

TOPIC 2

Integers

2.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

2.1.1 Why learn this?

Integers are whole numbers that can be positive or negative. There are many examples in everyday life where an understanding of integers is useful, such as money transactions and temperatures. A number line gives a picture of the world of numbers: positive, zero and negative. Many problems can be worked out using a number line.



2.1.2 What do you know?

assessment

1. **THINK** List what you know about positive and negative integers. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of positive and negative integers.

LEARNING SEQUENCE

- 2.1 Overview
- 2.2 Adding and subtracting integers
- 2.3 Multiplying integers
- 2.4 Dividing integers
- 2.5 Combined operations on integers
- 2.6 Review

2.2 Adding and subtracting integers

2.2.1 Integers

- **Integers** are positive whole numbers, negative whole numbers and zero.
- The group of integers is often referred to as the set Z .
- $Z = \{\dots -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$

2.2.2 Addition of integers

- A number line can be used to add integers.
- To add a positive integer, move to the right.
- To add a negative integer, move to the left.

WORKED EXAMPLE 1

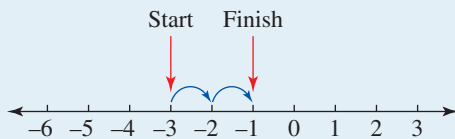
Calculate the value of each of the following.

a $-3 + +2$

b $-3 + -2$

THINK

- a 1** Start at -3 and move 2 units to the right, as this is the addition of a positive integer.



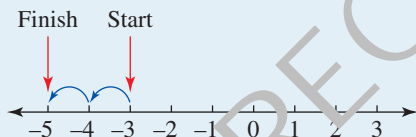
- 2** Write the answer.

WRITE

$-3 + +2$

$-3 + +2 = -1$

- b 1** Start at -3 and move 2 units to the left, as this is the addition of a negative integer.



- 2** Write the answer.

$-3 + -2$

$-3 + -2 = -5$

2.2.3 Subtraction of integers

- Subtracting an integer gives the same result as adding its opposite. For example, $-3 - 5 = -3 - +5 = -3 + -5 = -8$. Note that $+5$ and -5 are opposites.
- By developing and extending a pattern, we can show that subtracting a negative has the same effect as adding a positive. Look at the pattern shown at right. It can be seen from the table that subtracting a negative is the same as adding its inverse. For example, $8 - -4 = 8 + +4 = 12$.
- In mathematics, a number without a positive or negative sign is considered to be positive. So $8 + +4$ can be written as $8 + 4$ and $-5 - +1$ can be written as $-5 - 1$.

$8 - 3 =$	5
$8 - 2 =$	6
$8 - 1 =$	7
$8 - 0 =$	8
$8 - -1 =$	9
$8 - -2 =$	10
$8 - -3 =$	11

WORKED EXAMPLE 2

Calculate the value of each of the following.

a $-7 - +1$

b $-2 - -3$

THINK

- a 1** Subtracting an integer gives the same result as adding its opposite.
- 2** Using a number line, start at -7 and move 1 unit to the left.
- 3** Write the answer.
- b 1** Subtracting an integer gives the same result as adding its opposite.
- 2** Using a number line, start at -2 and move 3 units to the right.
- 3** Write the answer.

WRITE

a $-7 - +1$

$$= -7 + -1$$

$$= -8$$

b $-2 - -3$

$$= -2 + +3$$

$$= +1$$

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Watch this eLesson: Directed numbers

Searchlight ID: eles-0040



Complete this digital doc: SkillSHEET: Integers on the number line

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Complete this digital doc: SkillSHEET: Adding and subtracting integers

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Try out this interactivity: Directed number target

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Exercise 2.2 Adding and subtracting integers

assesson

Individual pathways

■ PRACTISE

Questions:

1–3, 4–d, 5a–d, 6a–i, 11, 12, 17, 18

■ CONSOLIDATE

Questions:

1–6, 7a, b, 9–14, 17, 18

■ MASTER

Questions:

3a–h, 4e–h, 5e–h, 6k–r, 7c, d, 8–11, 13–19

■ ■ ■ Individual pathway interactivity: int-4397

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Fluency

1. Which of the following numbers are integers?

$3, \frac{1}{2}, -4, 201, 20.1, -4.5, -62, -3\frac{2}{5}$

2. Copy and complete the following addition and subtraction number patterns by placing the correct integers in the boxes.

a. 6, 4, 2, , ,

b. -5, -10, -15, , ,

c. , , , -1, -3, -5

d. , , , -2, 0, 2

3. **WE1** Calculate the value of each of the following.

a. $-3 + 2$

b. $-7 + -3$

c. $6 + -7$

d. $-8 + -5$

e. $13 + +6$

f. $12 + -5$

g. $-25 + +10$

h. $16 + -16$

4. **WE2** Calculate the value of each of the following.

a. $7 - +2$

b. $-18 - +6$

c. $3 - +8$

d. $11 - +6$

e. $17 - -9$

f. $-28 - -12$

g. $14 - -8$

h. $-17 - -28$

5. Calculate the value of each of the following.

a. $-3 + -5$

b. $6 - -5$

c. $-17 + +3$

d. $-14 - -13$

e. $28 - -23$

f. $-48 + -3$

g. $-57 - -18$

h. $-32 - -40$

6. Simplify the following.

a. $-4 + -3$

b. $-6 - +3$

c. $5 + -2$

d. $17 - +5$

e. $-13 - -3$

f. $10 - -3$

g. $-26 + -14$

h. $25 + -7$

i. $32 - +5$

j. $-16 - +18$

k. $-26 - -15$

l. $124 - -26$

m. $-3 + -4 - -6$

n. $27 + -5 - -3$

o. $-10 + +3 - +6$

p. $23 + -15 - -14$

q. $15 - -4 + -10$

r. $-37 - -5 - -10$

Understanding

7. Copy and complete the following tables. For the subtraction table, subtract the number on the side from the number at the top.

a.

+	-8	+25	-18	32
-6	$-8 + -6 = -14$			
-13				
-16				
-19				

b.

-	+15	-17	-27	57
+7	$+15 - +7 = 8$			
-6				
-9				
+12				

c.

+	-11		13	
	-16			
+17		36		
		18	12	
-28				-35

d.

-	+9			42
	-17			
-14			-1	
		23		
+23		-2		

8. Design your own tables for addition and subtraction of integers like those in question 7. Fill in all answers in your tables.

9. In a kitchen, some food is stored at $-18\text{ }^{\circ}\text{C}$ in a freezer and some at $4\text{ }^{\circ}\text{C}$ in the fridge. A roast is cooking in the oven at a temperature of $180\text{ }^{\circ}\text{C}$.

Before answering each of the following questions, draw a number line to show the positions of the temperatures.

- What is the difference in temperature between the food stored in the freezer and the food stored in the fridge?
(Hint: Difference = largest value $-$ smallest value)
 - What is the difference in temperature between the food stored in the fridge and the roast cooking in the oven?
 - What is the difference in temperature between the food stored in the freezer and the roast cooking in the oven?
10. Calculate the difference between the two extreme temperatures recorded at Mawson Station in Antarctica in recent times.



11. Locate the button on your calculator that allows you to enter negative numbers. Use it to answer the following.

a. $-458 + 157$

b. $-5487 - 476$

c. $-248 - -658 - -120$

d. $-42 + 57 - -68 + -11$

12. Write out these equations, filling in the missing numbers.

a. $-7 + \square = 6$

b. $8 - \square = 12$

c. $-15 - \square = -26$

d. $42 - \square + -17 = 35$

e. $-7 - \square - -31 = -28$

f. $\square - 13 + 21 = 79$

13. The following is a homework sheet done by a student in Year 8. Correct her work for her and give her a mark out of six. Make sure you include the correct answer if her answer is wrong.

- a. $-3 + -7 = -10$
- b. $-4 - -10 = -6$
- c. $-7 - 8 = 15$
- d. $9 - -8 + -7 = 10$
- e. $42 + 7 - -11 = 60$
- f. $-17 + 4 - 8 = 23$



Reasoning

14. Evaluate and compare the following pairs of expressions.

- a. $-4 + 1$ and $+1 - 4$
- b. $-7 + 5$ and $+5 - 7$
- c. $-8 + 3$ and $+3 - 8$

15. What did you notice about the answers in question 14? Use a number line to help you explain why this is the case.

Problem solving

16. Evaluate and compare the following pairs of expressions.

- a. $-2 + -5$ and $-(2 + 5)$
- b. $-3 + -8$ and $-(3 + 8)$
- c. $-7 + -6$ and $-(7 + 6)$

17. What did you notice about the answers in question 16? Explain why this is the case.

18. Igor visited the Macau Casino. By using the following information and writing a loss as a negative number and a win as a positive number, find out how much money he had when he arrived at the casino. The currency used in the casino is the Hong Kong dollar (HKD).

Arriving at the casino at 10 pm, Igor went straight to the baccarat table, where he won 270 HKD. The roulette wheel that he played next was a disaster, costing 340 HKD in a very short space of time.

Igor moved to the 'vingt-et-un' table where he cleaned up, making 1450 HKD after a run of winning hands.

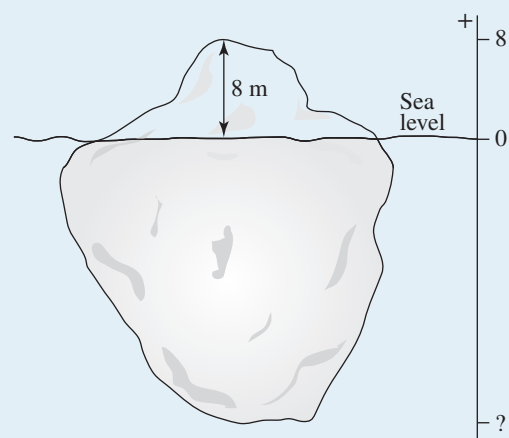
Little did Igor know that when he sat down at the stud poker table it would signal the end of his night at the casino. He lost everything in one hand, 2750 HKD, not even leaving enough for a taxi to his hotel.

Reflection

What strategy will you use to remember how to add and subtract integers?

CHALLENGE 2.1

The tip of this iceberg is 8 metres above sea level. If one-tenth of its total height is above the surface, represent the depth of its lowest point as an integer.



2.3 Multiplying integers

- Patterns in the answers in multiplication tables can be used to determine the product when two directed numbers are multiplied. Consider the following patterns.

$3 \times 3 = 9$	$-3 \times 3 = -9$
$3 \times 2 = 6$	$-3 \times 2 = -6$
$3 \times 1 = 3$	$-3 \times 1 = -3$
$3 \times 0 = 0$	$-3 \times 0 = 0$
$3 \times -1 = -3$	$-3 \times -1 = 3$
$3 \times -2 = -6$	$-3 \times -2 = 6$
$3 \times -3 = -9$	$-3 \times -3 = 9$
Answers go down by 3.	Answers go up by 3.

- When multiplying two directed numbers:
 - if their signs are the same, the answer is positive
 - if their signs are different, the answer is negative.

$+\times+=+$	$+\times-=-$
$-\times- = +$	$-\times+=-$

WORKED EXAMPLE 3

Evaluate each of the following.

a $-3 \times +7$

b -8×-7

THINK

a The two numbers have different signs, so the answer is negative ($7 \times 3 = 21$).

b The two numbers have the same signs, so the answer is positive ($8 \times 7 = 56$).

WRITE

a $-3 \times +7$
 $= -21$

b -8×-7
 $= 56$ (or $+56$)

2.3.1 Powers and square roots of directed numbers

- Calculating powers of negative numbers uses the same process as calculating powers of positive numbers.
- There are two possible answers when you take the square root of a number. For example:

$$\begin{aligned} 4^2 &= 4 \times 4 \\ &= 16 \end{aligned}$$

$$\begin{aligned} (-4)^2 &= (-4) \times (-4) \\ &= 16 \end{aligned}$$

Therefore when asked to take the square root of 16, the answer could be $\pm\sqrt{16}$. $\pm\sqrt{16} = -4$ or $+4$, which can also be written as ± 4 (positive or negative 4).

WORKED EXAMPLE 4

Evaluate each of the following.

a $(-5)^3$

b The square root of 64

THINK


- a 1** Write the question in expanded form.
- 2** Evaluate by working from left to right beginning with $-5 \times -5 = +25$.
- b** Look for the numbers that, when squared, result in 64 ($8 \times 8 = 64$ and $-8 \times -8 = 64$).

WRITE

$$\begin{aligned} \mathbf{a} \quad (-5)^3 &= (-5) \times (-5) \times (-5) \\ &= +25 \times (-5) \\ &= -125 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad \pm\sqrt{64} &= +8 \text{ or } -8 \\ &= \pm 8 \end{aligned}$$

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Searchlight ID: doc-6390

 Complete this digital doc: WorkSHEET 2.1
Searchlight ID: doc-6400

Exercise 2.3 Multiplying integers**assessment**

Individual pathways

PRACTISE

Questions:

1–3, 4a–c, 5a–d, 7, 8a–c, 9, 13, 15, 16

CONSOLIDATE

Questions:

2, 3, 4a–f, 5a–f, 7, 8a–d, 9, 10, 12, 13, 15, 16

MASTER

Questions:

2, 3c–e, 4f–i, 5f–i, 6–14, 16, 17

 Individual pathway interactivity: int-4098**learnon** ONLINE ONLY

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Fluency

- 1.**
- Copy and complete the following tables.

$4 \times 4 = \underline{\quad}$	$-5 \times 4 = \underline{\quad}$	$-6 \times -4 = \underline{\quad}$
$4 \times 3 = \underline{\quad}$	$-5 \times 3 = \underline{\quad}$	$-6 \times -3 = \underline{\quad}$
$4 \times 2 = \underline{\quad}$	$-5 \times 2 = \underline{\quad}$	$-6 \times -2 = \underline{\quad}$
$4 \times 1 = \underline{\quad}$	$-5 \times 1 = \underline{\quad}$	$-6 \times -1 = \underline{\quad}$
$4 \times 0 = \underline{\quad}$	$-5 \times 0 = \underline{\quad}$	$-6 \times 0 = \underline{\quad}$
$4 \times -1 = \underline{\quad}$	$-5 \times -1 = \underline{\quad}$	$-6 \times 1 = \underline{\quad}$
$4 \times -2 = \underline{\quad}$	$-5 \times -2 = \underline{\quad}$	$-6 \times 2 = \underline{\quad}$
$4 \times -3 = \underline{\quad}$	$-5 \times -3 = \underline{\quad}$	$-6 \times 3 = \underline{\quad}$
$4 \times -4 = \underline{\quad}$	$-5 \times -4 = \underline{\quad}$	$-6 \times 4 = \underline{\quad}$

- 2.**
- WE3**
- Evaluate each of the following.

a. -2×5	b. 3×-8	c. -6×-7	d. 2×-13
e. -8×-6	f. -7×6	g. -10×75	h. -115×-10
i. -7×9	j. $+9 \times -8$	k. -11×-5	l. 150×-2

- 3.**
- Use an appropriate method to evaluate the following.

a. $-2 \times 5 \times -8 \times -10$	b. $8 \times -1 \times 7 \times -2 \times 1$
c. $8 \times -4 \times -1 \times -1 \times 6$	d. $-3 \times -7 \times -2 \times -1 \times -1 \times -1$
e. $-5 \times -8 \times -2 \times -2$	

4. Complete the following equations.

a. $7 \times \underline{\quad} = -63$

b. $-3 \times \underline{\quad} = -21$

c. $16 \times \underline{\quad} = -32$

d. $\underline{\quad} \times -3 = 36$

e. $\underline{\quad} \times 7 = -42$

f. $\underline{\quad} \times -9 = -72$

g. $\underline{\quad} \times -4 = 80$

h. $-10 \times \underline{\quad} = 60$

i. $-11 \times \underline{\quad} = 121$

5. **WE4a** Evaluate each of the following.

a. $(-2)^3$

b. $(-3)^2$

c. $(-2)^4$

d. $(-3)^4$

e. $(-2)^5$

f. $(-4)^2$

g. $(-5)^3$

h. $(-4)^4$

i. $(-5)^4$

j. $(-6)^3$

6. Use your answers to question 5 to help complete the following statements.

a. If a negative number is raised to an even power, the answer is (positive/negative).

b. If a negative number is raised to an odd power, the answer is (positive/negative).

7. **WE4b** Evaluate the square root of the following numbers.

a. 25

b. 81

c. 49

d. 121

e. 100

8. If $a = -2$, $b = -6$, $c = 4$ and $d = -3$, calculate the values of the following expressions.

a. $a \times b \times c$

b. $a \times -b \times -d$

c. $b \times -c \times -d$

d. $c \times -a \times -a$

e. $d \times -(-c)$

f. $c \times d \times b \times c^2$

Understanding

9. For each of the following, write three possible sets of integers that can be placed in the boxes to make the equation a true statement.

a. $\square \times \square \times \square = -12$

b. $\square \times \square \times \square = 36$

c. $\square \times \square \times \square \times \square = -36$

10. For each of the following, determine whether the result is a positive or negative value. You do not have to work out the value.

a. $-25 \times 54 \times -47$

b. $-56 \times -120 \times -145$

c. $-a \times -b \times -c \times -d \times -e$

11. What happens when a number is multiplied by -1 ? Use some examples to illustrate your answer.

12. The notation $-(-3)$ is a short way of writing -1×-3 .

Write the expression represented by each of the following and then use an appropriate method to determine the answer.

a. $-(-2)$

b. $-(+3)$

c. $-(-5)$

d. $-(-(+5))$

e. $-(-(-7))$

f. $-(-(+4))$

Reasoning

13. Explain why the answer to this question is negative.

$$-2 \times -4 \times +3 \times -6 \times +4 \times +3$$

14. If the answer to $(-a)^n$, $n > 0$ is negative, establish whether a is positive or negative and whether n is odd or even. Give a reasoned explanation for your answer.

Problem solving

15. If $a = -3$ and $b = -4$, evaluate $a^3 \times b^2$.

16. Evaluate $(-1)^n \times (-1)^{n+1}$ if:

a. n is even

b. n is odd.

17. In a Year 12 Mathematics examination there are 30 multiple choice questions. A student scores 2 marks for a correct answer, -1 mark for an incorrect answer and zero marks for an unanswered question.

Mary scores a total of 33 marks in the multiple choice section. How could she have reached this total?

Reflection

Can we find square roots, cube roots, fourth roots and so on for negative numbers?

2.4 Dividing integers

- Division is the inverse operation of multiplication. We can use the multiplication facts for directed numbers to discover the division facts for directed numbers.

Multiplication fact	Division fact	Pattern
$2 \times 3 = 6$	$6 \div 3 = 2$ or $\frac{6}{3} = 2$ and $6 \div 2 = 3$ or $\frac{6}{2} = 3$	$\frac{\text{positive}}{\text{positive}} = \text{positive}$
$-2 \times -3 = 6$	$6 \div -3 = -2$ or $\frac{6}{-3} = -2$ and $6 \div -2 = -3$ or $\frac{6}{-2} = -3$	$\frac{\text{positive}}{\text{negative}} = \text{negative}$
$-2 \times 3 = -6$	$-6 \div 3 = -2$ or $\frac{-6}{3} = -2$ and $-6 \div -2 = 3$ or $\frac{-6}{-2} = 3$	$\frac{\text{negative}}{\text{positive}} = \text{negative}$ and $\frac{\text{negative}}{\text{negative}} = \text{positive}$

- When dividing two directed numbers:
 - if their signs are the same, the answer is positive
 - if their signs are different, the answer is negative.
- Remember that division statements can be written as fractions and then simplified. For example,

$$\begin{aligned}
 -12 \div -4 &= \frac{-12}{-4} \\
 &= \frac{12 \times -1}{4 \times -1} \\
 &= 3
 \end{aligned}$$

$\frac{+}{+} = +$	$\frac{+}{-} = -$
$\frac{-}{-} = +$	$\frac{-}{+} = -$

WORKED EXAMPLE 5

Evaluate each of the following.

a $-56 \div 8$

THINK

a The two numbers have different signs, so the answer is negative.

b Cancel the common factors (-1). The two numbers have the same signs, so the answer is positive.

WRITE

a $-56 \div 8$
 $= -7$

b $\frac{-36}{-9} = \frac{-1 \times 36}{-1 \times 9}$
 $= \frac{36}{9}$
 $= 4$

WORKED EXAMPLE 6

Evaluate the following.

a $234 \div -6$

b $-182 \div -14$

THINK

a 1 Complete the division as if both numbers were positive numbers.

2 Determine the sign of the answer. A positive number divided by a negative number is a negative number.

b 1 Complete the division as if both numbers were positive numbers.

2 Determine the sign of the answer. A negative number divided by a negative number is a positive number.

WRITE

a
$$\begin{array}{r} 39 \\ 6 \overline{)2354} \end{array}$$

$$234 \div -6 = -39$$

b
$$\begin{array}{r} 13 \\ 14 \overline{)182} \\ \underline{14} \\ 42 \\ \underline{42} \\ 0 \end{array}$$

$$-182 \div -14 = 13$$

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Exercise 2.4 Dividing integers

assesson

Individual pathways

PRACTISE

Questions:

1, 2, 3a-f, 5, 6, 7a-c, 8a-c, 13, 16

CONSOLIDATE

Questions:

1a-h, 2, 3a-i, 5a-f, 6, 7a-d, 8, 9a, 10a, 11, 13, 15, 16

MASTER

Questions:

1g-l, 2, 3g-l, 4, 5c-h, 6, 7c-f, 8c-f, 9-12, 14, 15

Individual pathway interactive activity: int-4399

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Fluency

1. WE5a Evaluate each of the following.

a. $-63 \div 9$

b. $8 \div -2$

c. $-8 \div 2$

d. $-6 \div -1$

e. $88 \div -11$

f. $0 \div -5$

g. $48 \div -3$

h. $-129 \div 3$

i. $-56 \div -7$

j. $+184 \div -4$

k. $-55 \div -11$

l. $304 \div -8$

2. WE5b Evaluate each of the following.

a. $\frac{-121}{-11}$

b. $\frac{-12}{3}$

c. $\frac{-36}{-12}$

d. $\frac{21}{-7}$

e. $\frac{-100}{-50}$

f. $-3 \times \frac{2}{-3}$

3. WE6 Evaluate the following.

a. $960 \div -8$

b. $-243 \div 9$

c. $-266 \div -7$

d. $-132 \div -4$

e. $-282 \div 6$

f. $1440 \div -9$

g. $324 \div -12$
j. $-2294 \div -37$

h. $-3060 \div 17$
k. $4860 \div 15$

i. $-6000 \div -24$
l. $-5876 \div -26$

Understanding

4. Write three different division statements, each of which has an answer of -8 .
5. Copy and complete the following by placing the correct integer in the box.
- a. $-27 \div \square = -9$ b. $-68 \div \square = 34$ c. $72 \div \square = -8$
d. $-18 \div \square = -6$ e. $\square \div 7 = -5$ f. $\square \div -4 = -6$
g. $-132 \div \square = 11$ h. $-270 \div \square = 27$
6. Calculate the value of each of the following by working from left to right.
- a. $-30 \div 6 \div -5$ b. $-120 \div 4 \div -5$ c. $-800 \div -4 \div -5 \div 2$
7. If $a = -12$, $b = 3$, $c = -4$ and $d = -6$, calculate the value of each of the following expressions.
- a. $a \div c$ b. $a \div b$ c. $a \div d$
d. $b \div c$ e. $b \div d$ f. $a \div b \div d$
8. If $a = -24$, $b = 2$, $c = -4$ and $d = -12$, calculate the value of each of the following expressions.
- a. $a \div b \times c$ b. $d \times c \div b \div c$ c. $b \div c \div a \times a$
d. $c \times a \div d \div b$ e. $a \times b \div d \div d$ f. $a \div d \times c \div b$
9. Copy and complete the following tables:

a.

\times			-6	$+8$
			18	
-10		-40		
	10		30	
-7				-56

b.

\times			-9	
-6	30			-42
		36		
		-55	99	
		-6	-18	

10. Copy and complete the following tables. Divide the number on the top by the number on the side.

a.

\div	4	-10	12	8
-2				
7				
-3				
-10				

b.

\div				-4
		-2		
-8	-4	3		
6			-6	
				1

Reasoning

11. The answer to $\frac{p \times a}{2 \times -5}$ is negative. What can you deduce about p and q ?
12. The answer to $\frac{(-a)^2}{(-b)^3} \div (-c)^4$ is positive. What can you deduce about b ?

Problem solving

13. If $a = 2$ and $b = -6$, evaluate $\frac{(-a)^3}{(-b)^4}$.
14. Evaluate $\frac{(-1)^{n+1}}{(-1)^{n+2}}$, $n > 0$, if:
- a. n is even
b. n is odd.

15. *Acceleration* is a measure of a change in speed or velocity. If you are increasing speed, acceleration is positive; if you are decreasing speed (slowing down) acceleration is negative. (You will need to do some research to answer this question.)
- Is it possible to have positive acceleration with a negative velocity? If so, give an example.
 - Is it possible to have negative acceleration with a positive velocity? If so, give an example.



16. My last five scores in a very silly computer game have been $+6$, -9 , -15 , $+8$ and -4 . What is my average score?

Reflection

Can you list 4 areas in real life where directed numbers are used?

2.5 Combined operations on integers

- The mathematical rules about order of operations apply when we work with directed numbers.
- BIDMAS helps us to remember the correct order in which we should perform the various operations. This means we do the brackets (B), first; then indices (I); then division (D) and multiplication (M), working from left to right, and finally addition (A) and subtraction (S), working from left to right.

WORKED EXAMPLE 1

Calculate the value of each of the following.

a $54 \div -6 + 8 \times -9 \div -4$

b $-8 \div 2 + (-2)^3$

THINK



- Write the question.
 - There are no brackets or powers, so, working from left to right, complete all multiplication and division, then the addition.
 - Write the answer.
- Write the question.
 - Simplify the cubed term.
 - Complete the division.
 - Write the answer.

WRITE

a $54 \div -6 + 8 \times -9 \div -4$
 $= -9 + 18$

$= 9$

b $-8 \div 2 + (-2)^3$
 $= -8 \div 2 + -8$
 $= -4 + -8$
 $= -12$

-  Complete this digital doc: SkillSHEET: Order of Operations and directed numbers
Searchlight ID: doc-6392
-  Complete this digital doc: WorkSHEET 2.2
Searchlight ID: doc-6407

Exercise 2.5 Combined operations on integers

assessment

Individual pathways

PRACTISE

Questions:
1–3, 4a, 8

CONSOLIDATE

Questions:
1–3, 4a, b, 8, 9

MASTER

Questions:
1–7, 9

Individual pathway interactivity: int-4400

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Fluency

1. **WE7a** Calculate the values of the following expressions.

a. $-4 - 6 - 2$	b. $-4 \times 2 + 1$	c. $8 \div (2 - 4) + 1$
d. $7 - (3 - 1) + 4$	e. $6 \times (4 + 1)$	f. $-3 - 40 \div 8 + 2$
g. $-4 + 5 - 6 - 7$	h. $-5 \times 12 + 2$	i. $12 \div (2 - 4) - 6$
j. $13 - (4 - 6) + 2$	k. $7 \times (6 + 2)$	l. $-6 - 36 \div 9 + 3$
2. **WE7b** Evaluate each of the following.

a. $-7 + 6 \times -2$	b. $-9 - 15 + 3$	c. $(-63 \div -7) \times -3 + -2$
d. $(-3)^3 - 3 \times -5$	e. $-5 \times -7 - [5 + (-8)^2]$	f. $[(-48 \div 8)^2 \times 36] \div -4$
3. Calculate the values of the following expressions.

a. $-3 + 15 - 26 - 27$	b. $-8 \times 11 + 12$	c. $52 \div (-9 - 4) - 8$
d. $23 - (16 - 4) + 7 - 3$	e. $15 \times (-6 + 2)$	f. $-6 - 64 \div -16 + 8$
g. $-3 \times -4 \times -1 \times 5$	h. $-6 \times (-13 + 5) + -4 + 2$	

Understanding

4. A class of Year 7 students were given the following question to evaluate.

$$4 + 8 \div -(2)^2 - 7 \times 2$$
 - a. A number of different answers were obtained, including -8 , -12 and -17 . Which one of these is the correct answer?
 - b. Using only brackets, change the question in two ways so that the other two answers would be correct.
5. In a particular adventure video game, a player loses and gains points based on who or what they come in contact with during the game. See the list below right for the number of 'hit' points associated with each contact. Use the table to calculate the number of points the player has at the end of each round of the game.



Round number	Points at the start of the round	Contacts during the round	Points at the end of the round
1	100	20 gnomes, 10 goblins and 3 healing potions	
2		3 gnomes, 5 goblins, 6 orcs and 5 healing potions	
3		3 orcs, 6 trolls and a cleric	
4		5 trolls, 1 balrog and a cleric	

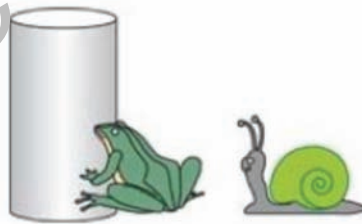
Character	'Hit' points
Balrog	-100
Troll	-10
Orc	-5
Goblin	-2
Gnome	-1
Healing potion	+20
Cleric	+50

Reasoning

6. The variables p and q have the same numerical value but the opposite sign; that is, one is positive and the other negative. If $-3 \times p + 4 \times q$ is positive, what can you say about p and q ?
7. Test your answer to question 6 if the numerical values of p and q , written as $|p|$ and $|q|$, are both equal to 7.

Problem solving

8. A frog and a snail are climbing an empty vertical pipe. The snail is 30 cm from the top of the pipe while the frog is 30 cm below the snail. At the start of the first hour, the frog climbs 40 cm up the pipe and rests. The snail crawls 20 cm during this time then also rests. While they are resting, both the frog and the snail slip back down the pipe. The frog slips back 20 cm while the snail slips back 10 cm. At the start of the second hour, the frog and the snail set off again and repeat the same process of climbing and resting until they reach the top of the pipe.



- a. Which reaches the top of the pipe first, the frog or the snail?
- b. During which hour do they reach the top of the pipe?
9. A classmate is recording the weather during July for a school project and wants help calculating information. He records the following data for one week.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Max (°C)	12.2	14.5	16.7	12.8	11.3	7.2	-0.3
Min (°C)	3.0	2.1	4.6	3.2	6.4	-2.9	-6.0

Round all answers correct to 1 d.p.

- a. What is the difference between the lowest temperature and the highest temperature recorded during this week?
- b. What is the average (mean) minimum temperature?

c. Your classmate tries to predict temperatures and says that the minimum temperature $= \frac{1}{4} \times$ maximum temperature.

He says this is the same as dividing by -4 . Explain why this may not be correct.

d. He then tries to make another prediction that involves taking the square root of the maximum temperature. Why might this not be a good idea?

e. On the Monday after this week, the temperature rises by 6.5°C from Sunday's minimum temperature. It then drops by 3.2°C overnight and rises by 8.9°C on the Tuesday. What is the minimum temperature on the Monday night and the maximum temperature on the Tuesday?

f. His last prediction involves subtracting 6 from the maximum temperature, then dividing by 2 to predict the minimum temperature. Calculate the predicted values for the minimum temperature for each of his three methods and discuss which method may be most accurate.



Reflection

What effects do directed numbers have on order of operations?

CHALLENGE 2.2

Arrange numbers in this magic square so that all rows, columns and diagonals sum to the same value. No numbers are used twice.

		2
3	-1	-5

2.6 Review

2.6.1 Review questions

Fluency

- True or false? The number -2.5 is called an integer.
- True or false? $-6 < -2$
- List the integers between -11 and -7 .
- Arrange these numbers in ascending order: $7, 0, -3, 10, -15$.
- Calculate the value of each of the following.

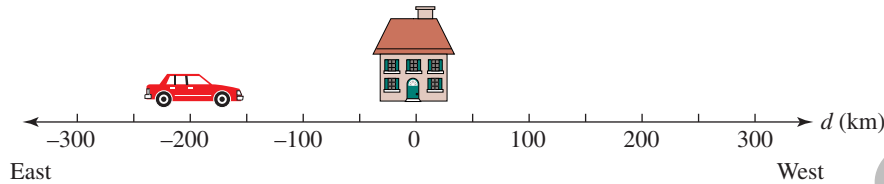
<ol style="list-style-type: none"> a. $-6 + -8$ c. $-3 - +7 + -2$ 	<ol style="list-style-type: none"> b. $16 - -5$ d. $-1 - -5 - +4$
---	---
- Write out the following equations and fill in the missing numbers.

<ol style="list-style-type: none"> a. $7 - \square = -14$ c. $\square - 13 - -12 = 10$ 	<ol style="list-style-type: none"> b. $-19 + \square = 2$ d. $-28 - \square = -17$
--	--

In science, directed numbers are often used to describe a direction or an increase or decrease in a measurement.

14. Directed numbers can describe the distance of an object from a reference point (known as the *displacement, d* , of the object). For example, if we are 200 km east of a town, and east is defined as a negative direction, we are -200 km from the town.

a. If a car travels 150 km in a westerly direction from -200 km, describe the displacement of the car from the town.



b. If a car travels from 300 km west of the town, describe the displacement of the car after it has travelled 450 km in a westerly direction.

15. Directed numbers can describe the direction that an object is travelling. For example, in question 6, travelling towards the west is travelling in a positive direction; towards the east is a negative direction. A car travelling east at 100 km/h goes at -100 km/h. Scientists use the term *velocity, v* , to mean a speed in a particular direction.

a. If a car travels past the town at -100 km/h, where will it be in 2 hours time?

b. If a car goes past the town travelling at -100 km/h, where was the car an hour ago?

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Try out this interactivity: Sudoku
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Complete this digital doc: Concept map
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Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

difference

directed number

integer

inverse operation

negative

notation

positive

power

product

quotient

square root

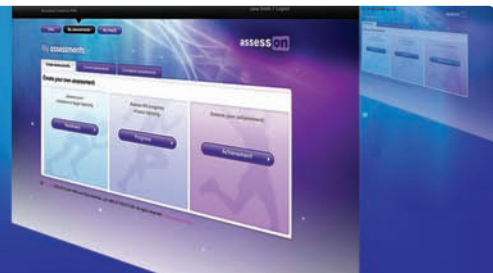
sum

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Investigation | Rich task

Directed numbers board game

Many board games engage the players in racing each other to the end of the board. Snakes and Ladders is an example of this style of game. You can climb ladders to get to the end quicker, but sliding down a snake means you get further away from the end.

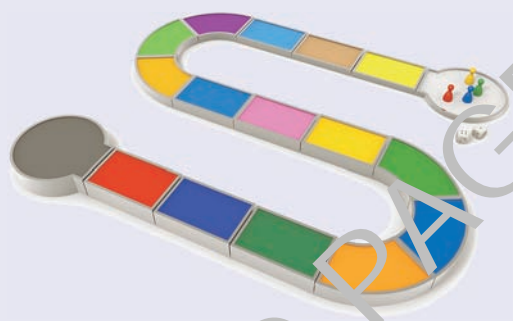
You are going to make a board game that will help you to practise addition and subtraction of directed numbers. This game is played with 2 dice and is a race to the end of the number line provided on the board. The diagram below shows part of a sample game, and can be used to explain the rules and requirements for your board game.

10	9	8	7	6		
					5	
					4	
-2	-1	0	1	2	3	
-3						
-4	-5	-6	-7	-8	-9	-10



Two dice are to be used in this board game. Die 1 is labelled with 3 N's and 3 P's, and die 2 is labelled with the integers -1 , -2 , -3 , $+1$, $+2$ and $+3$. (Stickers with these labels can be placed over two standard dice.) Rolling an N means you face the *negative* numbers; rolling a P means you face the *positive* numbers.

- Imagine that you are at 0, and that you roll an N and $+2$. N means that you face the negative numbers; $+2$ means that you move forward 2 places.
 - If you roll a P and $+1$, this means that you face the positive numbers and move forward 1 place.
 - If you roll a P and -2 , this means that you face the positive numbers and move backwards 2 places.
1. For the three examples listed above, state which square you would end up in if you started at 0 each time.
 2. Suppose that your first five turns at this game produced the following results on the dice. P and $+1$, P and $+3$, P and -2 , N and -1 , N and $+2$. If you started at 0, where were you after these five turns?



Your task is to design a board game similar to the one used in the example. The requirements for the game are listed below.

- The game is to be played with the two dice described earlier.
 - Two or more players are required, taking turns to throw the dice.
 - Start at 0 in the centre of the board. The race is on to get to either end of the board. The first person to reach an end is the winner.
 - For 0 to be at the centre, your board will require an odd number of columns.
 - At least 8 squares must have specific instructions — for example, ‘go back 5 squares’ or ‘miss a turn’. You may even wish to include ‘snakes and ladders’ style obstacles.
 - Use a sensible number of squares. If there are very few squares, the game will end too quickly; if there are many squares, they will be very small.
3. Work with a partner to make the two required dice and to design your board. Play with others to test it, and make necessary improvements if they are required. Be certain that all the requirements of the game are included.
 4. Present your game as a package that includes instructions and a list of contents. Include examples within your instructions to help players understand how to move throughout the game.

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Answers

Topic 2 Integers

Exercise 2.2 Adding and subtracting integers

1. 3, -4, 201, -62

2. a. 0, -2, -4

b. -20, -25, -30

c. 5, 3, 1

d. -8, -6, -4

3. a. -1 b. -10 c. -1 d. -13 e. 19

f. 7 g. -15 h. 0

4. a. 5 b. -24 c. -5 d. 5 e. 26

f. -16 g. 22 h. 11

5. a. -8 b. 11 c. -14 d. -1 e. 51

f. -51 g. -39 h. 8

6. a. -7 b. -9 c. 3 d. 12 e. -10

f. 13 g. -40 h. 18 i. 27 j. -34

k. -11 l. 150 m. -1 n. 25 o. -13

p. 22 q. 9 r. -22

7. a.

+	-8	25	-18	32
-6	-14	19	-24	26
-13	-21	12	-31	19
-16	-24	9	-34	16
-19	-27	6	-37	13

b.

-	15	-17	-27	57
7	8	-24	-34	50
-6	21	-11	-21	63
-9	24	-8	-18	66
12	3	-29	-39	45

c.

+	-11	19	13	-7
-5	-16	14	8	-12
17	6	36	30	10
-1	-12	18	12	-8
-28	-39	-9	-15	-35

d.

-	9	21	-15	4
26	-17	-5	-41	16
-14	23	35	-1	56
-2	11	23	-13	44
23	-14	-2	-38	19

8. Check with your teacher.

9. a. 22 °C

b. 176 °C

c. 198 °C

10. 37 °C

11. a. -301

b. -5963

c. 530

d. 72

12. a. 13

b. -4

c. 11

d. -10

e. 52

f. 71

13. a. Correct

b. Incorrect; 6

c. Incorrect; -15

d. Correct

e. Correct

f. Incorrect; -21

14. a. -3

b. -2

c. -5

15. The answers are the same.

16. a. $-2 + -5 = -7$; $-(2 + 5) = -7$

b. $-3 + -8 = -11$; $-(3 + 8) = -11$

c. $-7 + -6 = -13$; $-(7 + 6) = -13$

17. For each pair of expressions, the answers are the same: each number is negative, so putting brackets around the sum and then attaching a negative sign gives the same outcome.

18. 1370 HKD

Challenge 2.1

-72 metres

Exercise 2.3 Multiplying integers

1. $4 \times 4 = 16$ $-5 \times 4 = -20$ $-6 \times -4 = 24$
 $4 \times 3 = 12$ $-5 \times 3 = -15$ $-6 \times -3 = 18$
 $4 \times 2 = 8$ $-5 \times 2 = -10$ $-6 \times -2 = 12$
 $4 \times 1 = 4$ $-5 \times 1 = -5$ $-6 \times -1 = 6$
 $4 \times 0 = 0$ $-5 \times 0 = 0$ $-6 \times 0 = 0$
 $4 \times -1 = -4$ $-5 \times -1 = 5$ $-6 \times 1 = -6$
 $4 \times -2 = -8$ $-5 \times -2 = 10$ $-6 \times 2 = -12$
 $4 \times -3 = -12$ $-5 \times -3 = 15$ $-6 \times 3 = -18$
 $4 \times -4 = -16$ $-5 \times -4 = 20$ $-6 \times 4 = -24$
2. a. -10 b. -24 c. 42 d. -26 e. 48 f. -42
g. -750 h. 1150 i. -63 j. -72 k. 55 l. -300
3. a. -800 b. 112 c. -192 d. 42 e. 160
4. a. -9 b. 7 c. -2 d. -12 e. -6 f. 8
g. -20 h. -6 i. -11
5. a. -8 b. 9 c. 16 d. 81 e. -2 f. 16
g. -125 h. 256 i. 625 j. -216
6. a. Positive
b. Negative
7. a. ± 5 b. ± 9 c. ± 7 d. ± 11 e. ± 10
8. a. 48 b. -36 c. 72 d. 16 e. -12 f. -576
9. Check with your teacher.
10. a. Positive
b. Negative
c. Negative
11. If a positive number is multiplied by -1 , the number becomes negative.
If a negative number is multiplied by -1 , the number becomes positive.
12. a. 2 b. -3 c. 5 d. 5 e. -7 f. 4
13. Because there is an odd number of negatives.
14. $-a$ must be negative for the answer to be negative, thus a must be positive. The power must be odd to give a negative result, thus n is odd.
15. -432
16. a. -1
b. -1
17. One of the following: 17 correct, 1 incorrect; 18 correct, 3 incorrect; 19 correct, 5 incorrect; 20 correct, 7 incorrect; 21 correct, 9 incorrect

Exercise 2.4 Dividing integers

1. a. -7 b. -4 c. -4 d. 6 e. -8 f. 0 g. -16 h. -43 i. 8
j. -46 k. 5 l. -38
2. a. 11 b. -4 c. 3 d. -3 e. 2 f. 2
3. a. -120 b. -27 c. 38 d. 33 e. -47 f. -160 g. -27 h. -180 i. 250
j. 62 k. 324 l. 226
4. Check with your teacher.
5. a. 3 b. -2 c. -9 d. 3 e. -35 f. 24 g. -12 h. -10
6. a. 1 b. 6 c. -20
7. a. 3 b. -4 c. 2 d. $\frac{-3}{4}$ e. $\frac{-1}{2}$ f. $\frac{2}{3}$
8. a. 48 b. -6 c. -1 d. -4 e. $\frac{-1}{3}$ f. -4

9. a.

×	-2	4	-6	8
-3	6	-12	18	-24
-10	20	-40	60	-80
-5	10	-20	30	-40
-7	14	-28	42	-56

b.

×	5	-3	-9	-7
6	30	-18	-54	-42
-12	-60	36	108	84
-11	-55	33	99	77
2	10	-6	-18	-14

10. a.

÷	4	-10	12	-8
-2	-2	5	-6	4
7	$\frac{4}{7}$	$-\frac{10}{7}$	$\frac{12}{7}$	$-\frac{8}{7}$
-3	$-\frac{4}{3}$	$\frac{10}{3}$	-4	$\frac{8}{3}$
-10	$-\frac{2}{5}$	1	$-\frac{6}{5}$	$\frac{4}{5}$

b.

÷	32	-24	-36	-4
12	$\frac{8}{3}$	-2	-3	$-\frac{1}{3}$
-8	-4	3	$\frac{9}{2}$	$\frac{1}{2}$
6	$\frac{16}{3}$	-4	-6	$-\frac{2}{3}$
-4	-8	6	9	1

11. Same sign

12. Positive

13. $-\frac{1}{162}$

14. a. -1

b. -1

15. a. Yes; for example, a car is getting faster while travelling towards the east.

b. Yes; for example, a car is slowing down (negative acceleration) while travelling towards the west.

16. -2.8

Exercise 2.5 Combined operations on integers

1. a. -12 b. -7 c. -3 d. 9 e. 30 f. -6
 g. -12 h. -58 i. -12 j. 17 k. 56 l. -7
 2. a. -19 b. -21 c. -29 d. -12 e. -34 f. -324
 3. a. -41 b. -76 c. -12 d. 15
 e. -60 f. 6 g. -60 h. 46

4. a. -12

b. $(4 + 8) \div -(2)^2 - 7 \times 2 = -17$

$4 + 8 \div (-(2))^2 - 7 \times 2 = -8$

5.

Round number	Points at the start of the round	Contacts during the round	Points at the end of the round
1	100	20 gnomes, 10 goblins and 3 healing potions	120
2	120	3 gnomes, 5 goblins, 6 orcs and 5 healing potions	177
3	177	3 orcs, 6 trolls and a cleric	152
4	152	5 trolls, 1 balrog and a cleric	52

6. $1 < 2, 4 > 0$

7. Check with your teacher.

8. a. The frog and the snail get out of the pipe at exactly the same time.

b. They make it out of the pipe during the second hour.

9. a. 22.7 °C

b. 1.5 °C

c. Multiplying by $\frac{1}{4}$ is not the same as dividing by -4. However, multiplying by $\frac{1}{4}$ is the same as dividing by 4.

d. This is not a good idea because the square root ($\sqrt{\quad}$) of a number can be both positive and negative. Also, you cannot take the square root of a negative number, such as Sunday's maximum temperature.

