram is made of a 4-m-tall cylinder joined by a 3-m-tall cone. If the diameter of the cylinder is 5 m, what is the radius of the end of the cone if 0.75 m has been cut off the tip?

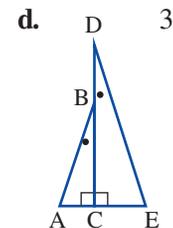
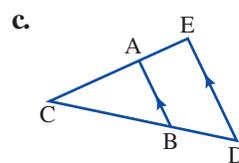
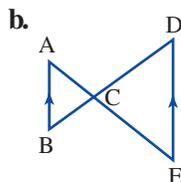
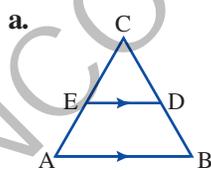


9. Calculate the values of the pronumerals.



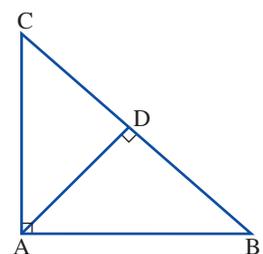
Reasoning

10. **WE5** Prove that $\triangle ABC$ is similar to $\triangle EDC$ in each of the following.

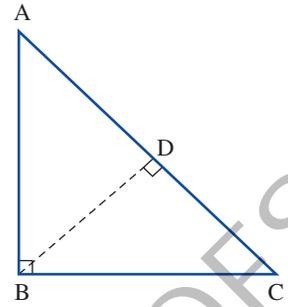


11. $\triangle ABC$ is a right-angled triangle. A line is drawn from A to D as shown so that $AD \perp BC$. Prove that:

- a. $\triangle ABD \sim \triangle ACB$
b. $\triangle ACD \sim \triangle ACB$.

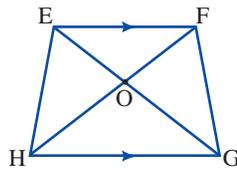


12. Explain why the AAA test cannot be used to prove congruence but can be used to prove similarity.
13. a. Prove Pythagoras' theorem, $AC^2 = AB^2 + BC^2$, using similar triangles.
 b. Show that the converse of Pythagoras' theorem holds true; that is, if the square on one side of a triangle equals the sum of the squares on the other two sides, then the angle between these other two sides is a right angle.

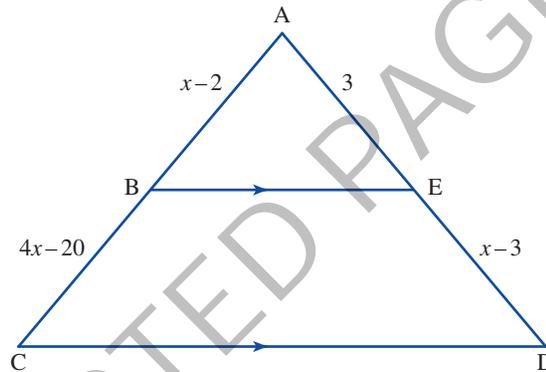


Problem solving

14. Prove that $\triangle EFO \sim \triangle GHO$.



15. Solve for x .

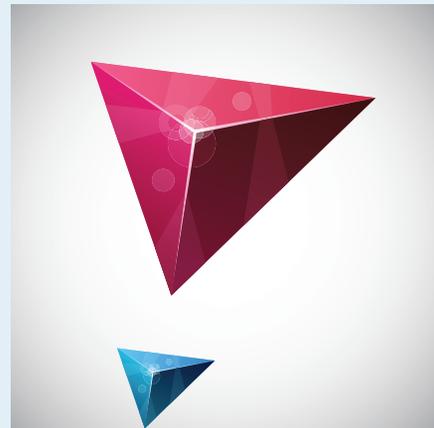


Reflection

How can you be certain that two figures are similar?

CHALLENGE 10.1

A tetrahedron (regular triangular-based pyramid) has an edge length of 2 cm. A similar tetrahedron has a total surface area of $36\sqrt{3}$ cm². What is the scale factor relationship between the side lengths of the two tetrahedra?

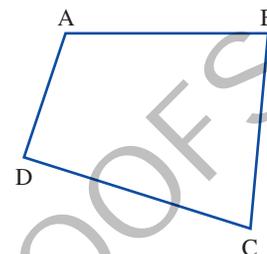


10.4 Quadrilaterals

- Quadrilaterals are four-sided plane shapes whose interior angles sum to 360° .

Theorem 5

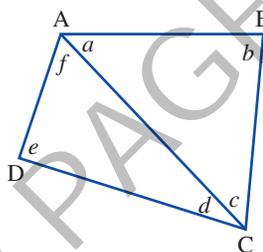
- **Theorem 5:** The sum of the interior angles in a quadrilateral is 360° .



Given: A quadrilateral ABCD

To prove: $\angle ABC + \angle BCD + \angle ADC + \angle BAD = 360^\circ$

Construction: Draw a line joining vertex A to vertex C. Label the interior angles of the triangles formed.



Proof:

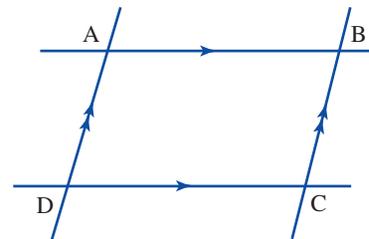
$$a + b + c = 180^\circ \quad (\text{sum of interior angles in a triangle is } 180^\circ)$$
$$d + e + f = 180^\circ \quad (\text{sum of interior angles in a triangle is } 180^\circ)$$
$$\Rightarrow a + b + c + d + e + f = 360^\circ$$
$$\therefore \angle ABC + \angle BCD + \angle ADC + \angle BAD = 360^\circ$$

10.4.1 Parallelograms

- A **parallelogram** is a quadrilateral with two pairs of parallel sides.

Theorem 6

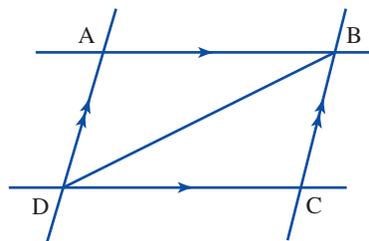
- **Theorem 6:** Opposite angles of a parallelogram are equal.



Given: $AB \parallel DC$ and $AD \parallel BC$

To prove: $\angle ABC = \angle ADC$

Construction: Draw a diagonal from B to D.



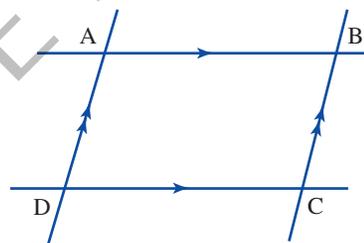
Proof:

$\angle ABD = \angle BDC$	(alternate angles)
$\angle ADB = \angle CBD$	(alternate angles)
$\angle ABC = \angle ABD + \angle CBD$	(by construction)
$\angle ADC = \angle BDC + \angle ADB$	(by construction)
$\therefore \angle ABC = \angle ADC$	

- Conversely, if each pair of opposite angles of a quadrilateral is equal then it is a parallelogram.

Theorem 7

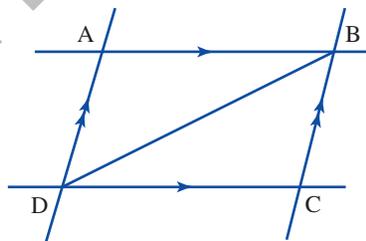
- Theorem 7:** Opposite sides of a parallelogram are equal.



Given: $AB \parallel DC$ and $AD \parallel BC$

To prove: $AB = DC$

Construction: Draw a diagonal from B to D.



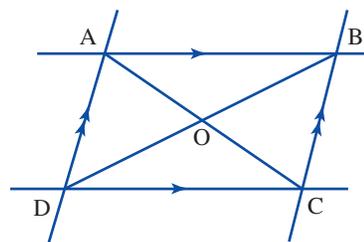
Proof:

$\angle ABD = \angle BDC$	(alternate angles)
$\angle ADB = \angle CBD$	(alternate angles)
BD is common to $\triangle ABD$ and $\triangle BCD$.	
$\Rightarrow \triangle ABD \cong \triangle BCD$ (ASA)	
$\therefore AB = DC$	

- Conversely, if each pair of opposite sides of a quadrilateral is equal then it is a parallelogram.

Theorem 8

- Theorem 8:** The diagonals of a parallelogram bisect each other.



Given: $AB \parallel DC$ and $AD \parallel BC$ with diagonals AC and BD

To prove: $AO = OC$ and $BO = OD$

Proof: In $\triangle AOB$ and $\triangle COD$,
 $\angle OAB = \angle OCD$ (alternate angles)
 $\angle OBA = \angle ODC$ (alternate angles)
 $AB = CD$ (opposite sides of a parallelogram)
 $\Rightarrow \triangle AOB \cong \triangle COD$ (ASA)
 $\Rightarrow AO = OC$ (corresponding sides in congruent triangles)
and $BO = OD$ (corresponding sides in congruent triangles)

10.4.2 Rectangles

- A rectangle is a parallelogram with four right angles.

Theorem 9

- Theorem 9:** A parallelogram with a right angle is a rectangle.



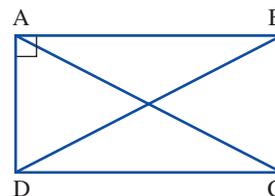
Given: Parallelogram $ABCD$ with $\angle BAD = 90^\circ$

To prove: $\angle BAD = \angle ABC = \angle BCD = \angle ADC = 90^\circ$

Proof: $AB \parallel CD$ (properties of a parallelogram)
 $\Rightarrow \angle BAD + \angle ADC = 180^\circ$ (co-interior angles)
But $\angle BAD = 90^\circ$ (given)
 $\Rightarrow \angle ADC = 90^\circ$
Similarly, $\angle BCD = \angle ADC = 90^\circ$
 $\therefore \angle BAD = \angle ABC = \angle BCD = \angle ADC = 90^\circ$

Theorem 10

- Theorem 10:** The diagonals of a rectangle are equal.



Given: Rectangle $ABCD$ with diagonals AC and BD

To prove: $AC = BD$

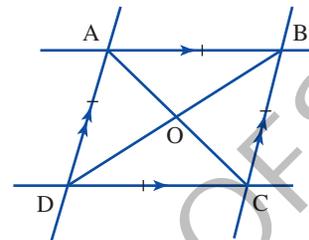
Proof: In $\triangle ADC$ and $\triangle BCD$,
 $AD = BC$ (opposite sides equal in a rectangle)
 $DC = CD$ (common)
 $\angle ADC = \angle BCD = 90^\circ$ (right angles in a rectangle)
 $\Rightarrow \triangle ADC \cong \triangle BCD$ (SAS)
 $\therefore AC = BD$

10.4.3 Rhombuses

- A **rhombus** is a parallelogram with four equal sides.

Theorem 11

- Theorem 11:** The diagonals of a rhombus are perpendicular.



Given: Rhombus ABCD with diagonals AC and BD

To prove: $AC \perp BD$

Proof: In $\triangle AOB$ and $\triangle BOC$,

$$AO = OC$$

(property of parallelogram)

$$AB = BC$$

(property of rhombus)

$$BO = OB$$

(common)

$$\Rightarrow \triangle AOB \cong \triangle BOC$$

(SSS)

$$\Rightarrow \angle AOB = \angle BOC$$

$$\text{But } \angle AOB + \angle BOC = 180^\circ$$

(supplementary angles)

$$\Rightarrow \angle AOB = \angle BOC = 90^\circ$$

$$\text{Similarly, } \angle AOD = \angle DOC = 90^\circ.$$

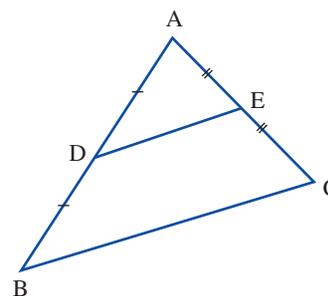
Hence, $AC \perp BD$

10.4.4 The midpoint theorem

- Now that the properties of quadrilaterals have been explored, the midpoint theorem can be tackled.

Theorem 12

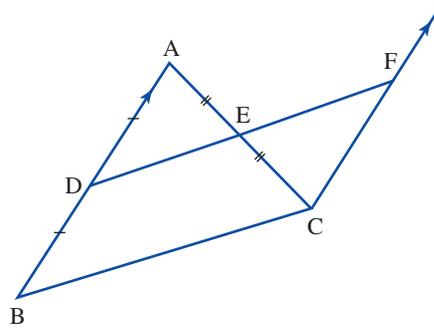
- Theorem 12:** The interval joining the midpoints of two sides of a triangle is parallel to the third side and half its length.



Given: $\triangle ABC$ in which $AD = DB$ and $AE = EC$

To prove: $DE \parallel BC$ and $DE = \frac{1}{2}BC$

Construction: Draw a line through C parallel to AB. Extend DE to F on the parallel line.



Proof:

In $\triangle ADE$ and $\triangle CEF$,

$$AE = EC$$

(E is the midpoint of AC, given)

$$\angle AED = \angle CEF$$

(vertically opposite angles)

$$\angle EAD = \angle ECF$$

(alternate angles)

$$\Rightarrow \triangle ADE \cong \triangle CEF$$

(ASA)

$$\therefore AD = CF \text{ and } DE = EF$$

(corresponding sides in congruent triangles)

$$\text{So, } AD = DB = CF.$$

We have $AB \parallel CF$

(by construction)

So BDFC is a parallelogram.

$$\Rightarrow DE \parallel BC$$

$$\text{Also, } BC = DF$$

(opposite sides in parallelogram)

$$\text{But } DE = DF$$

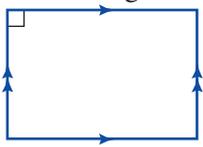
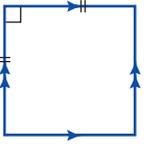
(sides in congruent triangles)

$$\Rightarrow DE = \frac{1}{2}BC$$

Therefore, $DE \parallel BC$ and $DE = \frac{1}{2}BC$.

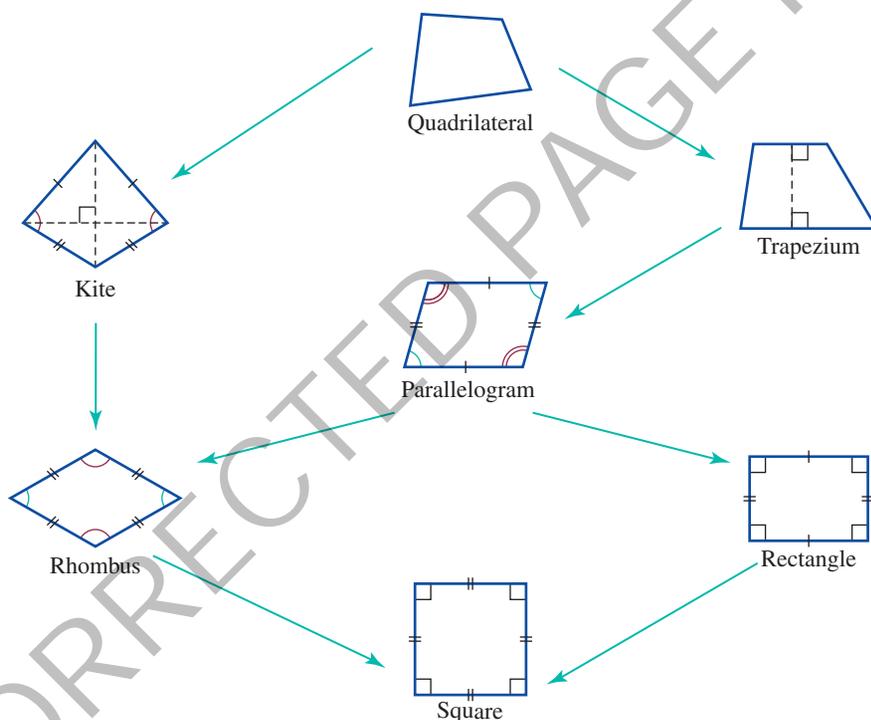
- Conversely, if a line interval is drawn parallel to a side of a triangle and half the length of that side, then the line interval bisects each of the other two sides of the triangle.
- A summary of the definitions and properties of quadrilaterals is shown in the table.

Shape	Definition	Properties
<p>Trapezium</p>	A trapezium is a quadrilateral with one pair of opposite sides parallel.	One pair of opposite sides is parallel but not equal in length.
<p>Parallelogram</p>	A parallelogram is a quadrilateral with both pairs of opposite sides parallel.	<ul style="list-style-type: none"> • Opposite angles are equal. • Opposite sides are equal. • Diagonals bisect each other.
<p>Rhombus</p>	A rhombus is a parallelogram with four equal sides.	<ul style="list-style-type: none"> • Diagonals bisect each other at right angles. • Diagonals bisect the angles at the vertex through which they pass.

Shape	Definition	Properties
<p>Rectangle</p> 	A rectangle is a parallelogram whose interior angles are right angles.	Diagonals are equal. Diagonals bisect each other.
<p>Square</p> 	A square is a parallelogram whose interior angles are right angles with four equal sides.	<ul style="list-style-type: none"> All angles are right angles. All side lengths are equal. Diagonals are equal in length and bisect each other at right angles. Diagonals bisect the vertex through which they pass (45°).

10.4.5 Relationships between quadrilaterals

- The flowchart below shows the relationships between quadrilaterals.



learn on RESOURCES – ONLINE ONLY

-  **Try out this interactivity:** Quadrilateral definitions
Searchlight ID: int-2786
-  **Complete this digital doc:** SkillSHEET: Identifying quadrilaterals
Searchlight ID: doc-5279
-  **Complete this digital doc:** WorkSHEET 10.2
Searchlight ID: doc-5283

Individual pathways

PRACTISE

Questions:
1–10

CONSOLIDATE

Questions:
1–14, 16

MASTER

Questions:
1–19

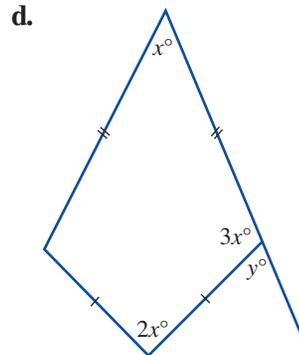
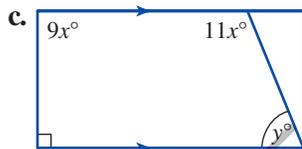
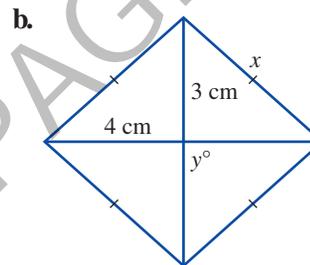
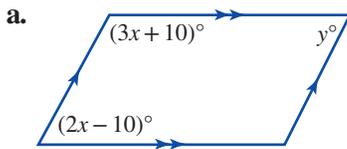
Individual pathway interactivity: int-4614

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

- Use the definitions of the five special quadrilaterals to decide if the following statements are true or false.
 - A square is a rectangle.
 - A rhombus is a parallelogram.
 - A square is a rhombus.
 - A rhombus is a square.
 - A square is a trapezium.
 - A parallelogram is a rectangle.
 - A trapezium is a rhombus.
 - A rectangle is a square.
- Determine the values of x and y in each of the following figures.

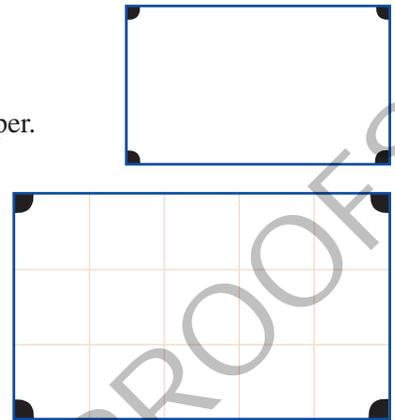


Understanding

- Draw three different trapeziums. Using your ruler, compass and protractor, decide which of the following properties are true in a trapezium.
 - Opposite sides are equal.
 - All sides are equal.
 - Opposite angles are equal.
 - All angles are equal.
 - Diagonals are equal in length.
 - Diagonals bisect each other.
 - Diagonals are perpendicular.
 - Diagonals bisect the angles they pass through.
- Draw three different parallelograms. Using your ruler and protractor to measure, decide which of the following properties are true in a parallelogram.
 - Opposite sides are equal.
 - All sides are equal.
 - Opposite angles are equal.
 - All angles are equal.
 - Diagonals are equal in length.
 - Diagonals bisect each other.
 - Diagonals are perpendicular.
 - Diagonals bisect the angles they pass through.

5. Name four quadrilaterals that have at least one pair of opposite sides that are parallel and equal.
6. Name a quadrilateral that has equal diagonals that bisect each other and bisect the angles they pass through.
7. Pool is played on a rectangular table. Balls are hit with a cue and bounce off the sides of the table until they land in one of the holes or pockets.

- a. Draw a rectangular pool table measuring 5 cm by 3 cm on graph paper. Mark the four holes, one in each corner.
- b. A ball starts at A. It is hit so that it travels at a 45° diagonal across the grid. When it hits the side of the table, it bounces off at a 45° diagonal as well. How many sides does the ball bounce off before it goes in a hole?

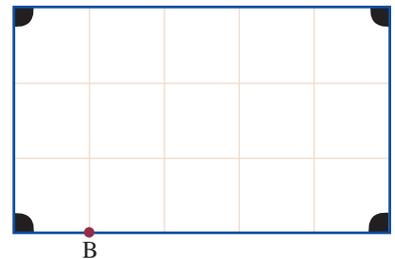


- c. A different size table is 7 cm by 2 cm. How many sides does a ball bounce off before it goes in a hole when hit from A?
- d. Complete the following table.

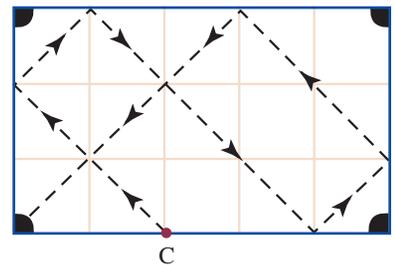
Table size	Number of sides hit
5 cm \times 3 cm	
7 cm \times 2 cm	
4 cm \times 3 cm	
4 cm \times 2 cm	
6 cm \times 3 cm	
9 cm \times 3 cm	
12 cm \times 4 cm	



- e. Can you see a pattern? How many sides would a ball bounce off before going in a hole when hit from A on an $m \times n$ table?
- f. The ball is now hit from B on a 5 cm \times 3 cm pool table. How many *different* paths can a ball take when hit along 45° diagonals? Do these paths all hit the same number of sides before going in a hole? Does the ball end up in the same hole each time? Justify your answer.

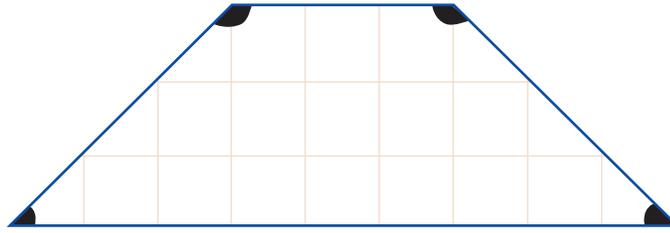


- g. The ball is now hit from C along the path shown. What type of triangles and quadrilaterals are formed by the path of the ball with itself and the sides of the table? Are any of the triangles congruent?



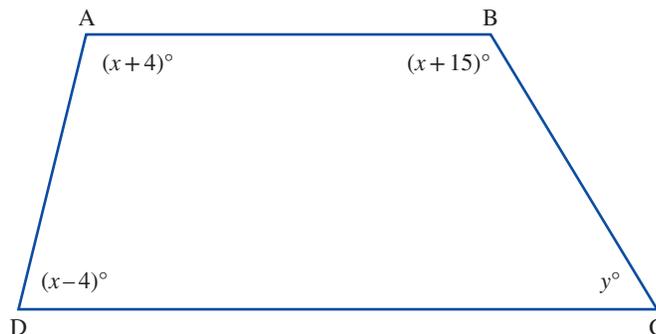
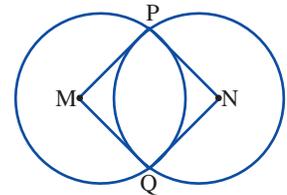
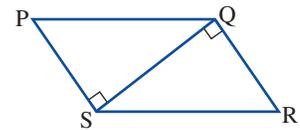
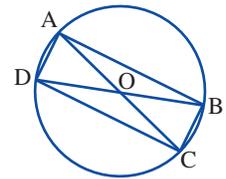
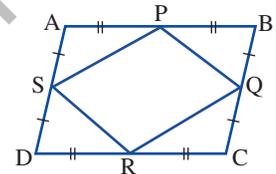
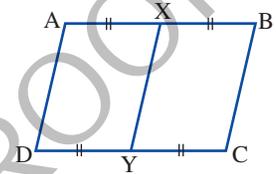
- h. A ball is hit from C on a 6 cm by 3 cm table. What shapes are formed by the path of the ball with itself and the sides of the table? Is there only one path possible?
- i. *Challenge:* A ball is hit from A along 45° diagonals. The table is $m \times n$. Can you find a formula to predict which hole the ball will go in?

j. *Challenge:* What would happen if the game was played on a trapezoidal table?



Reasoning

8. Prove that the diagonals of a rhombus bisect each other.
9. ABCD is a parallelogram. X is the midpoint of AB and Y is the midpoint of DC. Prove that AX YD is also a parallelogram.
10. ABCD is a parallelogram. P, Q, R and S are all midpoints of their respective sides of ABCD.
 - a. Prove $\triangle PAS \cong \triangle RCQ$.
 - b. Prove $\triangle SDR \cong \triangle PBQ$.
 - c. Hence, prove that PQRS is also a parallelogram.
11. AC and BD are diameters of a circle with centre O. Prove that ABCD is a rectangle.
12. The diagonals of a parallelogram meet at right angles. Prove that the parallelogram is a rhombus.
13. Two congruent right-angled triangles are arranged as shown. Show that PQRS is a parallelogram.
14. Two circles, centred at M and N, have equal radii and intersect at P and Q. Prove that PNQM is a rhombus.
15. Give reasons why a square is a rhombus, but a rhombus is not necessarily a square.
16. ABCD is a trapezium.

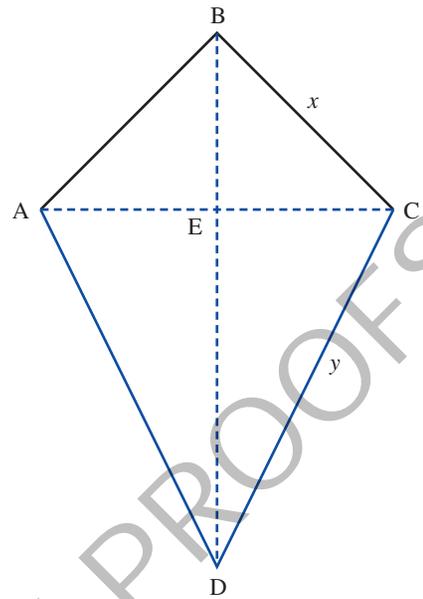


- a. What fact do you know about a trapezium?
- b. Find the values of x and y .

17. ABCD is a kite where $AC = 8$ cm, $BE = 5$ cm and $ED = 9$ cm.

Find the exact values of:

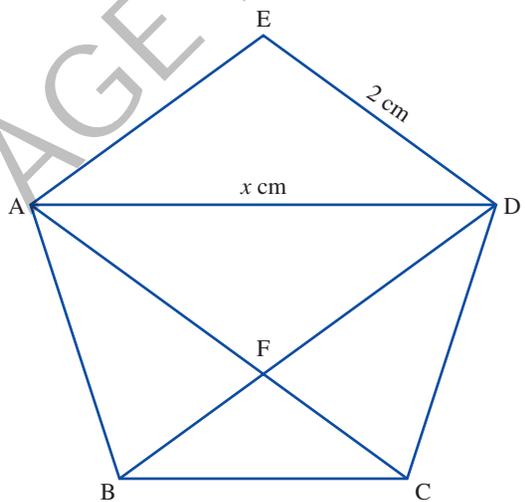
- a. i. x ii. y .
 b. Find angle BAD and hence angle BCD.



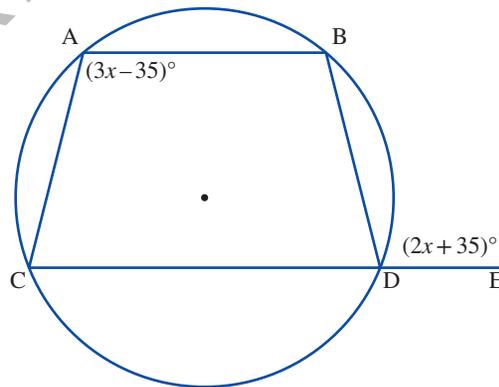
Problem solving

18. ABCDE is a regular pentagon whose side lengths are 2 cm. Each diagonal is x cm long.

- a. What kind of shape is AEDF and what is the length of FD?
 b. What kind of shape is ABCD?
 c. If $\angle EDA$ is 40° , find the value of $\angle ACB$, giving reasons for your findings.
 d. Which triangle is similar to AED?
 e. Explain why $FB = (x - 2)$ cm.
 f. Show that $x^2 - 2x - 4 = 0$.
 g. Solve the equation $x^2 - 2x - 4 = 0$, giving your answer as an exact value.



19. ABCD is called a cyclic quadrilateral because it is inscribed inside a circle.

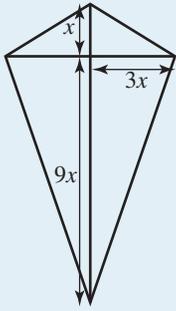


A characteristic of a cyclic quadrilateral is that the opposite angles are supplementary. Find the value of x .

Reflection

How do you know if a quadrilateral is a rhombus?

CHALLENGE 10.2

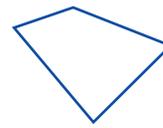


The perimeter of this kite is 80 cm. Determine the exact value of x .



10.5 Polygons

- **Polygons** are closed shapes that have three or more straight sides.



Irregular polygon



Not a polygon



Not a polygon

- **Regular polygons** are polygons with sides of the same length and interior angles of the same size, like the pentagon shown in the centre of the photo above.
- **Convex polygons** are polygons with no interior reflex angles.
- **Concave polygons** are polygons with at least one reflex interior angle. For example, the pentagon shown above is a concave polygon as well as a regular polygon.

10.5.1 Interior angles of a polygon

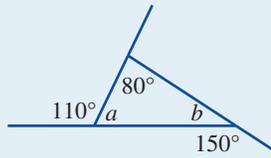
- The interior angles of a polygon are the angles inside the polygon at each vertex.
- The sum of the interior angles of a polygon is given by the formula:

$$\text{Angle sum} = 180^\circ \times (n - 2)$$

where n = the number of sides of the polygon

WORKED EXAMPLE 6

Calculate the value of the pronumerals in the figure below.



THINK

- 1 Angles a and 110° form a straight line and so are supplementary (add to 180°).
- 2 The interior angles of a triangle sum to 180° .
- 3 Substitute 70° for a and solve for b .

WRITE

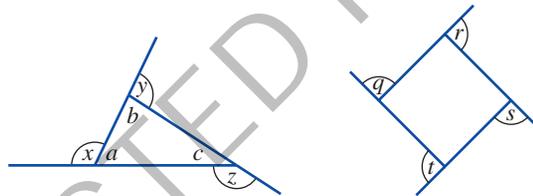
$$\begin{aligned}a + 110^\circ &= 180^\circ \\a + 110^\circ - 110^\circ &= 180^\circ - 110^\circ \\a &= 70^\circ\end{aligned}$$

$$b + a + 80^\circ = 180$$

$$\begin{aligned}b + 70^\circ + 80^\circ &= 180^\circ \\b + 150^\circ &= 180^\circ \\b &= 30^\circ\end{aligned}$$

10.5.2 Exterior angles of a polygon

- The exterior angles of a polygon are formed by the side of the polygon and an extension of its adjacent side. For example, x , y and z are external angles for the polygon (triangle) below.



- The exterior angle and interior angle at that vertex are supplementary (add to 180°). For example, $x + a = 180^\circ$.
- Exterior angles of polygons can be measured in a clockwise or anticlockwise direction.
- In a regular polygon, the size of the exterior angle can be found by dividing 360° by the number of sides.

$$\text{Exterior angle} = \frac{360^\circ}{n}$$

- The sum of the exterior angles of a polygon equals 360° .
- The exterior angle of a triangle is equal to the sum of the opposite interior angles.



RESOURCES – ONLINE ONLY

- Try out this interactivity: Angle sum of a polygon
Searchlight ID: int-0818
- Try out this interactivity: Exterior angles of a polygon
Searchlight ID: int-0819
- Complete this digital doc: WorkSHEET 10.3
Searchlight ID: doc-5284

Exercise 10.5 Polygons

Individual pathways

PRACTISE

Questions:
1–7, 9

CONSOLIDATE

Questions:
1–9, 12

MASTER

Questions:
1–12

Individual pathway interactivity: int-4615

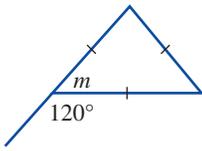
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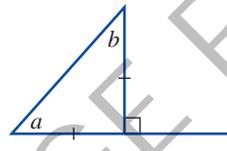
Fluency

- How are the internal and external angles of a polygon related to the number of sides in a polygon?
- WE6** Calculate the values of the pronumerals in the diagrams below.

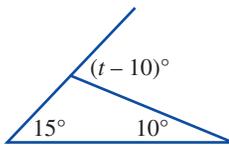
a.



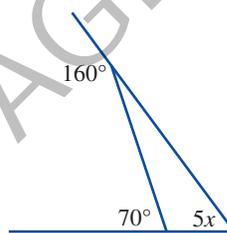
b.



c.

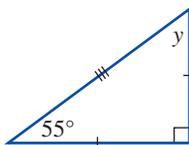


d.

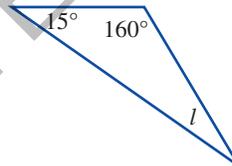


- For the five triangles below, evaluate the pronumerals and determine the size of the interior angles.

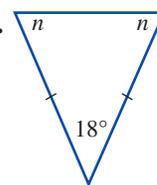
a.



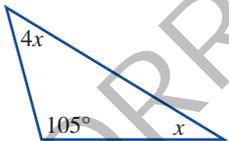
b.



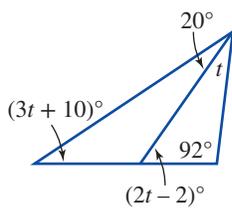
c.



d.

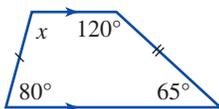


e.

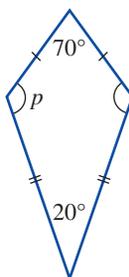


- For the five quadrilaterals below:
 - label the quadrilaterals as regular or irregular
 - determine the value of the pronumeral for each shape.

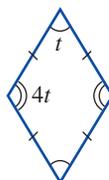
a.



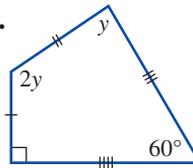
b.



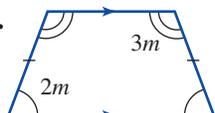
c.



d.



e.



Understanding

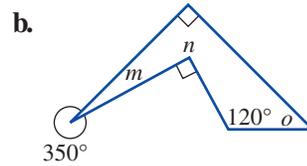
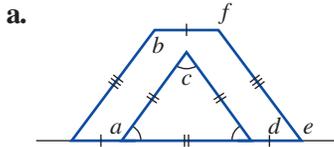
5. The photograph below shows a house built on the side of a hill. Use your knowledge of angles to calculate the values of the pronumerals.



6. Calculate the values of the four interior angles of the front face of the building in the photograph below.



7. Calculate the values of the pronumerals for the irregular polygons below.



8. Calculate the size of the exterior angle of a regular hexagon (6 sides).

Reasoning

9. A diagonal of a polygon joins two vertices.

a. Calculate the number of diagonals in a regular polygon with:

i. 4 sides

ii. 5 sides

iii. 6 sides

iv. 7 sides.

b. Write a formula that relates the number of diagonals for an n -sided polygon.

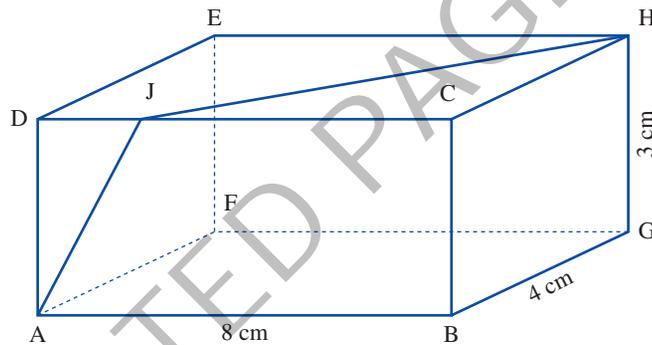
10. The external angle of a polygon can be calculated using the formula:

$$\text{exterior angle} = \frac{360^\circ}{n}$$

Use the relationship between internal and external angles of a polygon to write a formula for the internal angle of a regular polygon.

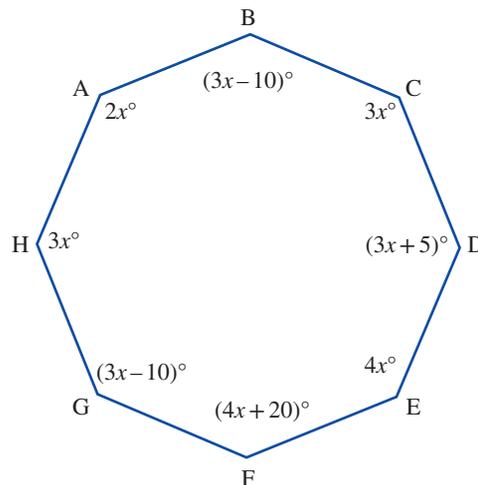
Problem solving

11.



A piece of string is fixed at A and H as shown. The string is tight and fixed to the surface of the cuboid. Locate the exact position of J on the edge CD.

12. ABCDEFGH is an octagon.



a. What is the sum of the interior angles of an octagon?

b. Find the value of x .

Reflection

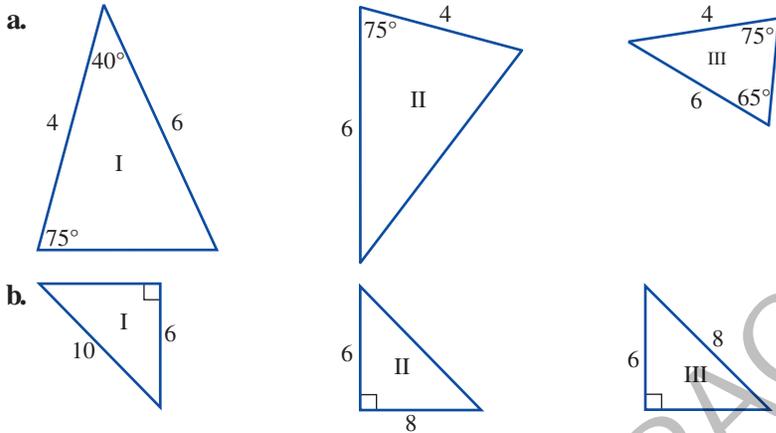
How are the angles associated with polygons related to each other and the polygon?

10.6 Review

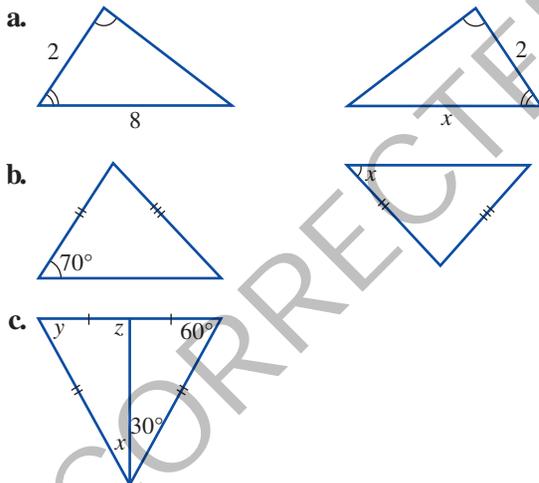
10.6.1 Review questions

Fluency

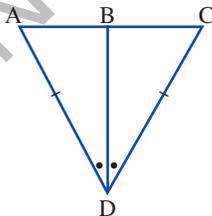
1. Select a pair of congruent triangles in each of the following sets of triangles, giving a reason for your answer. All angles are in degrees and side lengths in cm. (The figures are not drawn to scale.)



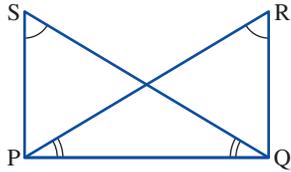
2. Find the value of the pronumeral in each pair of congruent triangles. All angles are given in degrees and side lengths in cm.



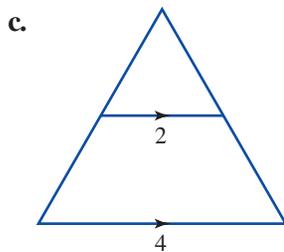
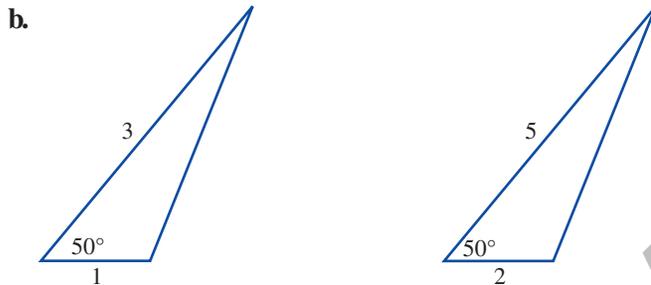
3. a. Prove that the two triangles shown in the diagram below are congruent.



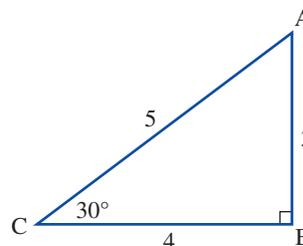
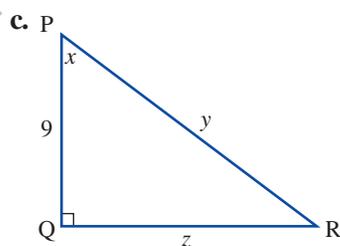
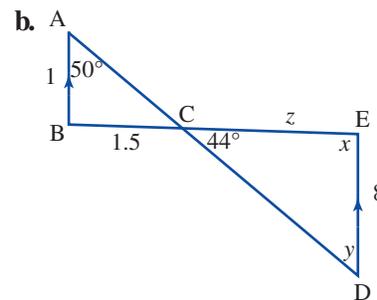
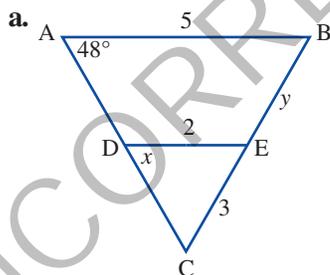
b. Prove that $\triangle PQR$ is congruent to $\triangle QPS$.



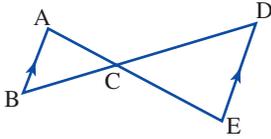
4. Test whether the following pairs of triangles are similar. For similar triangles find the scale factor. All angles are in degrees and side lengths in cm.



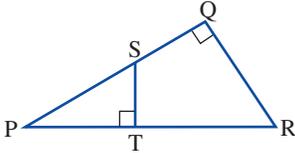
5. Find the value of the pronumeral in each pair of similar triangles. All angles are given in degrees and side lengths in cm.



6. Prove that $\triangle ABC \sim \triangle EDC$.



7. Prove that $\triangle PST \sim \triangle PRQ$.



8. Prove that the angles opposite the equal sides in an isosceles triangle are equal.

9. Two corresponding sides in a pair of similar octagons have lengths of 4 cm and 60 mm. The respective scale factor in length is:

a. 4 : 60

b. 6 : 40

c. 40 : 60

d. 60 : 40

10. A regular nonagon has side length x cm. Use a scale factor of $\frac{x+1}{x}$ to find the side length of a similar nonagon.

Problem solving

11. ABC is a triangle. D is the midpoint of AB, E is the midpoint of AC and F is the midpoint of BC. $DG \perp AB$, $EG \perp AC$ and $FG \perp BC$.

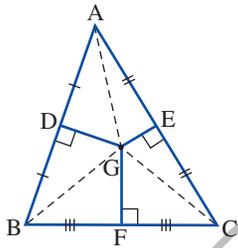
a. Prove that $\triangle GDA \cong \triangle GDB$.

b. Prove that $\triangle GDE \cong \triangle GCE$.

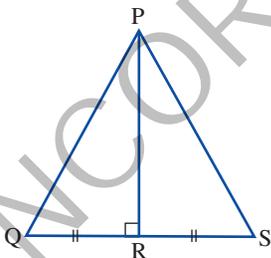
c. Prove that $\triangle GBF \cong \triangle GCF$.

d. What does this mean about AG, BG and CG?

e. A circle centred at G is drawn through A. What other points must it pass through?



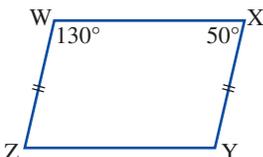
12. PR is the perpendicular bisector of QS. Prove that $\triangle PQS$ is isosceles.



13. Name any quadrilaterals that have diagonals that bisect the angles they pass through.

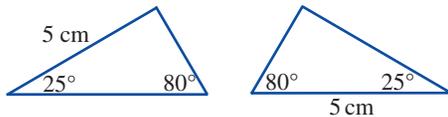
14. State three tests that can be used to show that a quadrilateral is a rhombus.

15. Prove that WXYZ is a parallelogram.



16. Prove that the diagonals in a rhombus bisect the angles they pass through.

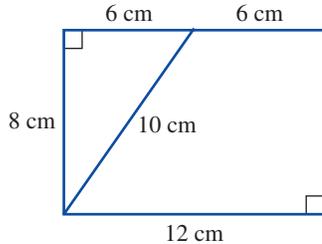
17. Explain why the triangles shown below are not congruent.



18. State the definition of a rhombus.

19. Name any quadrilaterals that have equal diagonals.

20. This 8 cm by 12 cm rectangle is cut into two sections as shown.



- a. Draw labelled diagrams to show how the two sections can be rearranged to form:
 - i. a parallelogram
 - ii. a right-angled triangle
 - iii. a trapezium.
- b. Comment on the perimeters of the figures.

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-  **Try out this interactivity:** Sudoku
Searchlight ID: int-3597
-  **Complete this digital doc:** Concept map
Searchlight ID: doc-13762

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

bisect
concave
congruency
convex
corresponding angles
corresponding sides

diagonal
exterior angle
interior angle
parallelogram
polygon
quadrilateral

rectangle
reflex
rhombus
similarity
square
trapezium



Investigation | Rich task

Enlargement activity

Enlargement is the construction of a bigger picture from a small one. The picture is identical to the other except that it is bigger. The new picture is often called the image. This can also be called creating a similar figure.



The geometrical properties shared by a shape and its image under enlargement can be listed as:

- lines are enlarged as lines
- sides are enlarged to corresponding sides by the same factor
- matching angles on the two shapes are equal.

In this activity, we will start with a small cartoon character, and then ‘blow it up’ to almost life-size.

Equipment: ruler, pencil, cartoon print, butcher’s paper or some other large piece of paper.

1. Do some research on the internet and select a cartoon character or any character of your choice.
2. Draw a grid of 2-cm squares over the small cartoon character.
Example: The small Casper is 9 squares wide and 7 squares tall.
3. Label the grids with letters across the top row and numbers down the first column, as shown in the example.
4. Get a large piece of paper and draw the same number of squares. You will have to work out the ratio of similitude (e.g. 2 cm : 8 cm).



5. If your small cartoon character stretches from one side of the 'small' paper (the paper the image is printed on) to the other, your 'large' Casper must stretch from one side of the 'big' paper to the other. Your large grid squares may have to be 8 cm by 8 cm or larger, depending on the paper size.
6. Draw this enlarged grid on your large paper. Use a metre ruler or some other long straight-edged tool. Be sure to keep all of your squares the same size.
 - At this point, you are ready to DRAW. Remember, you do NOT have to be an artist to produce an impressive enlargement.
 - All you do is draw EXACTLY what you see in each small cell into its corresponding large cell.
 - For example, in cell B3 of the Casper enlargement, you see the tip of his finger, so draw this in the big grid.
 - If you take your time and are very careful, you will produce an extremely impressive enlargement.
 - What you have used is called a RATIO OF SIMILITUDE. This ratio controls how large the new picture will be.

A 2 : 5 ratio will give you a smaller enlargement than a 2 : 7 ratio, because for every two units on the original you are generating only 5 units of enlargement instead of 7.

If Casper's ratio is 1 : 4, it produces a figure that has a linear measure that is four times bigger.

Big Casper's overall **area**, however, will be **16 times larger** than small Casper's. This is because area is found by taking length times width.

The length is 4 times longer and the width is 4 times longer.

Thus the **area** is $4 \times 4 = 16$ times **larger** than the original Casper.

His overall **volume** will be $4 \times 4 \times 4$ or **64 times larger!** This means that big Casper will weigh 64 times more than small Casper.



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Complete this digital doc: Code puzzle: Why was the archaeologist upset?
Searchlight ID: doc-15935

Answers

Topic 10 Deductive geometry

Exercise 10.2 Angles, triangles and congruence

- a. $a = 56^\circ$ b. $b = 30^\circ$ c. $c = 60^\circ$ d. $d = 120^\circ$ e. $e = 68^\circ$

a. I and III, SAS b. I and II, AAS c. II and III, RHS d. I and II, SSS
- a. $x = 6$, $y = 60^\circ$ b. $x = 80^\circ$, $y = 50^\circ$ c. $x = 32^\circ$, $y = 67^\circ$ d. $x = 45^\circ$, $y = 45^\circ$

a. $x = 3$ cm b. $x = 85^\circ$ c. $x = 80^\circ$, $y = 30^\circ$, $z = 70^\circ$

d. $x = 30^\circ$, $y = 7$ cm e. $x = 40^\circ$, $y = 50^\circ$, $z = 50^\circ$, $m = 90^\circ$, $n = 90^\circ$
- a. Use SAS b. Use SAS. c. Use ASA. d. Use ASA. e. Use SSS.
- C, D
- a. $x = 110^\circ$, $y = 110^\circ$, $z = 4$ cm, $w = 7$ cm b. $x = 70^\circ$

c. $x = 30^\circ$, $y = 65^\circ$
- The third sides are not necessarily the same.
- Corresponding sides are not the same.
- Use SSS.
- 11, 12 Check with your teacher.
13. Check with your teacher.
- $x = 20^\circ$, $y = 10^\circ$ and $z = 40^\circ$

Exercise 10.3 Similar triangles

- a. i and iii, RHS b. i and ii, SAS c. i and iii, SSS d. i and iii, AAA e. i and ii, SSS
- a. Triangles PQR and ABC b. Triangles ADB and ADC c. Triangles PQR and TSR

d. Triangles ABC and DEC e. Triangles ABC and DEC
- a. $\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$ b. $f = 9$, $g = 8$
- $x = 4$
- $x = 20^\circ$, $y = 2\frac{1}{4}$
- a. AAA b. $x = 3$, $y = 4$
- The slide is 7.23 m long.
- Radius = 0.625 m
- a. $x = 1$, $y = 7\sqrt{2}$ b. $x = 2.5$, $y = 3.91$
- Check with your teacher. One method would be to use Pythagorean triples.
- a. $\angle ABD = \angle ABC$ (common angle)
 $\angle ADB = \angle BAC = 90^\circ$
 $\triangle ABD \sim \triangle ACB$ (AAA)

b. $\angle ACD = \angle BCA$ (common angle)
 $\angle ADC = \angle CAB = 90^\circ$
 $\triangle ACD \sim \triangle ACB$ (AAA)
- Congruent triangles must be identical, that is, the angles must be equal and the side lengths must be equal. Therefore, it is not enough just to prove that the angles are equal.
- Check with your teacher.
- $\angle FEO = \angle OGH$ (alternate angles equal as $EF \parallel HG$)
 $\angle EFO = \angle OHG$ (alternate angles equal as $EF \parallel HG$)
 $\angle EOF = \angle HOG$ (vertically opposite angles equal)
 $\therefore \triangle EFO \sim \triangle GHO$ (equiangular)
- $x = 6$ or 11

Challenge 10.1

3

Exercise 10.4 Quadrilaterals

- a. True b. True c. True d. False e. False f. False g. False h. False
- a. $x = 36^\circ$, $y = 62^\circ$ b. $x = 5$ cm, $y = 90^\circ$ c. $x = 10^\circ$, $y = 70^\circ$ d. $x = 40^\circ$, $y = 60^\circ$
- None are true, unless the trapezium is a regular trapezium, then e is true.
- a, c, f

5. Parallelogram, rhombus, rectangle, square

6. Square

7. a.  b. 6 sides c. 7 sides

d.

Table size	Number of sides hit
5 cm × 3 cm	6
7 cm × 2 cm	7
4 cm × 3 cm	5
4 cm × 2 cm	1
6 cm × 3 cm	1
9 cm × 3 cm	2
12 cm × 4 cm	2

e. If the ratio of the sides is written in simplest form then the pattern is $m + n - 2$.

f. There are two routes for the ball when hit from B. Either 2 or 3 sides are hit. The ball does not end up in the same hole each time.

A suitable justification would be a diagram — student to draw.

g. Isosceles triangles and parallelograms. The triangles are congruent.

h. The shapes formed are parallelograms. There is only one possible path although the ball could be hit in either of two directions initially.

i. Given $m:n$ is the ratio length to width in simplest form. When m is even and n is odd the destination pocket will be the upper left. When m and n are both odd, the destination pocket will be the upper right. When m is odd and n is even the destination pocket will be the lower right.

j. Students to investigate.

8. Check with your teacher.

9. $AX \parallel DY$ because ABCD is a parallelogram

$AX = DY$ (given)

\therefore AXDY is a parallelogram since opposite sides are equal and parallel.

10. a. Use SAS. b. Use SAS. c. Opposite sides are equal.

11. $AC = DB$ (diameters of the same circle are equal)

$AO = OC$ and $OD = OB$ (radii of the same circle are equal)

\therefore ABCD is a rectangle. (Diagonals are equal and bisect each other.)

12. Check with your teacher.

13. $PS = QR$ (corresponding sides in congruent triangles are equal)

$PS \parallel QR$ (alternate angles are equal)

\therefore PQRS is a parallelogram since one pair of opposite sides are parallel and equal.

14. $MP = MQ$ (radii of same circle)

$PN = QN$ (radii of same circle) and circles have equal radii.

\therefore All sides are equal.

\therefore PNQM is a rhombus.

15. Check with your teacher.

16. a. One pair of opposite sides are parallel.

b. $x = 90^\circ$, $y = 75^\circ$

17. a. i. $x = \sqrt{41}$ ii. $y = \sqrt{97}$

b. $\angle BAD = \angle BCD = 117^\circ 23'$

18. a. Rhombus, 2 cm

b. Trapezium

c. 40°

d. Triangle BFC

e. Check with your teacher.

f. Check with your teacher.

g. $x = (1 + \sqrt{5})$ cm

19. 70°

Challenge 10.2

$x = \sqrt{10}$ cm

Exercise 10.5 Polygons

1. The sum of the interior angles is based on the number of sides of the polygon.

The size of the exterior angle can be found by dividing 360° by the number of sides.

2. a. $m = 60^\circ$

b. $a = 45^\circ$, $b = 45^\circ$

c. $t = 35^\circ$

d. $x = 10^\circ$

3. a. $y = 35^\circ$ b. $t = 5^\circ$ c. $n = 81^\circ$ d. $x = 15^\circ$ e. $t = 30^\circ$
 4. a. i. Irregular ii. $x = 95^\circ$ b. i. Irregular ii. $p = 135^\circ$
 c. i. Irregular ii. $t = 36^\circ$ d. i. Irregular ii. $y = 70^\circ$
 e. i. Irregular ii. $p = 36^\circ$
 5. $w = 75^\circ$, $x = 105^\circ$, $y = 94^\circ$, $z = 133^\circ$
 6. 82.5° , 82.5° , 97.5° , 97.5°
 7. a. $a = 120^\circ$, $b = 120^\circ$, $c = 60^\circ$, $d = 60^\circ$, $e = 120^\circ$, $f = 240^\circ$
 b. $m = 10^\circ$, $n = 270^\circ$, $o = 50^\circ$
 8. 60°
 9. a. i. 2 ii. 5 iii. 9 iv. 14

b. Number of diagonals = $\frac{1}{2}n(n - 3)$

10. Internal angle = $180^\circ - \frac{360^\circ}{n}$

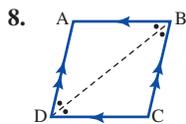
11. J is $\frac{24}{7}$ cm from D.

12. a. 1080° b. 43°

10.6 Review

Fluency

1. a. a I and III, ASA or SAS b. I and II, RHS
 2. a. $x = 8$ cm b. $x = 70^\circ$ c. $x = 30^\circ$, $y = 60^\circ$, $z = 90^\circ$
 3. a. Use SAS. b. Use ASA.
 4. a. Similar, scale factor = 1.5 b. Not similar c. Similar, scale factor = 2
 5. a. $x = 48^\circ$, $y = 4.5$ cm b. $x = 86^\circ$, $y = 50^\circ$, $z = 12$ cm c. $x = 60^\circ$, $y = 15$ cm, $z = 12$ cm
 6. Use the equiangular test.
 7. Use the equiangular test.



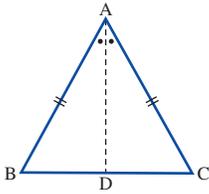
Bisect $\angle BAC$
 $AB = AC$ (given)
 $\angle BAD = \angle DAC$
 AD is common.
 $\therefore \triangle ABD \cong \triangle ACD$ (SAS)
 $\therefore \angle ABD = \angle ACD$ (corresponding sides in congruent triangles are equal)

9. C
 10. $x + 1$

Problem solving

11. a. Use SAS. b. Use SAS. c. Use SAS.
 d. They are all the same length. e. B and C
 12. Use SAS.
 $PQ = PS$ (corresponding sides in congruent triangles are equal)
 13. Rhombus, square
 14. A quadrilateral is a rhombus if:
 a. all sides are equal
 b. the diagonals bisect each other at right angles
 c. the diagonals bisect the angles they pass through.
 15. $WZ \parallel XY$ (co-interior angles are supplementary) and $WZ = XY$ (given)
 $\therefore WXYZ$ is a parallelogram since one pair of sides is parallel and equal.

16.



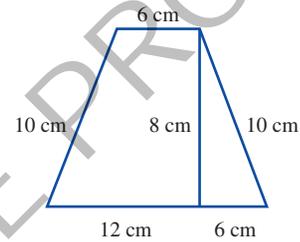
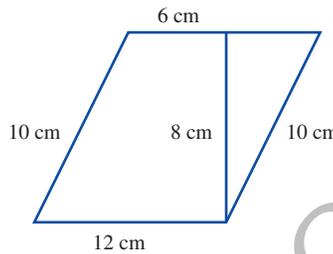
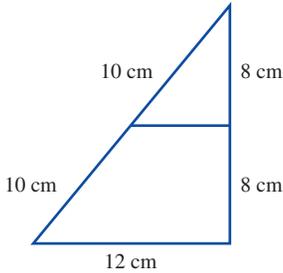
$\angle ABD = \angle ADB$ (angles opposite the equal sides in an isosceles triangle are equal)
 $\angle ABD = \angle BDC$ (alternate angles equal as $AB \parallel DC$)
 $\therefore \angle ADB = \angle BDC$
 \therefore Diagonals bisect the angles they pass through.

17. Corresponding sides are not the same.

18. A rhombus is a parallelogram with two adjacent sides equal in length.

19. Rectangle, square

20. a. i.



- b. Perimeter of rectangle = 40 cm
 Perimeter of parallelogram = 44 cm
 Perimeter of triangle = 48 cm
 Perimeter of trapezium = 44 cm
 The triangle has the largest perimeter, while the rectangle has the smallest.

Investigation – Rich task

Check with your teacher.

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ProjectsPLUS ICT Activity: Getting the budget in order

SEARCHLIGHT ID: PRO-0087

Scenario

Budgets are of great importance to everyone. From the federal budget down to the local council budget, they all impact on our lives. On a smaller scale, but of no less importance, is the family budget.

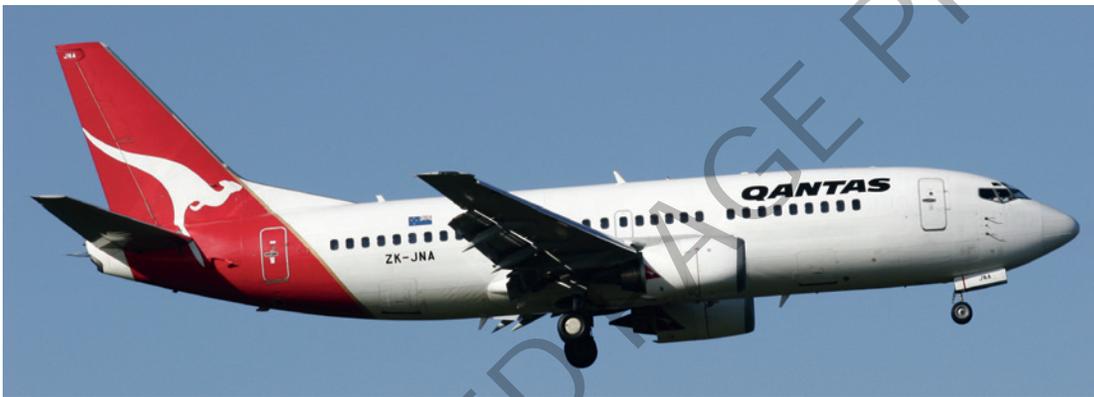
As a financial planner, you have been asked to prepare a budget for the Thompson family. The Thompson family lives in a major city in Australia and comprises Mr and Mrs Thompson and their two teenage children. They want to go on a holiday and they plan to save for a year. Taking their ongoing expenses into account, they want to see if and how they can afford this, following your recommendations.

You are to form a financial-planning group of three people. Each group member is to produce a budget plan that addresses the following holiday and ongoing expenses.

- Holiday: They want to be able to spend \$5000 on a holiday within Australia. Where can they go and for how long, taking flights, accommodation, meals and sightseeing expenses into account?

- Phone plans: Their two children have phone plans that each cost \$29 per month, currently paid by their parents. However, to help fund the holiday, their parents need them to pay back the \$29 per month, with either simple interest or compound interest charged. What is the best option for the children? How much do they need to work at their \$9.75-per-hour part-time jobs to cover the cost?
- Rent: The Thompson family lives in a rental house within 50 km from a major city. How much rent would they be paying?
- Taxes
- Utilities (electricity, water, gas, phone)
- Food
- Other expenses

Within the Thompson family, Mr Thompson takes care of the household duties and Mrs Thompson works in a salaried position, earning \$85 000 per annum. Their children have part-time jobs to repay the cost of their phone plans and to try and save for new clothes.



Task

You will produce an oral presentation and a written budget plan with recommendations for the Thompsons' family holiday. The budget plan will include the preferred method of interest charged to the children for the cost of their phone plan and the hours per week the children need to work at their part-time jobs to make this repayment. Your budget plan should also include value-for-money rentals located within 50 km of a major city, which will help the family assess how much they should be spending on rent. You also need to recommend a package holiday that includes flights and accommodation. Your presentation should explain why you have made each of your recommendations and give evidence to back up your decisions, taking all the expenses into account.

Process

- Open the ProjectsPLUS application for this chapter in your eBookPLUS. Watch the introductory video lesson, click the 'Start Project' button and then set up your project group. You can complete this project individually or invite other members of your class to form a group. Save your settings and the project will be launched.
- Navigate to your Research Forum. Here you will find a series of topics that will help you complete your task. Select the expenses you are researching or add new expenses you wish to include.
- Research. Make notes of important information and ideas that you discovered during your research. Enter your findings as articles under your topics in the Research forum. You should each find at least three sources of information (including off-line resources such as books and newspapers). You can view and comment on other group members' articles and rate the information they have entered.

When your research is complete, print your Research Report to hand in to your teacher.

- Visit your Media Centre and download the budget template and PowerPoint sample to help you prepare your presentation. Your Media Centre also includes images to help liven up your presentation.
- Use the budget template to give a clear overview of all the expenses taken into account and to give an overall summary of the whole family budget. Make sure you remember to address all the expenses that the Thompson family has requested you take into account.
- Use the PowerPoint template to develop your presentation. Remember that you are making recommendations that you believe are best for the Thompson family. Make sure you cover all the details they that have requested, and that your presentation will grab their attention.



SUGGESTED SOFTWARE

- ProjectsPLUS
- Microsoft Word
- PowerPoint, Prezi, Keynote or other presentation software
- Microsoft Excel
- CAS calculator (optional)