UNIT 2

AREA OF STUDY 1

CHAPTER 10 Analysing movement
CHAPTER 11 Forces in action
CHAPTER 12 Mechanical interactions

AREA OF STUDY 3

CHAPTER 13 Practical investigations

AREA OF STUDY 2

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The Options chapters 14–25 can be found in your eBookPLUS. A copy of each chapter can be downloaded from the Resource panel. A sample of each chapter can be found in the section of this book following Chapter 13. Students will study only one Option chapter.

CHAPTER 14 What are stars?
CHAPTER 15 Is there life beyond our solar system?
CHAPTER 16 How do forces act on the human body?
CHAPTER 17 How can AC electricity charge a DC device?
CHAPTER 18 How do heavy things fly?
CHAPTER 19 Are fission and fusion viable nuclear energy power sources?
CHAPTER 20 How is radiation used to maintain human health?
CHAPTER 21 How do particle accelerators and colliders work?
CHAPTER 22 How can human vision be extended?
CHAPTER 23 How do instruments make music?
CHAPTER 24 How can performance in ball sports be improved?
CHAPTER 25 How does the human body use electricity?

CHAPTER

10 Analysing movement

REMEMBER

Before beginning this chapter, you should be able to:

- know the units of distance and speed
- estimate distances and lengths
- estimate speeds
- use a graph to plot data.

KEY IDEAS

After completing this chapter, you should be able to:

- distinguish between vector and scalar quantities that describe motion
- use graphs to describe and analyse uniform and non-uniform motion
- analyse uniform motion along a straight line numerically and algebraically.

Whenever you drive a car, you need to be aware of, and to describe, your movement in terms of your position, speed, direction and acceleration.

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Unit 2 | AOS 1 | Topic 1 | Concept 1

Speed and velocity
Concept summary and practice questions

Describing movement

By observing the change in the position of an object during a measured time interval, you are able to describe the speed of the object and the direction in which it is travelling. Only by studying the way in which an object moves can you begin to understand the nature of forces, including those that you cannot see — like gravity, electrostatic forces and magnetic forces, which act without contact, and tension and compression, which act inside a material.

Distance and displacement

Distance is a measure of the length of the path taken by an object. It is a scalar quantity.

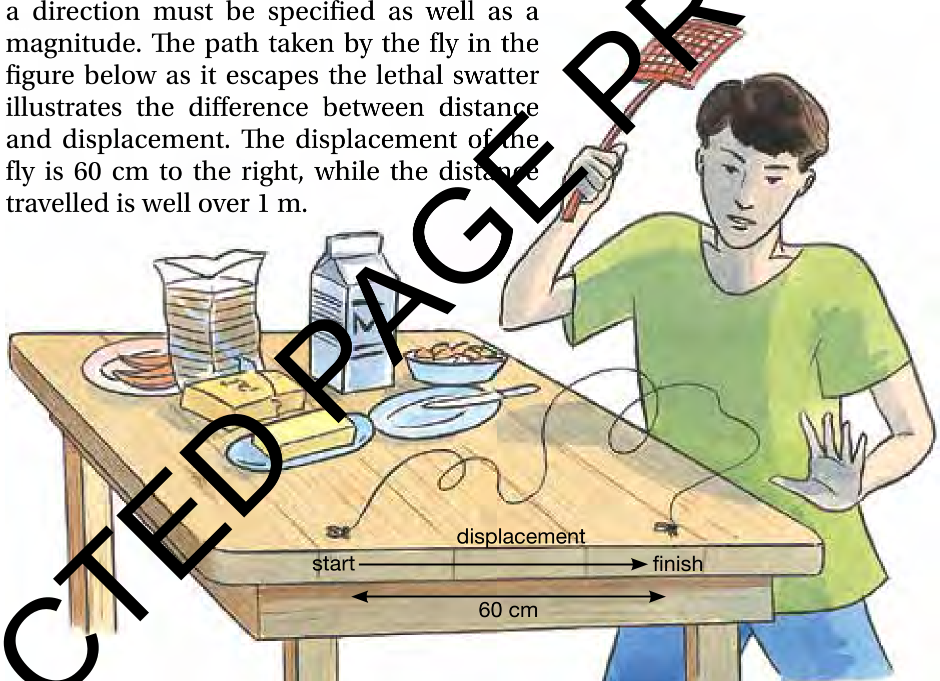
Scalar quantities specify magnitude (size) but not direction.

Displacement is a measure of the change in position of an object. It is a vector quantity.

A **vector** quantity specifies direction as well as magnitude (size).

Distance is a measure of the length of the path taken during the change in position of an object. Distance is a **scalar** quantity. It does not specify a direction.

Displacement is a measure of the change in position of an object. Displacement is a **vector** quantity. In order to describe a displacement fully, a direction must be specified as well as a magnitude. The path taken by the fly in the figure below as it escapes the lethal swatter illustrates the difference between distance and displacement. The displacement of the fly is 60 cm to the right, while the distance travelled is well over 1 m.



Distance and displacement are different quantities.

In a 100 m sprint, the magnitude of the displacement is the same as the distance. However, it is the displacement that fully describes the change in position of the runner because it specifies the direction.

In the case of movement in a straight line, the displacement of an object that has moved from position x_1 to position x_2 is expressed as:

$$\Delta x = x_2 - x_1.$$

Displacement can also be represented by the symbols x or s.

Sample problem 10.1

A hare and a tortoise decide to have a race along a straight 100 m stretch of highway. They both head due north. However, at the 80 m mark, the hare notices his girlfriend back at the 20 m mark. He heads back, gives her a quick kiss on the cheek, and resumes the race, arriving at the finishing line at the same time as the tortoise. (It was a very fast tortoise!)

(a) What was the displacement of the hare during the entire race?
(b) What was the distance travelled by the hare during the race?
(c) What was the distance travelled by the tortoise during the race?
(d) What was the displacement of the hare during his return to his girlfriend?

Solution: (a) Using the start as the reference point, the displacement was 100 m north. In symbols, this calculation can be done by denoting north as positive and south as negative. Thus:

$$\Delta \boldsymbol{x} = \boldsymbol{x}_2 - \boldsymbol{x}_1$$
$$= 100 \text{ m} - 0 \text{ m}$$
$$= 100 \text{ m}. \qquad \text{(representing 100 m north)}$$

(b) The distance is the length of the path taken. The hare travels a total distance of 80 m (before noticing his girlfriend) + 60 m (running back to the 20 m mark) + 80 m (from the 20 m mark to the finishing line). This gives a total of 220 m.

(c) The distance travelled by the tortoise is the length of the path taken, which is 100 m.

(d) The hare returns from a position 80 m north of the reference point (or start) back to a position 20 m north of the reference point. The displacement is 60 m south. Thus:

$$\Delta \boldsymbol{x} = \boldsymbol{x}_2 - \boldsymbol{x}_1$$
$$\Delta \boldsymbol{x} = 20 \text{ m} - 80 \text{ m}$$
$$= -60 \text{ m}. \qquad \text{(representing 60 m south)}$$

Revision question 10.1

(a) A jogger heads due north from his home and runs 400 m along a straight footpath before realising that he has forgotten his sunscreen and runs straight back to get it.

(i) What distance has the jogger travelled by the time he gets back home?

(ii) What was the displacement of the jogger when he started to run back home?

(iii) What was his displacement when he arrived back home to pick up the sunscreen?

(b) A cyclist rides 4.0 km due west from home, then turns right to ride a further 4.0 km due north. She stops, turns back and rides home along the same route.

(i) What distance did she travel during the entire ride?

(ii) What was her displacement at the instant that she turned back?

(iii) What was her displacement from the instant that she commenced her return journey until she arrived home?

(iv) What was her total displacement from the time she left home until the time she arrived back home?

Speed and velocity

Speed is a measure of the rate at which an object moves over a distance. Speed is a scalar quantity.

Speed is a measure of the rate at which an object moves over a distance. When you calculate the speed of a moving object, you need to measure the distance travelled over a time interval.

The average speed of an object can be calculated by dividing the distance travelled by the time taken, that is:

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time interval}}.$$

The speed obtained using this formula is the average speed during the time interval. Speed is a scalar quantity. The unit of speed is m s^{-1} if SI units are used for distance and time. However, it is often more convenient to use other units such as cm s^{-1} or km h^{-1}.

AS A MATTER OF FACT

A snail would lose a race with a giant tortoise! A giant tortoise can reach a top speed of 0.37 km h^{-1}. However, its 'cruising' speed is about 0.27 km h^{-1}. The world's fastest snail covers ground at the breathtaking speed of about 0.05 km h^{-1}. However, the common garden snail is more likely to move at a speed of about 0.02 km h^{-1}. Both of these creatures are slow compared with light, which travels through the air at 1080 million km h^{-1}, and sound, which travels through the air (at sea level) at about 1200 km h^{-1}.

How long would it take the snail, giant tortoise, light and sound respectively to travel once around the equator, a distance of 40 074 km?

Converting units of speed

It is often necessary to convert units that are not derived from SI units (such as km h^{-1}) to units that are derived from SI units, such as m s^{-1}.

To convert 60 km h^{-1} to m s^{-1}, the following procedure can be followed.

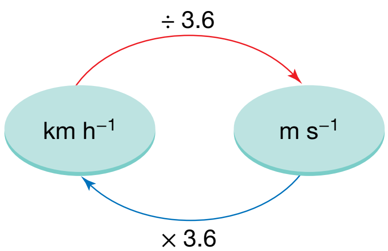
$$60\text{ km h}^{-1} = \frac{60\text{ km}}{1\text{ h}}$$

$$60\text{ km h}^{-1} = \frac{60\,000\text{ m}}{3600\text{ s}}$$

$$60\text{ km h}^{-1} = 16.7\text{ m s}^{-1}$$

In effect, the speed in km h^{-1} has been multiplied by $\frac{1000}{3600}$, or divided by 3.6.

To convert 30 m s^{-1} to km h^{-1}, a similar procedure can be followed.



$$30\text{ m s}^{-1} = \frac{30\text{ m}}{1\text{ s}}$$

$$= \frac{0.030\text{ km}}{\frac{1}{3600}\text{ h}}$$

$$= \frac{3600 \times 0.030\text{ km}}{1\text{ h}}$$

$$= 108\text{ km h}^{-1}$$

In effect, the speed in m s^{-1} has been multiplied by $\frac{3600}{1000}$, that is, by 3.6.

Sample problem 10.2

A plane carrying passengers from Melbourne to Perth flies at an average speed of 250 m s^{-1}. The flight takes 3.0 hours. Use this information to determine the approximate distance by air between Melbourne and Perth.

Solution:

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time interval}}$$

$\Rightarrow$ distance travelled = average speed × time interval (rearranging)

$= 250\text{ m s}^{-1} \times 3.0\text{ h}$

$= 900\text{ km h}^{-1} \times 3.0\text{ h}$ (× 3.6 to convert m s^{-1} to km h^{-1})

$= 2700\text{ km}$

Alternatively, the distance could be calculated in metres and then converted to kilometres, a more appropriate unit in this case.

distance travelled = average speed × time interval (rearranging)

$= 250\text{ m s}^{-1} \times 3.0\text{ h}$

$= 250\text{ m s}^{-1} \times 10\,800\text{ s}$ (× 3600 to convert h to s)

$= 2\,700\,000\text{ m}$

$= 2700\text{ km}$ (converting m to km)

Revision question 10.2

(a) **A car takes 8.0 hours to travel from Canberra to Ballarat at an average speed of 25 m s^{-1}. What is the road distance from Canberra to Ballarat?**

(b) **A jogger takes 30 minutes to cover a distance of 5.0 km. What is the jogger's average speed in:**
 (i) km h^{-1}
 (ii) m s^{-1}?

(c) **How long does it take for a car travelling at 60 km h^{-1} to cover a distance of 200 m?**

Velocity is a measure of the time rate of displacement, or the time rate of change in position. Velocity is a vector quantity.

In everyday language, the word *velocity* is often used to mean the same thing as speed. In fact, velocity is not the same quantity as speed. **Velocity** is a measure of the rate of displacement, or rate of change in position, of an object. Because displacement is a vector quantity, velocity is also a vector quantity. The velocity has the same direction as the displacement. The symbol $\boldsymbol{v}$ is used to denote velocity. (Unfortunately, the symbol v is often used to represent speed as well, which can be confusing.)

The average velocity of an object, $\boldsymbol{v}_{av}$, during a time interval Δt can be expressed as:

$$\boldsymbol{v}_{av} = \frac{\Delta x}{\Delta t}$$

where $\Delta \boldsymbol{x}$ represents the displacement (change in position).

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Investigation 10.1:
Going home
doc-16180

For motion in a straight line in one direction, the magnitude of the velocity is the same as the speed. The motion of the fly in the figure on page 153 illustrates the difference between velocity and speed. If the fly takes 2.0 seconds to complete its flight, its average velocity is:

$$\boldsymbol{v}_{av} = \frac{\Delta x}{\Delta t}$$

$$\boldsymbol{v}_{av} = \frac{60\ \text{cm to the right}}{2.0\ \text{s}}$$

$$\boldsymbol{v}_{av} = 30\ \text{cm s}^{-1}\ \text{to the right.}$$

The path taken by the fly is about 180 cm. Its average speed is:

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time interval}}$$

$$\text{average speed} = \frac{180\ \text{cm}}{2.0\ \text{s}}$$

$$\text{average speed} = 90\ \text{cm s}^{-1}.$$

Sample problem 10.3

Calculate the average speed and the average velocity of the hare in Sample problem 10.1 if it takes 20 s to complete the race.

Solution:

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time interval}} = \frac{220\ \text{m}}{20\ \text{s}} = 11\ \text{m s}^{-1}$$

$$\text{average velocity, } \boldsymbol{v}_{av} = \frac{\Delta \boldsymbol{x}}{\Delta t} = \frac{100\ \text{m north}}{20\ \text{s}} = 5\ \text{m s}^{-1}\ \text{north}$$

Revision question 10.3

During the final 4.2 km run stage of a triathlon, a participant runs 2.8 km east, then changes direction to run a further 1.4 km in the opposite direction, completing the stage in 20 minutes. What was the participant's:
(a) average speed
(b) average velocity?
Express both answers in m s^{-1}.

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Unit 2	**Position–time graph for motion**
AOS 1	Concept summary and practice questions
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Instantaneous speed and velocity — using graphs

Neither the average speed nor the average velocity provide information about movement at any particular instant of time. For example, when Jamaican athlete Usain Bolt broke the 100 m world record in 2009 with a time of 9.58 s, his average speed was 10.4 m s^{-1}. However, he was not travelling at that speed throughout his run. He would have taken a short time to reach his maximum speed and would not have been able to maintain it throughout the run. His maximum speed would have been much more than 10.4 m s^{-1}.

The speed at any particular instant of time is called the **instantaneous speed**. The velocity at any particular instant of time is, not surprisingly, called the **instantaneous velocity**. If an object moves with a constant velocity during a time interval, its instantaneous velocity throughout the interval is the same as its average velocity.

Instantaneous speed is the speed at a particular instant of time.

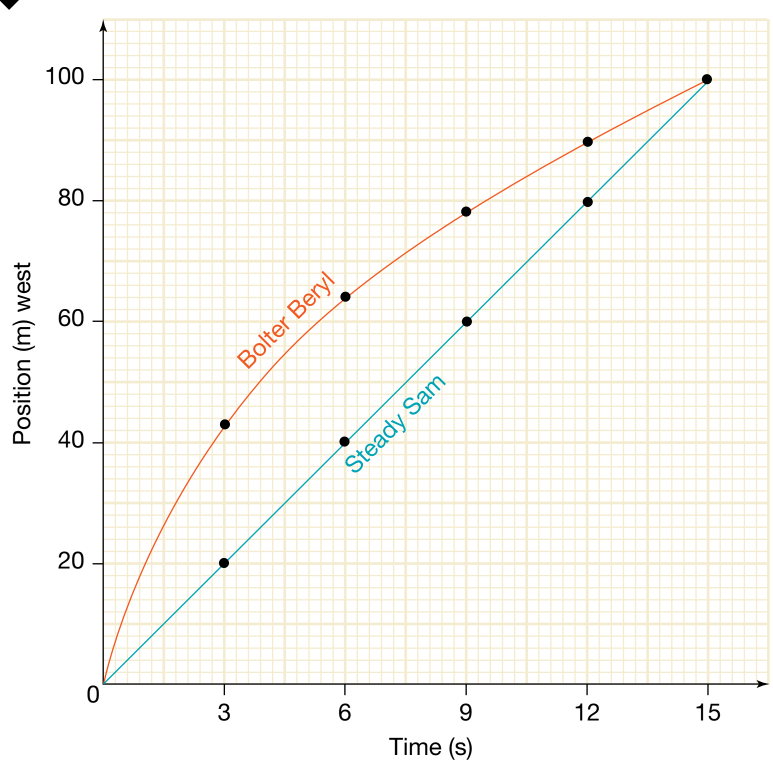
Instantaneous velocity is the velocity at a particular instant of time.

Graphing motion: position versus time

Bolter Beryl and Steady Sam decide to race each other on foot over a distance of 100 m. They run due west. Timekeepers are instructed to record the position of each runner after each 3.0 second interval.

TABLE 10.1 The progress of Bolter Beryl and Steady Sam

	Position (distance from starting line) in metres	
Time (seconds)	Bolter Beryl	Steady Sam
0.0	0	0
3.0	43	20
6.0	64	40
9.0	78	60
12.0	90	80
15.0	100	100



The graph of position versus time the race was run provides valuable information about the way.

The points indicating Bolter Beryl's position after each 3.0 s interval are joined with a smooth curve. It is reasonable to assume that her velocity changes gradually throughout the race.

A number of observations can be made from the graph of position versus time.

- Both runners reach the finish at the same time. The result is a dead heat. Bolter Beryl and Steady Sam each have the same average speed and the same average velocity.
- Steady Sam, who has an exceptional talent for steady movement, maintains a constant velocity throughout the race. In fact, his instantaneous velocity at every instant throughout the race is the same as his average velocity. Steady Sam's average velocity and instantaneous velocity are both equal to the gradient of the position-versus-time graph since:

$$\boldsymbol{v}_{av} = \frac{\Delta x}{\Delta t}$$

$$\boldsymbol{v}_{av} = \frac{100\,\text{m west}}{15\,\text{s}}$$

$$\boldsymbol{v}_{av} = \frac{\text{rise}}{\text{run}}$$

$$\boldsymbol{v}_{av} = \text{gradient.}$$

  Steady Sam's velocity throughout the race is 6.7 m s^{-1} west.
- Bolter Beryl, in her usual style, makes a flying start; however, after her initial 'burst', her instantaneous velocity decreases throughout the race as she tires. Her average velocity is also 6.7 m s^{-1} west.

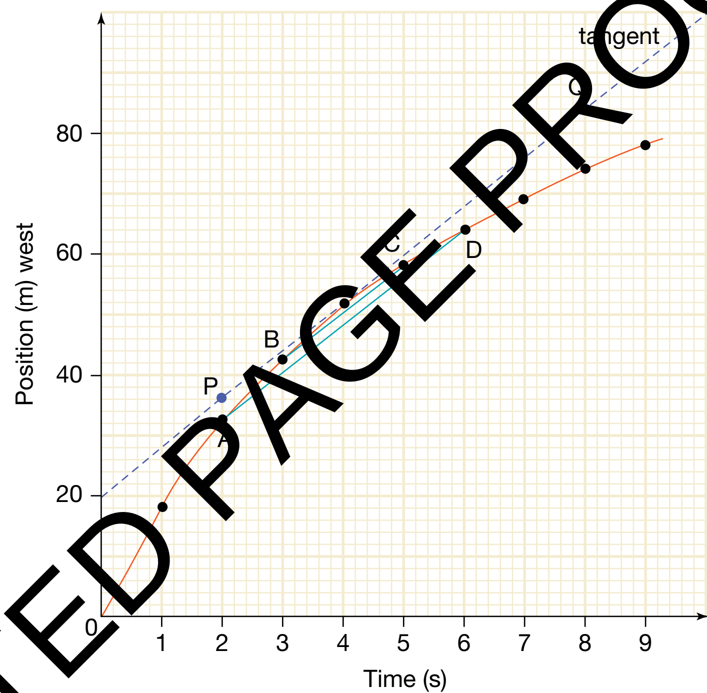
A more detailed description of Bolter Beryl's motion can be given by calculating her average velocity during each 3 s interval of the race (see the table below).

TABLE 10.2 Bolter Beryl's changing velocity

Time interval (s)	Displacement Δx (m west)	Average velocity during interval $v_{av} = \frac{\Delta x}{\Delta t}$ (m s^{-1} west)
0.0–3.0	43 – 0 = 43	14.0
3.0–6.0	64 – 43 = 21	7.0
6.0–9.0	78 – 64 = 14	4.7
9.0–12.0	90 – 78 = 12	4.0
12.0–15.0	100 – 90 = 10	3.3

The average velocity during each interval is the same as the gradient of the straight line joining the data points representing the beginning and end of the interval. An even more detailed description of Bolter Beryl's run could be obtained if the race was divided into, say, 100 time intervals. The average velocity during each time interval (and the gradient of the line joining the data points defining it) would be a very good estimate of the instantaneous velocity in the middle of the interval. In fact, if the race is progressively divided into smaller and smaller time intervals, the average velocity during each interval would become closer and closer to the instantaneous velocity in the middle of the interval.

The graph below shows how this process of using smaller time intervals can be used to find Bolter Beryl's instantaneous velocity at an instant 4.0 seconds from the start of the race. Her instantaneous velocity is not the same as the average velocity during the 3.0 to 6.0 s time interval shown in table 10.2. However, it can be estimated by drawing the line AD and finding its gradient. The gradient of the line BC would provide an even better estimate of the instantaneous velocity. If you continue this process of decreasing the time interval used to estimate the instantaneous velocity, you will eventually obtain a line which is a tangent to the curve. The gradient of the tangent to the curve is equal to the instantaneous velocity at the instant represented by the point at which it meets the curve.



The first 9.0 seconds of Bolter Beryl's run

The gradient of the tangent to the curve at 4.0 seconds in the figure above can be determined by using the points P and Q.

$$\begin{aligned}\text{gradient} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{(84-36)\text{ m}}{(8.0-2.0)\text{ s}} \\ &= \frac{48\text{ m}}{6.0\text{ s}} \\ &= 8.0\text{ m s}^{-1}\end{aligned}$$

Bolter Beryl's instantaneous velocity at 4.0 seconds from the start of the race is therefore 8.0 m s^{-1} west.

Just as the gradient of a position-versus-time graph can be used to determine the velocity of an object, a graph of distance versus time can be used to determine its speed. Because Bolter Beryl and Steady Sam were running in a straight line and in one direction only, their distance from the starting point is the magnitude of their change in position. Their speed is equal to the magnitude of their velocity.

Graphing motion: velocity versus time

The race between Bolter Beryl and Steady Sam described by the position-versus-time graph on page 157 can also be described by a graph of velocity versus time. Steady Sam's velocity is 6.7 m s^{-1} due west throughout the race. The curve describing Bolter Beryl's motion can be plotted by determining the instantaneous velocity at various times during the race. This can be done by drawing tangents at a number of points on the position-versus-time graph on page 157. Table 10.3 shows the data obtained using this method. The velocity-versus-time graph below describes the motion of Bolter Beryl and Steady Sam.

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Numerical model of motion 1: Finding speed from position–time data
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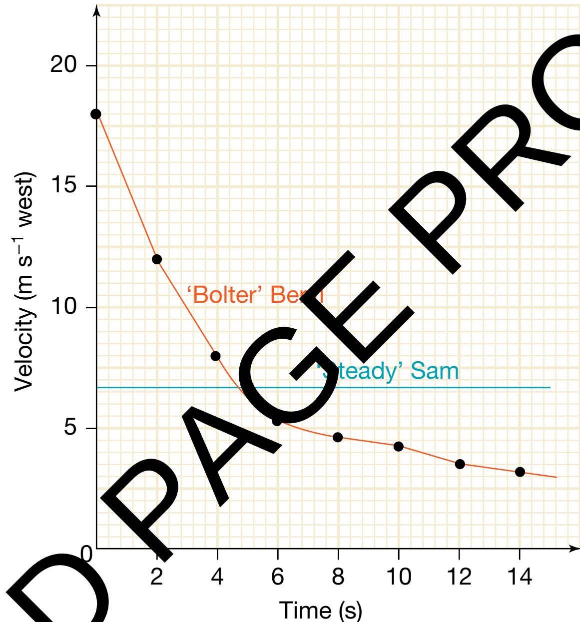
TABLE 10.3 Beryl's velocity during the race

Time (s)	Velocity (m s^{-1} west)
0.0	18.0
2.0	12.0
4.0	8.0
6.0	5.4
8.0	4.7
10.0	4.2
12.0	3.5
14.0	3.1



Graph of velocity versus time for the race

The velocity-versus-time graph confirms what you already knew by looking at the position-versus-time graph, namely that:

- Steady Sam's velocity is constant, and equal to his average velocity
- the magnitude of Bolter Beryl's velocity is decreasing throughout the race.

The velocity-versus-time graph allows you to estimate the velocity of each runner at any time. It provides a much clearer picture of the way that Bolter Beryl's velocity changes during the race, namely that:

- the magnitude of her velocity decreases rapidly at first, but less rapidly towards the end of the race
- for most of the duration of the race, she is running more slowly than Sam. In fact Bolter Beryl's speed (the magnitude of her velocity) drops below that of Steady Sam's after only 4.7 seconds.

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Unit 2 | AOS 1 | Topic 1 | Concept 4

Velocity–time graphs
Concept summary and practice questions

Displacement from a velocity-versus-time graph

In the absence of a position-versus-time graph, a velocity-versus-time graph provides useful information about the change in position, or displacement, of an object. Steady Sam's constant velocity, the same as his average velocity, makes it very easy to determine his displacement during the race.

$$\Delta \boldsymbol{x} = \boldsymbol{v}_{av}\,\Delta t \qquad \left(\text{since } \boldsymbol{v}_{av} = \frac{\Delta \boldsymbol{x}}{\Delta \boldsymbol{t}}\right)$$

$$\Delta \boldsymbol{x} = 6.7 \text{ m s}^{-1} \text{ west} \times 15 \text{ s}$$

$$\Delta \boldsymbol{x} = 100 \text{ m west}$$

This displacement is equal to the area of the rectangle under the graph depicting Steady Sam's motion.

$$\begin{aligned}\text{area} &= \text{length} \times \text{width} \\ &= 15\ \text{s} \times 6.7\ \text{m s}^{-1}\ \text{west} \\ &= 100\ \text{m west}\end{aligned}$$

Because the race was a dead heat, Bolter Beryl's average velocity was also 6.7 m s^{-1}. Her displacement during the race can be calculated in the same way as Steady Sam's.

$$\begin{aligned}\Delta \boldsymbol{x} &= \boldsymbol{v}_{av}\,\Delta t \\ &= 6.7\ \text{m s}^{-1} \times 15\ \text{s} \\ &= 100\ \text{m west}\end{aligned}$$

However, Bolter Beryl's displacement can also be found by calculating the area under the velocity-versus-time graph depicting her motion. This can be done by 'counting squares' or by dividing the area under the graph into rectangles and triangles as shown in the 'As a matter of fact' panel below. The area under Beryl's velocity-versus-time graph is, not surprisingly, 100 m.

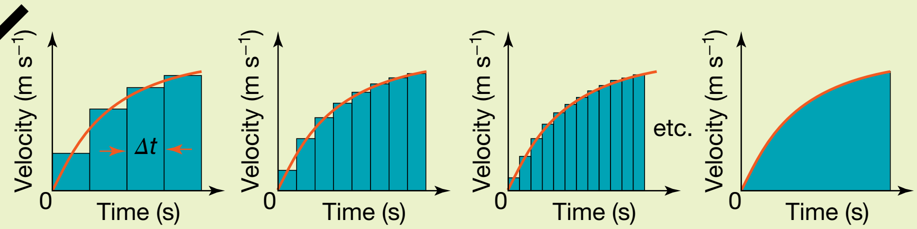
In fact, the area under any part of the velocity-versus-time graph is equal to the displacement during the interval represented by that part.

AS A MATTER OF FACT

When an object travels with a constant velocity, it is obvious that the displacement of the object is equal to the area under a velocity-versus-time graph of its motion. However, it is not so obvious when the motion is not constant. The graphs below describe the motion of an object that has an increasing velocity. The motion of the object can be approximated by dividing it into time intervals of Δt and assuming that the velocity during each time interval is constant. The approximate displacement during each time interval is equal to:

$$\Delta \boldsymbol{x} = \boldsymbol{v}_{av}\,\Delta t$$

which is the same as the area under each rectangle. The approximate total displacement is therefore equal to the total area of the rectangles.



By dividing the velocity-versus-time graph into rectangles representing small time intervals, the displacement can be estimated.

To better approximate the displacement, the graph can be divided into smaller time intervals. The total area of the rectangles is approximately equal to the displacement. By dividing the graph into even smaller time intervals, even better estimates of the displacement can be made. In fact, by continuing the process of dividing the graph into smaller and smaller time intervals, it can be seen that the displacement is, in fact, equal to the area under the graph.

Sample problem 10.4

In the race between Bolter Beryl and Steady Sam, how far ahead of Steady Sam was Bolter Beryl when her speed dropped below Sam's speed?

Solution: Although it is possible to answer this question using the position-versus-time graph on page 157 (you might like to explain how you would do this!), it is easier to use the velocity-versus-time graph (see the graph on page 160). It shows that Beryl's speed (and the magnitude of her velocity) drops below Steady Sam's 4.7 s after the race starts.

Steady Sam's displacement, after 4.7 s, is equal to the area under the line representing the first 4.3 s of his motion, that is, 4.7 s × 6.7 m s^{-1} west. Steady Sam is therefore 31 m west of the starting line after 4.7 s.

Bolter Beryl's displacement after 4.7 s equals the area under the curve representing the first 4.7 s of her motion. This area can be estimated by determining the shaded area of the triangle P and rectangle Q in the figure below.

$$\begin{aligned} \text{area} &= \text{area P} + \text{area Q} \\ &= \tfrac{1}{2} \times 4.7\,\text{s} \times 11\,\text{m s}^{-1}\ \text{west} + 4.7\,\text{s} \times 6.5\,\text{m s}^{-1}\ \text{west} \\ &= 25.85\ \text{m west} + 30.55\ \text{m west} \\ &= 56.40\ \text{m west} \end{aligned}$$

Bolter Beryl is therefore 56 m west of the starting line after 4.7 seconds. She is 25 m ahead of Steady Sam when her speed drops below his.

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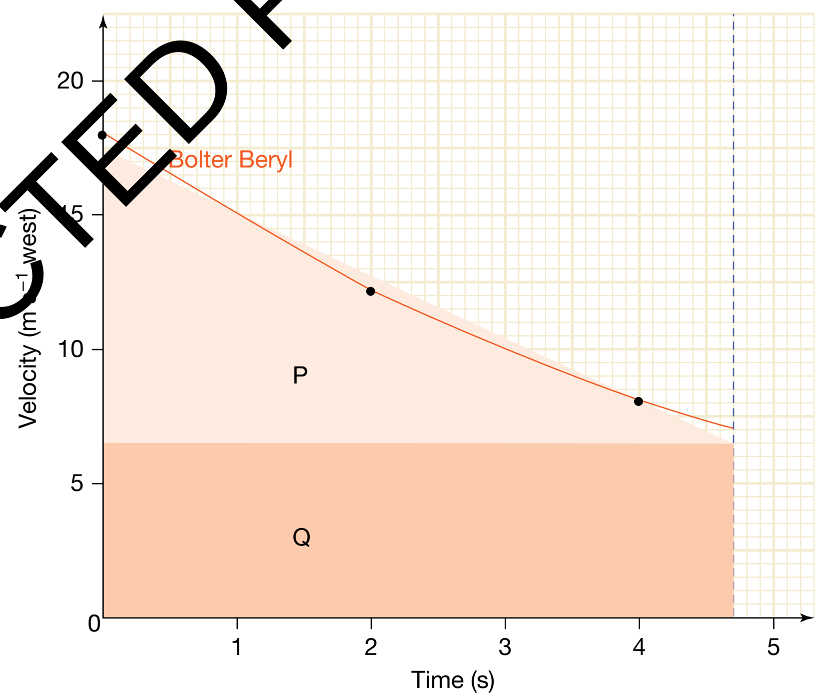

Revision question 10.4

(a) Use the graph above to estimate Bolter Beryl's displacement after 2.0 s.
(b) Use the graph on page 160 to determine how far ahead Bolter Beryl was 10 seconds into the race.

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Unit 2 | AOS 1 | Topic 1 | Concept 3

Acceleration
Concept summary and practice questions

Acceleration

When the velocity of an object changes, as it does in Bolter Beryl's run (see page 160), it is helpful to describe it in terms of the rate at which the velocity is changing. In everyday language, the word *accelerate* is used to mean 'speed up'. The word *decelerate* is used to mean 'slow down'. However, if you wish to describe motion precisely, these words are not adequate. The rate at which an object changes its velocity is called its **acceleration**. Acceleration is a vector quantity.

Acceleration is the rate of change of velocity.

A car starting from rest and reaching a velocity of 60 km h^{-1} north in 5 s has an average acceleration of 12 km h^{-1} per second or 12 (km h^{-1})s^{-1} north. This is expressed in words as 12 km per hour per second. In simple terms, it means that the car increases its speed in a northerly direction by an average of 12 km h^{-1} each second.

The average acceleration of an object, $\boldsymbol{a}_{av}$, can be expressed as:

$$\boldsymbol{a}_{av} = \frac{\Delta \boldsymbol{v}}{\Delta t}$$

where $\Delta \boldsymbol{v}$ = the change in velocity during the time interval Δt.

The direction of the average acceleration is the same as the direction of the change in velocity.

Sample problem 10.5

Spiro leaves home on his bicycle to post a letter to his sweetheart, Ying. He starts from rest and reaches a speed of 10 m s^{-1} in 4.0 s. He then cycles at a constant speed in a straight line to a letterbox. He brakes at the letterbox, coming to a stop in 2.0 s, posts the letter and returns home at a constant speed of 8.0 m s^{-1}. (He's tired!) On reaching home, he brakes, coming to rest in 2.0 s. The direction away from home towards the letterbox is assigned as positive.

(a) What is Spiro's average acceleration before he reaches his 'cruising speed' of 10 m s^{-1} on the way to the letterbox?
(b) What is Spiro's average acceleration as he brakes at the letterbox?
(c) What is Spiro's average acceleration as he brakes when arriving home?
(d) During which two parts of the trip is Spiro's acceleration negative?
(e) Does a positive acceleration always mean that the speed is increasing? Explain.

Solution: It is a good idea to start by sketching a graph describing the motion. In this case a velocity-versus-time graph would be appropriate. Although these questions can be answered without a graph, a graph provides an overview of the motion and allows you to check that your answers make sense.

(a) $$\boldsymbol{a}_{av} = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{+10\,\text{m}\,\text{s}^{-1}}{4.0\,\text{s}} = +2.5\,\text{m}\,\text{s}^{-2}$$

(b) $$\boldsymbol{a}_{av} = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{-10\,\text{m}\,\text{s}^{-1}}{2.0\,\text{s}} = -5.0\,\text{m}\,\text{s}^{-2}$$

(c) $\boldsymbol{a}_{av} = \dfrac{\Delta \boldsymbol{v}}{\Delta t}$

$= \dfrac{+8.0\ \text{m s}^{-1}}{2.0\ \text{s}}$

(Take care here! Spiro's velocity changes from -8.0 m s^{-1} to zero. That is a change of *positive* 8.0 m s^{-1}.)

$= 4.0\ \text{m s}^{-2}$

(d) Spiro's acceleration is negative while he brakes at the letterbox and when he sets off from the letterbox towards home.

(e) No. Spiro's acceleration is positive while his velocity is increasing in the positive direction. This occurs when he increases his speed as he leaves home and also when he decreases his speed as he returns home. A decreasing speed in the negative direction corresponds to a change in velocity in the positive direction.

Revision question 10.5

(a) A cheetah (the fastest land animal) takes 2.0 s to reach its maximum speed of 30 m s^{-1}. What is the magnitude of its average acceleration?

(b) A drag-racing car reaches a speed of 420 km h^{-1} from a standing start in 6.0 s. What is its average acceleration in

(i) km h^{-1} s^{-1}

(ii) m s^{-2}?

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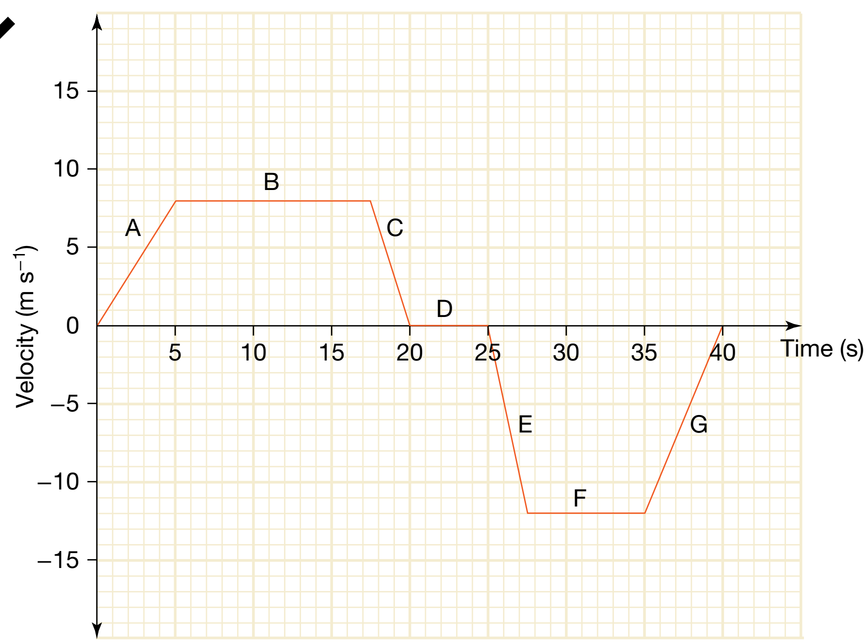
Acceleration from a velocity-versus-time graph

The graph that follows describes the motion of an elevator as it moves from the ground floor to the top floor and back down again. The elevator stops briefly at the top floor to pick up a passenger. For convenience, any upward displacement from the ground floor is defined as positive. The graph has been divided into seven sections labelled A–G.



The motion of an elevator

The acceleration at any instant during the motion can be determined by calculating the gradient of the graph. This is a consequence of the definition of acceleration. The gradient of a velocity-versus-time graph is a measure of the rate of change of velocity just as the gradient of a position-versus-time graph is a measure of the rate of change of position.

Throughout interval A (see the graph), the acceleration, $\boldsymbol{a}$, of the elevator is:

$$\begin{aligned}\boldsymbol{a} &= \frac{\text{rise}}{\text{run}}\\ &= \frac{+8.0\text{ m s}^{-1}}{5.0\text{ s}}\\ &= +1.6\text{ m s}^{-2}\text{ or }1.6\text{ m s}^{-2}\text{ up.}\end{aligned}$$

During intervals B, D and F, the velocity is constant and the gradient of the graph is zero. The acceleration during each of these intervals is, therefore, zero.

Throughout interval C, the acceleration is:

$$\begin{aligned}\boldsymbol{a} &= \frac{-8.0\text{ m s}^{-1}}{2.5\text{ s}}\\ &= -3.2\text{ m s}^{-2}\text{ or }3.2\text{ m s}^{-2}\text{ down.}\end{aligned}$$

Throughout interval E, the acceleration is:

$$\begin{aligned}\boldsymbol{a} &= \frac{-12\text{ m s}^{-1}}{2.5\text{ s}}\\ &= -4.8\text{ m s}^{-2}\text{ or }4.8\text{ m s}^{-2}\text{ down.}\end{aligned}$$

Throughout interval G, the acceleration is:

$$\begin{aligned}\boldsymbol{a} &= \frac{+12\text{ m s}^{-1}}{5.0\text{ s}}\\ &= +2.4\text{ m s}^{-2}\text{ or }2.4\text{ m s}^{-2}\text{ up.}\end{aligned}$$

Notice that during interval G the acceleration is positive (up) while the velocity of the elevator is negative (down). The direction of the acceleration is the same as the direction of the *change* in velocity.

The area under the graph is equal to the displacement of the elevator. Dividing the area into triangles and rectangles and working from left to right yields an area of:

$$\begin{aligned}&\left(\tfrac{1}{2}\times 5.0\text{ s}\times 8.0\text{ m s}^{-1}\right)+\left(12.5\text{ s}\times 8.0\text{ m s}^{-1}\right)+\left(\tfrac{1}{2}\times 2.5\text{ s}\times 8.0\text{ m s}^{-1}\right)+\\ &\left(\tfrac{1}{2}\times 2.5\text{ s}\times -12\text{ m s}^{-1}\right)+\left(7.5\text{ s}\times -12\text{ m s}^{-1}\right)+\left(\tfrac{1}{2}\times 5.0\text{ s}\times -12\text{ m s}^{-1}\right)\\ &= 20\text{ m} + 100\text{ m} + 10\text{ m} - 15\text{ m} - 90\text{ m} - 30\text{ m}\\ &= -5.0\text{ m.}\end{aligned}$$

This represents a downwards displacement of 5.0 m, which is consistent with the elevator finally stopping two floors below the ground floor.

Area under an acceleration-versus-time graph

Just as the area under a velocity-versus-time graph is equal to the change in position of an object, the area under an acceleration-versus-time graph is equal to the change in velocity of an object. The acceleration-versus-time graph of the motion of the elevator described previously is shown in the graph on the following page. The area under the part of the graph representing the entire upwards part of the journey is given by:

$$\begin{aligned}\text{area A} + \text{area C} &= 5.0\text{ s}\times 1.6\text{ m s}^{-2} + 2.5\text{ s}\times -3.2\text{ m s}^{-2}\\ &= +8.0\text{ m s}^{-1} + -8.0\text{ m s}^{-1}\\ &= 0.\end{aligned}$$

This indicates that change in velocity during the upward journey is zero. This is consistent with the fact that the elevator starts from rest and is at rest when it reaches the top floor. Similarly, the area under the whole graph is zero.

The change in velocity during intervals C, D and E is given by the sum of areas C, D and E. Thus:

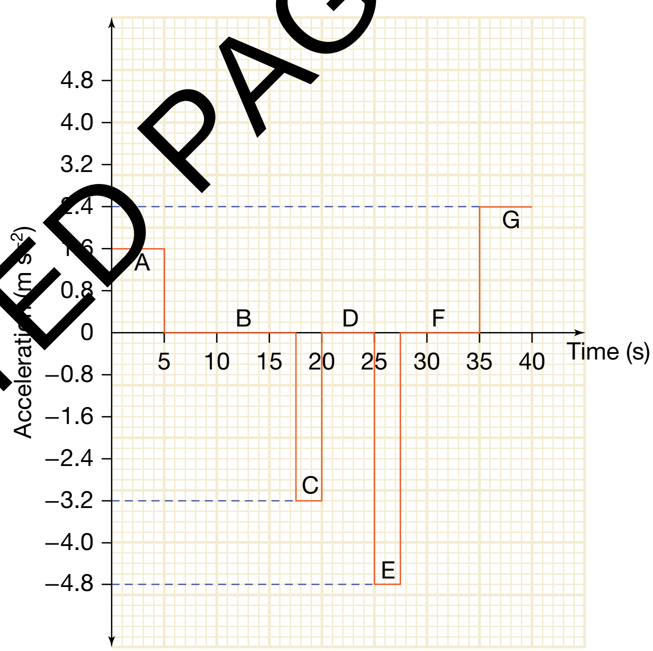
$$\begin{aligned}\text{area C} + \text{area D} + \text{area E} &= 2.5\text{ s} \times -3.2\text{ m s}^{-2} + 0 + 2.5\text{ s} \times -4.8\text{ m s}^{-2} \\ &= -8.0\text{ m s}^{-1} + -12\text{ m s}^{-1} \\ &= -20\text{ m s}^{-1}.\end{aligned}$$

The change in velocity is -20 m s^{-1}, or 20 m s^{-1} down.

At the beginning of time interval C, the velocity was 8.0 m s^{-1} upwards. A change of velocity of -20 m s^{-1} would result in a final velocity of 12 m s^{-1} downwards. This is consistent with the description of the motion in the velocity-versus-time graph on page 162. The symbol $\boldsymbol{u}$ is used to denote the initial velocity, while the symbol $\boldsymbol{v}$ is used to denote the final velocity.

In symbols, therefore:

$$\begin{aligned}\boldsymbol{v} &= \boldsymbol{u} + \Delta\boldsymbol{u} \qquad (\text{since } \Delta\boldsymbol{v} = \boldsymbol{v} - \boldsymbol{u}) \\ &= +8.0\text{ m s}^{-1} + -20\text{ m s}^{-1} \\ &= -12\text{ m s}^{-1}.\end{aligned}$$



An acceleration-versus-time graph for the elevator

Graphing motion in a nutshell

Position-versus-time graphs

- The instantaneous velocity of an object can be obtained from a graph of the object's position versus time by determining the gradient of the curve at the point representing that instant. This is a direct consequence of the fact that velocity is a measure of the rate of change of position.
- Similarly, the instantaneous speed of an object can be obtained by determining the gradient of a graph of the object's distance travelled from a reference point versus time.

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Constant acceleration app

Velocity-versus-time graphs

- The displacement of an object during a time interval can be obtained by determining the area under the velocity-versus-time graph representing that time interval. The actual position of an object at any instant during the time interval can be found only if the starting position is known.
- Similarly, the distance travelled by an object during a time interval can be obtained by determining the corresponding area under the speed-versus-time graph for the object.
- The instantaneous acceleration of an object can be obtained from a graph of the object's velocity versus time by determining the gradient of the curve at the point representing that instant. This is a direct consequence of the fact that acceleration is defined as the rate of change of velocity.

Acceleration-versus-time graphs

- The change in velocity of an object during a time interval can be obtained by determining the area under the acceleration-versus-time graph representing that time interval. The actual velocity of the object can be found at any instant during the time interval only if the initial velocity is known.

AS A MATTER OF FACT

A non-zero acceleration does not always result from a change in speed. Consider a car travelling at 60 km h^{-1} in a northerly direction turning right and continuing in an easterly direction at the same speed. Assume that the complete turn takes 10 s. The average acceleration during the time interval of 10 seconds is given by:

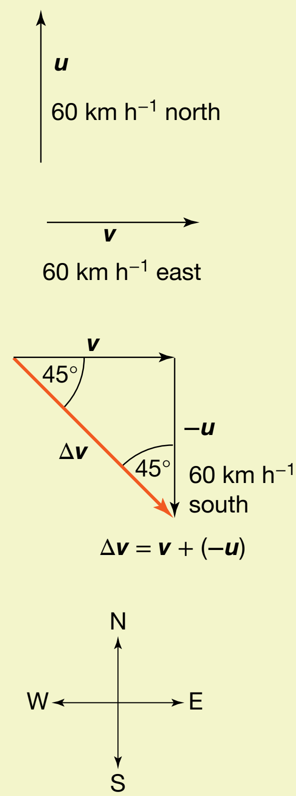
$$\boldsymbol{a}_{av} = \frac{\Delta \boldsymbol{v}}{\Delta t}.$$

The change in velocity must be determined first. Thus,

$$\Delta \boldsymbol{v} = \boldsymbol{v} - \boldsymbol{u}$$
$$= \boldsymbol{v} + -\boldsymbol{u}.$$

The vectors $\boldsymbol{v}$ and $-\boldsymbol{u}$ are added together to give the resulting change in velocity.

The magnitude of the change in velocity is calculated using Pythagoras' theorem or trigonometric ratios to be 85 km h^{-1}. Alternatively, the vectors can be added using a scale drawing and then measuring the magnitude and direction of the sum. The direction of the change in velocity can be seen in the figure at right to be south-east.



$$\boldsymbol{a}_{av} = \frac{\Delta \boldsymbol{v}}{\Delta t}$$
$$= \frac{85\ \text{km h}^{-1}\ \text{south-east}}{10\ \text{s}}$$
$$= 8.5\ \text{km h}^{-1}\ \text{s}^{-1}\ \text{south-east}$$

In fact, the direction of the average acceleration is the same as the direction of the average net force on the car during the 10 s interval. The steering wheel is used to turn the wheels to cause the net force to be in this direction. For more information on vectors, go to appendix 2.

A change in acceleration can occur even if there is no change in speed.

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Unit 2 | AOS 1 | Topic 1 | Concept 5

Motion with constant acceleration
Concept summary and practice questions

Constant acceleration without graphs

In the absence of a graphical representation, a number of formulae can be used to describe straight-line motion as long as the acceleration is constant. These formulae are expressed in terms of the quantities used to describe such motion. The terms are:

- initial velocity, u
- final velocity, v
- acceleration, a
- time interval, t
- displacement, s.

Because the formulae describe motion along a straight line, vector notation is not necessary. The displacement, velocity and acceleration can be expressed as positive or negative quantities.

The first formula is found by restating the definition of acceleration.

$$\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t}$$

where

Δv = the change in velocity

Δt = the time interval.

Thus,

$$a = \frac{v-u}{t}$$

$$\Rightarrow \quad v - u = at$$

$$\Rightarrow \quad v = u + at \qquad [1]$$

The second formula is found by restating the definition of average velocity.

$$v_{\text{av}} = \frac{\Delta s}{\Delta t}$$

where Δs = the change in position.

But $v_{\text{av}} = \dfrac{u+v}{2}$.

Thus,

$$\frac{u+v}{2} = \frac{s}{t}$$

$$\Rightarrow \quad s = \tfrac{1}{2}(u+v)t. \qquad [2]$$

Three more formulae are obtained by combining formulae [1] and [2].

$$s = \tfrac{1}{2}(u + u + at)t$$ (substituting $v = u + at$ from formula [1] into formula [2])

$$= \tfrac{1}{2}(2u + at)t$$

$$= \left(u + \tfrac{1}{2}at\right)t$$

$$\Rightarrow \quad s = ut + \tfrac{1}{2}at^2 \qquad [3]$$

$$s = \tfrac{1}{2}(v - at + v)t$$ (substituting $u = v - at$ from formula [1] into formula [2])

$$= \tfrac{1}{2}(2v - at)t$$

$$= \left(v - \tfrac{1}{2}at\right)t$$

$$\Rightarrow \quad s = vt - \tfrac{1}{2}at^2 \qquad [4]$$

A final formula can be found by eliminating t from formula [2].

$$s = \tfrac{1}{2}(u+v)t \qquad \text{(formula [2])}$$

But $t = \dfrac{v-u}{a}$. (rearranging formula [1])

$$\Rightarrow \quad s = \tfrac{1}{2}(u+v)\left(\frac{v-u}{a}\right)$$

$$= \tfrac{1}{2}\,\frac{v^2-u^2}{a} \qquad \text{(expanding the difference of two squares)}$$

$$\Rightarrow \quad 2as = v^2 - u^2$$

$$\Rightarrow \quad v^2 = u^2 + 2as \qquad [5]$$

In summary, the formulae for straight-line motion are:

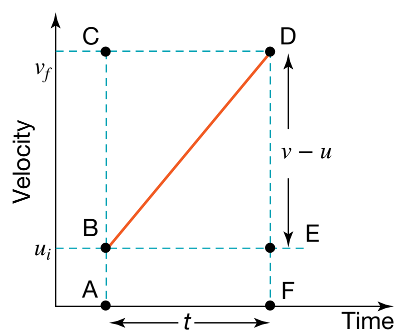
$$v = u + at \qquad [1]$$

$$s = \tfrac{1}{2}(u+v)t \qquad [2]$$

$$s = ut + \tfrac{1}{2}at^2 \qquad [3]$$

$$s = vt - \tfrac{1}{2}at^2 \qquad [4]$$

$$v^2 = u^2 + 2as \qquad [5]$$



A velocity-versus-time graph for an object travelling in a straight line with constant acceleration

Each of the five formulae derived here allow you to determine an unknown characteristic of straight-line motion with a constant acceleration as long as you know three other characteristics. Although the formulae have not been derived from graphs, they are entirely consistent with a graphical approach.

acceleration = gradient

$$= \frac{\text{rise}}{\text{run}}$$

$$= v - \frac{u}{t}$$

$$\Rightarrow \quad a = v - \frac{u}{t}$$

$$\Rightarrow \quad v = u + at \qquad [1]$$

displacement = area under graph

= area of trapezium ABDF

$$= \tfrac{1}{2}(u+v)t \qquad [2]$$

displacement = area under graph

= area of rectangle ABEF + area of triangle BDE

$$= ut + \tfrac{1}{2}t \times at \qquad (v - u = at \text{ from [1]})$$

$$= ut + \tfrac{1}{2}at^2 \qquad [3]$$

displacement = area under graph

= area of rectangle ACDF − area of triangle BCD

$$= vt - \tfrac{1}{2} \times t \times at \qquad (v - u = at \text{ from [1]})$$

$$= vt - \tfrac{1}{2}at^2 \qquad [4]$$

Formula [5] can be derived by combining formula [1] with any of formulae [2], [3] or [4].

Sample problem 10.6

Ying (hopelessly in love with Spiro) drops a coin into a wishing well and takes 3.0 s to make a wish. The coin splashes into the water just as she finishes making her wish. The coin accelerates towards the water at a constant 10 m s^{-2}.

(a) What is the coin's velocity as it strikes the water?
(b) How far does the coin fall before hitting the water?

Solution:

(a) $v = ?$

$u = 0$, $a = 10$ m s^{-2}, $t = 3.0$ s (assigning down as positive)

The appropriate formula here is $v = u + at$ because it includes the three known quantities and the unknown quantity v.

$v = 0 \text{ m s}^{-1} + 10 \text{ m s}^{-2} \times 3.0 \text{ s}$

$= 30 \text{ m s}^{-1}$

The coin is travelling at a velocity of 30 m s^{-1} down as it strikes the water.

(b) $s = ?$

The appropriate formula here is $s = ut + \frac{1}{2}at^2$ because it includes the three known quantities along with the unknown quantity s.

$s = 0 + \frac{1}{2} \times 10 \text{ m s}^{-2} \times (3.0 \text{ s})^2$

$= 45 \text{ m}$

The coin experiences a displacement of 45 m down during the fall.

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Revision question 10.6

A parked car with the handbrake off rolls down a hill in a straight line with a constant acceleration of 2.0 m s^{-2}. It stops after colliding with a brick wall at a speed of 12 m s^{-1}.

(a) For how long was the car rolling?
(b) How far did the car roll before colliding with the wall?

Sample problem 10.7

The driver of a car was forced to brake in order to prevent serious injury to a neighbour's cat. The car skidded in a straight line, stopping just 2 cm short of the startled but lucky cat. The driver (who happened to be a physics teacher) measured the length of the skid mark to be 12 m. His passenger (also a physics teacher with an exceptional skill for estimating small time intervals) estimated that the car skidded for 2.0 seconds.

(a) At what speed was the car travelling as it began to skid?
(b) What was the acceleration of the car during the skid?

Solution:

(a) $u = ?$

$s = 12$ m, $t = 2.0$ s, $v = 0$ (assigning forward as positive)

The appropriate formula here is:

$$s = \tfrac{1}{2}(u + v)t$$

$$12 \text{ m} = \tfrac{1}{2}(u + 0)2.0 \text{ s}$$

$u = 12 \text{ m s}^{-1}$. (rearranging and solving in one step)

The car was travelling at a speed of 12 m s^{-1}, about 43 km h^{-1}.

(b) $a = ?$

The appropriate formula here is $s = vt - \frac{1}{2}at^2$. Note that it is better to use the given data rather than calculated data. That way, if an error is made in an earlier part of the question, it will not affect this answer.

$$s = vt - \tfrac{1}{2}at^2$$

$$12\text{ m} = 0 - \tfrac{1}{2}a(2.0\text{ s})^2$$

$$\Rightarrow \quad 12\text{ m} = -2.0\text{ s}^2 \times a$$

$$\Rightarrow \quad a = \frac{-12\text{ m}}{2.0\text{ s}}$$

$$= -6.0\text{ m s}^{-2}$$

The car's acceleration was -6.0 m s^{-2}. In other words, it slowed down at the rate of 6.0 m s^{-1} each second.

Revision question 10.7

A car travelling at 24 m s^{-1} brakes to come to a stop in 1.5 s. If its acceleration (deceleration in this case) was constant, what was the car's:

(a) stopping distance

(b) acceleration?

It is worth noting that sample problems 10.6 and 10.7 could both have been solved without the use of the constant acceleration formulae. Both examples could have been completed with a graphical approach and a clear understanding of the definitions of velocity and acceleration. Why not go ahead and try to answer both problems without the formulae?

Practical investigations

- Place a small ruler over the edge of a desk. Hit the end so that it flies away. How far does it travel horizontally? What factors might affect this, and how? Investigate.
- How does the initial acceleration of a sprinter depend on the spacing between their feet on the blocks?
- Fill a bottle with some liquid. Lay it down on its side and give it a push. The bottle may first move forward and then oscillate before it comes to rest. Investigate the bottle's motion.
- Make a small parachute out of a piece of cloth, lengths of cotton and Blu-Tack. Drop it with the canopy open. The parachute accelerates, then maintains a steady speed. Investigate the motion and what factors affect the initial acceleration and the final speed.

Chapter review

study on

Unit 2 | AOS 1 | Topic 1

Concepts used to model motion

Sit Topic test

Summary

- Displacement is a measure of the change in position of an object. Displacement is a vector quantity.
- In order to fully describe any vector quantity, a direction must be specified as well as a magnitude.
- Speed is a measure of the rate at which an object moves over distance and is a scalar quantity. Velocity is the rate of displacement and is a vector quantity.
- Average speed $= \dfrac{\text{distance travelled}}{\text{time interval}}$
- Average velocity $= \dfrac{\text{displacement}}{\text{time interval}}$. The average velocity of an object, $\boldsymbol{v}_{av}$, during a time interval, t, can be expressed as $\boldsymbol{v}_{av} = \dfrac{\Delta s}{\Delta t}$.
- Instantaneous speed is the speed at a particular instant of time. Instantaneous velocity is the velocity at a particular instant of time.
- The instantaneous velocity of an object can be found from a graph of its displacement versus time by calculating the gradient of the graph. Similarly, the instantaneous speed can be found from a graph of distance versus time by calculating the gradient of the graph.
- The displacement of an object during a time interval can be found by determining the area under its velocity-versus-time graph. Similarly, the distance travelled by an object can be found by determining the area under its speed-versus-time graph.
- Acceleration is the rate at which an object changes its velocity. Acceleration is a vector quantity. The average acceleration of an object, $\boldsymbol{a}_{av}$, can be expressed as $\boldsymbol{a}_{av} = \dfrac{\Delta \boldsymbol{v}}{\Delta t}$ where $\Delta \boldsymbol{v}$ = the change in velocity during the time interval Δt.
- The instantaneous acceleration of an object can be found from a graph of its velocity versus time by calculating the gradient of the graph.
- When acceleration of an object is constant, the following formulae can be used to describe its motion:

$$v = u + at$$
$$s = \tfrac{1}{2}(u + v)t$$
$$s = ut + \tfrac{1}{2}at^2$$
$$s = vt - \tfrac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$

Questions

Describing movement

1. Which of the following are vector quantities?
   (a) distance
   (b) displacement
   (c) speed
   (d) velocity
   (e) acceleration
2. On the planet Znab, a Znabbian ran a distance of 5 znotters north, turned left and ran 12 znotters west. The total time taken was 0.5 znitters.
   (a) What distance was travelled by the Znabbian?
   (b) What was the displacement of the Znabbian?
   (c) Determine the average velocity and average speed of the Znabbian.
   (d) What is the unit of acceleration on the planet Znab?
3. The speed limit on Melbourne's suburban freeways is 100 km h^{-1}. Express this speed in m s^{-1}.
4. Leisel Jones's average speed while swimming a 100 m breaststroke race is about 1.5 m s^{-1}. Calculate what her average speed would be in km h^{-1}.
5. The speed limit on US freeways is 55 miles per hour. Express this speed in:
   (a) km h^{-1}
   (b) m s^{-1}.
   One mile is approximately equal to 1.6 km.
6. The world records for some men's track events (as at early 2015) are listed in the table below.

TABLE 10.4 World records for men's track events

Athlete	Event	Time
Usain Bolt (Jamaica)	100 m	9.58 s
Usain Bolt (Jamaica)	200 m	19.19 s
Michael Johnson (USA)	400 m	43.18 s
David Rudisha (Kenya)	800 m	1 min 40.91 s
Hicham el Guerrouj (Morocco)	1 500 m	3 min 26.00 s
Daniel Komen (Kenya)	3 000 m	7 min 20.67 s
Kenenisa Bekele (Ethiopia)	5 000 m	12 min 37.35 s
Kenenisa Bekele (Ethiopia)	10 000 m	26 min 17.53 s

   (a) Calculate the average speed (to three significant figures) of each of the athletes listed in the table.

(b) Why is there so little difference between the average speeds of the world-record holders of the 100 m and 200 m events despite the doubling of the distance?
(c) How long would it take Hicham el Guerrouj to complete the marathon if he could maintain his average speed during the 1500 m event for the entire 42.2 km course? (The world record for the men's marathon (set on 28 September 2014) is 2 h 2 min 57 s.)
(d) Which of the athletes in the table has an average speed that is the same as the magnitude of his average velocity? Explain.

7. In 2010, cyclist Sarah Hammer, of the USA, set a world record of 3 min, 22.269 s for the 3000 m pursuit.
(a) What was her average speed?
(b) How long would it take her to cycle from Melbourne to Bendigo, a distance of 151 km, if she could maintain her average speed for the 3000 m pursuit for the whole distance?
(c) How long does it take a car to travel from Melbourne to Bendigo if its average speed is 80 km h^{-1}?
(d) A car travels from Melbourne to Bendigo and back to Melbourne in 4.0 hours.
 (i) What is its average speed?
 (ii) What is its average velocity?

8. Once upon a time, a giant tortoise had a bet with a hare that she could beat him in a foot race over a distance of 1 km. The giant tortoise can reach a speed of about 7.5 cm s^{-1}. The hare can run as fast as 20 m s^{-1}. Both animals ran at their maximum speeds during the race. However, the hare was a rather arrogant creature and decided to have a little nap along the way. How long did the hare sleep if the result was a tie?

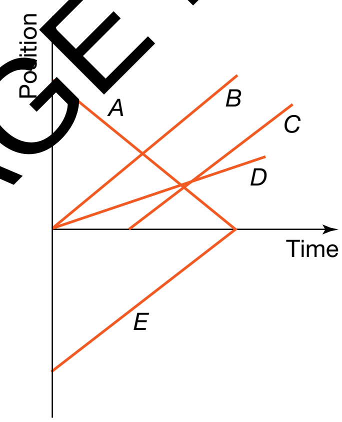
9. An unfit Year 11 student arrives at school late and attempts to run from the front gate of the school to the physics laboratory. He runs the first 120 m at an average speed of 6.0 m s^{-1}, the next 120 m at an average speed of 4.0 m s^{-1} and the final 120 m at an average speed of 2.0 m s^{-1}. What was the student's average speed during his attempt to arrive at his favourite class on time?

10. A holidaying physics teacher drives her old Volkswagen from Melbourne to Wodonga, a distance of 300 km. Her average speed was 80 km h^{-1}. She trades in her old Volkswagen and purchases a brand-new Toyota Prius. She proudly drives her new car back home to Melbourne at an average speed of 100 km h^{-1}.
(a) Make a quick prediction of her average speed for the whole trip.
(b) Calculate the average speed for the entire journey and explain any difference between the predicted and calculated average speed.

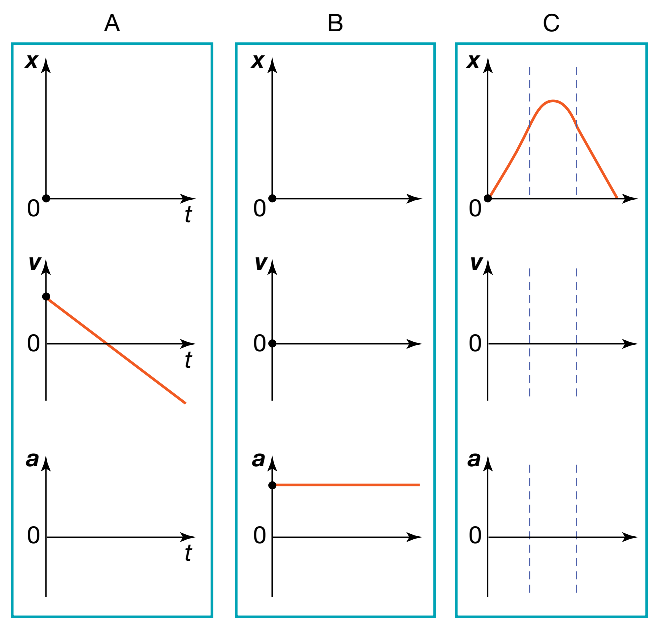
Instantaneous speed and velocity — using graphs

11. Why can't you ever measure the instantaneous velocity of an object with a stopwatch?

12. The position-versus-time graph shown above right describes the motion of five different objects which are labelled *A* to *E*.
(a) Which two objects start from the same position, but at different times?
(b) Which two objects start at the same position at the same time?
(c) Which two objects are travelling at the same speed as each other, but with different velocities?
(d) Which two objects are moving towards each other for the whole period shown on the graph?
(e) Which of the five objects has the lowest speed?



13. Describe in words the motion shown for each of scenarios A, B and C below. Copy the incomplete graphs for each scenario into your workbooks and then complete each graph.



14. Sketch a velocity-versus-time graph to illustrate the motion described in each of the following situations.
    (a) A bicycle is pedalled steadily along a road. The cyclist stops pedalling and allows the bicycle to come to a stop.
    (b) A parachutist jumps out of a plane and opens his parachute midway through the fall to the ground.
    (c) A ball is thrown straight up into the air and is caught at the same height from which it was thrown.
15. Sketch a position-versus-time graph for each scenario in question 8.

Acceleration

16. During motion with a constant acceleration, at what instant of time is the instantaneous velocity the same as the average velocity?
17. Determine (i) the change in speed and (ii) the change in velocity of each of the following situations.
    (a) The driver of a car heading north along a freeway at 100 km h^{-1} slows down to 60 km h^{-1} as the traffic gets heavier.
    (b) A fielder catches a cricket ball travelling towards him at 20 m s^{-1}.
    (c) A tennis ball travelling at 25 m s^{-1} is returned directly back to the server at a speed of 30 m s^{-1}.
18. A car travelling east at a speed of 10 m s^{-1} turns left to head south at the same speed. Has the car undergone an acceleration? Explain your answer with the aid of a diagram.
19. Estimate the acceleration of a car in m s^{-2} as it resumes its journey through the suburbs after stopping at traffic lights.
20. Use the data in table 10.4 on page 172 to help you estimate the average acceleration of a world-class 100 m sprinter at the beginning of a race.
21. How long does it take for:
    (a) a car to accelerate on a straight road at a constant 6.0 m s^{-2} from an initial speed of 60 km h^{-1} (17 m s^{-1}) to a final speed of 100 km h^{-1} (28 m s^{-1})
    (b) a downhill skier to accelerate from rest at a constant 2.0 m s^{-2} to a speed of 10 m s^{-1}?
22. In Acapulco, on the coast of Mexico, professional high divers plunge from a height of 36 m above the water. (The highest diving boards used in Olympic diving events are 10 m above the water.) Estimate:
    (a) the length of the time interval during which the divers fall through the air
    (b) the speed with which the divers enter the water.

    Assume that throughout their dive, the divers are falling vertically from rest with an acceleration of 10 m s^{-2}.
23. A skateboard rider travelling down a hill notices the busy road ahead and comes to a stop in 2.0 s over a distance of 12 m. Assume a constant negative acceleration.
    (a) What was the initial speed of the skateboard?
    (b) What was the acceleration of the skateboard as it came to a stop?
24. A car is travelling at a speed of 100 km h^{-1} when the driver sees a large fallen tree branch in front of her. At the instant that she sees the branch, it is 50 m from the front of her car. The car travels a distance of 48 m *after* the brakes are applied before coming to a stop.
    (a) What is the average acceleration of the car while the car is braking?
    (b) How long does the car take to stop once the brakes are applied?
    (c) What other information do you need in order to determine whether the car stops before it hits the branch? Make an estimate of the missing item of information to predict whether or not the car is able to stop in time.
25. A dancer in a school musical is asked to leap 80 cm into the air, taking off vertically on one beat of the music and landing with the next beat. If the music beats every 0.5 s, is the leap possible? The acceleration of the dancer during the leap can be assumed to be 10 m s^{-2} downwards.
26. A brand-new Rolls Royce rolls off the back of a truck as it is being delivered to its owner. The truck is travelling along a straight road at a constant speed of 60 km h^{-1}. The Rolls Royce slows down at a constant rate, coming to a stop over a distance of 240 m. It is a full minute before the truck driver realises that the precious load is missing. The driver brakes immediately, leaving a 25 m long skid mark on the road. The driver's reaction time (time interval between noticing the problem and depressing the brake) is 0.5 s.

    How far back is the Rolls Royce when the truck stops?
27. A girl at the bottom of a 100 m high cliff throws a tennis ball vertically upwards. At the same instant a boy at the very top of the cliff drops a golf ball so that it hits the tennis ball while both balls are still in the air. The

acceleration of both balls can be assumed to be 10 m s^{-2} downwards.

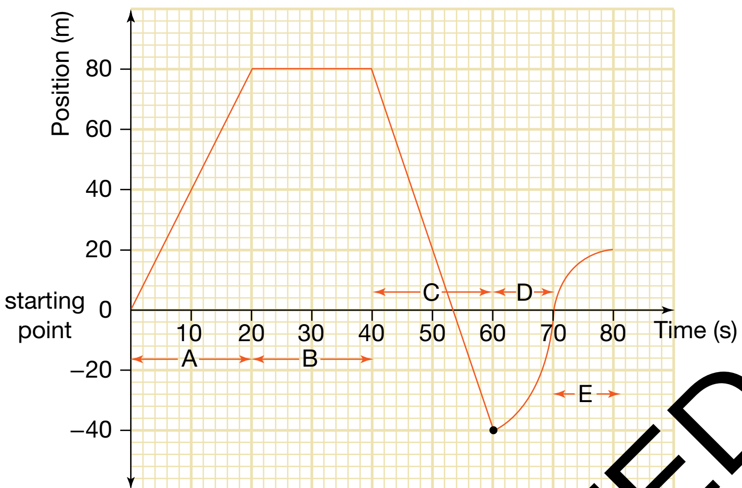
(a) With what speed is the tennis ball thrown so that the golf ball strikes it at the top of its path?

(b) What is the position of the tennis ball when the golf ball strikes it?

More questions on graphical analysis

28. The graph in the following figure is a record of the straight-line motion of a skateboard rider during an 80 s time interval. The time interval has been divided into sections labelled A to E.

The skateboarder initially moves north from the starting point.



(a) During which section of the interval was the skateboard rider stationary?

(b) During which sections of the interval was the skateboarder travelling north?

(c) At what instant did the skateboard rider first move back towards the starting line?

(d) What was the displacement of the skateboarder during the 80 s interval?

(e) What distance did the skateboarder travel during the 80 s interval?

(f) During which section of the interval was the skateboard rider speeding up?

(g) During which section of the interval was the skateboard rider slowing down?

(h) What was the skateboarder's average speed during the entire 80 s interval?

(i) What was the velocity of the skateboarder throughout section C?

(j) Estimate the velocity of the skateboarder 65 s into the interval.

29. The graph in the figure that follows is a record of the motion of a battery-operated toy robot during an 80 s time interval. The interval has been divided into sections labelled A to G.

(a) During which sections is the acceleration of the toy robot zero?

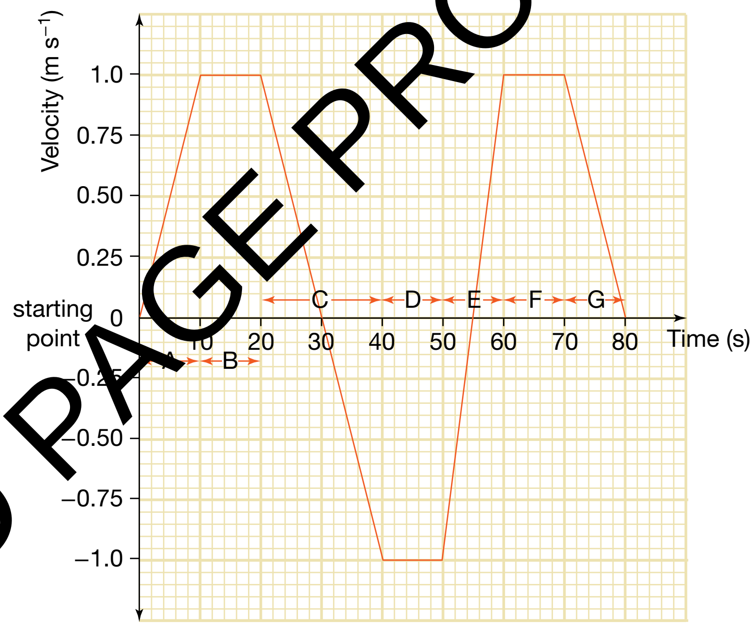
(b) What is the displacement of the toy robot during the 80 s interval?

(c) What is the average velocity of the toy robot during the entire interval?

(d) At what instant did the toy robot first reverse direction?

(e) At what instant did the toy robot first return to its starting point?

(f) During which intervals did the toy robot have a negative acceleration?



(g) During which intervals did the toy robot decrease its speed?

(h) Explain why your answers to (f) and (g) are different from each other.

(i) What is the acceleration of the toy robot throughout section E?

(j) What is the average acceleration during the first 20 s?

(k) Describe the motion of the toy robot in words.

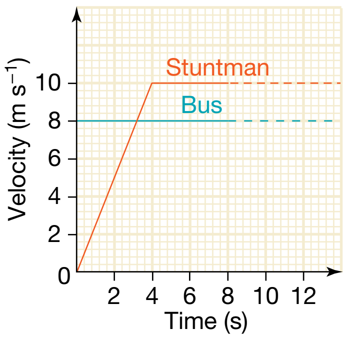
30. During the filming of a new movie, a stuntman has to chase a moving bus and jump into it. The stuntman is required to stand still until the bus passes him and then start chasing it. The velocity-versus-time graph in the figure that follows describes the motion of the stuntman and the bus from the instant that the bus door passes the stationary stuntman.

(a) At what instant did the stuntman reach the same speed as the bus?

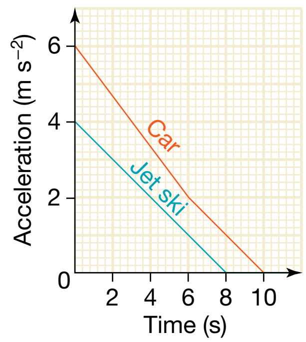
(b) What is the magnitude of the acceleration of the stuntman during the first 4.0 s?

(c) At what instant did the stuntman catch up with the bus door?

(d) How far did the stuntman run before he reached the door of the bus?



31. The figure that follows compares the straight-line motion of a jet ski and a car as they each accelerate from an initial speed of 5.0 m s^{-1}.



(a) Which is first to reach a constant speed — the jet ski or the car — and when does this occur?

(b) What is the final speed of:
 (i) the jet ski
 (ii) the car?

(c) Draw a speed-versus-time graph describing the motion of either the jet ski or the car.