CHAPTER

4 Gravitation

REMEMBER

Before beginning this chapter, you should be able to:

- model forces as vectors acting at the point of application (with magnitude and direction), labelling these forces using the convention 'force on A by B' or $F_{\text{on A by B}}$
- model the force due to gravity, F_g, as the force of gravity acting at the centre of mass of a body
- apply Newton's three laws of motion to a body on which forces act: $a = \frac{F_{\text{net}}}{m}$, $F_{\text{on A by B}} = -F_{\text{on B by A}}$
- analyse uniform circular motion in a horizontal plane
- resolve vectors into components
- apply the energy conservation model to energy transfers and transformations.

KEY IDEAS

After completing this chapter, you should be able to:

- apply Newton's Law of Universal Gravitation to the motion of planets and satellites
- describe gravitation using a field model
- describe the gravitational field around a point mass in terms of its direction and shape
- calculate the strength of the gravitational field at a point a distance, r, from a point mass
- analyse the motion of planets and satellites by modelling their orbits as uniform circular orbital motion
- describe potential energy changes of an object moving in the gravitational field of a point mass
- analyse energy transformations as objects change position in a changing gravitational field, using area under a force–distance graph and area under a field–distance graph multiplied by mass
- apply the concepts of force due to gravity, F_g, and normal reaction force, F_N, to satellites in orbit.

Understanding gravitational forces has allowed us to put satellites in orbit around the Earth.

Explaining the solar system

Isaac Newton (1642–1726) published his Law of Universal Gravitation in 1687. This law provided a mathematical and physical explanation for several important observations about the movement of planets in the solar system that had been made over the previous two centuries.

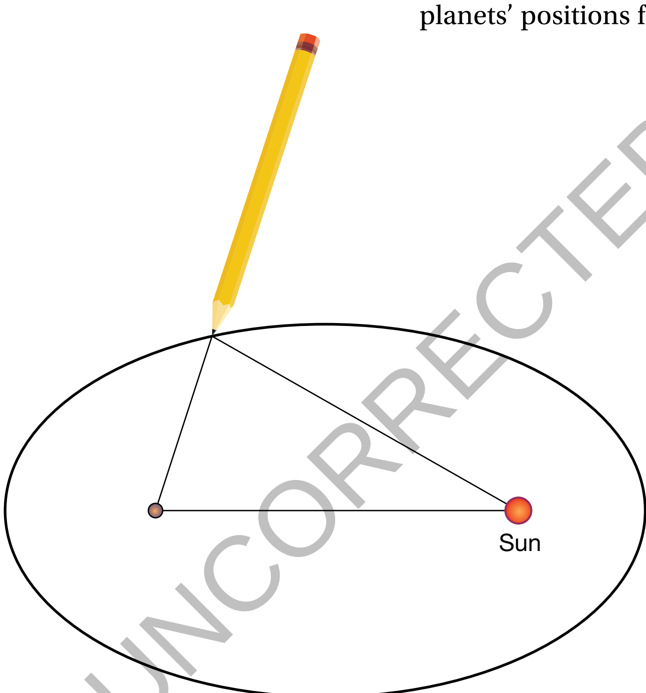
In 1542, Nicolas Copernicus (1473–1543) published 'On the Revolution of the Heavenly Orbs,' outlining an explanation for the observations of planetary motion with the Sun at the centre. In his explanation, the planets moved in circular orbits about the Sun. Copernicus's model became increasingly preferred over the geocentric model of Ptolemy because it made astronomical and astrological calculations easier. The publication had a significant scientific, social and political impact during the latter part of the 16th century.

Galileo Galilei (1564–1642) was a strong advocate for the view that the Copernican model was more than 'a set of mathematical contrivances, merely to provide a correct basis for calculation' and instead represented physical reality. (This had also been Copernicus's view, but he could not express this in print.) Galileo thought that astronomy could now ask questions about the structure, fabric and operation of the heavens, but as with so many of his scientific interests, Galileo did not pursue these questions further.

Johannes Kepler (1571–1630) decided his purpose in life was to reveal the fundamental coherence of a planetary system with the sun as its centre. In 1600–1601 he was working an assistant to Tycho Brahe (1546–1601), a Danish astronomer who had been compiling very precise measurements of the planets' positions for over twenty years. Brahe's data was so accurate that they are still valid today. Without the aid of a telescope, he was able to measure angles to an accuracy of half a minute of arc (for example $23°34' \pm 0.5'$).

Kepler was seeking to find patterns and relationships between motion of the various planets. He used the data to calculate the positions of the planets as they would be observed by someone outside the solar system, rather than from the revolving platform of the Earth. Initially he was looking for circular orbits, but Brahe's precise data did not fit such orbits. Eventually he tried other shapes, until in 1604 he formulated what is known as Kepler's First Law:

Each planet moves, not in a circle, but in an ellipse, with the sun, off centre, at a focus.



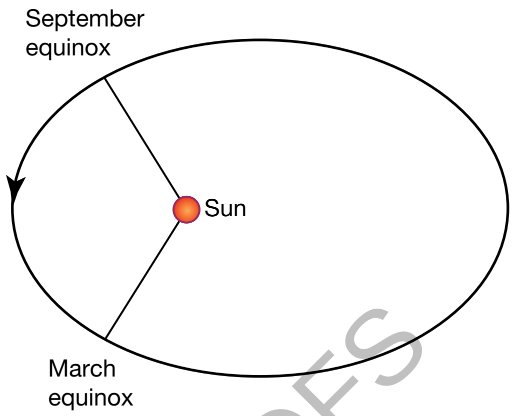
Drawing an ellipse

An ellipse is like a stretched circle. The shape can be drawn by placing two pins on the page several cm apart, with a loose piece of string tied between the pins. If a pencil is placed against the string to keep it tight and then the pencil is moved around the page, the drawn shape is an ellipse with a focus at each of the pins. The closer the two foci, the more like a circle the ellipse becomes.

Evidence of elliptical orbits

The equinoxes are the two days in the year when the sun is directly above the equator and the durations of night and day are equal. They occur when the line drawn from the Sun to the Earth is at right angles to the Earth's orbit. Because the Earth's orbit is an ellipse, when the Sun is off centre at one of the two foci, these two points are not directly opposite each other. This means the time for the Earth to go from the March equinox to the September equinox is longer by a few days than the time to go from the September equinox to the March equinox.

Look up the dates of the March and September equinoxes and determine the two times between them.



Location of the two equinoxes in the Earth's orbit

There was much speculation in Newton's time that gravitational attraction might vary inversely with the square of the distance, but it was Newton who was able to show mathematically, using a geometric proof, that the Earth's elliptical orbit means that the inverse square law applies to the attraction between the Sun and the Earth.

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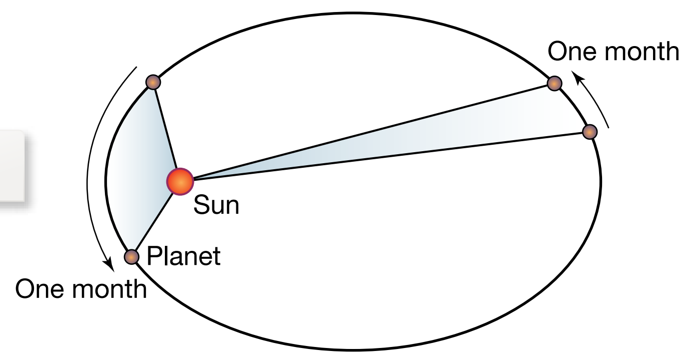
Kepler's Second Law

Kepler also looked at the speed of the planets in their orbits. His analysis of the data showed that speeds of the planets were not constant. The planets were slower when they were further away from the sun and faster when closer. He also found that their angular speed, the number of degrees a line from the sun to planet sweeps through every day, was not constant. Both results reinforced his first law. However, he did find in 1609 that the planets sweep out equal areas with time.

Kepler's Second Law: *The linear speed and angular speed of a planet are not constant, but the areal speed of each planet is constant.* That is, a line joining the sun to a planet sweeps out equal areas in equal times.



Kepler's Second Law

Newton was able to show mathematically that a constant areal velocity meant that the force acting on a planet must always act along the line joining the planet to the Sun.

Kepler's Third Law

Kepler was keen to find a mathematical relationship between the period of a planet's orbit around the sun and its average radius that gave the same result for each planet. He tried numerous possibilities and eventually in 1619 he found a relationship that fitted the data.

Kepler's Third Law: *For all planets, the cube of the average radius is proportional to the square of the orbital period; that is,* $\frac{R^3}{T^2}$ *is a constant for all planets going around the sun.*

Kepler was also able to show that the relationship held for the orbits of the moons of Jupiter.

Kepler had constructed as detailed a description of the solar system as was possible without a mechanism to explain the motion of the planets, although he did understand gravity as a reciprocal attraction. Kepler wrote, "Gravity is the mutual tendency between bodies towards unity or contact (of which the magnetic force also is), so that the Earth draws a stone much more than the stone draws the Earth . . ."

TABLE 4.1 The solar system: some useful data

Body	Mass (kg)	Radius of body (m)	Mean radius of orbit (m)	Period of revolution (s)
Sun	1.99×10^{30}	6.96×10^{8}	Not applicable	Not applicable
Earth	5.97×10^{24}	6.37×10^{6}	1.50×10^{11}	3.16×10^{7}
• Moon	7.35×10^{22}	1.74×10^{6}	3.84×10^{8}	2.36×10^{6}
Mercury	3.30×10^{23}	2.44×10^{6}	5.79×10^{10}	7.60×10^{6}
Venus	4.87×10^{24}	6.05×10^{6}	1.08×10^{11}	1.94×10^{7}
Mars	6.42×10^{23}	3.40×10^{6}	2.28×10^{11}	5.94×10^{7}
Jupiter	1.90×10^{27}	7.15×10^{7}	7.78×10^{11}	3.74×10^{8}
Saturn	5.68×10^{26}	6.03×10^{7}	1.43×10^{12}	9.29×10^{8}
Uranus	8.68×10^{25}	2.59×10^{7}	2.87×10^{12}	2.64×10^{9}
Neptune	1.02×10^{26}	2.48×10^{7}	4.50×10^{12}	5.17×10^{9}
Pluto*	1.46×10^{22}	1.18×10^{6}	5.90×10^{12}	7.82×10^{9}

*Pluto is no longer classified as a planet. Scientists have recently hypothesised that a ninth planet may exist, but it has not yet been directly observed.

Revision question 4.1

Use the data in table 4.1 to calculate the value of $\frac{R^3}{T^2}$ for each of the planets in the solar system and therefore confirm Kepler's Third Law.

Newton's Law of Universal Gravitation

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Unit 3	**Newton's Law of Universal Gravitation**
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Newton combined his deductions from Kepler's Laws with his own Laws of Motion to develop an expression for a law of universal gravitation.

From Kepler's first law, Newton had determined that the force on a planet was inversely proportional to the square of the distance.

$$F_{\text{on planet by Sun}} \propto \frac{1}{R^2}$$

Using his second law of motion, $F_{\text{net}} = ma$, Newton reasoned that the force $F_{\text{on planet by Sun}}$ depended on the mass of the planet. By using his third law of motion, $F_{\text{on planet by Sun}} = -F_{\text{on Sun by planet}}$, he reasoned that the force $F_{\text{on Sun by planet}}$ depended on the mass of the sun.

Combining these two statements produces:

Gravitational force between the sun and the planet $\propto \text{Mass}_{\text{Sun}} \times \frac{\text{Mass}_{\text{planet}}}{R^2}$.

In general,

$$F = \frac{Gm_1 m_2}{R^2}$$

where G is the universal gravitational constant and m_1 and m_2 are the masses of any two objects.

The value of G could not be determined at the time because the mass of the Earth was not known. It took another 130 years before Henry Cavendish was able to measure the gravitational attraction between two known masses and calculate the value of G.

The value of G is 6.674×10^{-11} N m^2 kg^{-2}. Alternatively, replacing newtons with kg m s^{-2}, G = 6.674×10^{-11} m^3 kg^{-1} s^{-2}.

The value of G is very small, which indicates that gravitation is quite a weak force. A large quantity of mass is need to produce a gravitational effect that is easily noticeable.

Sample problem 4.1

Calculate the force due to gravity of:

(a) Earth on a 70 kg person standing on the equator

(b) a 70 kg person standing on the equator on Earth.

Solution: (a) $m_{Earth} = 5.98 \times 10^{24}$ kg, $m_{person} = 70$ kg, radius$_{Earth} = 6.38 \times 10^6$ m,

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

$$F = \frac{G m_{Earth} m_{person}}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \times 5.98 \times 10^{24} \text{ kg} \times 70 \text{ kg}}{(6.38 \times 10^6 \text{ m})^2}$$

= 686 N towards the centre of Earth

(b) Newton's Third Law of Motion states that if one object exerts a force on another object, then the other object exerts an equal and opposite force on the first object. In this situation, if Earth is exerting a force of 686 N downwards on the person, then the person is exerting a 686 N force upwards on Earth! The same result could be calculated with the formula used in part (a).

Revision question 4.2

Use the data in table 4.1 to calculate the force due to gravity by:

(a) the Earth on the Moon

(b) the Moon on Earth.

The falling apple

Newton published his law of universal gravitation in 1687, in his famous book titled *Philosophiae Naturalis Principia Mathematica*. In this book he included an anecdote about observing an apple falling from a tree. There is no record of such an event in his earlier papers, so the story may just have served an explanatory purpose. Nevertheless, the story is instructive.

Newton said he observed that the falling apple had been pulled from the tree by the attractive force of the Earth, which had acted across the space between the Earth's surface and the apple, that is, 'action at a distance'. He speculated that the effect of the Earth's pull might reach higher into the atmosphere, possibly beyond the atmosphere to the moon. He had previously applied his Laws of Motion to circular motion and developed expressions for the inward acceleration, $a = \frac{v^2}{R}$ and $a = \frac{4\pi^2 R}{T^2}$.

The obvious questions that arise from this are:

1. How does the acceleration of the apple compare to that of the moon?
2. How are these two values related to their respective distances from the centre of the Earth?

TABLE 4.2 The relationship between the Earth, an apple and the Moon

Body	Apple	Moon
Distance to the centre of the Earth (m)	6.38×10^6	3.84×10^8
Period of orbit (s)		2.36×10^6
Acceleration towards the centre of the Earth ($m\,s^{-2}$)	$9.8\ m\,s^{-2}$	2.72×10^{-3}

With the data from the table, we can make the following calculations:

$$\frac{\text{distance of the Moon from the centre of the Earth}}{\text{distance of the apple from the centre of the Earth}} = \frac{3.84 \times 10^8}{6.38 \times 10^6}$$
$$= 60.1$$

$$\frac{\text{acceleration of the apple towards the centre of the Earth}}{\text{acceleration of the Moon towards the centre of the Earth}} = \frac{9.8}{2.72 \times 10^{-3}}$$
$$= 3603$$

The value of the second ratio, 3603, is very close to 60.1^2.

The ratio of the accelerations is the square of the ratio of the distances, but note that the Moon is in the numerator for the first ratio, while it is in the denominator for the second ratio. Newton used this calculation to show that the gravitational force is inversely proportional to the square of the separation of the two masses.

Newton's expression for the centripetal acceleration, $a = \frac{4\pi^2 R}{T^2}$, was used to confirm Kepler's Third Law, that $\frac{R^3}{T^2}$ is a constant for all planets or satellites orbiting a central body.

$$F_{net} = F_g$$

$$m_{planet} \times \frac{4\pi^2 R}{T^2} = \frac{GM_{Sun} m_{planet}}{R^2}$$

Cancelling m_{planet} and rearranging gives

$$\frac{R^3}{T^2} = \frac{GM_{Sun}}{4\pi^2}$$

which depends only on the mass of the Sun and thus has the same value for all planets orbiting the Sun.

Similarly, the Moon and all other satellites orbiting the Earth will have the same value for $\frac{R^3}{T^2}$, though in this case the value will equal $\frac{GM_{Earth}}{4\pi^2}$.

Sample problem 4.2

Calculate the value of $\frac{R^3}{T^2}$ for the Moon using the data in table 4.1 and use that value to calculate the mass of the Earth.

Solution: Radius of Moon's orbit, $R = 3.84 \times 10^8$ m; period, $T = 2.36 \times 10^6$ s;
$G = 6.67 \times 10^{-11}\ Nm^2\,kg^{-2}$; mass of Earth, $M_{Earth} = ?$

$$\frac{R^3}{T^2} = \frac{(3.84 \times 10^8\ m)^3}{(2.36 \times 10^6\ s)^2}$$
$$= 1.02 \times 10^{13}\ m^3\ s^{-2}$$

Using $\frac{R^3}{T^2} = \frac{GM_{Earth}}{4\pi^2}$,

$$M_{Earth} = \frac{R^3}{T^2} \times \frac{4\pi^2}{G}$$
$$= \frac{(3.84 \times 10^8 \text{ m})^3 \times 4\pi^2}{(2.36 \times 10^6 \text{ s})^2 \times 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}}$$
$$= 6.02 \times 10^{24} \text{ kg}.$$

Revision question 4.3

Find the average of the values of $\frac{R^3}{T^2}$ that you calculated in revision question 4.1. Use that average value to calculate the mass of the Sun.

Newton's Law of Universal Gravitation can also be used to calculate the average speed of planets around the Sun. This involves using the other expression for the centripetal acceleration.

$$F_{net} = F_g$$
$$m_{planet} \times \frac{v^2}{R} = \frac{GM_{Sun}m_{planet}}{R^2}$$

Cancelling m_{planet} and rearranging gives

$$v^2 = \frac{GM_{Sun}}{R}$$
$$v = \sqrt{\frac{GM_{Sun}}{R}}.$$

Sample problem 4.3

Calculate the average speed of the Earth around the Sun using the values in table 4.1.

Solution: Radius of Earth's orbit, $R = 1.50 \times 10^{11}$ m; mass of Sun, $M_{Sun} = 1.98 \times 10^{30}$ kg; $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}; speed, $v = ?$

Using $v = \sqrt{\frac{GM_{Sun}}{R}}$,

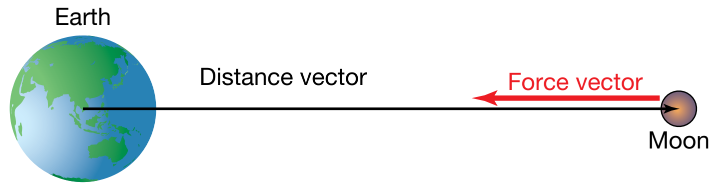
$$v = \sqrt{\frac{6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \times 1.98 \times 10^{30} \text{ kg}}{1.50 \times 10^{11} \text{ m}}}$$
$$= 2.97 \times 10^4 \text{ m s}^{-1}.$$

Revision question 4.4

Use the values in table 4.1 to calculate the average speed of the other planets around the Sun. Graph the average speed as a function of average orbital radius. Does the graph fit your expectation? Are there any outliers in the data? If so, suggest an explanation.

Graphing the gravitational force

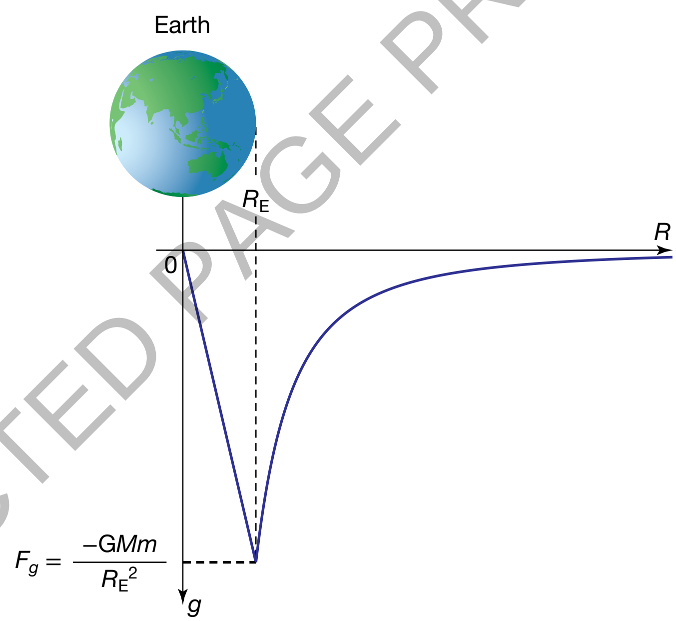
The gravitational force is an attractive force, whereas the force between electric charges can be either attractive or repulsive.



The Earth exerts gravitational force on the Moon.

For the force the Earth exerts on the Moon, there is a distance vector from the centre of the Earth to the centre of the Moon, whereas the force vector points in the opposite direction, back to the Earth. For this reason, the gravitational force equation should really have a negative sign and the force should be graphed under the distance axis. Thus, more correctly,

$$F_g = -\frac{Gm_1 m_2}{R^2}.$$



This diagram shows how the Earth's gravitational force varies with distance.

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The straight blue line in the graph shows how the gravitational force by the Earth on you would decrease if you were to drill down to the centre of the earth. Newton calculated that if you were inside a hollow sphere, the gravitational force from the mass in the shell would cancel out, no matter where you were inside the sphere. This means that if you were inside the Earth, only the mass in the inner sphere between you and the centre of the Earth would exert a gravitational force on you. This force will get smaller the closer to the centre you go, and at the centre of the Earth the gravitational force will be zero.

Gravitational fields

Newton's Law of Universal Gravitation describes the force between two masses. However, the solar system has many masses, each attracting each other. The sun, the heaviest object in the solar system, determines the orbits of all the other masses, but each planet can cause minor variations in the orbital paths of the other planets. Precise calculation of the path of a planet or comet becomes a complicated exercise with many gravitational forces needing to be considered.

Physicists after Newton realised it was easier to determine for each point in space the total force that would be experienced by a unit mass, that is, 1 kilogram, at that point. This idea slowly developed and in 1849 Michael Faraday, in explaining the interactions between electric charges and between magnets, formalised the concept, calling it a 'field'.

A field is more precisely defined as a physical quantity that has a value at each point in space. For example, a weather map showing the pressure across Australia could be described as a diagram of a pressure field. This is an example of a scalar field. In contrast, gravitational, electric and magnetic fields are vector fields; they give a value to the strength of the field at each point in space, and also a direction for that field at that point. For example, the arrows in the diagram of the Earth's gravitational field show the direction of the field, and the density of the lines (how close together the lines are) indicates the strength of the field.

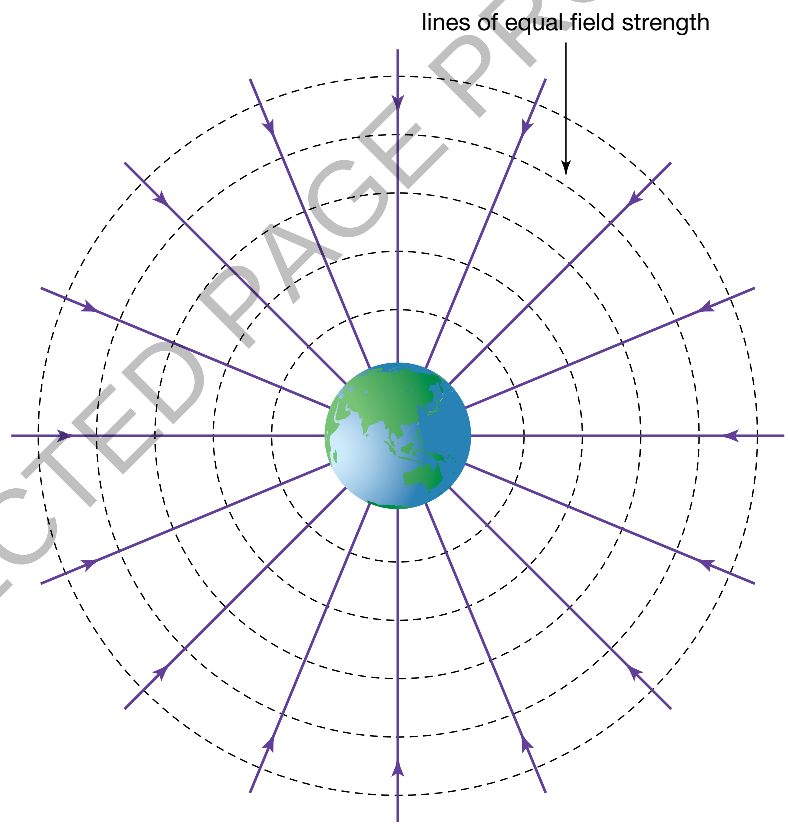

Diagram of the Earth's gravitational field

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Gravitational field strength
Concept summary and practice questions

A value for the strength of the gravitational field around a mass M can be determined from the value of the force on a unit mass in the field. If the mass m_2 in Newton's Universal Law of Gravitation equation is assigned a value of 1 kg, then the force expression will give the strength of the gravitational field.

$$\text{Gravitational field strength, } g = -\frac{GM}{R^2}$$

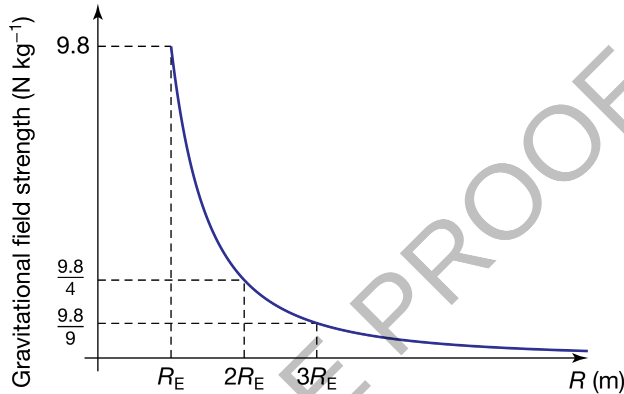
The unit of gravitational field strength is Newtons per kilogram, N kg^{-1}.

The strength of the gravitational field at the Earth's surface can be calculated using the values for the mass and radius of the Earth from table 4.1:

$$\text{Gravitational field strength, } g = -\frac{6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2} \times 5.98 \times 10^{24}}{(6.38 \times 10^6)^2}$$

$$= -9.80\ \text{N kg}^{-1}.$$

This is the acceleration due to gravity at the Earth's surface.



Graph of the magnitude of the strength of the Earth's gravitational field

Revision question 4.5

(a) Use the data in table 4.1 to calculate the gravitational field strength on the surface of the Moon. Show that it is about $\frac{1}{6}$ of the Earth's gravitational field at its surface.

(b) Determine which planet has the largest gravitational field strength at its surface. Table 4.1 is also available as a spreadsheet in your eBookPLUS.

At the time Newton developed his Law of Universal Gravitation, he knew it did not provide an explanation for how gravity works, that is, how 'action at a distance' was achieved.

> *It is inconceivable . . . that Gravity should be innate, inherent and essential to Matter, so that one body may act upon another at a distance thro' a Vacuum, without the Mediation of any thing else . . . is to me so great an Absurdity that I believe no Man who has in philosophical Matters a competent Faculty of thinking can ever fall into it. Gravity must be caused by an Agent acting constantly according to certain laws; but whether this Agent be material or immaterial, I have left to the Consideration of my readers.* Newton, 1692

The concept of a field now provides an explanation for action at a distance.

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Unit 3 | AOS 1 | Topic 1 | Concept 5

Gravitational potential energy
Concept summary and practice questions

Kinetic energy and potential energy in a gravitational field

Consider the following scenarios.

1. On 15 February 2013, an asteroid approached the Earth, gaining speed in the Earth's gravitational field. By the time it reached the atmosphere, it was travelling at a speed of 19 km s^{-1}. With a mass of about 1.2×10^7 kg, its kinetic energy was about 2.2×10^{15} J. It exploded about 30 km above Chelyabinsk in Russia.

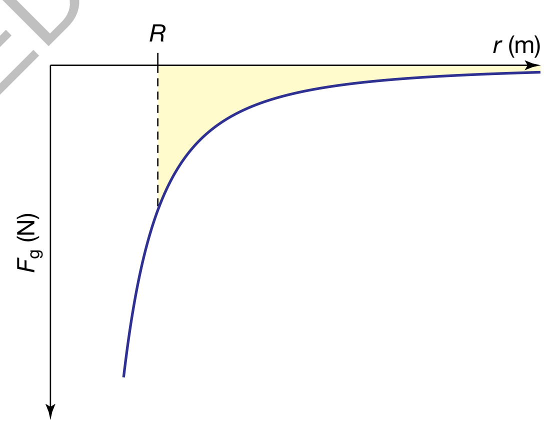
2. Edmund Halley used Newton's law of gravitation to calculate the effect of Jupiter and Saturn on the orbits of comets. He concluded that comet sightings in 1531 and 1607 were sightings of the same comet and that it should appear again in 1758. We now call it Halley's Comet.

   Halley's Comet orbits the sun every 75.3 years in a very stretched path. The closest it gets to the Sun is about 0.6 times the radius of the Earth's orbit, while its furthest distance is 35 times the radius. At its closest approach its kinetic energy is 1.6×10^{23} J with a speed of 38 km s^{-1}, but at its furthest it has only 4.5×10^{19} J, travelling at 0.64 km s^{-1}.
3. Ignoring the initial air resistance, a rock thrown at 11 km s^{-1} would eventually escape the effect of the Earth's gravitational pull, slowing down to zero only at an infinite distance away.

In each of these scenarios there are changes in speed and height, and thus in kinetic energy and gravitational potential energy. How do we describe these changes using Newton's law of gravitation?

The change in gravitational potential energy can be obtained from the area under a force–distance graph. Because gravitation is an attractive force, the force–distance graph is below the distance axis and the area under the graph has a negative value.

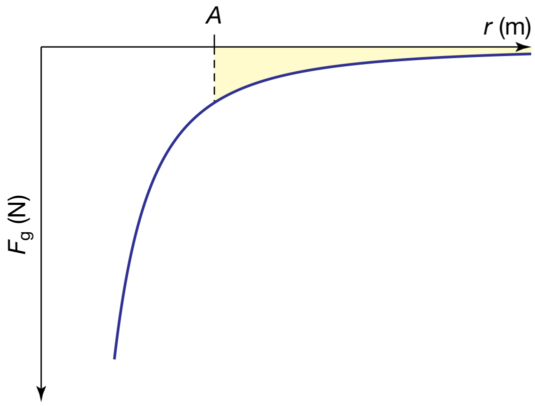
Change in potential energy needs a reference point or zero point. The Earth's surface is an obvious reference point for objects on or near the Earth; we can assume a constant value of 9.8 N kg^{-1} for the strength of the gravitational field at the Earth's surface. But out in space, with the gravitational force getting weaker with distance, the preferred reference point is at infinity where the gravitational force is zero. This means that the gravitational potential energy of a mass at a distance R from the Earth is the area under the graph from the distance R out to infinity.

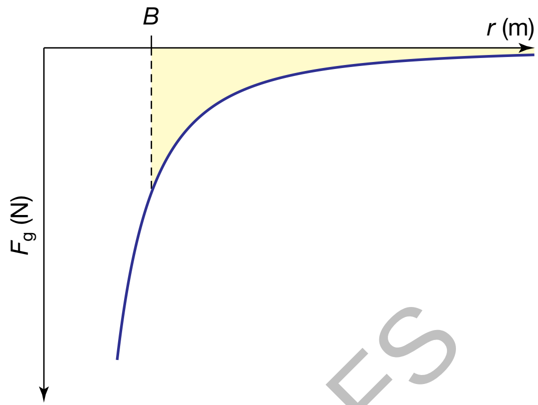


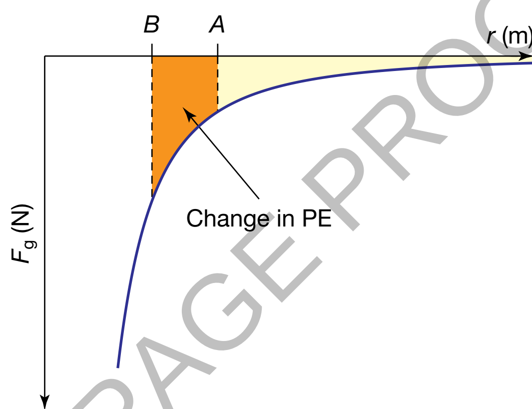
Graph of the strength of the Earth's gravitational force. The gravitational potential energy of a mass at distance R from the Earth is equal to the shaded area.

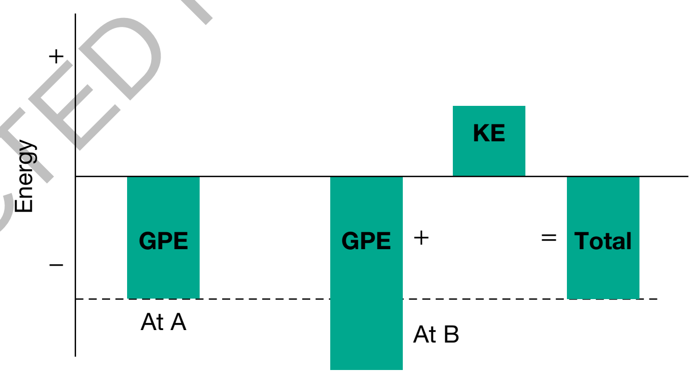
Let's look at situation 1, the Chelyabinsk asteroid. When the asteroid is some distance from Earth, its kinetic energy is relatively small. As it falls towards the Earth, its kinetic energy increases, its gravitational potential energy becomes more negative, and the total energy remains the same throughout. As the asteroid falls from A to B in the figures on the following page, the orange shaded area in the third graph is the gain in kinetic energy. That is,

change in GPE = change in KE.









The total energy at B equals the total energy at A.

Sample problem 4.4

A mass of 10 kg falls to the surface of the Earth from an altitude equal to two Earth radii. What is the gain in kinetic energy?

Solution: There are three methods, two of which give an approximate answer. The accuracy of each depends on the care you take.

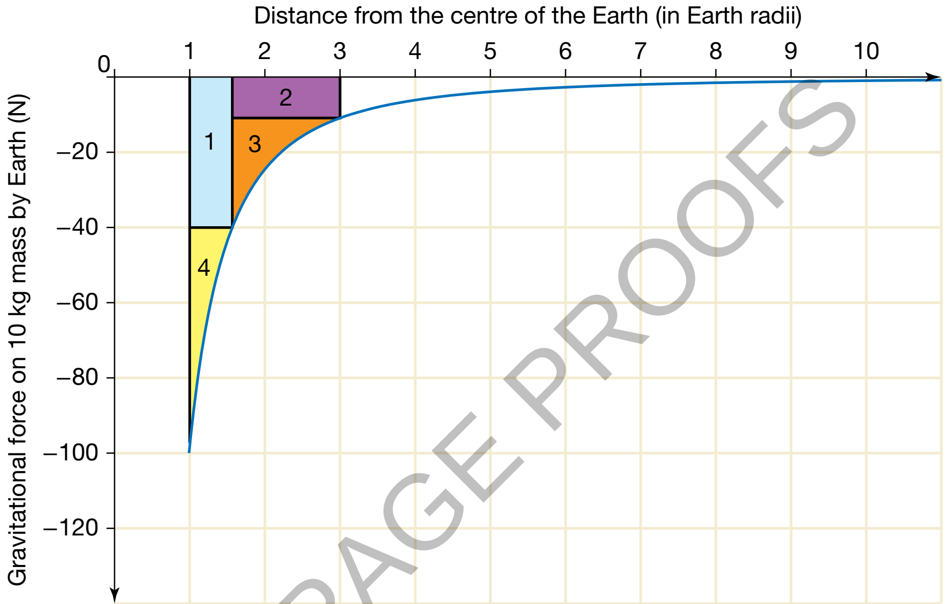
Method 1

Use this method if the graph has a relatively coarse grid.

- Divide the area up into simple geometric shapes such as rectangles and triangles.
- Calculate the area of each shape using graph-based units.

- Total the areas.
- Convert the total area to SI units for energy.

Take care in deciding the height of the rectangles or triangles so that their areas (approximating the area under the curve) will produce more representative results.



$$\begin{aligned}\text{Area 1 (blue)} &= 40 \times 0.5\\ &= 20 \text{ energy units}\\ \text{Area 2 (purple)} &= 10 \times 1.5\\ &= 15 \text{ energy units}\\ \text{Area 3 (orange)} &= \frac{1}{2} \times 24 \times 1.5\\ &= 18 \text{ energy units}\end{aligned}$$

(*Note:* The triangle with area $\frac{1}{2} \times 30 \times 1.5$ would be larger than the orange area, so the height of 30 was reduced to a level where the areas matched.)

$$\begin{aligned}\text{Area 4 (yellow)} &= \frac{1}{2} \times 53 \times 0.5\\ &= 13.25 \text{ energy units}\end{aligned}$$

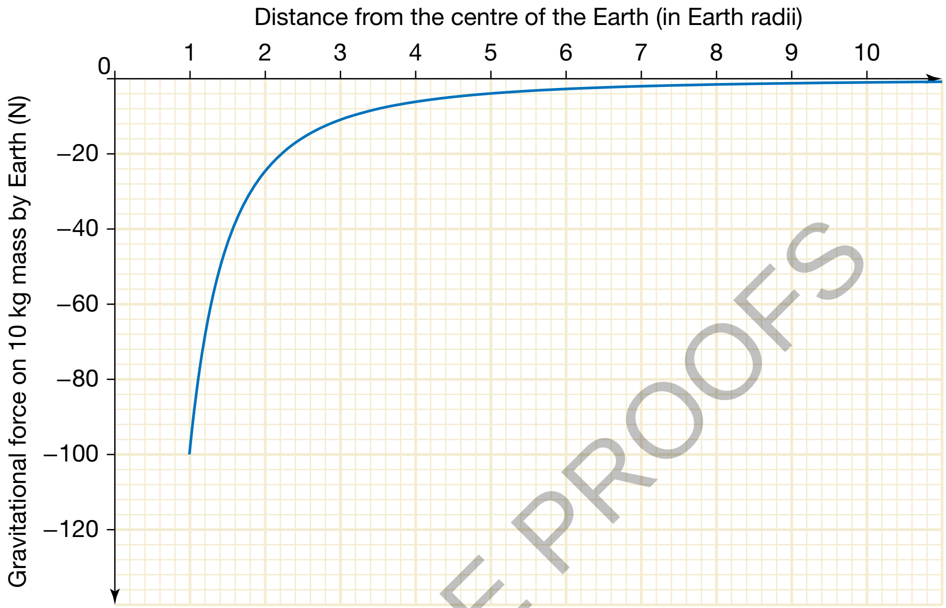
$$\begin{aligned}\text{Total area} &= 20 + 15 + 18 + 13.25\\ &= 66.25 \text{ energy units}\end{aligned}$$

$$\begin{aligned}1 \text{ energy unit} &= 1 \text{ N} \times 1 \text{ Earth radius}\\ &= 1 \text{ N} \times 6.38 \times 10^6 \text{ m}\\ &= 6.38 \times 10^6 \text{ J}\end{aligned}$$

$$\begin{aligned}\text{Therefore, the kinetic energy gained} &= 66.25 \times 6.38 \times 10^6\\ &= 4.23 \times 10^8 \text{ J.}\end{aligned}$$

Method 2

Use this method when the graph has a relatively fine grid.



- Count the number of small squares between the graph and the zero-value line or horizontal axis. Tick each one as you count it to avoid counting it twice. For partial squares, find two that add together to make one square and tick both.
- Calculate the area of one small square.
- Multiply the area of one small square by the number of small squares.

Number of small squares = 80.5

$$\begin{aligned}\text{Area of one small square} &= 4\,\text{N} \times 0.2 \times 1\ \text{Earth radius}\\ &= 4\,\text{N} \times 0.2 \times 6.38 \times 10^6\,\text{m}\\ &= 5.1 \times 10^6\ \text{J}\end{aligned}$$

Therefore, the gain in kinetic energy = $80.5 \times 5.1 \times 10^6$ J = 4.11×10^8 J.

Method 2 can be very accurate, but it is laborious.

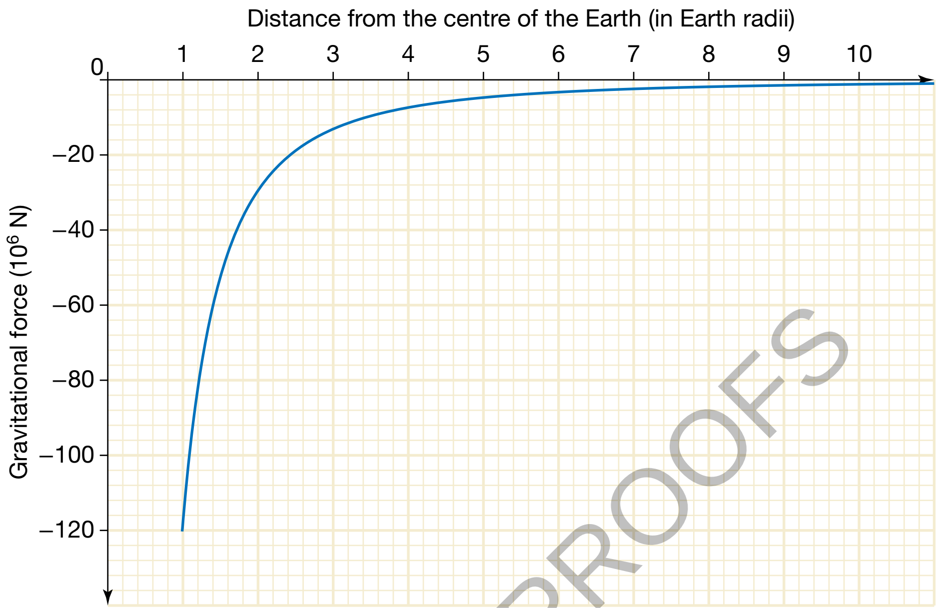
Method 3

- Print out the graph.
- Cut out the required shape.
- Measure the mass of the shape with a top-loading balance.
- Using the mass of a piece of the same paper with known dimensions, calculate the area of the cut-out shape.
- Use the scales on the axes of the graph to determine the value for the area under the graph.

Revision question 4.6

(a) Use the graph of the gravitational force on the Chelyabinsk asteroid (shown on the next page) to show that in moving from an altitude of two Earth radii down to an altitude of one Earth radius, it gained 1.25×10^{14} joules of kinetic energy.

(b) Use the graph to find, to the nearest whole number, approximately how much kinetic energy was gained from falling from an altitude of one Earth radius to the Earth's surface. Compare this value with your answer to (a) above.

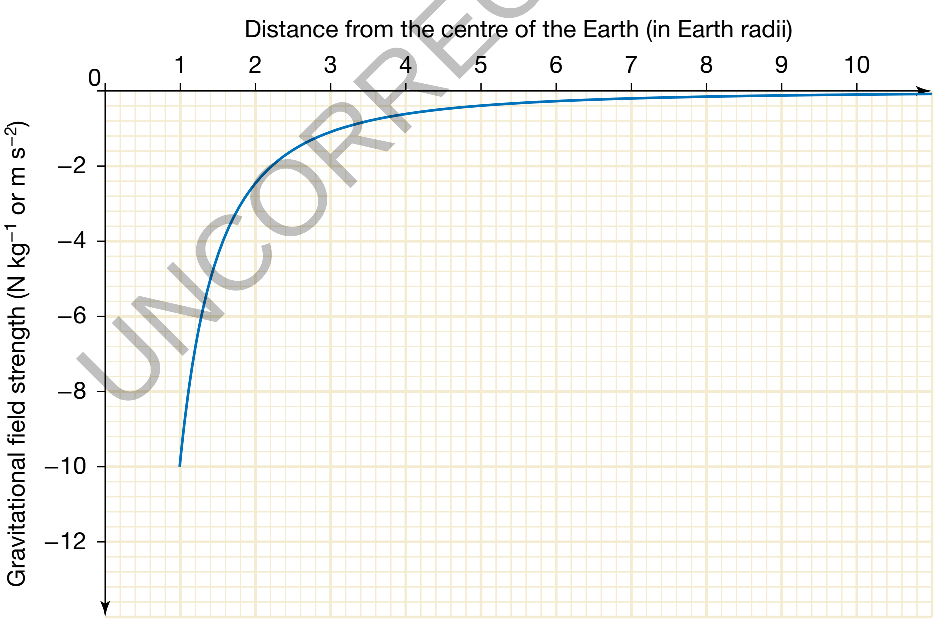


Gravitational force exerted on the Chelyabinsk asteroid by the Earth

Using the area under a field graph

The graphs for the 10 kg mass and the Chelyabinsk asteroid have different values on the force axis. To find the changes in energy of a rock escaping the Earth, a different graph would be needed, because the mass of the rock is different, and thus the gravitational force on it is different. It would be simpler if we could use the same graph for different objects regardless of their mass.

The graph that can be used for this purpose is a graph of the gravitational field against distance. The gravitational force on a mass at a point in space is just the value of the gravitational field at the point times its mass. Similarly, the change in energy for an object that moves from one point to another can obtained by multiplying the area under the graph of the gravitational field against distance by its mass.



Graph of the strength of the Earth's gravitational field

The unit for gravitational field is Newtons per kilogram. The unit for the area under a graph of gravitational field against distance is (Newtons per kilogram) × metre, hence Newton metre per kilogram or simply Joule per kilogram. The change in energy can be obtained from this area by multiplying by the mass of the object.

This method was also used in Chapter 2 on page 57 with the gravitational field close to the Earth's surface where the field strength is usually constant.

Revision question 4.7

If a rock of mass 1 kg was thrown upwards from the Earth's surface with sufficient kinetic energy to escape the Earth's gravitational field, the amount of kinetic energy required would be the area under the graph out to infinity. Using a distance of 10 Earth radii as an approximation for infinity, show that the required initial speed is about 11 km s^{-1}.

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Unit 3 | AOS 1 | Topic 1 | Concept 3

Satellites and gravitational force
Concept summary and practice questions

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Unit 3 | AOS 1 | Topic 1 | Concept 4

Satellite equations of motion
Concept summary and practice questions

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Unit 3 | AOS 1 | Topic 1 | Concept 4

Do more
How the speed and period changes with the radius of a satellite's orbit

Astronauts and satellites in orbit

As we saw earlier, Newton used his law of Universal Gravitation to show that Kepler's Third Law, $\frac{R^3}{T^2} = \frac{GM}{4\pi^2}$, applies to all satellites going around the same central mass. In the context of the Earth, this means that $\frac{R^3}{T^2}$ is the same for every single artificial satellite, regardless of the orientation of its orbit, as well as the Moon itself. Because we know the period and the radius of the Moon's orbit, we can use the method of ratios to calculate the characteristics of any other satellite:

$$\frac{{R_M}^3}{{T_M}^2} = \frac{{R_{sat}}^3}{{T_{sat}}^2}$$

$$\text{or } \left(\frac{R_M}{R_{sat}}\right)^3 = \left(\frac{T_M}{T_{sat}}\right)^2.$$

The benefit of this method is that because you are working with ratios, you don't need to use metres and seconds for your data. Earth radii and days can be used, making for simpler calculations.

The orbit of the Moon is slightly elliptical, but the average radius of the Moon's orbit is about 384 000 km or about 60 Earth radii.

The period of the Moon in relation to the stars is called the sidereal period and has been measured at 27.321 582 days (or approximately 2.36×10^6 seconds). For our purposes we can use 27.3 days. The period of the Moon in relation to the Sun, that is the time between full moons, is 29.5 days; this is longer than the sidereal period because in that time the Earth has moved further around the Sun.

Geostationary satellites

Artificial satellites are used for communication and exploration. Some transmit telephone and television signals around the world, some photograph cloud patterns to help weather forecasters, some are fitted with scientific equipment that enables them to collect data about X-ray sources in outer space, whereas others spy on our neighbours! The motion of an artificial satellite depends on what it is designed to do. Those satellites that are required to rotate so that they stay constantly above one place on Earth's surface are called **geostationary** satellites and they are said to be in geostationary orbit. In order to stay in position, a geostationary satellite must have the same period as the place it is above. Therefore, geostationary satellites have a period of 24 hours or 1 day.

A satellite in **geostationary** orbit is stationary relative to a point directly below it on Earth's surface. A geostationary orbit has the same period as the rotation of Earth.

Sample problem 4.5

What is the radius of the orbit of a geostationary satellite as a multiple of the Earth's radius and also in metres?

Solution: $R_M = 60 \times R_E$, $T_M = 27.3$ days, $T_{sat} = 1$ day, $R_{sat} = ?$

Rearranging $\left(\frac{R_M}{R_{sat}}\right)^3 = \left(\frac{T_M}{T_{sat}}\right)^2$ to find R_{sat}:

$$R_{sat}{}^3 = \left(\frac{T_{sat}}{T_M}\right)^2 \times R_M{}^3$$

$$= \left(\frac{1}{27.3}\right)^2 \times (60\,R_E)^3$$

$$= 289.8\,R_E{}^3$$

$$R_{sat} = 6.62\,R_E$$

In metres,

$$R_{sat} = 6.62 \times 6.38 \times 10^6 \text{ m}$$

$$= 4.22 \times 10^7 \text{ m}$$

Revision question 4.8

(a) Use a small coin to draw a circle in the middle of a blank page to represent the equator of the Earth.

(b) Using the answer from sample problem 4.5, put a dot on the page where you estimate a geostationary satellite would be.

(c) Draw two lines from the dot to the circle representing the Earth, to touch the circle at a tangent. The part of the circle facing the satellite between these two lines represents how much of the Earth's surface could receive signals from the satellite.

(d) Use your diagram to determine the minimum number of geostationary satellites required to cover all of the Earth's equator.

(e) Which parts of the Earth could not receive signals from any of these geostationary satellites?

Revision question 4.9

Global Positioning System (GPS) satellites are used for navigation. The Navstar 66 satellite, launched in 2011, has an orbital radius of about 20 100 km. What is the period of its orbit expressed in days?

AS A MATTER OF FACT

Why are geostationary satellites always above the equator? Why isn't there a geostationary satellite directly above central Australia?

Newton showed that the motion of a large object can be analysed as if all of its mass was concentrated at a single point, called its centre of mass. For a symmetrical object such as the Earth, the centre of mass is located at its geometric centre. Thus, all satellites around Earth are in orbit about the centre of the Earth.

If a satellite was to be directly above central Australia for 24 hours each day, the centre of its orbit would be at the centre of the matching circle of latitude, some distance away from the centre of the Earth.

If instead a satellite was to be only momentarily directly above central Australia, given the centre of its orbit is the centre of the Earth, the orbit would take it into the sky above the northern hemisphere for half the time.

This is why geostationary communication satellites orbit around the equator. A satellite dish has to be angled at the latitude of that point on the Earth to point towards one of these satellites.

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Unit 3 | AOS 1 | Topic 1 | Concept 6

Normal reaction forces in uniform orbits
Concept summary and practice questions

'Floating' in a spacecraft

An astronaut inside the International Space Station

The appearance of an astronaut floating around inside a spacecraft suggests that there is no force acting on them, leading some people to mistakenly think that there is no gravity in space. In fact, both the astronaut and the spacecraft are in a circular orbit about the Earth.

However, you also know that if an object is moving in a curved path, changing its direction, there must be an acceleration. If the path is circular the acceleration is directed towards the centre of that path.

The astronaut and the space craft are in the same gravitational field. They are at the same distance from the centre of the Earth. They are travelling at the same speed, taking the same time to orbit the earth. Therefore, their centripetal accelerations provided by the gravitational field are the same.

For the spacecraft:

$$\frac{GM_E m_s}{R^2} = m_s \times \frac{4\pi^2 R}{T^2}$$

$$\frac{GM_E}{R^2} = \frac{4\pi^2 R}{T^2}$$

For the astronaut:

$$\frac{GM_E m_a}{R^2} = m_a \times \frac{4\pi^2 R}{T^2}$$

$$\frac{GM_E}{R^2} = \frac{4\pi^2 R}{T^2}$$

There is no need for a normal reaction force by the spacecraft on the astronaut to explain the astronaut's motion. The astronaut inside the spacecraft circles the Earth as if the spacecraft was not there. Indeed, if the astronaut is outside the spacecraft doing a space walk, the astronaut's speed and acceleration around the Earth will be unchanged as they 'float' beside the spacecraft. Once back inside, their speed and acceleration are still unchanged, and this time they are 'floating' inside the spacecraft.

If the astronaut steps onto a set of bathroom scales, they will give a reading of zero. As shown in the second photo opposite, an astronaut running on a treadmill needs stretched springs attached to his waist to pull him down to the treadmill.

This astronaut is floating inside the International Space Station. Both the astronaut and the station are in orbit around the Earth.

The cloth-covered stretched springs are pulling the astronaut down so he can exercise on the treadmill.

Chapter review

studyon
Unit 3 | AOS 1 | Topic 1
Gravitational fields and forces
Sit Topic test

Summary

- The gravitational field strength g at a distance r from a body of mass M is given by the formula $g = \frac{GM}{R^2}$ where G is the gravitational constant. The force of gravity F on an object of mass m at a distance r from the same body is therefore given by $F = \frac{GMm}{R^2}$. This equation is referred to as Newton's Law of Universal Gravitation.
- For a given planetary or satellite system, $\frac{R^3}{T^2}$ is constant. The value of the constant is equal to $\frac{GM}{4\pi^2}$ where M is the mass of the central body.
- Gravitational attraction can be explained using a field model.
- The field model enables the gravitational field around a point mass to be described in terms of its direction and shape, and also its strength.
- The field model also enables descriptions of the changes in potential energy of an object moving in the gravitational field of a point mass.
- Changes in potential energy can be calculated from the area under a force–distance graph and from the area under a field–distance graph when multiplied by mass.
- Satellites in orbit and their occupants (who are also in orbit) experience no reaction force.

Questions

In answering the questions on the following pages, assume, where relevant, that the magnitude of the gravitational field at Earth's surface is 9.8 N kg^{-1}. Additional data required for questions relating to bodies in the solar system can be found in table 4.1 on page 110.

Modelling the motion of satellites

1. A gravitational field strength detector is released into the atmosphere and reports back a reading of 9.70 N kg^{-1}.
  (a) If the detector has a mass of 10 kg, what is the force of gravity acting on it?
  (b) If the detector is to remain stationary at this height, what upwards force must be exerted on the detector?
  (c) How far is the detector from the centre of Earth?

2. Use the information provided in table 4.1 on page 110 to calculate (i) the gravitational field strength and (ii) the weight of a 70 kg person at the surface of the following bodies of the solar system:
  (a) Earth
  (b) Mars
  (c) Venus
  (d) Pluto.

3. A space probe orbits a distance of 5.0×10^5 m from the centre of an undiscovered planet. It experiences a gravitational field strength of 4.3 N kg^{-1}. What is the mass of the planet?

4. Calculate the force of attraction between Earth and the Sun.

5. If the Earth expanded to twice its radius without any change in its mass, what would happen to your weight?

6. By how much would your reading on bathroom scales change with the Moon on the opposite side of the Earth to you, compared with being above you?

7. Determine the value of the ratio $\frac{F_{\text{on Moon by Sun}}}{F_{\text{on Moon by Earth}}}$. Assume the Moon is the same distance from the Sun as the Earth is.

8. How many Earth radii from the centre of the Earth must an object be for the gravitational force by the Earth on the object to equal the gravitational force that would be exerted by the Moon on the object if the object was on the Moon's surface?

9. A space station orbits at a height of 355 km above Earth and completes one orbit every 92 min.
  (a) What is the centripetal acceleration of the space station?
  (b) What gravitational field strength does the space station experience?
  (c) Your answers to (a) and (b) above should be the same. (i) Explain why. (ii) Explain any discrepancy in your answers.
  (d) If the mass of the space station is 1200 tonnes, what is its weight?
  (e) The mass of an astronaut and the special spacesuit he wears when outside the space station is 270 kg. If he is a distance of 10 m from the centre of mass of the space station, what is the force of attraction between the astronaut and the space station?

10. What is the centripetal acceleration of a person standing on Earth's equator due to Earth's rotation about its axis? (Radius of Earth is 6.38×10^6 m.) Would the centripetal acceleration be greater or less for a person standing in Victoria? Justify your answer.

11. In the future, it is predicted that space stations may rotate to simulate the gravitational field of Earth and therefore make life more normal for the occupants. Draw a diagram of such a space station. Include on your diagram:

- the axis of rotation
- the distance of the occupants from the axis
- arrows indicating which direction the occupants would consider as 'down'.

(Remember to consider the frame of reference of the occupants!) Make an estimate of the period of rotation your space station would need to simulate Earth's gravitational field.

12. Neutron stars are thought to rotate at about 1 revolution every second. What is the minimum mass for the neutron star so that a mass on the star's surface is in the same situation as a satellite in orbit, that is, the strength of the gravitational field equals the centripetal acceleration at the surface?
13. The Sun orbits the centre of our galaxy, the Milky Way, at a distance of 2.2×10^{20} m from the centre with a period of 2.5×10^{8} years. The mass of all the stars inside the Sun's orbit can be considered as being concentrated at the centre of the galaxy. The mass of the Sun is 2.0×10^{30} kg. If all the stars have the same mass as the Sun, how many stars are in the Milky Way?
14. The asteroid 243 Ida was discovered in 1884. The Galileo spacecraft, on its way to Jupiter, visited the asteroid in 1993. Search online for images of the flyby. The asteroid was the first to be found to have a natural satellite, that is, its own moon, now called Dactyl. Dactyl orbits Ida at a radius of 100 km and with a period of 27 hours. What is the mass of the asteroid?

Motion of the planets

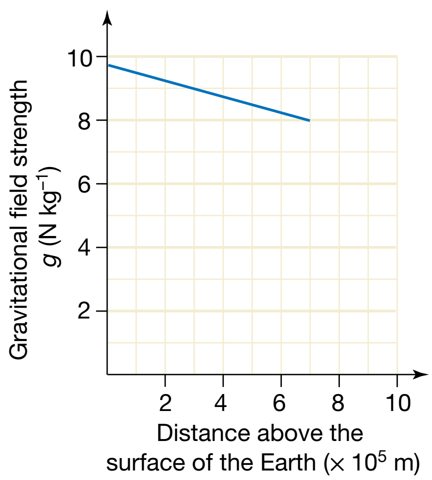
15. What force holds the solar system together? Explain how this results in the planets moving in roughly circular orbits.
16. Venus and Saturn both orbit the Sun. Using only information about the Sun and the periods of the two planets, calculate the value of the ratio:

$$\frac{\text{distance of Saturn from the Sun}}{\text{distance of Venus from the Sun}}.$$

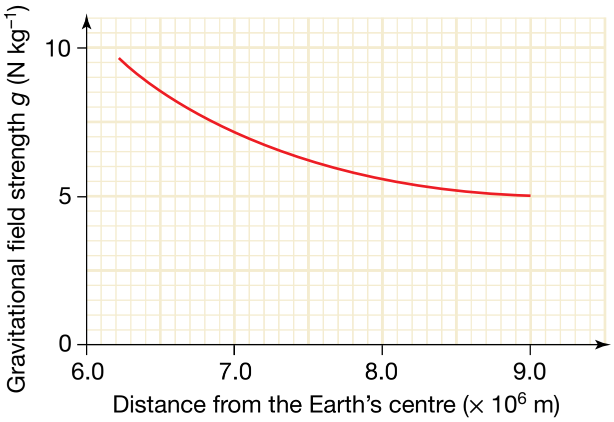
17. A spacecraft leaves Earth to travel to the Moon. How far from the centre of the Earth is the spacecraft when it experiences a net force of zero?
    Use the data in table 4.1 to determine where that point is, and draw a scale diagram to show its location.
18. A satellite is in a circular orbit around the Earth with a radius equal to half of the radius of the Moon's orbit. What is the satellite's period expressed as a fraction of the Moon's period about the Earth?

Satellites of the Earth

19. A geostationary satellite remains above the same position on Earth's surface. Once in orbit, the only force acting on the satellite is that of gravity towards the centre of Earth. Why doesn't the satellite fall straight back down to Earth?
20. A new geostationary satellite is to be launched. At what height above the centre of Earth must the satellite orbit?
21. Can a geostationary satellite remain above Melbourne? Why or why not?
22. Explain why the area under a gravitational force–distance graph gives the energy needed to launch a satellite, but the area under a gravitational field strength–distance graph gives the energy *per kilogram* needed to launch a satellite.
23. A space shuttle, orbiting Earth once every 93 mins at a height of 400 km above the surface, deploys a new 800 kg satellite that is to orbit a further 200 km away from Earth.
    (a) Use the following graph to estimate the work needed to deploy the satellite from the shuttle.
    (b) Use the mass and radius of Earth to assist you in determining the period of the new satellite.
    (c) Show how the period of the new satellite can be determined without knowledge of the mass of Earth.
    (d) If the new satellite was redesigned so that its mass was halved, how would your answers to (a) and (b) change?



24. A disabled satellite of mass 2400 kg is in orbit around Earth at a height of 2000 km above sea level. It falls to a height of 800 km before its built-in rocket system can be activated to stop the fall continuing.



(a) Calculate the gravitational force on the satellite while it is in its initial orbit.

(b) Calculate the loss of gravitational potential energy of the satellite during its fall.

(c) If the speed of the satellite during its initial orbit is 6900 m s^{-1}, what is its speed when the rocket system is activated?

25. In a space shuttle that is in orbit around Earth at an altitude of 360 km, what is the magnitude of:

(a) the gravitational field strength

(b) the reaction force by the shuttle on a 70 kg astronaut

(c) the gravitational force by the Earth on this astronaut?

26. Why does the gravitational force do no work on a satellite in orbit?