TOPIC 10 Indices

10.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

10.1.1 Why learn this?

Indices (the plural of index) give us a way of abbreviating multiplication, division and so on. They are most useful when working with very large or very small numbers. For calculations involving such numbers, we can use indices to simplify the process.

10.1.2 What do you know?

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1. **THINK** List what you know about indices. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class create a thinking tool such as a large concept map to show your class's knowledge of indices.

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10.2 Review of index laws

10.2.1 Index notation

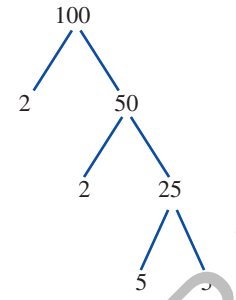
- The product of factors can be written in a shorter form called **index notation**.

Index, exponent

Base → $6^4 = 6 \times 6 \times 6 \times 6$ ← Factor form

$= 1296$

- Any composite number can be written as a product of powers of prime factors using a factor tree, or by other methods, such as repeated division.



$100 = 2 \times 2 \times 5 \times 5$

$= 2^2 \times 5^2$

WORKED EXAMPLE 1

TI | CASIO

Express 360 as a product of powers of prime factors using index notation.

THINK	WRITE
1 Express 360 as a product of a factor pair.	$360 = 6 \times 60$
2 Further factorise 6 and 60.	$= 2 \times 3 \times 4 \times 15$
3 Further factorise 4 and 15.	$= 2 \times 3 \times 2 \times 2 \times 3 \times 5$
4 There are no more composite numbers.	$= 2 \times 2 \times 2 \times 3 \times 3 \times 5$
5 Write the answer using index notation. *Note:* The factors are generally expressed with bases in ascending order.	$360 = 2^3 \times 3^2 \times 5$

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10.2.2 Multiplication using indices

- The **First Index Law states:** $a^m \times a^n = a^{m+n}$.
  That is, when multiplying terms with the same bases, add the indices.

WORKED EXAMPLE 2

Simplify $5e^{10} \times 2e^3$.

THINK	WRITE
1 The order is not important when multiplying, so place the coefficients first.	$5e^{10} \times 2e^3$ $= 5 \times 2 \times e^{10} \times e^3$
2 Simplify by multiplying the coefficients and applying the First Index Law (add the indices).	$= 10e^{13}$

- When more than one base is involved, apply the First Index Law to each base separately.

WORKED EXAMPLE 3 TI | CASIO

Simplify $7m^3 \times 3n^5 \times 2m^8n^4$.

THINK	WRITE
1 The order is not important when multiplying, so place the coefficients first and group the same pronumerals together.	$7m^3 \times 3n^5 \times 2m^8n^4$ $= 7 \times 3 \times 2 \times m^3 \times m^8 \times n^5 \times n^4$
2 Simplify by multiplying the coefficients and applying the First Index Law (add the indices).	$= 42m^{11}n^9$

10.2.3 Division using indices

- The **Second Index Law states:** $a^m \div a^n = a^{m-n}$.
  That is, when dividing terms with the same bases, subtract the indices.

WORKED EXAMPLE 4 TI | CASIO

Simplify $\dfrac{25v^6 \times 8w^9}{10v^4 \times 4w^5}$.

THINK	WRITE
1 Simplify the numerator and the denominator by multiplying the coefficients.	$\dfrac{25v^6 \times 8w^9}{10v^4 \times 4w^5}$ $= \dfrac{200v^6w^9}{40v^4w^5}$
2 Simplify further by dividing the coefficients and applying the Second Index Law (subtract the indices).	$= \dfrac{{}^{5}\cancel{200}}{{}^{1}\cancel{40}} \times \dfrac{v^6}{v^4} \times \dfrac{w^9}{w^5}$ $= 5v^2w^4$

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- When the coefficients do not divide evenly, simplify by cancelling.

WORKED EXAMPLE 5

Simplify $\frac{7t^3 \times 4t^8}{12t^4}$.

THINK	WRITE
1 Simplify the numerator by multiplying the coefficients.	$\frac{7t^3 \times 4t^8}{12t^4}$ $= \frac{28t^{11}}{12t^4}$
2 Simplify the fraction by dividing the coefficients by the highest common factor. Then apply the Second Index Law.	$= \frac{28}{12} \times \frac{t^{11}}{t^4}$ $= \frac{7t^7}{3}$

10.2.4 Zero index

- Any number divided by itself (except zero) is equal to 1.
- Therefore, $\frac{10}{10} = \frac{2.14}{2.14} = \frac{\pi}{\pi} = \frac{592^3}{592^3} = 1$.
- Similarly, $\frac{x^3}{x^3} = 1$. But using the Second Index Law, $\frac{x^3}{x^3} = x^0$. It follows that $x^0 = 1$.
- In the same way, $\frac{n^{10}}{n^{10}} = 1$, and $\frac{n^{10}}{n^{10}} = n^0$, so $n^0 = 1$.
- In general, any number (except zero) to the power zero is equal to 1.
- This is the **Third Index Law: $a^0 = 1$, where $a \neq 0$.**

WORKED EXAMPLE 6

Evaluate the following.

a t^0 **b** $(xy)^0$ **c** 17^0 **d** $5x^0$ **e** $(5x)^0 + 2$ **f** $5^0 + 3^0$

THINK	WRITE
a Apply the Third Index Law.	**a** $t^0 = 1$
b Apply the Third Index Law.	**b** $(xy)^0 = 1$
c Apply the Third Index Law.	**c** $17^0 = 1$
d Apply the Third Index Law.	**d** $5x^0 = 5 \times x^0$ $= 5 \times 1$ $= 5$
e Apply the Third Index Law.	**e** $(5x)^0 + 2 = 1 + 2$ $= 3$
f Apply the Third Index Law.	**f** $5^0 + 3^0 = 1 + 1$ $= 2$

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WORKED EXAMPLE 7

Simplify $\dfrac{9g^7 \times 4g^4}{6g^3 \times 2g^8}$.

THINK

1 Simplify the numerator and the denominator by applying the First Index Law.

2 Simplify the fraction further by applying the Second Index Law.

3 Simplify by applying the Third Index Law.

WRITE

$$\frac{9g^7 \times 4g^4}{6g^3 \times 2g^8}$$

$$= \frac{36g^{11}}{12g^{11}}$$

$$= \frac{{}^{3}\cancel{36}g^{11}}{{}_{1}\cancel{12}g^{11}}$$

$$= 3g^0$$
$$= 3 \times 1$$
$$= 3$$

10.2.5 Cancelling fractions

- Consider the fraction $\frac{x^3}{x^7}$. This fraction can be cancelled by dividing the denominator and the numerator by the highest common factor (HCF), x^3, so $\frac{x^3}{x^7} = \frac{1}{x^4}$.

  Note: $\frac{x^3}{x^7} = x^{-4}$ by applying the Second Index Law. We will study negative indices in a later section.

WORKED EXAMPLE 8

Simplify these fractions by cancelling.

a $\dfrac{x^5}{x^7}$ **b** $\dfrac{6x}{12x^8}$ **c** $\dfrac{30x^5y^6}{10x^7y^3}$

THINK

a Divide the numerator and denominator by the HCF, x^5.

b Divide the numerator and denominator by the HCF, $6x$.

c Divide the numerator and denominator by the HCF, $10x^5y^3$.

WRITE

a $\dfrac{x^5}{x^7} = \dfrac{1}{x^2}$

b $\dfrac{6x}{12x^8} = \dfrac{6}{12} \times \dfrac{x}{x^8}$
$= \dfrac{1}{2} \times \dfrac{1}{x^7}$
$= \dfrac{1}{2x^7}$

c $\dfrac{30x^5y^6}{10x^7y^3} = \dfrac{30}{10} \times \dfrac{x^5}{x^7} \times \dfrac{y^6}{y^3}$
$= \dfrac{3}{1} \times \dfrac{1}{x^2} \times \dfrac{y^3}{1}$
$= \dfrac{3y^3}{x^2}$

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Exercise 10.2 Review of index laws

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Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 5a–e, 6, 7a–e, 8–11, 13–18	Questions: 1–3, 4a–c, 5d–g, 6, 7d–g, 8–19	Questions: 1, 2, 3e–j, 4d–f, 5f–i, 6, 7f–j, 8–20

Individual pathway interactivity: int-4516

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE1 Express each of the following as a product of powers of prime factors using index notation.

a. 12 **b.** 72 **c.** 75
d. 240 **e.** 640 **f.** 9800

2. WE2 Simplify each of the following.

a. $4p^7 \times 5p^4$ **b.** $2x^2 \times 3x^$ **c.** $8y^6 \times 7y^4$
d. $3p \times 7p^7$ **e.** $12t^ \times t^2 \times 7t$ **f.** $6q^2 \times q^5 \times 5q^8$

3. WE3 Simplify each of the following.

a. $2a^2 \times 3a^4 \times e^3 \times e^4$ **b.** $4p^3 \times 2h^7 \times h^5 \times p^3$
c. $2m^3 \times 5m^2 \times 8m^4$ **d.** $2gh \times 3g^2h^5$
e. $5p^4q^2 \times 6p^2q^7$ **f.** $8u^3w \times 3uw^2 \times 2u^5w^4$
g. $9y^8d \times y^5d^3 \times 3y^5d^7$ **h.** $7b^3c^2 \times 2b^6c^4 \times 3b^5c^3$
i. $4r^2s^2 \times 3r^6s^{12} \times 2r^8s^4$ **j.** $10h^{10}v^2 \times 2h^8v^6 \times 3h^{20}v^{12}$

4. WE4 Simplify each of the following.

a. $\frac{15p^{12}}{5p^8}$ **b.** $\frac{18r^6}{3r^2}$ **c.** $\frac{45a^5}{5a^2}$
d. $\frac{60b^7}{20b^4}$ **e.** $\frac{100r^{10}}{5r^6}$ **f.** $\frac{9q^2}{q}$

5. WE5 Simplify each of the following.

a. $\frac{8p^6 \times 3p^4}{16p^5}$ **b.** $\frac{12b^5 \times 4b^2}{18b^2}$ **c.** $\frac{25m^{12} \times 4n^7}{15m^2 \times 8n}$
d. $\frac{27x^9y^3}{12xy^2}$ **e.** $\frac{16h^7k^4}{12h^6k}$ **f.** $\frac{12j^8 \times 6f^5}{8j^3 \times 3f^2}$
g. $\frac{8p^3 \times 7r^2 \times 2s}{6p \times 14r}$ **h.** $\frac{27a^9 \times 18b^5 \times 4c^2}{18a^4 \times 12b^2 \times 2c}$ **i.** $\frac{81f^{15} \times 25g^{12} \times 16h^{34}}{27f^9 \times 15g^{10} \times 12h^{30}}$

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6. WE6 Evaluate the following.

a. m^0 **b.** $6m^0$ **c.** $(6m)^0$ **d.** $(ab)^0$

e. $5(ab)^0$ **f.** w^0x^0 **g.** 85^0 **h.** $85^0 + 15^0$

i. $x^0 + 1$ **j.** $5x^0 - 2$ **k.** $\frac{x^0}{y^0}$ **l.** $x^0 + y^0$

m. $x^0 - y^0$ **n.** $3x^0 + 11$ **o.** $3a^0 + 3b^0$ **p.** $3(a^0 + b^0)$

7. WE7 Simplify each of the following.

a. $\frac{2a^3 \times 6a^2}{12a^5}$ **b.** $\frac{3c^6 \times 6c^3}{9c^9}$ **c.** $\frac{5b^7 \times 10b^5}{25b^{12}}$ **d.** $\frac{8f^3 \times 3f^7}{4f^5 \times 3f^{\cdot}}$

e. $\frac{9k^{12} \times 4k^{10}}{18k^4 \times k^{18}}$ **f.** $\frac{2h^4 \times 5k^2}{20h^2 \times k^2}$ **g.** $\frac{p^3 \times q^4}{5p^3}$ **h.** $\frac{m^7 \times n^3}{5m^{\cdot} \times m^4}$

i. $\frac{8u^9 \times v^2}{2u^5 \times 4u^4}$ **j.** $\frac{9x^6 \times 2y^{12}}{3y^{10} \times 3y^2}$

Understanding

8. WE8 Simplify the following by cancelling.

a. $\frac{x^7}{x^{10}}$ **b.** $\frac{m}{m^9}$ **c.** $\frac{m^3}{4m^9}$ **d.** $\frac{12x^6}{6x^8}$

e. $\frac{12x^8}{6x^6}$ **f.** $\frac{24t^{10}}{t^4}$ **g.** $\frac{5^{\cdot}}{10y^{\cdot}}$ **h.** $\frac{35x^2y^{10}}{20x^7y^7}$

i. $\frac{12m^2n^4}{30m^5n^8}$ **j.** $\frac{16m^5n^{10}}{8m^5n^{12}}$ **k.** $\frac{20x^4y^5}{10x^5y^4}$ **l.** $\frac{a^2b^4c^6}{a^6b^4c^2}$

9. Find the value of each of the following expressions if $a = 3$.

a. $2a$ **b.** a^2 **c.** $2a^2$

d. $a^2 + 2$ **e.** $a^2 + 2a$

Reasoning

10. Explain why x^2 and $2x$ are not the same number. Include an example to illustrate your reasoning.

11. MC **a.** $12a^8b^2c^4(de)^0f$ when simplified is equal to:

A. $12a^8b^2c^4$ **B.** $12a^8b^2c^4f$ **C.** $12a^8b^2f$ **D.** $12a^8b^2$

b. $\left(\frac{6}{11}a^2b^7\right)^0 \times \left(3a^2b^{11}\right)^0 + 7a^0b$ when simplified is equal to:

A. $7b$ **B.** $1 + 7b$ **C.** $-1 + 7ab$ **D.** $-1 + 7b$

c. You are told that there is an error in the statement $3p^7q^3r^5s^6 = 3p^7s^6$. To make the statement correct, what should the left-hand side be?

A. $(3p^7q^3r^5s^6)^0$ **B.** $(3p^7)^0q^3r^5s^6$ **C.** $3p^7(q^3r^5s^6)^0$ **D.** $3p^7(q^3r^5)^0s^6$

d. You are told that there is an error in the statement $\frac{8f^6g^7h^3}{6f^4g^2h} = \frac{8f^2}{g^2}$. To make the statement correct, what should the left-hand side be?

A. $\frac{8f^6(g^7h^3)^0}{(6)^0f^4g^2(h)^0}$ **B.** $\frac{8(f^6g^7h^3)^0}{(6f^4g^2h)^0}$ **C.** $\frac{8(f^6g^7)^0h^3}{(6f^4)^0g^2h}$ **D.** $\frac{8f^6g^7h^3}{(6f^4g^2h)^0}$

e. What does $\frac{6k^7m^2n^8}{4k^7(m^6n)^0}$ equal?

A. $\frac{6}{4}$ **B.** $\frac{3}{2}$ **C.** $\frac{3n^8}{2}$ **D.** $\frac{3m^2n^8}{2}$

12. Explain why $5x^5 \times 3x^3$ is not equal to $15x^{15}$.

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13. A multiple choice question requires a student to multiply 5^6 by 5^3. The student is having trouble deciding which of these four answers is correct: 5^{18}, 5^9, 25^{18} or 25^9.
a. Which is the correct answer?
b. Explain your answer by using another example to explain the First Index Law.

14. A multiple choice question requires a student to divide 5^{24} by 5^8. The student is having trouble deciding which of these four answers is correct: 5^{16}, 5^3, 1^{16} or 1^3.
a. Which is the correct answer?
b. Explain your answer by using another example to explain the Second Index Law.

15. a. What is the value of $\frac{5^7}{5^7}$?
b. What is the value of any number divided by itself?
c. Applying the Second Index Law dealing with exponents and division, $\frac{5^7}{5^7}$ should equal 5 raised to what index?
d. Explain the Third Index Law using an example.

Problem solving

16. a. For $x^2\,x^{\Delta} = x^{16}$ to be an identity, what number must replace the triangle?
b. For $x^{\Delta}\,x^{O}\,x^{\Diamond} = x^{12}$ to be an identity, there are 55 ways of assigning positive whole numbers to the triangle, circle, and diamond. Give at least four of these.

17. a. Can you find a pattern in the units digit for powers of 3?
b. The units digit of 3^6 is 9. What is the units digit of 3^{2001}?

18. a. Can you find a pattern in the units digit for powers of 4?
b. What is the units digit of 4^{105}?

19. a. Investigate the patterns in the units digit for powers of 2 to 9.
b. Predict the units digit for:
i. 2^{35} **ii.** 3^{16} **iii.** 8^{51}.

20. Write $4^{n+1} + 4^{n+1}$ as a single power of 2.

Reflection

How do the index laws aid calculations?

CHALLENGE 10.1

Find the value of each of the following.
a. $2^2 - 1^2$ **b.** $3^2 - 2^2$ **c.** $4^2 - 3^2$
Use your results to investigate a pattern and use it to determine the answer to $87^2 - 86^2$ without actually performing the calculation.

10.3 Raising a power to another power

- $(7^2)^3 = 7^2 \times 7^2 \times 7^2$
  $= 7^{2+2+2}$ (using the First Index Law)
  $= 7^{2\times 3}$
  $= 7^6$
- The indices are multiplied when a power is raised to another power. This is the **Fourth Index Law:** $(a^m)^n = a^{m\times n}$.
- The Fifth and Sixth Index Laws are extensions of the Fourth Index Law.

Fifth Index Law: $(a \times b)^m = a^m \times b^m$.

Sixth Index Law: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

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WORKED EXAMPLE 9

Simplify the following.
a $(7^4)^8$ **b** $(3a^2b^5)^3$

THINK	WRITE
a Simplify by applying the Fourth Index Law (multiply the indices).	**a** $(7^4)^8$ $= 7^{4 \times 8}$ $= 7^{32}$
b 1 Write the expression.	**b** $(3^1a^2b^5)^3$
2 Simplify by applying the Fifth Index Law for each term inside the brackets (multiply the indices).	$= 3^{1 \times 3}a^{2 \times 3}b^{5 \times 3}$ $= 3^3a^6b^{15}$
3 Write the answer.	$= 27a^6b^{15}$

WORKED EXAMPLE 10

Simplify $(2b^5)^2 \times (5b)^3$.

THINK	WRITE
1 Write the expression, including all indices.	$(2^1b^5)^2 \times (5^1b^1)^3$
2 Simplify by applying the Fifth Index Law.	$= 2^2b^{10} \times 5^3b^3$
3 Simplify further by applying the First Index Law.	$= 4 \times 125 \times b^{10} \times b^3$ $= 500b^{13}$

WORKED EXAMPLE 11

TI | CASIO

Simplify $\left(\frac{2a^5}{d^2}\right)^3$.

THINK	WRITE
1 Write the expression, including all indices.	$\left(\frac{2^1a^5}{d^2}\right)^3$
2 Simplify by applying the Sixth Index Law for each term inside the brackets.	$= \frac{2^3a^{15}}{d^6}$
3 Write the answer.	$= \frac{8a^{15}}{d^6}$

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Exercise 10.3 Raising a power to another power

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Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–f, 2a–f, 3a–d, 4–12, 14, 15	Questions: 1d–i, 2d–i, 3b–e, 4–12, 14–18	Questions: 1g–i, 2g–i, 3e–h, 4–18

Individual pathway interactivity: int-4517

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Fluency

1. WE9 Simplify each of the following.

a. $(e^2)^3$ **b.** $(f^8)^{10}$ **c.** $(p^{25})^4$
d. $(r^{12})^{12}$ **e.** $(a^2b^3)^4$ **f.** $(pq^3)^5$
g. $(g^3h^2)^{10}$ **h.** $(3w^9q^2)^4$ **i.** $(7e^5r^2q^4)^2$

2. WE10 Simplify each of the following.

a. $(p^4)^2 \times (q^3)^2$ **b.** $(r^5)^3 \times (w^3)^3$ **c.** $(b^5)^2 \times (c^3)^6$
d. $(j^6)^3 \times (g^4)^3$ **e.** $(q^2)^2 \times (r^4)^5$ **f.** $(h^3)^8 \times (j^2)^7$
g. $(f^4)^4 \times (a^7)^3$ **h.** $(t^5)^2 \times (u^4)^2$ **i.** $(i^3)^5 \times (j^2)^6$

3. WE11 Simplify each of the following.

a. $\left(\frac{3b^4}{d^3}\right)^2$ **b.** $\left(\frac{5h^{10}}{2j^2}\right)^2$ **c.** $\left(\frac{2k^5}{3t^8}\right)^3$ **d.** $\left(\frac{7p^9}{8q^{22}}\right)^2$
e. $\left(\frac{5y^7}{3z^{13}}\right)^3$ **f.** $\left(\frac{4a^3}{7c^5}\right)^4$ **g.** $\left(\frac{-4k^2}{7m^6}\right)^3$ **h.** $\left(\frac{-2g^7}{3h^{11}}\right)^4$

Understanding

4. Simplify each of the following.

a. $(2^3)^4 \times (2^4)^2$ **b.** $(t^3)^3 \times (t^3)^4$ **c.** $(a^4)^0 \times (a^3)^7$
d. $(b^6)^2 \times (b^4)^3$ **e.** $(e^8)^8 \times (e^5)^2$ **f.** $(g^7)^3 \times (g^9)^2$
g. $(3a^2)^4 \times (2a^6)^2$ **h.** $(2d^7)^3 \times (3d^2)^3$ **i.** $(10r^{12})^4 \times (2r^3)^2$

5. MC What does $(p^7)^2 \div p^2$ equal?

A. p^7 **B.** p^{12} **C.** p^{16} **D.** $p^{4.5}$

6. MC What does $\frac{(w^5)^2 \times (p^7)^3}{(w^2)^2 \times (p^3)^5}$ equal?

A. w^2p^6 **B.** $(wp)^6$ **C.** $w^{14}p^{36}$ **D.** w^2p^2

7. MC What does $(r^6)^3 \div (r^4)^2$ equal?

A. r^3 **B.** r^4 **C.** r^8 **D.** r^{10}

8. Simplify each of the following.

a. $(a^7)^3 \div (a^2)^3$ **b.** $(m^8)^2 \div (m^3)^4$ **c.** $(n^5)^3 \div (n^6)^2$
d. $(b^4)^5 \div (b^6)^2$ **e.** $(f^7)^3 \div (f^2)^2$ **f.** $(g^8)^2 \div (g^5)^2$
g. $(p^9)^3 \div (p^6)^3$ **h.** $(y^4)^4 \div (y^7)^2$ **i.** $\frac{(c^6)^5}{(c^5)^2}$
j. $\frac{(f^5)^3}{(f^2)^4}$ **k.** $\frac{(k^3)^{10}}{(k^2)^8}$ **l.** $\frac{(p^{12})^3}{(p^{10})^2}$

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Reasoning

9. a. Simplify each of the following.

i. $(-1)^{10}$ **ii.** $(-1)^{7}$ **iii.** $(-1)^{15}$ **iv.** $(-1)^{6}$

b. Write a general rule for the result obtained when -1 is raised to a positive power. Justify your solution.

10. a. Replace the triangle with the correct index for $4^7 \times 4^7 \times 4^7 \times 4^7 \times 4^7 = (4^7)^{\Delta}$.

b. The expression $(p^5)^6$ means to write p^5 as a factor how many times?

c. If you rewrote the expression from part **b** without any exponents, as $p \times p \times p \ldots$, how many factors would you need?

d. Explain the Fourth Index Law.

11. A multiple choice question requires a student to calculate $(5^4)^3$. The student is having trouble deciding which of these three answers is correct: 5^{64}, 5^{12} or 5^7.

a. Which is the correct answer?

b. Explain your answer by using another example to explain the Fourth Index Law.

12. Jo and Danni are having an algebra argument. Jo is sure that $-x^2$ is equivalent to $(-x)^2$, but Danni thinks otherwise. Explain who is correct and justify your answer.

13. a. Without using your calculator, simplify each side to the same base and solve each of the following equations.

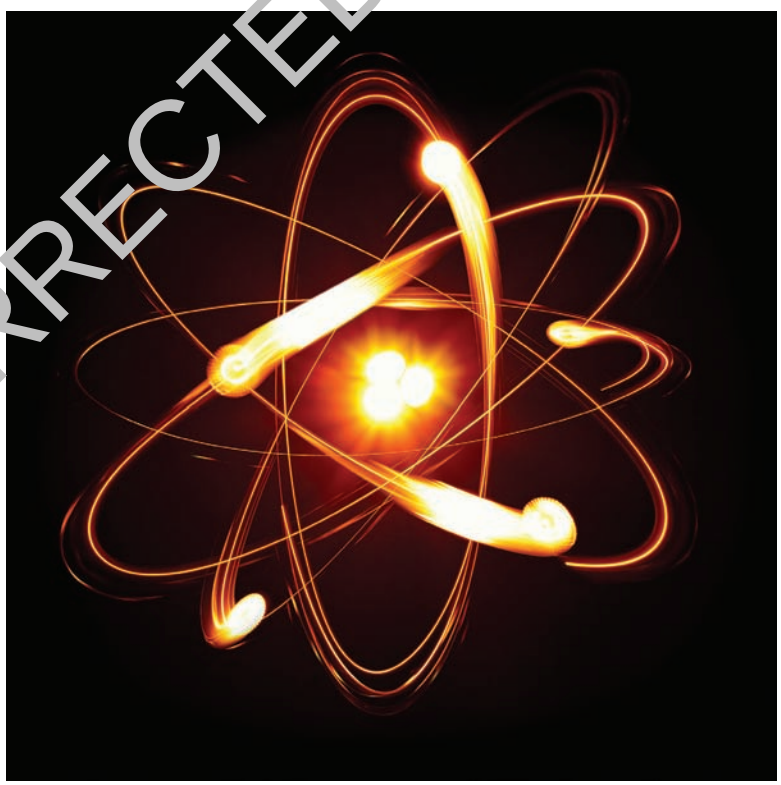
i. $8^x = 32$ **ii.** $27^x = 243$ **iii.** $1000^x = 100000$

b. Explain why all three equations have the same solution.

Problem solving

14. Consider the expression 4^{3^2}. Explain how you could get two different answers.

15. The diameter of a typical atom is so small that it would take about 10^8 of them, arranged in a line, to reach just one centimetre. Estimate how many atoms are contained in a cubic centimetre. Write this number as a power of 10.



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16. Writing a base as a power itself can be used to simplify an expression.
Copy and complete the following calculations.

a. $16^{\frac{1}{2}} = (4^2)^{\frac{1}{2}} =$ ..... **b.** $343^{\frac{2}{3}} = (7^3)^{\frac{2}{3}} =$ .....

17. Simplify the following using index laws.

a. $8^{\frac{1}{3}}$ **b.** $27^{\frac{4}{3}}$ **c.** $125^{-\frac{2}{3}}$ **d.** $512^{\frac{2}{9}}$

e. $16^{-\frac{1}{2}}$ **f.** $4^{-\frac{1}{2}}$ **g.** $32^{-\frac{1}{5}}$ **h.** $49^{-\frac{1}{2}}$

18. a. Use the index laws to simplify the following.

i. $(3^2)^{\frac{1}{2}}$ **ii.** $(4^2)^{\frac{1}{2}}$ **iii.** $(8^2)^{\frac{1}{2}}$ **iv.** $(11^2)^{\frac{1}{2}}$

b. Use your answers from part **a** to calculate the value of the following.

i. $9^{\frac{1}{2}}$ **ii.** $16^{\frac{1}{2}}$ **iii.** $64^{\frac{1}{2}}$ **iv.** $121^{\frac{1}{2}}$

c. Use your answers to parts **a** and **b** to write a sentence describing what raising a number to a power of one-half does.

Reflection

What difference, if any, is there between the operation of the index laws on numeric terms compared with similar operations on algebraic terms?

10.4 Negative indices

- As previously stated, $\frac{x^4}{x^6} = \frac{1}{x^2}$ if the numerator and denominator are both divided by the highest common factor, x^4.

  However, $\frac{x^4}{x^6} = x^{4-6} = x^{-2}$ if the Second Index Law is applied.

  It follows that $\boldsymbol{a^{-n} = \frac{1}{a^n}}$.

WORKED EXAMPLE 12

Evaluate the following.

a 5^{-2} **b** 7^{-1} **c** $\left(\frac{3}{5}\right)^{-1}$

THINK	WRITE
a 1 Apply the rule $a^{-n} = \frac{1}{a^n}$.	**a** $5^{-2} = \frac{1}{5^2}$
2 Simplify.	$= \frac{1}{25}$
b Apply the rule $a^{-n} = \frac{1}{a^n}$.	**b** $7^{-1} = \frac{1}{7^1}$
	$= \frac{1}{7}$
c 1 Apply the Sixth Index Law, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.	**c** $\left(\frac{3^1}{5^1}\right)^{-1} = \frac{3^{-1}}{5^{-1}}$
2 Apply the rule $a^{-n} = \frac{1}{a^n}$ to the numerator and denominator.	$= \frac{1}{3} \div \frac{1}{5}$
3 Simplify and write the answer.	$= \frac{1}{3} \times \frac{5}{1}$
	$= \frac{5}{3}$

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WORKED EXAMPLE 13

Write the following with positive indices.

a x^{-3} **b** $5x^{-6}$ **c** $\dfrac{x^{-3}}{y^{-2}}$

THINK

a Apply the rule $a^{-n} = \dfrac{1}{a^n}$.

b 1 Write in expanded form and apply the rule $a^{-n} = \dfrac{1}{a^n}$.

2 Simplify.

c 1 Write the fraction using division.

2 Apply the rule $a^{-n} = \dfrac{1}{a^n}$.

3 Simplify.

WRITE

a $x^{-3} = \dfrac{1}{x^3}$

b $5x^{-6} = 5 \times x^{-6}$

$= 5 \times \dfrac{1}{x^6}$

$= \dfrac{5}{x^6}$

c $\dfrac{x^{-3}}{y^{-2}} = x^{-3} \div y^{-2}$

$= \dfrac{1}{x^3} \div \dfrac{1}{y^2}$

$= \dfrac{1}{x^3} \times \dfrac{y^2}{1}$

$= \dfrac{y^2}{x^3}$

WORKED EXAMPLE 14

Simplify the following expressions, writing your answers with positive indices.

a $x^3 \times x^{-8}$

b $3x^{-2}y^{-3} \times 5xy^{-4}$

THINK

a 1 Apply the First Index Law, $a^n \times a^m = a^{m+n}$.

2 Write the answer with a positive index.

b 1 Write in expanded form and apply the First Index Law.

2 Apply the rule $a^{-n} = \dfrac{1}{a^n}$.

3 Simplify.

WRITE

a $x^3 \times x^{-8} = x^{3+-8}$

$= x^{-5}$

$= \dfrac{1}{x^5}$

b $3x^{-2}y^{-3} \times 5xy^{-4} = 3 \times 5 \times x^{-2} \times x^1 \times y^{-3} \times y^{-4}$

$= 15x^{-1}y^{-7}$

$= \dfrac{15}{1} \times \dfrac{1}{x} \times \dfrac{1}{y^7}$

$= \dfrac{15}{xy^7}$

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WORKED EXAMPLE 15

Simplify the following expressions, writing your answers with positive indices.

a $\dfrac{t^2}{t^{-5}}$ **b** $\dfrac{15m^{-5}}{10m^{-2}}$

THINK

a Apply the Second Index Law, $\dfrac{a^n}{a^m} = a^{n-m}$.

b 1 Apply the Second Index Law and simplify.

2 Write the answer with positive indices.

WRITE

a $\dfrac{t^2}{t^{-5}} = t^{2-(-5)}$
$= t^{2+5}$
$= t^7$

b $\dfrac{15m^{-5}}{10m^{-2}} = \dfrac{15}{10} \times \dfrac{m^{-5}}{m^{-2}}$
$= \dfrac{3}{2} \times m^{-5-(-2)}$
$= \dfrac{3}{2} \times m^{-3}$
$= \dfrac{3}{2} \times \dfrac{1}{m^3}$
$= \dfrac{3}{2m^3}$

Exercise 10.4 Negative indices

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–10, 13, 14	Questions: 1–11, 13–16	Questions: 1–17

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Fluency

1. Copy and complete the patterns below.

a. $3^5 = 243$
$3^4 = 81$
$3^3 = 27$
$3^2 =$
$3^1 =$
$3^0 =$
$3^{-1} = \dfrac{1}{3}$
$3^{-2} = \dfrac{1}{9}$
$3^{-3} =$
$3^{-4} =$
$3^{-5} =$

b. $5^4 = 625$
$5^3 =$
$5^2 =$
$5^1 =$
$5^0 =$
$5^{-1} =$
$5^{-2} =$
$5^{-3} =$
$5^{-4} =$

c. $10^4 = 10\,000$
$10^3 =$
$10^2 =$
$10^1 =$
$10^0 =$
$10^{-1} =$
$10^{-2} =$
$10^{-3} =$
$10^{-4} =$

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2. WE12 Evaluate each of the following expressions.

a. 2^{-5} **b.** 3^{-3} **c.** 4^{-1} **d.** 10^{-2}

e. 5^{-3} **f.** $\left(\frac{1}{7}\right)^{-1}$ **g.** $\left(\frac{3}{4}\right)^{-1}$ **h.** $\left(\frac{3}{4}\right)^{-2}$

i. $\left(\frac{1}{3}\right)^{-3}$ **j.** $\left(\frac{3}{2}\right)^{-1}$ **k.** $\left(2\frac{1}{4}\right)^{-2}$ **l.** $\left(\frac{2}{7}\right)^{-2}$

3. WE13 Write each expression with positive indices.

a. x^{-4} **b.** y^{-5} **c.** z^{-1} **d.** a^2b^{-3}

e. $7m^{-2}$ **f.** $m^{-2}n^{-3}$ **g.** $(m^2n^3)^{-1}$ **h.** $\frac{x^2}{y^{-2}}$

i. $\frac{5}{x^{-3}}$ **j.** $\frac{x^{-2}}{w^{-5}}$ **k.** $\frac{1}{x^{-2}y^{-2}}$ **l.** $a^2b^{-3}cd^{-4}$

m. $\frac{a^2b^{-2}}{c^2d^{-3}}$ **n.** $10x^{-2}y$ **o.** $3^{-1}x$ **p.** $\frac{m^{-3}}{x^2}$

Understanding

4. WE14 Simplify the following expressions, writing your answers with positive indices.

a. $a^3 \times a^{-8}$ **b.** $m^7 \times m^{-2}$

c. $m^{-3} \times m^{-4}$ **d.** $2x^{-2} \times 7x$

e. $x^5 \times x^{-8}$ **f.** $3x^2y^{-4} \times 2x^{-7}y$

g. $10x^5 \times 5x^{-2}$ **h.** $x^5 \times x^{-5}$

i. $10a^2 \times 5a^{-7}$ **j.** $10a^{10} \times a^{-}$

k. $16w^2 \times -2w^{-5}$ **l.** $4m^{-2} \times 4m^{-}$

m. $(3m^2n^{-4})^3$ **n.** $(a^2b^{5})^{-3}$

o. $(a^{-1}b^{-3})^{-2}$ **p.** $(5a^{-1})^2$

5. WE15 Simplify the following expressions, writing your answers with positive indices.

a. $\frac{x^3}{x^8}$ **b.** $\frac{x^{-3}}{x^8}$ **c.** $\frac{x^3}{x^{-8}}$ **d.** $\frac{x^{-3}}{x^{-8}}$

e. $\frac{10a^4}{5a^5}$ **f.** $\frac{6a^2c^5}{a^7c}$ **g.** $10a^2 \div 5a^8$ **h.** $5m^7 \div m^8$

i. $\frac{a^5b^6}{a^5b^7}$ **j.** $\frac{a^2b^8}{a^5b^{10}}$ **k.** $\frac{a^{-3}bc^3}{abc}$ **l.** $\frac{4^{-2}ab}{a^2b}$

m. $\frac{m^{-3} \times m^{-5}}{m^{-5}}$ **n.** $\frac{2t^2 \times 3t^{-5}}{4t^6}$ **o.** $\frac{t^3 \times t^{-5}}{t^{-2} \times t^{-3}}$ **p.** $\frac{(m^2n^{-3})^{-1}}{(m^{-2}n^3)^2}$

6. Write the following numbers as powers of 2.

a. 1 **b.** 8 **c.** 32

d. 64 **e.** $\frac{1}{8}$ **f.** $\frac{1}{32}$

7. Write the following numbers as powers of 4.

a. 1 **b.** 4 **c.** 64

d. $\frac{1}{4}$ **e.** $\frac{1}{16}$ **f.** $\frac{1}{64}$

8. Write the following numbers as powers of 10.

a. 1 **b.** 10 **c.** 10000

d. 0.1 **e.** 0.01 **f.** 0.00001

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Reasoning

9. a. The result of dividing 3^7 by 3^3 is 3^4. What is the result of dividing 3^3 by 3^7?
b. Explain what it means to have a negative index.
c. Explain how you write a negative index as a positive index.

10. Indices are encountered in science, where they help to deal with very small and large numbers. The diameter of a proton is 0.000 000 000 000 3 cm. Explain why it is logical to express this number in scientific notation as 3×10^{-13}.

11. a. When asked to find an expression that is equivalent to $x^3 + x^{-3}$, a student responded x^0. Is this answer correct? Explain why or why not.
b. When asked to find an expression that is equivalent to $(x^{-1} + y^{-1})^{-2}$, a student responded $x^2 + y^2$. Is this answer correct? Explain why or why not.

12. a. When asked to find an expression that is equivalent to $x^8 - x^{-5}$, a student responded x^3. Is this answer correct? Explain why or why not.
b. Another student said that $\frac{x^2}{x^8 - x^5}$ is equivalent to $\frac{1}{x^6} - \frac{1}{x^3}$. Is this answer correct? Explain why or why not.

Problem solving

13. What is the value of n in the following expressions?
a. $4793 = 4.793 \times 10^n$
b. $0.631 = 6.31 \times 10^n$
c. $134 = 1.34 \times 10^n$
d. $0.000\,56 = 5.6 \times 10^n$

14. Write the following numbers as basic numerals.
a. 4.8×10^{-2}
b. 7.6×10^3
c. 2.9×10^{-4}
d. 8.1×10^0

15. Find a half of 2^{20}.

16. Find one-third of 3^{21}.

17. Simplify the following expressions.
a. $(2^{-1} + 3^{-1})^{-1}$
b. $\frac{3^{400}}{3^{200}}$
c. $\sqrt{16x^{16}}$

Reflection

What strategy will you use to remember the index laws?

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10.5 Square roots and cube roots

10.5.1 Square root

- The symbol $\sqrt{\ }$ means square root — a number that multiplies by itself to give the original number.
- Each number actually has a positive and negative square root. For example, $(2)^2 = 4$ and $(-2)^2 = 4$. Therefore the square root 4 is +2 or –2. For this chapter, assume $\sqrt{\ }$ is positive unless otherwise indicated.
- The square root is the inverse of squaring (power 2).
- For this reason, a square root is equivalent to an index of $\frac{1}{2}$.
- In general, $\sqrt{a} = a^{\frac{1}{2}}$.

WORKED EXAMPLE 16

TI | CASIO

Evaluate $\sqrt{16p^2}$.

THINK	WRITE
1 We need to obtain the square root of both 16 and p^2.	$\sqrt{16p^2} = \sqrt{16} \times \sqrt{p^2}$
2 Which number is multiplied by itself to give 16? It is 4. Replace the square root sign with a power of $\frac{1}{2}$.	$= 4 \times (p^2)^{\frac{1}{2}}$
3 Use the Fourth Index Law.	$= 4 \times p^{2 \times \frac{1}{2}}$
	$= 4 \times p^1$
4 Simplify.	$= 4p$

10.5.2 Cube root

- The symbol $\sqrt[3]{\ }$ means cube root — a number that multiplies by itself three times to give the original number.
- The cube root is the inverse of cubing (power 3).
- For this reason, a square root is equivalent to an index of $\frac{1}{3}$.
- In general, $\sqrt[3]{a} = a^{\frac{1}{3}}$.

WORKED EXAMPLE 17

TI | CASIO

Evaluate $\sqrt[3]{8j^6}$.

THINK	WRITE
1 We need to obtain the cube root of both 8 and j^6.	$\sqrt[3]{8j^6} = \sqrt[3]{8} \times \sqrt[3]{j^6}$
2 Which number, written 3 times and multiplied gives 8? It is 2. Replace the cube root sign with a power of $\frac{1}{3}$.	$= 2 \times (j^6)^{\frac{1}{3}}$
3 Use the Fourth Index Law.	$= 2 \times j^{6 \times \frac{1}{3}}$
	$= 2 \times j^2$
4 Simplify.	$= 2j^2$

- In general terms, $a^{\frac{n}{m}} = \sqrt[m]{a^n}$.

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Exercise 10.5 Square roots and cube roots

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Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 10, 11	Questions: 1–8, 10–12	Questions: 1–13

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Fluency

1. Write the following in surd form.

a. $x^{\frac{1}{2}}$ **b.** $y^{\frac{1}{5}}$ **c.** $z^{\frac{1}{4}}$ **d.** $(2w)^{\frac{1}{3}}$ **e.** $7^{\frac{1}{2}}$

2. Write the following in index form.

a. $\sqrt{15}$ **b.** $\sqrt{m}$ **c.** $\sqrt[3]{t}$ **d.** $\sqrt[3]{v^2}$ **e.** $\sqrt[5]{n}$

3. WE16 Evaluate the following.

a. $49^{\frac{1}{2}}$ **b.** $4^{\frac{1}{2}}$ **c.** $27^{\frac{1}{3}}$ **d.** $125^{\frac{1}{3}}$

e. $1000^{\frac{1}{3}}$ **f.** $64^{\frac{1}{2}}$ **g.** 64 **h.** $128^{\frac{1}{7}}$

i. $243^{\frac{1}{5}}$ **j.** $1\,000\,000^{\frac{1}{2}}$ **k.** $1\,000\,000^{\frac{1}{3}}$ **l.** $(27^{\frac{1}{3}})^2$

Understanding

4. WE17 Simplify the following expressions.

a. $\sqrt{m^2}$ **b.** $\sqrt[3]{b^3}$ **c.** $\sqrt{36t^4}$ **d.** $\sqrt[3]{m^3n^6}$

e. $\sqrt[3]{125t^6}$ **f.** $\sqrt[5]{x^5y^{10}}$ **g.** $\sqrt[4]{a^8m^{40}}$ **h.** $\sqrt[3]{216y^6}$

i. $\sqrt[3]{64x^6y^6}$ **j.** $\sqrt{25a^2b^4c^6}$ **k.** $\sqrt[7]{b^{49}}$ **l.** $\sqrt[3]{b^3} \times \sqrt{b^4}$

5. MC **a.** What does $\sqrt[3]{8000m^6n^3p^3q^6}$ equal?

A. $2666.6m^2npq^2$ **B.** $20m^2npq^2$ **C.** $20m^3n^0p^0q^3$ **D.** $7997m^2npq^2$

b. What does $\sqrt[3]{3375a^9b^6c^3}$ equal?

A. $1125a^3b^2c$ **B.** $1125a^6b^3c^0$ **C.** $1123a^6b^3$ **D.** $15a^3b^2c$

c. What does $\sqrt[3]{5625f^3g^9h^9}$ equal?

A. $25fg^2h^3$ **B.** $25f^0g^3h^6$ **C.** $25g^3h^6$ **D.** $5208.3fg^2h^3$

Reasoning

6. a. Using the First Index Law, explain how $3^{\frac{1}{2}} \times 3^{\frac{1}{2}} = 3$.

b. What is another way that $3^{\frac{1}{2}}$ can be written?

c. Find $\sqrt{3} \times \sqrt{3}$.

d. How can $\sqrt[n]{a}$ be written in index form?

e. Without a calculator, solve:

i. $8^{\frac{1}{3}}$ **ii.** $32^{\frac{2}{5}}$.

7. a. Explain why calculating $z^{2.5}$ is a square root problem.

b. Is $z^{0.3}$ a cube root problem? Justify your reasoning.

8. Mark and Christina are having an algebra argument. Mark is sure that $\sqrt{x^2}$ is equivalent to x, but Christina thinks otherwise. Who is correct? Explain how you would resolve this disagreement.

9. Verify that $(-8)^{\frac{1}{3}}$ can be evaluated and explain why $(-8)^{\frac{1}{4}}$ cannot be evaluated.

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Problem solving

10. If $n^{\frac{3}{4}} = \frac{8}{27}$, what is the value of n?

11. The mathematician Augustus de Morgan enjoyed telling his friends that he was x years old in the year x^2. Find the year of Augustus de Morgan's birth, given that he died in 1871.

12. a. Investigate Johannes Kepler.

b. Kepler's Third Law describes the relationship between the distance of planets from the Sun and their orbital periods. It is represented by the equation $d^{\frac{1}{2}} = t^{\frac{1}{3}}$. Solve for:

i. d in terms of t **ii.** t in terms of d.

Johannes Kepler.

13. An unknown number is multiplied by 4 and then has five subtracted from it. It is now equal to the square root of the original unknown number squared.

a. Is this a linear algebra problem? Justify your answer.

b. How many solutions are possible? Explain why.

c. Find all possible values for the number.

Reflection

How would $\sqrt[n]{a^b}$ be written in index form?

CHALLENGE 10.2

The pronumeral i is used to represent $\sqrt{-1}$. This is the basis for imaginary numbers. It follows that the value of i^2 is -1. What is the value of i^{40}?

10.6 Review

10.6.1 Review questions

Fluency

1. State the base for each of the following.

a. 4^8 **b.** 5^9 **c.** a^4 **d.** x^7

2. State the power or index for each of the following.

a. 10^3 **b.** 6^{12} **c.** g^{89} **d.** 4^0

3. Write each of the following as a basic numeral.

a. 5^2 **b.** 4^2 **c.** 2^5 **d.** 10^3 **e.** 3^4 **f.** 12^8

4. Express each of the following as a product of powers of primes.

a. 100 **b.** 121 **c.** 104 **d.** 225 **e.** 588 **f.** 10 080

5. Simplify each of the following.

a. $b^7 \times b^3$ **b.** $m^9 \times m^2$ **c.** $k^3 \times k^5$ **d.** $f^2 \times f^8 \times f^4$

e. $h^4 \times h^5 \times h$ **f.** $2q^5 \times 3q^2 \times q^{10}$ **g.** $5w^3 \times 7w^{12} \times w^{14}$ **h.** $2e^2\, p^3 \times 6e^3\, p^5$

i. $5a^2\, b^4 \times 3a^8\, b^5 \times 7a^6\, b^8$

6. Simplify each of the following.

a. $a^5 \div a^2$ **b.** $t^5 \div t$ **c.** $r^{19} \div r^{12}$ **d.** $p^8 \div p^5$

e. $\frac{f^{17}}{f^{12}}$ **f.** $\frac{y^{100}}{y^{10}}$ **g.** $\frac{m^{24}}{m^{14}}$ **h.** $\frac{g^4 \times g^5}{g^2}$

i. $\frac{x^6 \times x^2 \times x}{x^8}$ **j.** $\frac{d^6 \times d^7 \times d^2}{d^8}$ **k.** $\frac{t^7 \times t \times t^3}{t^2 \times t^4}$ **l.** $\frac{p^5 \times p^3 \times p \times p^4}{p^2 \times p^4 \times p^2}$

m. $\frac{16k^{13}}{21} \div \frac{8k^9}{42}$ **n.** $\frac{22b^{15}}{c} \div \frac{2b^8}{c^6}$ **o.** $\frac{9d^8}{16e^{10}} \div \frac{2d^{10}}{e^{16}}$

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7. Simplify the following.

a. 5^0 **b.** 12^0 **c.** 345^0 **d.** q^0

e. r^0 **f.** ab^0 **g.** $3w^0$ **h.** $5q^0 - 2q^0$

i. $100s^0 + 99t^0$ **j.** a^7b^0 **k.** $v^{10}\,w^0$ **l.** prt^0

m. $a^9b^4c^0$ **n.** $j^8k^0m^3$ **o.** $4e^2f^0 - 36(a^2b^3)^0$ **p.** $-8(18x^2y^4z^6)^0$

q. $15 - 12x\left(\frac{3x}{8}\right)^0$ **r.** $-4p^0 \times 6(q^2r^3)^0 \div 8(-12q^2)^0$ **s.** $3(6w^0)^2 \div 2(5w^5)^0$

8. Raise each of the following to the given power.

a. $(b^4)^2$ **b.** $(a^8)^3$ **c.** $(k^7)^{10}$ **d.** $(j^{100})^2$

e. $(a^5b^2)^3$ **f.** $(m^7n^{12})^2$ **g.** $(st^6)^3$ **h.** $(qp^{30})^{10}$

9. What does $\left(\frac{4b^4}{d^2}\right)^3$ equal?

A. $\frac{4b^3}{d^3}$ **B.** $\frac{12b^{12}}{d^6}$ **C.** $\frac{64b^{12}}{d^6}$ **D.** $\frac{64b^{}}{d^{}}$

10. Write each of the following with positive indices.

a. a^{-1} **b.** k^{-4} **c.** $4m^3 \div 2m^7$ **d.** $7x^3y^{-4} \times 6x^{-3}y^{-1}$

11. Write each of the following using a negative index.

a. $\frac{1}{x}$ **b.** $\frac{2}{y^4}$ **c.** $z \div z^4$ **d.** $45p^2q^{-4} \times 3p^{-5}q$

12. Simplify each of the following.

a. $\sqrt{100}$ **b.** $\sqrt{36}$ **c.** $\sqrt{c^2}$ **d.** $\sqrt{b^2}$

e. $\sqrt{49f^4}$ **f.** $\sqrt[3]{27}$ **g.** $\sqrt[3]{1000}$ **h.** $\sqrt[3]{x^3}$

i. $\sqrt[3]{8d^3}$ **j.** $\sqrt[3]{64f^6g^3}$

Problem solving

The chessboard problem

Legend has it that an ancient Chinese king challenged the people around him to invent a game that would keep him amused for hours on end and would challenge him mentally. Days later the king was presented with the game of chess.

The king thought that this was a marvellous game and wanted to reward the inventor. The inventor said, *'I do not ask much for my king's pleasure. All I ask for is one grain of rice for the first square, two grains of rice for the second square, four grains of rice for the third square and so on.'*

The king was amazed that one could ask for so little for such a wonderful game. He sent his mathematician away to calculate exactly what the inventor was to be paid.

The mathematician came back and said that the inventor had asked for more than all the rice in the Kingdom of China!

Questions

13. The inventor asked for one grain of rice for the first square, two for the second, four for the third and so on. For the first row of the board, write down the number of grains of rice for each square.

14. We can write 4 as 2×2, 8 as $2 \times 2 \times 2$ and so on. Write each of the amounts of grain for the first row, in index form.

15. For each square on the board, write the number of grains of rice to be paid, in index notation.

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16. Complete the table below for the first row.

Square number	Grains paid	Total paid
1	1	1
2	2	3
3	4	7
4	8	
5		
6		
7		
8		

17. Can you see a pattern between the numbers in the total paid column and in the grains paid column?

18. Calculate the total number of rice grains to be paid.

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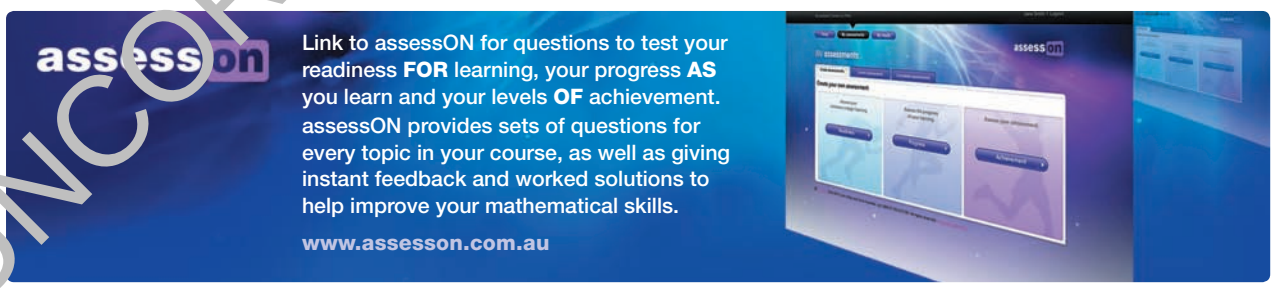
Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

coefficients
composite number
cube root
exponent
factor form
fractional index
index
index laws
index notation
indices
negative index
prime factors
square root
surd form
zero index



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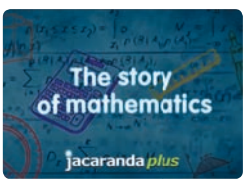

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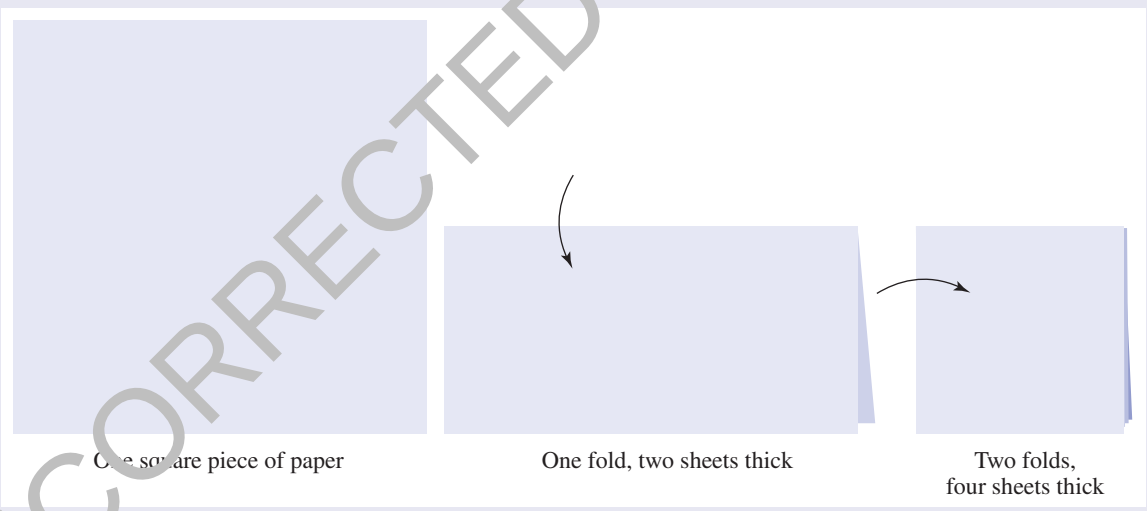
Investigation | Rich task

Paper folds

If you take a piece of paper and fold it a few times, you will soon notice that the area of the upper surface of the paper decreases with each fold. In this task, consider the relationships between the number of folds, the resulting thickness of paper and the upper surface area after each fold has been made.



Take a square piece of paper with a side length of 8 cm. The upper surface of this paper has an area of 64 cm^2. Fold the paper in half. The paper now shows an area of 32 cm^2, has 1 fold and is 2 sheets thick. Make another fold. The diagram above displays the changes that take place for 2 folds.

Continue with the folding process for up to 5 folds. The thickness of the paper and the surface area of the upper face change with each fold.

1. Write the dimensions of each upper surface after each fold.

2. Calculate the area (in cm^2) of each upper surface after each fold.

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3. Complete the following table to show the change in the upper surface area and the thickness after each fold.

Number of folds	0	1	2	3	4	5
Thickness of paper	1	2				
Area of upper surface after each fold (cm^2)	64	32				

4. Study the values recorded in the table in question **3**. Explain whether there is a linear relationship between the number of folds and the thickness of the paper, or between the number of folds and the area after each fold.

Let f represent the number of folds, t represent the thickness of the paper after each fold and a represent the area of the upper face after each fold. A relationship between the pronumerals may be more obvious if the values in the table are presented in a different form.

5. Complete the table below, presenting your values in index form with a base of 2.

Number of folds (f)	0	1	2	3	4	5
Thickness of paper (t)	2^0	2^1				
Area of upper surface after each fold (cm^2) (a)	2^6	2^5				

6. Consider the values in the table above to write a relationship between the following pronumerals.
- t and f
- a and t
- a and f

7. What difference, if any, would it make to these relationships if the original paper size had been a square with side length of 16 cm? Draw a table to show the change in area of each face and the thickness of the paper with each fold. Write formulas to describe these relationships.

8. Investigate these relationship with squares of different side lengths. Describe whether the relationship between the three features studied during this task can always be represented in index form.

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Complete this digital doc: Code puzzle: What is the name given to a boat with one sculler and two oarsmen?
Searchlight ID: doc-15903

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Answers

Topic 10 Indices

Exercise 10.2 Review of index laws

1. a. $2^2 \times 3$ **b.** $2^3 \times 3^2$ **c.** 3×5^2 **d.** $2^4 \times 3 \times 5$ **e.** $2^7 \times 5$ **f.** $2^3 \times 5^2 \times 7^2$

2. a. $20p^{11}$ **b.** $6x^8$ **c.** $56y^{10}$ **d.** $21p^8$ **e.** $84t^6$ **f.** $30q^{15}$

3. a. $6a^6e^7$ **b.** $8p^6h^{12}$ **c.** $80m^9$ **d.** $6g^3h^6$ **e.** $30p^6q^9$ **f.** $48u^9w^7$
g. $27d^{11}y^{17}$ **h.** $42b^{14}c^9$ **i.** $24r^{16}s^{18}$ **j.** $60h^{38}v^{20}$

4. a. $3p^4$ **b.** $6r^4$ **c.** $9a^3$ **d.** $3b^6$ **e.** $20r^4$ **f.** $9q$

5. a. $\frac{3p^5}{2}$ **b.** $\frac{8b^5}{3}$ **c.** $\frac{5m^{10}n^6}{6}$ **d.** $\frac{9x^8y}{4}$ **e.** $\frac{4hk^3}{3}$ **f.** i^5f^3
g. $\frac{4p^2rs}{3}$ **h.** $\frac{9a^5b^3c}{2}$ **i.** $\frac{20f^6g^2h^4}{3}$

6. a. 1 **b.** 6 **c.** 1 **d.** 1 **e.** 5 **f.** 1
g. 1 **h.** 2 **i.** 2 **j.** 3 **k.** 1 **l.** 2
m. 0 **n.** 14 **o.** 6 **p.** 6

7. a. 1 **b.** 2 **c.** 2 **d.** 2 **e.** 2 **f.** $\frac{h^2}{2}$
g. $\frac{q^4}{5}$ **h.** $\frac{n^3}{5}$ **i.** v^2 **j.** $2x^6$

8. a. $\frac{1}{x^3}$ **b.** $\frac{1}{m^8}$ **c.** $\frac{1}{4m^6}$ **d.** $\frac{2}{x^2}$ **e.** $2x^2$ **f.** $24t^6$
g. $\frac{1}{2y^5}$ **h.** $\frac{7y^3}{4x^5}$ **i.** $\frac{2}{5m^3n^4}$ **j.** $\frac{2}{c^2}$ **k.** $\frac{2y}{x}$ **l.** $\frac{c^4}{a^4}$

9. a. 6 **b.** 9 **c.** 18 **d.** 11 **e.** 15

10. Answers will vary.

11. a. B **b.** D **c.** D **d.** A **e.** D

12. Answers will vary.

13. a. 5^9 **b.** Answers will vary.

14. a. 5^{16} **b.** Answers will vary.

15. a. 1 **b.** 1 **c.** Zero **d.** Answers will vary.

16. a. $\Delta = 14$

b. Answers will vary, but $\Delta + O + \Diamond$ must sum to 12. Possible answers include: $\Delta = 3$, $O = 2$, $\Diamond = 7$; $\Delta = 1$, $O = 3$, $\Diamond = 8$; $\Delta = 4$, $O = 4$, $\Diamond = 4$; $\Delta = 5$, $O = 1$, $\Diamond = 6$.

17. a. The repeating pattern is 1, 3, 9, 7.

b. 3

18. a. The repeating pattern is 4, 6.

b. 4

19. a. Answers will vary.

b. i. 8 **ii.** 1 **iii.** 2

20. 2^{3n+3}

Challenge 10.1

a. 3 **b.** 5 **c.** 7

$87^2 - 86^2 = 173$

Exercise 10.3 Raising a power to another power

1. a. e^6 **b.** f^{80} **c.** p^{100} **d.** r^{144} **e.** a^8b^{12}
f. p^5q^{15} **g.** $g^{30}h^{20}$ **h.** $81w^{36}q^8$ **i.** $49e^{10}r^4q^8$

2. a. p^8q^6 **b.** $r^{15}w^9$ **c.** $b^{10}n^{18}$ **d.** $j^{18}g^{12}$ **e.** q^4r^{20}
f. $h^{24}j^{16}$ **g.** $a^{21}f^{16}$ **h.** $t^{10}u^8$ **i.** $i^{15}j^{12}$

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3. a. $\frac{9b^8}{d^6}$ **b.** $\frac{25h^{20}}{4j^4}$ **c.** $\frac{8k^{15}}{27t^{24}}$ **d.** $\frac{49p^{18}}{64q^{44}}$ **e.** $\frac{125y^{21}}{27z^{39}}$

f. $\frac{256a^{12}}{2401c^{20}}$ **g.** $\frac{-64k^6}{343m^{18}}$ **h.** $\frac{16g^{28}}{81h^{44}}$

4. a. 2^{20} **b.** t^{33} **c.** a^{21} **d.** b^{24} **e.** e^{66}

f. g^{39} **g.** $324a^{20}$ **h.** $216d^{27}$ **i.** $40\,000r^{54}$

5. B

6. B

7. D

8. a. a^6 **b.** m^4 **c.** n^3 **d.** b^8 **e.** f^{17} **f.** g^0

g. p^9 **h.** y^2 **i.** c^{20} **j.** f^7 **k.** k^{14} **l.** p^{16}

9. a. i. 1 **ii.** -1 **iii.** -1 **iv.** 1

b. $(-1)^{\text{even}} = 1(-1)^{\text{odd}} = -1$

10. a. 5 **b.** 6 **c.** 30 **d.** Answers will vary.

11. a. 5^{12} **b.** Answers will vary.

12. Danni is correct. Explanations will vary but should involve $(-x)(-x) = (-x)^2 = x^2$ and $-x^2 = -1 \times x^2 = -x^2$.

13. a. i. $x = \frac{5}{3}$ **ii.** $x = \frac{5}{3}$ **iii.** $x = \frac{5}{3}$

b. When equating the powers, $3x = 5$.

14. Answers will vary. Possible answers are 4096 and 262 144.

15. $10^8 \times 10^8 \times 10^8 = (10^8)^3$ atoms

16. a. 4^1 **b.** 7^2

17. a. 2^1 **b.** 3^4 **c.** $\frac{1}{5^2}$ **d.** 2^{-3}

e. $\frac{1}{2^2}$ **f.** $\frac{1}{2}$ **g.** $\frac{1}{2}$ **h.** $\frac{1}{7}$

18. a. i. 3 **ii.** 4 **iii.** 8 **iv.** 11

b. i. 3 **ii.** 4 **iii.** 8 **iv.** 11

c. Raising a number to a power of one-half is the same as finding the square root of that number.

Exercise 10.4 Negative indices

1. a. $3^5 = 243$, $3^4 = 81$, $3^3 = 27$, $3^2 = 9$, $3^1 = 3$, $3^0 = 1$, $3^{-1} = \frac{1}{3}$, $3^{-2} = \frac{1}{9}$, $3^{-3} = \frac{1}{27}$, $3^{-4} = \frac{1}{81}$, $3^{-5} = \frac{1}{243}$

b. $5^4 = 625$, $5^3 = 125$, $5^2 = 25$, $5^1 = 5$, $5^0 = 1$, $5^{-1} = \frac{1}{5}$, $5^{-2} = \frac{1}{25}$, $5^{-3} = \frac{1}{125}$, $5^{-4} = \frac{1}{625}$

c. $10^4 = 10\,000$, $10^3 = 1000$, $10^2 = 100$, $10^1 = 10$, $10^0 = 1$, $10^{-1} = \frac{1}{10}$, $10^{-2} = \frac{1}{100}$, $10^{-3} = \frac{1}{1000}$, $10^{-4} = \frac{1}{10000}$

2. a. $\frac{1}{32}$ **b.** $\frac{1}{27}$ **c.** $\frac{1}{4}$ **d.** $\frac{1}{100}$ **e.** $\frac{1}{125}$ **f.** 7

g. $\frac{4}{3}$ **h.** $\frac{16}{9}$ **i.** 27 **j.** $\frac{2}{3}$ **k.** $\frac{16}{81}$ **l.** $\frac{49}{4}$

3. a. $\frac{1}{x^4}$ **b.** $\frac{1}{y^5}$ **c.** $\frac{1}{z}$ **d.** $\frac{a^2}{b^3}$ **e.** $\frac{7}{m^2}$ **f.** $\frac{1}{m^2n^3}$

g. $\frac{1}{m^2n^3}$ **h.** x^2y^2 **i.** $5x^3$ **j.** $\frac{w^5}{x^2}$ **k.** x^2y^2 **l.** $\frac{a^2c}{b^3d^4}$

m. $\frac{a^2d^3}{b^2c^2}$ **n.** $\frac{10y}{x^2}$ **o.** $\frac{x}{3}$ **p.** $\frac{1}{m^3x^2}$

4. a. $\frac{1}{a^5}$ **b.** m^5 **c.** $\frac{1}{m^7}$ **d.** $\frac{14}{x}$ **e.** $\frac{1}{x^3}$ **f.** $\frac{6}{x^5y^3}$

g. $50x^3$ **h.** 1 **i.** $\frac{50}{a^5}$ **j.** $10a^4$ **k.** $\frac{-32}{w^3}$ **l.** $\frac{16}{m^4}$

m. $\frac{27m^6}{n^{12}}$ **n.** $\frac{1}{a^6b^{15}}$ **o.** a^2b^6 **p.** $\frac{25}{a^2}$

5. a. $\frac{1}{x^5}$ **b.** $\frac{1}{x^{11}}$ **c.** x^{11} **d.** x^5 **e.** $\frac{2}{a}$ **f.** $\frac{6c^4}{a^2}$

g. $\frac{2}{a^6}$ **h.** $\frac{5}{m}$ **i.** $\frac{1}{b}$ **j.** $\frac{1}{a^3b^2}$ **k.** $\frac{c^2}{a^4}$ **l.** $\frac{1}{16a}$

m. $\frac{1}{m^3}$ **n.** $\frac{3}{2t^9}$ **o.** t^3 **p.** $\frac{m^2}{n^3}$

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6. a. 2^0 **b.** 2^3 **c.** 2^5 **d.** 2^6 **e.** 2^{-3} **f.** 2^{-5}

7. a. 4^0 **b.** 4^1 **c.** 4^3 **d.** 4^{-1} **e.** 4^{-2} **f.** 4^{-3}

8. a. 10^0 **b.** 10^1 **c.** 10^4 **d.** 10^{-1} **e.** 10^{-2} **f.** 10^{-5}

9. a. $3^{-4} = \frac{1}{3^4}$

b. Answers will vary but should convey that if dividing, the power in the numerator is lower than that in the denominator.

c. Answers will vary.

10. Answers will vary but should convey that the negative 13 means the decimal point is moved 13 places to the left of 3. Using scientific notation allows the number to be expressed more concisely.

11. a. No. The equivalent expression with positive indices is $\frac{x^6 + 1}{x^3}$.

b. No. The equivalent expression with positive indices is $\frac{(xy)^2}{(x + y)^2}$.

12. a. No. The equivalent expression with positive indices is $\frac{x^{13} - 1}{x^5}$.

b. No. The correct equivalent expression is $\frac{1}{x^6 - x^3}$.

13. a. 3 **b.** –1 **c.** 2 **d.** –4

14. a. 0.048 **b.** 7600 **c.** 0.00029 **d.** 8.1

15. 2^{19}

16. 3^{20}

17. a. $\frac{6}{5}$ **b.** $\left(\frac{3}{2}\right)^{200}$ **c.** $4x^8$

Exercise 10.5 Square roots and cube roots

1. a. $\sqrt{x}$ **b.** $\sqrt[5]{y}$ **c.** $\sqrt[4]{z}$ **d.** $\sqrt[3]{w}$ **e.** $\sqrt{7}$

2. a. $15^{\frac{1}{2}}$ **b.** $m^{\frac{1}{2}}$ **c.** $t^{\frac{1}{3}}$ **d.** $w^2)^{\frac{1}{3}}$ **e.** $n^{\frac{1}{5}}$

3. a. 7 **b.** 2 **c.** 3 **d.** 5 **e.** 10 **f.** 8

g. 4 **h.** 2 **i.** 3 **j.** 1000 **k.** 100 **l.** 9

4. a. m **b.** b **c.** $6t^2$ **d.** mn^2 **e.** $5t^2$ **f.** xy^2

g. a^2m^{10} **h.** $6y^2$ **i.** x^2y^2 **j.** $5ab^2c^3$ **k.** b^7 **l.** b^3

5. a. B **b.** D **c.** A

6. a. $3^{\frac{1}{2}+\frac{1}{2}} = 3^1$ **b.** $\sqrt{3}$ **c.** 1 **d.** $a^{\frac{1}{n}}$

e. i. 2 **ii.** 4

7. a. $z^{2.5} = z^{\frac{5}{2}} = (z^5)^{\frac{1}{2}} = \sqrt{z^5}$

b. No, it is the tenth root: $z^{\frac{3}{10}} = z^{10} = (z^3)^{\frac{1}{10}} = \sqrt[10]{z^3}$.

8. Mark is correct: $\sqrt{x^2} = x^{\frac{1}{2}2} = x^1 = x$; x can be a positive or negative number.

9. $(-2^3)^{\frac{1}{3}} = -2$; answers will vary but should include that we cannot take the fourth root of a negative number.

10. $\frac{16}{81}$

11. $\sqrt{1871} \approx 43.25$

$42^2 = 1764$

$43^2 = 1849$

He was 43 years old in 1849. Therefore, he was born in $1849 - 43 = 1806$.

12. a. Answers will vary.

b. i. $d = t^{\frac{2}{3}}$ **ii.** $t = d^{\frac{3}{2}}$

13. a. No, since it has x^2 and $\sqrt{\ }$.

b. 2, because the square root of a number has a positive and a negative answer

c. $\frac{5}{3}$, 1

Challenge 10.2

$i^{40} = (i^2)^{20} = (-1)^{20} = 1$

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10.6 Review

1. a. 4 **b.** 5 **c.** a **d.** x

2. a. 3 **b.** 12 **c.** 89 **d.** 0

3. a. 25 **b.** 16 **c.** 32 **d.** 1000
e. 81 **f.** 429 981 696

4. a. $2^2 \times 5^2$ **b.** 11^2 **c.** $2^3 \times 13$ **d.** $3^2 \times 5^2$
e. $2^2 \times 3 \times 7^2$ **f.** $2^5 \times 3^2 \times 5 \times 7$

5. a. b^{10} **b.** m^{11} **c.** k^8 **d.** f^{14} **e.** h^{10} **f.** $6q^{17}$
g. $35w^{29}$ **h.** $12e^5p^8$ **i.** $105a^{16}b^{17}$

6. a. a^3 **b.** t^4 **c.** r^7 **d.** p^3 **e.** f^5 **f.** y^{90}
g. m^{10} **h.** g^7 **i.** x **j.** d^7 **k.** t^5 **l.** p^5
m. $4k^4$ **n.** $11b^7c^5$ **o.** $\dfrac{9e^6}{32d^2}$

7. a. 1 **b.** 1 **c.** 1 **d.** 1 **e.** 1
f. a **g.** 3 **h.** 3 **i.** 199 **j.** a^7
k. v^{10} **l.** pr **m.** a^9b^4 **n.** j^8m^3 **o.** $4c^2 - 36$
p. -8 **q.** $15 - 12x$ **r.** -3 **s.** 54

8. a. b^8 **b.** a^{24} **c.** k^{70} **d.** j^{200}
e. $a^{15}b^6$ **f.** $m^{14}n^{24}$ **g.** s^3t^{18} **h.** $q^{10}p^{300}$

9. C

10. a. $\dfrac{1}{a}$ **b.** $\dfrac{1}{k^4}$ **c.** $\dfrac{2}{m^4}$ **d.** $\dfrac{12}{j^5}$

11. a. x^{-1} **b.** $2y^{-4}$ **c.** z^{-3} **d.** $15p^{-3}q^{-3}$

12. a. 10 **b.** 6 **c.** a **d.** b **e.** $7f^2$ **f.** 3
g. 10 **h.** x **i.** $2d$ **j.** $4f^2g$

13.

1	2	4	8	16	32	64	128

14.

2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7

15.

2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7
2^8	2^9	2^{10}	2^{11}	2^{12}	2^{13}	2^{14}	2^{15}
2^{16}	2^{17}	2^{18}	2^{19}	2^{20}	2^{21}	2^{22}	2^{23}
2^{24}	2^{25}	2^{26}	2^{27}	2^{28}	2^{29}	2^{30}	2^{31}
2^{32}	2^{33}	2^{34}	2^{35}	2^{36}	2^{37}	2^{38}	2^{39}
2^{40}	2^{41}	2^{42}	2^{43}	2^{44}	2^{45}	2^{46}	2^{47}
2^{48}	2^{49}	2^{50}	2^{51}	2^{52}	2^{53}	2^{54}	2^{55}
2^{56}	2^{57}	2^{58}	2^{59}	2^{60}	2^{61}	2^{62}	2^{63}

16.

Square number	Grains paid	Total paid
1	1	1
2	2	3
3	4	7
4	8	15
5	16	31
6	32	63
7	64	127
8	128	255

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17. The total paid is 1 less than twice the number of grains paid.
Total paid = 2(Grains paid)−1

18. $2(2^{63}) - 1 = 18\,446\,744\,073\,709\,551\,615$ or $1.844\,674\,407 \times 10^{19}$

Investigation — Rich task

1. Fold 1, 8 cm × 4 cm; fold 2, 4 cm × 4 cm; fold 4, 2 cm × 2 cm; fold 5, 2 cm × 1 cm

2. Fold 1, 32 cm^2; fold 2, 16 cm^2; fold 4, 4 cm^2; fold 5, 2 cm^2

3.

Number of folds	0	1	2	3	4	5
Thickness of paper	1	2	4	8	16	32
Area of surface after each fold (cm^2)	64	32	16	8	4	2

4. There is no linear relationship.

5.

Number of folds (f)	0	1	2	3	4	5
Thickness of paper (t)	2^0	2^1	2^2	2^3	2^4	2^5
Area of surface after each fold (cm^2) (a)	2^6	2^5	2^4	2^3	2^2	2^1

6. $t = 2^f$, $at = 2^6$, $a = 2^{6-f}$

7. $t = 2^f$, $at = 2^8$, $a = 2^{8-f}$

8. Check with your teacher.

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