TOPIC 8 Proportion and rates

8.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

8.1.1 Why learn this?

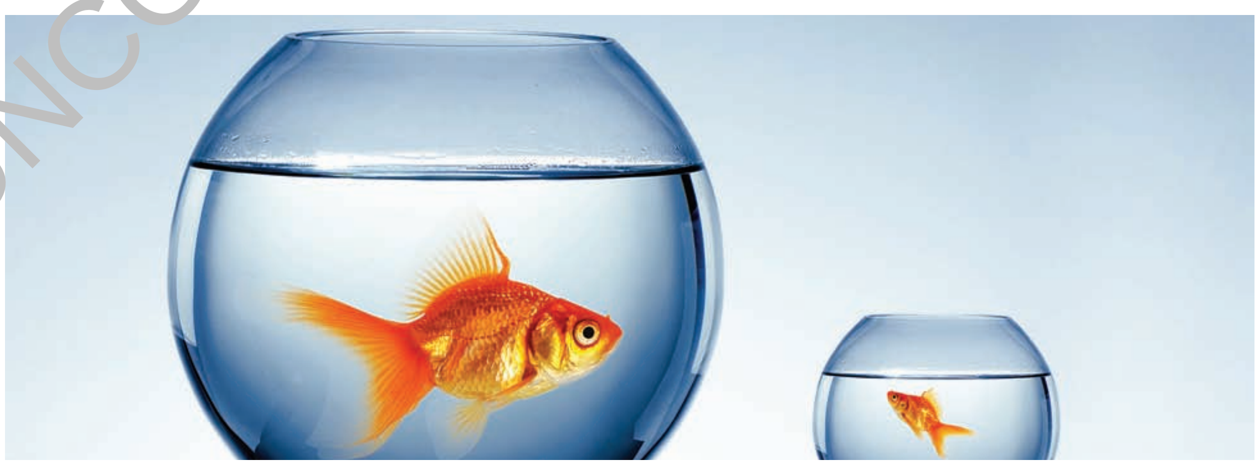
Proportion and rates are often used to compare quantities. Kilometres per hour, prices per kilogram, dollars per litre, and pay per hour or per week are all examples of rates. Dividing quantities according to a particular rule requires an understanding of ratios and rates.

8.1.2 What do you know?

assesson

1. **THINK** List what you know about proportion and rates. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then a small group.
3. **SHARE** As a class, use a thinking tool such as a large concept map to show your class's knowledge of proportion and rates.

LEARNING SEQUENCE



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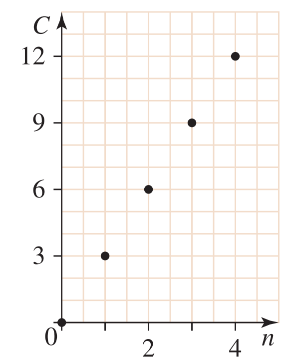
Watch this video: The story of mathematics: The Golden Ratio
Searchlight ID: eles-1695

8.2 Direct proportion

- Suppose that ice-creams cost \$3 each and that you are to buy some for your friends. There is a relationship between the cost of the ice-creams (C) and the number of the ice-creams that you buy (n).

The relationship can be illustrated in a table or a graph.

n	0	1	2	3	4
C (\$)	0	3	6	9	12



- This relationship has some important characteristics:
  - As n increases, so does C.
  - When $n = 0$, $C = 0$.
  - The graph of the relationship is a straight line passing through the origin.

  When all of these characteristics apply, the relationship is called **direct proportion**.
- We say that 'C is directly proportional to n' or 'C varies directly as n.' This is written as $C \propto n$.

WORKED EXAMPLE 1

Does direct proportion exist between these variables?

a The height of a stack of photocopy paper (h) and the number of sheets (n) in the stack

b Your Mathematics mark (m) and the number of hours of Maths homework you have completed (n)

THINK

a When n increases, so does h.
When $n = 0$, $h = 0$.
If graphed, the relationship would be linear.

WRITE

a $h \propto n$

THINK

b As n increases, so does m.
When $n = 0$, I may get a low mark at least, so $n \neq 0$.

WRITE

b m is not directly proportional to n.

WORKED EXAMPLE 2

Do the following relationships show direct proportion?

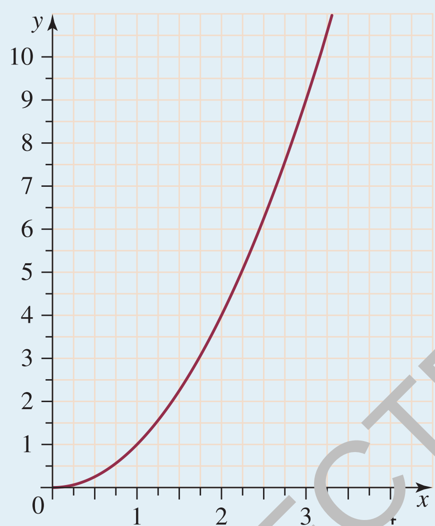
a

t	0	1	2	3	4
y	0	1	3	7	15

b

n	1	2	3	4
c	5	10	15	20

c



THINK

a From the table, when t increases, so does y.

When $t = 0$, $y = 0$. The t-values increase by a constant amount but the y-values do not, so the relationship is not linear.

When t is doubled, y is not.

WRITE

a y is not directly proportional to t.

THINK

b From the table, as n increases, so does m.

Extending the pattern gives $n = 0$, $C = 0$. The n-values and C-values increase by constant amounts, so the relationship is linear.

WRITE

b $C \propto n$

THINK

c When x increases, so does y.

When $x = 0$, $y = 0$.

The graph is not a straight line.

WRITE

c y is not directly proportional to x.

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Try out this interactivity: Direct proportion
Searchlight ID: int-2767

Complete this digital doc: SkillSHEET: Measuring the rise and the run
Searchlight ID: doc-6174

Exercise 8.2 Direct proportion

assess on

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2	Questions: 1, 2, 4, 5	Questions: 1–7

Individual pathway interactivity: int-4510 learnON ONLINE ONLY

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Fluency

1. **WE1** For each of the following pairs of variables, state whether direct proportion exists. If it does not exist, give a reason.
   a. The distance (d) travelled in a car travelling at 60 km/h and the time taken (t)
   b. The speed of a swimmer (s) and the time the swimmer takes to complete one lap of the pool (t)
   c. The cost of a bus ticket (c) and the distance travelled (d)
   d. The perimeter (p) of a square and the side length (l)
   e. The area of a square (A) and the side length (l)
   f. The total cost (C) of buying n boxes of pencils
   g. The weight of an object in kilograms (k) and the weight in pounds (p)
   h. The distance (d) travelled in a taxi and the cost (c)
   i. A person's height (h) and their age (a)

Understanding

2. **WE2** For each of the following, determine whether direct proportion exists between the variables. If it does not, give a reason.

a.

x	0	1	2	3	4
y	0	1	3	8	15

b.

a	0	1	2	3	6
M	0	8	16	24	48

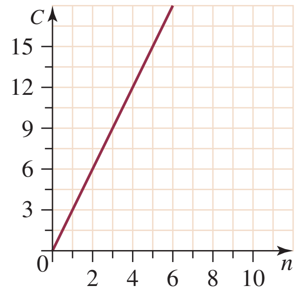
c.

t	0	1	2	4	8
d	0	3	6	9	12

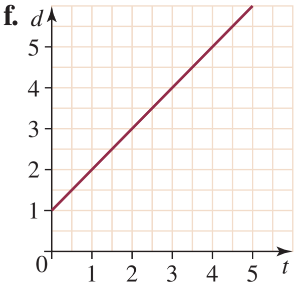
d.

n	0	1	2	3	4
C	10	20	30	40	50

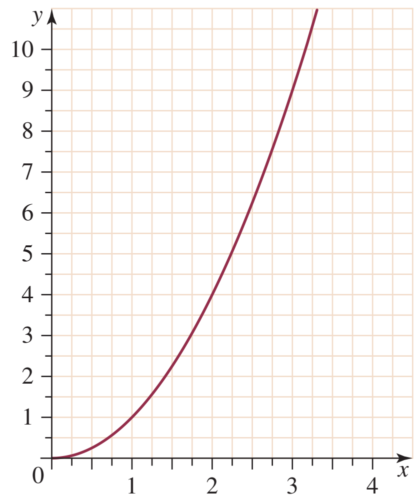
e.



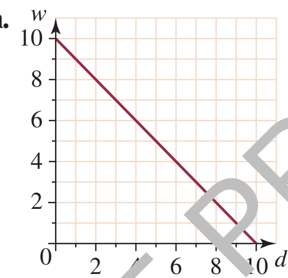
f.



g.



h.



3. List five pairs of real-life variables that exhibit direct proportion.

Reasoning

4. Which point must always exist in a table of values if the two variables exhibit direct proportionality?

5. If direct proportion exists between two variables, m and n, fill out the table and explain your reasoning.

m	0	2	5
n	0		20

Problem solving

6. Mobile phone calls are charged at 17 cents per 30 seconds

a. Does direct proportion exist between the cost of a phone bill and the number of 30-second time periods?

b. If a call went for 7.5 minutes, how much would the call cost?

7. Bruce is building a pergola and needs to buy treated pine timber. He wants 4.2-metre and 5.4-metre lengths of timber. If it costs $23.10 for the 4.2-metre length and $29.16 for the 5.4-metre length, does direct proportion exist between the cost of the timber and the length of the timber per metre? Explain.

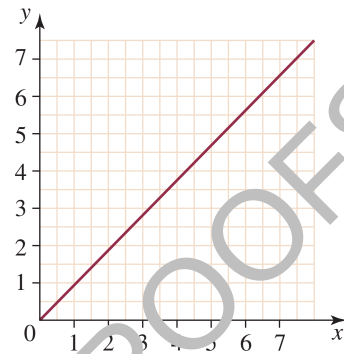
Reflection

How do you know when two quantities are directly proportional?

8.3 Direct proportion and ratio

8.3.1 Constant of proportionality

- A direct proportion relationship ($y \propto x$) is shown by the graph at right. Because the graph passes through the origin, the y-intercept is equal to zero, and the graph has the equation $y = kx$, where k is the gradient of the graph.
- In the equation $y = kx$:
  - k is a constant, and is called the **constant of proportionality** (or the constant of variation)
  - $y \propto x$ is equivalent to the equation $y = kx$
  - y is called the dependent variable and is normally placed in the bottom row of a table
  - x is called the independent variable and is normally placed in the top row of a table.



WORKED EXAMPLE 3

Given that $y \propto x$, and $y = 12$ when $x = 3$, find the constant of proportionality and state the rule linking y and x.

THINK	WRITE
1 $y \propto x$, so write the linear rule.	$y = kx$
2 Substitute $y = 12$, $x = 3$ into $y = kx$.	$12 = 3k$
3 Find the constant of proportionality by solving for k.	$k = 4$
4 State the rule.	$y = 4x$

WORKED EXAMPLE 4

The weight (W) of $1 coins in a bag varies directly as the number of coins (n). Twenty coins weigh 180 g.
a Find the relationship between W and n.
b How much will 57 coins weigh?
c How many coins weigh 252 g?

THINK

Summarise the information given in a table.

WRITE

$W = kn$

n	20	57	
W	180		252

THINK	WRITE
a 1 Substitute $n = 20$, $W = 180$ into $W = kn$.	**a** $180 = 20k$
2 Solve for k.	$k = 9$

3 State the relationship between W and n. — $W = 9n$

b 1 State the rule. — **b** $W = 9n$

2 Substitute 57 for n to find W.

$$\begin{aligned} W &= 9 \times 57 \\ &= 513 \end{aligned}$$

3 State the answer. — Fifty-seven coins will weigh 513 g.

c 1 State the rule. — **c** $W = 9n$

2 Substitute 252 for W. — $252 = 9n$

3 Solve for n.

$$\begin{aligned} n &= \frac{252}{9} \\ &= 28 \end{aligned}$$

4 State the answer. — Twenty-eight coins weigh 252 g.

8.3.2 Ratio

- If $y \propto x$, then $y = kx$, where k is constant.
  Transposing this formula gives $\frac{y}{x} = k$.
  In other words, for any pair of values (x, y) the ratio $\frac{y}{x}$ is constant and equals the constant of proportionality.
- For example, this table shows that $v \propto t$.

t	1	2	3	4
v	5	10	15	20

It is clear that $\frac{20}{4} = \frac{15}{3} = \frac{10}{2} = \frac{5}{1} = 5.$

WORKED EXAMPLE 5

Sharon works part time and is paid at a fixed rate per hour. If she earns $135 for 6 hours work, how much will she earn for 11 hours?

THINK / **WRITE**

1 Sharon's payment is directly proportional to the number of hours worked. Write the rule. — $P = kn$

2 Summarise the information given. The value of x is to be found.

n	6	11
P	135	x

3 Since $P = kn$, $\frac{P}{n}$ is constant.
Solve for x.

$$\frac{135}{6} = \frac{x}{11}$$

$$x = \frac{135 \times 11}{6}$$

$$x = 247.5$$

4 State the answer. — Sharon earns $247.50 for working 11 hours.

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Searchlight ID: doc-6175

Complete this digital doc: WorkSHEET 8.1
Searchlight ID: doc-6181

Exercise 8.3 Direct proportion and ratio

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Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6, 9–11, 14	Questions: 1–5, 7, 9, 10, 12, 14, 15	Questions: 1–5, 8–10, 12–17

Individual pathway interactivity: int-4511 learnON ONLINE ONLY

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Fluency

1. WE3 If a is directly proportional to b, and $a = 30$ when $b = 5$, find the constant of proportionality and state the rule linking a and b.
2. If $a \propto b$, and $a = 2.5$ when $b = 5$, find the rule linking a and b.
3. If $C \propto t$, and $C = 100$ when $t = 8$, find the rule linking C and t.
4. If $v \propto t$, and $t = 20$ when $v = 10$, find the rule linking v and t.
5. If $F \propto a$, and $a = 40$ when $F = 100$, find the rule linking a and F.

Understanding

6. WE4 Springs are often used to weigh objects, because the extension of a spring (E) is directly proportional to the weight (W) of the object hanging from the spring.
A 4-kg load stretches a spring by 2.5 cm.
   a. Find the relationship between E and W.
   b. What load will stretch the spring by 12 cm?
   c. How much will 7 kg extend the spring?
7. Han finds that 40 shelled almonds weigh 52 g.
   a. Find the relationship between the weight (W) and the number of almonds (n).
   b. How many almonds would there be in a 500-g bag?
   c. How much would 250 almonds weigh?
8. Petra knows that her bicycle wheel turns 40 times when she travels 100 m.
   a. Find the relationship between the distance travelled (d) and the number of turns of the wheel (n).
   b. How far does she go if her wheel turns 807 times?
   c. How many times does her wheel turn if she travels 5 km?

9. Fiona, who operates a plant nursery, uses large quantities of potting mix.
Last week she used 96 kg of potting mix to place 800 seedlings in medium-sized pots.

a. Find the relationship between the mass of potting mix (M) and the number of seedlings (n).
b. How many seedlings can she pot with her remaining 54 kg of potting mix?
c. How much potting mix will she need to pot 3000 more seedlings?

10. WE5 Tamara is paid at a fixed rate per hour. If she earns \$136 for 5 hours work, how much will she earn for working 8 hours?
11. It costs \$158 to buy 40 packets of cards. How much will 55 packets cost?
12. If 2.5 L of lawn fertiliser will cover an area of 150 m^2, how much fertiliser is needed to cover an area of 800 m^2?

Reasoning

13. Paul paid \$68.13 for 45 L of fuel. At the same rate, how much would he pay for 70 L? Justify your answer.
14. Rose gold is an alloy of gold and copper that is used to make high-quality musical instruments. If it takes 45 g of gold to produce 60 g of rose gold, how much gold would be needed to make 500 g of rose gold? Justify your answer.

Problem solving

15. If Noah takes a group of friends to the movies for his birthday and it would cost \$62.50 for five tickets, how much would it cost if there were 12 people (including Noah) in the group?
16. Sharyn enjoys quality chocolate, so she makes a trip to her favourite chocolate shop. She is able to select her favourite chocolates for \$7.50 per 150 grams. Since Sharyn loves her chocolate, she decides to purchase 675 grams. How much did she spend?
17. Anthony drives to Mildura, covering an average of 75 kilometres in 45 minutes. He has to travel 610 kilometres to get to Mildura.
    a. Find the relationship between the distance travelled in kilometres, D, and his driving time in hours, T.
    b. How long will it take Anthony to complete his trip?
    c. If he stops at Bendigo, 220 kilometres from his starting position, how long does it take him to reach Bendigo?
    d. How much longer will it take him to arrive at Mildura after leaving Bendigo?

Reflection

What is ratio?

CHALLENGE 8.1

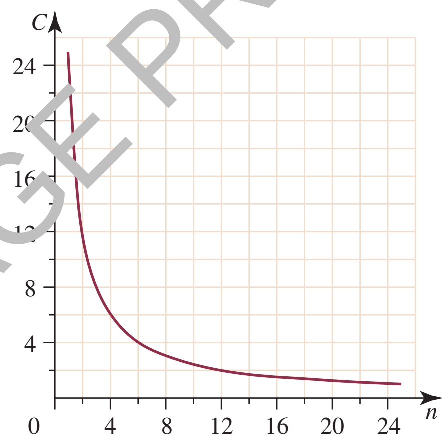
The volume of a bird's egg can be determined by the formula $V = kl^3$, where V is the volume in cm^3, l is the length of the egg in cm and k is a constant. A typical ostrich egg is 15 cm long and has a volume of 7425 cm^3. What is the volume of a typical 5 cm long chicken egg?

8.4 Inverse proportion

- If 24 sweets are shared between 4 children, then each child will receive 6 sweets. If the sweets are shared by 3 children, then each will receive 8 sweets.
- The relationship between the number of children (C) and the number sweets for each child (n) can be given in a table.

C	1	2	3	4	6	8	12
n	24	12	8	6	4	3	2

- As the number of children (C) increases, the number of sweets for each child (n) decreases. This is an example of **inverse proportion** or inverse variation.
- We say that 'n is inversely proportional to C' or 'n varies inversely as C'.
  This is written as $C \propto \frac{1}{n}$, or $C = \frac{k}{n}$, where k is a constant (the constant of proportionality). This formula can be rearranged to $Cn = k$. Note that multiplying any pair of values in the table (3×8, 12×2) gives the same result.
- The relationship has some important characteristics:
  – As C increases, n decreases, and vice versa.
  – The graph of the relationship is a **hyperbola**.



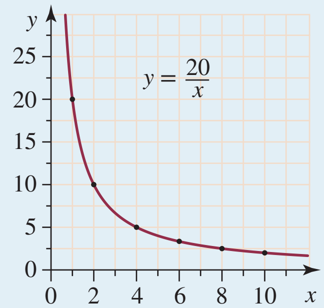
WORKED EXAMPLE 6

y is inversely proportional to x and $y = 10$ when $x = 2$.
a Calculate the constant of proportionality, k, and hence the rule relating x and y.
b Plot a graph of the relationship between x and y, for values of x from 2 to 10.

THINK	WRITE/DRAW
a 1 Write the relationship between the variables.	**a** $y \propto \frac{1}{x}$
2 Rewrite as an equation using k, the constant of proportionality.	$y = \frac{k}{x}$
3 Substitute $y = 10$, $x = 2$ into $y = \frac{k}{x}$ and solve for k.	$10 = \frac{k}{2}$ $k = 20$
4 Write the rule by substituting $k = 20$ into $y = \frac{k}{x}$.	$y = \frac{20}{x}$
b 1 Use the rule $y = \frac{20}{x}$ to set up a table of values for x and y, taking values for x which are positive factors of k so that only whole number values of y are obtained. For example, $x = 4$, $y = \frac{20}{4} = 5$.	**b** (see table below)

x	2	4	6	8	10
y	10	5	3.3	2.5	2

2 Plot the points on a clearly labelled set of axes and join the points with a smooth curve. Label the graph.



WORKED EXAMPLE 7

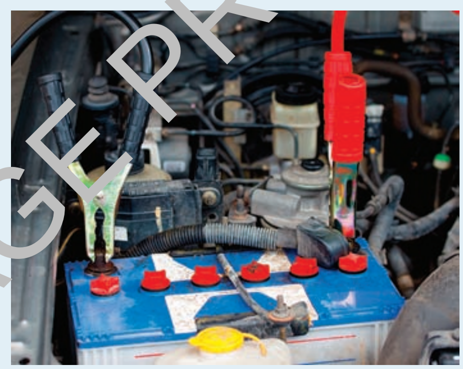
When a wire is connected to a power source, the amount of electrical current (I) passing through the wire is inversely proportional to the resistance (R) of the wire. If a current of 0.2 amperes flows through a wire of resistance 60 ohms:

a find the constant of proportionality

b determine the rule relating R and I

c find the resistance if the current equals 5 amperes

d find the current that will flow through a wire of resistance 20 ohms.



THINK

Summarise the information in a table.

WRITE

R	60		20
I	0.2	5	

THINK: $I \propto \dfrac{1}{R}$

Write the rule.

WRITE: $I = \dfrac{k}{R}$

a 1 Substitute $R = 60$, $I = 0.2$ into $I = \dfrac{k}{R}$.

a $0.2 = \dfrac{k}{60}$

2 Solve for k.

$0.2 \times 60 = k$

$k = 12$

b Write the rule using $k = 12$.

b $I = \dfrac{12}{R}$

c 1 Substitute $I = 5$ into $I = \dfrac{12}{R}$.

c $5 = \dfrac{12}{R}$

2 Solve for R.

$5R = 12$

$R = \dfrac{12}{5}$

$= 2.4$

3 Write the answer.

The resistance equals 2.4 ohms.

d 1 Substitute $R = 20$ into $I = \dfrac{12}{R}$.

d $I = \dfrac{12}{20}$

$= 0.6$

2 Write the answer.

The current will be 0.6 amperes.

Exercise 8.4 Inverse proportion

assessON

Individual pathways

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1–10	Questions: 1–13	Questions: 1–13

■■■ **Individual pathway interactivity:** int-4512

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Fluency

1. Decide whether inverse proportion exists between each pair of variables. If it does exist, write an equation to describe the relationship.

a. The speed of a car (s) and the time (t) it takes to complete one lap of a race circuit

b. The amount of money (D) that I have and the number (n) of cards that I can buy

c. The time (t) that it takes to make a pair of jeans and the number of pairs (p) that can be made in one day

d. The price (P) of petrol and the amount (L) that can be bought for $80

e. The price (P) of petrol and the cost (C) of buying 80 L

f. The number of questions (n) in a test and the amount of time (t) available to answer each one

2. List three examples of inverse proportion.

3. WE6 y varies inversely as x and $y = 100$ when $x = 10$.

a. Calculate the constant of proportionality, k, and hence the rule relating x and y.

b. Plot a graph of the relationship between x and y, for values of x that are positive factors of k less than 21.

4. p is inversely proportional to q and $p = 12$ when $q = 4$.

a. Calculate the constant of proportionality, k, and hence the rule relating p and q.

b. Plot a graph of the relationship between q and p, for values of q that are positive factors of k less than 11.

5. y varies inversely as x and $y = 42$ when $x = 1$.

a. Calculate the constant of proportionality, k, and hence the rule relating x and y.

b. Plot a graph of the relationship between x and y, for values of x from 1 to 10.

Understanding

6. WE7 When a constant force is applied to an object, its acceleration is inversely proportional to its mass. When the acceleration of an object is 40 m/s^2, the corresponding mass is 100 kg.

a. Calculate the constant of proportionality.

b. Determine the rule relating mass and acceleration.

c. Determine the acceleration of a 200-kg object.

d. Determine the acceleration of a 1000-kg object.

7. The number of colouring pencils sold is inversely proportional to the price of each pencil.

Two thousand pencils are sold when the price is $0.25 each.

a. Calculate the constant of proportionality.

b. Determine the number of pencils that could be sold for $0.20 each.

c. Determine the number of pencils that could be sold for $0.50 each.

8. The time taken to complete a journey is inversely proportional to the speed travelled. A trip is completed in 4.5 hours travelling at 75 km per hour.

a. Calculate the constant of proportionality.
b. Determine how long, to the nearest minute, the trip would take if the speed was 85 km per hour.
c. Determine the speed required to complete the journey in 3.5 hours, correct to 1 decimal place.
d. Determine the distance travelled in each case.

9. The cost per person travelling in a charter plane is inversely proportional to the number of people in the charter group. It costs \$350 per person when 50 people are travelling.

a. Calculate the constant of variation.
b. Determine the cost per person, to the nearest cent, if there are 75 people travelling.
c. Determine how many people are required to reduce the cost to \$250 per person.
d. Determine the total cost of hiring the charter plane.

Reasoning

10. The electrical current in a wire is inversely proportional to the resistance of the wire to that current. There is a current of 10 amperes when the resistance of the wire is 20 ohms.

a. Calculate the constant of proportionality.
b. Determine the current possible when the resistance is 200 ohms.
c. Determine the resistance of the wire when the current is 15 amperes
d. Justify your answer to parts **b** and **c** using a graph.

11. Two equations relating the time of a trip, T, and the speed at which they travel, S, are given. For both cases the time is inversely proportional to the speed. $T_1 = \frac{5}{S_1}$ and $T_2 = \frac{7}{S_2}$. Explain what impact the different constants of proportionality have on the time of the trip.

Problem solving

12. The time it takes to pick a field of strawberries is inversely proportional to the number of pickers. It takes 2 people 5 hours to pick all of the strawberries in a field.

a. Calculate the constant of proportionality.
b. Determine the rule relating time (T) and the number of pickers (P).
c. Determine the time spent if there are 6 pickers.

13. For a constant distance covered by a sprinter, the sprinter's speed is inversely proportional to their time. If a sprinter runs at a speed of 10.4 m/s, the corresponding time is 9.62 seconds.

a. Calculate the constant of variation.
b. Determine the rule relating speed (V) and time (T).
c. Determine the time, correct to 2 decimal places, if they ran at a speed of 10.44 m/s.
d. Determine the time, correct to 2 decimal places, if they ran at a speed of 6.67 m/s.

Reflection

Explain what is meant by inverse proportion.

8.5 Introduction to rates

- The word **rate** occurs commonly in news reports and conversation.
  'Home ownership rates are falling.'
  'People work at different rates.'
  'What is the current rate of inflation?'
  'Do you offer a student rate?'
  'The crime rate seems to be increasing.'
- Rate is a word often used when referring to a specific ratio.

8.5.1 Average speed

- If you travel 160 km in 4 hours, your average speed is 40 km per hour.
  Speed is an example of a rate, and its unit of measurement (kilometres per hour) contains a formula.
  The word 'per' can be replaced with 'divided by', so speed $= \dfrac{\text{distance (in km)}}{\text{time (in hours)}}$.
- It is important to note the units involved. For example, an athlete's speed is often measured in metres per second (m/s) rather than km/h.

8.5.2 Rates

- In general, to find a rate (or special ratio), one quantity is divided by another.

WORKED EXAMPLE 8

Calculate the rates suggested by these statements.
a A shearer shears 1110 sheep in 5 days.
b Eight litres of fuel costs $12.56
c A cricket team scored 152 runs in 20 overs.

THINK	WRITE
a The rate suggested is 'sheep per day'. Write the ratio (rate).	**a** $\text{Rate} = \dfrac{\text{number of sheep}}{\text{number of days}} = \dfrac{1110}{5} = 222 \text{ sheep/day}$
b The rate suggested is 'dollars per litre'. Write the ratio (rate).	**b** $\text{Rate} = \dfrac{\text{number of dollars}}{\text{number of litres}} = \dfrac{12.56}{8} = \$1.57 \text{ per litre}$
c The rate suggested is 'runs per over'. Write the ratio (rate).	**c** $\text{Rate} = \dfrac{\text{number of runs}}{\text{number of overs}} = \dfrac{152}{20} = 7.6 \text{ runs per over}$

WORKED EXAMPLE 9

The concentration of a solution is measured in g/L (grams per litre). What is the concentration of the solution when 10 g of salt is dissolved in 750 mL of water?

THINK	WRITE
Concentration is measured in g/L, which means that concentration = number of grams (mass) ÷ number of litres (volume). Write the ratio (rate).	$\text{Concentration} = \dfrac{\text{mass}}{\text{volume}}$ $= \dfrac{10}{0.75}$ $= 13.3 \text{ g/L}$

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Exercise 8.5 Introduction to rates

assess**on**

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5	Questions: 1–11	Questions: 1–11

Individual pathway interactivity: int-4513

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Fluency

1. Hayden drove from Hay to Bee. He covered a total distance of 96 km and took 1.5 hours for the trip. What was Hayden's average speed for the journey?

2. Find the average speed in km/h for each of the following.

a. 90 km in 45 min

b. 5500 km in 3 h 15 min (correct to 1 decimal place)

3. WE8 Calculate the rates suggested by these statements, giving your answers to 2 decimal places where necessary.

a. It costs \$736 for 8 theatre tickets.

b. Penelope decorated 72 small cakes in 3 hours.

c. Usain Bolt has a 100 m world sprint record of 9.58 seconds.

d. It takes 30 hours to fill the swimming pool to a depth of 90 cm.

e. Peter received \$260 for 15 hours work.

f. Yan received \$300 for assembling 6 air conditioners.

Understanding

4. A metal bolt of volume 25 cm^3 has a mass of 100 g. Find its density (mass per unit of volume).

5. WE9 One hundred and twenty grams of sugar is dissolved in 200 mL of water. What is the concentration of this solution in g/L? Explain your answer.

Reasoning

6. In the race between the tortoise and the hare, the hare ran at 72 km per hour while the giant tortoise moved at 240 cm per minute. Compare and explain the difference in the speed of the two animals.

7. The average speed of a car is determined by the distance of the journey and the time the journey takes. Explain the two ways in which the speed can be increased.

Problem solving

8. In the 2000 Sydney Olympics, Cathy Freeman won gold in the 400-metre race. Her time was 49.11 seconds. In the 2008 Beijing Olympics, Usain Bolt set a new world record for the men's 100-metre race. His time was 9.69 seconds. Calculate the average speed of the winner for each race in kilometres per hour.
9. A school had 300 students in 2013 and 450 students in 2015. What was the average rate of growth in students per year?
10. Mt Feathertop is Victoria's second highest peak. To walk to the top involves an increase in height of 1500 m over a horizontal distance of 10 km (10 000 m). Calculate the average gradient of the track.

11. Beaches are sometimes unfit for swimming if heavy rain has washed pollution into the water. A beach is declared unsafe for swimming if the concentration of bacteria is more than 5000 organisms per litre. A sample of 20 millilitres was tested and found to contain 55 organisms. Calculate the concentration in the sample (in organisms/litre) and state whether or not the beach should be closed.

Reflection

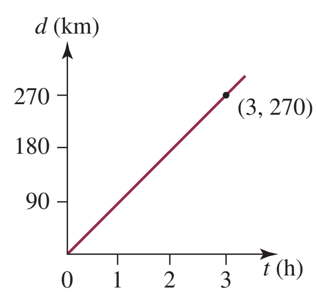
What is an example of a rate that is not listed in this section?

8.6 Constant and variable rates

8.6.1 Constant rates

- Consider a car travelling along a highway at a constant speed (rate) of 90 km/h. After 1 hour it will have travelled 90 km, as shown in the table below.

Time (h)	0	1	2	3
Distance (km)	0	90	180	270



- The distance–time graph is shown at right.

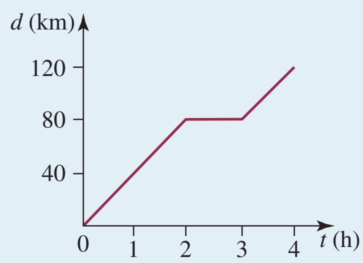
  The gradient of the graph is $\dfrac{270 - 0}{3 - 0} = 90$.

  The equation of the graph is $d = 90t$, and the gradient is equal to the speed, or rate of progress.

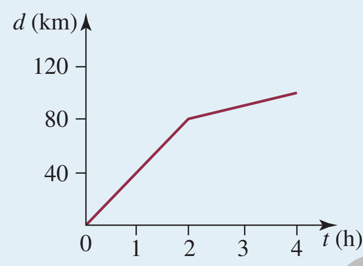
WORKED EXAMPLE 10

Each diagram below illustrates the distance travelled by a car over time. Describe the journey, including the speed of the car.

a



b



THINK	WRITE
a There are three distinct sections.	**a**
1 In the first 2 hours, the car travels 80 km: $\frac{80}{2} = 40$ km/h.	
2 In the next hour, the car does not move.	
3 In the fourth hour, the car travels 40 km.	The car travels at a speed of 40 km/h for 2 hours, and then stops for 1 hour. After this it travels for 1 hour at 40 km/h.
b There are two distinct sections.	**b**
1 In the first 2 hours, the car travels 80 km: $\frac{80}{2} = 40$ km/h.	
2 In the next 2 hours, the car travels 20 km: $\frac{20}{2} = 10$ km/h.	The car travels at a speed of 40 km/h for 2 hours, and then travels at 10 km/h for a further 2 hours.

WORKED EXAMPLE 11

Draw a distance–time graph to illustrate the following journey.
A cyclist travels for 1 hour at a constant speed of 10 km/h, and then stops for a 30-minute break before riding a further 6 km for half an hour at a constant speed.

THINK

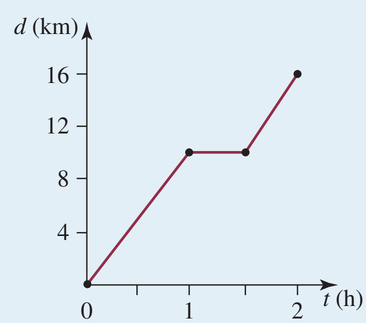
There are three phases to the journey.
The graph starts at (0, 0).

1 In the first hour, the cyclist travels 10 km. Draw a line segment from (0, 0) to (1, 10).

2 For the next half-hour, the cyclist is stationary, so draw a horizontal line segment from (1, 10) to (1.5, 10).

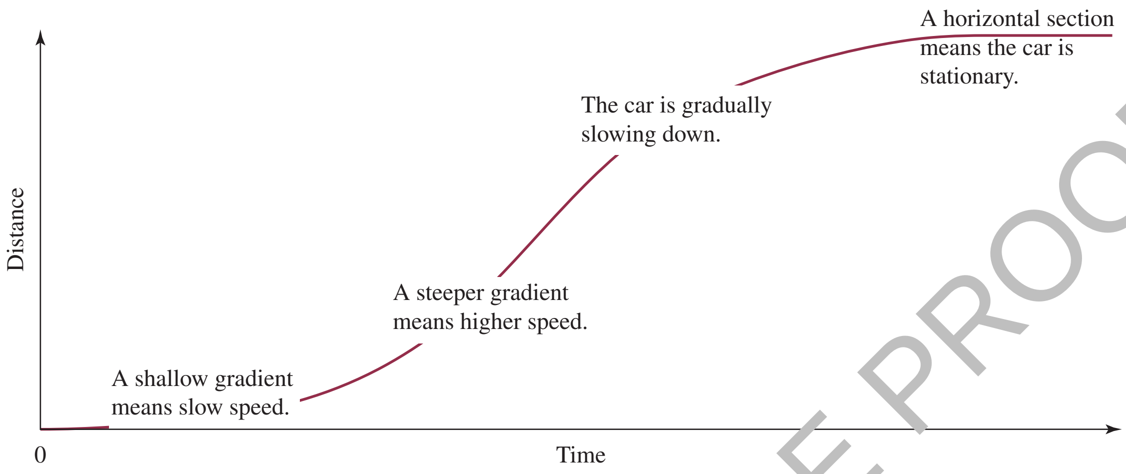
3 In the next half-hour, the cyclist travels 6 km. Draw a line segment from (1.5, 10) to (2, 16).

DRAW



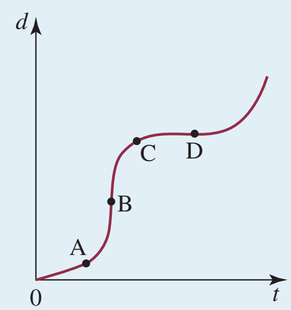
8.6.2 Variable rates

- In reality, a car tends not to travel at constant speed. It starts from rest and gradually picks up speed.
- When the speed is low, the distance–time graph will have a small gradient, and when the speed is high, the gradient will be steep.



WORKED EXAMPLE 12

The diagram at right illustrates the distance travelled by a car over time. Describe what is happening, in terms of speed, at each of the marked points.



THINK	WRITE
From the graph: At A the gradient is small but becoming steeper.	At A the car is travelling slowly but accelerating.
At B the gradient is at its steepest and is not changing.	At B the car is at its greatest speed.
At C the graph is becoming flatter.	At C the car is slowing down.
At D the graph is horizontal.	At D the car is stationary.

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Searchlight ID: doc-10844

Complete this digital doc: SkillSHEET: Equivalent rates
Searchlight ID: doc-10845

Exercise 8.6 Constant and variable rates

assess**on**

Individual pathways

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1–7	Questions: 1–13	Questions: 1–10, 13

■■■ **Individual pathway interactivity:** int-4514

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Fluency

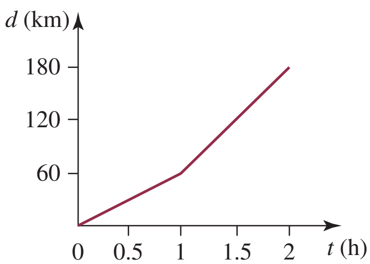
1. Two friends take part in a 24-kilometre mini-marathon. They run at constant speed. Ali takes 2 hours and Beth takes 3 hours to complete the journey.

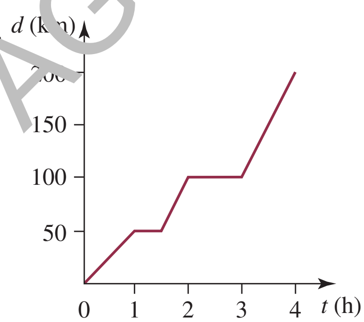
a. On the same diagram, draw a distance–time graph for each runner.

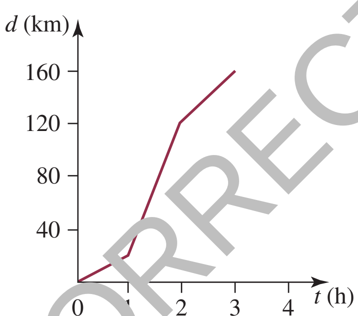
b. i. What is the equation for each graph?

ii. What is the difference between the two graphs?

2. WE10 Each diagram below illustrates the distance travelled by a car over time. Describe the journey, including the speed of the car.

a.


b.


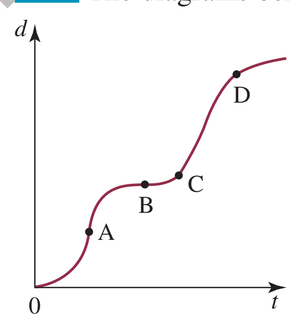
c.


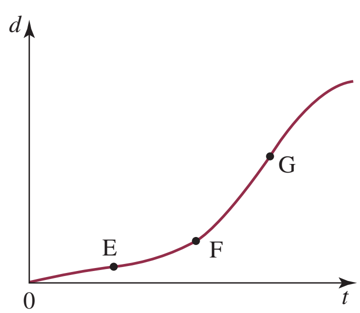
3. WE11 Draw a distance–time graph to illustrate each of the following journeys.

a. A cyclist rides at 40 km/h for 30 minutes, stops for a 30-minute break, and then travels another 20 km at a speed of 15 km/h.

b. Zelko jogs at a speed of 10 km/h for one hour, and then at half the speed for another hour.

4. WE12 The diagrams below illustrate the distances travelled by two cars over time.

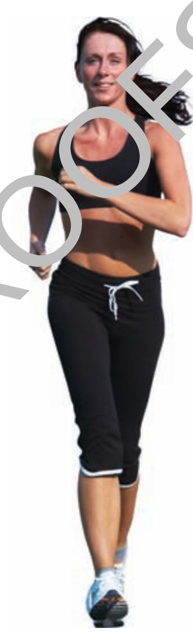




a. Describe what is happening in terms of speed at each of the marked points.
b. For each diagram:
  i. at which point is the speed the greatest?
  ii. at which point is the speed the least?
  iii. at which point is the car stationary?

Understanding

5. The table below shows the distance travelled, D, as a person runs for R minutes.



R **(minutes)**	10	20	50
D **(km)**	2	4	10

a. Find the rate in km/minute between the time 10 minutes and 20 minutes.
b. Find the rate in km/minute between the distance 4 km and 10 km.
c. Is the person's speed constant? Why?

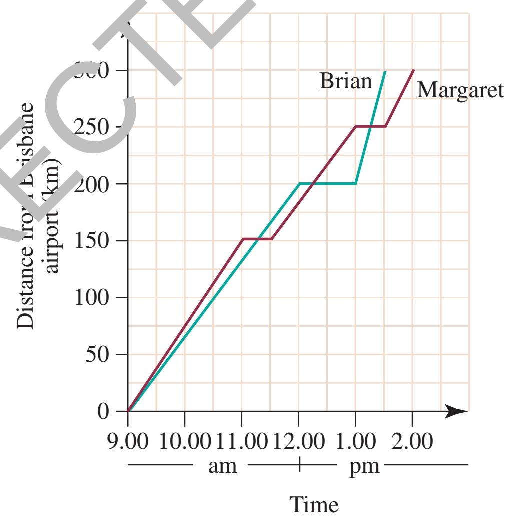
6. The table below shows the water used, W, after the start of the shower, where T is the time after the shower was started.

T **(minutes)**	0	1	2	4
W **(litres)**	0	20	30	100

a. Find the rate of water usage in L/minute for the 4 stages of the shower.
b. Was the rate of water usage constant?

Reasoning

7. Margaret and Brian left Brisbane airport at 9.00 am. They travelled separately but on the same road and in the same direction. Their journeys are represented by the travel graph below.



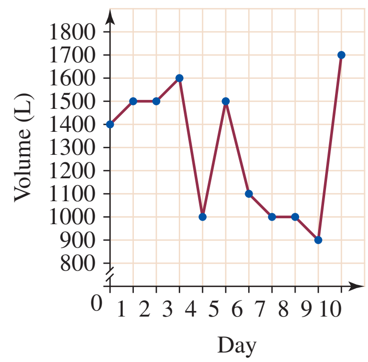
a. At what distance from the airport did their paths cross?
b. How far apart were they at 1.00 pm?
c. For how long did each person stop on the way?
d. What was the total time spent driving and the total distance for each person?
e. Calculate the average speed while driving for each person.

8. Hannah rode her bike along the bay one morning. She left home at 7.30 am and covered 12 km in the first hour. She felt tired and rested for half an hour. After resting she completed another 8 km in the next hour to reach her destination.
   a. How long did Hannah take for the entire journey?
   b. What is the total distance for which she actually rode her bike?
   c. Draw a travel graph for Hannah's journey.

9. Rebecca and Joanne set off at the same time to jog 12 kilometres. Joanne ran the entire journey at constant speed and finished at the same time as Rebecca. Rebecca set out at 12 km/h, stopping after 30 minutes to let Joanne catch up, then she ran at a steady rate to complete the distance in 2 hours.
   a. Show the progress of the two runners on a distance–time graph.
   b. How long did Rebecca wait for Joanna to catch up?

Problem solving

10. Use the graph showing the volume of water in a rainwater tank to answer the following questions.
    a. During which day(s) was the rate of change positive?
    b. During which day(s) was the rate of change negative?
    c. During which day(s) was the rate of change zero?
    d. On which day did the volume of water increase at the fastest rate?
    e. On which day did the volume of water decrease at the fastest rate?

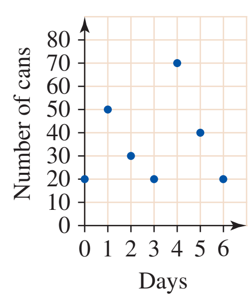


11. An internet service provider charges $30 per month plus $0.10 per megabyte downloaded. The table of monthly cost versus download amount is shown.

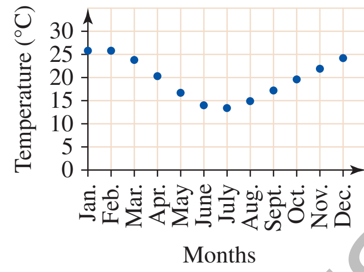
Download (MB)	0	100	200	300	400	500
Cost ($)	30	40	50	60	70	80

    a. By how much does the cost increase when the download amount increases from:
       i. 0 to 100 MB ii. 100 to 200 MB iii. 200 to 300 MB?
    b. Is the cost increasing at a constant rate?

12. The graph shown at right shows the number of soft-drink cans in a vending machine at the end of each day.
    a. By how much did the number of soft-drink cans change in the first day?
    b. By how much did the number of soft-drink cans change in the fifth day?
    c. Is the number of soft-drink cans changing at a constant or variable rate?



13. The table below and graph at right show Melbourne's average daily maximum temperature over the year.



	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Mean maximum (°C)	25.8	25.8	23.8	20.3	16.7	14.0	13.4	14.9	17.2	19.6	21.9	24.2

Source: Weatherzone, www.weatherzone.com.au.

a. What is the average maximum temperature in:
  i. February
  ii. June?

b. What is the change in temperature from:
  i. January to August
  ii. November to December?

c. Is the temperature changing at a constant rate?

Reflection

How can you tell the difference between constant and variable rates?

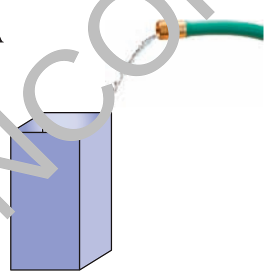
CHALLENGE 8.2

An early method of catching speeding motorists was the amphometer. It consisted of two airtight tubes placed across the road at a separation of 15 m. The tubes were connected to a timing device that detected the change in pressure as a car crossed them. In a 60 km/h zone, what was the minimum time for the car to travel across the device without being booked for speeding?

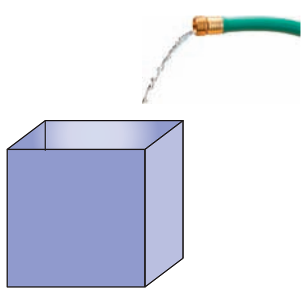
8.7 Rates of change

- Imagine that water is flowing out of a hose at a steady rate and will be used to fill several containers.

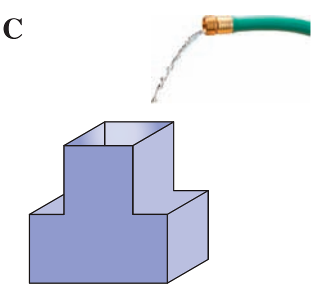
A



B



C



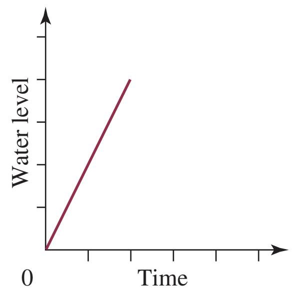
Because they are of different widths, they will fill at different rates. The narrow container will fill at a faster rate than the other two.

- Consider a graph of the water level against time for the three containers.

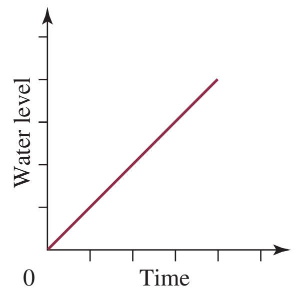
Container A is narrow, so the water level will rise quickly.

Container B is wide and will fill at a slower rate.

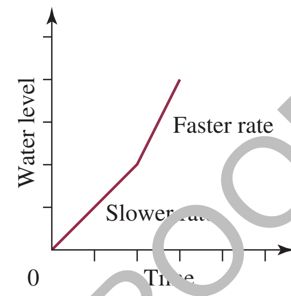
Container C is wide at the bottom, so the water will rise slowly at first, then quickly when it reaches the narrow part.



The water level changes at a constant rate.

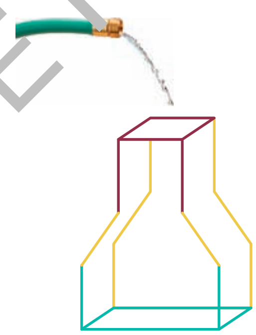


The water level changes at a constant rate.

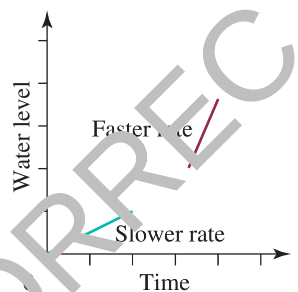


The water level changes at two different but constant rates.

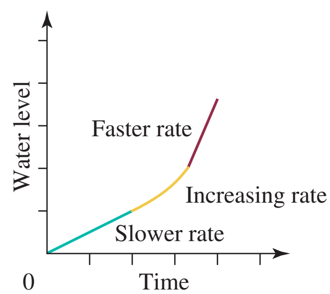
- Here is a more complex container, with three distinct sections.



The bottom section fills slowly and the top section fills quickly.



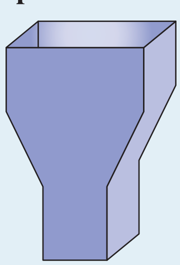
In between, the rate changes steadily from slow to fast. In this section, the rate is increasing.



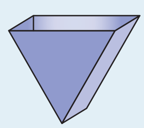
WORKED EXAMPLE 13

Each of these containers is being filled with water at a steady rate. For each container, sketch a graph of the water level against time.

a



b

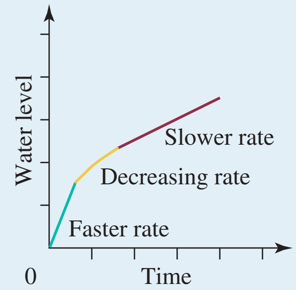


THINK

DRAW

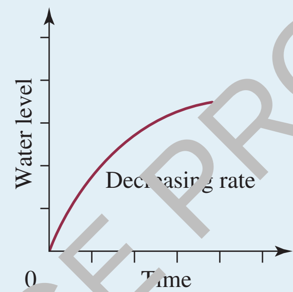
a The container has three distinct sections.
The bottom section will fill quickly.
The top section will fill slowly.
In the middle section, the rate will increase gradually from slow to quick.

a



b At the bottom of the container the water will rise rapidly, slowing down as the container becomes wider.

b



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Searchlight ID: doc-10846

Complete this digital doc: SkillSHEET: Plotting from a table of values
Searchlight ID: doc-10847

Complete this digital doc: WorkSHEET 8.3
Searchlight ID: doc-6187

Exercise 8.7 Rates of change

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6	Questions: 1–8	Questions: 1–9

Individual pathway interactivity: int-4515

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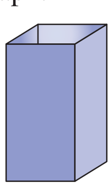
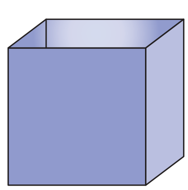
Fluency

1. These containers are filled with water at a steady rate. Match each container with the appropriate graph.

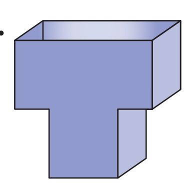
a.



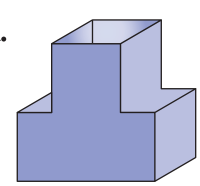
b.



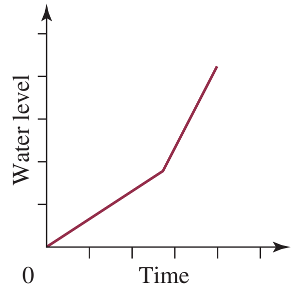
c.



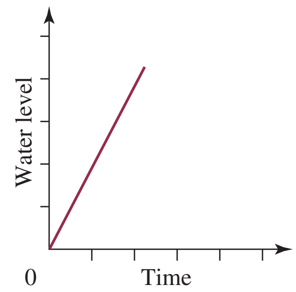
d.



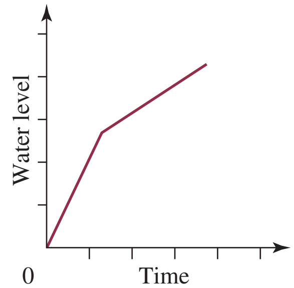
A.



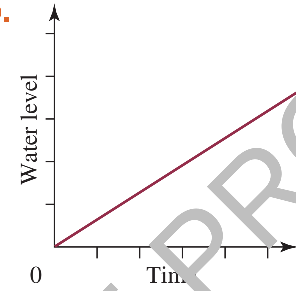
B.



C.

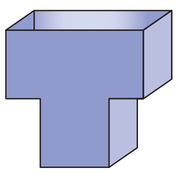


D.

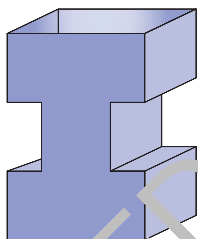


2. These containers are being filled with water at a steady rate. Sketch a graph of the water level against time.

a.



b.



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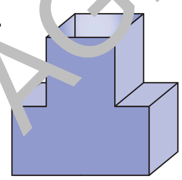

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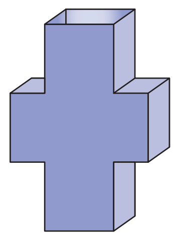

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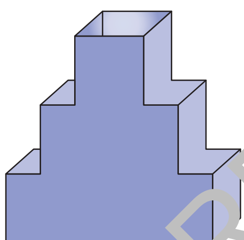

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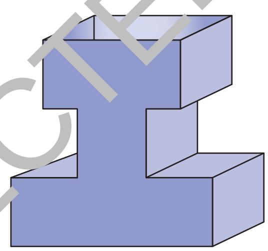

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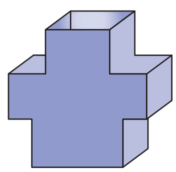

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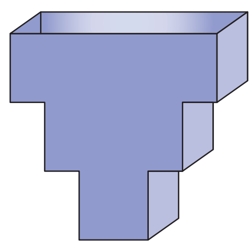
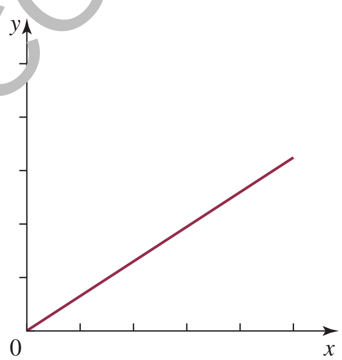


Understanding

3. Which of these graphs show:

a. a steady rate **b.** an increasing rate **c.** a decreasing rate?

A.



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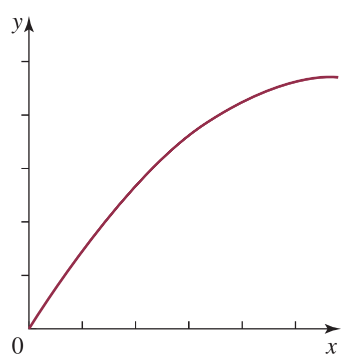

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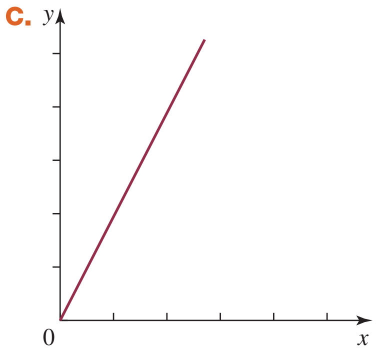

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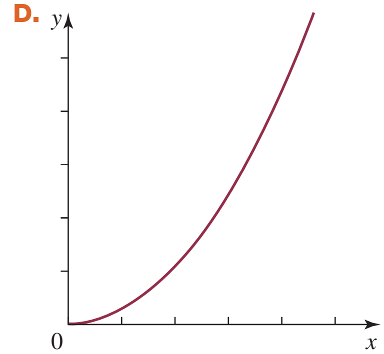
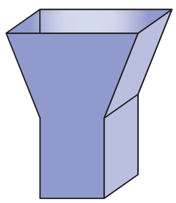


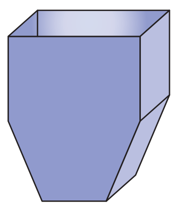
Reasoning

4. **WE13** These containers are being filled with water at a steady rate. For each container, sketch a graph of water level against time and explain your reasons.

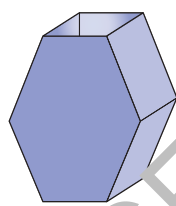
a.



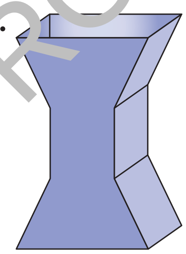
b.



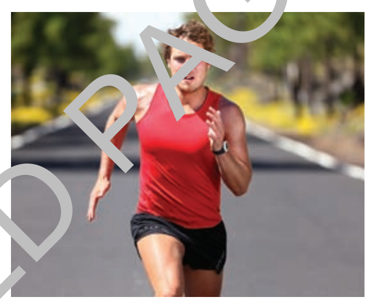
c.

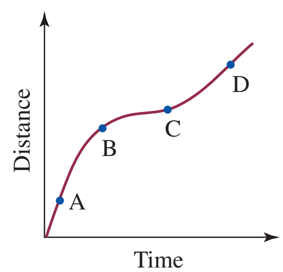


d.



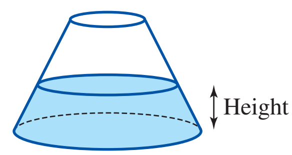
5. The distance versus time graph for a runner is shown at right. Four times during the run are marked as A, B, C and D on the graph. State, with reasoning, at which of the points the runner is:
   - **a.** travelling fastest
   - **b.** travelling slowest
   - **c.** slowing down
   - **d.** speeding up
   - **e.** travelling at constant speed.



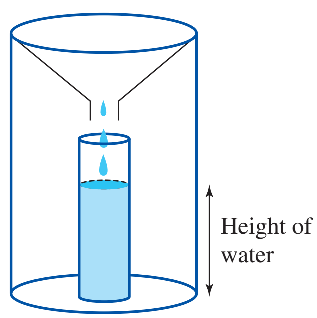


Problem solving

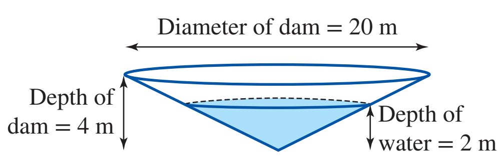
6. Water is poured at a constant rate into the container shown. Sketch the graph of height of water against time.



7. A rain gauge is made up of an inner cylinder and an outer cylinder, as shown in the diagram. When the inside cylinder fills and overflows, then the outside cylinder fills, so there are two scales on the device. The inner cylinder measures smaller quantities accurately and the outer cylinder measures larger quantities. For a day when the rain falls steadily, sketch a graph of the height of the water versus time.



8. A dam can be thought of as an inverted cone with a large radius and small height, as shown in the diagram. If the depth of water is half of the depth of the dam, at what percentage of its capacity is the dam? The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.



Reflection

How can you tell from a graph whether the rate of change is constant or variable?

8.8 Review

8.8.1 Review questions

Fluency

1. t is directly proportional to s and the gradient $m = 1.5$. When $s = 2$, t is equal to:
A. 1.5 **B.** 0.66 **C.** 0.75 **D.** 3

2. y is directly proportional to x and $y = 450$ when $x = 15$. The rule relating x and y is:
A. $y = 0.0333x$ **B.** $y = 30x$ **C.** $y = 60x$ **D.** $y = 6750x$

3. If $y \propto x$ and $y = 10$ when $x = 50$, the constant of proportionality is:
A. 10 **B.** 5 **C.** 1 **D.** 0.2

4. If $y \propto x$ and $y = 10$ when $x = 50$, the value of x when $y = 12$ is:
A. 6 **B.** 60 **C.** 40 **D.** 2.4

5. If y is inversely proportional to x, then which of the following statements is true?
A. $x + y$ is a constant value **B.** $y \div x$ is a constant value
C. $y \times x$ is a constant value **D.** $y - x$ is a constant value

6. The number of calculators a company sells is inversely proportional to the selling price. If a company can sell 1000 calculators when the price is \$22, how many could they sell if they reduced the price to \$16?
A. 2000 **B.** 727 **C.** 6000 **D.** 1375

7. Maya is sharing her collection of 60 fuchsia plants among three members of the family in the ratio of 2 : 3 : 5. The difference between the largest and smallest share of fuchsias is:
A. 12 **B.** 18 **C.** 36 **D.** 42

8. A speed of 60 km/h is equivalent to approximately:
A. 17 m/min **B.** 1 km/s **C.** 1 m/s **D.** 17 m/s

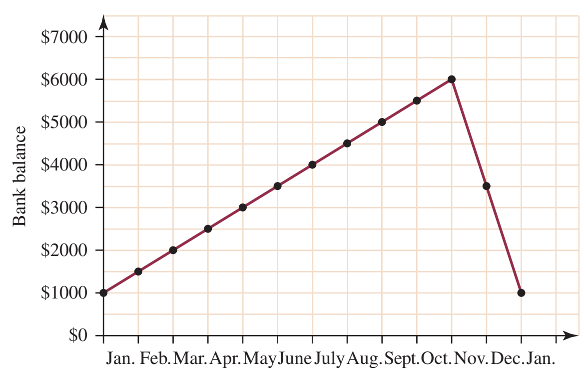
9. A metal part has a density of 37 g/mm^3. If its volume is 6 mm^3, it has a mass of:
A. 6.1 g **B.** 6.1 kg **C.** 222 kg **D.** 222 g

10. Which of the following is not a rate?
A. 50 km/h **B.** 70 beats/min **C.** 40 kg **D.** Gradient

11. How long does it take to travel 240 km at an average of 60 km/hour?
A. 0.25 hours **B.** 240 minutes **C.** 180 hours **D.** 14 400 hours

12. A plane takes 2 hours to travel 1600 km. How long does it take to travel 1000 km?
A. 150 minutes **B.** 48 minutes **C.** 2 hours **D.** 75 minutes

13. The graph at right is of Sandy's bank balance. How many months during the year has she been saving?
A. 2 months **B.** 5 months
C. 10 months **D.** 12 months



14. If $w \propto v$, and $w = 7.5$ when $v = 5$, calculate k, the constant of proportionality.

15. The gear ratio for front and back sprockets of a bicycle is 10 : 3. If the front (large) sprocket has 40 teeth, how many teeth does the back sprocket have?

16. Calculate the missing quantities in the table below.

Mass	Volume	Density
500 g	20 cm^3	
1500 g		50g/cm^3
	120 cm^3	17g/cm^3

17. a. A 2000-litre water tank takes 2 days to fill. Express this rate in litres per hour with 2 decimal places.
b. How far do you have to travel vertically to travel 600 metres horizontally if the gradient of a track is 0.3?
c. How long does it take for a 60-watt (60 joules/second) light globe to use 100 kilojoules?
d. Find the cost of 2.3 m^3 of sand at $40 per m^3.

18. The graph below shows the partial variation between the height of a child over time. What is the rate of change of height with respect to age in cm/year?

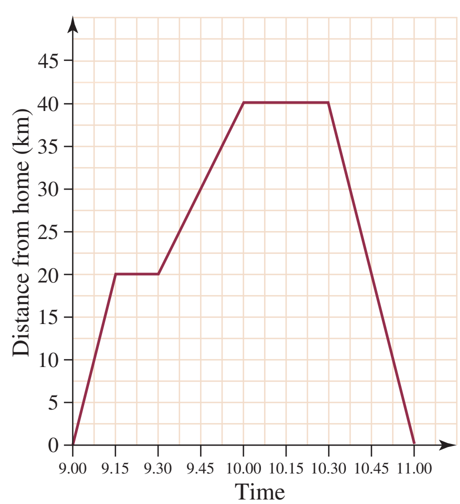
Problem solving

19. The authors of a Physics textbook are going to share royalties from sales of the book in the ratio proportional to the number of chapters each has written. Miss Alan wrote 4 chapters, Mr Bradley wrote 3 chapters, Mrs Cato wrote 7 chapters and Ms Dawn wrote 6 chapters. If the expected amount to be shared is $28 000, how much money will each author get?

20. Lisa drove to the city from her school. She covered a distance of 180 km in 2 hours.
a. What is Lisa's average speed?
b. Lisa travelled back at an average speed of 60 km/h. How long did she take?

21. Seventy grams of ammonium sulfate crystals are dissolved in 0.5 L of water.
a. What is the concentration of the solution in g/mL?
b. Another 500 mL of water is added. What is the concentration of the solution now?

22. Karina left home at 9.00 am. She spent some time at a friend's house then travelled to the airport to pick up her sister. She then travelled straight back home. Her journey is shown by the travel graph at right.
a. How far from Karina's house is her friend's house?
b. How much time did Karina spend at her friend's place?
c. How far is the airport from Karina's house?
d. How much time did Karina spend at the airport?
e. How much time did Karina take to drive home?



f. Find the average speed of Karina's journey:

i. from her home to the friend's place

ii. from her friend's place to the airport

iii. from the airport to her home.

23. A skyscraper can be built at a rate of 4.5 storeys per month.

a. How many storeys will be built after 6 months?

b. How many storeys will be built after 24 months?

24. A certain kind of eucalyptus tree grows at a linear rate for its first 2 years of growth. If the growth rate is 5 cm per month, how long will it take to grow to be 1.07 m tall?

25. The pressure inside a boiler increases as the temperature increases. For each 1 °C, the pressure increases by 10 units. At a temperature of 100 °C the pressure is 600 units. If the boiler can withstand a pressure of 2000 units, at what temperature does this occur?

26. Hector has a part-time job as a waiter at a local café and is paid \$8.50 per hour. Complete the table of values relating the amount of money received to the number of hours worked.

Number of hours	0	2	4	6	8	10
Pay						

27. A fun park charges a \$10 entry fee and an additional \$3 per ride. Complete the following table of values relating the total cost to the number of rides.

Rides	0	2	4	6	8	10
Cost						

28. Speed is a measure of the distance covered in a period of time. Distance is a length measurement and the units can be kilometres, metres, centimetres, millimetres or other smaller metric units. The length measurements could also be in imperial units, such as miles, yards, feet or inches. Time can be measured in years, days, hours, minutes or seconds, just to name a few. This means that speed can be quoted in units such as kilometres per hour, metres per minute, miles per hour, feet per second and the like. In Australia, we use the metric measurement of length (kilometres, metres, centimetres, millimetres) and this is the case in most parts of the world. However, the United States uses imperial units of length (miles, yards, feet, inches) while the United Kingdom uses a combination of imperial and metric. How can you compare, for instance, the speed of a car travelling at 100 km/h in Australia with one travelling at 100 miles/h (or mph) in the United States?

To help compare speed in different units, a length conversion chart is useful. Time is measured in the same units throughout the world, so a time conversion chart is not necessary.

Length conversion chart

Imperial unit	**Conversion factor**	**Metric unit**
inches (ins)	25.4	mm
feet (ft)	30.5	cm
yards (yds)	0.915	m
miles	1.61	km

To convert an imperial length unit into its equivalent metric unit, multiply by the conversion factor. Divide by the conversion factor when converting from metric to imperial units.

a. Use the table to complete each of the following.

i. 12 ins = ______ mm **ii.** 3 ft = ______ mm

iii. 1 m = = ______ yd **iv.** 1 km = ______ miles

b. Which is the faster car — the one travelling at 100 km/h, or the one travelling at 100 miles/h?

29. The speed limit of 60 km/h in Australia would be equivalent to a speed limit of ____ miles/h in USA. Give your answer correct to 3 decimal places.
30. The first supersonic land speed record for wheeled vehicles on ground was set by Andy Green of the United States in 1997. He reached a speed of 763.035 miles/h. How fast is this in km/h? Give your answer correct to 3 decimal places.
31. A commercial aircraft covers a distance of 1700 km in 2 hours 5 minutes. How many metres does the aircraft travel each second?
32. A launched rocket covers a distance of 17 miles in 10 seconds. Calculate its speed in km/h.
33. Here are some record speeds for moving objects.

Motorcycle	149 m/sec
Train	302 miles/h
Human skiing	244 km/h
Bullet from 38-calibre revolver	4000 ft/sec

Convert these speeds to the same unit. Then place them in order from fastest to slowest.

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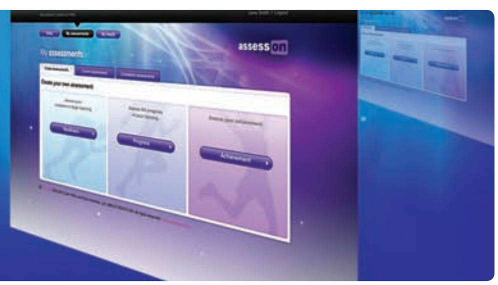
Complete this digital doc: Concept map
Searchlight ID: doc-10799

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

average rate
constant rate
constant of proportionality
dependent variable
direct proportion
gradient
independent variable
inverse proportion
origin
stationary
variable rate
varies directly as speed



Investigation | Rich task

Fastest speeds

To calculate the average speed of a journey, we can use the speed formula:

$$\text{speed} = \frac{\text{distance}}{\text{time}}.$$

Speed is a *rate* because it compares two quantities of different units. Speed can be expressed in units such as km/h, km/min and m/s. The units of the quantities substituted into the numerator and denominator determine the final units of speed. In order to compare the speeds of different events, it is useful to convert them to the same units.

In 2008, the summer and winter Olympics were held. At the summer Olympics, sprinter Usain Bolt ran 100 m in 9.69 seconds. At the winter Olympics, cross-country skier Petter Northug covered 50 km in 2 hours and 5 minutes, and speed skater Mika Poutala covered 0.5 km in 34.86 seconds. Which competitor was the fastest? In order to answer this question, we need to determine the

speed of each competitor. Since the information is quoted in a variety of units, we need to decide on a common unit for speed.

1. Calculate the speed of each athlete in m/s, correct to 1 decimal place.

2. Which athlete was the fastest?

Consider the speed of objects in the world around you. This task requires you to order the following objects from fastest to slowest, assuming that each object is travelling at its fastest speed possible:

electric car, submarine, diesel train, car ferry, skateboard, bowled cricket ball, windsurfer, served tennis ball, solar-powered car, motorcycle, aircraft carrier, Jaguar motorcar, helicopter, airliner, rocket-powered car, the Concorde supersonic airliner.

3. From your personal understanding and experience, order the above list from fastest to slowest.

We can make a more informed judgement if we have facts available about the movement of each of these objects. Consider the following facts:

When travelling at its fastest speed, the Concorde can cover a distance of about 7000 km in 3 hours. The fastest airliner can cover a distance of 5174 km in 2 hours and a helicopter 600 km in 1.5 hours. It would take 9.5 hours for a rocket-powered car to travel 9652 km, 3.25 hours for an electric car to travel 1280 km, 1.5 hours for a Jaguar car to travel 525 km and (on a sunny day) a solar-powered car can travel 39 km in half an hour. In 15 minutes, a car ferry can travel 26.75 km, an aircraft carrier 14 km and a submarine 18.5 km. It takes the fastest diesel train 0.42 hours to travel 100 km and a motorcyclist just 12 minutes to travel the same distance. In perfect weather conditions, a skateboarder can travel 30 km in 20 minutes and a windsurfer can travel 42 km in 30 minutes. A bowled cricket ball can travel 20 m in about 0.45 seconds while a served tennis ball can travel 25 m in about 0.4 seconds.

4. Using the information above, decide on a common unit of speed and determine the speed of each of the objects.

5. Order the above objects from fastest to slowest.

6. Compare the order from question 5 with the list you made in question **3**.

7. Conduct an experiment with your classmates and record times that objects take to cover a certain distance. For example, record the time it takes to run 100 m or throw a ball 20 m. Compare the speeds with the speeds of the objects calculated during this investigation.

Answers

Topic 8 Proportion and rates

Exercise 8.2 Direct proportion

1. a. Yes
   b. No; as speed increases, time decreases.
   c. No; doubling distance doesn't double cost.
   d. Yes
   e. No; doubling the side length doesn't double the area.
   f. Yes
   g. Yes
   h. No; doubling distance doesn't double cost (due to the initial fee).
   i. No; doubling age doesn't double height.

2. a. No; as x doubles, y does not.
   b. Yes
   c. No; as t doubles, d does not.
   d. No; when $n = 0$, d does not.
   e. Yes
   f. No; when $t = 0$, d does not.
   g. No; as x doubles, y does not.
   h. No; when $d = 0$, w does not.

3. Answers will vary.

4. The point (0, 0) must always exist in a table of values of two variables exhibiting direct proportionality.

5. The missing value is 8. The relationship between n and m can be calculated by the values 5 and 20 in the table ($n = 4m$).

6. a. Yes, direct proportion does exist.
   b. $2.55

7. Direct proportion does not exist. The price per metre for the 4.2 – metre length is $5.50, and the price per metre for the 5.4 – metre length is $5.40.

Exercise 8.3 Direct proportion and ratio

1. $k = 6$, $a = 6b$

2. $a = 0.5b$

3. $C = 12.5t$

4. $v = 0.5t$

5. $F = 2.5a$

6. a. $E = 0.625W$ **b.** 19.2 kg **c.** 4.375 cm

7. a. $W = 1.3n$ **b.** ≈ 385 almonds **c.** 325 g

8. a. $d = 2.5n$ **b.** 2017.5 km **c.** 2000 turns

9. a. $M = 0.$[illegible]$2n$ **b.** 450 seedlings **c.** 360 kg

10. $217.6[illegible]

11. $217.25

12. 13.33 L

13. $1[illegible].98

14. 375 g

15. $150

16. $33.75

17. a. $D = 100T$
   b. 6 hours 6 minutes
   c. 2 hours 12 minutes
   d. 3 hours 54 minutes

Challenge 8.1

275 cm^3

Exercise 8.4 Inverse proportion

1. a. $s = \dfrac{k}{t}$ **b.** No

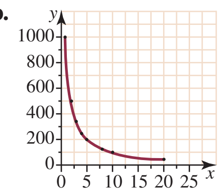
c. $t = \dfrac{k}{p}$ or $p = \dfrac{k}{t}$ **d.** $L = \dfrac{k}{p}$ or $p = \dfrac{k}{L}$

e. No **f.** $t = \dfrac{k}{n}$ or $n = \dfrac{k}{t}$

2. Answers will vary.

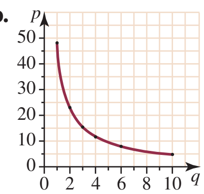
3. a. $k = 1000$, $y = \dfrac{1000}{x}$

b.



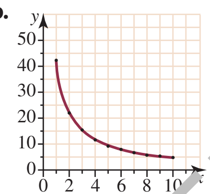
4. a. $k = 48$, $p = \dfrac{48}{q}$

b.



5. a. $k = 42$, $y = \dfrac{42}{x}$

b.



6. a. 4000 **b.** $a = \dfrac{4000}{m}$ **c.** 20 m/s^2 **d.** 4 m/s^2

7. a. 500 **b.** 2500 pencils **c.** 1000 pencils

8. a. 337.5 **b.** 3.97 hours = 3 h 58 min **c.** 96.4 km/h **d.** 337.5 km

9. a. 17 500 **b.** $233.33 **c.** 70 people **d.** $17 500

10. a. 200 **b.** 1 ampere **c.** 13.3 ohms **d.** Check with your teacher.

11. The constant of proportionality represents the distance of the trip; therefore, when this value is smaller, the time taken to complete the trip at the same speed is also smaller.

12. a. 10 **b.** $T = \dfrac{10}{P}$ **c.** 1 hour 40 minutes

13. a. 100.048 **b.** $T = \dfrac{100.048}{V}$ **c.** 9.58 seconds **d.** 15.00 seconds

Exercise 8.5 Introduction to rates

1. 64 km/h

2. a. 120 km/h **b.** 1692.3 km/h

3. a. $92 per ticket **b.** 24 cakes per hour **c.** 10.44 m/s

d. 3 cm/h **e.** $17.33/h **f.** $50 per air conditioner

4. 4 g/cm^3 **5.** 600 g/L

6. The hare runs at 120 000 cm per minute; the tortoise runs at 0.144 km per hour. The hare runs 500 times faster than the tortoise.

7. Either the distance of the journey decreases and the time remains constant, or the distance of the journey remains constant and the time decreases.

8. Cathy Freeman: 29.32 km/h; Usain Bolt: 37.15 km/h

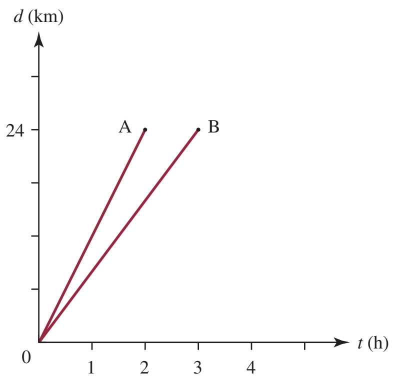
9. 75 students/year

10. 0.15

11. 2750 organisms/litre. The beach should not be closed.

Exercise 8.6 Constant and variable rates

1. a.



b. i. A: $d = 12t$ B: $d = 8t$

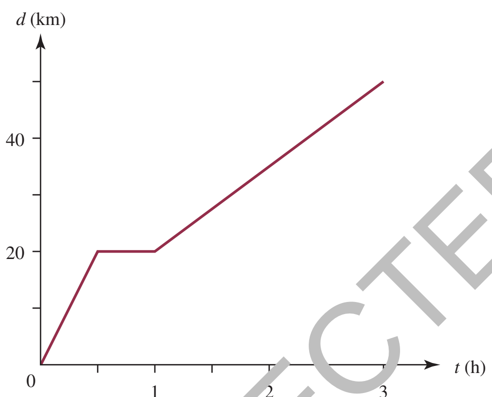
ii. A has a steeper gradient than B.

2. a. 1 hour at 60 km/h, then 1 hour at 120 km/h

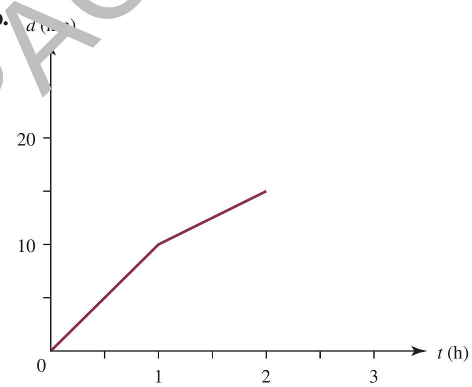
b. 1 hour at 50 km/h, a 30-minute stop, 30 minutes at 100 km/h, a 1-hour stop, then 1 hour at 100 km/h

c. 1 hour at 20 km/h, 1 hour at 100 km/h, 1 hour at 40 km/h

3. a.



b.



4. a. **A** The car is moving with steady speed.

B The car is momentarily stationary.

C The speed is increasing.

D The car is slowing down.

E The car is moving at a slow steady speed.

F The speed is increasing.

G The car is moving at a faster steady speed.

b. i. A, G **ii.** B, E **iii.** B

5. a. 0.2 km/min **b.** 0.2 km/min **c.** Yes, both rates are the same.

6. a. 20, 10, 35, 0 L/min **b.** No

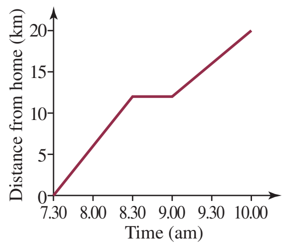
7. a. 150 km, 200 km and 250 km **b.** 50 km **c.** Both stop for 1 hour.

d. Brian — 300 km, 3.5 h; Margaret — 300 km, 4 h

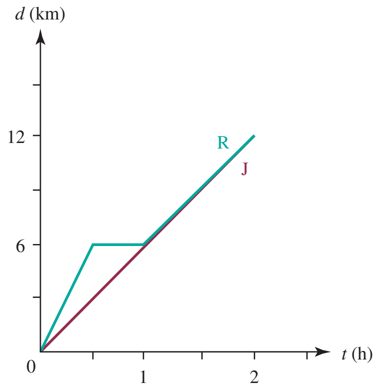
e. Brian — 85.7 km/h; Margaret — 75 km/h

8. a. 2.5 hours **b.** 20 km

c.



9. a.



b. 30 minutes

10. a. 1, 3, 5, 10 **b.** 4, 6, 7, 9 **c.** 2, 8 **d.** 10 **e.** 4

11. a. i. \$10 **ii.** \$10 **iii.** \$10

b. Yes

12. a. 30 **b.** 30 **c.** Variable

12. a. i. 25.8 °C **ii.** 14.0 °C

b. i. −10.9 °C **ii.** 2.3 °C

c. No

Challenge 8.2

0.9 seconds

Exercise 8.7 Rates of change

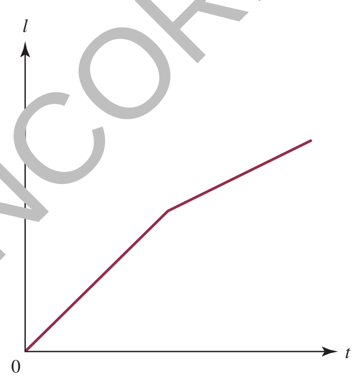
1. a. B

b. D

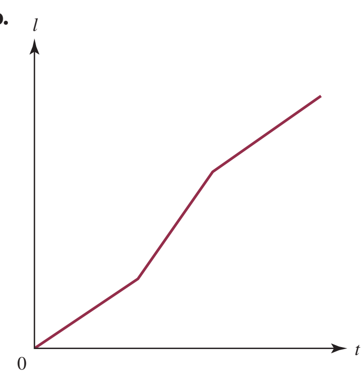
c. C

d. A

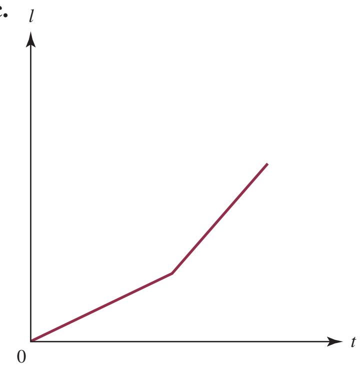
2. a.



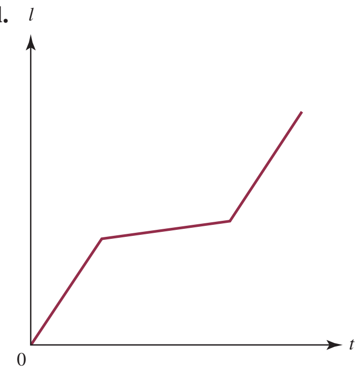
b.



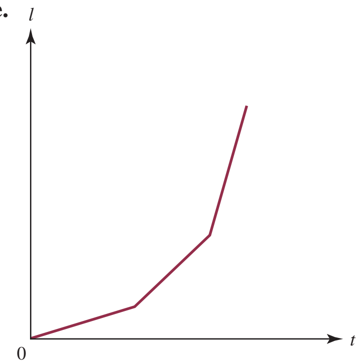
c.



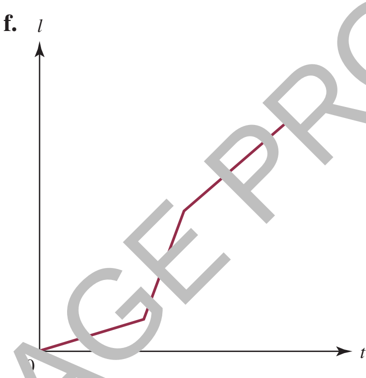
d.



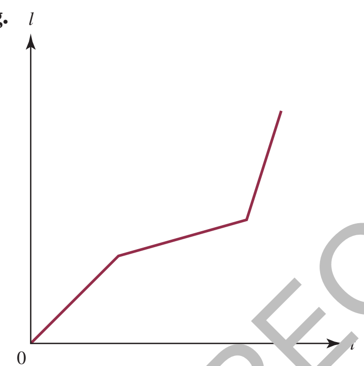
e.



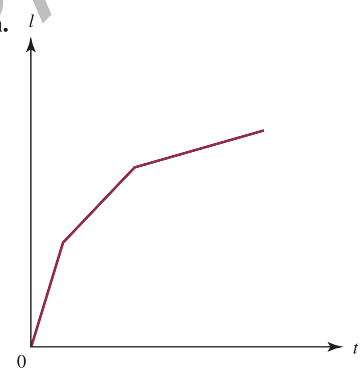
f.



g.

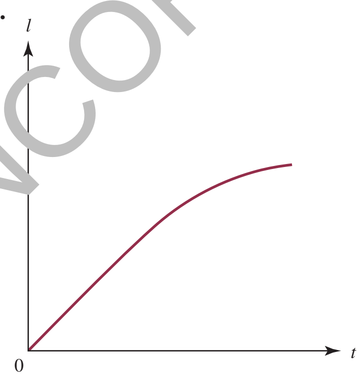


h.

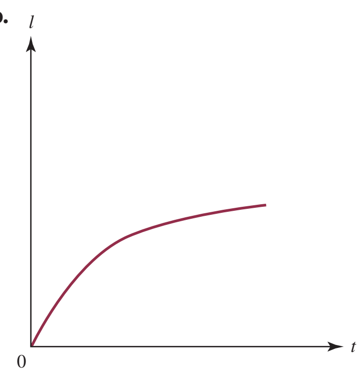


3. a. A, C **b.** D **c.** B

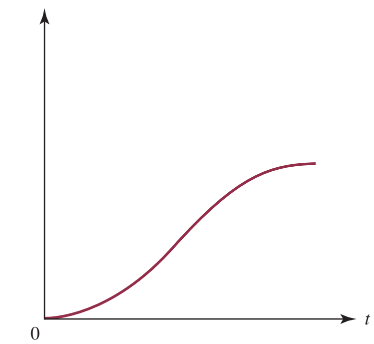
4. a.



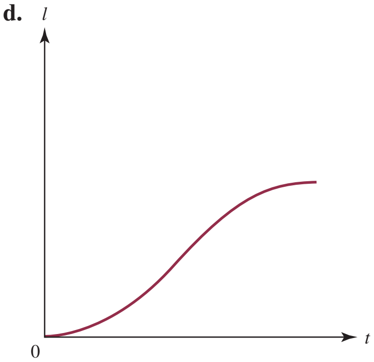
b.



c.

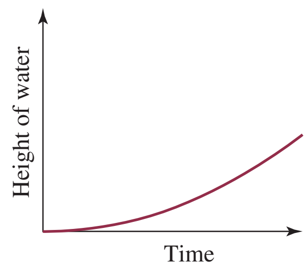


d.

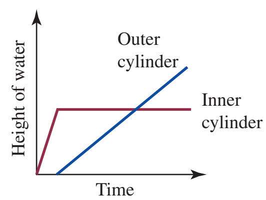


5. a. Point A — the gradient is greatest.
b. Point C — the gradient is least.
c. Point B — the gradient is decreasing.
d. Point C — the gradient is increasing.
e. Points A and D — the gradient is constant.

6.



7.



8. 12.5%

8.8 Review

1. D **2.** B **3.** D **4.** B **5.** C
6. D **7.** D **8.** D **9.** D **10.** C
11. B **12.** D **13.** C **14.** 1.5 **15.** 12 teeth

16.

Mass	Volume	Density
500 g	20 cm^3	25 g/cm^3
1500 g	30 cm^3	50 g/cm^3
2040 g	120 cm^3	17 g/cm^3

17. a. 41.67 L/h **b.** 180 m **c.** 28 minutes **d.** $92
18. 18 cm/year
19. Mrs Alan $5600, Mr Bradley $4200, Mrs Cato $9800, Ms Dawn $8400
20. a. 90 km/h **b.** 3 h
21. a. 0.14 g/mL **b.** 0.07 g/mL
22. a. 20 km **b.** 15 min **c.** 40 km **d.** 30 min **e.** 30 min
f. i. 80 km/h **ii.** 40 km/h **iii.** 80 km/h
23. a. 27 **b.** 108
24. 21.4 months
25. 240 °C

26.

Number of hours	0	2	4	6	8	10
Pay	$0	$17	$34	$51	$68	$85

27.

Rides	0	2	4	6	8	10
Cost	$10	$16	$22	$28	$34	$40

28. a. i. 304.8 mm **ii.** 915 mm **iii.** 1.09 yd **iv.** 0.62 miles

b. The one travelling at 100 miles/h

29. 37.267 miles/h

30. 1228.486 km/h

31. 13 226$\frac{2}{3}$ m/sec

32. 9853.2 km/h

33. The conversions may vary but the order from fastest to slowest is:

Bullet from 38-calibre revolver

Motor cycle

Train

Human skiing

Investigation — Rich task

1. Usain Bolt = 10.3 m/s

Petter Northug = 6.7 m/s

Mika Poutala = 14.3 m/s

2. Mika Poutala

3. Answers will vary.

4. Concorde: 2333.3 km/h

Airliner: 2587 km/h

Helicopter: 400 km/h

Rocket-powered car: 1016 km/h

Electric car: 393.8 km/h

Jaguar car: 350 km/h

Solar-powered car: 78 km/h

Car ferry: 107 km/h

Aircraft carrier: 56 km/h

Submarine: 74 km/h

Diesel train: 238.1 km/h

Motorcyclist: 500 km/h

Skateboarder: 90 km/h

Windsurfer: 84 km/h

Bowled cricket ball: 160 km/h

Served tennis ball: 225 km/h

5. Airliner, Concorde, rocket-powered car, motorcyclist, helicopter, electric car, Jaguar car, diesel train, served tennis ball, bowled cricket ball, oar ferry, skateboarder, windsurfer, solar-powered car, submarine, aircraft carrier

6. Answers will vary.

7. Answers will vary.