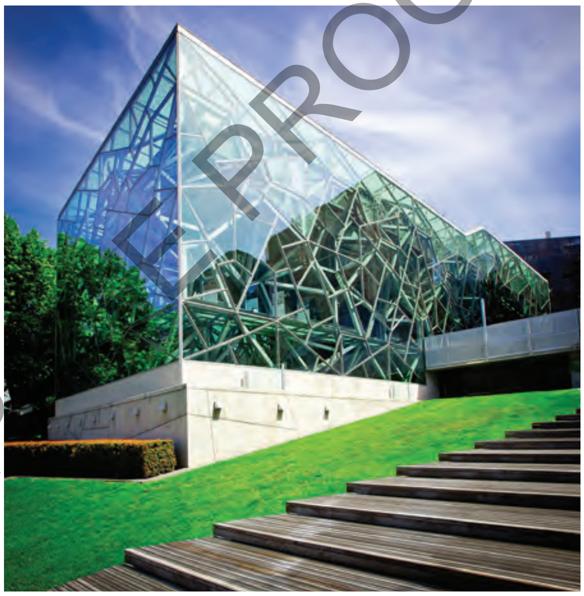
TOPIC 14 Properties of geometrical figures

[Stages 5.2 and 5.3]

14.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.



14.1.1 Why learn this?

assess on

Learning about geometry includes being able to reason deductively and to prove logically that certain mathematical statements are true. It is important to be able to prove theories meticulously and step by step in order to show that the conclusions reached are soundly based. Mathematicians spend most of their time trying to prove new theories, and they rely heavily on all the proofs that have gone before. Reasoning skills, and hence the ability to prove theories, can be developed and learned through practice and application.

DISCUSSION

Euclid is known as 'the father of geometry'. Who are the fathers of some of the other branches of mathematics and the sciences?

LEARNING SEQUENCE

LEARNING OUTCOMES

A student:

- selects appropriate notations and conventions to communicate mathematical ideas and solutions MA5.2-1WM
- interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems MA5.2-2WM
- constructs arguments to prove and justify results MA5.2-3WM
- calculates the angle sum of any polygon and uses minimum conditions to prove triangles are congruent or similar MA5.2-14MG
- uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures MA5.3-1WM
- generalises mathematical ideas and techniques to analyse and solve problems efficiently MA5.3-2WM
- uses deductive reasoning in presenting arguments and formal proofs MA5.3-3WM
- proves triangles are similar, and uses formal geometric reasoning to establish properties of triangles and quadrilaterals MA5.3-16MG

CONTENT DESCRIPTIONS

Students:
Formulate proofs involving congruent triangles and angle properties (ACMMG243)
Use the enlargement transformations to explain similarity and to develop the conditions for triangles to be similar (ACMMG220)
Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes (ACMMG244)
Source: NSW Syllabus for the Australian Curriculum

learnon RESOURCES — ONLINE ONLY

eLesson: The story of mathematics: Euclid (eles-1849)

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

14.2 Angles, triangles and congruence [Stages 5.2 and 5.3]

14.2.1 Proofs and theorems

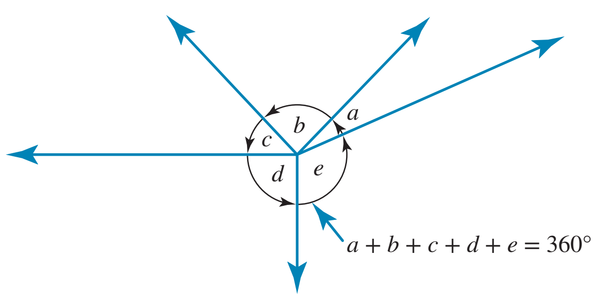
- Euclid (c. 300 BC) was the mathematician who developed a systematic approach to geometry, now referred to as Euclidean geometry, that relied on mathematical proofs.
- A **proof** is an argument that shows why a statement is true.
- A **theorem** is a statement that can be demonstrated to be true. To demonstrate that a statement is proved, formal language needs to be used. It is conventional to use the following structure when setting out a theorem.
  - **Given:** a summary of the information given
  - **To prove:** a statement that needs to be proved
  - **Construction:** a description of any additions to the diagram given
  - **Proof:** a sequence of steps that can be justified and form part of a formal mathematical proof.

DISCUSSION

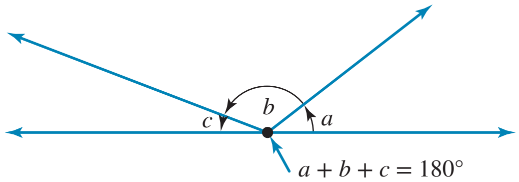
Can you explain the difference between a proof and a theorem to a friend?

14.2.2 Angles at a point

- The sum of the angles at a point is 360°.
  $a + b + c + d + e = 360°$



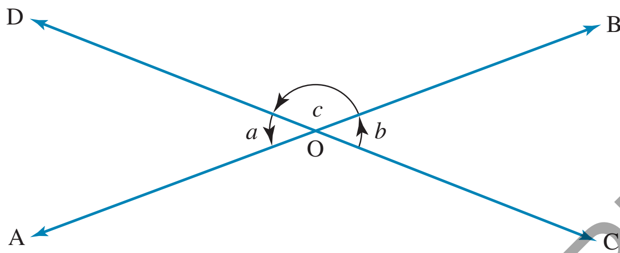
14.2.3 Supplementary angles



- The sum of the angles on a straight line is 180°.
- Angles a, b and c are **supplementary angles**.
  $a + b + c = 180°$
- The sum of the angles at a point is 360°.

14.2.4 Vertically opposite angles

- **Theorem 1:** **Vertically opposite angles** are equal.

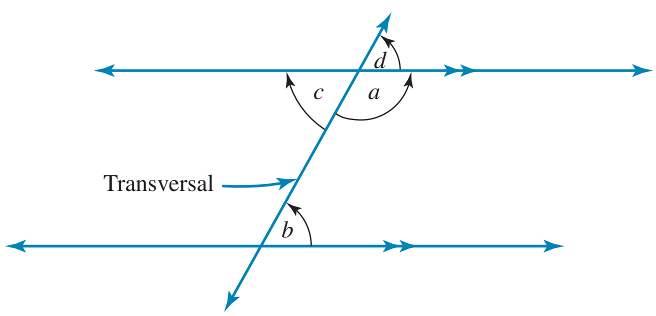


Given: Straight lines AB and CD intersect at O.
To prove: $\angle AOD = \angle BOC$ and $\angle BOD = \angle AOC$
Construction: Label $\angle AOD$ as a, $\angle BOC$ as b and $\angle BOD$ as c.
Proof: Let $\angle AOD = a°$, $\angle BOC = b°$ and $\angle BOD = c°$.

$a + c = 180°$ (supplementary angles)
$b + c = 180°$ (supplementary angles)
$\therefore a + c = b + c$
$\therefore a = b$
So, $\angle AOD = \angle BOC$.
Similarly, $\angle BOD = \angle AOC$.

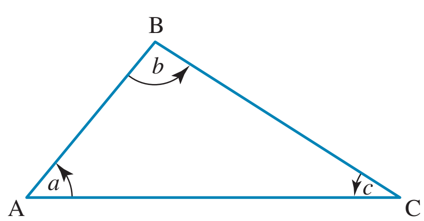
14.2.5 Parallel lines

- If two lines are parallel and cut by a **transversal**, then:
  - co-interior angles are supplementary. For example, $a + b = 180°$.
  - corresponding angles are equal. For example, $b = d$.
  - alternate angles are equal. For example, $b = c$.
  - opposite angles are equal. For example, $c = d$.



14.2.6 Angle properties of triangles

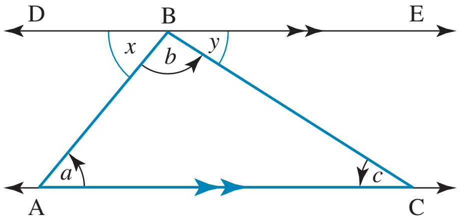
- **Theorem 2:** The sum of the interior angles of a triangle is 180°.



Given: ΔABC with interior angles a, b and c

To prove: $a + b + c = 180°$

Construction: Draw a line parallel to AC, passing through B, and label it DE as shown. Label $\angle ABD$ as x and $\angle CBE$ as y.



Proof:

$a = x$ (alternate angles)

$c = y$ (alternate angles)

$x + b + y = 180°$ (supplementary angles)

$\therefore a + b + c = 180°$

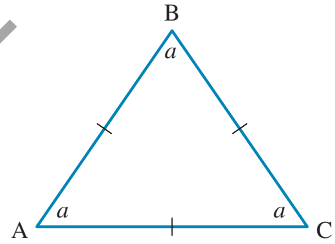
14.2.7 Equilateral triangles

- It follows from Theorem 2 that each interior angle of an **equilateral triangle** is 60°, and, conversely, if the three angles of a triangle are equal, then the triangle is equiangular.

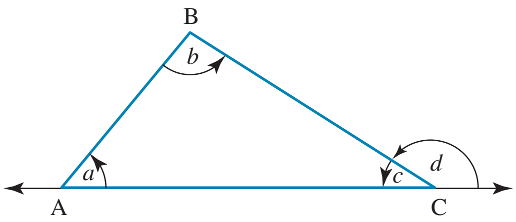
$a + a + a = 180°$ (sum of interior angles in a triangle is 180°)

$3a = 180°$

$a = 60°$



- **Theorem 3:** The exterior angle of a triangle is equal to the sum of the opposite interior angles.



Given: ΔABC with the exterior angle labelled d

To prove: $d = a + b$

Proof:

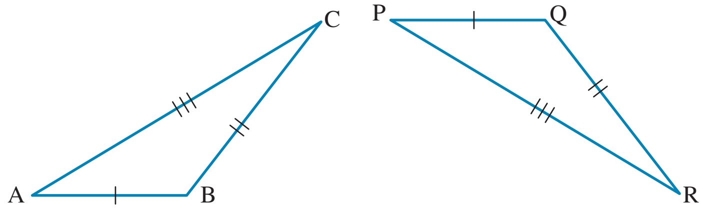
$c + d = 180°$ (supplementary angles)

$a + b + c = 180°$ (sum of interior angles in a triangle is 180°)

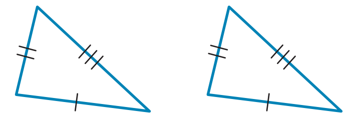
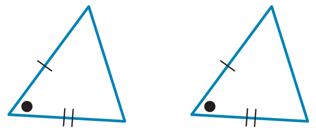
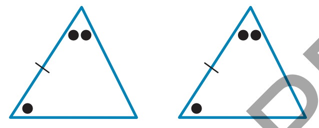
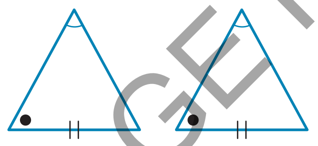
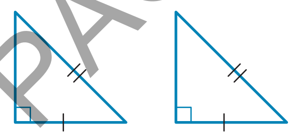
$\therefore d = a + b$

14.2.8 Congruent triangles

- **Congruent triangles** have the same size and the same shape; that is, they are identical in all respects.
- The symbol used for congruency is $\cong$.
- For example, ΔABC in the diagram below is congruent to ΔPQR. This is written as $\Delta ABC \cong \Delta PQR$.



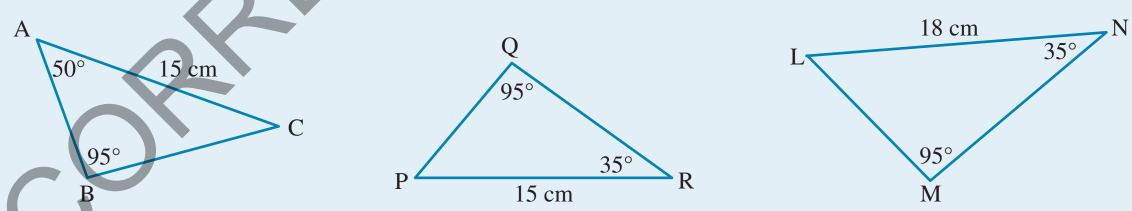
- Note that the vertices of the two triangles are written in corresponding order.
- There are five tests designed to check whether triangles are congruent. The tests are summarised in the table below.

Test	Diagram	Abbreviation
All three sides in one triangle are equal in length to the corresponding sides in the other triangle.		SSS
Two corresponding sides and the included angle are the same in both triangles.		SAS
Two corresponding angles and a pair of corresponding sides are the same in both triangles.		ASA
A pair of corresponding angles and a non-included side are equal in both triangles.		AAS
The hypotenuse and one pair of the other corresponding sides in two right-angled triangles are the same in two right-angled triangles.		RHS

- In each of the tests we need to show three equal measurements about a pair of triangles in order to show they are congruent.

WORKED EXAMPLE 1

Select a pair of congruent triangles from the diagrams below, giving a reason for your answer.



THINK

1 In each triangle the length of the side opposite the 95° angle is given. If triangles are to be congruent, the sides opposite the angles of equal size must be equal in length. Draw your conclusion.

WRITE

All three triangles have equal angles, but the sides opposite the angle 95° are not equal.
AC = PR = 15 and LN = 18 cm

THINK

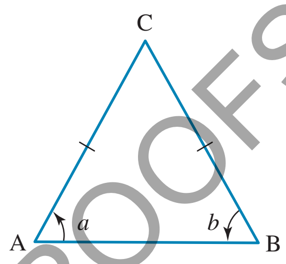
2 To test whether ΔABC is congruent to ΔPQR, first find the angle C.

WRITE

ΔABC: $\angle A = 50°$, $\angle B = 95°$,
$\angle C = 180° - 50° - 95°$
$= 35°$

3 Apply a test for congruence. Triangles ABC and PQR have a pair of corresponding sides equal in length and 2 pairs of angles the same, so draw your conclusion.	A pair of corresponding angles $\angle B = \angle Q$ and $\angle C = \angle R$ and a non-included side (AP = PR) are equal. $\Delta ABC \cong \Delta PQR$ (AAS)

14.2.9 Isosceles triangles

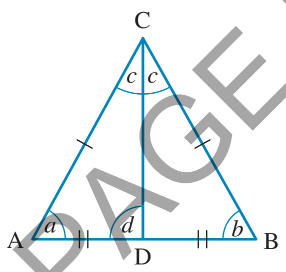


- A triangle is isosceles if the lengths of two sides are equal but the third side is not equal.
- **Theorem 4:** The angles at the base of an **isosceles triangle** are equal.

Given: AC = CB

To prove: $\angle BAC = \angle CBA$

Construction: Draw a line from the vertex C to the midpoint of the base AB and label the midpoint D. CD is the bisector of $\angle ACB$.



Proof: In ΔACD and ΔBCD,

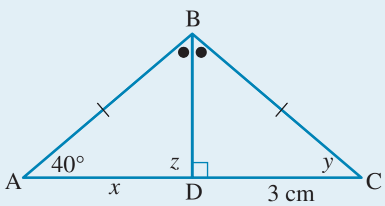
CD = CD	(common side)
AD = DB	(construction, D is the midpoint of AB)
AC = CB	(given)
$\Rightarrow \Delta ACD \cong \Delta BCD$	(SSS)
$\therefore \angle BAC = \angle CBA$	

- Conversely, if two angles of a triangle are equal, then the sides opposite those angles are equal.
- It also follows that $\angle ADC = \angle BDC = d$

and that $2d = 180°$ (supplementary)
$\Rightarrow d = 90°$

WORKED EXAMPLE 2

Given that $\Delta ABD \cong \Delta CBD$, find the values of the pronumerals in the figure below.

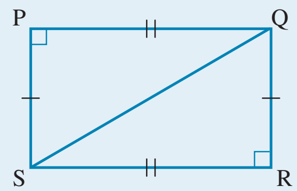


THINK	WRITE
1 In congruent triangles corresponding sides are equal in length. Side AD (marked x) corresponds to side DC, so state the value of x.	$\Delta ABD \cong \Delta CBD$ AD = CD, AD = x, CD = 3 So $x = 3$ cm.

2 Since the triangles are congruent, corresponding angles are equal. State the angles corresponding to y and z and hence find the values of these pronumerals.	$\angle BAD = \angle BCD$ $\angle BAD = 40°, \angle BCD = y$ So $y = 40°$. $\angle BDA = \angle BDC$ $\angle BDA = z, \angle BDC = 90°$ So $z = 90°$.

WORKED EXAMPLE 3

Prove that ΔPQS is congruent to ΔRSQ.



THINK	WRITE
1 Write the information given.	Given: Rectangle PQRS with diagonal QS.
2 Write what needs to be proved.	To prove: that ΔPQS is congruent to ΔRSQ. QP = SR (given) $\angle SPQ = \angle SRQ = 90°$ (given) QS is common.
3 Select the appropriate congruency test for proof. (In this case it is RHS because the triangles have an equal side, a right angle and a common hypotenuse.)	So $\Delta PQS \cong \Delta RSQ$ (RHS).

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Interactivity: Angles at a point (int-6157)
Interactivity: Supplementary angles (int-6158)
Interactivity: Vertically opposite and adjacent angles (int-3968)
Interactivity: Co-interior angles (int-3970)
Interactivity: Corresponding angles (int-3969)
Interactivity: Alternate angles (int-3971)
Interactivity: Angles in a triangle (int-3965)
Interactivity: Interior and exterior angles of a triangle (int-3966)
Interactivity: Congruent triangles (int-3754)
Interactivity: Congruency tests (int-3755)
Interactivity: Angles in an isosceles triangle (int-6159)
Digital doc: SkillSHEET: Substitution into quadratic equations (doc-5266)
Digital doc: SkillSHEET: Equation of a vertical line (doc-5267)

Exercise 14.2 Angles, triangles and congruence

assess**on**

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5, 7, 9, 11	Questions: 1–5, 6, 8–10, 12, 13	Questions: 1–14

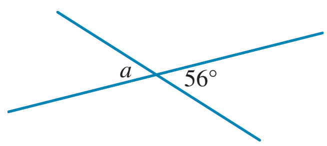
Individual pathway interactivity: int-4612 learn**on** ONLINE ONLY

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

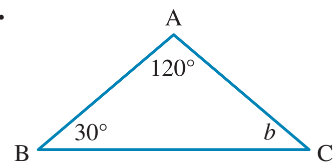
Understanding and fluency

1. Determine the values of the unknown in each of the following.

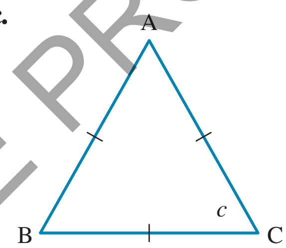
a.



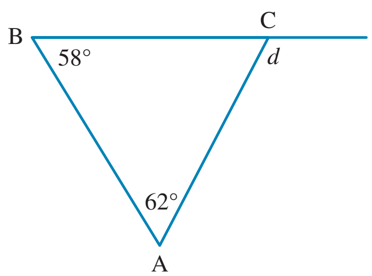
b.



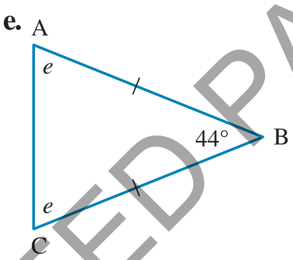
c.



d.

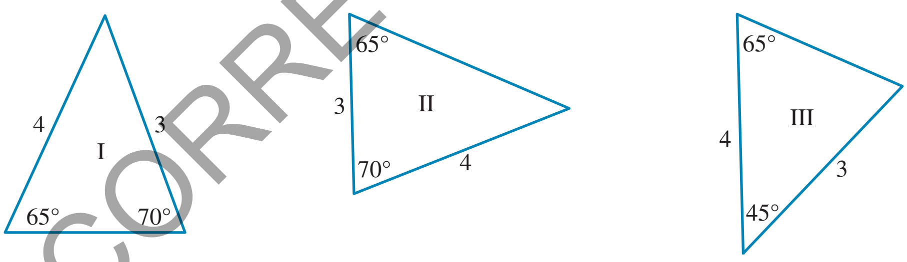


e.



2. WE1 Select a pair of congruent triangles in each of the following, giving a reason for your answer. All side lengths are in cm.

a.



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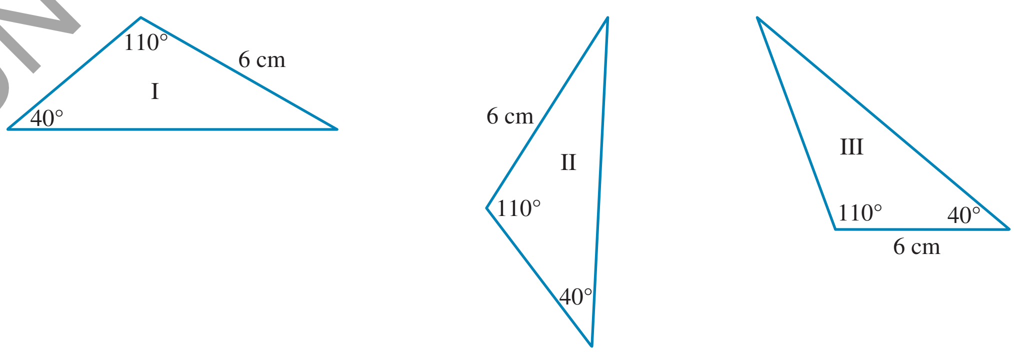

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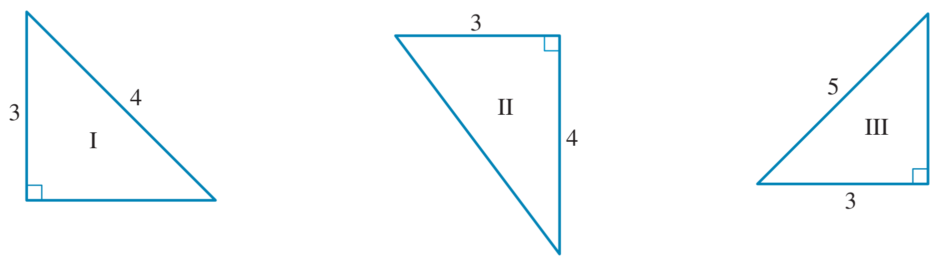

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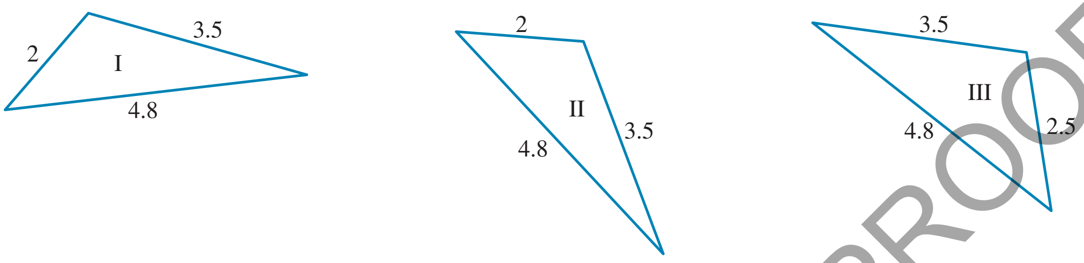
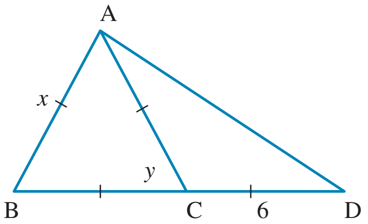
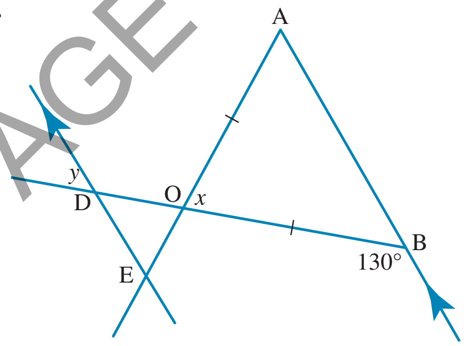


3. Find the missing values of x and y in each of the following diagrams. Give reasons for your answers.

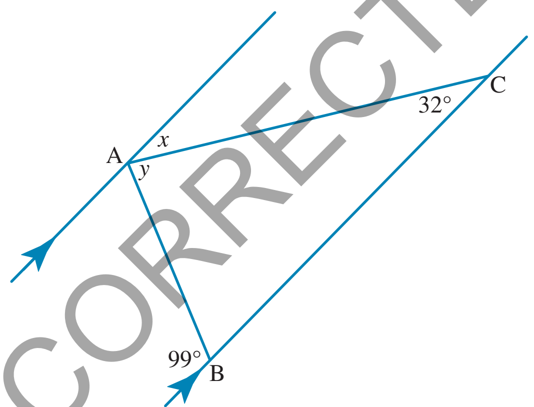
a.



b.



c.



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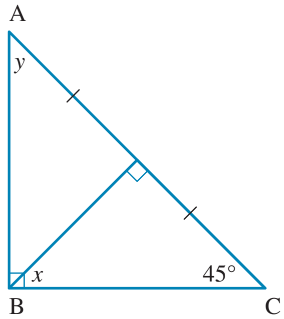
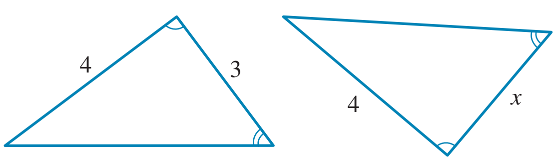
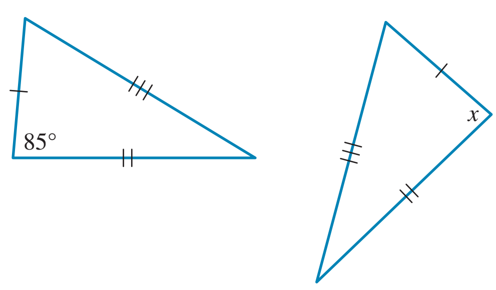


4. WE2 Find the value of the pronumeral in each of the following pairs of congruent triangles. All side lengths are in cm.

a.



b.



c.

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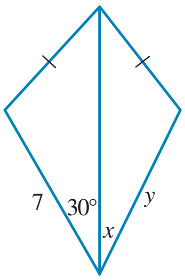

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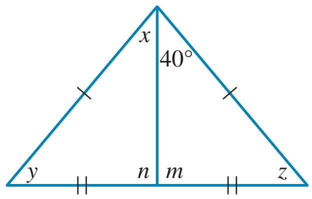
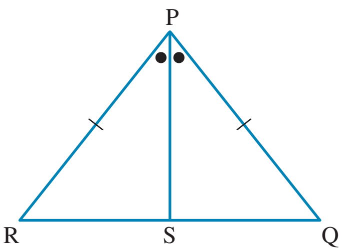


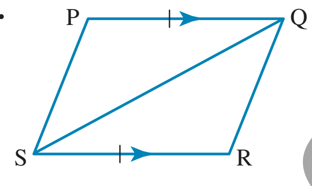
Communicating, reasoning and problem solving

5. WE3 Prove that each of the following pairs of triangles are congruent.

a.



b.



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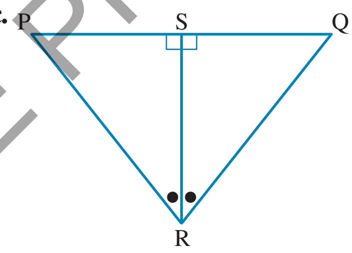

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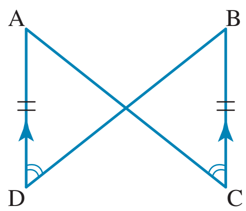

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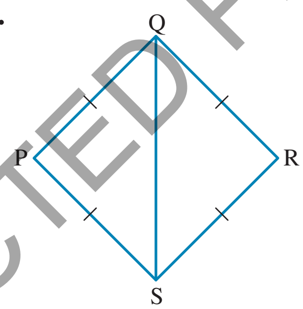
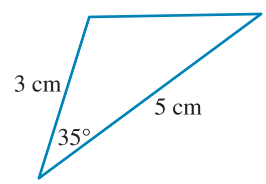
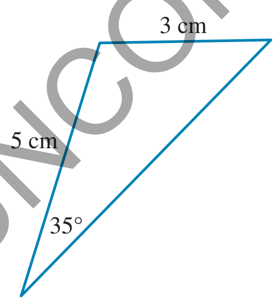


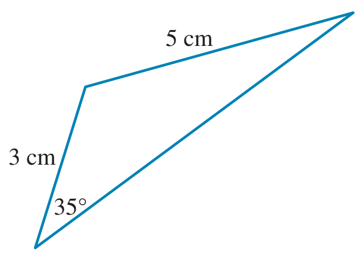
6. MC Which of the following is congruent to the triangle shown?
Note: There may be more than one correct answer.



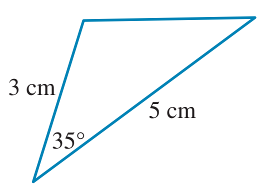
A.



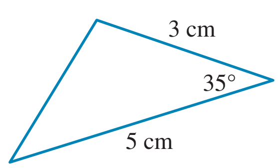
B.



C.

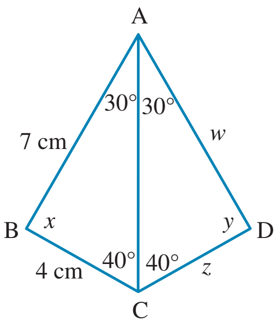


D.

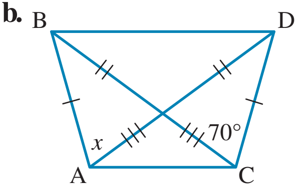


7. Prove that $\Delta ABC \cong \Delta ADC$ and hence find the values of the pronumerals in each of the following.

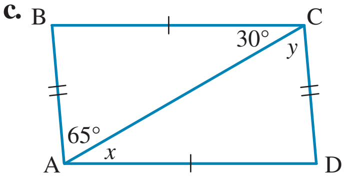
a.



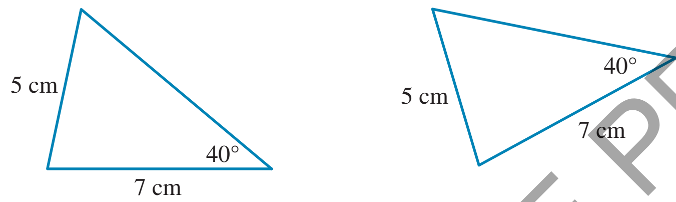
b.



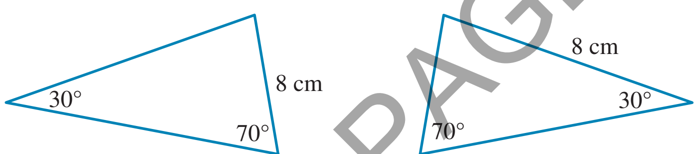
c.



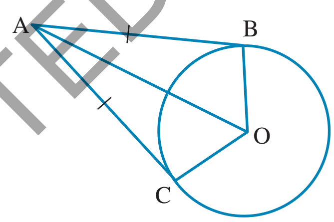
8. Explain why the triangles shown are not necessarily congruent.



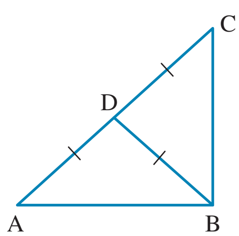
9. Explain why the triangles shown are not congruent.



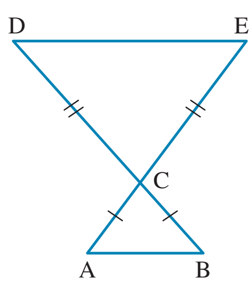
10. Show that $\Delta ABO \cong \Delta ACO$, if O is the centre of the circle.



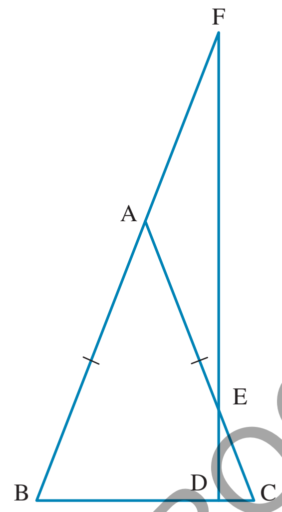
11. If DA = DB = DC, prove that $\angle ABC$ is a right angle.



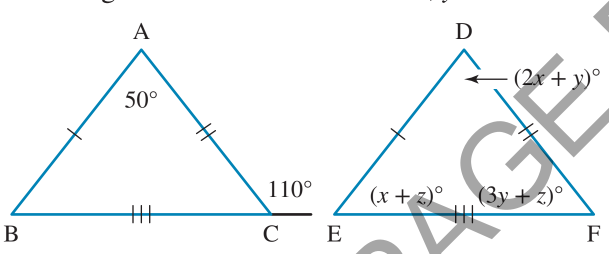
12. If AC = CB and DC = CE in the diagram shown, prove that AB || DE.



13. ABC is an isosceles triangle in which AB and AC are equal in length. BDF is a right-angled triangle. Show that triangle AEF is an isosceles triangle.



14. Triangles ABC and DEF are congruent. Find the values of x, y and z.



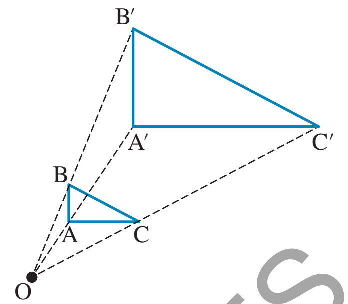
15. Use congruent triangle results to prove the following.
   a. If two sides of a triangle are equal in length, prove that the angels opposite the equal sides are equal.
   b. If two angles of a triangle are equal, prove that the sides opposite those angles are equal.
   c. If three sides of a triangle are equal, prove that each interior angle is 60°.

14.3 Similar triangles [Stages 5.2 and 5.3]

14.3.1 Similar figures

- Two geometric shapes are **similar** when one is an **enlargement** or reduction of the other shape.

– An enlargement increases the length of each side of a figure in all directions by the same factor. For example, in the diagram shown, triangle A′B′C′ is an enlargement of triangle ABC by a factor of 3 from its **centre of enlargement** at O.



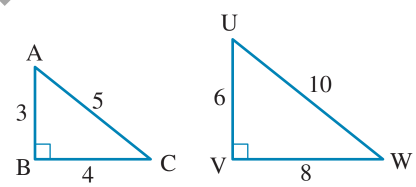
- The symbol for similarity is ~ and is read as 'is similar to'.
- The **image** of the original object is the enlarged or reduced shape.
- To create a similar shape, use a **scale factor** to enlarge or reduce the original shape.
- The scale factor can be found using the formula below and the lengths of a pair of corresponding sides.

$$\text{Scale factor} = \frac{\text{image side length}}{\text{object side length}}$$

- If the scale factor is less than 1, the image is a reduced version of the original shape. If the scale factor is greater than 1, the image is an enlarged version of the original shape.

14.3.2 Similar triangles

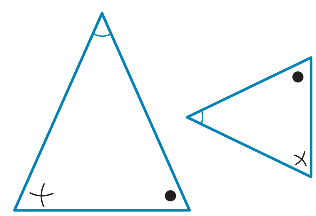
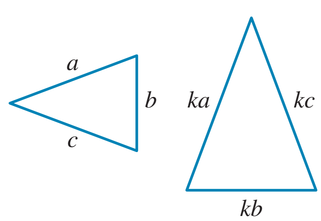
- Two triangles are similar if:
  – the angles are equal, or
  – the corresponding sides are proportional.
- Consider the pair of **similar triangles** shown.



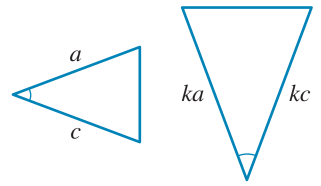
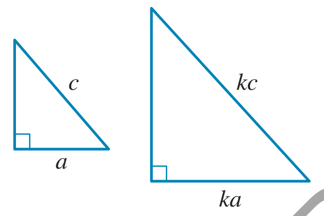
- The following statements are true for these triangles.
  – Triangle UVW is similar to triangle ABC or, using symbols, $\Delta UVW \sim \Delta ABC$.
  – The corresponding angles of the two triangles are equal in size: $\angle CAB = \angle WUV$, $\angle ABC = \angle UVW$ and $\angle ACB = \angle UWV$.
  – The corresponding sides of the two triangles are in the same ratio. $\frac{UV}{AB} = \frac{VW}{BC} = \frac{UW}{AC} = 2$; that is, ΔUVW has each of its sides twice as long as the corresponding sides in ΔABC.
  – The scale factor is 2.

14.3.3 Testing triangles for similarity

- Triangles can be checked for similarity using one of the tests described in the table below.

Test	Diagram	Abbreviation
Two angles of a triangle are equal to two angles of another triangle. This implies that the third angles are equal, as the sum of angles in a triangle is 180°.		**AAA**
The three sides of a triangle are proportional to the three sides of another triangle.		**SSS**

(*continued*)

Test	Diagram	Abbreviation
Two sides of a triangle are proportional to two sides of another triangle, and the included angles are equal.		SAS
The hypotenuse and a second side of a right-angled triangle are proportional to the hypotenuse and a second side of another right-angled triangle.		RHS

- *Note:* When using the equiangular test, only two corresponding angles have to be checked. Since the sum of the interior angles in any triangle is a constant number (180°), the third pair of corresponding angles will automatically be equal, provided that the first two pairs match exactly.

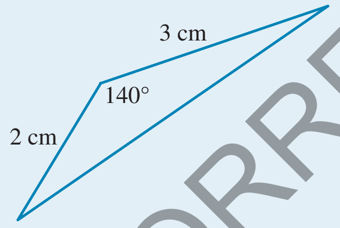
DISCUSSION

Explain the differences between the tests for similarity and congruency for triangles.

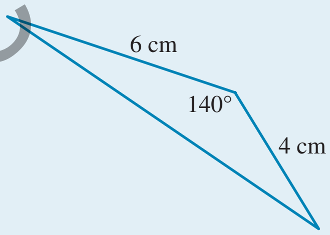
WORKED EXAMPLE 4

Find a pair of similar triangles among those shown. Give a reason for your answer.

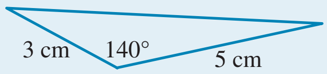
a



b



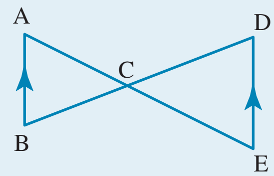
c



THINK	WRITE
1 In each triangle the lengths of two sides and the included angle are known, so the SAS test can be applied. Since all included angles are equal (140°), we need to the find ratios of corresponding sides, taking two triangles at a time.	For triangles **a** and **b**: $\frac{6}{3} = \frac{4}{2} = 2$ For triangles **a** and **c**: $\frac{5}{3} = 1.6, \frac{3}{2} = 1.5$ For triangles **b** and **c**: $\frac{5}{6} = 0.83, \frac{3}{4} = 0.75$
2 Only triangles **a** and **b** have corresponding sides in the same ratio (and included angles are equal). State your conclusion, specifying the similarity test that has been used.	Triangle **a** ~ triangle **b** (SAS)

WORKED EXAMPLE 5

Prove that ΔABC is similar to ΔEDC.



THINK

1 Write the information given. AB is parallel to DE. Transversal BD forms two alternate angles: ∠ABC and ∠EDC.

2 Write what is to be proved.

3 Write the proof.

WRITE

1 Given:
ΔABC and ΔDCE
AB || DE
C is common.

2 To prove: ΔABC ~ ΔEDC

3 Proof:
∠ABC = ∠EDC (alternate angles)
∠BAC = ∠DEC (alternate angles)
∠BCA = ∠DCE (vertically opposite angles)
∴ ΔABC ~ ΔEDC (equiangular, AAA)

learnON RESOURCES — ONLINE ONLY

Interactivity: Scale factors (int-6041)
Interactivity: Angle-angle-angle condition of similarity (AAA) (int-6042)
Interactivity: Side-side-side condition of similarity (SSS) (int-6448)
Interactivity: Side-angle-side condition of similarity (SAS) (int-6447)
eLesson: Similar triangles (eles-1925)
Digital doc: SkillSHEET: Writing similarity statements (doc-5278)
Digital doc: SkillSHEET: Calculating unknown side lengths in a pair of similar triangles (doc-5281)
Digital doc: WorkSHEET: Deductive geometry 1 (doc-5282)

Exercise 14.3 Similar triangles

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–10	Questions: 1–11, 15	Questions: 1–15

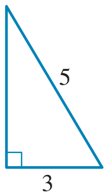
Individual pathway interactivity: int-4613

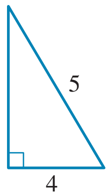
learnON ONLINE ONLY

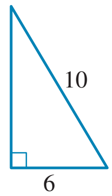
To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

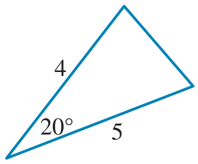
Understanding and fluency

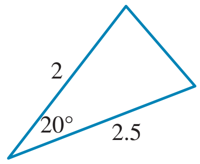
1. **WE4** Find a pair of similar triangles among those shown in each part. Give a reason for your answer.

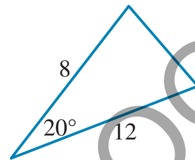
a. i.


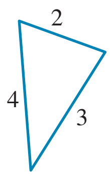
ii.


iii.


b. i.


ii.


iii.


c. i.


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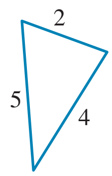

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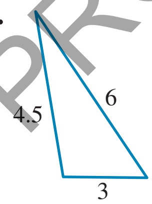
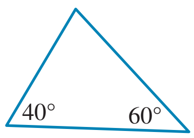
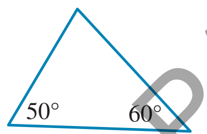


d. i.


ii.


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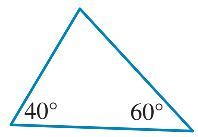
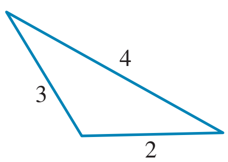
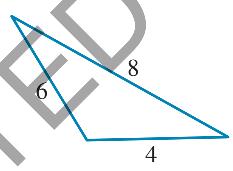
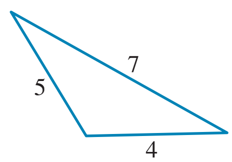
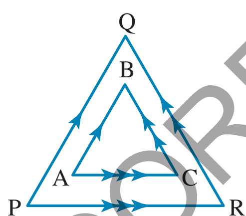


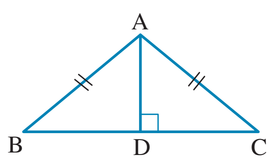
e. i.


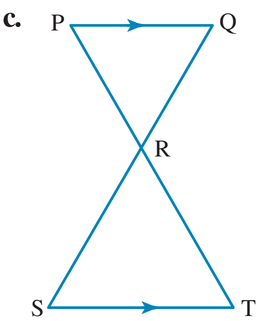
ii.


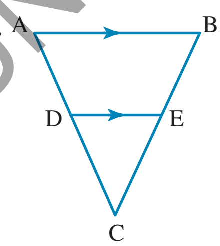
iii.


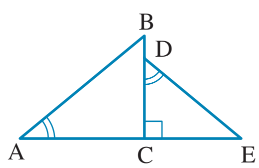
2. Name two similar triangles in each of the following figures.

a.


b.


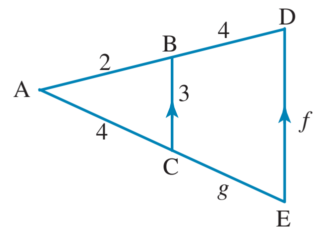
c.


d.


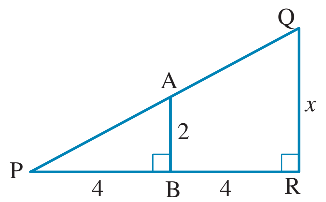
e.


3. **a.** Complete this statement: $\dfrac{AB}{AD} = \dfrac{BC}{} = \dfrac{}{AE}$.

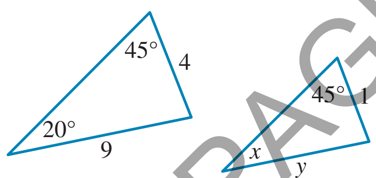
b. Find the value of the pronumerals.



4. Find the value of the pronumeral in the diagram shown.

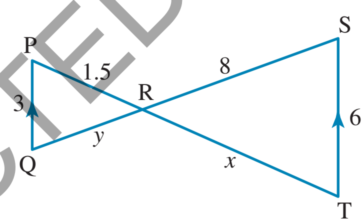


5. The triangles shown are similar. Find the value of x and y.

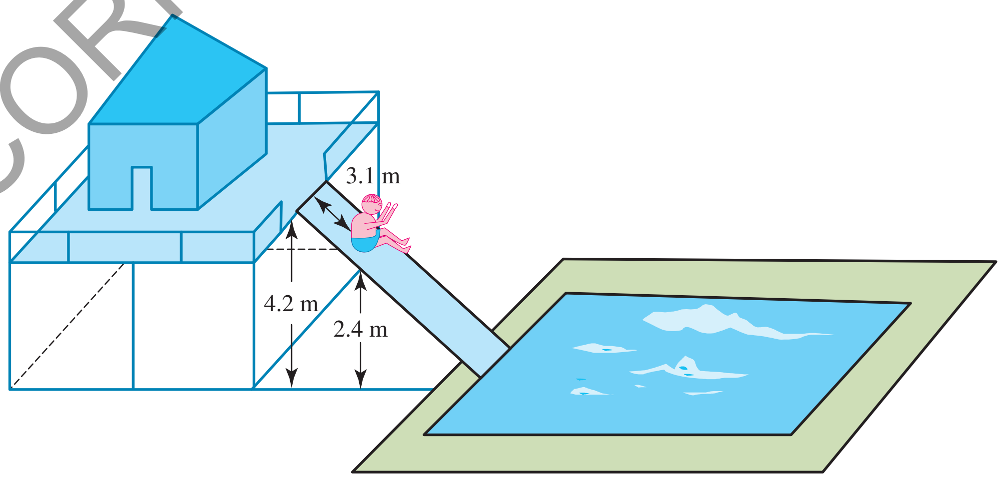


6. **a.** State why these two triangles shown are similar.

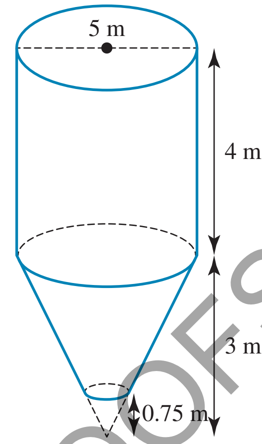
b. Find the values of x and y in the diagram.



7. A waterslide is 4.2 m high and has a support 2.4 m tall. If a student reaches this support when she is 3.1 m down the slide, how long is the slide?

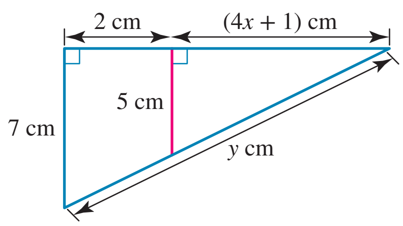


8. A storage tank as shown in the diagram is made of a 4-m-tall cylinder joined by a 3-m-tall cone. If the diameter of the cylinder is 5 m, what is the radius of the end of the cone if 0.75 m has been cut off the tip?

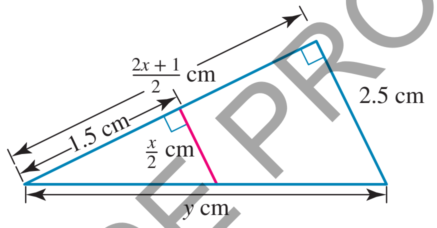


9. Calculate the values of the pronumerals in the following diagrams.

a.



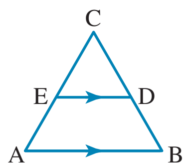
b.



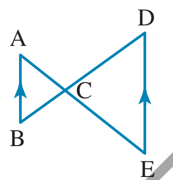
Communicating, reasoning and problem solving

10. **WE5** Prove that ΔABC is similar to ΔEDC in each of the following.

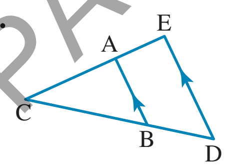
a.



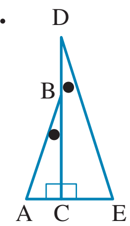
b.



c.



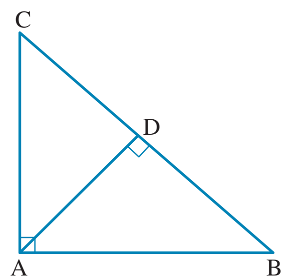
d.



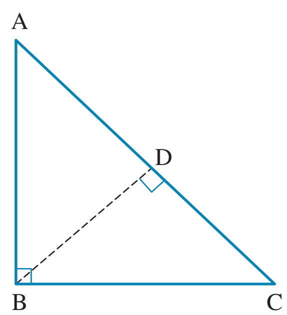
11. ΔABC is a right-angled triangle. A line is drawn from A to D as shown so that AD ⊥ BC. Prove that:

a. ΔABD ~ ΔACB

b. ΔACD ~ ΔACB.

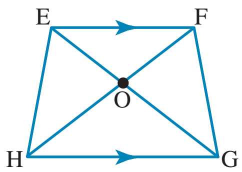


12. Explain why the AAA test cannot be used to prove congruence but can be used to prove similarity.

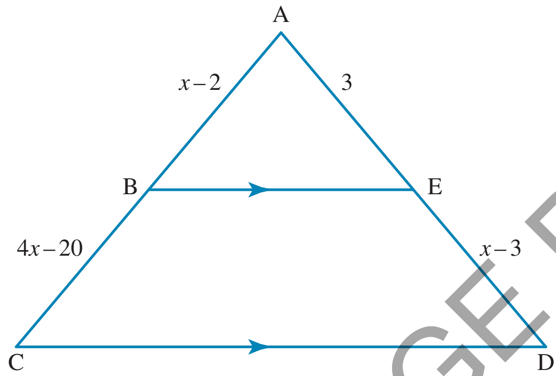


13. **a.** Prove Pythagoras' theorem, $AC^2 = AB^2 + BC^2$, using similar triangles.
b. Show that the converse of Pythagoras' theorem holds true; that is, if the square on one side of a triangle equals the sum of the squares on the other two sides, then the angle between these other two sides is a right angle.

14. Prove that $\Delta EFO \sim \Delta GHO$.



15. Solve for x.



16. How can you be certain that two figures are similar?

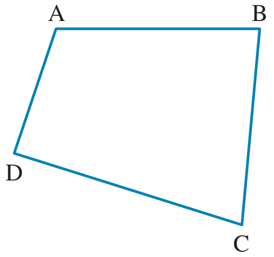
17. A tetrahedron (regular triangular-based pyramid) has an edge length of 2 cm. A similar tetrahedron has a total surface area of $36\sqrt{3}$ cm^2. What is the scale factor relationship between the side lengths of the two tetrahedra?

18. Explain why the remaining (third) angles must also be equal if two angles of a triangle are equal to two angles of another triangle.

14.4 Quadrilaterals [Stages 5.2 and 5.3]

14.4.1 Definition of a quadrilateral

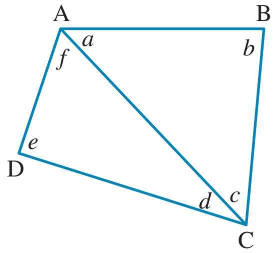
- A quadrilateral is a four-sided plane shape whose interior angles sum to 360°.
- **Theorem 5:** The sum of the interior angles in a quadrilateral is 360°.



Given: A quadrilateral ABCD

To prove: $\angle ABC + \angle BCD + \angle ADC + \angle BAD = 360°$

Construction: Draw a line joining vertex A to vertex C. Label the interior angles of the triangles formed.



Proof:

$a + b + c = 180°$ (sum of interior angles in a triangle is 180°)

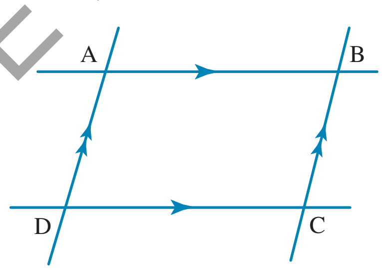
$d + e + f = 180°$ (sum of interior angles in a triangle is 180°)

$\Rightarrow a + b + c + d + e + f = 360°$

$\therefore \angle ABC + \angle BCD + \angle ADC + \angle BAD = 360°$

14.4.2 Parallelograms

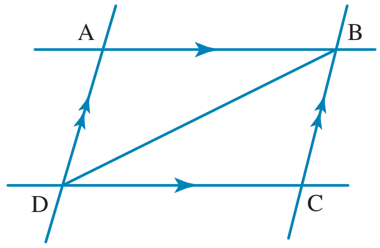
- A **parallelogram** is a quadrilateral with two pairs of parallel sides.
- **Theorem 6:** Opposite angles of a parallelogram are equal.



Given: AB || DC and AD || BC

To prove: $\angle ABC = \angle ADC$

Construction: Draw a diagonal from B to D.



Proof:

$\angle ABD = \angle BDC$ (alternate angles)

$\angle ADB = \angle CBD$ (alternate angles)

$\angle ABC = \angle ABD + \angle CBD$ (by construction)

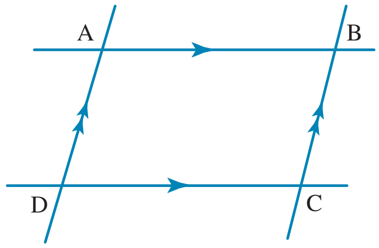
$\angle ADC = \angle BDC + \angle ADB$ (by construction)

$\therefore \angle ABC = \angle ADC$

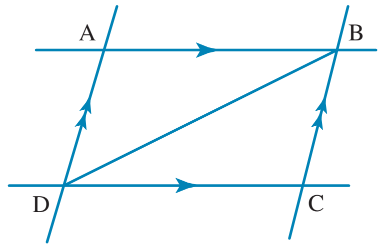
- Conversely, if each pair of opposite angles of a quadrilateral is equal, then it is a parallelogram.

14.4.3 Theorem 7

- **Theorem 7:** Opposite sides of a parallelogram are equal.



Given: AB || DC and AD || BC
To prove: AB = DC
Construction: Draw a diagonal from B to D.

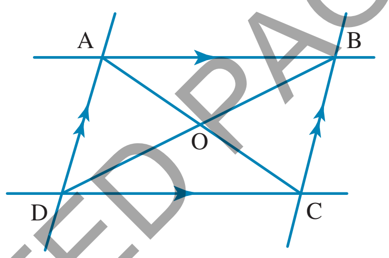


Proof:
$\angle ABD = \angle BDC$ (alternate angles)
$\angle ADB = \angle CBD$ (alternate angles)
BD is common to ΔABD and ΔBCD.
$\Rightarrow \Delta ABD \cong \Delta BCD$ (ASA)
$\therefore AB = DC$

- Conversely, if each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.

14.4.4 Theorem 8

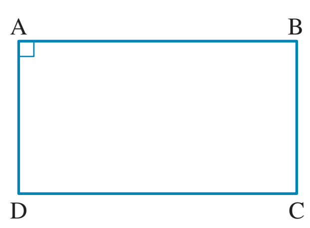
- **Theorem 8:** The diagonals of a parallelogram bisect each other.



Given: AB || DC and AD || BC with diagonals AC and BD
To prove: AO = OC and BO = OD
Proof: In ΔAOB and ΔCOD,
$\angle OAB = \angle OCD$ (alternate angles)
$\angle OBA = \angle ODC$ (alternate angles)
$AB = CD$ (opposite sides of a parallelogram)
$\Rightarrow \Delta AOB \cong \Delta COD$ (ASA)
$\Rightarrow AO = OC$ (corresponding sides in congruent triangles)
and $BO = OD$ (corresponding sides in congruent triangles)

14.4.5 Rectangles

- A rectangle is a parallelogram with four right angles.
- **Theorem 9:** A parallelogram with a right angle is a rectangle.



Given: Parallelogram ABCD with $\angle BAD = 90°$

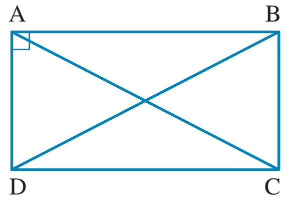
To prove: $\angle BAD = \angle ABC = \angle BCD = \angle ADC = 90°$

Proof:

$AB \parallel CD$	(properties of a parallelogram)
$\Rightarrow \angle BAD + \angle ADC = 180°$	(co-interior angles)
But $\angle BAD = 90°$	(given)
$\Rightarrow \angle ADC = 90°$	
Similarly, $\angle BCD = \angle ADC = 90°$	
$\therefore \angle BAD = \angle ABC = \angle BCD = \angle ADC = 90°$	

14.4.6 Theorem 10

- **Theorem 10:** The diagonals of a rectangle are equal.



Given: Rectangle ABCD with diagonals AC and BD

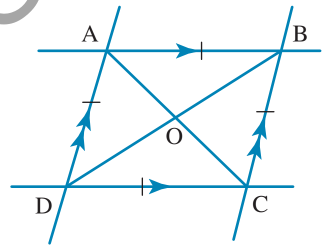
To prove: $AC = BD$

Proof:

In ΔADC and ΔBCD,	
$AD = BC$	(opposite sides equal in a rectangle)
$DC = CD$	(common)
$\angle ADC = \angle BCD = 90°$	(right angles in a rectangle)
$\Rightarrow \Delta ADC \cong \Delta BCD$	(SAS)
$\therefore AC = BD$	

14.4.7 Rhombuses

- A **rhombus** is a parallelogram with four equal sides.
- **Theorem 11:** The diagonals of a rhombus are perpendicular.



Given: Rhombus ABCD with diagonals AC and BD

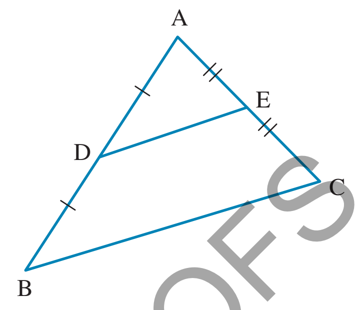
To prove: $AC \perp BD$

Proof:

In ΔAOB and ΔBOC,	
$AO = OC$	(property of parallelogram)
$AB = BC$	(property of rhombus)
$BO = OB$	(common)
$\Rightarrow \Delta AOB \cong \Delta BOC$	(SSS)
$\Rightarrow \angle AOB = \angle BOC$	
But $\angle AOB + \angle BOC = 180°$	(supplementary angles)
$\Rightarrow \angle AOB = \angle BOC = 90°$	
Similarly, $\angle AOD = \angle DOC = 90°$.	
Hence, $AC \perp BD$.	

14.4.8 The midpoint theorem

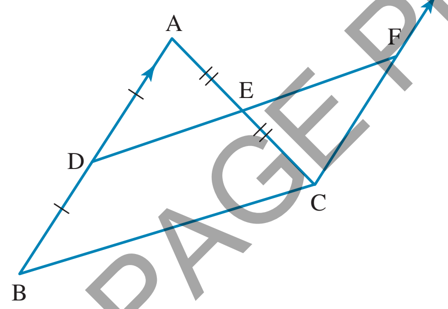
- Now that the properties of quadrilaterals have been explored, the midpoint theorem can be tackled.
- **Theorem 12:** The interval joining the midpoints of two sides of a triangle is parallel to the third side and half its length.



Given: ΔABC in which AD = DB and AE = EC

To prove: DE $\|$ BC and DE = $\frac{1}{2}$BC

Construction: Draw a line through C parallel to AB. Extend DE to F on the parallel line.

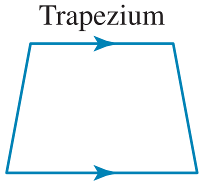


Proof: In ΔADE and ΔCEF,

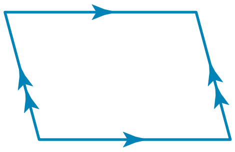
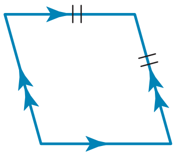
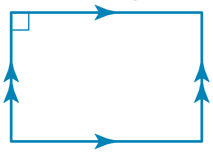
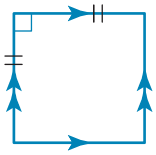
AE = EC	(E is the midpoint of AC, given)
$\angle$AED = $\angle$CEF	(vertically opposite angles)
$\angle$EAD = $\angle$ECF	(alternate angles)
$\Rightarrow$ ΔADE $\cong$ ΔCEF	(ASA)
$\therefore$ AD = CF and DE = EF	(corresponding sides in congruent triangles)
So, AD = DB = CF.	
We have AB $\|$ CF	(by construction)
So BDFC is a parallelogram.	
$\Rightarrow$ DE $\|$ BC	
Also, BC = DF	(opposite sides in parallelogram)
But DE = DF	(sides in congruent triangles)
$\Rightarrow$ DE = $\frac{1}{2}$BC	

Therefore, DE $\|$ BC and DE = $\frac{1}{2}$BC.

- Conversely, if a line interval is drawn parallel to a side of a triangle and half the length of that side, then the line interval bisects each of the other two sides of the triangle.
- A summary of the definitions and properties of quadrilaterals is shown in the table.

Shape	Definition	Properties
Trapezium 	A trapezium is a quadrilateral with one pair of opposite sides parallel.	• One pair of opposite sides is parallel but not equal in length.

(*continued*)

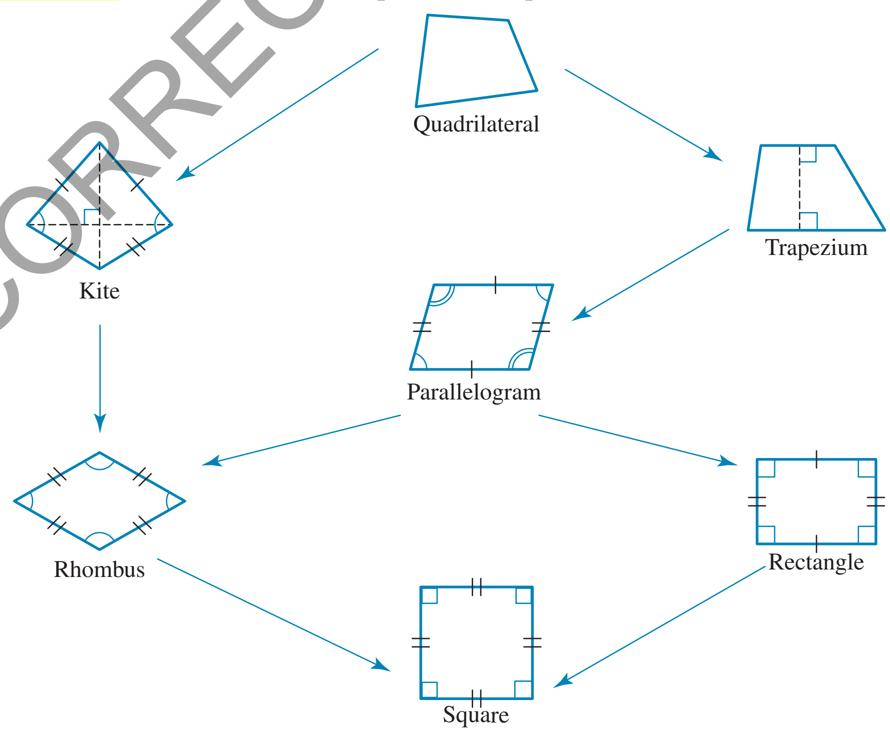
Shape	Definition	Properties
Parallelogram 	A parallelogram is a quadrilateral with both pairs of opposite sides parallel.	• Opposite angles are equal. • Opposite sides are equal. • Diagonals bisect each other.
Rhombus 	A rhombus is a parallelogram with four equal sides.	• Diagonals bisect each other at right angles. • Diagonals bisect the angles at the vertex through which they pass.
Rectangle 	A rectangle is a parallelogram whose interior angles are right angles.	• Diagonals are equal in length and bisect each other.
Square 	A square is a parallelogram whose interior angles are right angles with four equal sides.	• All angles are right angles. • All side lengths are equal. • Diagonals are equal in length and bisect each other at right angles. • Diagonals bisect the vertex through which they pass (45°).

DISCUSSION

What is the minimum information needed to identify a figure as a rhombus or a trapezium?

14.4.9 Relationships between quadrilaterals

- The flowchart below shows the relationships between quadrilaterals.



learnON RESOURCES — ONLINE ONLY

Interactivity: Angles in a quadrilateral (int-3967)
Interactivity: Opposite angles of a parallelogram (int-6160)
Interactivity: Opposite sides of a parallelogram (int-6161)
Interactivity: Diagonals of a parallelogram (int-6162)
Interactivity: Diagonals of a rectangle (int-6163)
Interactivity: Diagonals of a rhombus (int-6164)
Interactivity: The midpoint theorem (int-6165)
Interactivity: Quadrilaterals (int-3756)
Interactivity: Quadrilateral definitions (int-2786)
Digital doc: SkillSHEET: Identifying quadrilaterals (doc-5279)
Digital doc: WorkSHEET: Deductive geometry 2 (doc-5283)

Exercise 14.4 Quadrilaterals

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–10	Questions: 1–14, 16	Questions: 1–19

Individual pathway interactivity: int-4614

learnON ONLINE ONLY

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

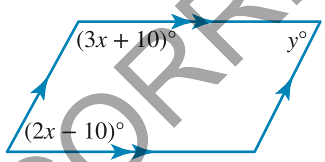
Understanding and fluency

1. Use the definitions of the five special quadrilaterals to decide if the following statements are true or false.

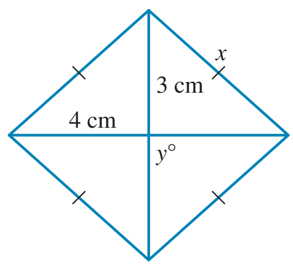
a. A square is a rectangle.
b. A rhombus is a parallelogram.
c. A square is a rhombus.
d. A rhombus is a square.
e. A square is a trapezium.
f. A parallelogram is a rectangle.
g. A trapezium is a rhombus.
h. A rectangle is a square.

2. Determine the values of x and y in each of the following figures.

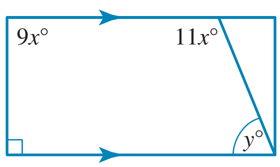
a.



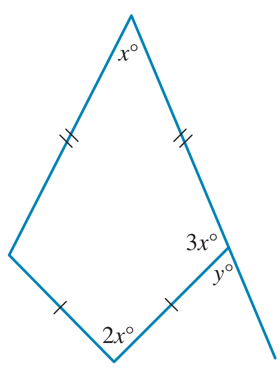
b.



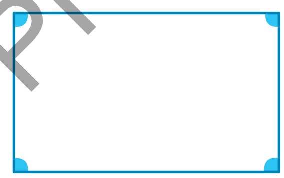
c.



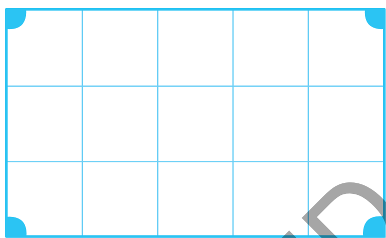
d.



3. Draw three different trapeziums. Using your ruler, compass and protractor, decide which of the following properties are true in a trapezium.
   a. Opposite sides are equal.
   b. All sides are equal.
   c. Opposite angles are equal.
   d. All angles are equal.
   e. Diagonals are equal in length.
   f. Diagonals bisect each other.
   g. Diagonals are perpendicular.
   h. Diagonals bisect the angles they pass through.
4. Draw three different parallelograms. Using your ruler and protractor to measure, decide which of the following properties are true in a parallelogram.
   a. Opposite sides are equal.
   b. All sides are equal.
   c. Opposite angles are equal.
   d. All angles are equal.
   e. Diagonals are equal in length.
   f. Diagonals bisect each other.
   g. Diagonals are perpendicular.
   h. Diagonals bisect the angles they pass through.
5. Name four quadrilaterals that have at least one pair of opposite sides that are parallel and equal.
6. Name a quadrilateral that has equal diagonals that bisect each other and bisect the angles they pass through.
7. Pool is played on a rectangular table. Balls are hit with a cue and bounce off the sides of the table until they land in one of the holes or pockets.



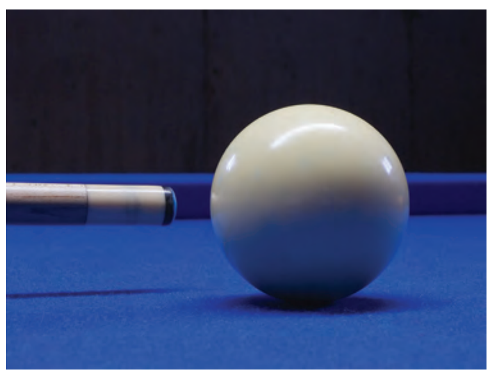
   a. Draw a rectangular pool table measuring 5 cm by 3 cm on graph paper. Mark the four holes, one in each corner.



A

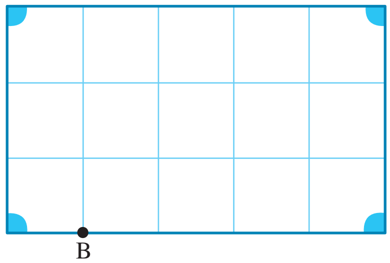
   b. A ball starts at A. It is hit so that it travels at a 45° diagonal across the grid. When it hits the side of the table, it bounces off at a 45° diagonal as well. How many sides does the ball bounce off before it goes in a hole?
   c. A different table is sized 7 cm by 2 cm. How many sides does a ball bounce off before it goes in a hole when hit from A?
   d. Complete the following table.

Table size	Number of sides hit
5 cm × 3 cm	
7 cm × 2 cm	
4 cm × 3 cm	
4 cm × 2 cm	
6 cm × 3 cm	
9 cm × 3 cm	
12 cm × 4 cm	

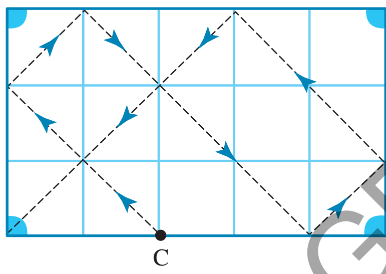


   e. Can you see a pattern? How many sides would a ball bounce off before going in a hole when hit from A on an $m \times n$ table?

f. The ball is now hit from B on a 5 cm × 3 cm pool table. How many *different* paths can a ball take when hit along 45° diagonals? Do these paths all hit the same number of sides before going in a hole? Does the ball end up in the same hole each time? Justify your answer.



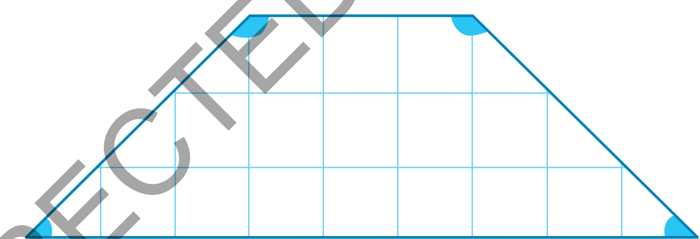
g. The ball is now hit from C along the path shown. What type of triangles and quadrilaterals are formed by the path of the ball with itself and the sides of the table? Are any of the triangles congruent?



h. A ball is hit from C on a 6 cm by 3 cm table. What shapes are formed by the path of the ball with itself and the sides of the table? Is there only one path possible?

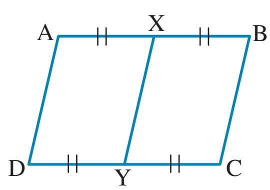
i. *Challenge:* A ball is hit from A along 45° diagonals. The table is $m \times n$. Can you find a formula to predict which hole the ball will go in?

j. *Challenge:* What would happen if the game was played on a trapezoidal table?

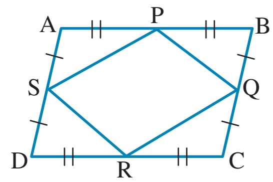


Communicating, reasoning and problem solving

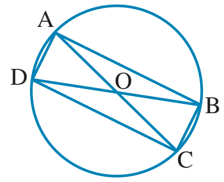
8. Prove that the diagonals of a rhombus bisect each other.
9. ABCD is a parallelogram. X is the midpoint of AB and Y is the midpoint of DC. Prove that AXYD is also a parallelogram.



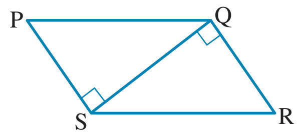
10. ABCD is a parallelogram. P, Q, R and S are all midpoints of their respective sides of ABCD.
    a. Prove $\Delta PAS \cong \Delta RCQ$.
    b. Prove $\Delta SDR \cong \Delta PBQ$.
    c. Hence, prove that PQRS is also a parallelogram.



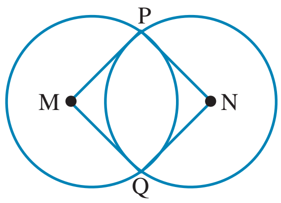
11. AC and BD are diameters of a circle with centre O. Prove that ABCD is a rectangle.



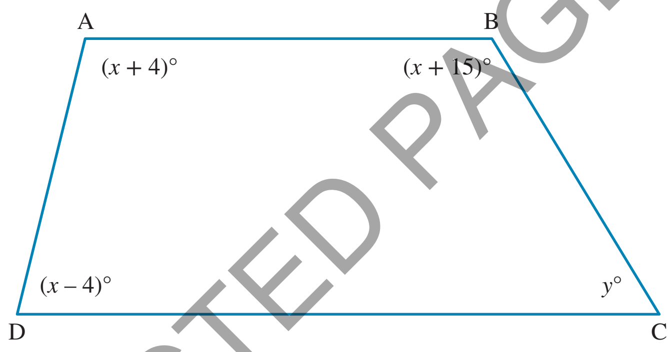
12. The diagonals of a parallelogram meet at right angles. Prove that the parallelogram is a rhombus.
13. Two congruent right-angled triangles are arranged as shown. Show that PQRS is a parallelogram.



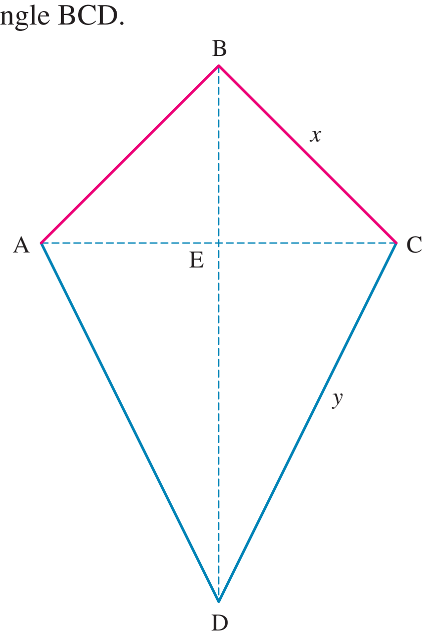
14. Two circles, centred at M and N, have equal radii and intersect at P and Q. Prove that PNQM is a rhombus.



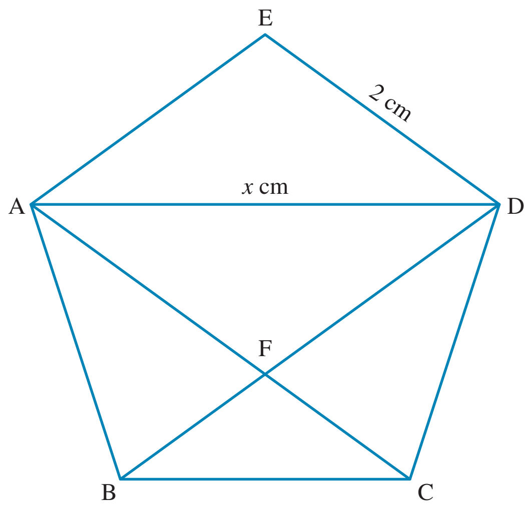
15. Give reasons why a square is a rhombus but a rhombus is not necessarily a square.
16. ABCD is a trapezium.



   a. What fact do you know about a trapezium?
   b. Find the values of x and y.
17. ABCD is a kite where AC = 8 cm, BE = 5 cm and ED = 9 cm. Find the exact values of:
   a. i. x ii. y
   b. Find angle BAD and hence angle BCD.

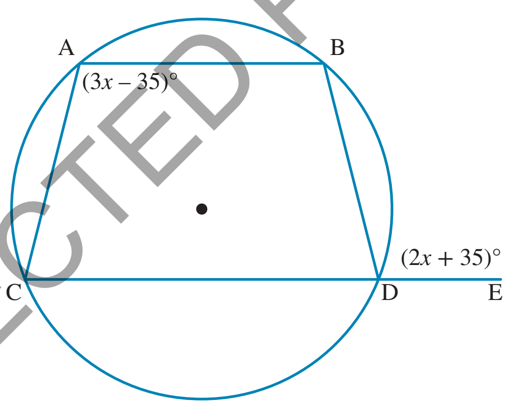


18. ABCDE is a regular pentagon whose side lengths are 2 cm. Each diagonal is x cm long.



a. What kind of shape is AEDF and what is the length of FD?
b. What kind of shape is ABCD?
c. If $\angle$EDA is 40°, find the value of $\angle$ACB, giving reasons for your findings.
d. Which triangle is similar to AED?
e. Explain why FB = $(x - 2)$ cm.
f. Show that $x^2 - 2x - 4 = 0$.
g. Solve the equation $x^2 - 2x - 4 = 0$, giving your answer as an exact value.

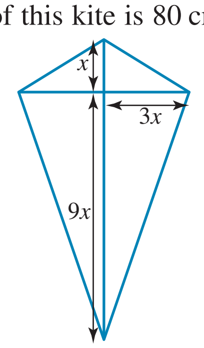
19. ABCD is called a cyclic quadrilateral because it is inscribed inside a circle.

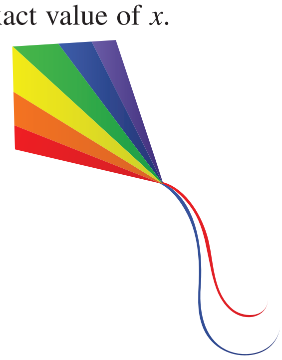


A characteristic of a cyclic quadrilateral is that the opposite angles are supplementary. Determine the value of x.

20. How do you know if a quadrilateral is a rhombus?

21. The perimeter of this kite is 80 cm. Determine the exact value of x.

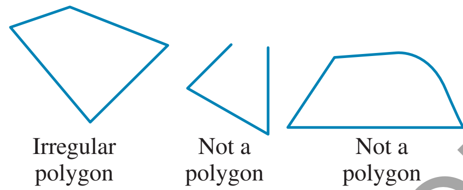




14.5 Polygons [Stages 5.2 and 5.3]

14.5.1 Definition of a polygon

- A **polygon** is a closed shape that has three or more straight sides.



- **Regular polygons** are polygons with sides of the same length and interior angles of the same size, like the pentagon shown in the centre of the photo above.
- **Convex polygons** are polygons with no interior reflex angles.
- **Concave polygons** are polygons with at least one reflex interior angle. For example, the pentagon shown above is a concave polygon as well as a regular polygon.

14.5.2 Interior angles of a polygon

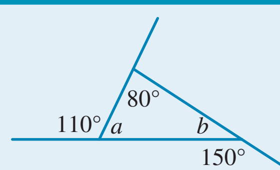
- The interior angles of a polygon are the angles inside the polygon at each vertex.
- The sum of the interior angles of a polygon is given by the formula:

$$\text{Angle sum} = 180° \times (n - 2)$$

where n = the number of sides of the polygon

WORKED EXAMPLE 6

Calculate the values of the pronumerals in the figure below.



THINK	WRITE
1 Angles a and 110° form a straight line, so they are supplementary (add to 180°).	$a + 110° = 180°$ $a + 110° - 110° = 180° - 110°$ $a = 70°$

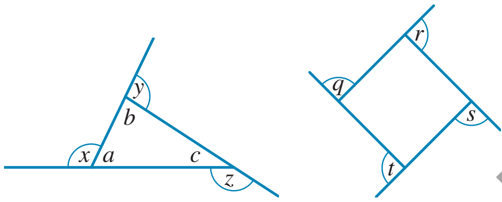
2 The interior angles of a triangle sum to 180°. $b + a + 80° = 180°$

3 Substitute 70° for a and solve for b.

$$b + 70° + 80° = 180°$$
$$b + 150° = 180$$
$$b = 30°$$

14.5.3 Exterior angles of a polygon

- The exterior angles of a polygon are formed by the side of the polygon and an extension of its adjacent side. For example, x, y and z are external angles for the polygon (triangle) below.



- The exterior angle and interior angle at that vertex are supplementary (add to 180°). For example, $x + a = 180°$.
- Exterior angles of polygons can be measured in a clockwise or anticlockwise direction.
- In a regular polygon, the size of the exterior angle can be found by dividing 360° by the number of sides.

$$\text{Exterior angle} = \frac{360°}{n}$$

- The sum of the exterior angles of a polygon equals 360°.
- The exterior angle of a triangle is equal to the sum of the opposite interior angles.

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Interactivity: Interior angles of a polygon (int-6166)
Interactivity: Exterior angles of a polygon (int-6167)
Interactivity: Angle sum of a polygon (int-0818)
Interactivity: Exterior angles of a polygon (int-0819)
Digital doc: WorkSHEET: Deductive geometry 3 (doc-5284)

Exercise 14.5 Polygons

assesson

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 9	Questions: 1–9, 12	Questions: 1–12

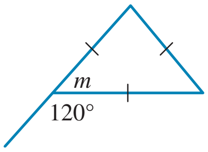
Individual pathway interactivity: int-4615

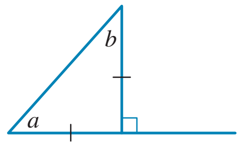
learnon ONLINE ONLY

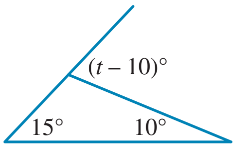
To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

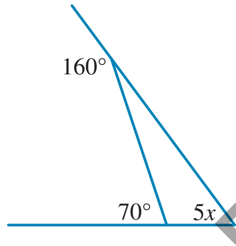
Understanding and fluency

1. How are the internal and external angles of a polygon related to the number of sides in a polygon?
2. WE6 Calculate the values of the pronumerals in the diagrams below.

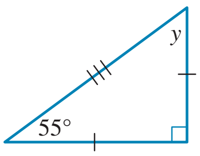
a.


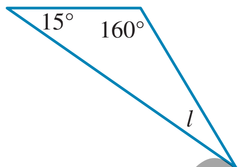
b.


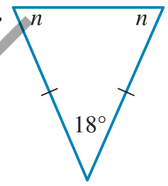
c.


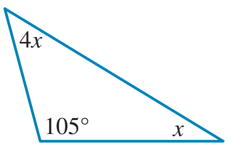
d.


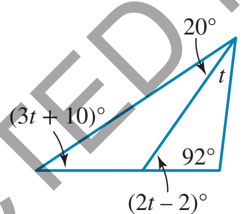
3. For the five triangles below, evaluate the pronumerals and determine the size of the interior angles.

a.


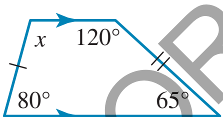
b.


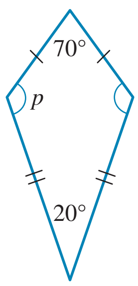
c.


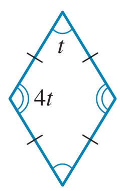
d.


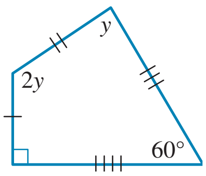
e.


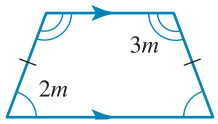
4. For the five quadrilaterals below:
   i. label the quadrilaterals as regular or irregular
   ii. determine the value of the pronumeral for each shape.

a.


b.


c.


d.


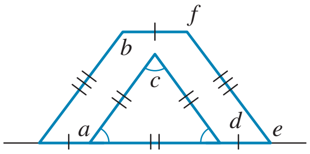
e.


5. The photograph below shows a house built on the side of a hill. Use your knowledge of angles to calculate the values of the pronumerals.

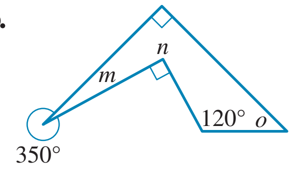
6. Calculate the values of the four interior angles of the front face of the building in the photograph below.

7. Calculate the values of the pronumerals for the irregular polygons below.

a.



b.



8. Calculate the size of the exterior angle of a regular hexagon (6 sides).

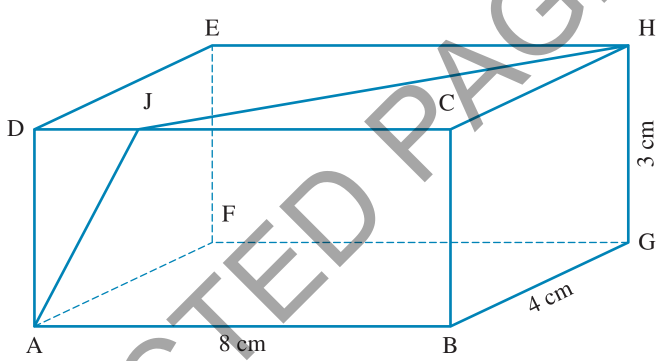
Communicating, reasoning and problem solving

9. A diagonal of a polygon joins two vertices.
   a. Calculate the number of diagonals in a regular polygon with:
      i. 4 sides ii. 5 sides iii. 6 sides iv. 7 sides.
   b. Write a formula that relates the number of diagonals for an n-sided polygon.
10. The external angle of a polygon can be calculated using the formula:

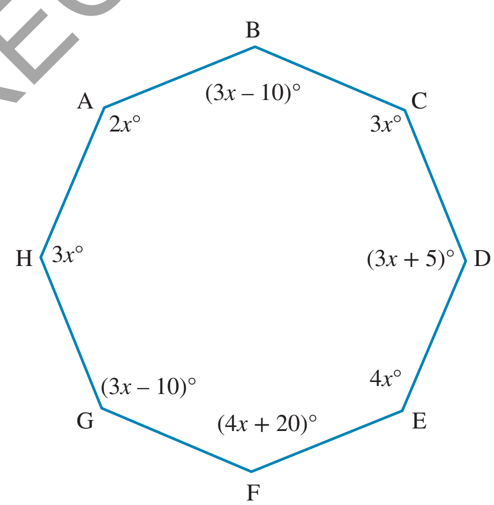
$$\text{exterior angle} = \frac{360°}{n}$$

Use the relationship between internal and external angles of a polygon to write a formula for the internal angle of a regular polygon.

11. A piece of string is fixed at A and H as shown. The string is tight and fixed to the surface of the cuboid. Locate the exact position of J on the edge CD.



12. ABCDEFGH is an octagon.



   a. What is the sum of the interior angles of an octagon?
   b. Find the value of x.
13. How are the angles associated with polygons related to each other and the polygon?

14.6 Review

Investigation | Rich task

Enlargement activity

Enlargement is the construction of a bigger picture from a small one. The picture is identical to the other except that it is bigger. The new picture is often called the image. This can also be called creating a similar figure.

The geometrical properties shared by a shape and its image under enlargement can be listed as:

- lines are enlarged as lines
- sides are enlarged to corresponding sides by the same factor
- matching angles on the two shapes are equal.

In this activity, we will start with a small cartoon character, and then 'blow it up' to almost life-size.

Equipment: ruler, pencil, cartoon print, butcher's paper or some other large piece of paper.

1. Do some research on the internet and select a cartoon character or any character of your choice.

2. Draw a grid of 2-cm squares over the small cartoon character.

Example: The small Casper is 9 squares wide and 7 squares tall.

3. Label the grids with letters across the top row and numbers down the first column, as shown in the example.

4. Get a large piece of paper and draw the same number of squares. You will have to work out the ratio of similitude (e.g. 2: 8).
5. If your small cartoon character stretches from one side of the 'small' paper (the paper the image is printed on) to the other, your 'large' Casper must stretch from one side of the 'big' paper to the other. Your large grid squares may have to be 8 cm by 8 cm or larger, depending on the paper size.
6. Draw this enlarged grid on your large paper. Use a metre ruler or some other long straight-edged tool. Be sure to keep all of your squares the same size.
   - At this point, you are ready to DRAW. Remember, you do NOT have to be an artist to produce an impressive enlargement.
   - All you do is draw EXACTLY what you see in each small cell into its corresponding large cell.
   - For example, in cell B3 of the Casper enlargement, you see the tip of his finger, so draw this in the big grid.
   - If you take your time and are very careful, you will produce an extremely impressive enlargement.
   - What you have used is called a RATIO OF SIMILITUDE. This ratio controls how large the new picture will be.

A 2 : 5 ratio will give you a smaller enlargement than a 2 : 7 ratio, because for every two units on the original you are generating only 5 units of enlargement instead of 7.

If Casper's ratio is 1 : 4, it produces a figure that has a linear measure that is four times bigger.

Big Casper's overall area, however, will be 16 times larger than small Casper's. This is because area is found by taking length times width.

The length is 4 times longer and the width is 4 times longer.

Thus the area is $4 \times 4 = 16$ times larger than the original Casper.

His overall volume will be $4 \times 4 \times 4$ or 64 times larger! This means that big Casper will weigh 64 times more than small Casper.

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Interactivity: Word search: Topic 14 (int-2853)
Interactivity: Crossword: Topic 14 (int-2854)
Interactivity: Sudoku: Topic 14 (int-3597)
Digital doc: Code puzzle: Why was the archaeologist upset? (doc-15935)
Digital doc: Summary (doc-22912)

Exercise 14.6 Review questions

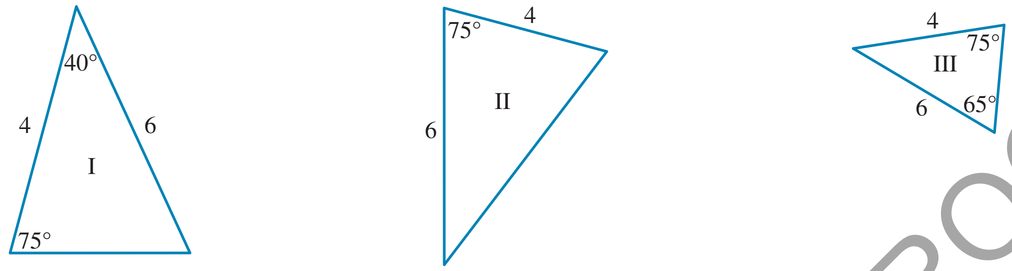
assess**on**

To answer questions online and to receive immediate feedback and fully worked solutions for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

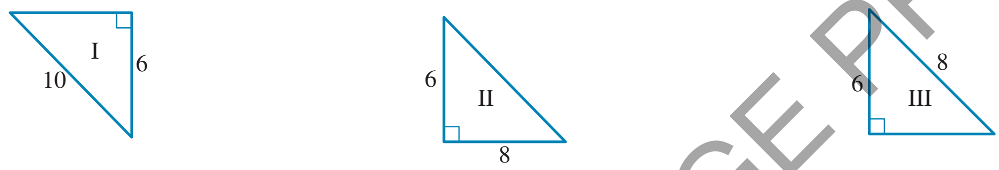
Understanding and fluency

1. Select a pair of congruent triangles in each of the following sets of triangles, giving a reason for your answer. All angles are in degrees and side lengths in cm. (The figures are not drawn to scale.)

a.



b.

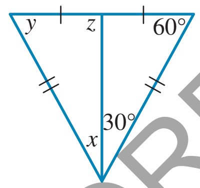


2. Find the value of the pronumeral in each pair of congruent triangles. All angles are given in degrees and side lengths in cm.

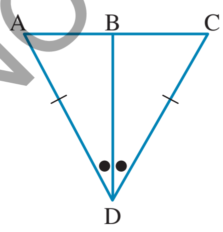
a.

b.

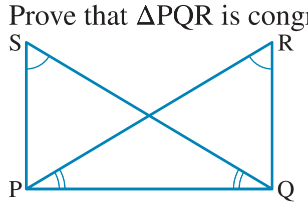
c.



3. a. Prove that the two triangles shown in the diagram below are congruent.

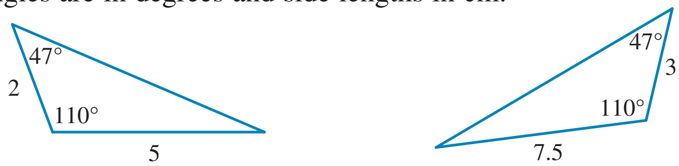


b. Prove that ΔPQR is congruent to ΔQPS.

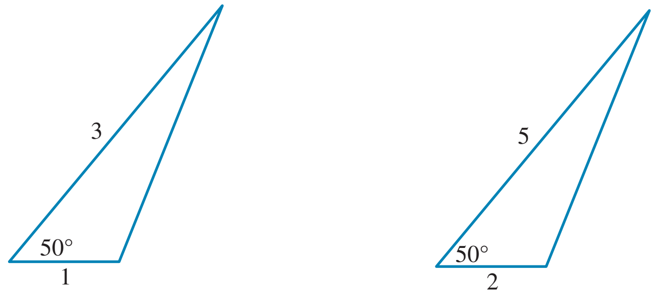


4. Test whether the following pairs of triangles are similar. For similar triangles find the scale factor. All angles are in degrees and side lengths in cm.

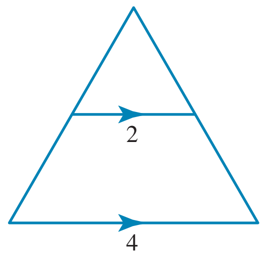
a.



b.

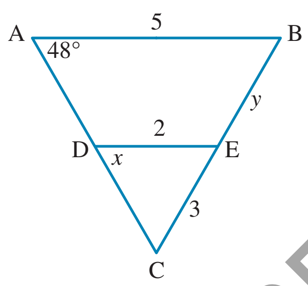


c.

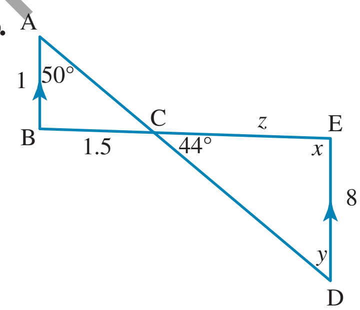


5. Find the value of the pronumeral in each pair of similar triangles. All angles are given in degrees and side lengths in cm.

a.

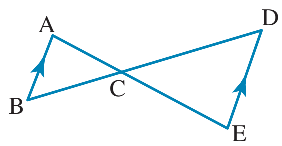


b.

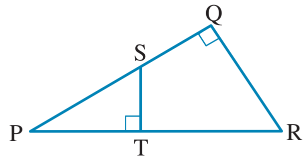


c.

6. Prove that $\Delta ABC \sim \Delta EDC$.



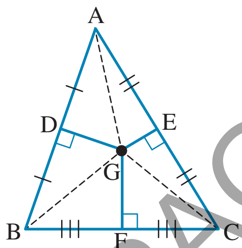
7. Prove that $\Delta PST \sim \Delta PRQ$.



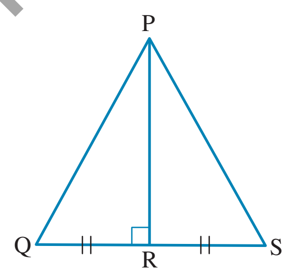
8. Prove that the angles opposite the equal sides in an isosceles triangle are equal.
9. **MC** Two corresponding sides in a pair of similar octagons have lengths of 4 cm and 60 mm. The respective scale factor in length is:
   A. 4 : 60 **B.** 6 : 40 **C.** 40 : 60 **D.** 60 : 40 **E.** 40 : 6
10. A regular nonagon has side length x cm. Use a scale factor of $\frac{x+1}{x}$ to find the side length of a similar nonagon.

Communicating, reasoning and problem solving

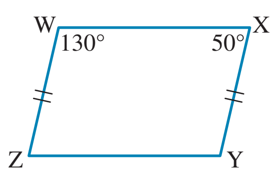
11. ABC is a triangle. D is the midpoint of AB, E is the midpoint of AC and F is the midpoint of BC. DG ⊥ AB, EG ⊥ AC and FG ⊥ BC.



   a. Prove that $\Delta GDA \cong \Delta GDB$.
   b. Prove that $\Delta GDE \cong \Delta GCE$.
   c. Prove that $\Delta GBF \cong \Delta GCF$.
   d. What does this mean about AG, BG and CG?
   e. A circle centred at G is drawn through A. What other points must it pass through?
12. PR is the perpendicular bisector of QS. Prove that ΔPQS is isosceles.



13. Name any quadrilaterals that have diagonals that bisect the angles they pass through.
14. State three tests that can be used to show that a quadrilateral is a rhombus.
15. Prove that WXYZ is a parallelogram.



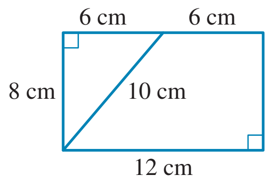
16. Prove that the diagonals in a rhombus bisect the angles they pass through.

17. Explain why the triangles shown are not congruent.

18. State the definition of a rhombus.

19. Name any quadrilaterals that have equal diagonals.

20. This 8 cm by 12 cm rectangle is cut into two sections as shown.



a. Draw labelled diagrams to show how the two sections can be rearranged to form:
   i. a parallelogram
   ii. a right-angled triangle
   iii. a trapezium.

b. Comment on the perimeters of the figures.

Answers

Topic 14 Properties of geometrical figures

Exercise 14.2 Angles, triangles and congruence

1. a. $a = 56°$ **b.** $b = 30°$ **c.** $c = 60°$ **d.** $d = 120°$ **e.** $e = 68°$

2. a. I and III, SAS **b.** I and II, AAS **c.** II and III, RHS **d.** I and II, SSS

3. a. $x = 6$, $y = 60°$ **b.** $x = 80°$, $y = 50°$ **c.** $x = 32°$, $y = 67°$ **d.** $x = 45°$, $y = 45°$

4. a. $x = 3$ cm

b. $x = 85°$

c. $x = 80°$, $y = 30°$, $z = 70°$

d. $x = 30°$, $y = 7$ cm

e. $x = 40°$, $y = 50°$, $z = 50°$, $m = 90°$, $n = 90°$

5. a. Use SAS **b.** Use SAS. **c.** Use ASA. **d.** Use ASA. **e.** Use SSS.

6. C, D

7. a. $x = 110°$, $y = 110°$, $z = 4$ cm, $w = 7$ cm

b. $x = 70°$

c. $x = 30°$, $y = 65°$

8. The third sides are not necessarily the same.

9. Corresponding sides are not the same.

10. Use SSS.

11, 12, 13. Check with your teacher.

14. $x = 20°$, $y = 10°$, $z = 40°$

15. Check with your teacher.

Exercise 14.3 Similar triangles

1. a. i and **iii**, RHS **b. i** and **ii**, SAS **c. i** and **iii**, SSS **d. i** and **iii**, AAA **e. i** and **ii**, SSS

2. a. Triangles PQR and ABC **b.** Triangles ADB and ADC **c.** Triangles PQR and TSR

d. Triangles ABC and DEC **e.** Triangles ABC and DEC

3. a. $\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$ **b.** $f = 9$, $g = 8$

4. $x = 4$

5. $x = 20°$, $y = 2\frac{1}{4}$

6. a. AAA **b.** $x = 3$, $y = 4$

7. The slide is 7.23 m long.

8. Radius = 0.625 m

9. a. $x = 1$, $y = 7\sqrt{2}$ **b.** $x = 2.5$, $y = 3.91$

10. Check with your teacher. One method would be to use Pythagorean triples.

11. a. $\angle ABD = \angle ABC$ (common angle)

$\angle ADB = \angle BAC = 90°$

$\Delta ABD \sim \Delta ACB$ (AAA)

b. $\angle ACD = \angle BCA$ (common angle)

$\angle ADC = \angle CAB = 90°$

$\Delta ACD \sim \Delta ACB$ (AAA)

12. Congruent triangles must be identical; that is, the angles must be equal and the side lengths must be equal. Therefore, it is not enough just to prove that the angles are equal.

13. Check with your teacher.

14. $\angle FEO = \angle OGH$ (alternate angles equal as EF || HG)

$\angle EFO = \angle OHG$ (alternate angles equal as EF || HG)

$\angle EOF = \angle HOG$ (vertically opposite angles equal)

$\therefore \Delta EFO \sim \Delta GHO$ (equiangular)

15. $x = 6$ or 11

16. Similar triangles have equal angles or corresponding sides are proportional.

17. 3

Exercise 14.4 Quadrilaterals

1. a. True **b.** True **c.** True **d.** False

e. False **f.** False **g.** False **h.** False

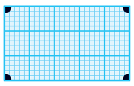
2. a. $x = 36°$, $y = 62°$ **b.** $x = 5$ cm, $y = 90°$ **c.** $x = 10°$, $y = 70°$ **d.** $x = 40°$, $y = 60°$

3. None are true, unless the trapezium is a regular trapezium, then e is true.

4. a, **c**, **f**

5. Parallelogram, rhombus, rectangle, square

6. Square

7. a. 

b. 6 sides

c. 7 sides

d.

Table size	Number of sides hit
5 cm × 3 cm	6
7 cm × 2 cm	7
4 cm × 3 cm	5
4 cm × 2 cm	1
6 cm × 3 cm	1
9 cm × 3 cm	2
12 cm × 4 cm	2

e. If the ratio of the sides is written in simplest form, then the pattern is $m + n - 2$.

f. There are two routes for the ball when hit from B. Either 2 or 3 sides are hit. The ball does not end up in the same hole each time.

A suitable justification would be a diagram — student to draw.

g. Isosceles triangles and parallelograms. The triangles are congruent.

h. The shapes formed are parallelograms. There is only one possible path although the ball could be hit in either of two directions initially.

i. Given: $m : n$ is the ratio length to width in simplest form.

When m is even and n, is odd the destination pocket will be the upper left.

When m and n are both odd, the destination pocket will be the upper right.

When m is odd and n is even, the destination pocket will be the lower right.

j. Students to investigate.

8. Check with your teacher.

9. AX || DY because ABCD is a parallelogram.

AX = DY (given)

∴ AXYD is a parallelogram since opposite sides are equal and parallel.

10. a. Use SAS. **b.** Use SAS. **c.** Opposite sides are equal.

11. AC = DB (diameters of the same circle are equal)

AO = OC and OD = OB (radii of the same circle are equal)

∴ ABCD is a rectangle (diagonals are equal and bisect each other).

12. Check with your teacher.

13. PS = QR (corresponding sides in congruent triangles are equal)

PS || QR (alternate angles are equal)

∴ PQRS is a parallelogram since one pair of opposite sides are parallel and equal.

14. MP = MQ (radii of same circle)
PN = QN (radii of same circle) and circles have equal radii.
∴ All sides are equal.
∴ PNQM is a rhombus.
15. Check with your teacher.
16. **a.** One pair of opposite sides is parallel.
b. $x = 90°$, $y = 75°$
17. **a. i.** $x = \sqrt{41}$ **ii.** $y = \sqrt{97}$
b. $\angle BAD = \angle BCD = 117°23'$
18. **a.** Rhombus, 2 cm **b.** Trapezium **c.** 40° **d.** Triangle BFC
e. Check with your teacher. **f.** Check with your teacher. **g.** $x = (1 + \sqrt{5})$ cm
19. 70°
20. Rhombuses have four equal sides.
21. $x = \sqrt{10}$ cm

Exercise 14.5 Polygons

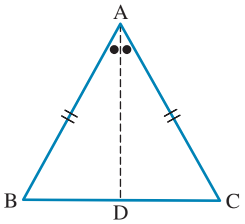
1. The sum of the interior angles is based on the number of sides of the polygon.
The size of the exterior angle can be found by dividing 360° by the number of sides.
2. **a.** $m = 60°$ **b.** $a = 45°$, $b = 45°$ **c.** $t = 35°$ **d.** $t = 30°$
3. **a.** $y = 35°$ **b.** $x = 10°$ **c.** $t = 5°$ **d.** $n = 81°$ **e.** $x = 15°$ **f.** $t = 30°$
4. **a. i.** Irregular **ii.** $x = 95°$
b. i. Irregular **ii.** $p = 135°$
c. i. Irregular **ii.** $t = 36°$
d. i. Irregular **ii.** $y = 70°$
e. i. Irregular **ii.** $p = 36°$
5. $w = 75°$, $x = 105°$, $y = 94°$, $z = 133°$
6. 82.5°, 82.5°, 97.5°, 97.5°
7. **a.** $a = 120°$, $b = 120°$, $c = 60°$, $d = 60°$, $e = 120°$, $f = 240°$
b. $m = 10°$, $n = 270°$, $o = 50°$
8. 60°
9. **a. i.** 2 **ii.** 5 **iii.** 9 **iv.** 14
b. Number of diagonals $= \frac{1}{2}n(n - 3)$
10. Internal angle $= 180° - \dfrac{360°}{n}$
11. J is $\frac{24}{7}$ cm from D.
12. **a.** 1080° **b.** 43°
13. Discuss with teacher

Investigation | Rich task

Check with your teacher.

Exercise 14.6 Review questions

1. **a.** I and III, ASA or SAS **b.** I and II, RHS
2. **a.** $x = 8$ cm **b.** $x = 70°$ **c.** $x = 30°$, $y = 60°$, $z = 90°$
3. **a.** Use SAS. **b.** Use ASA.
4. **a.** Similar, scale factor = 1.5
b. Not similar
c. Similar, scale factor = 2
5. **a.** $x = 48°$, $y = 4.5$ cm **b.** $x = 86°$, $y = 50°$, $z = 12$ cm **c.** $x = 60°$, $y = 15$ cm, $z = 12$ cm
6. Use the equiangular test.
7. Use the equiangular test.

8. 

Bisect ∠BAC.

AB = AC (given)

∠BAD = ∠DAC

AD is common.

∴ ΔABD ≅ ΔACD (SAS)

∴ ∠ABD = ∠ACD (corresponding sides in congruent triangles are equal)

9. C

10. $x + 1$

11. a. Use SAS. **b.** Use SAS. **c.** Use SAS.

d. They are all the same length.

e. B and C

12. Use SAS.

PQ = PS (corresponding sides in congruent triangles are equal)

13. Rhombus, square

14. A quadrilateral is a rhombus if:

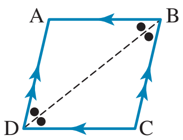
a. all sides are equal

b. the diagonals bisect each other at right angles

c. the diagonals bisect the angles they pass through.

15. WZ || XY (co-interior angles are supplementary) and WZ = XY (given)

∴ WXYZ is a parallelogram since one pair of sides is parallel and equal.

16. 

∠ABD = ∠ADB (angles opposite the equal sides in an isosceles triangle are equal)

∠ABD = ∠BDC (alternate angles equal as AB || DC)

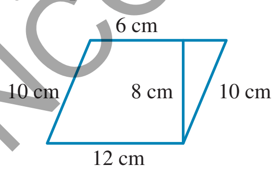
∴ ∠ADB = ∠BDC

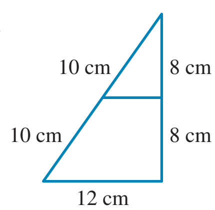
∴ Diagonals bisect the angles they pass through.

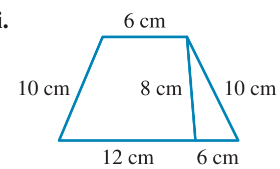
17. Corresponding sides are not the same.

18. A rhombus is a parallelogram with two adjacent sides equal in length.

19. Rectangle, square

20. a. i. 

ii. 

iii. 

b. Perimeter of rectangle = 40 cm

Perimeter of parallelogram = 44 cm

Perimeter of triangle = 48 cm

Perimeter of trapezium = 44 cm

The triangle has the largest perimeter, and the rectangle has the smallest.