3.1 Overview

Why learn this?
What did you weigh as a baby, and how tall were you? Did you grow at a steady (linear) rate, or were there periods in your life when you grew rapidly? What is the relationship between your height and your weight? We constantly seek to find relationships between variables, and coordinate geometry provides a picture, a visual clue as to what the relationships may be.

What do you know?

1 THINK List what you know about linear graphs and their equations. Use a thinking tool such as a concept map to show your list.
2 PAIR Share what you know with a partner and then with a small group.
3 SHARE As a class, create a large concept map that shows your class’s knowledge of linear graphs and their equations.

Learning sequence

3.1 Overview
3.2 Sketching linear graphs
3.3 Determining linear equations
3.4 The distance between two points
3.5 The midpoint of a line segment
3.6 Parallel and perpendicular lines
3.7 Review
WATCH THIS VIDEO
The story of mathematics: Descartes

Searchlight ID: eles-1842
3.2 Sketching linear graphs

- If a series of points \((x, y)\) is plotted using the rule \(y = mx + c\), then the points always lie in a straight line whose gradient equals \(m\) and whose \(y\)-intercept equals \(c\).
- The rule \(y = mx + c\) is called the equation of a straight line written in ‘gradient–intercept’ form.

Plotting linear graphs

- To plot a linear graph, complete a table of values to determine the points.

**WORKED EXAMPLE 1**

Plot the linear graph defined by the rule \(y = 2x - 5\) for the \(x\)-values \(-3, -2, -1, 0, 1, 2\) and \(3\).

**THINK**

1. Create a table of values using the given \(x\)-values.

2. Find the corresponding \(y\)-values by substituting each \(x\)-value into the rule.

3. Plot the points on a Cartesian plane and rule a straight line through them. Since the \(x\)-values have been specified, the line should only be drawn between the \(x\)-values of \(-3\) and \(3\).

**WRITE/DRAW**

<table>
<thead>
<tr>
<th>(x)</th>
<th>(-3)</th>
<th>(-2)</th>
<th>(-1)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>(-11)</td>
<td>(-9)</td>
<td>(-7)</td>
<td>(-5)</td>
<td>(-3)</td>
<td>(-1)</td>
<td>1</td>
</tr>
</tbody>
</table>

**SKETCHING STRAIGHT LINES**

- A minimum of two points are necessary to plot a straight line.
- Two methods can be used to plot a straight line:
  - Method 1: The \(x\)- and \(y\)-intercept method.

**METHOD 1: SKETCHING A STRAIGHT LINE USING THE \(x\)- AND \(y\)-INTERCEPTS**

- As the name implies, this method involves plotting the \(x\)- and \(y\)-intercepts, then joining them to sketch the straight line.
- The line cuts the \(y\)-axis where \(x = 0\) and the \(x\)-axis where \(y = 0\).
### WORKED EXAMPLE 2

Sketch graphs of the following linear equations by finding the \(x\)- and \(y\)-intercepts.

\[ \text{a} \quad 2x + y = 6 \]
\[ \text{b} \quad y = -3x - 12 \]

#### THINK

**a**
1. Write the equation.
2. Find the \(x\)-intercept by substituting \(y = 0\).
3. Find the \(y\)-intercept by substituting \(x = 0\).
4. Plot both points and rule the line.
5. Label the graph.

**b**
1. Write the equation.
2. Find the \(x\)-intercept by substituting \(y = 0\).
   - Add 12 to both sides of the equation.
   - Divide both sides of the equation by \(-3\).
3. Find the \(y\)-intercept. The equation is in the form \(y = mx + c\), so compare this with our equation to find the \(y\)-intercept, \(c\).
4. Plot both points and rule the line.
5. Label the graph.

#### WRITE/DRAW

**a**
- \(2x + y = 6\)
  - \(x\)-intercept: when \(y = 0\),
    \[ \begin{align*}
    2x + 0 &= 6 \\
    2x &= 6 \\
    x &= 3
    \end{align*} \]
  - \(x\)-intercept is \((3, 0)\).

- \(y\)-intercept: when \(x = 0\),
  \[ \begin{align*}
    2(0) + y &= 6 \\
    y &= 6
    \end{align*} \]
  - \(y\)-intercept is \((0, 6)\).

**b**
- \(y = -3x - 12\)
  - \(x\)-intercept: when \(y = 0\),
    \[ \begin{align*}
    -3x - 12 &= 0 \\
    -3x &= 12 \\
    x &= -4
    \end{align*} \]
  - \(x\)-intercept is \((-4, 0)\).
  - \(c = -12\)
  - \(y\)-intercept is \((0, -12)\).
Method 2: Sketching a straight line using the gradient–intercept method

- This method is often used if the equation is in the form \( y = mx + c \), where \( m \) represents the gradient (slope) of the straight line, and \( c \) represents the \( y \)-intercept.
- The steps below outline how to use the gradient–intercept method to sketch a linear graph.

**Step 1:** Plot a point at the \( y \)-intercept.

**Step 2:** Write the gradient in the form \( m = \frac{\text{rise}}{\text{run}} \). (To write a whole number as a fraction, place it over a denominator of 1.)

**Step 3:** Starting from the \( y \)-intercept, move up the number of units suggested by the rise (move down if the gradient is negative).

**Step 4:** Move to the right the number of units suggested by the run and plot the second point.

**Step 5:** Rule a straight line through the two points.

Sketch the graph of \( y = \frac{2}{3}x - 3 \) using the gradient–intercept method.

**THINK**

1. Write the equation of the line.
2. Identify the value of \( c \) (that is, the \( y \)-intercept) and plot this point.
3. Write the gradient, \( m \), as a fraction.
4. \( m = \frac{\text{rise}}{\text{run}} \), note the rise and run.
5. Starting from the \( y \)-intercept at \((0, -3)\), move 2 units up and 5 units to the right to find the second point \((5, -1)\). We have still not found the \( x \)-intercept.

**WRITE/DRAW**

\[ y = \frac{2}{3}x - 3 \]
\[ c = -3 \text{, so } y \text{-intercept: } (0, -3). \]
\[ m = \frac{2}{5} \]
So, rise = 2; run = 5.

Sketching linear graphs of the form \( y = mx \)

- Graphs given by \( y = mx \) pass through the origin \((0, 0)\), since \( c = 0 \).
- A second point may be determined using the rule \( y = mx \) by substituting a value for \( x \) to find \( y \).

**WORKED EXAMPLE 4**

Sketch the graph of \( y = 3x \).

**THINK**

1. Write the equation.
2. Find the \( x \)- and \( y \)-intercepts.

**Note:** By recognising the form of this linear equation, \( y = mx \) you can simply state that the graph passes through the origin, \((0, 0)\).

**WRITE/DRAW**

\( y = 3x \)
\( x \)-intercept: when \( y = 0 \),
\( 0 = 3x \)
\( x = 0 \)
\( y \)-intercept: \((0, 0)\)
Both the \( x \)- and \( y \)-intercepts are at \((0, 0)\).
3 Find another point to plot by finding the y-value when \( x = 1 \).

When \( x = 1 \), \( y = 3 \times 1 = 3 \)

Another point on the line is \( (1, 3) \).

4 Plot the two points \((0, 0)\) and \((1, 3)\) and rule a straight line through them.

5 Label the graph.

Sketching linear graphs of the form \( y = c \) and \( x = a \)

- The line \( y = c \) is parallel to the \( x \)-axis, having a gradient of zero and a \( y \)-intercept of \( c \).
- The line \( x = a \) is parallel to the \( y \)-axis and has an undefined (infinite) gradient.

**WORKED EXAMPLE 5**

**Sketch graphs of the following linear equations.**

a \( y = -3 \)

**THINK**

a 1 Write the equation.

2 The \( y \)-intercept is \(-3\). As \( x \) does not appear in the equation, the line is parallel to the \( x \)-axis, such that all points on the line have a \( y \)-coordinate equal to \(-3\). That is, this line is the set of points \((x, -3)\) where \( x \) is an element of the set of real numbers.

3 Sketch a horizontal line through \((0, -3)\).

**WRITE/DRAW**

a \( y = -3 \)

\( y \)-intercept \( = -3 \), \((0, -3)\)

4 Label the graph.
b Write the equation.

2 The $x$-intercept is 4. As $y$ does not appear in the equation, the line is parallel to the $y$-axis, such that all points on the line have an $x$-coordinate equal to 4. That is, this line is the set of points $(4, y)$ where $y$ is an element of the set of real numbers.

3 Sketch a vertical line through $(4, 0)$.

4 Label the graph.

Using linear graphs to model real-life contexts

- If a real-life situation involves a constant increase or decrease at regular intervals, then it can be modelled by a linear equation. Examples include water being poured from a tap into a container at a constant rate, or money being deposited into a savings account at regular intervals.
- To model a linear situation, we first need to determine which of the two given variables is the independent variable and which is the dependent variable.
- The independent variable does not depend on the value of the other variable, whereas the dependent variable takes its value depending on the value of the other variable. When plotting a graph of a linear model, the independent variable will be on the $x$-axis (horizontal) and the dependent variable will be on the $y$-axis (vertical).
- The following table contains a list of situations, with the independent and dependent variable being identified in each instance.

<table>
<thead>
<tr>
<th>Situation</th>
<th>Independent variable</th>
<th>Dependent variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money being deposited into a savings account at regular intervals</td>
<td>Time</td>
<td>Money in account</td>
</tr>
<tr>
<td>The age of a person in years and their height in cm</td>
<td>Age in years</td>
<td>Height in cm</td>
</tr>
<tr>
<td>The temperature at a snow resort and the depth of the snow</td>
<td>Temperature</td>
<td>Depth of snow</td>
</tr>
<tr>
<td>The length of Pinocchio’s nose and the amount of lies he told</td>
<td>Amount of lies</td>
<td>Length of Pinocchio’s nose</td>
</tr>
<tr>
<td>The number of workers building a house and the time taken to complete the project</td>
<td>Number of workers</td>
<td>Time</td>
</tr>
</tbody>
</table>
• Note that if time is one of the variables, it will usually be the independent variable. The final example above is a rare case of time being the dependent variable. Also, some of the above cases can’t be modelled by linear graphs, as the increases or decreases aren’t necessarily happening at constant rates.

**WORKED EXAMPLE 6**

Water is leaking from a bucket at a constant rate. After 1 minute there is 45 litres in the bucket; after 3 minutes there is 35 litres in the bucket; after 5 minutes there is 25 litres in the bucket; and after 7 minutes there is 15 litres in the bucket.

a Define two variables to represent the given information.
b Determine which variable is the independent variable and which is the dependent variable.
c Represent the given information in a table of values.
d Plot a graph to represent how the amount of water in the bucket is changing.
e Use your graph to determine how much water was in the bucket at the start and how long it will take for the bucket to be empty.

**THINK**

a Determine which two values change in the relationship given.
b The dependent variable takes its value depending on the value of the independent variable. In this situation the amount of water depends on the amount of time elapsed, not the other way round.
c The independent variable should appear in the top row of the table of values, with the dependent variable appearing in the second row.
d The values in the top row of the table represent the values on the horizontal axis, and the values in the bottom row of the table represent the values on the vertical axis. As the value for time can’t be negative and there can’t be a negative amount of water in the bucket, only the first quadrant needs to be drawn for the graph. Plot the 4 points and rule a straight line through them. Extend the graph to meet the vertical and horizontal axes.
e The amount of water in the bucket at the start is the value at which the line meets the vertical axis, and the time taken for the bucket to be empty is the value at which the line meets the horizontal axis.

**WRITE/DRAW**

a The two variables are ‘time’ and ‘amount of water in bucket’.
b Independent variable = time
Dependent variable = amount of water in bucket

c Time (minutes) | 1  | 3  | 5  | 7  |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount of water in bucket (litres)</td>
<td>45</td>
<td>35</td>
<td>25</td>
<td>15</td>
</tr>
</tbody>
</table>

d

e There was 50 litres of water in the bucket at the start, and it will take 10 minutes for the bucket to be empty.
Exercise 3.2 Sketching linear graphs

INDIVIDUAL PATHWAYS

PRACTISE Questions: 1, 2, 3a–h, 4a–e, 5a–d, 6a–f, 7a–d, 8a–d, 9, 10, 12

CONSOLIDATE Questions: 1, 2, 3f–m, 4a–e, 5a–d, 6a–f, 7c–f, 8a–f, 9–12

MASTER Questions: 1, 2, 3h–o, 4d–i, 5c–f, 6e–i, 7d–h, 8c–h, 9–13

FLUENCY

1. **WE1** Generate a table of values and then plot the linear graphs defined by the following rules for the given range of \( x \)-values.

<table>
<thead>
<tr>
<th>Rule</th>
<th>( x )-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a ) ( y = 10x + 25 )</td>
<td>(-5, -4, -3, -2, -1, 0, 1)</td>
</tr>
<tr>
<td>( b ) ( y = 5x - 12 )</td>
<td>(-1, 0, 1, 2, 3, 4)</td>
</tr>
<tr>
<td>( c ) ( y = -0.5x + 10 )</td>
<td>(-6, -4, -2, 0, 2, 4)</td>
</tr>
<tr>
<td>( d ) ( y = 100x - 240 )</td>
<td>(0, 1, 2, 3, 4, 5)</td>
</tr>
<tr>
<td>( e ) ( y = -5x + 3 )</td>
<td>(-3, -2, -1, 0, 1, 2)</td>
</tr>
<tr>
<td>( f ) ( y = 7 - 4x )</td>
<td>(-3, -2, -1, 0, 1, 2)</td>
</tr>
</tbody>
</table>

2. Plot the linear graphs defined by the following rules for the given range of \( x \)-values.

<table>
<thead>
<tr>
<th>Rule</th>
<th>( x )-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a ) ( y = -3x + 2 )</td>
<td>(x)-values</td>
</tr>
<tr>
<td>( b ) ( y = -x + 3 )</td>
<td>(x)-values</td>
</tr>
<tr>
<td>( c ) ( y = -2x + 3 )</td>
<td>(x)-values</td>
</tr>
</tbody>
</table>

3. **WE2** Sketch graphs of the following linear equations by finding the \( x \)- and \( y \)-intercepts.

<table>
<thead>
<tr>
<th>Rule</th>
<th>( x )- and ( y )-intercepts</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a ) ( 5x - 3y = 10 )</td>
<td>(x)-intercept: ( -2 ), ( y )-intercept: ( 1 )</td>
</tr>
<tr>
<td>( b ) ( 5x + 3y = 10 )</td>
<td>(x)-intercept: ( 2 ), ( y )-intercept: ( -2 )</td>
</tr>
<tr>
<td>( c ) ( -5x + 3y = 10 )</td>
<td>(x)-intercept: ( -2 ), ( y )-intercept: ( 5 )</td>
</tr>
<tr>
<td>( d ) ( -5x - 3y = 10 )</td>
<td>(x)-intercept: ( 2 ), ( y )-intercept: ( 5 )</td>
</tr>
<tr>
<td>( e ) ( 2x - 8y = 20 )</td>
<td>(x)-intercept: ( 10 ), ( y )-intercept: ( -0.25 )</td>
</tr>
<tr>
<td>( f ) ( 10x + 30y = -150 )</td>
<td>(x)-intercept: ( -3 ), ( y )-intercept: ( 5 )</td>
</tr>
<tr>
<td>( g ) ( -x + 6y = 120 )</td>
<td>(x)-intercept: ( -120 ), ( y )-intercept: ( 20 )</td>
</tr>
<tr>
<td>( h ) ( -2x + 8y = -20 )</td>
<td>(x)-intercept: ( 10 ), ( y )-intercept: ( -2.5 )</td>
</tr>
<tr>
<td>( i ) ( 6x - 4y = -24 )</td>
<td>(x)-intercept: ( -8 ), ( y )-intercept: ( 6 )</td>
</tr>
<tr>
<td>( j ) ( 5x + 30y = -150 )</td>
<td>(x)-intercept: ( -30 ), ( y )-intercept: ( -5 )</td>
</tr>
<tr>
<td>( k ) ( -9x + 4y = 36 )</td>
<td>(x)-intercept: ( -4 ), ( y )-intercept: ( 9 )</td>
</tr>
<tr>
<td>( l ) ( 4x + 4y = 40 )</td>
<td>(x)-intercept: ( 10 ), ( y )-intercept: ( 10 )</td>
</tr>
<tr>
<td>( m ) ( 10x + 30y = -150 )</td>
<td>(x)-intercept: ( -15 ), ( y )-intercept: ( -5 )</td>
</tr>
<tr>
<td>( n ) ( y = -5x + 20 )</td>
<td>(x)-intercept: ( 4 ), ( y )-intercept: ( 20 )</td>
</tr>
</tbody>
</table>

4. **WE3** Sketch graphs of the following linear equations using the gradient–intercept method.

<table>
<thead>
<tr>
<th>Rule</th>
<th>( x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a ) ( y = 4x + 1 )</td>
<td>( x = 0 )</td>
</tr>
<tr>
<td>( b ) ( y = 3x - 7 )</td>
<td>( x = 0 )</td>
</tr>
<tr>
<td>( c ) ( y = -2x + 3 )</td>
<td>( x = 0 )</td>
</tr>
<tr>
<td>( d ) ( y = -5x - 4 )</td>
<td>( x = 0 )</td>
</tr>
<tr>
<td>( e ) ( y = \frac{1}{2}x - 2 )</td>
<td>( x = 0 )</td>
</tr>
<tr>
<td>( f ) ( y = -\frac{2}{3}x + 3 )</td>
<td>( x = 0 )</td>
</tr>
<tr>
<td>( g ) ( y = 0.6x + 0.5 )</td>
<td>( x = 0 )</td>
</tr>
<tr>
<td>( h ) ( y = 8x )</td>
<td>( x = 0 )</td>
</tr>
<tr>
<td>( i ) ( y = x - 7 )</td>
<td>( x = 0 )</td>
</tr>
</tbody>
</table>
5. **WE4** Sketch the graphs of the following linear equations.
   a. $y = 2x$
   b. $y = 5x$
   c. $y = -3x$
   d. $y = \frac{1}{2}x$
   e. $y = \frac{2}{3}x$
   f. $y = -\frac{5}{2}x$

6. **WE5** Sketch the graphs of the following linear equations.
   a. $y = 10$
   b. $y = -10$
   c. $x = 10$
   d. $x = -10$
   e. $y = 100$
   f. $y = 0$
   g. $x = 0$
   h. $x = -100$
   i. $y = -12$

7. Transpose each of the equations to standard form (that is, $y = mx + c$). State the $x$- and $y$-intercept for each.
   a. $5(y + 2) = 4(x + 3)$
   b. $5(y - 2) = 4(x - 3)$
   c. $2(y + 3) = 3(x + 2)$
   d. $10(y - 20) = 40(x - 2)$
   e. $4(y + 2) = -4(x + 2)$
   f. $2(y - 2) = -(x + 5)$
   g. $-5(y + 1) = 4(x - 4)$
   h. $5(y + 2.5) = 2(x - 3.5)$
   i. $2.5(y - 2) = -6.5(x - 1)$

8. **UNDERSTANDING**
   Find the $x$- and $y$-intercepts of the following lines.
   a. $-y = 8 - 4x$
   b. $6x - y + 3 = 0$
   c. $2y - 10x = 50$

9. Explain why the gradient of a horizontal line is equal to zero and the gradient of a vertical line is undefined.

10. **REASONING**
    Determine whether $\frac{x}{3} - \frac{y}{2} = \frac{7}{6}$ is the equation of a straight line by rearranging into an appropriate form and hence sketch the graph, showing all relevant features.

11. **WE6** Your friend loves to download music. She earns $50 and spends some of it buying music online at $1.75 per song. She saves the remainder. Her saving is given by the function $f(x) = 50 - 1.75x$.
    a. Determine which variable is the independent variable and which is the dependent variable.
    b. Sketch the function.
    c. How many songs can she buy and still save $25? 

12. **PROBLEM SOLVING**
    A straight line has a general equation defined by $y = mx + c$. This line intersects the lines defined by the rules $y = 7$ and $x = 3$. The lines $y = mx + c$ and $y = 7$ have the same $y$-intercept while $y = mx + c$ and $x = 3$ have the same $x$-intercept.
    a. On the one set of axes, sketch all three graphs.
    b. Determine the $y$-axis intercept for $y = mx + c$.
    c. Determine the gradient for $y = mx + c$.
    d. **MC** The equation of the line defined by $y = mx + c$ is:
       A. $x + y = 3$
       B. $3x + 3y = 21$
       C. $3x + 7y = 21$
       D. $x + y = 7$
       E. $7x + 3y = 7$

13. Water is flowing from a tank at a constant rate. The equation relating the volume of water in the tank, $V$ litres, to the time the water has been flowing from the tank, $t$ minutes, is given by $V = 80 - 4t, t \geq 0$.
    a. Determine which variable is the independent variable and which is the dependent variable.
    b. How much water is in the tank initially?
    c. Why is it important that $t \geq 0$?
    d. At what rate is the water flowing from the tank?
    e. How long does it take for the tank to empty?
    f. Sketch the graph of $V$ versus $t$. 

---

**Topic 3 • Coordinate geometry**

81
3.3 Determining linear equations
Finding a linear equation given two points

- The gradient of a straight line can be calculated from the coordinates of two points \((x_1, y_1)\) and \((x_2, y_2)\) that lie on the line.

\[
\text{Gradient } m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}
\]

- The equation of the straight line can then be found in the form \(y = mx + c\), where \(c\) is the \(y\)-intercept.

**WORKED EXAMPLE 7**

Find the equation of the straight line shown in the graph.

**THINK**

1. There are two points given on the straight line: the \(x\)-intercept \((3, 0)\) and the \(y\)-intercept \((0, 6)\).

2. Find the gradient of the line by applying the formula

\[
m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } (x_1, y_1) = (3, 0) \text{ and } (x_2, y_2) = (0, 6).
\]

3. The graph has a \(y\)-intercept of 6, so \(c = 6\). Substitute \(m = -2\), and \(c = 6\) into \(y = mx + c\) to find the equation.

**WRITE**

\((3, 0), (0, 6)\)

\[
m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}
\]

\[
= \frac{6 - 0}{0 - 3}
\]

\[
= \frac{6}{-3}
\]

\[
= -2
\]

The gradient \(m = -2\).

\[
y = -2x + 6
\]
WORKED EXAMPLE 8

Find the equation of the straight line shown in the graph.

THINK
1. There are two points given on the straight line: the $x$- and $y$-intercept $(0, 0)$ and another point $(2, 1)$.
2. Find the gradient of the line by applying the formula $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$, where $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (2, 1)$.
3. The $y$-intercept is 0, so $c = 0$. Substitute $m = \frac{1}{2}$ and $c = 0$ into $y = mx + c$ to determine the equation.

WRITE
$(0, 0), (2, 1)$

$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{2 - 0} = \frac{1}{2}$
The gradient $m = \frac{1}{2}$.

$y = \frac{1}{2}x + 0$

WORKED EXAMPLE 9

Find the equation of the straight line passing through $(-2, 5)$ and $(1, -1)$.

THINK
1. Write the general equation of a straight line.
2. Write the formula for calculating the gradient of a line between two points.
3. Let $(x_1, y_1)$ and $(x_2, y_2)$ be the two points $(-2, 5)$ and $(1, -1)$ respectively. Substitute the values of the pronumerals into the formula to calculate the gradient.
4. Substitute the value of the gradient into the general rule.
5. Select either of the two points, say $(1, -1)$, and substitute its coordinates into $y = -2x + c$.
6. Solve for $c$; that is, add 2 to both sides of the equation.
7. State the equation by substituting the value of $c$ into $y = -2x + c$.

WRITE

$y = mx + c$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{1 - (-2)} = \frac{-6}{3} = -2$
y = $-2x + c$

Point $(1, -1)$:

$-1 = -2 \times 1 + c$

$-1 = -2 + c$

$1 = c$

The equation of the line is $y = -2x + 1$. 

CASIO

TI

WORKED EXAMPLE 8

Find the equation of the straight line shown in the graph.

THINK
1. There are two points given on the straight line: the $x$- and $y$-intercept $(0, 0)$ and another point $(2, 1)$.
2. Find the gradient of the line by applying the formula $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$, where $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (2, 1)$.
3. The $y$-intercept is 0, so $c = 0$. Substitute $m = \frac{1}{2}$ and $c = 0$ into $y = mx + c$ to determine the equation.

WRITE
$(0, 0), (2, 1)$

$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{2 - 0} = \frac{1}{2}$
The gradient $m = \frac{1}{2}$.

$y = \frac{1}{2}x + 0$

WORKED EXAMPLE 9

Find the equation of the straight line passing through $(-2, 5)$ and $(1, -1)$.

THINK
1. Write the general equation of a straight line.
2. Write the formula for calculating the gradient of a line between two points.
3. Let $(x_1, y_1)$ and $(x_2, y_2)$ be the two points $(-2, 5)$ and $(1, -1)$ respectively. Substitute the values of the pronumerals into the formula to calculate the gradient.
4. Substitute the value of the gradient into the general rule.
5. Select either of the two points, say $(1, -1)$, and substitute its coordinates into $y = -2x + c$.
6. Solve for $c$; that is, add 2 to both sides of the equation.
7. State the equation by substituting the value of $c$ into $y = -2x + c$.

WRITE

$y = mx + c$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{1 - (-2)} = \frac{-6}{3} = -2$
y = $-2x + c$

Point $(1, -1)$:

$-1 = -2 \times 1 + c$

$-1 = -2 + c$

$1 = c$

The equation of the line is $y = -2x + 1$. 

CASIO

TI
Finding the equation of a straight line using the gradient and one point

- If the gradient of a line is known, only one point is needed to determine the equation of the line.

**WORKED EXAMPLE 10**

Find the equation of the straight line with gradient of 2 and y-intercept of $-5$.

**THINK**
1. Write the known information. The other point is the y-intercept, which makes the calculation of $c$ straightforward.
2. State the values of $m$ and $c$.
3. Substitute these values into $y = mx + c$ to find the equation.

**WRITE**
- Gradient = 2, y-intercept = $-5$
- $m = 2$, $c = -5$
- $y = mx + c$
- $y = 2x - 5$

**WORKED EXAMPLE 11**

Find the equation of the straight line passing through the point $(5, -1)$ with a gradient of 3.

**THINK**
1. Write the known information.
2. State the values of $m$, $x$ and $y$.
3. Substitute the values $m = 3$, $x = 5$ and $y = -1$ into $y = mx + c$ and solve to find $c$.
4. Substitute $m = 3$ and $c = -16$ into $y = mx + c$ to determine the equation.

**WRITE**
- Gradient = 3, point $(5, -1)$.
- $m = 3$, $(x, y) = (5, -1)$
- $y = mx + c$
- $-1 = 3(5) + c$
- $-1 = 15 + c$
- $-16 = c$
- The equation of the line is $y = 3x - 16$.

**A simple formula**
- The diagram shows a line of gradient $m$ passing through the point $(x_1, y_1)$.
- If $(x, y)$ is any other point on the line, then:

\[
m = \frac{\text{rise}}{\text{run}} = \frac{y - y_1}{x - x_1}
\]

\[
m(x - x_1) = y - y_1
\]

- The formula $y - y_1 = m(x - x_1)$ can be used to write down the equation of a line, given the gradient and the coordinates of one point.
WORKED EXAMPLE 12

Find the equation of the line with a gradient of \(-2\) which passes through the point \((3, -4)\). Write the equation in general form, that is in the form \(ax + by + c = 0\).

**THINK**

1. Use the formula \(y - y_1 = m(x - x_1)\). Write the values of \(x_1, y_1,\) and \(m\).
2. Substitute for \(x_1, y_1,\) and \(m\) into the equation.
3. Transpose the equation into the form \(ax + by + c = 0\).

**WRITE**

\[ m = -2, x_1 = 3, y_1 = -4 \]
\[ y - y_1 = m(x - x_1) \]
\[ y - (-4) = -2(x - 3) \]
\[ y + 4 = -2x + 6 \]
\[ y + 4 + 2x - 6 = 0 \]
\[ 2x + y - 2 = 0 \]

WORKED EXAMPLE 13

A printer prints pages at a constant rate. It can print 165 pages in 3 minutes and 275 pages in 5 minutes.

\(a\) Determine which variable is the independent variable (\(x\)) and which is the dependent variable (\(y\)).

\(b\) Determine the gradient of the equation and explain what this means in the context of the question.

\(c\) Write an equation, in algebraic form, linking the independent and dependent variables.

\(d\) Rewrite your equation in words.

\(e\) Using the equation, determine how many pages can be printed in 11 minutes.

**THINK**

\(a\) The dependent variable takes its value depending on the value of the independent variable.

In this situation the number of pages depends on the time elapsed, not the other way round.

\(b\) Determine the two points given by the information in the question.

\(x_1, y_1 = (3, 165)\)
\(x_2, y_2 = (5, 275)\)

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ = \frac{275 - 165}{5 - 3} \]
\[ = \frac{110}{2} \]
\[ = 55 \]

\(c\) The gradient states how much the dependent variable increases for each increase of 1 unit in the independent variable.

**WRITE/DRAW**

\(a\) Independent variable = time

Dependent variable = number of pages

\(b\)

\(x_1, y_1 = (3, 165)\)
\(x_2, y_2 = (5, 275)\)

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ = \frac{275 - 165}{5 - 3} \]
\[ = \frac{110}{2} \]
\[ = 55 \]

In the context of the question, this means that each minute 55 pages are printed.
The graph travels through the origin, as the time elapsed for the printer to print 0 pages is 0 seconds. Therefore, the equation will be in the form $y = mx$.
Substitute in the value of $m$.

**Replace $x$ and $y$ in the equation with the independent and dependent variables.**

**1** Substitute $x = 11$ into the equation.

**2** Write the answer in words.

---

**Exercise 3.3 Determining linear equations**

**INDIVIDUAL PATHWAYS**

- **PRACTISE**
  Questions: 1a–d, 2, 3, 4, 5a–d, 7

- **CONSOLIDATE**
  Questions: 1a–f, 2, 3, 4, 5c–g, 7, 9

- **MASTER**
  Questions: 1d–h, 2, 3, 4, 5e–j, 6–10

**FLUENCY**

**1** Determine the equation for each of the straight lines shown.

- **a**
- **b**
- **c**
- **d**
- **e**
- **f**
- **g**
- **h**
2. **WE8** Determine the equation of each of the straight lines shown.

   a. \( y = 2x + 4 \)

   b. \( y = -3x + 12 \)

   c. \( y = -2x - 2 \)

   d. \( y = -4x - 8 \)

3. **WE9** Find the equation of the straight line that passes through each pair of points.

   a. \((1, 4)\) and \((3, 6)\)

   b. \((0, -1)\) and \((3, 5)\)

   c. \((-1, 4)\) and \((3, 2)\)

   d. \((3, 2)\) and \((-1, 0)\)

   e. \((-4, 6)\) and \((2, -6)\)

   f. \((-3, -5)\) and \((-1, -7)\)

4. **WE10** Find the linear equation given the information in each case below.

   a. Gradient = 3, y-intercept = 3

   b. Gradient = -3, y-intercept = 4

   c. Gradient = -4, y-intercept = 2

   d. Gradient = 4, y-intercept = 2

   e. Gradient = -1, y-intercept = -4

   f. Gradient = 0.5, y-intercept = -4

   g. Gradient = 5, y-intercept = 2.5

   h. Gradient = -6, y-intercept = 3

   i. Gradient = -2.5, y-intercept = 1.5

   j. Gradient = 3.5, y-intercept = 6.5

5. **WE11, 12** For each of the following, find the equation of the straight line with the given gradient and passing through the given point.

   a. Gradient = 5, point = \((5, 6)\)

   b. Gradient = -5, point = \((5, 6)\)

   c. Gradient = -4, point = \((-2, 7)\)

   d. Gradient = 4, point = \((8, -2)\)

   e. Gradient = 3, point = \((10, -5)\)

   f. Gradient = -3, point = \((3, -3)\)

   g. Gradient = -2, point = \((20, -10)\)

   h. Gradient = 2, point = \((2, -0.5)\)

   i. Gradient = 0.5, point = \((6, -16)\)

   j. Gradient = -0.5, point = \((5, 3)\)

**UNDERSTANDING**

6. **WE13** a. Determine which variable (time or cost) is the independent variable and which is the dependent variable.

   b. If \( t \) represents the time in hours and \( C \) represents cost ($), construct a table of values for 0–3 hours for the cost of playing ten-pin bowling at the new alley.

   c. Use your table of values to plot a graph of time versus cost. (Hint: Ensure your time axis (horizontal axis) extends to 6 hours and your cost axis (vertical axis) extends to $40.)

   Save $$$ with Supa-Bowl!!!
   NEW Ten-Pin Bowling Alley
   *Shoe rental just $2 (fixed fee)*
   Rent a lane for ONLY $6/hour!
d  i  What is the y-intercept?
  ii  What does the y-intercept represent in terms of the cost?

e  Calculate the gradient and explain what this means in the context of the question.

f  Write a linear equation to describe the relationship between cost and time.

g  Use your linear equation from part f to calculate the cost of a 5-hour tournament.

h  Use your graph to check your answer to part g.

REASONING

7  When using the gradient to draw a line, does it matter if you rise before you run or run before you rise? Explain your answer.

8  a  Using the graph below, write a general formula for the gradient \( m \) in terms of \( x, y \) and \( c \).

  b  Transpose your formula to make \( y \) the subject. What do you notice?

PROBLEM SOLVING

9  The points A \((x_1, y_1)\), B \((x_2, y_2)\) and P \((x, y)\) are co-linear. P is a general point that lies anywhere on the line. Show that an equation relating these three points is given by

\[
y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).\]

10  Show that the quadrilateral ABCD is a parallelogram.
3.4 The distance between two points

- The distance between two points can be calculated using Pythagoras’ theorem.
- Consider two points \( A(x_1, y_1) \) and \( B(x_2, y_2) \) on the Cartesian plane as shown below.

\[
\begin{align*}
\text{AC} &= x_2 - x_1 \\
\text{BC} &= y_2 - y_1
\end{align*}
\]

By Pythagoras’ theorem:

\[
AB^2 = AC^2 + BC^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2
\]

Hence

\[
AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

The distance between two points \( A(x_1, y_1) \) and \( B(x_2, y_2) \) is:

\[
AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

- This distance formula can be used to calculate the distance between any two points on the Cartesian plane.
- The distance formula has many geometric applications.

**WORKED EXAMPLE 14**

Find the distance between the points \( A \) and \( B \) in the figure below. Answer correct to two decimal places.

**THINK**

1. From the graph, locate points \( A \) and \( B \).
2. Let \( A \) have coordinates \((x_1, y_1)\).

**WRITE**

\( A(-3, 1) \) and \( B(3, 4) \)

Let \((x_1, y_1) = (-3, 1)\)
Let \( B \) have coordinates \((x_2, y_2)\).

Find the length \( AB \) by applying the formula for calculating the distance between two points.

\[ AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

Let \((x_2, y_2) = (3, 4)\)

\[ AB = \sqrt{(3 - (-3))^2 + (4 - 1)^2} \]
\[ = \sqrt{36 + 9} \]
\[ = \sqrt{45} \]
\[ = 3\sqrt{5} \]
\[ \approx 6.71 \text{ (correct to 2 decimal places)} \]

Note: If the coordinates were named in the reverse order, the formula would still give the same answer. Check this for yourself using \((x_1, y_1) = (3, 4)\) and \((x_2, y_2) = (-3, 1)\).

WORKED EXAMPLE 15

Find the distance between the points \( P (-1, 5) \) and \( Q (3, -2) \).

THINK

Let \( P \) have coordinates \((x_1, y_1)\).

Let \( Q \) have coordinates \((x_2, y_2)\).

Find the length \( PQ \) by applying the formula for the distance between two points.

WRITE

Let \((x_1, y_1) = (-1, 5)\)

Let \((x_2, y_2) = (3, -2)\)

\[ PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ = \sqrt{(3 - (-1))^2 + (-2 - 5)^2} \]
\[ = \sqrt{4^2 + (-7)^2} \]
\[ = \sqrt{16 + 49} \]
\[ = \sqrt{65} \]
\[ = 8.06 \text{ (correct to 2 decimal places)} \]

WORKED EXAMPLE 16

Prove that the points \( A (1, 1), B (3, -1) \) and \( C (-1, -3) \) are the vertices of an isosceles triangle.

THINK

Plot the points and draw the triangle.

Note: For triangle \( ABC \) to be isosceles, two sides must have the same magnitude.

WRITE/DRAW

\[ AC = \sqrt{[1 - (-1)]^2 + [1 - (-3)]^2} \]
\[ = \sqrt{2^2 + 4^2} \]
\[ = \sqrt{20} \]
\[ = 2\sqrt{5} \]
3 Find the length BC.
B (3, −1) = (x_2, y_2)
C (−1, −3) = (x_1, y_1)

\[ BC = \sqrt{(3 - (-1))^2 + (-1 - (-3))^2} \]
\[ = \sqrt{(4)^2 + (2)^2} \]
\[ = \sqrt{20} \]
\[ = 2\sqrt{5} \]

4 Find the length AB.
A (1, 1) = (x_1, y_1)
B (3, -1) = (x_2, y_2)

\[ AB = \sqrt{(3 - (1))^2 + (-1 - (1))^2} \]
\[ = \sqrt{(2)^2 + (-2)^2} \]
\[ = \sqrt{4 + 4} \]
\[ = \sqrt{8} \]
\[ = 2\sqrt{2} \]

5 State your proof.
Since AC = BC ≠ AB, triangle ABC is an isosceles triangle.

Exercise 3.4 The distance between two points

**INDIVIDUAL PATHWAYS**

<table>
<thead>
<tr>
<th>PRACTISE</th>
<th>CONSOLIDATE</th>
<th>MASTER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Questions: 1, 2a–d, 5, 8, 9</td>
<td>Questions: 1, 2c–f, 5, 7, 9, 11</td>
<td>Questions: 1, 2a–h, 3–7, 9–12</td>
</tr>
</tbody>
</table>

**FLUENCY**

1 **WE14** Find the distance between each pair of points shown at right.

2 **WE15** Find the distance between the following pairs of points.
   a (2, 5), (6, 8)
   b (−1, 2), (4, 14)
   c (−1, 3), (−7, −5)
   d (5, −1), (10, 4)
   e (4, −5), (1, 1)
   f (−3, 1), (5, 13)
   g (5, 0), (−8, 0)
   h (1, 7), (1, −6)
   i \((a, b), (2a, −b)\)
   j \((−a, 2b), (2a, −b)\)

**UNDERSTANDING**

3 **MC** If the distance between the points (3, b) and (−5, 2) is 10 units, then the value of b is:
   A −8  B −4  C 4  D 0  E 2

**REFLEXION** How could you use the distance formula to show that a series of points lay on the circumference of a circle with centre C?
4 MC A rhombus has vertices A (1, 6), B (6, 6), C (−2, 2) and D (x, y). The coordinates of D are:
A (2, −3) B (2, 3) C (−2, 3) D (3, 2) E (3, −2)
5 The vertices of a quadrilateral are A (1, 4), B (−1, 8), C (1, 9) and D (3, 5).
   a Find the lengths of the sides.
   b Find the lengths of the diagonals.
   c What type of quadrilateral is it?

REASONING
6 WE16 Prove that the points A (0, −3), B (−2, −1) and C (4, 3) are the vertices of an isosceles triangle.
7 The points P (2, −1), Q (−4, −1) and R (−1, 3√3 − 1) are joined to form a triangle. Prove that triangle PQR is equilateral.
8 Prove that the triangle with vertices D (5, 6), E (9, 3) and F (5, 3) is a right-angled triangle.
9 A rectangle has vertices A (1, 5), B (10.6, z), C (7.6, −6.2) and D (−2, 1). Find:
   a the length of CD
   b the length of AD
   c the length of the diagonal AC
   d the value of z.
10 Show that the triangle ABC with coordinates A (a, a), B (m, −a) and C (−a, m) is isosceles.

PROBLEM SOLVING
11 Triangle ABC is an isosceles triangle where AB = AC, B is the point (−1, 2), C is the point (6, 3) and A is the point (a, 3a). Find the value of the integer constant a.

12 ABCD is a parallelogram.
   a Find the gradients of AB and BC.
   b Find the coordinates of the point D (x, y).
   c Show that the diagonals AC and BD bisect each other.
3.5 The midpoint of a line segment

**Midpoint of a line segment**

- The **midpoint** of a line segment is the halfway point.
- The $x$- and $y$-coordinates of the midpoint are halfway between those of the coordinates of the end points.
- The following diagram shows the line interval $AB$ joining points $A (x_1, y_1)$ and $B (x_2, y_2)$.
  
  The midpoint of $AB$ is $P$, so $AP = PB$.

  Points $C (x, y_1)$ and $D (x_2, y)$ are added to the diagram and are used to make the two right-angled triangles $\triangle APC$ and $\triangle PBD$.

  The two triangles are congruent:

  $AP = PB$ (given)
  $\angle APC = \angle PBD$ (corresponding angles)
  $\angle CAP = \angle DPB$ (corresponding angles)

  So $\triangle APC \equiv \triangle PBD$ (ASA)

  This means that $AC = PD$:

  i.e. $x - x_1 = x_2 - x$ (solve for $x$)
  i.e. $2x = x_1 + x_2$
  $x = \frac{x_1 + x_2}{2}$

  In other words $x$ is simply the average of $x_1$ and $x_2$.

  Similarly, $y = \frac{y_1 + y_2}{2}$.

  In general, the coordinates of the midpoint of a line segment joining the points $(x_1, y_1)$ and $(x_2, y_2)$ can be found by averaging the $x$- and $y$-coordinates of the end points, respectively.

  The coordinates of the midpoint of the line segment joining $(x_1, y_1)$ and $(x_2, y_2)$ are: $\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.
WORKED EXAMPLE 17

Find the coordinates of the midpoint of the line segment joining (−2, 5) and (7, 1).

THINK

1. Label the given points \((x_1, y_1)\) and \((x_2, y_2)\).

2. Find the \(x\)-coordinate of the midpoint.

3. Find the \(y\)-coordinate of the midpoint.

4. Give the coordinates of the midpoint.

WRITE

Let \((x_1, y_1) = (−2, 5)\) and \((x_2, y_2) = (7, 1)\)

\[
x = \frac{x_1 + x_2}{2} = \frac{−2 + 7}{2} = \frac{5}{2} = 2.5
\]

\[
y = \frac{y_1 + y_2}{2} = \frac{5 + 1}{2} = \frac{6}{2} = 3
\]

The midpoint is \((2.5, 3)\).

WORKED EXAMPLE 18

The coordinates of the midpoint, \(M\), of the line segment \(AB\) are \((7, 2)\). If the coordinates of \(A\) are \((1, −4)\), find the coordinates of \(B\).

THINK

1. Let the start of the line segment be \((x_1, y_1)\) and the midpoint be \((x, y)\).

2. The average of the \(x\)-coordinates is 7. Find the \(x\)-coordinate of the end point.

3. The average of the \(y\)-coordinates is 2. Find the \(y\)-coordinate of the end point.

WRITE/DRAW

Let \((x_1, y_1) = (1, −4)\) and \((x, y) = (7, 2)\)

\[
x = \frac{x_1 + x_2}{2}
\]

\[
7 = \frac{1 + x_2}{2}
\]

\[
14 = 1 + x_2
\]

\[
x_2 = 13
\]

\[
y = \frac{y_1 + y_2}{2}
\]

\[
2 = \frac{−4 + y_2}{2}
\]

\[
4 = −4 + y_2
\]

\[
y_2 = 8
\]
Give the coordinates of the end point. The coordinates of the point B are (13, 8).

Check that the coordinates are feasible by drawing a diagram.

Exercise 3.5 The midpoint of a line segment

**INDIVIDUAL PATHWAYS**

**PRACTISE**
Questions: 1a–d, 2, 3a, 4, 9, 11

**CONSOLIDATE**
Questions: 1a–d, 2–6, 9, 11

**MASTER**
Questions: 1a–f, 2–12

**FLUENCY**

1. **WE17** Use the formula method to find the coordinates of the midpoint of the line segment joining the following pairs of points.
   - a. (−5, 1), (−1, −8)
   - b. (4, 2), (11, −2)
   - c. (0, 4), (−2, −2)
   - d. (3, 4), (−3, −1)
   - e. (a, 2b), (3a, −b)
   - f. (a + 3b, b), (a − b, a − b)

2. **WE18** The coordinates of the midpoint, M, of the line segment AB are (2, −3). If the coordinates of A are (7, 4), find the coordinates of B.

**UNDERSTANDING**

3. A square has vertices A (0, 0), B (2, 4), C (6, 2) and D (4, −2). Find:
   - a. the coordinates of the centre
   - b. the length of a side
   - c. the length of a diagonal

4. **MC** The midpoint of the line segment joining the points (−2, 1) and (8, −3) is:
   - A (6, −2)
   - B (5, 2)
   - C (6, 2)
   - D (3, −1)
   - E (5, −2)

5. **MC** If the midpoint of AB is (−1, 5) and the coordinates of B are (3, 8), then A has coordinates:
   - A (1, 6.5)
   - B (2, 13)
   - C (−5, 2)
   - D (4, 3)
   - E (7, 11)

6. a. The vertices of a triangle are A (2, 5), B (1, −3) and C (−4, 3). Find:
   - i. the coordinates of P, the midpoint of AC
   - ii. the coordinates of Q, the midpoint of AB
   - iii. the length of PQ
   - b. Show that BC = 2 PQ.

7. a. A quadrilateral has vertices A (6, 2), B (4, −3), C (−4, −3) and D (−2, 2). Find:
   - i. the midpoint of the diagonal AC
   - ii. the midpoint of the diagonal BD.
   - b. What can you infer about the quadrilateral?
8 a The points A (−5, 3.5), B (1, 0.5) and C (−6, −6) are the vertices of a triangle. Find:
   i the midpoint, P, of AB
   ii the length of PC
   iii the length of AC
   iv the length of BC.
   b Describe the triangle. What does PC represent?

**REASONING**

9 Find the equation of the straight line that passes through the midpoint of A (−2, 5) and B (−2, 3), and has a gradient of −3.

10 Find the equation of the straight line that passes through the midpoint of A (−1, −3) and B (3, −5), and has a gradient of \( \frac{2}{3} \).

**PROBLEM SOLVING**

11 The points A (2m, 3m), B (5m, −2m) and C (−3m, 0) are the vertices of a triangle. Show that this is a right-angled triangle.

12 Write down the coordinates of the midpoint of the line joining the points (3k − 1, 4 − 5k) and (5k − 1, 3 − 5k). Show that this point lies on the line with equation 5x + 4y = 9.

### 3.6 Parallel and perpendicular lines

**Parallel lines**

- Lines that have the same gradient are parallel lines. The three lines on the graph at right all have a gradient of 1 and are parallel to each other.
Show that AB is parallel to CD given that A has coordinates \((-1, -5)\), B has coordinates \((5, 7)\), C has coordinates \((-3, 1)\) and D has coordinates \((4, 15)\).

**THINK**

1. Find the gradient of AB by applying the formula \(m = \frac{y_2 - y_1}{x_2 - x_1}\).

2. Find the gradient of CD.

3. Draw a conclusion. (Note: || means ‘is parallel to’.)

**WRITE**

Let \(A (-1, -5) = (x_1, y_1)\) and \(B (5, 7) = (x_2, y_2)\).

Since \(m = \frac{y_2 - y_1}{x_2 - x_1}\),

\[m_{AB} = \frac{7 - (-5)}{5 - (-1)} = \frac{12}{6} = 2\]

Let \(C (-3, 1) = (x_1, y_1)\) and \(D (4, 15) = (x_2, y_2)\).

Since \(m_{CD} = \frac{15 - 1}{4 - (-3)} = \frac{14}{7} = 2\),

Since \(m_{AB} = m_{CD} = 2\), then \(AB \parallel CD\).

**Collinear points**

- **Collinear points** are points that all lie on the same straight line.
- If \(A\), \(B\) and \(C\) are collinear, then \(m_{AB} = m_{BC}\).

**WORKED EXAMPLE 20**

Show that the points \(A (2, 0)\), \(B (4, 1)\) and \(C (10, 4)\) are collinear.

**THINK**

1. Find the gradient of AB.

**WRITE**

Let \(A (2, 0) = (x_1, y_1)\) and \(B (4, 1) = (x_2, y_2)\).

Since \(m = \frac{y_2 - y_1}{x_2 - x_1}\),

\[m_{AB} = \frac{1 - 0}{4 - 2} = \frac{1}{2}\]
2 Find the gradient of BC.

Let \( B(4, 1) = (x_1, y_1) \) and \( C(10, 4) = (x_2, y_2) \)

\[
m_{BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{10 - 4} = \frac{3}{6} = \frac{1}{2}
\]

3 Show that A, B and C are collinear.

Since \( m_{AB} = m_{BC} = \frac{1}{2} \) and B is common to both line segments, A, B and C are collinear.

Perpendicular lines

- There is a special relationship between the gradients of two perpendicular lines. The graph at right shows two perpendicular lines. What do you notice about their gradients?
- Consider the diagram shown below, in which the line segment AB is perpendicular to the line segment BC, AC is parallel to the \( x \)-axis, and BD is the perpendicular height of the resulting triangle ABC.

In \( \Delta ABD \), let \( m_{AB} = m_1 \)

\[
m_1 = \frac{a}{b} = \tan(\theta)
\]

In \( \Delta BCD \), let \( m_{BC} = m_2 \)

\[
m_2 = \frac{a}{c} = -\tan(\alpha)
\]

In \( \Delta ABD \), \( \tan(\alpha) = \frac{b}{a} \)

So \( m_2 = \frac{b}{a} \)

\[
m_2 = \frac{-1}{m_1}
\]

Hence \( m_2 = \frac{-1}{m_1} \)

or \( m_1m_2 = -1 \)

- Hence, if two lines are perpendicular to each other, then the product of their gradients is \(-1\).
  Two lines are perpendicular if and only if:

\[
m_1m_2 = -1
\]

- If two lines are perpendicular, then their gradients are \( \frac{a}{b} \) and \( -\frac{b}{a} \) respectively.
Show that the lines $y = -5x + 2$ and $5y - x + 15 = 0$ are perpendicular to one another.

**THINK**

1. Find the gradient of the first line.
   - $y = -5x + 2$
   - Hence $m_1 = -5$

2. Find the gradient of the second line.
   - $5y - x + 15 = 0$
   - Rewrite in the form $y = mx + c$:
     - $5y = x - 15$
     - $y = \frac{x}{5} - 3$
   - Hence $m_2 = \frac{1}{5}$
   - $m_1m_2 = -5 \times \frac{1}{5} = -1$

3. Test for perpendicularity. (The two lines are perpendicular if the product of their gradients is $-1$.)
   - Hence, the two lines are perpendicular.

**Determined the equation of a line parallel or perpendicular to another line**

- The gradient properties of parallel and perpendicular lines can be used to solve many problems.

**WORKED EXAMPLE 22**

Find the equation of the line that passes through the point $(3, -1)$ and is parallel to the straight line with equation $y = 2x + 1$.

**THINK**

1. Write the general equation.
   - $y = mx + c$

2. Find the gradient of the given line.
   - The two lines are parallel, so they have the same gradient.
   - $y = 2x + 1$ has a gradient of 2
   - Hence $m = 2$

3. Substitute for $m$ in the general equation.
   - $y = 2x + c$

4. Substitute the given point to find $c$.
   - $(x, y) = (3, -1)$
   - $-1 = 2(3) + c$
   - $-1 = 6 + c$
   - $c = -7$
   - $y = 2x - 7$
   - or $2x - y - 7 = 0$
Find the equation of the line that passes through the point \((0, 3)\) and is perpendicular to a straight line with a gradient of 5.

**THINK**
1. For perpendicular lines, \(m_1 \times m_2 = -1\). Find the gradient of the perpendicular line.
2. Use the equation \(y - y_1 = m(x - x_1)\) where \(m = -\frac{1}{5}\) and \((x_1, y_1) = (0, 3)\).

**WRITE**

Given \(m_1 = 5\)  
\(m_2 = -\frac{1}{5}\)  

Since \(y - y_1 = m(x - x_1)\) and \((x_1, y_1) = (0, 3)\), then \(y - 3 = -\frac{1}{5} (x - 0)\)  
\[y - 3 = -\frac{x}{5}\]  
\[5(y - 3) = -x\]  
\[5y - 15 = -x\]  
\[x + 5y - 15 = 0\]

**Horizontal and vertical lines**

- Horizontal lines are parallel to the \(x\)-axis, have a gradient of zero, are expressed in the form \(y = c\) and have no \(x\)-intercept.
- Vertical lines are parallel to the \(y\)-axis, have an undefined (infinite) gradient, are expressed in the form \(x = a\) and have no \(y\)-intercept.

**WORKED EXAMPLE 24**

Find the equation of:

- **a** the vertical line that passes through the point \((2, -3)\)
- **b** the horizontal line that passes through the point \((-2, 6)\).

**THINK**

- **a** The equation of a vertical line is \(x = a\). The \(x\)-coordinate of the given point is 2.
- **b** The equation of a horizontal line is \(y = c\). The \(y\)-coordinate of the given point is 6.

**WRITE**

- **a** \(x = 2\)
- **b** \(y = 6\)
WORKED EXAMPLE 25

Find the equation of the perpendicular bisector of the line joining the points (0, −4) and (6, 5). (A bisector is a line that crosses another line at right angles and cuts it into two equal lengths.)

THINK

1. Find the gradient of the line joining the given points by applying the formula.
   \[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

2. Find the gradient of the perpendicular line.
   \[ m_1 \times m_2 = -1 \]

3. Find the midpoint of the line joining the given points.
   \[ M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]
   \[ (x_1, y_1) = (0, -4) \text{ and } (x_2, y_2) = (6, 5). \]

4. Find the equations of the line with gradient \(-\frac{2}{3}\) that passes through \(\left(3, \frac{1}{2}\right)\).

5. Simplify by removing the fractions.
   Multiply both sides by 3.
   Multiply both sides by 2.

WRITE

Let \((0, -4) = (x_1, y_1)\).
Let \((6, 5) = (x_2, y_2)\).

\[ m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-4)}{6 - 0} = \frac{9}{6} = \frac{3}{2} \]

\[ m_2 = \frac{\text{gradient of perpendicular line}}{m_1} = \frac{-1}{\frac{3}{2}} = -\frac{2}{3} \]

\[ x = \frac{x_1 + x_2}{2} = \frac{0 + 6}{2} = 3 \]
\[ y = \frac{y_1 + y_2}{2} = -\frac{4 + 5}{2} = \frac{-9}{2} = \frac{-1}{2} \]

Hence \(\left(3, \frac{1}{2}\right)\) are the coordinates of the midpoint.

Since \(y - y_1 = m(x - x_1)\),
then \(y - \frac{1}{2} = -\frac{2}{3}(x - 3)\)

\[ 3(y - \frac{1}{2}) = -2(x - 3) \]
\[ 3y - \frac{3}{2} = -2x + 6 \]
\[ 6y - 3 = -4x + 12 \]
\[ 4x + 6y - 15 = 0 \]

Exercise 3.6 Parallel and perpendicular lines

INDIVIDUAL PATHWAYS

<table>
<thead>
<tr>
<th>PRACTICE</th>
<th>CONSOLIDATE</th>
<th>MASTER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Questions: 1a–d, 2, 5, 6a–c, 7, 8, 9a–c, 12, 13, 16a–b, 18, 20a, 21, 23, 26a, 27</td>
<td>Questions: 1a–d, 2–5, 6c–d, 7, 8, 9a–c, 12, 13, 15, 16a–b, 17a, 18, 20a, 21–23, 26–28, 30, 32</td>
<td>Questions: 1c–f, 2, 3, 4, 5, 6e–f, 7–19, 20b, 21, 22, 24–31, 33–37</td>
</tr>
</tbody>
</table>

REFLECTION

How could you use coordinate geometry to design a logo for an organisation?
FLUENCY

1. WE19 Find whether AB is parallel to CD given the following sets of points.
   a. A (4, 13), B (2, 9), C (0, -10), D (15, 0)
   b. A (2, 4), B (8, 1), C (-6, -2), D (2, -6)
   c. A (-3, -10), B (1, 2), C (1, 10), D (8, 16)
   d. A (1, -1), B (4, 11), C (2, 10), D (-1, -5)
   e. A (1, 0), B (2, 5), C (3, 15), D (7, 35)
   f. A (1, -6), B (-5, 0), C (0, 0), D (5, -4)

2. Which pairs of the following straight lines are parallel?
   a. 2x + y + 1 = 0
   b. y = 3x - 1
   c. 2y - x = 3
   d. y = 4x + 3
   e. y = \frac{x}{2} - 1
   f. 6x - 2y = 0
   g. 3y = x + 4
   h. 2y = 5 - x

3. WE20 Show that the points A (0, -2), B (5, 1) and C (-5, -5) are collinear.

4. Show that the line that passes through the points (-4, 9) and (0, 3) also passes through the point (6, -6).

5. WE21 Show that the lines y = 6x - 3 and x + 6y - 6 = 0 are perpendicular to one another.

6. Determine whether AB is perpendicular to CD, given the following sets of points.
   a. A (1, 6), B (3, 8), C (4, -6), D (-3, 1)
   b. A (2, 12), B (-1, -9), C (0, 2), D (7, 1)
   c. A (1, 3), B (4, 18), C (-5, 4), D (5, 0)
   d. A (1, -5), B (0, 0), C (5, 11), D (-10, 8)
   e. A (-4, 9), B (2, -6), C (5, 8), D (10, 14)
   f. A (4, 4), B (-8, 5), C (-6, 2), D (3, 11)

7. WE22 Find the equation of the line that passes through the point (4, -1) and is parallel to the line with equation y = 2x - 5.

8. WE23 Find the equation of the line that passes through the point (-2, 7) and is perpendicular to a line with a gradient of \( \frac{2}{3} \).

9. Find the equations of the following lines.
   a. Gradient 3 and passing through the point (1, 5)
   b. Gradient -4 and passing through the point (2, 1)
   c. Passing through the points (2, -1) and (4, 2)
   d. Passing through the points (1, -3) and (6, -5)
   e. Passing through the point (5, -2) and parallel to x + 5y + 15 = 0
   f. Passing through the point (1, 6) and parallel to x - 3y - 2 = 0
   g. Passing through the point (-1, -5) and perpendicular to 3x + y + 2 = 0

10. Find the equation of the line that passes through the point (-2, 1) and is:
    a. parallel to the line with equation 2x - y - 3 = 0
    b. perpendicular to the line with equation 2x - y - 3 = 0.

11. Find the equation of the line that contains the point (1, 1) and is:
    a. parallel to the line with equation 3x - 5y = 0
    b. perpendicular to the line with equation 3x - 5y = 0.
12 **WE24** Find the equation of:
   a the vertical line that passes through the point (1, −8)
   b the horizontal line that passes through the point (−5, −7).

13 **MC** a The vertical line passing through the point (3, −4) is given by:
   A $y = −4$ 
   B $x = 3$ 
   C $y = 3x − 4$
   D $y = −4x + 3$ 
   E $x = −4$
   b Which of the following points does the horizontal line given by the equation $y = −5$ pass through?
   A (−5, 4) 
   B (4, 5) 
   C (3, −5)
   D (5, −4) 
   E (5, 5)
   c Which of the following statements is true?
   A Vertical lines have a gradient of zero.
   B The $y$-coordinates of all points on a vertical line are the same.
   C Horizontal lines have an undefined gradient.
   D The $x$-coordinates of all points on a vertical line are the same.
   E A horizontal line has the general equation $y = a$.
   d Which of the following statements is false?
   A Horizontal lines have a gradient of zero.
   B The line joining the points (1, −1) and (−7, −1) is vertical.
   C Vertical lines have an undefined gradient.
   D The line joining the points (1, 1) and (−7, 1) is horizontal.
   E A horizontal line has the general equation $y = c$.

14 The triangle $ABC$ has vertices $A (9, −2)$, $B (3, 6)$, and $C (1, 4)$.
   a Find the midpoint, $M$, of $BC$.
   b Find the gradient of $BC$.
   c Show that $AM$ is the perpendicular bisector of $BC$.
   d Describe triangle $ABC$.

15 **WE25** Find the equation of the perpendicular bisector of the line joining the points (1, 2) and (−5, −4).

16 Find the equation of the perpendicular bisector of the line joining the points (−2, 9) and (4, 0).

17 $ABCD$ is a parallelogram. The coordinates of $A$, $B$ and $C$ are (4, 1), (1, −2) and (−2, 1) respectively. Find:
   a the equation of $AD$
   b the coordinates of $D$.
   c the equation of $DC$.

**UNDERSTANDING**

18 In each of the following, show that $ABCD$ is a parallelogram.
   a $A (2, 0)$, $B (4, −3)$, $C (2, −4)$, $D (0, −1)$
   b $A (2, 2)$, $B (0, −2)$, $C (−2, −3)$, $D (0, 1)$
   c $A (2.5, 3.5)$, $B (10, −4)$, $C (2.5, −2.5)$, $D (−5, 5)$

19 In each of the following, show that $ABCD$ is a trapezium.
   a $A (0, 6)$, $B (2, 2)$, $C (0, −4)$, $D (−5, −9)$
   b $A (26, 32)$, $B (18, 16)$, $C (1, −1)$, $D (−3, 3)$
   c $A (2, 7)$, $B (1, −1)$, $C (−0.6, −2.6)$, $D (−2, 3)$
20 **MC** The line that passes through the points \((0, -6)\) and \((7, 8)\) also passes through:
- A \((4, 3)\)
- B \((5, 4)\)
- C \((-2, 10)\)
- D \((1, -8)\)
- E \((1, 4)\)

21 **MC** The point \((-1, 5)\) lies on a line parallel to \(4x + y + 5 = 0\). Another point on the same line as \((-1, 5)\) is:
- A \((2, 9)\)
- B \((4, 2)\)
- C \((4, 0)\)
- D \((-2, 3)\)
- E \((3, -11)\)

22 Find the equation of the straight line given the following conditions.
- a Passes through the point \((-1, 3)\) and parallel to \(y = -2x + 5\)
- b Passes through the point \((4, -3)\) and parallel to \(3y + 2x = -3\)

23 Determine which pairs of the following lines are perpendicular.
- a \(x + 3y - 5 = 0\)
- b \(y = 4x - 7\)
- c \(y = x\)
- d \(2y = x + 1\)
- e \(y = 3x + 2\)
- f \(x + 4y - 9 = 0\)
- g \(2x + y = 6\)
- h \(x + y = 0\)

24 Find the equation of the straight line that cuts the \(x\)-axis at 3 and is perpendicular to the line with equation \(3y - 6x = 12\).

25 Calculate the value of \(m\) for which lines with the following pairs of equations are perpendicular to each other.
- a \(2y - 5x = 7\) and \(4y + 12 = mx\)
- b \(5x - 6y = -27\) and \(15 + mx = -3y\)

26 **MC** The gradient of the line perpendicular to the line with equation \(3x - 6y = 2\) is:
- A \(3\)
- B \(-6\)
- C \(2\)
- D \(\frac{1}{2}\)
- E \(-2\)

27 **MC** Triangle ABC has a right angle at B. The vertices are A\((-2, 9)\), B\((2, 8)\) and C\((1, z)\). The value of \(z\) is:
- A \(8\frac{3}{4}\)
- B \(4\)
- C \(12\)
- D \(7\frac{3}{4}\)
- E \(-4\)

**REASONING**

28 The map shows the proposed course for a yacht race. Buoys have been positioned at A \((1, 5)\), B \((8, 8)\), C \((12, 6)\), and D \((10, w)\).
- a How far is it from the start, O, to buoy A?
- b The race marshall boat, M, is situated halfway between buoys A and C. What are the coordinates of the boat’s position?
- c Stage 4 of the race (from C to D) is perpendicular to stage 3 (from B to C). What is the gradient of CD?
- d Find the linear equation that describes stage 4.
- e Hence determine the exact position of buoy D.
- f An emergency boat is to be placed at point E, \(7, 3\). How far is the emergency boat from the hospital, located at H, 2 km north of the start?

29 Show that the following sets of points form the vertices of a right-angled triangle.
- a A \((1, -4)\), B \((2, -3)\), C \((4, -7)\)
- b A \((3, 13)\), B \((1, 3)\), C \((-4, 4)\)
- c A \((0, 5)\), B \((9, 12)\), C \((3, 14)\)

30 Prove that the quadrilateral ABCD is a rectangle when A is \((2, 5)\), B\((6, 1)\), C \((3, -2)\) and D \((-1, 2)\).
31 Prove that the quadrilateral ABCD is a rhombus, given A (2, 3), B (3, 5), C (5, 6) and D (4, 4).

_Hint:_ A rhombus is a parallelogram with diagonals that intersect at right angles.

32 a A square has vertices at (0, 0) and (2, 0). Where are the other 2 vertices? (There are 3 sets of answers.)
b An equilateral triangle has vertices at (0, 0) and (2, 0). Where is the other vertex? (There are 2 answers.)
c A parallelogram has vertices at (0, 0) and (2, 0) and (1, 1). Where is the other vertex? (There are 3 sets of answers.)

33 A is the point (0, 0) and B is the point (0, 2).

a Find the perpendicular bisector of AB.
b Show that any point on this line is equidistant from A and B.

**Questions 34 and 35 relate to the diagram.**

M is the midpoint of OA.

N is the midpoint of AB.
P is the midpoint of OB.

34 A simple investigation:

a Show that MN is parallel to OB.
b Is PN parallel to OA?
c Is PM parallel to AB?

35 A difficult investigation:

a Find the perpendicular bisectors of OA and OB.
b Find the point W where the two bisectors intersect.
c Show that the perpendicular bisector of AB also passes through W.
d Explain why W is equidistant from O, A and B.
e W is called the circumcentre of triangle OAB. Using W as the centre, draw a circle through O, A and B.

**PROBLEM SOLVING**

36 The lines $l_1$ and $l_2$ are at right angles to each other. The line $l_1$ has the equation $px + py + r = 0$. Show that the distance from M to the origin is given by $\frac{r}{\sqrt{p^2 + p^2}}$. 

**Diagram:**

- A diagram showing a coordinate plane with points A (4, 6), B (2, 4), C (5, 6) and D (4, 4) forming a rhombus.
- A graph showing lines $l_1$ and $l_2$ at right angles with point M on the line $l_2$.

**Graph:**

- A graph with axes labeled x and y, showing points A (4, 6), B (2, 4), C (5, 6) and D (4, 4).
- Lines $l_1$ and $l_2$ intersecting at right angles, with point M on the line $l_2$. 

**Diagram:**

- A grid with labeled axes and a point M labeled.
- Lines $l_1$ and $l_2$ intersecting at right angles with point M on the line $l_2$. 

**Graph:**

- A graph with axes labeled x and y, showing lines $l_1$ and $l_2$ at right angles.
37 Line A is parallel to the line with equation $2x - y = 7$ and passes through the point $(2, 3)$. Line B is perpendicular to the line with equation $4x - 3y + 3 = 0$ and also passes through the point $(2, 3)$. Line C intersects with line A where it cuts the y-axis and intersects with line B where it cuts the x-axis.

a Determine the equations for all three lines. Give answers in the form $ax + by + c = 0$.

b Sketch all three lines on the one set of axes.

c Determine whether the triangle formed by the three lines is scalene, isosceles or equilateral.

**CHALLENGE 3.2**
The first six numbers of a particular number pattern are 1, 2, 3, 6, 11 and 20. Given that this pattern continues, what will be the next four numbers? Describe the pattern.
3.7 Review

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:
• **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
• **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

<table>
<thead>
<tr>
<th>axes</th>
<th>horizontal</th>
<th>rise</th>
</tr>
</thead>
<tbody>
<tr>
<td>bisect</td>
<td>independent variable</td>
<td>run</td>
</tr>
<tr>
<td>Cartesian plane</td>
<td>linear graph</td>
<td>segment</td>
</tr>
<tr>
<td>collinear</td>
<td>midpoint</td>
<td>substitute</td>
</tr>
<tr>
<td>coordinates</td>
<td>origin</td>
<td>trapezium</td>
</tr>
<tr>
<td>dependent variable</td>
<td>parallel</td>
<td>vertical</td>
</tr>
<tr>
<td>diagonal</td>
<td>parallelogram</td>
<td>vertices</td>
</tr>
<tr>
<td>general form</td>
<td>perpendicular</td>
<td>x-intercept</td>
</tr>
<tr>
<td>gradient</td>
<td>quadrilateral</td>
<td>y-intercept</td>
</tr>
<tr>
<td>gradient–intercept form</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The story of mathematics

is an exclusive Jacaranda video series that explores the history of mathematics and how it helped shape the world we live in today.

Descartes (eles-1842) tells the story of Rene Descartes, a French philosopher and mathematician who brought the concepts of geometry and algebra together, developing the two-dimensional grid we know as the Cartesian plane.

Link to assessON for questions to test your readiness FOR learning, your progress AS you learn and your levels OF achievement.

assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

www.assesson.com.au
On computer hardware, and on many different software applications, a broad range of symbols is used. These symbols help us to identify where things need to be plugged into, what buttons we need to push, or what option needs to be selected. The main focus of this task involves constructing a common symbol found on the computer. The instructions are given below. Grid lines have been provided on the opposite page for you to construct the symbol.

The construction part of this task requires you to graph nine lines to reveal a common computer symbol. Draw the scale of your graph to accommodate $x$- and $y$-values in the following ranges: $-10 \leq x \leq 16$ and $-10 \leq y \leq 16$. Centre the axes on the grid lines provided.

- Line 1 has an equation $y = x - 1$. Graph this line in the range $-7 \leq x \leq -2$.
- Line 2 is perpendicular to line 1 and has a $y$-intercept of $-5$. Determine the equation of this line, and then draw the line in the range $-8 \leq x \leq -1$.
- Line 3 is parallel to line 1, with a $y$-intercept of $3$. Determine the equation of the line, and then graph the line in the range $-9 \leq x \leq -4$.
- Line 4 is parallel to line 1, with a $y$-intercept of $-3$. Determine the equation of the line, and then graph the line in the range $-1 \leq x \leq 2$. 

What common computer symbol is this?
• Line 5 has the same length as line 4 and is parallel to it. The point \((-2, 3)\) is the starting point of the line, which decreases in both \(x\)- and \(y\)-values from there.
• Line 6 commences at the same starting point as line 5, and then runs at right angles to line 5. It has an \(x\)-intercept of 1 and is the same length as line 2.
• Line 7 commences at the same starting point as both lines 5 and 6. Its equation is \(y = 6x + 15\). The point \((-1, 9)\) lies at the midpoint.
• Line 8 has the equation \(y = -x + 15\). Its midpoint is the point \((7, 8)\) and its extremities are the points where the line meets line 7 and line 9.
• Line 9 has the equation \(6y - x + 8 = 0\). It runs from the intersection of lines 4 and 6 until it meets line 8.

1 What common computer symbol have you drawn?
2 The top section of your figure is a familiar geometric shape. Use the coordinates on your graph, together with the distance formula to determine the necessary lengths to calculate the area of this figure.
3 Using any symbol of interest to you, draw your symbol on grid lines and provide instructions for your design. Ensure that your design involves aspects of coordinate geometry that have been used throughout this task.
Who won the inaugural 875 km Sydney to Melbourne marathon in 1983?

The equations of the straight lines, in the form \( y = mx + c \), that fit the given information and the letter beside each give the puzzle’s answer code.

- **A**: gradient of –2 and \( y \)-intercept of 2
- **B**: passes through (2, 5) and (0, 3)
- **C**: gradient of 1 and passes through (5, 4)
- **D**: \( m = -1 \) and \( c = 1 \)
- **E**: \( m = 4 \) and passes through (–2, –3)
- **F**: \( m = -1 \) and \( c = -3 \)
- **G**: gradient of 3 and \( y \)-intercept of 4
- **H**: passes through (0, –3) and (3, 0)
- **I**: passes through (2, 7) and (–3, 2)
- **J**: \( m = 1 \) and \( c = -2 \)
- **K**: passes through (2, 5) and (0, 3)
- **L**: passes through (2, 7) and (–3, 2)
- **M**: \( m = -4 \) and passes through (1, 0)
- **N**: passes through (5, 9) and (0, –1)
- **O**: \( m = 2 \) and passes through (–2, –1)
- **P**: \( m = -3 \) and \( c = 1 \)
- **Q**: passes through (3, –8) and has a gradient of –1
- **R**: passes through (1, 2) and (3, 10)
- **S**: \( m = -3 \) and \( c = -1 \)
- **T**: passes through (2, –3) with a gradient of –2
- **U**: passes through (0, 2) and (2, 0)
- **V**: \( y = -x - 5 \)
- **W**: \( y = -x + 3 \)
- **X**: \( y = -2x + 1 \)
- **Y**: \( y = -4x - 2 \)
- **Z**: \( y = x + 2 \)
- **AA**: \( y = 2x - 1 \)
- **AB**: \( y = -x + 1 \)
- **AC**: \( y = -x + 2 \)
- **AD**: \( y = 2x + 1 \)
- **AE**: \( y = 2x + 1 \)
- **AF**: \( y = 2x + 1 \)
- **AG**: \( y = 2x + 1 \)
- **AH**: \( y = 2x + 1 \)
- **AI**: \( y = 2x + 1 \)
- **AJ**: \( y = 2x + 1 \)
- **AK**: \( y = 2x + 1 \)
- **AL**: \( y = 2x + 1 \)
- **AM**: \( y = 2x + 1 \)
- **AN**: \( y = 2x + 1 \)
- **AO**: \( y = 2x + 1 \)
- **AP**: \( y = 2x + 1 \)
- **AQ**: \( y = 2x + 1 \)
- **AR**: \( y = 2x + 1 \)
- **AS**: \( y = 2x + 1 \)
- **AT**: \( y = 2x + 1 \)
- **AU**: \( y = 2x + 1 \)
- **AV**: \( y = 2x + 1 \)
- **AW**: \( y = 2x + 1 \)
- **AX**: \( y = 2x + 1 \)
- **AY**: \( y = 2x + 1 \)
- **AZ**: \( y = 2x + 1 \)
3.1 Overview

**Video**
- The story of mathematics (eles-1842)

**3.2 Sketching linear graphs**

**Interactivity**
- IP interactivity 3.2 (int-4572): Sketching linear graphs

**Digital docs**
- SkillSHEET (doc-5197): Describing the gradient of a line
- SkillSHEET (doc-5198): Plotting a line using a table of values
- SkillSHEET (doc-5199): Stating the $y$-intercept from a graph
- SkillSHEET (doc-5200): Solving linear equations that arise when finding $x$- and $y$-intercepts
- SkillSHEET (doc-5201): Using Pythagoras’ theorem
- SkillSHEET (doc-5202): Substitution into a linear rule
- SkillSHEET (doc-5203): Transposing linear equations to standard form

**3.3 Determining linear equations**

**Interactivity**
- IP interactivity 3.3 (int-4573): Determining linear equations

**Digital docs**
- SkillSHEET (doc-5196): Measuring the rise and the run
- SkillSHEET (doc-5204): Finding the gradient given two points
- WorkSHEET 3.1 (doc-13849): Gradient

**3.4 The distance between two points**

**Interactivity**
- IP interactivity 3.4 (int-4574): The distance between two points

**Digital docs**
- Spreadsheet (doc-5206): Distance between two points

**3.5 The midpoint of a line segment**

**Interactivity**
- IP interactivity 3.5 (int-4575): The midpoint of a line segment

**Digital docs**
- Spreadsheet (doc-5207): Midpoint of a segment
- WorkSHEET 3.2 (doc-13850): Midpoint of a line segment

**3.6 Parallel and perpendicular lines**

**Interactivities**
- Parallel and perpendicular lines (int-2779)
- IP interactivity 3.6 (int-4576): Parallel and perpendicular lines

**Digital docs**
- Spreadsheet (doc-5209): Perpendicular checker
- Spreadsheet (doc-5210): Equation of a straight line

**3.7 Review**

**Interactivities**
- Word search (int-2832)
- Crossword (int-2833)
- Sudoku (int-3590)

**Digital docs**
- Topic summary (doc-13713)
- Concept map (doc-13714)
Answers

**TOPIC 3 Coordinate geometry**

**Exercise 3.2 — Sketching linear graphs**

1 a

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>−5</td>
<td>−25</td>
</tr>
<tr>
<td>−4</td>
<td>−15</td>
</tr>
<tr>
<td>−3</td>
<td>−5</td>
</tr>
<tr>
<td>−2</td>
<td>5</td>
</tr>
<tr>
<td>−1</td>
<td>15</td>
</tr>
<tr>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td>1</td>
<td>35</td>
</tr>
</tbody>
</table>

1 b

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>−1</td>
<td>−17</td>
</tr>
<tr>
<td>0</td>
<td>−12</td>
</tr>
<tr>
<td>1</td>
<td>−7</td>
</tr>
<tr>
<td>2</td>
<td>−2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

1 c

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>−6</td>
<td>13</td>
</tr>
<tr>
<td>−4</td>
<td>12</td>
</tr>
<tr>
<td>−2</td>
<td>11</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

1 d

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>−240</td>
</tr>
<tr>
<td>1</td>
<td>−140</td>
</tr>
<tr>
<td>2</td>
<td>−40</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
</tr>
<tr>
<td>4</td>
<td>160</td>
</tr>
<tr>
<td>5</td>
<td>260</td>
</tr>
</tbody>
</table>

1 e

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>−3</td>
<td>18</td>
</tr>
<tr>
<td>−2</td>
<td>13</td>
</tr>
<tr>
<td>−1</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>−2</td>
</tr>
<tr>
<td>2</td>
<td>−7</td>
</tr>
</tbody>
</table>

1 f

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>−3</td>
<td>19</td>
</tr>
<tr>
<td>−2</td>
<td>15</td>
</tr>
<tr>
<td>−1</td>
<td>11</td>
</tr>
<tr>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>−1</td>
</tr>
</tbody>
</table>

2 a

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>−6</td>
<td>20</td>
</tr>
<tr>
<td>−4</td>
<td>14</td>
</tr>
<tr>
<td>−2</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>−4</td>
</tr>
<tr>
<td>4</td>
<td>−10</td>
</tr>
<tr>
<td>6</td>
<td>−16</td>
</tr>
</tbody>
</table>

2 b

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>−3</td>
<td>6</td>
</tr>
<tr>
<td>−2</td>
<td>5</td>
</tr>
<tr>
<td>−1</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>
Exercise 3.3 — Determining linear equations

1 a \( y = 2x + 4 \) 
   b \( y = -3x + 12 \) 
   c \( y = -x + 5 \) 
   d \( y = 2x - 8 \) 
   e \( y = \frac{1}{2}x + 3 \) 
   f \( y = \frac{1}{4}x - 4 \) 
   g \( y = 7x - 5 \) 
   h \( y = -3x - 15 \) 

2 a \( y = 2x \) 
   b \( y = -3x \) 
   c \( y = \frac{1}{2}x \) 
   d \( y = -\frac{3}{2}x \) 

3 a \( y = x + 3 \) 
   b \( y = 2x - 1 \) 
   c \( y = \frac{1}{2}x + \frac{7}{2} \) 
   d \( y = \frac{3}{2}x + \frac{1}{2} \) 
   e \( y = -2x - 2 \) 
   f \( y = -x - 8 \) 

4 a \( y = 3x + 3 \) 
   b \( y = -4x + 2 \) 
   c \( y = -x - 4 \) 
   d \( y = 5x + 2.5 \) 
   e \( y = -2.5x + 1.5 \) 
   f \( y = -3x + 4 \) 
   g \( y = 4x + 2 \) 
   h \( y = 0.5x - 4 \) 
   i \( y = 6x + 3 \) 
   j \( y = 3.5x + 6.5 \)
Exercise 3.4 — The distance between two points

1. \((-3, -3\frac{1}{2})\)  
2. \((-3, -10)\)

3. \((3, 1)\)  
4. \(D\)

5. \(C\)

6. Answers will vary.

7. Answers will vary.

8. Answers will vary.

Exercise 3.5 — The midpoint of a line segment

1. \((-3, -3\frac{1}{2})\)  
2. \((0, 1\frac{1}{2})\)

3. \((-2, 2)\)

4. \((3, 1)\)  
5. \(C\)

6. Answers will vary.

7. Answers will vary.

8. Answers will vary.

Exercise 3.6 — Parallel and perpendicular lines

1. No  
2. Yes  
3. No  
4. Answers will vary.

5. Answers will vary.

6. No  
7. Answers will vary.

8. Answers will vary.

9. Answers will vary.

10. Answers will vary.

11. Teacher to check

12. \((4k - 1, 3.5 - 5k)\)

Exercise 3.7 — Quadrilaterals

1. Answers will vary.

2. Answers will vary.

3. Answers will vary.

4.Answers will vary.

5. Answers will vary.

6. Answers will vary.

7. Answers will vary.

8. Answers will vary.

9. Answers will vary.

10. Answers will vary.

11. Teacher to check

12. \((4k - 1, 3.5 - 5k)\)

Exercise 3.8 — Coordinate geometry

1. \((0, 1/2)\)  
2. \((3, 1)\)

3. \((2, 5)\)  
4. \(B\)

5. Answers will vary.

6. Answers will vary.

7. Answers will vary.

8. Answers will vary.

9. Answers will vary.

10. Answers will vary.

11. Teacher to check

12. \((4k - 1, 3.5 - 5k)\)
35  a  OA: 2x + 3y − 13 = 0; OB: x = 3
  b  (3, \frac{7}{2})
  c, d  Answers will vary.
36  Teacher to check
37  a  Line A: 2x − y − 1 = 0, Line B: 3x + 4y − 18 = 0, Line C: x − 6y − 6 = 0

**Investigation — Rich task**

1  The symbol is the one used to represent a speaker.
2  The shape is a trapezium.
   Area = \frac{1}{2} \left( \text{length line 6} + \text{length line 8} \right) \times \text{perpendicular distance between these lines.}
   = \frac{1}{2} \left( 4\sqrt{2} + 14\sqrt{2} \right) \times 7\sqrt{2}
   = 126 \text{ units}^2
3  Teacher to check

**Code puzzle**
Sixty-one-year-old potato farmer Cliff Young in five days and fifteen hours.

**Challenge 3.2**
37, 68, 125, 230. To find the next number, add the three preceding numbers.