Why learn this?
Geometry is a study of points, lines and angles and how they combine to make different shapes. Similarity and congruence between two figures are important concepts in geometry. Recognising and using congruent and similar shapes can make calculations and design more effective.

What do you know?
1 THINK List what you know about congruent and similar figures. Use a thinking tool such as a concept map to show your list.
2 PAIR Share what you know with a partner and then with a small group.
3 SHARE As a class, create a thinking tool such as a large concept map to show your class's knowledge of congruent and similar figures.

Learning sequence
5.1 Overview
5.2 Ratio and scale
5.3 Congruent figures
5.4 Similar figures
5.5 Area and volume of similar figures
5.6 Review
WATCH THIS VIDEO
The story of mathematics: Mathematics in art
5.2 Ratio and scale

Ratio

- **Ratios** are used to compare quantities of the same kind, measured in the same unit.
- The ratio ‘1 is to 4’ can be written in two ways: as 1 : 4 or as \(\frac{1}{4}\).
- The order of the numbers in a ratio is important.

**WORKED EXAMPLE 1**

A lighthouse is positioned on an 80 m high cliff. A ship at sea is 3600 m from the base of the cliff.

**A** Write the following ratios in simplest form.

i Height of the cliff to the distance of the ship from shore

\[ \text{Height of the cliff : distance of ship from shore} = 80 : 3600 \]

\[ = \frac{80}{3600} \]

\[ = \frac{1}{45} \]

ii Distance of the ship from shore to the height of the cliff

\[ \text{Distance of ship from shore : height of cliff} = \frac{3600}{80} \]

\[ = \frac{45}{1} \]

**THINK**

a i 1 The height and distance are in the same unit (m). Write the height first.

2 Simplify the ratio by dividing both terms by the highest common factor (80).

ii 1 Write the distance from the ship to the shore first.

2 Simplify.

**Note:** Do not write \(\frac{45}{1}\) as 45, because a ratio is a comparison of two numbers.

**b** Compare the distance of the ship from shore with the height of the cliff.

\[ \text{The distance of the ship from shore is 45 times the height of the cliff.} \]

**WORKED EXAMPLE 2**

Express each of the following ratios in simplest form.

\[ a \ 24 : 8 \]

\[ b \ 3.6 : 8.4 \]

\[ c \ \frac{4}{5} : \frac{2}{3} \]

**THINK**

a Divide both terms by the highest common factor (8).

**WRITE**

a 24 : 8

\[ = \frac{24}{8} \]

\[ = 3 : 1 \]
b 1 Multiply both terms by 10 to obtain whole numbers.

b 3.6 : 8.4
   = 36 : 84
   = 3 : 7

2 Divide both terms by the highest common factor, (12).

3.6 : 8.4
   = 3 : 7

3 Change both mixed numbers into improper fractions.

\[ 1\frac{4}{9} : 1\frac{2}{3} \]
\[ = \frac{13}{9} : \frac{5}{3} \]
\[ = 13 : 15 \]

A proportion is a statement that indicates that two ratios are equal. A proportion can be written in two ways: \( 4 : 7 = x : 15 \) or \( \frac{2}{3} = \frac{11.5}{x} \).

**WORKED EXAMPLE 3**

Find the value of \( x \) in the proportion \( 4 : 9 = 7 : x \).

**THINK**

1 Write the ratios as equal fractions.
\[ \frac{4}{9} = \frac{7}{x} \]

2 Multiply both sides by \( x \).
\[ \frac{4x}{9} = 7 \]

3 Solve the equation.
\[ 4x = 63 \]
\[ x = 15.75 \]

**Scale**

- Ratios are used when producing scale drawings or maps.
- Consider the case where we want to enlarge triangle ABC (called the object) by a scale factor of 2, that is, to make it twice its size.

Here is one method that we can use.
1. Mark a point O somewhere outside the triangle and draw the lines OA, OB and OC as shown.

2. Measure the length of OA and mark in the point A' (called the image of A) so that the distance OA' is twice that of OA.

3. In the same way, mark in points B' and C' (OB' = 2 × OB, and OC' = 2 × OC.)

4. Joining A'B'C' gives a triangle that has side lengths double those of ΔABC. ΔA'B'C' is called the image of ΔABC.

- By definition:

\[
\text{scale factor} = \frac{\text{image length}}{\text{object length}}
\]
Enlarge triangle ABC by a scale factor of 3, with the centre of enlargement at point O.

THINK

1. Join each vertex of the triangle to the centre of enlargement O with straight lines and extend them.
2. Locate points A', B' and C' along the lines, OA' = 3OA, OB' = 3OB and OC' = 3OC.
3. Join points A', B' and C' to complete the image.

• Enlargements have the following properties.
  – The corresponding side lengths of the enlarged figure are changed in a fixed ratio (that is, the same ratio).
  – The corresponding angles are the same.
  – A scale factor greater than 1 produces an enlarged figure.
  – If the scale factor is a positive number less than 1, the image is smaller than the object (reduction has taken place).

WORKED EXAMPLE 5

A triangle PQR has been enlarged to triangle P'Q'R'. PQ = 4 cm, PR = 6 cm, P'Q' = 10 cm and Q'R' = 20 cm. Calculate:

a. the scale factor for the enlargement
b. the length of P'R'
c. the length of QR.

THINK

a. 1. Draw a diagram.

WRITE/DRAW

2. Find two corresponding sides. P'Q' corresponds to PQ.

\[
\text{Scale factor} = \frac{\text{image length}}{\text{object length}} = \frac{P'Q'}{PQ} = \frac{10}{4} = 2.5
\]
b 1 Apply the scale factor.
P’R’ = 2.5 \times PR

2 Write the answer.

b 1 P’R’ = 2.5 \times PR

\[ P’R’ = 2.5 \times 6 = 15 \]
P’R’ is 15 cm long.

2 Write the answer.

2

c 1 Apply the scale factor.
Q’R’ = 2.5 \times QR

2 Write the answer.

\[ Q’R’ = 2.5 \times QR \]
\[ QR = \frac{20}{2.5} = 8 \]

QR is 8 cm long.

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**Exercise 5.2 Ratio and scale**

**INDIVIDUAL PATHWAYS**

**PRACTISE**
Questions:
1, 2, 3a–f, 4a–f, 5a–e, 6a–f, 7–16, 19

**CONSOLIDATE**
Questions:
1, 2, 3d–i, 4e–j, 5b–g, 6e–j, 7–14, 16, 17, 20–22

**MASTER**
Questions:
1, 2, 3f–j, 4g–l, 5f–i, 6g–l, 7–14, 17, 18, 20, 23, 24

**FLUENCY**

1 **WE1** This horse track is 1200 m long and 35 m wide.

a Write the following ratios in simplest form.

i Track length to track width

\[ a = \frac{1200}{35} \]

ii Track width to track length

\[ b = \frac{35}{1200} \]

b Compare the distance of the length of the track with the width of the track.

2 A dingo perched on top of a cliff spots an emu on the ground below.

a Write the following ratios in simplest form.

i Cliff height to distance from cliff base to emu

\[ c = \frac{20}{8} \]

ii Distance of emu from cliff base cliff height

\[ d = \frac{8}{20} \]

b Compare the height of the cliff with the ground distance from the base of the cliff to the emu.
3. **WE2a** Express each of the following ratios in the simplest form.
   - a. 12 : 18
   - b. 8 : 56
   - c. 9 : 27
   - d. 14 : 35
   - e. 88 : 66
   - f. 16 : 60
   - g. 200 : 155
   - h. 144 : 44
   - i. 32 : 100
   - j. 800 : 264

4. **WE2b** Express each of the following ratios in simplest form.
   - a. 1.2 : 0.2
   - b. 3.9 : 4.5
   - c. 9.6 : 2.4
   - d. 18 : 3.6
   - e. 1.8 : 3.6
   - f. 4.4 : 0.66
   - g. 0.9 : 5.4
   - h. 0.35 : 0.21
   - i. 6 : 1.2
   - j. 12.1 : 5.5
   - k. 8.6 : 4
   - l. 0.07 : 14

5. **WE2c** Write each of the following ratios in the simplest form.
   - a. $\frac{1}{2} : 2$
   - b. $2 : \frac{1}{3}$
   - c. $\frac{1}{2} : 2$
   - d. $\frac{1}{2} : \frac{1}{4}$
   - e. $\frac{3}{5} : \frac{2}{5}$
   - f. $\frac{4}{7} : 2$
   - g. $5 : \frac{1}{2}$
   - h. $23\frac{3}{4} : 1\frac{1}{5}$
   - i. $3\frac{2}{6} : 2\frac{1}{2}$
   - j. $1\frac{3}{5} : \frac{3}{2}$

6. **WE3** Find the value of the pronumeral in each of the following proportions.
   - a. $a : 15 = 3 : 5$
   - b. $b : 18 = 4 : 3$
   - c. $24 : c = 3 : 4$
   - d. $21 : d = 49 : 4$
   - e. $e : 33 = 5 : 44$
   - f. $6 : f = 5 : 12$
   - g. $3 : 4 = g : 5$
   - h. $9 : 8 = 5 : h$
   - i. $11 : 3 = i : 8$
   - j. $7 : 20 = 3 : j$
   - k. $15 : 13 = 12 : k$
   - l. $3 : 4 = l : 15$

7. **WE4** Enlarge each of the following figures by the given scale factor and the centre of enlargement marked O. Show the image of each figure.
   - a. 
     ![Diagram a]
     - SF = 2
   - b. 
     ![Diagram b]
     - SF = 3
   - c. 
     ![Diagram c]
     - SF = 1.5
   - d. 
     ![Diagram d]
     - SF = $\frac{1}{2}$

8. **WE5** A quadrilateral ABCD is enlarged to A'B'C'D'.
   - AB = 7 cm, AD = 4 cm, A'B' = 21 cm, B'C' = 10.5 cm.
   - Find:
     - a. the scale factor for enlargement
     - b. A'D'
     - c. BC.

9. ΔABC is scaled down to ΔA'B'C'. By measuring the side lengths, determine the scale factor.

   ![Diagram e]
UNDERSTANDING

10 The estimated volume of the Earth’s salt water is $1285\,600\,000$ cubic kilometres. The estimated volume of fresh water is about $35\,000\,000$ cubic kilometres.
   a What is the ratio of fresh water to salt water (in simplest form)?
   b Find the value of $x$, to the nearest whole number, when the ratio found in a is expressed in the form $1 : x$.

11 Super strength glue comes in two tubes which contain Part A and Part B pastes. These pastes have to be mixed in the ratio $1 : 4$ for maximum strength. How many mL of Part A would be needed for 10 mL of Part B?

12 A recipe states that butter and flour must be combined in the ratio $2 : 7$. How many grams of butter would be necessary for 3.5 kg of flour?

13 The diagram below shows the ground plan of a house. Bedroom 1 is $8 \times 4$ m.

   a Using the dimensions given for bedroom 1, find the scale factor when the actual house (object) is built from the plan (image).
   b Give an estimate of the dimensions of:
      i bedroom 3
      ii the kitchen.

REASONING

14 Pure gold is classed as 24-carat gold. This is too soft to use as jewellery, so it is combined with other metals to form an alloy. 18-carat gold contains gold and other metals in the ratio $18 : 6$. The composition of 18-carat rose gold is 75% gold, 22.25% copper and 2.75% silver.
   a Show the mass of silver in a 2.5-gram rose gold bracelet is 0.07 g.
   b Give the composition of a rose gold bracelet which has 0.5 g of copper.
15 The angles of a triangle are in the ratio 3 : 4 : 5. Show the sizes of the three angles are 45°, 60° and 75°.

16 The dimensions of a rectangular box are in the ratio 2 : 3 : 5 and its volume is 21 870 cm². Show the dimensions of the box are 18, 27 and 45 cm.

17 Tyler, Dylan and Aaron invested money in the ratio 11 : 9 : 4. If the profits are shared in the ratio 17 : 13 : 6, comment if this is fair for each person. Explain.

18 Five pens cost the same as 2 pens and 6 pencils and the same as 6 sharpeners and a pencil. Show a relationship between the cost of each item.

**PROBLEM SOLVING**

19 Gordon, a tourist at Kakadu National Park, takes a picture of a two-metre crocodile beside a cliff. When he develops his pictures, the two-metre crocodile is 2.5 cm long and the cliff is 8.5 cm high. What was the actual height of the cliff in cm?

20 Find the ratio of $y : z$ if $2x = 3y$ and $3x = 4z$.

21 The ratio of boys to girls among the students who signed up for a basketball competition is 4 : 3. If 3 boys drop out of the competition and 4 girls join, there will be the same number of boys and girls. How many students have signed up for the basketball competition?

22 Two quantities $P$ and $Q$ are in the ratio 2 : 3. If $P$ is reduced by 1, the ratio is $\frac{1}{2}$. Find the values of $P$ and $Q$.

23 In the group of students who voted in a Year 9 school leader election, the ratio of girls to boys is 2 : 3. If 10 more girls and 5 more boys had voted, the ratio would have been 3 : 4. How many students voted altogether?

24 Two cylinders are such that the ratio of their base radii is 2 : 1 and the ratio of their heights is 3 : 1. Find the ratio of their respective volumes.

### 5.3 Congruent figures

- **Congruent figures** are identical figures; that is, they have exactly the same shape and size.
- They can be superimposed exactly on top of each other, using reflection, rotation and translation.

- The symbol for congruence is $\cong$. This is read as ‘is congruent to’.
- For the diagrams shown on the previous page, $ABCD \cong A'B'C'$ and $ABCDE \cong PQRST$.
- When writing congruence statements, the vertices of the figures are named in corresponding order.
Select a pair of congruent shapes from the following set.

**Think**
Figures a and c are identical in shape and size; they just have different orientation.

**Write**
Shape a ≅ shape c

**Testing triangles for congruence**

- It is not necessary to know that all three sides and all three angles of one triangle are equal to the corresponding sides and angles of another triangle to ensure that the two triangles are congruent. There are certain minimum conditions that will guarantee that this is so.

**Side-side-side condition of congruence (SSS)**

- If two triangles have equal corresponding sides, the angles opposite these corresponding sides will also be equal in size. This means that these two triangles are congruent.
- This is the side-side-side (SSS) condition of congruence.

**Side-angle-side condition of congruence (SAS)**

- In this situation, two pairs of corresponding sides are equal. If the angles between these sides are equal, then the triangles are congruent.
- This is the side-angle-side (SAS) condition of congruence.

**Angle-side-angle condition of congruence (ASA)**

- Two pairs of corresponding angles are equal in these triangles. (The third pair of angles will also be equal.)
- If one pair of corresponding sides is equal, then the triangles are congruent.
- This is the angle-side-angle (ASA) condition of congruence.

**Right angle-hypotenuse-side condition of congruence (RHS)**

- In a right-angled triangle, if the hypotenuse and one other side are equal, then the triangles are congruent.
- This is the right angle-hypotenuse-side (RHS) condition of congruence.
Summary of congruence tests

<table>
<thead>
<tr>
<th>Test Description</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>All corresponding sides are equal in length.</td>
<td>SSS (side–side–side)</td>
</tr>
<tr>
<td>Two corresponding sides are equal in length and the included angles are equal in size.</td>
<td>SAS (side–angle–side)</td>
</tr>
<tr>
<td>Two angles are equal in size and there is one pair of corresponding sides of equal length.</td>
<td>ASA (angle–side–angle)</td>
</tr>
<tr>
<td>In a right-angled triangle, the hypotenuse and one pair of corresponding sides are equal in length.</td>
<td>RHS (right angle–hypotenuse–side)</td>
</tr>
</tbody>
</table>

**WORKED EXAMPLE 7**

Which of the following triangles are congruent? Give reasons for your answer.

**THINK**

In all three triangles two given sides are of equal length (2 cm and 5 cm). Triangles ABC and KLM also have the included angle of equal size (60°). B corresponds to L, and A corresponds to M.

**WRITE**

\[ \triangle ABC \cong \triangle MLK \ (SAS) \]

**WORKED EXAMPLE 8**

Given that \( \triangle ABD \cong \triangle CBD \), find the values of the pronumerals in the figure at right.
 THINK

1. In congruent triangles corresponding sides are equal in length. Side AD (marked $x$) corresponds to side CD.

2. Since triangles are congruent, corresponding angles are equal.

WRITE

$\triangle ABD \cong \triangle CBD$
AD = CD
$x = 3$ cm

$\angle A = \angle C$
$y = 40^\circ$

$\angle BDA = \angle BDC$
z = 90^\circ

WORKED EXAMPLE 9

Prove that $\triangle PQS$ is congruent to $\triangle RQS$.

THINK

1. Study the diagram and state which sides and/or angles are equal.

QP = QR (given)
PS = RS (given)
QS is common.

2. This fits the SSS test and proves congruence.

WRITE

$\triangle PQS \cong \triangle RQS$ (SSS)

Exercise 5.3 Congruent figures

INDIVIDUAL PATHWAYS

REFLECTION
What is the easiest way to determine if two figures are congruent?

PRACTISE
Questions: 1–7, 10, 11

CONSOLIDATE
Questions: 1–8, 10–13

MASTER
Questions: 1–15

FLUENCY

1 WE6 Select a pair of congruent shapes from the figures in each part of the following question.

a i ii iii iv

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UNDERSTANDING

2 MC Which of the following is congruent to the triangle shown at right?

A

B

C

D

3 WE7 In each part of the question, which of the triangles are congruent? Give a reason for your answer.

a

b
4. Find the value of the pronumeral in each of the following pairs of congruent triangles. All side lengths are in centimetres.

\[ \begin{align*}
\text{(a)} & \quad 4 \quad 3 \\
\text{(b)} & \quad 85^\circ \\
\text{(c)} & \quad 80^\circ \\
\text{(d)} & \quad 30^\circ \\
\text{(e)} & \quad 40^\circ 
\end{align*} \]
5 Find the length of the side marked with the pronumeral using congruent triangles.

a

\[ \triangle ABC \cong \triangle PQR \]

b

\[ \triangle ABD \cong \triangle CAE \]

c

\[ \triangle PQR \cong \triangle PSR \]

d

\[ \triangle PQR \cong \triangle PSR \]

6 Do congruent figures have the same area? Explain.

REASONING

7 For each of the following, prove that:

a

\[ \triangle ABC \cong \triangle ADC \]

b

\[ \triangle PQR \cong \triangle PSR \]

c

\[ \triangle DBA \cong \triangle DCA \]

8 Give an example to show that triangles with two angles of equal size and a pair of non-corresponding sides of equal length may not be congruent.

9 ABCD is a trapezium with both AD and BC perpendicular to AB. If a right-angled triangle DEC is constructed with an angle \( \angle ECD \) equal to 45°, prove that \( \triangle EDA \cong \triangle ECB \).
PROBLEM SOLVING

10 If two congruent triangles have a right angle, is the reason always ‘right angle, hypotenuse and corresponding side’? Justify your answer.

11 A teacher asked his class to each draw a triangle that has side lengths of 5 cm and 4 cm, and an angle of 45 degrees that is not formed at the point joining the 5 cm and 4 cm side. Would the triangles drawn by every member of the class be congruent? Explain why.

12 Make 5 congruent triangles from 9 matchsticks.

13 Make 7 congruent triangles from 9 matchsticks.

14 Show how this figure can be cut into four congruent pieces.

15 Find the ratio of the outer (unshaded) area to the inner (shaded) area of this six-pointed star.

5.4 Similar figures

- Similar figures have identical shape but different size. The corresponding angles in similar figures are equal in size and the corresponding sides are in the same ratio, given by the scale factor.
- The symbol used to denote similarity is ~, which is read as ‘is similar to’.
- Similar figures can be obtained as a result of enlargement or reduction.
- If an enlargement (or a reduction) takes place, the original figure can be called the object and the enlarged (or reduced) figure called the image.
  It can also be said that the object maps to the image.
- For any two similar figures, the scale factor can be obtained using the following formula:

\[
\text{scale factor} = \frac{\text{length of the image}}{\text{length of the object}}
\]

Note: The size of the scale factor indicates whether the original object has been enlarged or reduced.
• If the scale factor is greater than 1, an enlargement has occurred.
• If the scale factor is positive but less than 1, a reduction has occurred.
Consider the pair of similar triangles below.

\[ \triangle UVW \sim \triangle ABC. \]

- The corresponding angles of the two triangles are equal in size:
  \[ \angle CAB = \angle WUV, \angle ABC = \angle UVW \text{ and } \angle ACB = \angle UWV. \]
- The corresponding sides of the two triangles are in the same ratio.
  \[ \frac{UV}{AB} = \frac{VW}{BC} = \frac{UW}{AC} = 2; \] that is, the side lengths of \( \triangle UVW \) are twice as long as the corresponding sides in \( \triangle ABC \).
- The scale factor is 2.
- The original figure, \( \triangle ABC \), can be called the object, while \( \triangle UVW \), obtained as the result of enlargement, is the image.
- It can be said that \( \triangle ABC \) maps to \( \triangle UVW \).

WORKED EXAMPLE 10

Enlarge the shape at right by a factor of 2.

THINK

1. Select a point, O, somewhere inside the given shape and join it with straight-line segments to each vertex. Extend the lines beyond the shape.

2. Measure the distance OA and mark in the point A’ so that OA’ = 2 × OA. Repeat this for the other vertices.
Testing triangles for similarity

• As with congruent triangles, it is not necessary to know that all pairs of corresponding sides are in the same ratio and that all corresponding angles are equal to ensure that two triangles are similar. There are certain minimum conditions which will guarantee that this is so.

Angle-angle-angle condition of similarity (AAA)

• If the angles of one triangle are the same as the angles of a second triangle, then the triangles are similar.

This is the angle-angle-angle (AAA) condition for similarity. From the diagram above, \( \triangle ABC \sim \triangle RST \) (AAA).

Side-side-side condition for similarity (SSS)

• In the diagram at right, the ratio of pairs of corresponding sides is constant.

That is, \( \frac{9}{6} = \frac{15}{10} = \frac{10.5}{7} = 1.5 \).

This is enough to show that the triangles are similar.
• This is the side-side-side (SSS) condition for similarity.

In this case, \( \triangle ABC \sim \triangle RST \) (SSS).
Side-angle-side condition for similarity (SAS)

- In the diagram at right, two pairs of sides are in the same ratio; that is, \( \frac{9}{6} = \frac{15}{10} = 1.5 \), and the included angles are equal as well. This is enough to show that the triangles are similar.
- This is the side-angle-side (SAS) condition for similarity. In this case \( \triangle ABC \sim \triangle RST \) (SAS).

Right angle-hypotenuse-side condition for similarity (RHS)

- With right-angled triangles, a special condition can apply.
- If the hypotenuse and one other pair of sides are in the same ratio (e.g. in the diagram at right, \( \frac{12}{6} = \frac{10}{5} \)), then the triangles are similar.
- This is the right angle-hypotenuse-side (RHS) condition for similarity. In this case \( \triangle ABC \sim \triangle RST \) (RHS).

Summary of similarity tests

- Triangles can be checked for similarity using one of the tests described in the table below.

<table>
<thead>
<tr>
<th>Test description</th>
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<tbody>
<tr>
<td>All corresponding angles are equal in size</td>
<td>AAA (angle–angle–angle)</td>
</tr>
<tr>
<td>All corresponding sides are in the same ratio</td>
<td>SSS (side–side–side)</td>
</tr>
<tr>
<td>Two pairs of corresponding sides are in the same ratio and the included angles are equal in size</td>
<td>SAS (side–angle–side)</td>
</tr>
<tr>
<td>In right-angled triangles, the hypotenuses and one other pair of sides are in the same ratio.</td>
<td>RHS (right angle–hypotenuse–side)</td>
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</table>

- Note: When using the AAA test, it is sufficient to show that two pairs of corresponding angles are equal. Since the sum of the interior angles in any triangle is 180°, the third pair of angles will automatically be equal.
Find a pair of similar triangles from those shown. Give a reason for your answer.

**WORKED EXAMPLE 11**

1. In each triangle we know the size of two sides and the included angle, so the SAS test can be applied. Since all included angles are equal (30°), we need to find ratios of corresponding sides, taking two triangles at a time.

2. Write the answer.

**WRITE**

For triangles a and b:
\[
\frac{15}{10} = \frac{9}{6} = 1.5
\]

For triangles a and c:
\[
\frac{20}{10} = 2, \quad \frac{15}{6} = 2.5
\]

Triangle a ~ triangle b (SAS)

**WORKED EXAMPLE 12**

Prove that ΔABC is similar to ΔEDC.

**WRITE**

\[\angle ABC = \angle EDC \text{ (alternate angles)}\]

\[\angle BAC = \angle DEC \text{ (alternate angles)}\]

\[\angle BCA = \angle DCE \text{ (vertically opposite angles)}\]

\[\Delta ABC \sim \Delta EDC \text{ (AAA)}\]

- The ratio of the corresponding sides in similar figures can be used to calculate missing side lengths or angles in these figures.
A 1.5-metre pole casts a shadow 3 metres long, as shown. Find the height of a building that casts a shadow 15 metres long at the same time of the day.

**THINK**

1. Represent the given information on a diagram. 
   \[ \angle BAC = \angle EDC; \quad \angle BCA = \angle ECD \]

2. Triangles ABC and DEC are similar. Therefore, the ratios of corresponding sides are the same. Write the ratios.

3. Solve the equation for \( h \).

4. Write the answer in words, including units.

**WRITE/DRAW**

\[ \triangle ABC \sim \triangle DEC \text{ (AAA)} \]

\[ \frac{h}{1.5} = \frac{15}{3} \]

\[ h = \frac{15 \times 1.5}{3} = 7.5 \]

The height of the building is 7.5 m.

**Exercise 5.4 Similar figures**

**INDIVIDUAL PATHWAYS**

**PRACTISE**
Questions: 1–4, 5, 8, 11–14, 17, 18

**CONSOLIDATE**
Questions: 1–4, 6, 9, 11–14, 16, 17, 19, 21, 22

**MASTER**
Questions: 1–4, 7, 10–16, 18, 20, 21, 23, 24

**REFLECTION**
Do similar objects have the same perimeters?

**FLUENCY**

1. **WE10** Enlarge (or reduce) the following shapes by the scale factor given.

   a. Scale factor = 3

   \[
   \begin{array}{c}
   \text{8 cm} \\
   \text{2 cm}
   \end{array}
   \]
2. Find a pair of similar triangles among those shown in each part. Give a reason for your answer.

a. i. \( \angle 40^\circ, 60^\circ \) ii. \( \angle 50^\circ, 60^\circ \) iii. \( \angle 40^\circ, 60^\circ \)

b. i. \( \frac{3}{2} \) ii. \( \frac{6}{4} \) iii. \( \frac{5}{4} \)

c. i. \( \frac{4}{20} \) ii. \( \frac{2}{20} \) iii. \( \frac{8}{20} \)

d. i. \( \frac{5}{3} \) ii. \( \frac{5}{4} \) iii. \( \frac{10}{6} \)

e. i. \( \frac{2}{1} \) ii. \( \frac{2}{4} \) iii. \( \frac{6}{4.5} \)

3. Name two similar triangles in each of the following figures, ensuring that vertices are listed in the correct order.

a. \( A \rightarrow B \rightarrow D \rightarrow E \rightarrow C \)

b. \( Q \rightarrow B \rightarrow R \rightarrow A \rightarrow C \)

c. \( P \rightarrow Q \rightarrow R \rightarrow S \rightarrow T \)
4 In the diagram at right C is the centre of the circle. Complete this statement: ΔABC is similar to ...

5 ABCDEF is a regular hexagon, similar to PQRSTU.
   a What is the length of:
      i AB
      ii RS?
   b What is the scale factor for enlargement?
   Note: You need to measure the lengths.

6 a Complete this statement: \( \frac{AB}{AD} = \frac{BC}{AE} = \frac{DE}{EF} \)
   b Find the value of the pronumerals.

7 a Find the values of \( h \) and \( i \).
   b Find the values of \( j \) and \( k \).

8 Find the value of the pronumeral in the diagram below.
9 If the triangles shown at right are similar, find the values of \(x\) and \(y\).

![Diagram](image)

10 Find the values of \(x\) and \(y\) in the diagram below.

![Diagram](image)

**REASONING**

11 Prove that \(\triangle ABC\) is similar to \(\triangle EDC\) in each of the following.

(a) \[
\begin{align*}
A & \quad 4 \text{ cm} \\
B & \quad 3 \text{ cm} \\
C & \quad 6 \text{ cm} \\
D & \quad 7.5 \text{ cm}
\end{align*}
\]

(b) \[
\begin{align*}
A & \quad 4 \text{ cm} \\
B & \quad 3 \text{ cm} \\
C & \quad 6 \text{ cm} \\
D & \quad 7.5 \text{ cm}
\end{align*}
\]

(c) \[
\begin{align*}
A & \quad 4 \text{ cm} \\
B & \quad 3 \text{ cm} \\
C & \quad 6 \text{ cm} \\
D & \quad 7.5 \text{ cm}
\end{align*}
\]

(d) \[
\begin{align*}
A & \quad 4 \text{ cm} \\
B & \quad 3 \text{ cm} \\
C & \quad 6 \text{ cm} \\
D & \quad 7.5 \text{ cm}
\end{align*}
\]

12 Find the value of each pronumeral in these triangles. Show how you arrived at your answers.

(a) \[
\begin{align*}
A & \quad 4 \text{ cm} \\
B & \quad 3 \text{ cm} \\
C & \quad 6 \text{ cm} \\
D & \quad 7.5 \text{ cm}
\end{align*}
\]

(b) \[
\begin{align*}
A & \quad 4 \text{ cm} \\
B & \quad 3 \text{ cm} \\
C & \quad 6 \text{ cm} \\
D & \quad 7.5 \text{ cm}
\end{align*}
\]

(c) \[
\begin{align*}
A & \quad 4 \text{ cm} \\
B & \quad 3 \text{ cm} \\
C & \quad 6 \text{ cm} \\
D & \quad 7.5 \text{ cm}
\end{align*}
\]
13 **WE13** A ladder just touches a bench and leans on a wall 4 metres above the ground, as shown. If the bench is 50 centimetres high and is 1 metre from the base of the ladder, show that the base of the ladder is 8 metres from the wall.

14 Natalie, whose height is 1.5 metres, casts a shadow 2 metres long at a certain time of the day. If Alex is 1.8 metres tall, show that his shadow would be 2.4 m long.

15 A string 50 metres long is pegged to the ground and tied to the top of a flagpole. It just touches the head of Maureen, who is 5 metres away from the point where the string is held to the ground. If Maureen is 1.5 metres tall, show that the height, \( h \), of the flagpole is 14.37 m.

16 Using diagrams or otherwise, explain whether the following statements are true or false.
   a. All equilateral triangles are similar.
   b. All isosceles triangles are similar.
   c. All right-angled triangles are similar.
   d. All right-angled isosceles triangles are similar.
PROBLEM SOLVING

17 Penny and Paul play tennis at night under floodlights. When Penny stands 2.5 m from the base of the floodlight, her shadow is 60 cm long.
   a If Penny is 1.3 m tall, how high is the floodlight in metres, correct to 2 decimal places?
   b If Paul, who is 1.6 m tall, stands in the same place, how long will his shadow be in cm?

18 To determine the height of a flagpole, Jenna and Mia decided to measure the shadow cast by the flagpole. They place a 1 m ruler at a distance of 3 m from the base of the flagpole and measure the shadows that both the ruler and flagpole cast. Both shadows finished at the same point. After measuring the shadow of the flagpole, Jenna and Mia calculate that the height of the flagpole is 5 m. Determine the length of the shadow cast by the flagpole, in metres, as measured by Jenna and Mia.

19

![Diagram]

Use the diagram above to find the value of $a$ if $XZ = 8$ cm, $X'Z' = 12$ cm, $X'X = a$ cm and $XY = (a + 1)$ cm.

20 PQ is a diameter of this circle with a centre at S. R is any point on the circumference. T is the midpoint of PR.

![Diagram]

a Write down everything you know about this figure.
b Explain why $\triangle PTS$ is similar to $\triangle PRQ$.
c Find the length of TS if RQ is 8 cm.
d Find the length of every other side also given that PT is 3 cm and angle PRQ is a right angle.

21 AB and CD are parallel lines in the figure below.

![Diagram]

a State the similar triangles.
b Calculate the values of $x$ and $y$. 
22 For the diagram given, show that if the base of the triangle is raised to half the height of the triangle, the length of the base of the newly formed triangle will be half of its original length.

23 AB is a straight line. The fraction of the large rectangle that is shaded is \(\frac{12}{25}\). Find the ratio \(a:b\).

**5.5 Area and volume of similar figures**

**Units of length**
- Metric units of length include millimetres (mm), centimetres (cm), metres (m) and kilometres (km).
- To convert between the units of length, we use the conversion chart shown below.

- When converting from a large unit to a smaller unit, multiply by the conversion factor; when converting from a smaller unit to a larger unit, divide by the conversion factor.
Units of area

- Area is measured in square units, such as square millimetres (mm²), square centimetres (cm²), square metres (m²) and square kilometres (km²).
- Area units can be converted using the chart below.

- Area units are the squares of the corresponding length units.

Units of volume

- Volume is measured in cubic units such as cubic millimetres (mm³), cubic centimetres (cm³) and cubic metres (m³).
- Volume units can be converted using the chart shown below.

- Volume units are the cubes of the corresponding length units.

Area and surface area of similar figures

- If the side lengths in any figure are increased by a scale factor of \( n \), then the area of similar figures increases by a scale factor of \( n^2 \).

For example, consider the following squares:

- Area of square A = \( 2 \times 2 = 4 \text{ cm}^2 \)
- Area of square B = \( 4 \times 4 = 16 \text{ cm}^2 \)
- Area of square C = \( 6 \times 6 = 36 \text{ cm}^2 \)
The scale factors for the side lengths and the scale factors for the areas are calculated below.

<table>
<thead>
<tr>
<th>Squares</th>
<th>Scale factor for side length</th>
<th>Scale factor for area</th>
</tr>
</thead>
<tbody>
<tr>
<td>A and B</td>
<td>( \frac{4}{2} = 2 )</td>
<td>( \frac{16}{4} = 4 = 2^2 )</td>
</tr>
<tr>
<td>A and C</td>
<td>( \frac{6}{2} = 3 )</td>
<td>( \frac{36}{4} = 9 = 3^2 )</td>
</tr>
<tr>
<td>B and C</td>
<td>( \frac{6}{4} = \frac{3}{2} )</td>
<td>( \frac{36}{16} = \frac{9}{4} = \left( \frac{3}{2} \right)^2 )</td>
</tr>
</tbody>
</table>

- If the side lengths in any figure are increased by a scale factor of \( n \), then the surface area of similar figures increases by a scale factor of \( n^2 \).

Consider the cubes below.

Surface area = \( 6 \times 4 \) = 24 cm\(^2\)
Surface area = \( 6 \times 16 \) = 96 cm\(^2\)
Surface area = \( 6 \times 36 \) = 216 cm\(^2\)

The scale factors for the side lengths and the scale factors for the surface areas are calculated below.

<table>
<thead>
<tr>
<th>Cubes</th>
<th>Scale factor for side length</th>
<th>Scale factor for surface area</th>
</tr>
</thead>
<tbody>
<tr>
<td>A and B</td>
<td>( \frac{4}{2} = 2 )</td>
<td>( \frac{96}{24} = 4 = 2^2 )</td>
</tr>
<tr>
<td>A and C</td>
<td>( \frac{6}{2} = 3 )</td>
<td>( \frac{216}{24} = 9 = 3^2 )</td>
</tr>
<tr>
<td>B and C</td>
<td>( \frac{6}{4} = \frac{3}{2} )</td>
<td>( \frac{216}{96} = \frac{9}{4} = \left( \frac{3}{2} \right)^2 )</td>
</tr>
</tbody>
</table>

**Volume of similar figures**

- If the side lengths in any solid are increased by a scale factor of \( n \), then the volume of similar solids increases by a scale factor of \( n^3 \).
• Once again consider the cubes shown earlier. The scale factors for the side lengths and the scale factors for the volumes are calculated below.

\[
\begin{align*}
\text{Volume of } A &= 2 \times 2 \times 2 = 8 \text{ cm}^3 \\
\text{Volume of } B &= 4 \times 4 \times 4 = 64 \text{ cm}^3 \\
\text{Volume of } C &= 6 \times 6 \times 6 = 216 \text{ cm}^3
\end{align*}
\]

<table>
<thead>
<tr>
<th>Cubes</th>
<th>Scale factor for side length</th>
<th>Scale factor for volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>A and B</td>
<td>(\frac{4}{2} = 2)</td>
<td>(\frac{64}{8} = 8 = 2^3)</td>
</tr>
<tr>
<td>A and C</td>
<td>(\frac{6}{2} = 3)</td>
<td>(\frac{216}{8} = 27 = 3^3)</td>
</tr>
<tr>
<td>B and C</td>
<td>(\frac{6}{4} = \frac{3}{2})</td>
<td>(\frac{216}{64} = \left(\frac{3}{2}\right)^3)</td>
</tr>
</tbody>
</table>

**WORKED EXAMPLE 14**

The side lengths of a box have been increased by a factor of 3.

a Find the surface area of the new box if the original surface area is 94 cm\(^2\).
b Find the volume of the new box if the original volume is 60 cm\(^3\).

**THINK**

a 1. State the scale factor for side length used to produce the new box.
   2. The scale factor for surface area is the square of the scale factor for length.
   3. Calculate the surface area of the new box.

b 1. The scale factor for volume is the cube of the scale factor for length.
   2. Calculate the volume of the new box.

**WRITE**

a Scale factor for side length = 3
   
   Scale factor for surface area
   \(= 3^2\)
   \(= 9\)
   
   Surface area of new box
   \(= 94 \times 9\)
   \(= 846 \text{ cm}^2\)

b Scale factor for volume
   \(= 3^3\)
   \(= 27\)
   
   Volume of new box
   \(= 60 \times 27\)
   \(= 1620 \text{ cm}^3\)
Exercise 5.5 Area and volume of similar figures

**INDIVIDUAL PATHWAYS**

<table>
<thead>
<tr>
<th>PRACTISE</th>
<th>CONSOLIDATE</th>
<th>MASTER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Questions: 1, 2, 4, 6, 8, 13</td>
<td>Questions: 1–4, 6, 8, 13–16</td>
<td>Questions: 1, 3, 5–18</td>
</tr>
</tbody>
</table>

**FLUENCY**

1 **WE14a** The side lengths of the following shapes have all been increased by a factor of 3. Copy and complete the following table.

<table>
<thead>
<tr>
<th>Original surface area</th>
<th>Enlarged surface area</th>
</tr>
</thead>
<tbody>
<tr>
<td>a 100 cm²</td>
<td></td>
</tr>
<tr>
<td>b 7.5 cm²</td>
<td></td>
</tr>
<tr>
<td>c 95 mm²</td>
<td>918 cm²</td>
</tr>
<tr>
<td>d</td>
<td>45 m²</td>
</tr>
<tr>
<td>e</td>
<td>225 mm²</td>
</tr>
</tbody>
</table>

2 **WE14b** The side lengths of the following shapes have all been increased by a factor of 3. Copy and complete the following table.

<table>
<thead>
<tr>
<th>Original volume</th>
<th>Enlarged volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>a 200 cm³</td>
<td></td>
</tr>
<tr>
<td>b 12.5 cm³</td>
<td></td>
</tr>
<tr>
<td>c 67 mm³</td>
<td>2700 cm³</td>
</tr>
<tr>
<td>d</td>
<td>67.5 m³</td>
</tr>
<tr>
<td>e</td>
<td>27 mm³</td>
</tr>
</tbody>
</table>

3 A rectangular box has a surface area of 96 cm² and volume of 36 cm³. Find the volume and surface area of a similar box that has side lengths double the size of the original.

**UNDERSTANDING**

4 The area of a bathroom on a house plan is 5 cm². Find the area of the bathroom if the map has a scale of 1 : 100.

5 The area of a kitchen is 25 m².
   a Change 25 m² to cm².
   b Find the area of the kitchen on a plan if the scale of the plan is 1 : 120. (Give your answer correct to 1 decimal place.)

6 The volume of a swimming pool from its construction plan is calculated to be 20 cm³. Find the volume of the pool if the plan has a scale of 1 : 75.

7 The total surface area of the wings on a 747 aircraft is 120 m³.
   a Change 120 m³ to cm³.
   b Find the total surface area of the wings on a scale model built using the scale 1 : 80.
REASONING

8 A cube has a surface area of 253.5 cm². (Give answers correct to 1 decimal place where appropriate.)
   a Show that the side length of the cube is 6.5 cm.
   b Show that the volume of the cube is 274.625 cm³.
   c Find the volume of a similar cube that has side lengths twice as long.
   d Find the volume of a similar cube that has side lengths half as long.
   e Find the surface area of a similar cube that has side lengths one third as long.

9 In the diagram at right a light is shining through a hole, resulting in a circular bright spot with a radius of 5 cm on the screen. The hole is 10 mm wide. If the light is 1 m behind the hole, show that the light is 10 m from the screen.

10 A triangle ABC maps to triangle A'B'C' under an enlargement,
   \[ AB = 7 \text{ cm}, \ AC = 5 \text{ cm}, \ A'B' = 21 \text{ cm}, \ B'C' = 30 \text{ cm}. \]
   a Show that the scale factor for enlargement is 3.
   b Find BC.
   c Find A'C'.
   d If the area of \( \Delta ABC \) is 9 cm², show that the area of \( \Delta A'B'C' \) is 81 cm².

11 Two rectangles are similar. If the width of one rectangle is twice of width of the other, prove that the ratio of their areas is 4 : 1.

12 A pentagon has an area of 20 cm². If all the side lengths are doubled, show that the area of the enlarged pentagon is 80 cm².

PROBLEM SOLVING

13 a Calculate the areas of squares with sides 2 cm, 5 cm, 10 cm and 20 cm.
   b State in words how the ratio of the areas is related to the ratio of the side lengths.

14 The areas of two similar trapeziums are 9 and 25. What is the ratio of a pair of corresponding side lengths?

15 Two cones are similar. The ratio of volumes is 27 : 64. Find the ratio of:
   a the perpendicular heights
   b the areas of the bases.

16 Rectangle A has the dimensions 5 by 4, rectangle B has the dimensions 4 by 3 and rectangle C has the dimensions 3 by 2.4.
   a Which rectangles are similar? Explain.
   b Find the area scale factor for the similar rectangles.

17 A balloon in the shape of a sphere has an initial volume of 840 cm³ and is increased to a volume of 430 080 cm³. What is the increase in the radius of the balloon?

18 Part of an egg timer in the shape of a cone has sand poured into it as shown in the diagram.
   Find the ratio of the volume of sand in the cone to the volume of empty space in the bottom half of the egg timer.
5.6 Review

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

Download the Review questions document from the links found in your eBookPLUS.

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology,

<table>
<thead>
<tr>
<th>alternate angles</th>
<th>object</th>
<th>rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>congruent figures</td>
<td>proportion</td>
<td>scale factor</td>
</tr>
<tr>
<td>corresponding sides</td>
<td>ratio</td>
<td>similar figures</td>
</tr>
<tr>
<td>enlargement</td>
<td>reduction</td>
<td>translation</td>
</tr>
<tr>
<td>image</td>
<td>reflection</td>
<td>vertices</td>
</tr>
</tbody>
</table>

The story of mathematics

is an exclusive Jacaranda video series that explores the history of mathematics and how mathematics helped shape the world we live in today.

*Mathematics in art* (eles-1692) explores the historical relationship between mathematics and art, and how mathematics still underpins many facets of art today, from paintings to sculpture, architecture and beyond.
What’s this object?

When using geometrical tools to construct shapes, we have to ensure that the measurements taken are precise. Small errors in each step of the measuring process can result in the creation of an incorrect shape. The object that you have to create for this task is one that has generated a great deal of interest over the years. It is made by combining three congruent shapes. Instructions to make the congruent shape are as follows.
Part 1 The congruent shape
1 Using a ruler, protractor, pencil and a pair of compasses, follow the instructions below to construct the first part of the object. Draw the shape in your workbook.
   • Measure a horizontal line AB that is 9.5 cm long. To ensure that there is enough space for the entire object, draw the line close to the bottom of the space.
   • At point B, construct an angle of 120° above the line AB. Extend the line to point C, making BC 2 cm long.
   • At point C, construct an angle of 60° on the same side of the line BC as point A. Measure the line CD to be 7.5 cm long.
   • At point D and above the line DC, construct an angle of 60°. Measure the length DE to be 3.5 cm.
   • On the line DE, at point E, construct an angle of 120° above the line DE. Let the measurement from E to F be 2 cm. Join point F to point A.
2 What is the length of the line joining point A to point F?
3 What do you notice about the size of the angles FAB and AFE?
4 Shade the shape with any colour you wish.

Part 2 The object
You have now constructed the shape that is to be used three times to make the final shape. To do this, follow the instructions below.
   • Trace the shape onto a piece of tracing paper twice and cut around the edges. Label the shapes with the letters used in its construction.
   • Place the line AF of your trace against the line CD of your drawn shape. Reproduce this shape on your object. Shade this section using a different colour.
   • Place the line DC of your second trace against the line FA of your drawn shape. Reproduce this shape on your object. Again, shade this section with a different colour.
5 Describe the object you have created.
6 Using the internet, library or other references, investigate other impossible objects drawn as two-dimensional shapes. Recreate them on a separate sheet of paper and include reasons why they are termed impossible.
Why does a giraffe have a long neck?

The lengths of the lettered sides in the pairs of similar triangles give the puzzle's answer code.
Activities

5.1 Overview

Video
• The story of mathematics: Mathematics in art (eles-1692)

5.2 Ratio and scale

Interactivity
• IP interactivity 5.2 (int-4494): Ratio and scale

Digital docs
• SkillSHEET (doc-6190): Simplifying fractions
• SkillSHEET (doc-6191): Simplifying ratios
• SkillSHEET (doc-6192): Finding and converting to the lowest common denominator
• SkillSHEET (doc-6193): Solving equations of the type \( a = \frac{x}{b} \) to find \( x \)
• SkillSHEET (doc-6194): Solving equations of the type \( a = \frac{D}{X} \) to find \( x \)
• WorkSHEET 5.1 Ratio and scale (doc-6198)

5.3 Congruent figures

Interactivity
• IP interactivity 5.3 (int-4495): Congruent figures

Digital docs
• SkillSHEET (doc-6195): Naming angles
• SkillSHEET (doc-6196): Complementary and supplementary angles
• SkillSHEET (doc-6197): Angles in a triangle
• WorkSHEET 5.2 Congruent figures (doc-6201)

5.4 Similar figures

Interactivity
• IP interactivity 5.4 (int-4496): Similar figures

Digital doc
• WorkSHEET 5.3: Similar figures (doc-6202)

5.5 Area and volume of similar figures

Interactivities
• Similar figures (int-2768): Learn more about similar figures
• IP interactivity 5.5 (int-4497): Area and volume of similar figures

5.6 Review

Interactivities
• Word search (int-2692)
• Crossword (int-2693)
• Sudoku (int-3205)

Digital docs
• Topic summary (doc-10783)
• Concept map (doc-10796)

To access eBookPLUS activities, log on to www.jacplus.com.au
Answers

**TOPIC 5 Congruence and similarity**

### Exercise 5.2 — Ratio and scale
1. a) 240 : 7
   b) The track is $34\frac{2}{7}$ times as long as it is wide.
2. a) 5 : 2
   b) The cliff is 2.5 times as high as the distance from the base of the cliff to the emu.
3. a) 2 : 3
   b) 1 : 7
   c) 1 : 3
   d) 2 : 5
   e) 4 : 3
   f) 4 : 15
   g) 40 : 31
   h) 36 : 11
   i) 8 : 25
   j) 100 : 33
4. a) 6 : 1
   b) 13 : 15
   c) 4 : 1
   d) 5 : 1
   e) 1 : 2
   f) 20 : 3
   g) 1 : 6
   h) 5 : 3
   i) 5 : 1
   j) 11 : 5
   k) 43 : 20
   l) 1 : 200
5. a) 3 : 4
   b) 8 : 7
   c) 2 : 3
   d) 28 : 25
   e) 9 : 2
   f) 2 : 7
   g) 10 : 3
   h) 33 : 16
   i) 23 : 15
   j) 16 : 65
6. a) $a = 9$
   b) $b = 24$
   c) $c = 32$
   d) $d = \frac{15}{7}$
   e) $e = \frac{3^3}{2}$
   f) $f = 14\frac{7}{9}$
   g) $g = \frac{3^2}{2}$
   h) $h = 4\frac{5}{9}$
   i) $i = 29\frac{1}{9}$
   j) $j = 8\frac{1}{7}$
   k) $k = 10\frac{2}{9}$
   l) $l = 11\frac{1}{4}$

### Exercise 5.3 — Congruent figures
7. a) $90^\circ$
   b) $90^\circ$
   c) $90^\circ$
   d) $90^\circ$
8. a) 3 cm
   b) 12 cm
   c) 3.5 cm
   d) 6 m x 6 m
   e) 5 m x 5 m
9. $\frac{7}{2}$
10. a) 175 : 6428
    b) 37
11. 2.5 mL
12. 1000 g
13. a) 200
    b) 6 m x 6 m
    c) 5 m x 5 m
14. a) Check with your teacher.
    b) 1.69 g gold, 0.5 g copper, 0.00 g silver
15. a) 3 + 4 + 5 = 12
    b) 180 ÷ 12 = 15
    c) 3 x 15 = 45; 4 x 15 = 60; 5 x 15 = 75
    The three angles are 45°, 60° and 75°.
16. a) $2k \times 3k \times 5k = 30k^3$
    b) $k^3 = 21,870$
    c) $k = 29$
    d) $k = 9$

Substituting $k$ into the ratio ($2k : 3k : 5k$), the dimensions are 18 cm, 27 cm and 45 cm.
17. The profits aren’t shared in a fair ratio. Tyler gets more profit than his share and Dylan gets less profit than his share. Only Aaron gets the correct share of the profit.
18. a) A pen costs twice as much as a pencil.
    b) 2 sharpeners cost the same as 3 pencils.
    c) 4 sharpeners cost the same as 3 pens.

Pen : pencil : sharpener = 1 : 2 : 3
19. 680 cm
20. $y:z = 9:8$
21. 49 students
22. $P = 4, Q = 6$
23. 125 students
24. 12 : 1

### Exercise 5.3 — Congruent figures
1. a) SSS
    b) SAS
    c) SSS
    d) SAS
    e) SAS
    f) SAS
2. a) $x = 3 cm$
    b) $x = 85^\circ$
    c) $x = 80^\circ, y = 30^\circ, z = 70^\circ$
    d) $x = 30^\circ, y = 70^\circ$
    e) $x = 40^\circ, y = 50^\circ, z = 50^\circ, m = 90^\circ, n = 90^\circ, M = 90^\circ$
3. a) $2 cm$
    b) $3 cm$
    c) $6 mm$
    d) $7 mm$
4. Yes, because they are identical.
5. a) SSS
    b) SAS
    c) SAS
6. Check with your teacher.
7. a) SSS
    b) SAS
    c) SAS
8. a) $15 cm$
    b) $15 cm$
    c) $15 cm$
    d) $89^\circ$
    e) $20^\circ$
9. a) $20^\circ$
    b) $20^\circ$
    c) $20^\circ$
10. No; it could be ASA.
11. Because the angle is not between the two given sides, the general shape of the triangle is not set; therefore, many shapes are possible.
12. This can be done with an equilateral triangle and a regular tetrahedron.
13. This can be done with a double regular tetrahedron.
14 Each piece is similar to the original shape.

15 \[ \frac{1}{1} \]

Challenge 5.1

Exercise 5.4 — Similar figures

1. a. \[ \frac{24}{6} \] b. \[ \frac{10}{3} \] c. \[ \frac{1.25}{2} \]

2. a. i and iii, AAA b. i and ii, SAS c. i and ii, SSS d. i and iii, RHS e. i and iii, SSS

3. a. Triangles ABC and DEC b. Triangles PQR and ABC c. Triangles PQR and TSR d. Triangles ABC and DEC e. Triangles ADB and ADC

4. a. \( \triangle EDC \)

Answers may vary due to inconsistencies in measurement.

- a. i 1.3 cm ii 2.6 cm
- b. 2
- c. \( \frac{AB}{BC} = \frac{AC}{AD} \)
- d. \( f = 9, g = 8 \)
- e. \( h = 3.75, i = 7.5 \)
- f. \( j = 2.4, k = 11.1 \)
- g. \( x = 4 \)
- h. \( x = 20^\circ, y = 31^\circ \)
- i. \( x = 3, y = 4 \)
- j. Check with your teacher.
- k. \( x = 7.1 \) b. \( x = 3.1 \) c. \( x = 7.5, y = 7.7 \)
- l. Answers will vary.
- m. Answers will vary.
- n. Answers will vary.
- o. Answers will vary.
- p. Answers will vary.
- q. Answers will vary.
- r. Answers will vary.
- s. Answers will vary.
- t. Answers will vary.
- u. Answers will vary.
- v. Answers will vary.
- w. Answers will vary.
- x. Answers will vary.
- y. Answers will vary.
- z. Answers will vary.

17 The new radius is 8 times the old radius.

18 The ratio is 7 : 1.

Investigation — Rich task

1. a. 7.5 cm

2. a. \( \angle FAB = 60^\circ, \angle AFE = 60^\circ \)

4. a. \( \triangle EDC \)

5. The impossible triangle

6. Teacher to check.

Code puzzle

To join its head to its body.