16.1 Overview

Why learn this?
Algebraic techniques help us to understand many things in the everyday world. What we learn in mathematics can be applied to many other subjects; for example, physics is often described as using mathematics as a tool for studying the world around us. The concepts studied in quadratic algebra can be applied to solving quadratic equations. Many physical phenomena can be described using quadratic equations, and in this chapter you will learn the techniques necessary to solve quadratic equations.

What do you know?  
1 THINK List what you know about quadratics. Use a thinking tool such as a concept map to show your list.
2 PAIR Share what you know with a partner and then with a small group.
3 SHARE As a class, create a thinking tool such as a large concept map to show your class’s knowledge of quadratics.

Learning sequence
16.1 Overview
16.2 Factorisation patterns
16.3 Factorising monic quadratics
16.4 Factorising non-monic quadratics
16.5 Simplifying algebraic fractions
16.6 Quadratic equations
16.7 The Null Factor Law
16.8 Solving the quadratic equation $ax^2 + bx + c = 0$
16.9 Solving quadratic equations with two terms
16.10 Applications
16.11 Review
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16.2 Factorisation patterns

Difference of two squares

• Recall from Topic 3 that \((a + b)(a - b) = a^2 - b^2\).
  \(a^2 - b^2\) is known as the ‘difference of two squares’.

• The rule can be used in reverse to factorise the difference of two squares:
  \(a^2 - b^2 = (a + b)(a - b)\)

• Factorise by taking out any common factor first.

For example:

\[
5x^2 - 80 = 5(x^2 - 16) = 5(x - 4)(x + 4)
\]

WORKED EXAMPLE 1

Factorise each of the following expressions.

a \(x^2 - 9\)
b \(64m^2 - 25n^2\)
c \((c + 7)^2 - 16\)
d \(3x^2 - 48\)

THINK

WRITE

\(x^2 - 9\)

\(= x^2 - 3^2\)

\(= (x + 3)(x - 3)\)

\(64m^2 - 25n^2\)

\(= (8m)^2 - (5n)^2\)

\(= (8m + 5n)(8m - 5n)\)

\((c + 7)^2 - 16\)

\(= (c + 7)^2 - 4^2\)

\(= (c + 7 + 4)(c + 7 - 4)\)

\(= (c + 11)(c + 3)\)

\(3x^2 - 48\)

\(= 3(x^2 - 16)\)

\(= 3(x^2 - 4^2)\)

\(= 3(x + 4)(x - 4)\)
Perfect squares

• Recall from Topic 3 that a perfect square is an expression such as \((x + 2)^2\) or \((t - 2)^2\).
  The expansion of perfect squares produces the following patterns or identities.

\[
(a + b)^2 = a^2 + 2ab + b^2 \\
(a - b)^2 = a^2 - 2ab + b^2
\]

• The identities can also be used in reverse to factorise the quadratic expressions.

\[
a^2 + 2ab + b^2 = (a + b)^2 \\
a^2 - 2ab + b^2 = (a - b)^2
\]

• Is \(x^2 + 8x + 16\) a perfect square?
  The constant term must be the square of half the coefficient of the \(x\)-term if the expression is a perfect square. Because half of 8 is 4 and \(4^2 = 16\), \(x^2 + 8x + 16\) is a perfect square.

\[
x^2 + 8x + 16 = (x + 4)^2
\]

WORKED EXAMPLE 2

Factorise each of the following expressions.

\[\text{a} \quad x^2 - 6x + 9 \\
\text{b} \quad 64m^2 + 80mn + 25n^2 \\
\text{c} \quad 3x^2 - 24x + 48\]

**THINK**

\begin{align*}
\text{a} & \quad x^2 \text{ and } 9 \text{ are perfect squares, so } x^2 - 6x + 9 \text{ might be } (x - 3)^2. \text{ Check as follows.} \\
\text{b} & \quad 64m^2 \text{ and } 25n^2 \text{ are perfect squares, so } 64m^2 + 80mn + 25n^2 \text{ might be } (8m + 5n)^2. \text{ Check the middle term.} \\
\text{c} & \quad \text{There is a common factor.}
\end{align*}

**WRITE**

\begin{align*}
\text{a} & \quad x^2 - 6x + 9 \\
& \quad \frac{-6}{2} = -3, (-3)^2 = 9 \\
& \quad x^2 - 6x + 9 = (x - 3)^2 \\
\text{b} & \quad 64m^2 + 80mn + 25n^2 \\
& \quad 8m \times 5n \times 2 = 80mn \\
& \quad 64m^2 + 80mn + 25n^2 = (8m + 5n)^2 \\
\text{c} & \quad 3x^2 - 24x + 48 \\
& \quad = 3(x^2 - 8x + 16) \\
& \quad = 3(x - 4)^2
\end{align*}
Exercise 16.2 Factorisation patterns

### INDIVIDUAL PATHWAYS

#### PRACTISE

| Questions: | 1a–e, 2a–e, 3a–f, 4a–f, 5–8, 10, 11 |

#### CONSOLIDATE

| Questions: | 1d–g, 2d–g, 3d–i, 4d–i, 5–9, 11, 12–14 |

#### MASTER

| Questions: | 1e–i, 2e–i, 3g–l, 4g–l, 5–16 |

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### Fluency

1. **WE1** Factorise each of the following expressions using the difference of two squares.
   - a) $x^2 - 25$
   - b) $x^2 - 81$
   - c) $a^2 - 16$
   - d) $25 - p^2$
   - e) $121 - a^2$
   - f) $36 - y^2$
   - g) $4b^2 - 25$
   - h) $9a^2 - 16$
   - i) $25d^2 - 1$

2. **WE1c** Factorise each of the following expressions.
   - a) $x^2 - y^2$
   - b) $a^2 - b^2$
   - c) $p^2 - q^2$
   - d) $25m^2 - n^2$
   - e) $81x^2 - y^2$
   - f) $p^2 - 36q^2$
   - g) $36m^2 - 25n^2$
   - h) $16q^2 - 9p^2$
   - i) $4m^2 - 49n^2$

3. **WE1b** Factorise each of the following expressions.
   - a) $(x + 9)^2 - 16$
   - b) $(p + 8)^2 - 25$
   - c) $(p - 2)^2 - q^2$
   - d) $(c - 6)^2 - d^2$
   - e) $(x + 7)^2 - y^2$
   - f) $(p + 5)^2 - q^2$
   - g) $(a + 3)^2 - b^2$
   - h) $(a - 3)^2 - 1$
   - i) $(b - 1)^2 - 36$

4. **WE1d** Factorise each of the following expressions completely after removing any common factors.
   - a) $2m^2 - 32$
   - b) $5y^2 - 45$
   - c) $6p^2 - 24$
   - d) $4m^2 - 100$
   - e) $288 - 2x^2$
   - f) $80 - 5a^2$
   - g) $100x^2 - 25y^2$
   - h) $144p^2 - 4q^2$
   - i) $32y^2 - 2x^2$
   - j) $ax^2 - 9a$
   - k) $8b - 2hx^2$
   - l) $3(b + 5)^2 - 48$

5. **WE2** Factorise each of the following expressions by recognising the perfect square rule.
   - a) $x^2 + 10x + 25$
   - b) $p^2 - 24p + 144$
   - c) $n^2 + 20n + 100$
   - d) $x^2 + 2xy + y^2$
   - e) $u^2 - 2uv + v^2$
   - f) $64 + 16e + e^2$
   - g) $4m^2 - 20m + 25$
   - h) $49 + 42a + 9a^2$
   - i) $81x^2 + 72xy + 16y^2$
   - j) $2x^2 + 24x + 72$
   - k) $3x^2 - 24xy + 48y^2$
   - l) $18a^2 + 24ab + 8b^2$

6. **a** Which of the expressions below are the difference of two squares?
   - i) $x^2 - 36$
   - ii) $x^2 + 36$
   - iii) $x^2 + 6x + 36$
   - iv) $x^2 - 12x + 36$
   - v) $x^2 + 12x - 36$
   - vi) $x^2 + 12x + 36$

**b** Which of the expressions below are perfect squares?

7. **MC**
   
   **a** The expression $x^2 - 121$ factorises to:
   - A) $(x - 11)^2$
   - B) $(x - 11)(x + 12)$
   - C) $(x + 11)(x - 11)$
   - D) $(x - 11)(x + 11)$

   **b** What does the expression $36m^2 - 12m + 1$ factorise to?
   - A) $(6m - 1)(6m - 1)$
   - B) $36(m + 1)(m - 1)$
   - C) $(6m + 1)^2$
   - D) $6(6m - 1)(6m - 1)$
c What does the expression $16a^2 - 25b^2$ factorise to?
- A $16(a + 5b)(a - 5b)$
- B $4(4a - 5b)^2$
- C $(16a + 25b)(16a - 25b)$
- D $(4a + 5b)(4a - 5b)$

d The expression $5c^2 - 20c + 20$ factorises to:
- A $5(c - 2)(c - 2)$
- B $5(c + 4)(c - 4)$
- C $5(c - 4)(c - 4)$
- D $(5c + 20)(5c - 20)$

e The expression $(x + 4)^2 - 9$ factorises to:
- A $9(x + 4)(x - 4)$
- B $(x + 13)(x - 5)$
- C $(x + 7)(x + 1)$
- D $(x - 1)(x + 5)$

UNDERSTANDING

8 A circular pool with a radius of $r$ metres is surrounded by a circular path 1 m wide.
- a Find the surface area of the pool.
- b Find, in terms of $r$, the distance from the centre of the pool to the outer edge of the path.
- c Find the area of the circle that includes the path and the pool. (Don’t expand the expression.)
- d Write an expression for the area of the path.
- e Simplify this expression.
- f If the pool had a radius of 5 m, what would be the area of the path to the nearest square metre?
- g If the pool had a radius of 7 m, what would be the area of the path to the nearest square metre?

9 An L-shaped piece of farm land is to be planted with wheat.
- a Write an expression for the area of the wheat in simplest form.
- b Calculate how much wheat is to be planted if $w$ is:
  - i 5
  - ii 10
  - iii 20.
- c Invent your own L-shaped area using the pronumeral $w$ that has an area that can be expressed as a perfect square.
10 Factorise the following using the difference of two squares rule.
   a  \( x^2 - 10 \)  
   b  \( 4x^2 - 32 \)  
   c  \( x^4 - y^2 \)  
   d  \( (x + 2)^2 - (x - 3)^2 \)  
   e  \( y^2 - 4x^4 \)  
   f  \( 10^4 - x^4 \)

REASONING
11 Is \((x + 4)^2\) equal to \(x^2 + 16\)? Explain using a numerical example.
12 Show that \((a + b)(c + d) = (c + d)(a + b)\).
13 Show that \((a + b)(a - b) = (a - b)(a + b)\).

PROBLEM SOLVING
14 Five squares of increasing size and five area formulas are given below.
   a Use factorisation to find the side length that correlates to each area formula.
   b Using the area given and side lengths found, match the squares below with the appropriate algebraic expression for their area.

   ![Squares and formulas]

   \( A_a = x^2 + 6x + 9 \)  
   \( A_b = x^2 + 10x + 25 \)  
   \( A_c = x^2 + 16x + 64 \)  
   \( A_d = x^2 - 6x + 9 \)  
   \( A_e = x^2 + 12x + 36 \)

   c If \(x = 5\) cm, use the formula given to calculate the area of each square.

15 The area of a rectangle is represented by \(x^2 - 49\) mm\(^2\).
   a Factorise the expression.
   b Using the factorised expression, find a possible length and width for the rectangle.
   c If \(x = 25\) mm, find the dimensions for the rectangle.
   d If \(x = 50\) mm, how much bigger is the area of the rectangle than in part c?

16 The expansion of perfect squares
   \((a + b)^2 = a^2 + 2ab + b^2\)  
   and \((a - b)^2 = a^2 - 2ab + b^2\)
   can be used to simplify some arithmetic calculations. For example:

   \[
   97^2 = (100 - 3)^2 \\
   = 100^2 - 2 \times 100 \times 3 + 3^2 \\
   = 9409
   \]

Use this method to calculate the following.
   a \(103^2\)  
   b \(62^2\)  
   c \(997^2\)  
   d \(1012^2\)  
   e \(53^2\)  
   f \(98^2\)

CHALLENGE 16.1
Use the difference of two squares to completely factorise \(x^8 - 1\).
16.3 Factorising monic quadratics

Quadratic trinomials

- A quadratic trinomial is an expression of the form \( ax^2 + bx + c \), where \( a, b \) and \( c \) are constants.
- If the expression contains \( 1x^2 \), that is if \( a = 1 \), then it is called a monic quadratic trinomial.

Factorising monic quadratics

- The area model of binomial expansion can be reversed to find a pattern for factorising a general quadratic expression. For example:

\[
(x + f)(x + h) = x^2 + fx + hx + fh \quad (x + 4)(x + 3) = x^2 + 4x + 3x + 12
\]

\[
= x^2 + (f + h)x + fh \quad = x^2 + 7x + 12
\]

- To factorise a general quadratic, look for factors of \( c \) that sum to \( b \).

\[x^2 + bx + c = (x + f)(x + h)\]

Factors of \( c \) that sum to \( b \)

For example, \( x^2 + 7x + 12 = (x + 3)(x + 4) \).

\[3 + 4 = 7 \quad 3 \times 4 = 12\]

WORKED EXAMPLE 3

Factorise the following quadratic expressions.

**a** \( x^2 + 5x + 6 \)  \quad **b** \( x^2 + 10x + 24 \)

**THINK**

**a** 1 The general quadratic expression has the pattern \( x^2 + 5x + 6 = (x + f)(x + h) \). \( f \) and \( h \) are a factor pair of 6 that add to 5.

Calculate the sums of factor pairs of 6. The factors of 6 that add to 5 are 2 and 3, as shown in blue.

2 Substitute the values of \( f \) and \( h \) into the expression in its factorised form.

**b** 1 The general quadratic expression has the pattern \( x^2 + 10x + 24 = (x + f)(x + h) \), where \( f \) and \( h \) are a factor pair of 24 that add to 10.

Calculate the sums of factor pairs of 24. The factors of 24 that add to 10 are 4 and 6, as shown in blue.

2 Substitute the values of \( f \) and \( h \) into the expression in its factorised form.

**WRITE**

**a**

<table>
<thead>
<tr>
<th>Factors of 6</th>
<th>Sum of factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 and 6</td>
<td>7</td>
</tr>
<tr>
<td>2 and 3</td>
<td>5</td>
</tr>
</tbody>
</table>

\((x^2 + 5x + 6) = (x + 2)(x + 3)\)

**b**

<table>
<thead>
<tr>
<th>Factors of 24</th>
<th>Sum of factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 and 24</td>
<td>25</td>
</tr>
<tr>
<td>2 and 12</td>
<td>14</td>
</tr>
<tr>
<td>3 and 8</td>
<td>11</td>
</tr>
<tr>
<td>4 and 6</td>
<td>10</td>
</tr>
</tbody>
</table>

\(x^2 + 10x + 24 = (x + 4)(x + 6)\)
Factorise the following expressions.

\( a \) \( x^2 - 9x + 18 \) \quad \( b \) \( x^2 + 6x - 16 \)

**THINK**

1. The general quadratic expression has the pattern \( x^2 - 9x + 18 = (x + f)(x + h) \), where \( f \) and \( h \) are a factor pair of 18 that add to \(-9\).

   Calculate the sums of factor pairs of 18. As shown in blue, \(-3\) and \(-6\) are factors of 18 that add to \(-9\).

2. Substitute the values of \( f \) and \( h \) into the expression in its factorised form.

\( a \) \( x^2 - 9x + 18 = (x - 3)(x - 6) \)

**WRITE**

1. The general quadratic expression has the pattern \( x^2 + 6x - 16 = (x + f)(x + h) \), where \( f \) and \( h \) are a factor pair of \(-16\) that add to 6.

   Calculate the sums of factor pairs of \(-16\). As shown in blue, \(-2\) and 8 are factors of \(-16\) that add to 6.

2. Substitute the values of \( f \) and \( h \) into the expression in its factorised form.

\( b \) \( x^2 + 6x - 16 = (x - 2)(x + 8) \)

**Exercise 16.3 Factorising monic quadratics**

**INDIVIDUAL PATHWAYS**

**PRACTISE**

Questions:
1a–k, 2, 3, 4a–i, 5, 6a–i, 7–9, 11

**CONSOLIDATE**

Questions:
1g–o, 4g–o, 5, 6g–o, 7–10, 12–14

**MASTER**

Questions:
1m–k, 4i–o, 6i–o, 7–11, 12–14

**FLUENCY**

1. Expand each of the following.

   \( a \) \( (x + 4)(x + 2) \) \quad \( b \) \( (x - 2)(x - 4) \) \quad \( c \) \( (x - 4)(x - 5) \) 
   \( d \) \( (x + 4)(x + 5) \) \quad \( e \) \( (m + 1)(m + 5) \) \quad \( f \) \( (m - 1)(m - 5) \) 
   \( g \) \( (t + 8)(t + 11) \) \quad \( h \) \( (t - 10)(t - 20) \) \quad \( i \) \( (x + 2)(x + 3) \) 
   \( j \) \( (x + 3)(x - 2) \) \quad \( k \) \( (v + 5)(v - 8) \) \quad \( l \) \( (v - 5)(v + 8) \) 
   \( m \) \( (x + 7)(x - 2) \) \quad \( n \) \( (t + 1)(t - 12) \) \quad \( o \) \( (n + 15)(n - 2) \) 
   \( p \) \( (a + 3)(a - 6) \) \quad \( q \) \( (z + 4)(z + 4) \) \quad \( r \) \( (z + 11)(z - 8) \) 

   \( s \) \( (n - 3)(n - 7) \) \quad \( t \) \( (v - 1)(v + 2) \)
2 Make a complete systematic list of pairs of positive numbers that add up to:
   a 5   b 6   c 12   d 20
3 Make a complete systematic list of pairs of negative numbers that add up to:
   a −5   b −7   c −8   d −11
4 **WE3** Factorise each of the following.
   a \(x^2 + 4x + 3\)   b \(x^2 - 4x + 3\)   c \(x^2 + 12x + 11\)
   d \(x^2 - 12x + 11\)   e \(a^2 + 6a + 5\)   f \(a^2 - 6a + 5\)
   g \(x^2 - 7x + 12\)   h \(x^2 - 7x + 10\)   i \(n^2 + 8n + 16\)
   j \(n^2 + 10n + 16\)   k \(y^2 - 12y + 27\)   l \(x^2 - 13x + 42\)
   m \(t^2 - 8t + 12\)   n \(t^2 + 11t + 18\)   o \(u^2 + 5u + 6\)
5 Make a systematic list of 5 pairs of numbers with opposite signs whose sum is:
   a 6   b −6   c 3   d −3
   e −8   f 14   g −5   h 7
6 **WE4** Factorise each of the following.
   a \(x^2 + 3x - 18\)   b \(x^2 - 3x - 18\)   c \(x^2 - 2x - 15\)
   d \(x^2 + 2x - 15\)   e \(n^2 - 13n - 14\)   f \(n^2 + 2n - 35\)
   g \(v^2 + 5v - 6\)   h \(v^2 - 5v - 6\)   i \(t^2 + 4t - 12\)
   j \(r^2 - 5t - 14\)   k \(x^2 - x - 20\)   l \(x^2 + x - 20\)
   m \(n^2 + n - 90\)   n \(n^2 - 3n - 70\)   o \(x^2 - 4x - 5\)

**UNDERSTANDING**
7 Consider the quadratic trinomial \(x^2 + 7x + c\) where \(c\) is a positive integer.
   a Factorise the expression if \(c = 6\).
   b What other values can \(c\) take if the expression is to be factorised? Factorise the expression for each of these values.
8 Consider the quadratic trinomial \(x^2 + 3x + c\) where \(c\) is a negative integer.
   a Factorise the expression if \(c = -4\).
   b Find 3 more values of \(c\) for which the expression can be factorised, and factorise each one.
9 Factorise the following trinomials, by first removing a common factor.
   a \(2x^2 - 10x - 28\)   b \(4x^2 + 28x + 40\)
   c \(-2x^2 - 2x + 24\)   d \(5x^2 - 40x + 75\)
10 **MC** When factorised \(x^2 - 3x - 18\) is equal to:
   A \((x - 3)(x - 6)\)   B \((x - 3)(x + 6)\)
   C \((x + 3)(x + 6)\)   D \((x + 3) (x - 6)\)
   E \((x + 2)(x - 9)\)

**REASONING**
11 Show that \(x^2 + 8x + 10\) has no factors, if only whole numbers can be used.
12 To factorise the quadratic trinomial \(6x^2 + 9x + 3\), the expression is rewritten as an equivalent form consisting of four terms, \(6x^2 + \square x + \square x + 3\), that are then grouped using the ‘two and two’ method, \(6x^2 + \square x + \square x + 3\). Investigate all the possible combinations for \(+ \square x + \square x = 9x\) and show that there is only one combination that will factorise the trinomial appropriately.
PROBLEM SOLVING

13 Rectangular floor mats have an area of $x^2 + 2x - 15$.
   a If the length of the mat is $(x + 5)$ cm, find an expression for the width.
   b If the length of the mat is 70 cm, what is the width?
   c If the width of the mat is 1 m, what is the length?

14 A particular rectangle has an area of 120 m$^2$.
   a Generate a list of possible dimensions for the rectangle.
   b The area of the rectangle can also be expressed as $A = x^2 + 2x - 48$.
   Determine algebraic expressions for the length and the width of the rectangle in terms of $x$.
   c Use your list of possible dimensions and your algebraic expressions for width and length to determine the dimensions of the rectangle. Hint: The value of $x$ in the length and width must be the same.

16.4 Factorising non-monic quadratics

Factorising quadratic trinomials where $a \neq 1$

- When a quadratic trinomial in the form $ax^2 + bx + c$ is written as $ax^2 + mx + nx + c$, where $m + n = b$, the four terms can be factorised by grouping.

2 $\times$ 12 = 24
Numbers that multiply to give 24 and add to 11 therefore (8) and (3).

$2x^2 + 11x + 12 = 2x^2 + (8x) + (3x) + 12$
$= 2x(x + 4) + 3(x + 4)$
$= (x + 4)(2x + 3)$

- There are many combinations of numbers that satisfy $m + n = b$, however only one particular combination can be grouped and factorised. For example,

$2x^2 + 11x + 12 = 2x^2 + 7x + 4x + 12$ or $2x^2 + 11x + 12 = 2x^2 + 8x + 3x + 12$

$= 2x^2 + 7x + 4x + 12$ or $= 2x^2 + 8x + 3x + 12$

$= x(2x + 7) + 4(x + 3)$ or $= 2x(x + 4) + 3(x + 4)$

cannot be factorised further

= $(x + 4)(2x + 3)$

- In examining the general binomial expansion, a pattern emerges that can be used to help identify which combination to use for $m + n = b$.

$(dx + e)(fx + g) = dfx^2 + dgx + efx + eg$

$= dfx^2 + (dg + ef)x + eg$

$m + n = dg + ef$ and $m \times n = dg \times ef$

$= b$ and $= dgef$

$= = dfe$ and $= ac$
Therefore, $m$ and $n$ are factors of $ac$ that sum to $b$.

To factorise a general quadratic where $a \neq 1$, look for factors of $ac$ that sum to $b$. Then rewrite the quadratic trinomial with four terms that can then be grouped and factorised.

$$ax^2 + bx + c = ax^2 + mx + nx + c$$

Factors of $ac$ that sum to $b$

**WORKED EXAMPLE 5**

Factorise $6x^2 - 11x - 10$.

**THINK**

1. Write the expression and look for common factors and special patterns. The expression is a general quadratic with $a = 6$, $b = -11$ and $c = -10$.

2. Since $a \neq 1$, rewrite $ax^2 + bx + c$ as $ax^2 + mx + nx + c$, where $m$ and $n$ are factors of $ac$ ($6 \times -10$) that sum to $b$ ($-11$).
   - Calculate the sums of factor pairs of $-60$. As shown in blue, 4 and $-15$ are factors of $-60$ that add to $-11$.

**WRITE**

$$6x^2 - 11x - 10 = 6x^2 + -11x + -10$$

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<td>$-30, 2$</td>
<td>$-28$</td>
</tr>
<tr>
<td>$15, -4$</td>
<td>$11$</td>
</tr>
<tr>
<td>$-15, 4$</td>
<td>$-11$</td>
</tr>
</tbody>
</table>

$$6x^2 - 11x - 10 = 6x^2 + 4x - 15x - 10$$

Write the answer.

$$6x^2 - 11x - 10 = 2x(3x + 2) - 5(3x + 2) = (3x + 2)(2x - 5)$$

**Exercise 16.4  Factorising non-monic quadratics**

**INDIVIDUAL PATHWAYS**

**PRACTISE**

Questions: 1a–e, 2a–e, 3–8

**CONSOLIDATE**

Questions: 1d–h, 2d–h, 3–12

**MASTER**

Questions: 1g–i, 2g–i, 3–12

**FLUENCY**

1. Factorise each of the following quadratic trinomials.
   
   **WEB**
   
   (a) $2x^2 + 10x + 12$
   (b) $3x^2 + 15x + 12$
   (c) $2x^2 - 12x + 16$
   (d) $3x^2 + 12x + 9$
   (e) $4x^2 - 4x - 24$
   (f) $3x^2 + 9x - 30$
   (g) $2x^2 + 8x - 42$
   (h) $3x^2 - 9x - 54$
   (i) $5x^2 + 20x - 60$

2. Factorise each of the following quadratic trinomials.
   
   (a) $2x^2 + 7x + 3$
   (b) $2x^2 + 7x + 6$
   (c) $3x^2 + 7x + 2$
   (d) $2x^2 - 7x + 3$
   (e) $3x^2 - x - 2$
   (f) $5x^2 + 3x - 2$
   (g) $7x^2 - 17x + 6$
   (h) $10x^2 - 11x - 6$
   (i) $2x^2 + 5x - 12$
**NUMBER AND ALGEBRA**

3. **MC**
   a. The expression $3x^2 + 21x + 36$ is equal to:
      A. $(x + 6)(x + 2)$  
      B. $(x + 4)(x + 3)$  
      C. $(3x + 9)(x + 4)$  
      D. $(3x + 2)(x + 18)$
   b. The expression $2x^2 - 16x + 14$ is equal to:
      A. $(x - 1)(x - 7)$  
      B. $(2x - 1)(x - 7)$  
      C. $(2x - 7)(x - 2)$  
      D. $(2x - 2)(x - 7)$
   c. The expression $12x^2 + 58x - 10$ is equal to:
      A. $(12x - 2)(x - 5)$  
      B. $(2x + 1)(x - 5)$  
      C. $(2x + 5)(x - 1)$  
      D. $(2x - 6)(x + 1)$

**UNDERSTANDING**

4. Factorise the following trinominals using the cross-product method.
   a. $2x^2 + 3x - 2$  
   b. $8x^2 + 2x - 3$  
   c. $6x^2 + x - 2$  
   d. $2x^2 + 9x + 10$

5. Factorise the following trinominals using both the grouping terms and the cross-product methods. Discuss which method is simpler to perform.
   a. $3x^2 - x - 4$  
   b. $6x^2 + 5x - 21$

6. Factorise:
   a. $6x^2 - 5x - 6$  
   b. $4x^2 + 13x + 10$  
   c. $4x^2 - 19x + 15$  
   d. $6x^2 + 5x - 4$

7. Factorise $3x^2 + 15x + 12$:
   a. by first taking out a common factor
   b. by first separating the middle term, then grouping.

**REASONING**

8. Complete the statements below by writing in the second factor. Check your answer by expanding.
   a. $4x^2 - 17x - 15 = (4x + 3)(x - 5)$  
   b. $3x^2 + 10x + 3 = (x + 3)(3x + 1)$
   c. $10x^2 - 9x - 9 = (5x + 3)(2x - 3)$  
   d. $15x^2 + x - 2 = (3x - 1)(5x + 2)$

9. a. Explain why the quadratic $4x^2 - 9x - 9$ can be written as $4x^2 + 3x - 12x - 9$.
   b. Factorise the quadratic $4x^2 - 9x - 9$.

**PROBLEM SOLVING**

10. a. Factorise $12x^2 - 4x - 5$.
   b. Assume that the factors represent the length and width of a rectangle.
      i. What is the area of the rectangle?
      ii. For what value of $x$ is the rectangle a square?
      iii. For what values of $x$ is one side twice as long as the other?

11. The area formulas below relate to either squares or rectangles.
    i. $9x^2 + 48x + 64$  
    ii. $25x^2 - 4$  
    iii. $s^2 + 4s + 3$  
    iv. $4s^2 - 24s - 36$  
    a. Without completing any algebraic operations, examine the formulas and determine which belong to squares and which to rectangles. Explain your answer.
    b. Factorise each formula and classify it as a square or rectangle. Check this against your answer to part a.

12. Factorise the following expressions.
    a. $9b^2 + 48b + 64$  
    b. $(ac)^2 - d^2$  
    c. $a^2 + ab - 182b^2$  
    d. $b^2 - 2abc + a^2c^2$
16.5 Simplifying algebraic fractions

Cancelling common factors

• Algebraic fractions are fractions that have algebraic expressions in the numerator and/or denominator; for example \( \frac{a}{2} \), \( \frac{b + 3}{4} \) and \( \frac{x^2 + 3x}{4x + 2} \).

• Just as fractions can be simplified by cancelling common factors, so too can algebraic fractions.

• A fraction can only be simplified if the numerator and denominator are both products.

• A factor can be cancelled if it is a factor of the whole of the numerator and the whole of the denominator. For example:

\[
\frac{2a}{ab} = \frac{2}{b} \quad \text{and} \quad \frac{2(x + 3)}{5a(x - 3)} = \frac{2}{5a}
\]

THINK Write the expression.

WRITE

WORKED EXAMPLE 6

Simplify each of the following.

a \( \frac{4}{x + 3} \times \frac{x + 3}{5} \)  

b \( \frac{(z - 3)(z - 4)}{(z - 5)(z + 1)} \times \frac{(z + 1)(z - 1)}{(z + 3)(z - 3)} \)

THINK

1 Write the expression.

2 Cancel the common factors.

3 Simplify the answer.

WRITE

a \( \frac{4}{x + 3} \times \frac{x + 3}{5} \)

\[= \frac{4}{1(x + 3)} \times \frac{(x + 3)^1}{5} \]

\[= \frac{4}{5} \]

b \( \frac{(z - 3)(z - 4)}{(z - 5)(z + 1)} \times \frac{(z + 1)(z - 1)}{(z + 3)(z - 3)} \)

\[= \frac{1(z - 3)(z - 4)}{(z - 5)(z + 1)} \times \frac{1(z + 1)(z - 1)}{(z + 3)(z - 3)} \]

\[= \frac{(z - 4)(z - 1)}{(z - 5)(z + 3)} \]

WORKED EXAMPLE 7

Simplify the following.

a \( \frac{1}{x - 3} \div \frac{3}{x - 3} \)  

b \( \frac{(z + 2)(z - 3)}{(z - 4)} \div \frac{z - 3}{z - 4} \)

THINK

1 Write the expression.

WRITE

a \( \frac{1}{x - 3} \div \frac{3}{x - 3} \)
2 Change to a multiplication problem by changing the ÷ to a × and turning the second fraction upside down.

\[ \frac{1}{x - 3} \times \frac{x - 3}{3} \]

3 Cancel the common factors.

\[ = \frac{1}{x - 3} \times \frac{(x - 3)^1}{3} \]

4 Simplify the fraction.

\[ = \frac{1}{3} \]

b 1 Write the expression.

b \[ \frac{(z + 2)(z - 3)}{(z - 4)} \div \frac{z - 3}{z - 4} \]

2 Change to multiplication.

\[ = \frac{(z + 2)(z - 3)}{(z - 4)} \times \frac{(z - 4)}{(z - 3)} \]

3 Cancel the common factors.

\[ = \frac{(z + 2)}{1} \]

4 Simplify the resultant fraction.

\[ = z + 2 \]

WORKED EXAMPLE 8

Simplify each of the following fractions by first factorising the numerator and then the denominator and cancelling as appropriate.

a \[ \frac{3x + 9}{15} \]

b \[ \frac{5}{10x + 20} \]

c \[ \frac{10x + 15}{6x + 9} \]

d \[ \frac{x^2 + 4x}{x^2 - 5x} \]

THINK

WRITE

a 1 Write the fraction.

a \[ \frac{3x + 9}{15} \]

2 Factorise both the numerator and denominator.

\[ = \frac{3(x + 3)}{3 \times 5} \]

3 Cancel the common factors.

\[ = \frac{1 \times 3(x + 3)}{5} \]

4 Simplify the resulting fraction.

\[ = \frac{x + 3}{5} \]

b 1 Write the fraction.

b \[ \frac{5}{10x + 20} \]

2 Factorise both the numerator and denominator.

\[ = \frac{5 \times 1}{10(x + 2)} \]

3 Cancel the common factors.

\[ = \frac{5 \times 1}{10 \times (x + 2)} \]

4 Simplify the resulting fraction.

\[ = \frac{1}{2(x + 2)} \]
c 1 Write the fraction.

2 Factorise both the numerator and denominator.

3 Cancel the common factors.

d 1 Write the fraction.

2 Factorise both the numerator and denominator.

4 Cancel the common factors.

WORKED EXAMPLE 9

Simplify each of the following algebraic fractions.

\[
\begin{align*}
a & \quad \frac{x^2 + 3x - 4}{x - 1} \\
b & \quad \frac{x^2 - 7x - 8}{x^2 + 3x + 2} \\
c & \quad \frac{2x^2 - 6x + 5}{x^2 - 16x + 30}
\end{align*}
\]

THINK

a 1 Write the expression.

2 Factorise the numerator and denominator and cancel the common factors.

3 Simplify.

b 1 Write the original fraction.

2 Factorise the numerator and denominator.

3 Cancel any common factors and simplify.

c 1 Write the original fraction.

2 Factorise the numerator and denominator.

3 Cancel any common factors and simplify.
Note: A fraction such as $\frac{x - 1}{x - 3}$ cannot be simplified any further.

It may be tempting to cancel the $x$ in the numerator and denominator, but $x$ is neither a factor of $(x - 1)$ nor $(x - 3)$, so it cannot be cancelled.

**WORKED EXAMPLE 10**

Simplify $\frac{2x + 4}{3} \times \frac{3x - 3}{x + 2}$ by factorising the numerator and denominator and cancelling as appropriate.

**THINK**

1. Write the expression.
   
   $\frac{2x + 4}{3} \times \frac{3x - 3}{x + 2}$

2. Factorise each numerator and denominator and cancel common factors.
   
   $\frac{2(x+2)}{3} \times \frac{1(3)(x-1)}{1(x+2)}$

   
   $2(x - 1)$

**Exercise 16.5  Simplifying algebraic fractions**

**INDIVIDUAL PATHWAYS**

**INDIVIDUAL PATHWAYS**

**PRACTISE** Questions:
1a, c, e, k, 2a, c, e, k, 3a–h, 4a–h, 5a–h, 6a, c, e, g, 7

**CONSOLIDATE** Questions:
1a, c, g, i, k, 2a–k, 3e–i, 4e–i, 5e–i, 6a–i, 7–10

**MASTER** Questions:
1b–l, 2b–l, 3i–p, 4i–p, 5i–p, 6b–j, 7–10

**FLUENCY**

1. Simplify the following.

   a. $\frac{x + 1}{2} \times \frac{1}{x + 1}$
   
   b. $\frac{y - 1}{3} \times \frac{2}{y - 1}$
   
   c. $\frac{p - 2}{4} \times \frac{4}{p - 2}$
   
   d. $\frac{b + 5}{3} \times \frac{9}{b + 5}$
   
   e. $\frac{m + 4}{m} \times \frac{m}{m + 4}$
   
   f. $\frac{3p}{p - 6} \times \frac{p - 6}{p}$
   
   g. $\frac{y - 7}{4y} \times \frac{y}{y - 7}$
   
   h. $\frac{ha}{a - 2} \times \frac{a - 2}{2a}$
   
   i. $\frac{p - 3}{p + 3} \times \frac{5(p + 3)}{p - 2}$
   
   j. $\frac{(x + 2)(x - 7)}{x + 1} \times \frac{x + 1}{x - 7}$
   
   k. $\frac{(b - 2)(b + 3)}{(b + 5)(b - 4)} \times \frac{b - 4}{b - 2}$
   
   l. $\frac{x - 3}{x + 2} \times \frac{(x + 2)(x + 1)}{(x - 6)(x - 3)}$
2. **WE7** Simplify the following.

   a. \( \frac{x + 2}{3} \div \frac{x + 2}{x} \)

   b. \( \frac{m - 3}{4} \div \frac{m - 3}{8} \)

   c. \( \frac{5}{x - 3} \div \frac{10}{x - 3} \)

   d. \( \frac{12}{a + 2} \div \frac{4}{a + 2} \)

   e. \( \frac{m}{m - 6} \div \frac{m}{m - 6} \)

   f. \( \frac{p + 2}{p - 1} \div \frac{p - 2}{p - 1} \)

   g. \( \frac{s - 6}{3(s + 1)} \div \frac{s}{s + 1} \)

   h. \( \frac{(a + 1)(a - 3)}{a + 2} \div \frac{a + 1}{a + 2} \)

   i. \( \frac{8m}{(m + 2)(m - 5)} \div \frac{2m}{m - 5} \)

   j. \( \frac{(a + 1)(a - 3)}{(a - 5)(a + 4)} \div \frac{a + 1}{a + 4} \)

   k. \( \frac{m(m - 2)}{(m + 1)(m + 5)} \div \frac{m}{m + 1} \)

   l. \( \frac{p}{p + 7} \div \frac{p(p - 4)}{(p + 2)(p + 7)} \)

3. **WE8** Simplify each of the following by first factorising the numerator and then the denominator and cancelling as appropriate.

   a. \( \frac{2a + 2}{2} \)

   b. \( \frac{3a + 6}{9} \)

   c. \( \frac{4a - 4}{4} \)

   d. \( \frac{7p - 21}{7} \)

   e. \( \frac{4x + 8}{8} \)

   f. \( \frac{3x - 12}{6} \)

   g. \( \frac{9x + 54}{3} \)

   h. \( \frac{12x - 96}{4} \)

   i. \( \frac{12x - 24}{18} \)

   j. \( \frac{9x - 45}{15} \)

   k. \( \frac{2}{2x + 10} \)

   l. \( \frac{-6}{6x - 24} \)

   m. \( \frac{2x + 2}{3x + 3} \)

   n. \( \frac{3x - 6}{4x - 8} \)

   o. \( \frac{5x - 15}{6x - 18} \)

   p. \( \frac{9x + 18}{3x + 6} \)

**UNDERSTANDING**

4. **WE8** Simplify each of the following fractions by first factorising the numerator and then the denominator and cancelling as appropriate.

   a. \( \frac{x^2 + 3x}{x} \)

   b. \( \frac{p^2 - 5p}{p} \)

   c. \( \frac{y^2 - 4y}{y} \)

   d. \( \frac{a^2 + 10a}{a} \)

   e. \( \frac{7x}{5x - x^2} \)

   f. \( \frac{5b}{5b - b^2} \)

   g. \( \frac{9y}{y^2 - 9y} \)

   h. \( \frac{4m}{m^2 - 4m} \)

   i. \( \frac{7x}{x - 2x^2} \)

   j. \( \frac{3m}{2m^2 - m} \)

   k. \( \frac{a - 2a^2}{a^2 + 3a} \)

   l. \( \frac{2b^2 + b}{5b - 8b^2} \)

   m. \( \frac{3x^2 + x}{9x - x^2} \)

   n. \( \frac{7b - 2b^2}{b^2 + 5b} \)

   o. \( \frac{9m^2 + 18m}{12m^2 - 3m} \)

   p. \( \frac{32a - 16a^2}{24a + 16a^2} \)
5 **WE9** Simplify each of the following algebraic fractions by first factorising the numerator, then the denominator.

a \[
\frac{x^2 + 5x + 6}{x + 3}
\]

b \[
\frac{x^2 + 7x + 12}{x + 4}
\]

c \[
\frac{x^2 - 9x + 20}{x - 5}
\]

d \[
\frac{b^2 - 8b + 7}{b - 1}
\]

e \[
\frac{a^2 - 18a + 81}{a - 9}
\]

f \[
\frac{a^2 - 22a + 121}{a - 11}
\]

g \[
\frac{x^2 - 49}{x - 7}
\]

h \[
\frac{m^2 - 64}{m - 8}
\]

i \[
\frac{a^2 - 7a + 12}{a^2 - 16}
\]

j \[
\frac{p^2 - 4p - 5}{p^2 - 25}
\]

k \[
\frac{x^2 + 6x + 9}{x^2 + 2x - 3}
\]

l \[
\frac{m^2 - 2m + 1}{m^2 + 5m - 6}
\]

m \[
\frac{y^2 - 4y - 12}{y^2 - 36}
\]

n \[
\frac{x^2 - 4x + 4}{x^2 - 4}
\]

o \[
\frac{x^2 + 3x - 40}{x^2 + 6x - 16}
\]

p \[
\frac{x^2 + 3x - 18}{x^2 - 6x + 9}
\]

6 **WE10** Simplify each of the following by factorising the numerator and denominator and cancelling as appropriate.

a \[
\frac{20x + 40}{2x + 8} \times \frac{x + 4}{x + 2}
\]

b \[
\frac{9a + 27}{3a - 3} \times \frac{a - 1}{a + 3}
\]

c \[
\frac{m + 5}{m - 2} \times \frac{8m - 16}{2m + 10}
\]

b \[
\frac{9a + 27}{3a - 3} \times \frac{a - 1}{a + 3}
\]

d \[
\frac{q - 3}{q + 1} \times \frac{15q + 15}{5q - 15}
\]

e \[
\frac{3x - 6}{x + 6} \times \frac{x + 6}{4x - 8}
\]

f \[
\frac{3a + 6}{2a - 6} \div \frac{a + 3}{8a - 24}
\]

\[
\frac{10x - 5}{4x + 28} \div \frac{20x - 10}{6x + 1}
\]

\[
\frac{m^2 + 8m + 15}{m^2 + 7m + 10} \times \frac{m^2 + 6m + 8}{m^2 + 10m + 21}
\]

i \[
\frac{x^2 + 6x + 8}{x^2 + 5x + 6} \times \frac{x^2 + 8x + 15}{x^2 + 7x + 12}
\]

j \[
\frac{x^2 + x - 6}{x^2 + 5x - 14} \times \frac{x^2 + 9x + 14}{x^2 - x - 12}
\]

**REASONING**

7 Explain why \[
\frac{x + 5}{x + 1}
\]
does not equal \[
\frac{5}{1}
\]. Illustrate your answer with an example.

8 Show that \[
\frac{3x + 11}{x + 2} = 3 + \frac{5}{x + 2}
\].

**PROBLEM SOLVING**

9 Simplify the following fractions.

a \[
\frac{c^2 + 14c + 24}{c^2 - 16c - 36}
\]

b \[
\frac{m^2 + 2m + 1}{m^2 + 9m + 8}
\]

c \[
\frac{f^2 - 81}{f^2 - 3f - 54}
\]

d \[
\frac{p^2 - 28r + 196}{p^2 + 28r + 196}
\]

e \[
\frac{w^2 + 10w + 25}{w^2 + 25w + 100}
\]

f \[
\frac{p^2 - 49}{p^2 - 64}
\]
10 A proposed new flag for Australian schools will have the Australian flag in the top left-hand corner. The dimensions given are in metres.

\[
\begin{array}{c}
\text{x} \\
\text{x+2} \\
\text{2x}
\end{array}
\]

a Write an expression, in factorised form, for the area of:
   i the Australian flag
   ii the proposed flag.

b Use the answers to part a to write the area of the Australian flag as a fraction of the school flag and then simplify this fraction.

c Use the fraction from part b to express the area of the Australian flag as a percentage of the proposed school flag.

d Using the formula for the percentage of area taken up by the Australian flag, find the percentages for the following school flag widths.
   i 4 m
   ii 4.5 m
   iii 4.8 m

**CHALLENGE 16.2**
Consider two numbers; 2 and \( x \). When \( x \) is subtracted from 2 and the result is squared, the answer is the same as the difference of the two numbers squared. Form possible equations to match this description and attempt to solve them.

Simplify the expression \( 1 + \frac{1}{x} = \frac{1}{1 + \frac{1}{x}} \).

**16.6 Quadratic equations**
A quadratic equation is an equation of the general form \( ax^2 + bx + c = 0 \).
For example, \( x^2 + 2x - 7 = 0 \) is a quadratic equation, and so are \( 2x^2 = 8 \) and \( x^2 = 5x \).

**WORKED EXAMPLE 11**
Rearrange the following quadratic equations so that they are in general form and state the values of \( a, b \) and \( c \).

a \( 5x^2 - 2x + 3 = 2x^2 + 4x - 12 \)

b \( \frac{x^2}{2} - \frac{1}{6} = x \left( \frac{x}{3} \right) - 4 \)

c \( x(3 - 2x) = 4(x - 6) \)
**THINK**

| a | 1. Write the equation. |
|   | 2. Subtract $2x^2$ from both sides of the equation. |
|   | 3. Subtract $4x$ from both sides of the equation. |
|   | 4. Add 12 to both sides of the equation. The equation is now in the general form $ax^2 + bx + c = 0$. |
|   | 5. Write the values of $a$, $b$ and $c$. |
| b | 1. Write the equation. |
|   | 2. Expand the bracket. Lowest common denominator (LCD) = 6. |
|   | 4. Simplify each fraction. |
|   | 5. Collect like terms on the left-hand side. The equation is now in the general form $ax^2 + bx + c = 0$. |
|   | 6. Write the values of $a$, $b$ and $c$. |
| c | 1. Write the equation. |
|   | 2. Expand the brackets. |
|   | 3. To collect like terms on the left-hand side of the equation, subtract $4x$ from both sides. |
|   | 4. Add 24 to both sides. |
|   | 5. Multiply all terms by $-1$ to make the $x^2$ term positive. The equation is now in the general form $ax^2 + bx + c = 0$. |
|   | 6. Write the values of $a$, $b$ and $c$. |

**WRITE**

| a | $5x^2 - 2x + 3 = 2x^2 + 4x - 12$  | $3x^2 - 2x + 3 = 4x - 12$ |
|   | $3x^2 - 6x + 3 = -12$ |
|   | $3x^2 - 6x + 15 = 0$ |
| b | $a = 3$, $b = -6$, $c = 15$ |
|   | $\frac{x^2}{2} - \frac{1}{6} = x \left(\frac{x}{3}\right) - 4$ |
|   | $\frac{x^2}{2} - \frac{1}{6} = \frac{x^2}{3} - 4$ |
|   | $\frac{x^2}{2} \times \frac{6}{1} - \frac{1}{6} \times \frac{6}{1} = \frac{x^2}{3} \times \frac{6}{1} - 4 \times 6$ |
|   | $3x^2 - 1 = 2x^2 - 24$ |
|   | $x^2 - 1 = -24$ |
|   | $x^2 + 23 = 0$ |
| c | $a = 1$, $b = 0$, $c = 23$ |
|   | $x(3 - 2x) = 4(x - 6)$ |
|   | $3x - 2x^2 = 4x - 24$ |
|   | $-2x^2 - x = -24$ |
|   | $-2x^2 - x + 24 = 0$ |
|   | $2x^2 + x - 24 = 0$ |
|   | $a = 2$, $b = 1$, $c = -24$ |

**Solving the equation $ax^2 + c = 0$**

- Solving a quadratic equation means finding the values of $x$ that satisfy the equation.
- Some quadratic equations have two solutions, some have only one solution, and some have no solutions.
Solve the following equations.

a \[ 2x^2 - 18 = 0 \]

b \[ x^2 + 9 = 0 \]

c \[ 3x^2 + 4 = 4 \]

d \[ 3(x + 2)^2 = 27 \]

THINK

WRITE

a 1 Write the equation. The aim is to make \( x \) the subject.

\[ 2x^2 = 18 \]

\[ x^2 = 9 \]

\[ x = 3, -3 \]

Two solutions: \( x = 3, x = -3 \)

b 1 Write the equation. The aim is to make \( x \) the subject.

\[ x^2 = -9 \]

−9 has no square root.

The equation has no solution.

c 1 Write the equation. The aim is to make \( x \) the subject.

\[ 3x^2 = 0 \]

\[ x^2 = 0 \]

\[ x = 0 \]

One solution: \( x = 0 \)

d 1 Write the equation. The aim is to make \( x \) the subject.

\[ (x + 2)^2 = 9 \]

\[ x + 2 = 3 \text{ or } x + 2 = -3 \]

\[ x = 1, x = -5 \]

Confirming the solutions to quadratic equations

• Solutions to quadratic equations are confirmed by substituting the values into the equation.

WORKED EXAMPLE 13

Determine whether any of the following are solutions to the quadratic equation \( x^2 = 4x - 3 \).

a \( x = 1 \) \hspace{1cm} b \( x = 2 \) \hspace{1cm} c \( x = 3 \)

THINK

WRITE

a 1 Substitute \( x = 1 \) into both sides of the equation.

\[ x^2 = 4x - 3 \]

\[ LHS = (1)^2 \]

\[ = 1 \]
3 Evaluate the right-hand side.
\[ \text{RHS} = 4(1) - 3 \\
= 4 - 3 \\
= 1 \]
x = 1 is a solution to the equation.

4 Since the left-hand side equals the right-hand side, \( x = 1 \) is a solution.
Write the answer.

\[ x = 1 \text{ is a solution to the equation.} \]

b Substitute \( x = 2 \) into both sides of the equation.

2 Evaluate the left-hand side.
\[ \text{LHS} = (2)^2 \\
= 4 \]

3 Evaluate the right-hand side.
\[ \text{RHS} = 4(2) - 3 \\
= 8 - 3 \\
= 5 \]
x = 2 is not a solution to the equation.
Write the answer.

\[ x = 2 \text{ is not a solution to the equation.} \]

c Substitute \( x = 3 \) into both sides of the equation.

2 Evaluate the left-hand side.
\[ \text{LHS} = (3)^2 \\
= 9 \]

3 Evaluate the right-hand side.
\[ \text{RHS} = 4(3) - 3 \\
= 12 - 3 \\
= 9 \]
x = 3 is a solution to the equation.
Write the answer.

\[ x = 3 \text{ is a solution to the equation.} \]

Exercise 16.6 Quadratic equations

**INDIVIDUAL PATHWAYS**

- **PRACTISE**
  Questions: 1–3, 5, 7, 9, 10

- **CONSOLIDATE**
  Questions: 1, 3, 5, 7–12

- **MASTER**
  Questions: 2, 4–12

**FLUENCY**

1 **WE11** Rearrange each of the following quadratic equations so that they are in the form \( ax^2 + bx + c = 0 \).

- a \( 3x - x^2 + 1 = 5x \)
- b \( 5(x - 2) = x(4 - x) \)
- c \( x(5 - 2x) = 6(5 - x^2) \)
- d \( 4x^2 - 5x + 2 = 6x - 4x^2 \)
- e \( 5(x - 12) = x(4 - 2x) \)
- f \( x^2 - 4x - 16 = 1 - 4x \)

2 **MC** a Which of the following is a quadratic equation?

- A \( 2x - 1 = 0 \)
- B \( 2^x - 1 = 0 \)
- C \( x^2 - x = 1 + x^2 \)
- D \( x^2 - x = 1 - x^2 \)
b Which of the following is not a quadratic equation?
A \( x(x - 1) = 2x - 1 \)  
B \( -3x^2 + 2x = 1 \)  
C \( 3(x + 2) + 5(x + 3) = 2(x + 1) \)  
D \( 2(x - 1) + 3x = x(2x - 3) \)

3 **WE12** a, b, c Solve the following quadratic equations.

\[ a \quad 2x^2 + 18 = 0 \]
\[ b \quad 3x^2 + 2x = x(x + 2) \]
\[ c \quad 2(x^2 + 7) = 16 \]
\[ d \quad x^2 + 17 = 13 \]
\[ e \quad -3x^2 + 17 = 5 \]

4 **WE12** Solve the following quadratic equations.

\[ a \quad 4(x + 2)^2 - 4 = 0 \]
\[ b \quad 4(x + 2)^2 + 4 = 0 \]
\[ c \quad 2(x + 3)^2 - 32 = 0 \]
\[ d \quad 2(x - 4)^2 - 32 = 0 \]
\[ e \quad (x + 2)^2 - 9 = 0 \]
\[ f \quad -(x - 2)^2 - 9 = 0 \]
\[ g \quad 3(x + 5)^2 - 10 = 2 \]
\[ h \quad 3(x - 4)^2 + 48 = 36 \]
\[ i \quad 2(x - 8)^2 + 4 = 4 \]

5 **WE10** Determine whether \( x = 4 \) is a solution to the following equations.

\[ a \quad x^2 = x + 12 \]
\[ b \quad x^2 = 3x + 1 \]
\[ c \quad x^2 = 4x \]

6 Determine whether \( x = -3 \) is a solution to the following equations.

\[ a \quad x^2 = x + 12 \]
\[ b \quad x^2 = 3x + 1 \]
\[ c \quad x^2 = 4x \]

7 Determine whether \( x = 0 \) is a solution to the following equations.

\[ a \quad x^2 = x + 12 \]
\[ b \quad x^2 = 3x + 1 \]
\[ c \quad x^2 = 4x \]

8 Determine whether either of the following are solutions to the equation \( 5(x - 1)^2 + 7 = 27 \).

\[ a \quad x = -1 \]
\[ b \quad x = 1 \]

**REASONING**

9 a Show that the shaded area can be represented by \( A = x^2(4 - \pi) \).

b If the shaded area = 10 cm\(^2\), find the value of \( x \) correct to 3 significant figures.

c Why can’t \( x \) be equal to -3.41?

10 Explain why \( x^2 + 7x + 4 = 7x \) has no solutions.

**PROBLEM SOLVING**

11 The sum of 5 and the square of a number is 41.

a What is the number?

b Is there more than one possible number? If so, what is the other number and why is there more than one?

12 Two less than a number is squared and its result is tripled. The difference between this and 7 is 41.

a Write the algebraic equation.

b What is the number? Give both possible answers.

**16.7 The Null Factor Law**

- The **Null Factor Law** states that if the product of two or more factors is zero, then **at least one** of the factors must be zero.

\[
\text{If } a \times b = 0 \\
\text{then:} \\
a = 0 \\
b = 0 \text{ or} \\
a = b = 0.
\]
For example, if \((x - 5)(x - 2) = 0\) then:
\[(x - 5) = 0 \text{ or } (x - 2) = 0 \text{ or both } x - 5 = x - 2 = 0.\]

If \(x - 5 = 0\)  
If \(x - 2 = 0\)
\[x = 5 \quad x = 2\]
Both \(x = 5\) and \(x = 2\) make the equation true.

- This can be checked by substituting the values into the original equation.

For example:

if \(x = 5\), then:  
if \(x = 2\), then:
\[(x - 5)(x - 2) = (5 - 5)(x - 2) = 0(x - 2) = 0 \quad (x - 5)(2 - 2) = (x - 5)0 = 0\]

Solve each of the following quadratic equations.

a \((x - 2)(2x + 1) = 0\)  
b \((4 - 3x)(6 + 11x) = 0\)  
c \(x(x - 3) = 0\)  
d \((x - 1)^2 = 0\)

**WORKED EXAMPLE 14**

Solve each of the following quadratic equations.

a \((x - 2)(2x + 1) = 0\)  
b \((4 - 3x)(6 + 11x) = 0\)  
c \(x(x - 3) = 0\)  
d \((x - 1)^2 = 0\)

THINK  
WRITE

**a** The product of 2 factors is zero, so apply the Null Factor Law.  
\((x - 2)(2x + 1) = 0\)

1 One of the factors must equal zero.  
Either \(x - 2 = 0\) or \(2x + 1 = 0\).

2 Solve the equations.  
\[x = 2 \quad 2x = -1\]
\[x = -\frac{1}{2} \quad x = 2\]

3 Write the solutions.  
x = \(-\frac{1}{2}\), \(x = 2\)

**b** The product of 2 factors is zero, so apply the Null Factor Law.  
\((4 - 3x)(6 + 11x) = 0\)

1 One of the factors must equal zero.  
Either \(4 - 3x = 0\) or \(6 + 11x = 0\).

2 Solve the equations.  
\[-3x = -4 \quad 11x = -6\]
\[x = \frac{4}{3} \quad x = \frac{-6}{11}\]

3 Write the solutions.  
x = \(\frac{4}{3}\), \(x = \frac{-6}{11}\)

**c** The product of 2 factors is zero, so apply the Null Factor Law.  
\(x(x - 3) = 0\)

1 One of the factors must equal zero.  
Either \(x = 0\) or \(x - 3 = 0\).

2 Solve the equations.  
x = 0

3 Write the solutions.  
x = 0, 3
Exercise 16.7 The Null Factor Law

**INDIVIDUAL PATHWAYS**

**FLUENCY**

1. **WE14** Solve each of the following quadratic equations.
   - a. \((x - 2)(x + 3) = 0\)
   - b. \((2x + 4)(x - 3) = 0\)
   - c. \((x + 2)(x - 3) = 0\)
   - d. \((x - 1)^2 = 0\)
   - e. \((x + 5)(4x + 3) = 0\)
   - f. \((x + 4)(2x + 1) = 0\)
   - g. \((2x + 1)(3 - x) = 0\)
   - h. \((1 - x)(3x - 1) = 0\)
   - i. \(x(x - 2) = 0\)
   - j. \((x + \frac{1}{2})(2x - \frac{1}{2}) = 0\)
   - k. \((5x - 1.5)(x + 2.3) = 0\)
   - l. \((2x + \frac{1}{3})(2x - \frac{1}{3}) = 0\)
   - m. \((x - 2)^2 = 0\)
   - n. \(x(4x - 15) = 0\)
   - o. \((x + 4)^2 = 0\)

2. The Null Factor Law can be extended to products of more than 2 factors. Use this to find all the solutions to the following equations.
   - a. \((x - 2)(x + 2)(x + 3) = 0\)
   - b. \((x + 2)(x + 2)(2x - 5) = 0\)
   - c. \((x + 2)(x + 2)(x + 4) = 0\)
   - d. \(x(x + 2)(3x + 12) = 0\)
   - e. \((2x - 2.2)(x + 2.4)(x + 2.6) = 0\)
   - f. \((2x + 6)(x + \frac{1}{2})(9x - 15) = 0\)
   - g. \(3(x - 3)^2 = 0\)
   - h. \((x + 1)(x - 2)^2 = 0\)

3. **MC** a. The solutions to \((2x - 4)(x + 7) = 0\) are:
   - A. \(x = 4, x = 7\)
   - B. \(x = 4, x = -7\)
   - C. \(x = 2, x = 7\)
   - D. \(x = 2, x = -7\)

   b. The Null Factor Law cannot be applied to the equation \(x(x + \frac{1}{2})(x - 1) = 1\) because:
   - A. there are more than 2 factors
   - B. the right-hand side equals 1
   - C. the first factor is a simple \(x\)-term
   - D. the third term has \(x\) in a fraction

**UNDERSTANDING**

4. Rewrite the following equations so that the Null Factor Law can be used. Then solve the resulting equation.
   - a. \(x^2 + 10x = 0\)
   - b. \(2x^2 - 14x = 0\)
   - c. \(25x^2 - 40x = 0\)
REASONING
5  a Explain why \( x = 2 \) is not a solution of the equation \((2x - 2)(x + 2) = 0\).
     b What are the solutions to the equation?
6  What is the maximum number of solutions that a quadratic such as \( x^2 - x - 56 = 0 \) can have? Explain why.

PROBLEM SOLVING
7  A bridge is constructed with a supporting structure in the shape of a parabola as shown in the diagram. The origin \((0, 0)\) is at the left-hand edge of the bridge, which is 100 m long.
   a What is the maximum height of the bridge support?
   b If the equation of the support is \( y = ax(b - x) \), determine the values of \( a \) and \( b \).
     \((Hint: \ Let \ y = 0.\)\)
   c What is the height of the support when \( x = 62 \)?
8  Consider a ball thrown upwards so that it reaches a height of \( h \) metres after \( t \) seconds. The expression \( 20t - 4t^2 \) represents the height of the ball, in metres, after \( t \) seconds.
   a Factorise the expression for height.
   b How high is the ball after:
      i 1 second
      ii 5 seconds?
   c How did factorising the expression make it easier to evaluate?
9  The product of 7 more than a number and 14 more than that same number is 330. What are the possible values of the number?

16.8 Solving the quadratic equation
\[ ax^2 + bx + c = 0 \]
- If \( ax^2 + bx + c \) can be factorised, then the quadratic equation \( ax^2 + bx + c = 0 \) can be solved using the Null Factor Law.
Solve the following quadratic equations.

\( ax^2 - 5x - 6 = 0 \)
\( x^2 + 14x = 15 \)
\( 2x^2 + 7x + 3 = 0 \)

**THINK**

1. Write the equation.
2. Factorise \( x^2 - 5x - 6 \).
3. Apply the Null Factor Law.
4. Solve the equations.
5. Write the solutions.

**WRITE**

\[ a \quad x^2 - 5x - 6 = 0 \]
\[ (x - 6)(x + 1) = 0 \]
\[ x - 6 = 0 \quad \text{or} \quad x + 1 = 0 \]
\[ x = 6, \quad x = -1 \]
\[ x = -1, \quad x = 6 \]

\[ b \quad x^2 + 14x = 15 \]
\[ x^2 + 14x - 15 = 0 \]
\[ (x + 15)(x - 1) = 0 \]
\[ x + 15 = 0 \quad \text{or} \quad x - 1 = 0 \]
\[ x = -15, \quad x = 1 \]

\[ c \quad 2x^2 + 7x + 3 = 0 \]
\[ 2x^2 + 6x + x + 3 = 0 \]
\[ 2(x + 3) + 1(x + 3) = 0 \]
\[ (2x + 1)(x + 3) = 0 \]
\[ x + 1 = 0 \quad \text{or} \quad x + 3 = 0 \]
\[ x = -1, \quad x = -3 \]

**Exercise 16.8 Solving the quadratic equation**

\[ ax^2 + bx + c = 0 \]

**INDIVIDUAL PATHWAYS**

**PRACTISE**

Questions: 1a, c, g, k, o, 2–5, 7

**CONSOLIDATE**

Questions: 1b, d, f, j, n, 2–5, 7–9

**MASTER**

Questions: 1b, d, f, h, j, l, n, 2–11
FLUENCY
1 Solve each of the following quadratic equations.
   a $x^2 - 6x + 8 = 0$
   b $x^2 + 6x + 8 = 0$
   c $x^2 + 6x + 5 = 0$
   d $x^2 + x - 6 = 0$
   e $x^2 + 2x - 15 = 0$
   f $x^2 + 4x + 4 = 0$
   g $x^2 + 2x - 24 = 0$
   h $x^2 - 5x - 24 = 0$
   i $x^2 - x - 12 = 0$
   j $x^2 + 13x + 12 = 0$
   k $x^2 - 10x = 11$
   l $x^2 + x = 20$
   m $0 = x^2 - 2x - 8$
   n $x^2 - 15x = -50$

2 Solve each of the following quadratic equations.
   a $2x^2 + 7x + 3 = 0$
   b $2x^2 + x - 1 = 0$
   c $2x^2 - x - 15 = 0$
   d $2x^2 + 8x + 6 = 0$
   e $3x^2 + 13x + 14 = 0$
   f $3x^2 + 5x + 2 = 0$
   g $5x^2 - 22x + 21 = 0$
   h $5x^2 - 17x + 6 = 0$
   i $7x^2 - 2x - 5 = 0$
   j $7x^2 - 33x + 20 = 0$
   k $4x^2 + 4x - 3 = 0$
   l $6x^2 + 17x + 5 = 0$

3 MC a The quadratic equation $x^2 + 2x - 8 = 0$ has solutions:
   A $x = 4$ and $x = -2$
   B $x = -4$ and $x = 2$
   C $x = -4$ and $x = 4$
   D $x = -2$ and $x = 2$

   b The quadratic equation $x^2 - 7x - 8 = 0$ has solutions:
   A $x = 1$ and $x = 8$
   B $x = -1$ and $x = -8$
   C $x = -8$ and $x = 1$
   D $x = -1$ and $8$

   c The quadratic equation $4a^2 + 13a + 3$ has solutions:
   A $a = 3$ and $a = 1$
   B $a = -3$ and $a = -1$
   C $a = \frac{1}{4}$ and $a = 3$
   D $a = -3$ and $a = -\frac{1}{4}$

UNDERSTANDING
4 Solve the following for $x$.
   $2x^2 + 4x + 5 = 3x^2 + 9x - 1$

5 The rectangle shown has an area of 45 cm$^2$. By solving a quadratic equation, find the dimensions of the rectangle.

6 Find the dimensions of this rectangle, which has an area of 48 m$^2$. 
REASONING
7 A small rectangular pool, 6m by 10m, is surrounded by a paved pathway of uniform width as shown. How wide is the pavement if it covers an area of 57 m²?

8 The number of diagonals in a polygon is given by the formula \( D = \frac{n}{2}(n - 3) \), where \( n \) is the number of sides in the polygon.
   a How many diagonals are there in:
      i a triangle?
      ii a square?
      iii a decagon?
   b What type of polygon has:
      i 20 diagonals?
      ii 170 diagonals?

PROBLEM SOLVING
9 a Write algebraic expressions for each of the parts below. Your friend picks a number, \( n \).
   i He adds 1 to it.
   ii He adds 7 to it.
   iii He multiplies these two new numbers together.
   iv He then divides the product in part iii by the product of 2 more than the original number, \( n \), and 1 more than the original number, \( n \).
   b The result of the division is 2. What number, \( n \), did he begin with?

10 Calculate the value(s) of \( x \) for which the expressions \( 4x^2 + 13x - 16 \) and \( 3x^2 + 8x - 2 \) are equal by following the steps outlined below:
   a Equate the expressions.
   b Rewrite the resulting equation by collecting like terms so that the equation is equal to zero.
   c Factorise the quadratic equation.
   d Apply the Null Factor Law and solve.

11 A girl, her brother and her teacher are shown below. The product of the teacher’s and brother’s ages is 256.
   a Write an algebraic expression in terms of \( g \) to represent the teacher’s age.
   b Use the algebraic expressions to write an expression for the product of the brother’s and teacher’s ages and expand this product.
   c Write an equation using your answer to part b for the product of their ages.
      Hint: An equation has an ‘is equal to’ part to it.
   d Rearrange your answer to part c by collecting like terms and making it equal to zero.
Find the value of $g$ by solving the equation in part d using the Null Factor Law. 

Note: Discard any value that does not make sense.

State the ages of the girl, her brother and the teacher.

If I am $g$ years old, my brother is $g - 6$.

I am 18 years older than my students.

I am 6 years younger than my sister.

16.9 Solving quadratic equations with two terms

Solving quadratic equations of the form $ax^2 + c = 0$

Earlier in this topic, the equation $2x^2 - 18 = 0$ was solved using algebraic techniques. It can also be solved using the Null Factor Law.

WORKED EXAMPLE 16

Solve each of the following quadratic equations.

<p>| | |</p>
<table>
<thead>
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</tr>
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<tbody>
<tr>
<td><strong>a</strong></td>
<td><strong>b</strong></td>
</tr>
<tr>
<td>$x^2 - 1 = 0$</td>
<td>$2x^2 - 18 = 0$</td>
</tr>
</tbody>
</table>

**THINK**

**WRITE**

**a**

1. Write the equation: $x^2 - 1$ is the difference of two squares.
2. Factorise the left-hand side using the difference of two squares rule.
3. Apply the Null Factor Law.
4. Solve the equations.
5. State the solutions.

$x + 1 = 0$  \( x - 1 = 0 \)

$x = -1$  \( x = 1 \)

$x = 1, x = -1$

(This can be abbreviated to $x = \pm 1$.)

**b**

1. Write the equation.
2. Take out the common factor.
3. Divide both sides of the equation by 2.

$2(x^2 - 9) = 0$

$x^2 - 9 = 0$
4 Factorise the left-hand side using the difference of two squares rule.

\[(x + 3)(x - 3) = 0\]

5 Apply the Null Factor Law.

\[x + 3 = 0 \quad x - 3 = 0\]  
\[x = -3 \quad x = 3\]

6 Solve the equations.

7 State the solutions.

\[x = \pm 3\]

Solving quadratic equations of the form \(ax^2 + bx = 0\)

- The equation \(ax^2 + bx = 0\) can be solved easily using the Null Factor Law.

**WORKED EXAMPLE 17**

Solve each of the following equations.

\[a \quad x^2 + 4x = 0 \quad b \quad -2x^2 - 4x = 0\]

**THINK**

**WRITE**

\[a \quad x^2 + 4x = 0\]

1 Write the equation.

\[x(x + 4) = 0\]

2 Factorise by taking out a common factor of \(x\).

\[x = 0 \quad x + 4 = 0\]  
\[x = 0 \quad x = -4\]

3 Apply the Null Factor Law.

4 Solve the equations.

5 Write the solutions.

\[x = -4, x = 0\]

\[b \quad -2x^2 - 4x = 0\]

1 Write the equation.

\[-2x(x + 2) = 0\]

2 Factorise by taking out the common factor of \(-2x\).

\[-2x = 0 \quad x + 2 = 0\]  
\[x = 0 \quad x = -2\]

3 Apply the Null Factor Law.

4 Solve the equations.

5 Write the solutions.

\[x = -2, x = 0\]

**WORKED EXAMPLE 18**

If the square of a number is multiplied by 5, the answer is 45. Find the number.

**THINK**

**WRITE**

1 Define the number.

Let \(x\) be the number.

2 Write an equation that can be used to find the number.

\[5x^2 = 45\]

3 Transpose to make the right-hand side equal to zero.

\[5x^2 - 45 = 0\]
Solve the equation.

\[ 5(x^2 - 9) = 0 \]
\[ x^2 - 9 = 0 \]
\[ x^2 - 3^2 = 0 \]
\[ (x + 3)(x - 3) = 0 \]
\[ x + 3 = 0 \quad \text{or} \quad x - 3 = 0 \]
\[ x = -3 \quad \text{or} \quad x = 3 \]

4 Write the answer in a sentence.

The number is either 3 or −3.

Exercise 16.9 Solving quadratic equations with two terms

INDIVIDUAL PATHWAYS

<table>
<thead>
<tr>
<th>PRACTISE</th>
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<tr>
<td>Questions: 1–7</td>
<td>Questions: 1–9, 11, 12</td>
<td>Questions: 1–13</td>
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FLUENCY

1 WE16 Solve each of the following quadratic equations using the Null Factor Law.
   a \[ x^2 - 9 = 0 \]  b \[ x^2 - 16 = 0 \]  c \[ 2x^2 - 18 = 0 \]  d \[ 2x^2 - 50 = 0 \]
   e \[ 100 - x^2 = 0 \]  f \[ 49 - x^2 = 0 \]  g \[ 3x^2 - 27 = 0 \]  h \[ 5x^2 - 20 = 0 \]
   i \[ x^2 + 6 = 0 \]  j \[ 2x^2 + 18 = 0 \]  k \[ -x^2 + 9 = 0 \]  l \[ -3x^2 + 48 = 0 \]
   m \[ -4x^2 + 100 = 0 \]  n \[ x^2 = 0 \]  o \[ -x^2 = 0 \]

2 WE17 Solve each of the following equations.
   a \[ x^2 + 6x = 0 \]  b \[ x^2 - 8x = 0 \]  c \[ x^2 + 9x = 0 \]  d \[ x^2 - 11x = 0 \]
   e \[ 2x^2 - 12x = 0 \]  f \[ 2x^2 - 15x = 0 \]  g \[ 3x^2 - 2x = 0 \]  h \[ 4x^2 + 7x = 0 \]
   i \[ 2x^2 - 5x = 0 \]  j \[ x^2 + x = 0 \]  k \[ 4x^2 - x = 0 \]  l \[ -x^2 - 5x = 0 \]
   m \[ -2x^2 - 24x = 0 \]  n \[ -x^2 + 18x = 0 \]  o \[ x^2 - 2.5x = 0 \]

3 MC a The solutions to \[ 4x^2 - 36 = 0 \] are:
   A \[ x = 3 \] and \[ x = -3 \]  B \[ x = 9 \] and \[ x = -9 \]
   C \[ x = 1 \] and \[ x = -1 \]  D \[ x = 2 \] and \[ x = -2 \]
   b What are the solutions to \[ x^2 - 5x = 0 \]?
   A \[ x = 1 \] and \[ x = 5 \]  B \[ x = 0 \] and \[ x = -5 \]
   C \[ x = 0 \] and \[ x = 5 \]  D \[ x = -1 \] and \[ x = 5 \]

UNDERSTANDING

4 WE18 If the square of a number is multiplied by 2, the answer is 32. Find the number.

5 A garden has two vegetable plots. One plot is a square; the other plot is a rectangle with one side 3 m shorter than the side of the square and the other side 4 m longer than the side of the square. Both plots have the same area. Sketch a diagram and find the dimensions of each plot.

6 The square of a number is equal to 10 times the same number. What is the number?
REASONING
7 Explain why the equation $x^2 + 9 = 0$ cannot be solved using the Null Factor Law.
8 Solve the equation $m^2x^2 - n^2 = 0$:
   a by working backwards  
   b using the Null Factor Law.
9 a Explain why the equation $x^2 + bx = 0$ cannot be solved by working backwards.
   b Solve the equation using the Null Factor Law.
10 A quadratic expression may be written as $ax^2 + bx + c$. Using examples, explain why
   $b$ and $c$ may take any values, but $a$ cannot equal zero.

PROBLEM SOLVING
11 The square of a number tripled is equal to 24 times the same number. What is the number?
12 The sum of 6 times the square of a number and 72 times the same number is equal to zero. What is the number?
13 A rectangular horse paddock has a width of $x$ and a length that is 30 metres longer than its width. The area of the paddock is $50x$ square metres.
   a Write the algebraic equation for the area described.
   b Factorise the equation.
   c Solve the equation.
   d Can you use all your solutions? Explain why or why not.

16.10 Applications
• There are many situations in science, engineering, economics and other fields where quadratic equations can be applied to finding solutions to problems.

WORKED EXAMPLE 19
The distance travelled by an accelerating skier is given by the formula $d = 3t + t^2$, where $t$ is the time in seconds and $d$ is the distance in metres. If the distance travelled was 130 m, for how long was the skier travelling?
THINK

1 Write the equation.  
   \[ d = 3t + t^2 \]
2 Substitute the given value, \( d = 130 \), into the equation.  
   \[ 130 = 3t + t^2 \]
3 Rearrange the equation so that it is in the form \( ax^2 + bx + c = 0 \).  
   \[ 0 = 3t + t^2 - 130 \]
4 Factorise the left-hand side of the equation, using the factors of \(-130\) that add up to 3.  
   \[ -130: -10 + 13 = 3 \]  
   \[ (t - 10)(t + 13) = 0 \]
5 Solve the equation by using the Null Factor Law.  
   Either \( t - 10 = 0 \) or \( t + 13 = 0 \)  
   \[ t = 10 \]  
   \[ t = -13 \]
6 Evaluate the result.  
   The only feasible answer is \( t = 10 \) because time is always positive.  
   The skier was travelling for 10 seconds.

WORKED EXAMPLE 20

When 10 is added to the square of a positive number the result is equal to 3 times the number subtracted from twice its square. Find the number.

THINK

1 Define the unknown quantity.  
   Let \( x \) be the positive number.
2 Translate each part of the sentence into an algebraic expression or term.  
   The square of a positive number: \( x^2 \)  
   Add 10 to the (positive number)\(^2\): \( x^2 + 10 \)  
   Three times the number: \( 3x \)  
   Three times the number is subtracted from twice its square: \( 2x^2 - 3x \)
3 Form a quadratic equation and rearrange it so that it is in the form \( ax^2 + bx + c = 0 \).  
   \[ x^2 + 10 = 2x^2 - 3x \]
   \[ 10 = x^2 - 3x \]
   \[ x^2 - 3x - 10 = 0 -10: -5 + 2 = -3 \]
   \[ (x - 5)(x + 2) = 0 \]
   \[ x - 5 = 0 \] or \( x + 2 = 0 \)
   \[ x = 5 \] \( \quad x = -2 \]
5 Evaluate the result.  
   The number is positive so \( x = 5 \) is the only valid solution.  
   The number is 5.

6 Answer the question in a sentence.
Exercise 16.10  Applications

**INDIVIDUAL PATHWAYS**

**PRACTISE**
Questions: 1, 3, 5, 7, 9

**CONSOLIDATE**
Questions: 1, 3–5, 7, 8, 10

**MASTER**
Questions: 2, 4–10

**REFLECTION**
Describe a situation that might require the application of quadratic equations.

**FLUENCY**

1. The product of two consecutive numbers is 42. What are the numbers?

2. The product of two consecutive odd numbers is 143. What are the numbers?

**UNDERSTANDING**

3. If the shaded area is 180 square units, find the value of \(x\).

4. If these triangles have the same area, what is the value of \(x\)?

5. Find the side lengths of this right-angled triangle. Show your working.

6. Find the side lengths of this right-angled triangle. Show your working.
PROBLEM SOLVING

7 A sheet of A4 paper is approximately 30 cm by 21 cm. Four corners are cut from a sheet as shown, and the remaining paper is folded to make a box with a base area of 286 cm². What are the dimensions of the box?

![Diagram of A4 paper with four corners cut and folded into a box]

8 A gardener maintains a large rectangular rose garden of length 15 m and width 12 m. He plans to lay a wide pathway around the garden that will require 160 m² of paving. What will be the width of the pathway?

9 A cricket ball is struck high into the air. After \( t \) seconds its height, \( h \), is given by the formula \( h = 20t - 5t^2 \).

   a. What is its height after 1 second?
   b. When will the ball strike the ground?

10 Each term in the sequence \(-3, -1, 1, 3, 5 \ldots\) is 2 more than the previous term. The sum \( S \) of the first \( n \) terms in the sequence is given by the formula \( S = n^2 - 4n \).

   a. What is the sum of the first 4 terms?
   b. How many terms are needed to make a sum of 77?
16.11 Review

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

**Language**

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

<table>
<thead>
<tr>
<th>algebraic fraction</th>
<th>difference of two squares</th>
<th>numerator</th>
</tr>
</thead>
<tbody>
<tr>
<td>common factor</td>
<td>expand</td>
<td>perfect square</td>
</tr>
<tr>
<td>constant term</td>
<td>factorise</td>
<td>quadratic equation</td>
</tr>
<tr>
<td>denominator</td>
<td>Null Factor Law</td>
<td>quadratic trinomial</td>
</tr>
</tbody>
</table>

**The story of mathematics**

is an exclusive Jacaranda video series that explores the history of mathematics and how it helped shape the world we live in today.

*Planetary orbits* (eles-1702) shows how the mathematician Johannes Kepler was able to model the orbits of the planets around the Sun over 400 years ago, and how his discoveries helped us to better understand our solar system.

**assessON**

Link to assessON for questions to test your readiness FOR learning, your progress AS you learn and your levels OF achievement.

assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au
Suspension bridges

When you hold up a chain at both ends, it forms a curve. Its shape is actually a catenary — a trigonometric function with a shape similar to that of a quadratic equation whose shape is a parabola. When constructing a suspension bridge, the main cables are attached to a tower at both ends and the curve is a catenary. However, when vertical cables are attached from the main cable to the deck, the curve takes the shape of a parabola. A catenary curves under its own weight; the bridge’s main cables are curving not just under their own weight, but also curving from holding up the weight of the deck.

There are many well-known suspension bridges around the world, for example, the Golden Gate Bridge in San Francisco.

A sketch of one side of the bridge can be represented as follows (we will consider just one side of the bridge in our calculations). Note that there are no vertical cables at the ends where the main cable is attached to the towers.

The shape of the cable can be represented by the equation \( v = 0.000832 \cdot h^2 + 3 \) where \( v \) represents the vertical distance between the main cable and the deck (in metres) and \( h \) represents the horizontal distance from the centre point of the main cable (in metres). The curve is symmetrical around this central point.
Let us consider a suspension bridge with 55-m high towers at the ends and vertical cables evenly spaced every 50 m.

1. Substitute into the equation to calculate the horizontal distance of the towers from the centre of the main cable.
2. Determine the length of the deck.
3. Draw a diagram displaying the spacing of all the vertical cables on the bridge.
4. How many vertical cables are there in the structure?
5. Use the equation to calculate the length (to the nearest metre) of each of these vertical cables.
6. What is the total length of cable required for these vertical cables?

The Golden Gate Bridge consists of a main suspension span in the centre with side suspension spans at both ends.

Here are some facts about the structure of the bridge.

- Length of suspension span including main span and side spans: 1966 m
- Horizontal distance between towers: 1280 m
- Clearance above mean high water: 67 m
- Height of tower above roadway: 152 m
- The bridge has two main cables that pass over the tops of the two main towers and are secured at either end. Length of each main cable: 2332 m
- Total length of wire used in main cables and vertical cables: 129 000 km

7. Draw a sketch of the bridge showing as much detail as possible.
Inventions and their inventors

The solutions to the equations give the puzzle’s answer code.

1792–

\[
(N - 3)(G - 16) = 0
\]

\[
2(3I - 15)(U - 14) = 0
\]

\[
(7P - 7)(2S - 24) = 0
\]

1892–

\[
5(C - 4)(M - 18) = 0
\]

\[
(2Z - 30)(3E - 6) = 0
\]

1903–

\[
2(H - 17)(2L - 12)(A - 11) = 0
\]

\[
(\frac{F}{3} - 3)(W - 19) = 0
\]

\[
(2D - 42)(T - 8) = 0
\]

\[
3(V - 4)(O - 7) = 0
\]

1926–

\[
(Y - 13)(\frac{R}{2} - 5) = 0
\]

1948–

\[
3(\frac{V}{5} - 4)(O - 7) = 0
\]
Activities

16.1 Overview

Video
- The story of mathematics: Planetary orbits (eles-1702)

16.2 Factorisation patterns

Digital doc
- SkillSHEET (doc-10980): Factorising difference of two squares expressions

Interactivity
- IP interactivity 16.2 (int-4545) Factorisation patterns

16.3 Factorising monic quadratics

Digital docs
- SkillSHEET (doc-10981): Factorising quadratic trinomials
- SkillSHEET (doc-10982): Finding the factor pair that adds to a given number

Interactivities
- Factorising quadratic equations (int-2775)
- Factorise trinomials (int-0748)
- IP interactivity 16.3 (int-4546) Factorising monic quadratics

16.4 Factorising non-monic quadratics

Digital doc
- WorkSHEET 16.1 (doc-10986): Quadratics I

Interactivity
- IP interactivity 16.4 (int-4547) Factorising non-monic quadratics

16.5 Simplifying algebraic fractions

Digital doc
- SkillSHEET (doc-10983): Simplifying algebraic fractions

Interactivity
- IP interactivity 16.5 (int-4548) Simplifying algebraic fractions

16.6 Quadratic equations

Interactivity
- IP interactivity 16.6 (int-4549) Quadratic equations

16.7 The Null Factor Law

Digital doc
- SkillSHEET (doc-10984): Solving linear equations

Interactivity
- IP interactivity 16.7 (int-4550) The Null Factor Law

16.8 Solving the quadratic equation \( ax^2 + bx + c = 0 \)

Digital doc
- WorkSHEET 16.2 (doc-10987): Quadratics II

Interactivity
- IP interactivity 16.8 (int-4551) Solving the quadratic equation \( ax^2 + bx + c = 0 \)

16.9 Solving quadratic equations with two terms

Digital doc
- SkillSHEET (doc-10985): Factorising expressions of the type \( ax^2 + bx \)

Interactivity
- IP interactivity 16.9 (int-4552) Solving quadratic equations with two terms

16.10 Applications

Interactivity
- IP interactivity 16.10 (int-4553) Applications

16.11 Review

Interactivities
- Word search (int-0693)
- Crossword (int-0707)
- Sudoku (int-3214)

Digital docs
- Topic summary (doc-10988)
- Concept map (doc-10805)

To access eBookPLUS activities, log on to www.jacplus.com.au

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Skills Lesson 16.2 Factorisation patterns

Exercise 16.2 Factorisation patterns

1. $(x + 5)(x - 5)$
2. $(x + 4)(x - 4)$
3. $(x + 9)(x - 9)$
4. $(x + 8)(x - 8)$
5. $(x + 10)(x - 10)$
6. $(x + 12)(x - 12)$
7. $(x + 14)(x - 14)$
8. $(x + 16)(x - 16)$
9. $(x + 18)(x - 18)$
10. $(x + 20)(x - 20)$

Challenge 16.1

1. $(x - 3)(x + 3)$
2. $(x - 4)(x + 4)$
3. $(x - 5)(x + 5)$
4. $(x - 6)(x + 6)$
5. $(x - 7)(x + 7)$
6. $(x - 8)(x + 8)$
7. $(x - 9)(x + 9)$
8. $(x - 10)(x + 10)$
9. $(x - 11)(x + 11)$
10. $(x - 12)(x + 12)$

Exercise 16.3 Factorising monic quadratics

1. $x^2 + 2x - 8 = (x + 4)(x - 2)$
2. $x^2 + 3x - 10 = (x + 5)(x - 2)$
3. $x^2 + 5x - 6 = (x + 6)(x - 1)$
4. $x^2 + 6x - 16 = (x + 8)(x - 2)$
5. $x^2 + 7x - 12 = (x + 4)(x - 3)$
6. $x^2 + 8x - 15 = (x + 5)(x - 3)$
7. $x^2 + 9x - 20 = (x + 5)(x - 4)$
8. $x^2 + 10x - 21 = (x + 7)(x - 3)$
9. $x^2 + 11x - 24 = (x + 8)(x - 3)$
10. $x^2 + 12x - 35 = (x + 7)(x - 5)$

Exercise 16.4 Factorising nonmonic quadratics

1. $(2x^2 + 8x + 4)$
2. $(3x^2 - 15x + 18)$
3. $(4x^2 + 20x + 25)$
4. $(5x^2 - 35x + 50)$
5. $(6x^2 + 30x + 36)$
6. $(7x^2 - 28x + 28)$
7. $(8x^2 - 32x + 32)$
8. $(9x^2 - 45x + 45)$
9. $(10x^2 - 50x + 50)$
10. $(11x^2 - 55x + 55)$

Exercise 16.5 Solving quadratic equations

1. $x^2 - 5x + 6 = 0$ (Factors: $(x - 2)(x - 3)$)
2. $x^2 + 6x - 16 = 0$ (Factors: $(x + 8)(x - 2)$)
3. $x^2 - 3x - 10 = 0$ (Factors: $(x - 5)(x + 2)$)
4. $x^2 + 5x + 4 = 0$ (Factors: $(x + 1)(x + 4)$)
5. $x^2 - 7x + 12 = 0$ (Factors: $(x - 3)(x - 4)$)
6. $x^2 + 2x - 8 = 0$ (Factors: $(x + 4)(x - 2)$)
7. $x^2 - 9x + 20 = 0$ (Factors: $(x - 5)(x - 4)$)
8. $x^2 + 6x - 16 = 0$ (Factors: $(x + 8)(x - 2)$)
9. $x^2 - 12x + 20 = 0$ (Factors: $(x - 2)(x - 10)$)
10. $x^2 + 3x - 10 = 0$ (Factors: $(x + 5)(x - 2)$)

Exercise 16.6 Solving quadratic inequalities

1. $x^2 - 5x + 6 > 0$ (Solutions: $x < 2$ or $x > 3$)
2. $x^2 + 6x - 16 < 0$ (Solutions: $-8 < x < 2$)
3. $x^2 - 3x - 10 < 0$ (Solutions: $-2 < x < 5$)
4. $x^2 + 5x + 4 < 0$ (Solutions: $-4 < x < -1$)
5. $x^2 - 7x + 12 > 0$ (Solutions: $x < 3$ or $x > 4$)
6. $x^2 + 2x - 8 < 0$ (Solutions: $-4 < x < 2$)
7. $x^2 - 9x + 20 < 0$ (Solutions: $4 < x < 5$)
8. $x^2 + 6x - 16 > 0$ (Solutions: $x < -8$ or $x > 2$)
9. $x^2 - 12x + 20 < 0$ (Solutions: $2 < x < 10$)
10. $x^2 + 3x - 10 < 0$ (Solutions: $-5 < x < 2$)
11 Answers will vary.
12 \(3x + 6x = 9x\)
\(6x^2 + 3x + 6x + 3 = 3(2x + 1) + 3(2x + 1)\)
\(= (3x + 3)(2x + 1)\)

13 a \(w = \frac{x^2 + 2x - 15}{x + 5}\)
b \(w = 62\) cm
c \(l = 103\) cm

14 a \(1 \times 120\) m
\(2 \times 60\) m
\(3 \times 40\) m
\(4 \times 30\) m
\(5 \times 24\) m
\(6 \times 20\) m
\(8 \times 15\) m
\(10 \times 12\) m
b \(l = x + 8\), \(w = x - 6\)
c \(6 \times 20\) m

Exercise 16.4 Factorising non-monic quadratics
1 a \(2(x + 2)(x + 3)\)
b \(3(x + 4)(x + 1)\)
c \(2(x - 2)(x - 3)\)
d \(4(x + 3)(x - 1)\)
e \(6x(x - 3)(x - 5)\)
f \(6(x - 4)(x - 5)\)
g \(4(x + 3)(x - 5)\)
h \(6(x - 4)(x - 5)\)

2 a \((x + 3)(x + 2)\)
b \((x + 2)(x + 3)\)
c \((x + 2)(x + 3)\)
d \((x - 1)(x + 2)\)
e \((x + 3)(x + 2)\)
f \((x + 3)(x + 2)\)
g \((x - 2)(x + 3)\)
h \((x - 3)(x + 2)\)

3 a \((x + 2)^2\)
b \((x + 3)^2\)
c \((x - 2)^2\)
d \((x - 3)^2\)

4 a \((x + 3)(x + 2)\)
b \((x + 2)(x + 3)\)
c \((x + 2)(x + 3)\)
d \((x - 1)(x + 2)\)
e \((x + 3)(x + 2)\)
f \((x + 3)(x + 2)\)
g \((x - 2)(x + 3)\)
h \((x - 3)(x + 2)\)

5 a \((x + 2)^2\)
b \((x + 3)^2\)
c \((x - 2)^2\)
d \((x - 3)^2\)

6 a \((x + 2)^2\)
b \((x + 3)^2\)
c \((x - 2)^2\)
d \((x - 3)^2\)

7 a \((x + 2)^2\)
b \((x + 3)^2\)
c \((x - 2)^2\)
d \((x - 3)^2\)

8 a \((x + 2)(x + 3)\)
b \((x + 3)(x + 2)\)
c \((x - 2)(x + 3)\)
d \((x - 3)(x + 2)\)

9 a \((x + 2)^2\)
b \((x + 3)^2\)
c \((x - 2)^2\)
d \((x - 3)^2\)

10 a \((x + 2)^2\)
b \((x + 3)^2\)
c \((x - 2)^2\)
d \((x - 3)^2\)

Exercise 16.5 Simplifying algebraic fractions
1 a \(\frac{x}{2}\)
b \(\frac{y}{2}\)
c \(1\)
d \(3\)
e \(1\)
f \(3\)

2 a \(\frac{x}{3}\)
b \(\frac{2}{3}\)
c \(1\)
d \(3\)
e \(1\)

3 a \(\frac{3}{4}\)
b \(\frac{3}{4}\)
c \(\frac{3}{4}\)
d \(3\)
e \(1\)

4 a \(\frac{x - 6}{3x}\)
b \(\frac{a - 3}{a - 5}\)
c \(\frac{m - 2}{m + 5}\)
d \(\frac{p + 2}{p - 4}\)
e \(\frac{4}{m + 2}\)

5 a \(\frac{a + 1}{3}\)
b \(\frac{a + 2}{3}\)
c \(\frac{a - 1}{3}\)
d \(\frac{p - 3}{2}\)
e \(\frac{p - 4}{2}\)

6 a \(\frac{x + 3}{7}\)
b \(\frac{p - 5}{7}\)
c \(\frac{y - 4}{7}\)
d \(\frac{7}{5 - x}\)
e \(\frac{5 - b}{5 - b}\)

7 a \(\frac{x + 2}{7}\)
b \(\frac{x + 3}{7}\)
c \(\frac{x - 4}{7}\)
d \(\frac{a - 9}{7}\)
e \(\frac{f - 11}{7}\)

8 a \(\frac{x + 2}{8}\)
b \(\frac{x + 3}{8}\)
c \(\frac{x - 2}{8}\)
d \(\frac{a + 4}{8}\)
e \(\frac{m + 6}{8}\)

9 a \(\frac{c + 12}{c - 18}\)
b \(\frac{m + 4}{m + 8}\)
c \(\frac{f + 9}{f + 6}\)
d Cannot be simplified

e \(\frac{w + 5}{w + 20}\)

10 a \(\frac{x(x - 2)}{2x}\)
b \(\frac{m^2}{2x}\)
c \(\frac{50(x - 2)^2}{x}\)
d \(\frac{25\%}{x}\)
e \(\frac{27.8\%}{x}\)

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Exercise 16.6 Quadratic equations

1. a) $a = 1, b = 2, c = -1$
   b) $a = 1, b = 1, c = -10$
   c) $a = 4, b = 5, c = -30$
   d) $a = 8, b = -11, c = 2$
   e) $a = 2, b = 1, c = -60$
   f) $a = 1, b = 0, c = -1$

2. a) $D$
   b) $C$

3. a) $x = 4, x = -4$
   b) There is no solution.
   c) $x = 0$
   d) $x = 1, x = -1$
   e) There is no solution.
   f) $x = 2, x = -2$
   g) $x = -1, x = -3$
   h) There is no solution.
   i) $x = 0, x = 8$
   j) $x = 1, x = -7$
   k) $x = 1, x = -5$
   l) There is no solution.
   m) $x = -3, x = -7$
   n) There is no solution.
   o) $x = 4, x = -2$

5. a) $Y$
   b) $N$
   c) $Y$
   d) $Y$
   e) $Y$
   f) $b$
   g) $b$
   h) $b$
   i) $b$
   j) $b$

9. a) Check your teacher.
   b) $x = 3.41$
   c) A negative number is not appropriate for a length.
   d) $x = 3.41$

11. a) $6$ or $-6$
   b) There are two possible numbers, as the square root of a positive number, in this case 36, has two possible solutions, one positive and one negative.

12. a) $3(x - 2)^2 - 7 = 41$
   b) $-2$ and $6$

Exercise 16.7 The Null Factor Law

1. a) $x = -3, x = 2$
   b) $x = -2, x = 3$
   c) $x = -2, x = 3$
   d) $x = -2, x = -3$
   e) $x = 4, x = -4$
   f) $x = 3, x = -3$
   g) $x = 1, x = 2$
   h) $x = 0, x = 2$
   i) $x = 1, x = -1$
   j) $x = 2, x = -2$
   k) $x = -2.3, x = 0.3$
   l) $x = 0, x = 15$
   m) $x = 2$
   n) $x = 0, x = 15$

4. a) $x = -4$
   b) $x = 2, x = -2, x = -3$
   c) $x = -2, x = 2.5$
   d) $x = -2, x = -4$
   e) $x = 1.1, x = -2.4, x = -2.6$
   f) $x = -3, x = -1.5, x = 1.5$
   g) $x = 2$
   h) $x = 2$
   i) $x = 2$

5. a) When $x = 2$, the first bracket equals 2 and the second bracket equals 8; therefore, the product is 9.
   b) $-2$ and $1$

A quadratic can have a maximum of two solutions, because a quadratic can at most be factorised into two separate pairs of brackets, each of which represent one solution.

7. a) 10 metres
   b) $y = \frac{1}{50}x(100 - x)$; $a = \frac{1}{50}$, $b = 100$
   c) 9.42 m

Exercise 16.8 Solving the quadratic equation

ax^2 + bx + c = 0

1. a) $x = 2, x = 4$
   b) $x = -2, x = -4$
   c) $x = -1, x = -5$
   d) $x = 2, x = -3$
   e) $x = 3, x = -5$
   f) $x = -2$
   g) $x = 4, x = -6$
   h) $x = 8, x = -3$
   i) $x = -3, x = 4$
   j) $x = -12, x = -1$
   k) $x = 11, x = -1$
   l) $x = 4, x = -5$
   m) $x = -25, x = -4$
   n) $x = 5, x = 10$
   o) $x = 4, x = -2$

2. a) $-3, -\frac{1}{2}$
   b) $-1, -\frac{1}{2}$
   c) $-2, -\frac{1}{2}$
   d) $-3, -1$
   e) $-2, -2$
   f) $-1, -\frac{2}{3}$
   g) $-\frac{1}{2}, -3$
   h) $\frac{2}{3}, -1$
   i) $-\frac{3}{4}, 1$
   j) $-\frac{1}{2}, \frac{1}{2}$
   k) $-\frac{1}{2}, \frac{1}{2}$
   l) $-\frac{1}{3}, -\frac{1}{3}$

3. a) $B$
   b) $D$
   c) $C$

4. a) $x = 1, -6$
   b) $9$ by $5$
   c) $8$ by $6$
   d) $1.5$

5. a) $i = 0$
   b) $i = Octagon$
   c) $i = Icosahedron$

9. a) $i = n + 1$
   b) $n = 3$
   c) $i = n + 7$
   d) $iv (n + 7)$

Exercise 16.9 Solving quadratic equations with two terms

1. a) $x = 3, x = 3$
   b) $x = -4, x = 4$
   c) $x = 3, x = 3$
   d) $x = -5, x = 5$
   e) $x = 10, x = 10$
   f) $x = 7, x = 7$
   g) $x = -3, x = 3$
   h) $x = -2, x = 2$

3. a) $x = 0$
   b) $x = 0, x = 8$
   c) $x = 0, x = 8$
   d) $x = 0, x = 11$
   e) $x = 0, x = 6$
   f) $x = 0, x = 7.5$
   g) $x = 0, x = \frac{1}{2}$
   h) $x = 0, x = -1$

5. The square plot is 12 m x 12 m; the rectangular plot is 16 m x 9 m.
6 The number is 0 or 10.
7 \(x^2 + 9\) cannot be factorised.
8 a \(x = \pm \frac{n}{m}\)  
    b \(x = \pm \frac{n}{m}\)
9 a \(x\) cannot be isolated.
    b \(x = 0, -b\)
10 If \(a = 0\), then the expression is not quadratic.
11 8
12 -12 or 0
13 a \(x(x + 30) = 50x\)
    b \(x(x - 20) = 0\)
    c \(x = 0\) or \(x = 20\)
    d No, \(x\) cannot be 0, because the width has to have a positive value.

Exercise 16.10 Applications
1 6 and 7 or -7 and -6
2 11 and 13 or -13 and -11
3 \(x = 9\)
4 \(x = 10\)
5 3, 4, 5
6 5, 12, 13
7 Length = 22 cm, width = 13 cm, height = 4 cm
8 2.5 m
9 a 15 m  
    b 4 seconds
10 a 0  
    b 11 terms

Investigation — Rich task
1 \(h = 250\) m
2 \(500\) m
3 
   \[55 \text{ m} \quad 50 \text{ m}\]
4 18
5 3 m; 5 m; 11 m; 22 m; 36 m
6 305 m
7 
   \[152 \text{ m} \quad 232 \text{ m}\]
   \[1290 \text{ m} \quad 67 \text{ m} \quad 1966 \text{ m}\]

Code puzzle
Pencil, Conte, France
Electric iron, Seely, US
Safety razor, Gillete, US
Aerosol spray, Rotheim, Norway
Velcro, Mestral, Switzerland