REMEMBER
Before beginning this chapter, you should be able to:
- use the equation $c = f \lambda$ for light
- equate the work done, $W = Vq$, with the change in kinetic energy, $\Delta E_k$
- apply simple wave and particle models to explain the behaviour of light.

KEY IDEAS
After completing this chapter, you should be able to:
- interpret the photoelectric effect as evidence for the particle-like nature of light
- describe why the wave model for light cannot account for the experimental results produced by the photoelectric effect
- calculate the kinetic energy, $E_k$, of a charged particle, $q$, having passed through a voltage, $V$, as a measure of the work done, $W$: $W = Vq = \Delta E_k$
- calculate the energy of a photon of light using the equation $E = hf$
- explain how the intensity of incident radiation affects the emission of photoelectrons from an irradiated electrode
- use the Einstein interpretation of the photoelectric effect and equation $E_{k_{\text{max}}} = hf - W$
- calculate the momentum of a photon of light using the equation $p = \frac{h}{\lambda}$
- use information sources to assess risk in the use of light sources, lasers and related equipment.

The work of Albert Einstein is central to our present-day understanding of the photoelectric effect.
Physics before the observation of the photoelectric effect

By the latter half of the nineteenth century, the ability of Newtonian mechanics to predict and explain much of the material world was unquestioned. At the same time, discoveries in chemistry showed that the world consisted of many elements, each made up of identical atoms, and compounds made up of combinations of atoms in fixed proportion. Most scientists believed that all matter was made up of particles, and that the universe was governed by deterministic mechanical laws. That is, they thought the universe was like a big machine. Newtonian mechanics allowed them to explain the working of the universe in terms of energy transformations, momentum transfer, and the conservation of energy and momentum due to the action of well-understood forces.

The modelling of light was also progressing well, with many experiments indicating light was a wave of some type. James Clerk Maxwell developed a set of equations that were able to explain all the existing observations of light at the time based on the premise that light was an electromagnetic wave, making an assertion as to the nature of light itself. Light came to be modelled as a transverse wave consisting of perpendicular electric and magnetic fields.

Thomas Young had shown that the behaviour of light passing through narrow slits could be explained using ideas of waves. He had even measured the wavelengths of light in the visible spectrum, but he did not know what sort of wave light might be. James Clerk Maxwell provided the answer in 1864. He began with the ideas of electric and magnetic interactions that you will have explored in electric power. From these ideas he developed a theory predicting that an oscillating electric charge would produce an oscillating electric field, together with a magnetic field oscillating at right angles to the electric field. These inseparable fields would travel together through a vacuum. Maxwell predicted their speed, using known electric and magnetic properties of a vacuum, to be $3 \times 10^8 \text{ m s}^{-1}$. This is the speed of light! Maxwell had produced a theory that explained how light was produced and travelled through space as electromagnetic waves. This applied not only to visible light, but also to other radiation that we cannot see, such as infra-red and ultraviolet radiation.

Maxwell’s theoretical wave model for light was able to show that the energy associated with electromagnetic waves was related to the size or amplitude of the wave. The more intense the wave the greater the amplitude and hence the energy it contained. He was also able to show that an electromagnetic wave had momentum and was thus capable in principle of exerting forces on other objects. According to Maxwell’s model the amount of momentum contained in an electromagnetic wave $p$ is related to the energy contained in the wave $E$ by the simple equation $p = \frac{E}{c}$ or $E = pc$.

At the same time, Max Planck was trying to understand how hot objects emit electromagnetic waves. That is, he was studying light emitted by incandescent objects such as the sun, light bulbs or a wood fire. He could make his mathematical models fit the available data only if he conceded that the energy associated with the electromagnetic radiation emitted was directly proportional to the frequency of radiation and, importantly, that the energy came in bundles that he called quanta. Thus $E = hf$, where $h$ is a constant and has come to be known as ‘Planck’s constant’. Planck’s constant is equal to $6.63 \times 10^{-34} \text{ J s}$.

What all of this meant was not clear — Maxwell’s wave model for light worked extremely well and yet understanding incandescent objects required a model that concentrated energy into localised packets called quanta that were more like particles.
A pair of problems existed. One question was how matter could convert some of its kinetic and potential energy into light. Max Planck and other scientists were working on this problem as part of their efforts to understand black body radiation (that is, radiation emitted by incandescent objects). The other question was how light could transfer its energy to matter. This process became known as the photoelectric effect.

Planck’s conclusion about a particle nature for light did not fit comfortably with the successful wave model of light proposed by Maxwell. It would be for Albert Einstein to interpret this apparent quandary with other experimental data over a decade later. In reward for his success, he won the Nobel Prize for Physics in 1921. Einstein's interpretation asserted that light is best thought of as a stream of particles, now called photons, with each photon carrying energy

\[ E_{\text{photon}} = hf \]

and capable of transferring this energy to other particles such as electrons.

**Sample problem 11.1**

(a) Blue light has a frequency of \( 6.7 \times 10^{14} \) Hz.

(i) Calculate the energy associated with a bundle of blue light.

(ii) Find the momentum associated with a quantum of blue light.

**Solution:**

(a) (i) The energy of the blue light \( E \) is given by:

\[
E = hf = 6.63 \times 10^{-34} \times 6.7 \times 10^{14} = 4.4 \times 10^{-19} \text{ J.}
\]

(ii) The momentum \( p \) is given by:

\[
p = \frac{E}{c} = \frac{4.4 \times 10^{-19}}{3 \times 10^8} = 1.5 \times 10^{-27} \text{ N s.}
\]

(b) From the wavelength we can find the frequency. From the frequency we can find the energy. From the energy we can find the momentum. We can combine these three steps into one.

\[
f = \frac{c}{\lambda} \Rightarrow E = hf \Rightarrow E = \frac{hc}{\lambda}
\]

Now \( p = \frac{E}{c} \Rightarrow p = \frac{hc}{\lambda c} \Rightarrow p = \frac{h}{\lambda} \)

\[
p = \frac{h}{\lambda} = 6.63 \times 10^{-34} = 6.5 \times 10^{-27} \text{ N s}
\]

**Revision question 11.1**

A quantum of light has a momentum of \( 9.8 \times 10^{-28} \) N s. Calculate the frequency of the light.
Sample problem 11.2

(a) What is the energy of each photon emitted by a source of green light having a wavelength of 515 nm?

(b) How many photons per second are emitted by a light source emitting a power of 0.3 W as 515 nm light? (This power is similar to the power emitted by a 40 W fluorescent tube in the wavelength range 515 ± 0.5 nm.)

Solution: (a) The photon energy can be found as follows:

\[ E_{\text{photon}} = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \text{ J s} \times 2.9979 \times 10^8 \text{ m s}^{-1}}{515 \times 10^{-9} \text{ m}} = 3.86 \times 10^{-19} \text{ J.} \]

(b) The power emitted by the globe is:

\[ \text{power} = \frac{\text{energy emitted}}{\text{time interval}} = \frac{E}{\Delta t} = \frac{N E_{\text{photon}}}{\Delta t} \]

where

\( N \) is the number of photons emitted in the time interval \( \Delta t \).

So

\[ N = \frac{\text{power} \times \Delta t}{E_{\text{photon}}} = \frac{0.3 \text{ W} \times 1 \text{ s}}{3.86 \times 10^{-19} \text{ J}} = 8 \times 10^{17} \text{ s}^{-1}. \]

Since each photon carries a tiny amount of energy, huge numbers of photons are emitted from quite ordinary light sources in each second.

Revision question 11.2

A radio station has a 1000 W transmitter and transmits electromagnetic radiation with a frequency 104.6 MHz. Calculate the number of photons emitted per second by the transmitter.

A mysterious radiation

A mysterious sort of radiation discovered in 1895 was given a mysterious-sounding name: X-rays. Wilhelm Röntgen was studying the behaviour of cathode rays. These rays travel from the negative electrode, the cathode, to the positive electrode, the anode, of an evacuated tube. These rays could travel the length of the evacuated tube but could not penetrate the end of the tube. Röntgen had completely covered the cathode ray tube with black cardboard and turned the lights off so he could check that the covering was opaque. He was amazed to see a weak glow, just like fluorescent paint.
about a metre away from the tube. By the light of a match he identified a fluorescent screen as the source of the only glow. The glow could not have occurred spontaneously because fluorescent materials glow as a result of the energy received when absorbing other radiation. Röntgen realised there must have been other radiation striking the fluorescent materials, but the room was completely dark, there were no ultraviolet sources and cathode rays could not cross a metre of air. He reasoned that there must be another form of radiation, produced by the tube, which could pass through the glass tube, through air and cross the room. After using a magnet to deflect the cathode rays it became clear that the new rays were produced at the point where the cathode rays struck the end of the tube. He called the radiation X-rays to indicate that they were a new form of radiation whose properties were not known.

Röntgen measured the penetration of these new rays through various substances, including his own hand, and noted their lack of deflection by magnetic and electric fields, and the absence of observable interference effects with usual optical diffraction gratings.

Röntgen performed his experiments in a completely dark room. There were no ultraviolet sources and the cathode rays could not cross a metre of air. He reasoned that there must be another form of radiation, produced by the tube, which could pass through the glass tube, through air and cross the room. After using a magnet to deflect the cathode rays it became clear that the new rays were produced at the point where the cathode rays struck the end of the tube. He called the radiation X-rays to indicate that they were a new form of radiation whose properties were not known.

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The key question was: Are the X-rays particles or waves? Their straight paths through magnetic fields and electric fields eliminated the possibility of charged particles. Neutral particles or electromagnetic radiation were the remaining options, but the lack of observable interference seemed to rule out electromagnetic radiation.

X-radiation is electromagnetic radiation. Röntgen did not observe interference effects because of the diffraction grating he used. A grating is needed with ‘slits’ that are separated by a distance similar to the wavelength of X-rays, only $10^{-10}$ m. Confirmation of the wave behaviour of X-rays was finally produced by experiments in which the ‘slits’ were provided by the regular layers of atoms of crystals. These layers are commonly separated by $10^{-10}$ m, ideal to form a diffraction grating for X-rays. Max von Laue recommended, and his colleagues Friedrich and Knipping performed, the first demonstration of this wave behaviour when they directed a beam of X-rays through a thin crystal towards a photographic plate. After many hours of exposure the developed plate showed a delightfully symmetric pattern of bright spots on a dark background. These bright ‘Laue spots’ were evidence of constructive interference — X-rays were electromagnetic waves. This confirmation was not achieved until 1912.

The Coolidge tube, invented in 1913, became the standard method of producing X-rays. Electrons from a heated cathode are accelerated by high voltage towards the anode whose face is angled at 45° to the electron beam. Their collision with atoms in the anode, a high melting point material, produces X-rays. The anode must be cooled.

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Some preliminaries — measuring the energy of light and the energy of electrons

In order to appreciate the results of the photoelectric effect, it is necessary to be able to calculate both the energy associated with light and the energy associated with a moving particle such as an electron.

The energy associated with light, $E$, provided it is treated as a localised object as necessitated by Planck, can be equated to the product of the frequency and Planck’s constant: $E = hf$. The speed of light is related to the frequency and wavelength: $c = f\lambda$, in accordance with a wave model for light. For completeness, since $E = pc$, the momentum associated with light, $p$, can be related to the wavelength $\lambda$ by the equation $p = \frac{h}{\lambda}$. It needs to be mentioned at this stage that both a wave model for light and a particle model for light have been used simultaneously. This usage of two models simultaneously came to be known as the wave–particle duality, and for many years it remained an unresolved component in physics. With the development of quantum mechanics in the 1920s, a consistent mathematical model incorporating both aspects emerged.

Potential differences can be used to accelerate and decelerate charged particles. Let us now review how the kinetic energy of a charged particle can be related to the electrical potential difference through which it can be made to move. Understanding this relationship will make understanding
the photoelectric effect easier. It will be also useful to know how the kinetic energy of matter is related to its momentum, just as in the case for light.

The simplest way to accelerate electrons is with two parallel metal plates in an evacuated chamber (see the figure). The two plates are connected to a DC power supply (similar to a capacitor connected to a battery). An electron will experience an electric force anywhere in the region between the plates: it will be attracted by the positively charged plate and repelled by the negatively charged plate. Both of these forces act in the same direction.

The size of this force will also be the same throughout this region. At point A, the downward repulsive force on an electron from the negatively charged plate will be greater than the downward attractive force of the positive plate. At point B, the downward attractive force will be greater. However, the combined effect of the two forces will be the same at each point.

This constant electric force on a charge placed between the plates can be compared to the constant gravitational force on a mass located above the ground. In gravitation, where the force acts on the mass of an object:

\[ W = mg \]

With an electric force, the force acts on the electric charge of an object:

\[ F = Eq \]

The electric field, \( E \), can be expressed as electric force, \( F \), divided by electric charge, \( q \):

\[ E = \frac{F}{q} \]

This equation is also applied to the magnitude of the electric field. That is, \( E = \frac{F}{q} \). The unit of electric field is newtons per coulomb (\( \text{N C}^{-1} \)), in the same way that the gravitational field can be measured in newtons per kilogram (\( \text{N kg}^{-1} \)). However, the magnitude of the electric field can also be shown to be:

\[ E = \frac{V}{d} \]

These two relationships for the electric field \( (E = \frac{F}{q} \text{ and } E = \frac{V}{d}) \) give it two equivalent units: newtons per coulomb (\( \text{N C}^{-1} \)) and volts per metre (\( \text{V m}^{-1} \)). These two relationships can also be linked by considering energy. The gain in energy of the electron can be obtained by calculating the work done on the charge to move it from one plate to the other. It can also be obtained by recalling that the voltage across a battery equals the energy gained by one coulomb of charge. So:

\[ \text{work} = \text{force} \times \text{distance} = \text{voltage} \times \text{electric charge} \]

\[ \Rightarrow F \times d = V \times q \]
The work done by the potential difference, $V$, on a free electron is equal to the change in the kinetic energy of the electron, $\Delta E_k$. Since kinetic energy is given by the expression $\frac{1}{2}mv^2$, and by further making the assumption that the initial kinetic energy of an electron emitted by a filament is zero, we then get a useful non-relativistic equation:

$$E_k = Vq = \frac{1}{2}mv^2$$

We interpret this equation in the following way. For a given voltage, $V$, acting on an electron (mass $m = 9.1 \times 10^{-31}$ kg and charge $q = 1.6 \times 10^{-19}$ C), we are able to calculate both the speed of the electron and hence its momentum ($p = mv$), as well as its energy, $E_k$.

Thus, an arrangement of negative and positive charged plates can be used to accelerate a charged particle in a straight line. This arrangement came to be known as an electron gun. By reversing the polarity of charge on the plates, electrons with energy can be decelerated. The voltage required to achieve this stopping of electrons with energy is known as a stopping voltage.

**Measuring the energy of photoelectrons**

In the photoelectric effect, energy is transferred from light to electrons. Lenard was able to measure the maximum kinetic energy of photoelectrons by applying a retarding voltage to stop them. Recall that the work done on a charge, $q$, passing through a potential difference, $V$, is equal to $qV$. That is, an electron passing through a potential difference of 3.0 V would have $1.6 \times 10^{-19} \times 3.0 \times 10^{-19} = 4.8 \times 10^{-19}$ J of work done on it. If the voltage is arranged so that the emitted electrons leave the positive terminal and are collected at a negative terminal, then electrons lose 4.8 $\times 10^{-19}$ J of energy. In the graph on page 12, the voltage, $V$, can be measured when the photocurrent drops to zero. This indicates that all the electrons which absorbed energy from light striking the electrode have been stopped. At this voltage — the so-called stopping voltage, $V_0$ — the photoelectrons have had all their kinetic energy removed. Thus the kinetic energy that the photoelectrons left the surface with, $E_K$, is $qV_0$. In general, photoelectrons with a kinetic energy $E_K$ will be stopped by a stopping voltage $V_0$ such that $E_K = qV_0$.

The energy unit the joule is many orders of magnitude too large to be useful in describing energy changes in atoms. Instead we frequently use the electron volt, abbreviated to eV.

**Sample problem 11.3**

An electron gun uses a 500 V potential difference to accelerate electrons evaporated from a tungsten filament. Model the evaporated electrons as having zero kinetic energy.

(a) How much work is done on an electron moved across a potential difference of 500 V?

(b) What type of energy is this work transformed into?

(c) Calculate the kinetic energy of the electrons in electron volt and joule.

(d) Using the equation for the kinetic energy, $E_k$, of a particle with mass $m$, determine the speed, $v$, of these electrons.

(e) Calculate the momentum of these electrons.

**Solution:**

(a) Use $W = Vq = 500 \times 1.6 \times 10^{-19} = 8.0 \times 10^{-17}$ J or 500 eV.

(b) Potential energy available is transformed into the kinetic energy of the electron: $W = Vq = \Delta E_k$. 

**An electron gun** is a device to provide free electrons for a linear accelerator. It usually consists of a hot wire filament with a current supplied by a low-voltage source.

An electron volt is the quantity of energy acquired by an elementary charge ($e = 1.6 \times 10^{-19}$ C) passing through a potential difference of 1 V. Thus $1.6 \times 10^{-19}$ J = 1 eV.
(c) Assuming the initial kinetic energy of the electrons evaporated from a tungsten filament is 0, the kinetic energy of the electrons is equal to the work done: \( E_k = W = 8.0 \times 10^{-17} \text{ J} \) or 500 eV.

(d) \( E_k = \frac{1}{2} mv^2 = 8.0 \times 10^{-17} \text{ J} \), provided the electron speed is sufficiently small to ignore relativistic effects. Take the mass of an electron to be \( m = 9.1 \times 10^{-31} \text{ kg} \) and solve equation for \( v \). Thus:

\[
v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 8.0 \times 10^{-17}}{9.1 \times 10^{-31}}} = 1.33 \times 10^7 \text{ m s}^{-1}
\]

This is substantially slower than the speed of light; therefore, we can ignore relativistic effects.

(e) \( p = mv = 9.1 \times 10^{-31} \times 1.33 \times 10^7 = 1.2 \times 10^{-23} \text{ N s} \)

**Revision question 11.3**

An electron in a beam of electrons generated by an electron gun has energy \( 1.26 \times 10^{-17} \text{ J} \).

(a) Calculate the energy of this electron in electron volts.

(b) State the potential difference required to stop electrons with this energy, that is to remove their kinetic energy and bring them to rest.

(c) Determine the speed of the electron, assuming that its kinetic energy is given by the equation \( E_k = \frac{1}{2} mv^2 \).

(d) Use your answer to (c) to calculate the momentum of this electron.

**Sample problem 11.4**

(a) Electrons are emitted from a surface with a kinetic energy of \( 2.6 \times 10^{-19} \text{ J} \). What is the size of the stopping voltage that will remove all of this energy from the electrons?

(b) What energy electrons will a 4.2 V stopping voltage stop?

**Solution:**

(a) The kinetic energy of each electron is \( 2.6 \times 10^{-19} \text{ J} \). The charge on an electron is \( 1.6 \times 10^{-19} \text{ C} \).

\[
E_k = qV_0
2.6 \times 10^{-19} \text{ J} = 1.6 \times 10^{-19} \text{ C} \times V_0
V_0 = \frac{2.6 \times 10^{-19} \text{ J}}{1.6 \times 10^{-19} \text{ C}}
= 1.62 \text{ V}
= 1.6 \text{ V (accurate to 2 significant figures)}
\]

A stopping voltage of 1.6 V will stop the electrons emitted from the surface.

(b) The stopping voltage is 4.2 V. The charge of an electron is \( 1.6 \times 10^{-19} \text{ C} \).

\[
E_k = qV_0
= 1.6 \times 10^{-19} \text{ C} \times 4.2 \text{ V}
= 6.72 \times 10^{-19} \text{ J}
= 6.7 \times 10^{-19} \text{ J (accurate to 2 significant figures)}
\]

A stopping voltage of 4.2 V will stop electrons with energy \( 6.7 \times 10^{-19} \text{ J} \).
Electrons are emitted from the surface of a photocell with $4.8 \times 10^{-19}$ J of kinetic energy. What is the size of the stopping voltage that will remove all of this energy from the electrons?

Remember that a joule is the electric potential energy change that occurs when one coulomb of charge moves through a potential difference of one volt.

$$1 \text{ V} = \frac{1 \text{ J}}{1 \text{ C}}$$

$$\Rightarrow 1 \text{ J} = 1 \text{ C} \times 1 \text{ V}$$

An electron volt is defined as the electric potential energy change that occurs when one electronic charge, $q_e = 1.6021 \times 10^{-19}$ C, moves through one volt.

$$1 \text{ eV} = 1 q_e \times 1 \text{ V}$$

where $q_e$ is the magnitude of charge of an electron

$$\Rightarrow 1 \text{ eV} = 1.6021 \times 10^{-19} \text{ C} \times 1 \text{ V}$$

$$\Rightarrow 1 \text{ eV} = 1.6021 \times 10^{-19} \text{ J}.$$  

We now have some calculating tools for working with light, although it is modelled at this stage rather ambiguously as something like a particle — a localised packet with energy $E = hf$ and momentum $p = \frac{E}{c}$ — but propagating like a wave with speed $c = f\lambda$, which further implies a momentum $p = \frac{h}{\lambda}$ for a localised packet. This localised packet, as we will see, is now known as a photon — a particle of light.

We also have some calculating tools for working with electrons, modelling them as particles. These particles have kinetic energy $E_k = \frac{1}{2}mv^2$ and momentum $p = mv$. We can also write the kinetic energy in terms of the momentum:

$$E_k = \frac{p^2}{2m}.$$  

This equation, in particular, will prove to be useful later. With the right experimental apparatus we can either give or take energy from charged particles by allowing a potential difference $V$ to do work $W$ on a charge $q$ according to the equation $W = Vq$. Electrons can thus be accelerated or decelerated by a potential difference depending solely on the polarity of the potential difference attached to the equipment. This equipment is generically referred to as an electron gun. We are now ready to learn about the photoelectric effect and to interpret data arising from experiments.

**The photoelectric effect**

The nineteenth century view of light was developed as a result of the success of the wave model in explaining refraction, diffraction and interference. The wave model did a great job!

The first signs of behaviour that could not be explained using a wave model almost went unnoticed in 1887. Heinrich Hertz was in the middle of the experimental work which would show that radio waves and light were really the same thing — electromagnetic waves. He produced radio waves with a frequency of about $5 \times 10^8$ hertz (yes, the unit for frequency was named after him) by creating a spark across the approximately one centimetre gap between two small metal spheres. The radio waves were detected up to several hundred metres away, by the spark they excited across another air gap, this time between the pointed ends of a circular piece of wire. Hertz was able to show that the radio waves travelled at the speed of light. Although Hertz was not aware of it, this was the beginning of radio communication.
During his experiments Hertz noticed that the spark showing the arrival of the radio waves at the receiver became brighter whenever the gap was simultaneously exposed to ultraviolet radiation. He was puzzled, and made note of it, but did not follow it up. Now we know that the reason for the brighter spark was that the ultraviolet radiation ejected electrons from the metal points of the detector. The presence of these electrons reduced the electrical resistance of the air gap, so a spark flashed brighter than usual whenever the radio waves were being detected.

This ejection of electrons by light is called the **photoelectric effect**. Following up Hertz’s observations of this effect led to a breakthrough in the way we view the behaviour of light.

**The experiment**

Fifteen years passed before Philipp Lenard, a German physicist, performed careful experiments to investigate the effect. Lenard replaced Hertz’s spark gap with two metal electrodes on opposite sides of an evacuated chamber. He investigated the energies of electrons ejected from one of these electrodes when light shone on it. The experimental arrangement used in 1902 by Lenard is shown overhead, top left. Lenard designed his experiment so that he could vary several features of this arrangement.

- **The frequency and intensity of the light** could be varied. Light from an electric discharge arcing between two electrodes was introduced into the chamber through a window. The arc produced a spectrum of several different frequencies characteristic of the electrode material. Filters in front of the window were used as frequency selectors to ensure that light of a single chosen frequency reached the electrode X. Light sources that emit light of only one frequency are called **monochromatic** light sources. Lenard varied the light intensity either by changing the arc current, or by moving the light source to a different distance from the window.

- **The potential difference between the electrodes in the chamber** could be varied by changing the position of the slide contact on the coiled resistor. By varying the contact position to both right and left of Z, the potential difference could be made either accelerating or retarding for electrons.

- Lenard could vary the **distance between the electrode receiving light, X, and the second electrode, Y.**
First, Lenard used a fixed intensity light source and a fixed accelerating voltage while he varied the distance between the electrodes. He found that the current of photoelectrons, called the photocurrent, increased to a maximum when the electrodes were about 5 mm apart. He reasoned that after being ejected by light the electrons flew out in different directions, and that at this short distance the second electrode was collecting all electrons. This separation was used for all the later experiments.

Now he was ready to explore the effects of the light on this photoelectric effect. The results of Lenard’s further experiments are summarised in the graphs of photocurrent as a function of the potential difference between the electrodes for several light intensities shown below.

The graphs above illustrate several important parts of Lenard’s investigations. The numbers on the diagrams refer to the numbered points below.

1. **Keeping the light frequency constant**, Lenard investigated how the maximum photocurrent depended on light intensity. Higher intensity light produced greater values of the maximum photocurrent, as shown in the figure above. In fact Lenard’s results showed that the maximum photocurrent was directly proportional to the light intensity. To his surprise this proportionality held true over a wide intensity range, right down to light of a tiny $3 \times 10^{-7}$ of the highest intensity light he could produce.

2. When Lenard applied a retarding voltage between the electrodes, the current decreased as the magnitude of the voltage increased. This was not surprising. It was expected that when the electric field between the plates exerted a force opposing the motion of the electrons, they would slow down and probably reverse direction before reaching the opposite electrode. The kinetic energy of the electrons would be converted into electric potential energy. Only the very slow electrons would reverse direction before being collected at the electrode Y when the voltage between the plates was low. So, only a few electrons would then be removed from the stream contributing to the photocurrent. As the magnitude of the voltage was increased, more and more electrons would turn around before reaching the electrode, until at a particular voltage no electrons completed the crossing and the current dropped to zero. This minimum voltage which causes all electrons to turn back is called the stopping voltage.

3. Lenard found that the stopping voltage did not depend on the intensity of the light being used. Brighter light did not increase the kinetic energy of the electrons emitted from the cathode. The same potential difference was required to convert all of the kinetic energy of the electron into electric potential energy, no matter how bright the light.
4. The stopping voltage, however, depended on both the frequency of the light (see the following figure) and on the material of the electrode. In fact, for each material there was a minimum frequency required for electrons to be ejected. Below this cut-off frequency no electrons were ever ejected, no matter how intense the light or how long the electrode was exposed to the light. Above this frequency a photocurrent could always be detected. The photocurrent could be detected as quickly as $10^{-9}$ s after turning on the light source. This time interval was independent of the brightness of the light source.

These experiments provided evidence that the energy of light is bundled into packets whose energy depends on the light frequency. In explaining these experiments, the behaviour of light is best described as a stream of particles — very reminiscent of Newton's view! Albert Einstein, in 1905, first proposed the model to explain the photoelectric effect. For this work he won the Nobel Prize in 1921, even though he is now better known for his theories of relativity, explaining the behaviour of objects travelling at speeds close to the speed of light. Lenard had already won the Nobel Prize in 1905 for his experimental investigations.

**Sample problem 11.5**

The diagram below shows the current-versus-stopping voltage curve for a typical photoelectric cell using green light.

The colour is changed to blue, but with a lower intensity. Sketch the curve that would result from these changes.

**Solution:** Because blue light has a higher frequency than green light, the stopping voltage would be greater. The lower intensity would make the photocurrent smaller. This is shown in the diagram below.
A photon is a discrete bundle of electromagnetic radiation. Photons can be thought of as discrete packets of light energy with zero mass and zero electric charge.

**Revision question 11.5**

Consider the same arrangement as in Sample problem 11.5 except this time yellow light is used but sufficient to cause the photoelectric effect to occur. The intensity of the light is greater than with the green light. Sketch the curve that would result from this change.

To help understand Einstein’s explanation of the photoelectric effect, it is helpful to have a mental picture of how the wave and particle models describe a light bulb and its intensity. We will then return to the photoelectric effect.

### The particle model view of a light bulb

The particle model describes a light bulb as an object emitting large numbers of light particles each second. These light particles are now called photons. The photons from a monochromatic light source all have the same energy, whereas a white light source emits photons having a range of energies. An intense monochromatic light source emits a greater number of photons per second than a dim light source emitting the same colour light.

Each photon has an energy that is characteristic of the frequency of the light. The relationship between photon energy, $E_{\text{photon}}$, and frequency, $f$, is:

$$E_{\text{photon}} = hf$$

where $h$ is Planck’s constant, named after Max Planck who first proposed that light was emitted in fixed quantities of energy related to frequency. The value of $h$ is $6.63 	imes 10^{-34}$ J s, or $4.15 \times 10^{-15}$ eV. Since wave speed, frequency and wavelength are related by the equation $c = f \lambda$, we can also write:

$$E_{\text{photon}} = \frac{hc}{\lambda}$$

where

- $c =$ the speed of light in a vacuum
- $\lambda =$ the wavelength of the light.

It is paradoxical that the photon energy, a particle characteristic of light, is related to wavelength, which arises from its wave behaviour.

(a) dim light source  
(b) more intense light source

(a) A dim and (b) a more intense light source. The reduced size of the art does not indicate that the photons here have been drawn as fuzzy blobs. A fuzzy blob has been used to indicate that a photon is not a particle like a billiard ball. It does not have definite edges.
**Sample problem 11.6**

An electron is ejected from an atom with a kinetic energy of 1.9 eV. A retarding voltage of 1.2 V causes it to slow down during a photoelectric effect experiment (see figure below left). Describe the energy changes and calculate their values, in both eV and J.

**Solution:** Energy is transformed from kinetic energy to electric potential energy. Let $q_e$ represent the magnitude of the charge on the electron. The increase in electric potential energy is:

$$\Delta E_{ep} = -q_e V = -q_e \times -1.2 \text{ V} = 1.2 \text{ eV}.$$  

The electric potential energy has increased by 1.2 eV. The kinetic energy will have decreased from 1.9 eV to 0.7 eV.

Converting the unit of this increase of electric potential energy to joules:

$$1.2 \text{ eV} = 1.2 \text{ eV} \times 1.6021 \times 10^{-19} \text{ J eV}^{-1} = 1.9 \times 10^{-19} \text{ J}.$$  

In one step:

$$\Delta E_{ep} = -q_e V = -1.6021 \times 10^{-19} \text{ C} \times -1.2 \text{ V} = 1.9 \times 10^{-19} \text{ J}.$$  

**Revision question 11.6**

An electron is ejected from an atom with kinetic energy $E$. A retarding voltage of 1.8 V causes it to slow down so that its kinetic energy is 0.50 eV.

(a) Calculate the initial kinetic energy $E$ of the electron in eV.
(b) Convert this energy into joules.

**Sample problem 11.7**

The energy of a photon of 515 nm light is $3.86 \times 10^{-19}$ J. How many eV is that?

**Solution:** To convert energy in J to eV, divide by $1.6021 \times 10^{-19}$ J eV$^{-1}$.

$$3.86 \times 10^{-19} \text{ J} = \frac{3.86 \times 10^{-19} \text{ J}}{1.6021 \times 10^{-19} \text{ J eV}^{-1}} = 2.41 \text{ eV}.$$  

Clearly the eV unit is much more convenient.

**Revision question 11.7**

What is the energy in joules of a photon whose energy is 13.6 eV?

**A wave model view of a light bulb**

Now we turn to thinking about a light bulb as a source of waves. The waves are moving oscillations of linked electric and magnetic fields, as shown in the figure on page 2. Spherical wavefronts spread out from the light bulb. If the light bulb is monochromatic, it emits light of a single frequency, and because all light travels at the same speed in a vacuum this frequency determines the...
wavelength. The intensity of the light affects the amplitude of the wave, not its frequency. When more intense light passes a point there is a greater difference between the maximum and minimum values of the electric field, and the magnetic field, occurring at that point as the light passes.

Wave model of two light sources emitting light of the same frequency but with different intensities. Imagine a water surface being regularly disturbed by an object dipping into the water. The water level could represent the electric field of the light wave.

The particle model and the photoelectric effect

Now that we have an idea of the wave and particle model descriptions of intensity, let’s consider how each of the observations of the photoelectric effect experiment could be explained using a particle model, and why a wave model is not as successful in this situation. Remember, a close inspection of the evidence should be able to allow us to decide whether electrons are being hit by particles or waves.

The next figure illustrates the two models. In both models light transfers energy to the electrons, enabling them to escape from the overall attractive force exerted by the metal electrode. In the particle model description, the entire energy of a single photon is transferred to a single electron; the photon is gone. (One photon — two electron processes are very rare.) Some of the photon energy is required to enable the electron to escape from the electrode. This transferred energy, which enables an electron to escape the attraction of a material, is called its ionisation energy. Electrons in the metal have a range of energy levels, so they also have a range of ionisation energies. The minimum ionisation energy is called the work function of the material. The photon energy which is ‘left over’ becomes the kinetic energy of the electron. Naturally, the electrons requiring the least energy to enable them to escape will leave with the greatest kinetic energy.

**Ionisation energy** is the amount of energy required to be transferred to an electron to enable it to escape from a material.

The **work function** is the minimum energy required to release an electron from the surface of a material.
The photoelectric effect

**Concept 9**

(c) Electron escapes, with maximum photon energy allows an electron to escape.

Four identical photons deliver their energy to four electrons.

(a) Electron escapes, with maximum $E_k = hf - W$.

(b) Photon energy is just enough for electron to escape, but electron $E_k$ is zero.

(c) Electron escapes, with $E_k = hf - E_{ionisation}$; $E_k < \text{maximum} E_k$.

(d) Photon energy is insufficient to enable electron to escape.

The kinetic energy of each photoelectron is given by:

$$E_k = E_{\text{photon}} - E_{\text{ionisation}} = hf - E_{\text{ionisation}}$$

The maximum kinetic energy of photoelectrons, $E_{k_{\text{max}}}$, is given by:

$$E_{k_{\text{max}}} = E_{\text{photon}} - W = hf - W$$

where $W$ is the work function.

**An energy perspective**

An energy picture of the effect can also be useful. Note that the vertical axis in the figure below is not the depth of the electron in the material, but the electron energy. Electrons in the metal have a range of energies, depending on how strongly they are bound to the metal. Electrons having higher energies are more loosely held by the material and need to receive less energy to escape than electrons at lower energy.
Explaining Lenard’s experimental observations

Here is how the particle model explains Lenard’s experimental observations. The numbering here matches the number of these observations earlier in the chapter. (See pages 12–13.)

1. **Maximum photocurrent is proportional to intensity.**

Doubling the intensity without changing frequency doubles the number of photons reaching the electrode each second, but not their energy. This doubles the rate of electron emission without changing the energy transferred to each electron, and therefore doubles the maximum photocurrent.

2. **Retarding voltage reduces photocurrent. A stopping voltage exists above which no electrons reach the second electrode.**

Ejected electrons have a variety of energies, depending on the photon energy and their ionisation energy. A low retarding voltage turns back only the electrons having low kinetic energies. Increasing the retarding voltage will turn back electrons with higher kinetic energies, until at the stopping voltage none can reach the second electrode.

3. **Stopping voltage is independent of light intensity.**

Changing the light intensity only does not change its frequency, so the photon energy is not changed. Photoelectrons will have the same range of energies, and so the same retarding voltage is needed to reduce the photocurrent to zero.

4. **Stopping voltage depends on light frequency and material: a cut-off frequency exists.**

Since the stopping voltage reverses the direction of all electrons, it is the voltage required to entirely transform the kinetic energy of the fastest electrons into electric potential energy.

\[
E_{kmax} = \text{magnitude of change in electron’s electrical potential energy} = q_e V_0
\]

where \( q_e \) here is the magnitude of the electronic charge.

Our photon model tells us that:

\[
E_{kmax} = E_{\text{photon}} - W = hf - W
\]

So \( q_e V_0 = hf - W \).

Clearly \( V_0 \) depends on the light frequency, \( f \), and also on the electrode material through its work function, \( W \). A photon whose energy, \( hf \), is less than the work function, \( W \), cannot supply enough energy for an electron to escape. The electron remains trapped by the electrode.

### Sample problem 11.8

Light with a wavelength of 425 nm strikes a clean metallic surface and photoelectrons are emitted. A voltage of 1.25 V is required to stop the most energetic electrons emitted from the photocell.

(a) Calculate the frequency of a photon of light whose wavelength is 425 nm.

(b) Calculate the energy in joules and also in electron volts of a photon of light whose wavelength is 425 nm.

(c) State the energy of the emitted electron in both electron volts and joules.

(d) Calculate the work function \( W \) of the metal in eV and J.

(e) Determine threshold frequency \( f_0 \) and consequently the maximum wavelength of a photon that will just free a surface electron from the metal.
(f) Light of a wavelength 390 nm strikes the same metal surface. Calculate the stopping voltage.

**Solution:**

(a) \[ f = \frac{c}{\lambda} \]
\[ = \frac{3.0 \times 10^8}{4.25 \times 10^{-7}} \]
\[ = 7.06 \times 10^{14} \text{ Hz} \]
\[ = 7.1 \times 10^{14} \text{ Hz} \]

(b) \[ E = hf \]
\[ = 6.63 \times 10^{-34} \times 7.06 \times 10^{14} \]
\[ = 4.68 \times 10^{-19} \text{ J} \]

To convert energy in joules into energy in electron volts, divide by \(1.6 \times 10^{-19}\) joules eV\(^{-1}\).

\[ E = \frac{4.68 \times 10^{-19}}{1.6 \times 10^{-19}} \]
\[ = 2.92 \text{ eV} \]
\[ = 2.9 \text{ eV} \]

(c) Since the stopping voltage is 1.25 V, the energy of the emitted electron is 1.25 eV. The energy in joules can be found by multiplying by \(1.6 \times 10^{-19}\).

Thus the energy is:

\[ 1.25 \times 1.6 \times 10^{-19} = 2.00 \times 10^{-19} \text{ J} \]

(d) Using the equation \(E_{k_{\max}} = hf - W\), the work function can be found. We know that when the photon energy \(hf\) equals 2.92 eV the electrons have an energy of 1.25 eV. Thus \(1.25 = 2.92 - W\). Thus:

\[ W = 2.92 - 1.25 = 1.67 \text{ eV} = 2.67 \times 10^{-19} \text{ J} = 2.7 \times 10^{-19} \text{ J} \]

(e) Again use the equation \(E_{k_{\max}} = hf - W\). The threshold frequency \(f_0\) is the frequency below which the photoelectric effect does not occur. At this frequency electrons are just not able to leave the surface. This model implies \(0 = hf_0 - W\). Rearrange this equation to give the useful result:

\[ f_0 = \frac{W}{h} \]
\[ = \frac{2.67 \times 10^{-19}}{6.63 \times 10^{-34}} \]
\[ = 4.03 \times 10^{14} \text{ Hz} \]

The maximum wavelength is thus:

\[ \lambda = \frac{c}{f_0} \]
\[ = \frac{3.0 \times 10^8}{4.03 \times 10^{14}} \]
\[ = 7.4 \times 10^{-7} \text{ m or 740 nm} \]
(f) Use the equation \( E_{k_{\text{max}}} = \frac{hc}{\lambda} - W \) to find the energy of the emitted electrons. When this is known the stopping voltage can be readily found. It is convenient to use eV here.

\[
E_{k_{\text{max}}} = \frac{4.15 \times 10^{-15} \times 3.0 \times 10^8}{3.90 \times 10^{-7}} - 1.67 \\
= 3.19 - 1.67 \\
= 1.52 \text{ eV} \\
= 1.5 \text{ eV}
\]

A stopping voltage of 1.5 V is required to stop the emitted electrons.

**Revision question 11.8**

A new photocell with a different metallic surface is used. Again light of wavelength 425 nm strikes a clean metallic surface and photoelectrons are emitted. This time, a stopping voltage of 0.87 V is required to stop the most energetic electrons emitted from the photocell.

(a) State the highest energy of the emitted electrons in both electron volts and joules.

(b) Calculate the work function \( W \) of the metal.

(c) Determine threshold frequency \( f_0 \) and, consequently, the maximum wavelength of a photon that will just free a surface electron from the metal.

(d) Light of a wavelength 650 nm strikes the same metal surface. Explain what happens.

**Sample problem 11.9**

The table below gives some data collected by students investigating the photoelectric effect using a photocell with a lithium cathode. This cell is illustrated in the schematic diagram on the left.

<table>
<thead>
<tr>
<th>Wavelength of light used (nm)</th>
<th>Frequency of light used ( \times 10^{14} ) (Hz)</th>
<th>Photon energy of light used, ( E_{\text{photon}} ) (eV)</th>
<th>Stopping voltage readings (V)</th>
<th>Maximum photo-electron energy ( E_{\text{max}} ) (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>663</td>
<td>6.14</td>
<td>0.45</td>
<td></td>
<td>( 1.84 \times 10^{-19} )</td>
</tr>
</tbody>
</table>

(a) Complete the table.
(b) Using only the two data points supplied in the table, plot a graph of maximum photo-electron energy in joules versus photon frequency in hertz for the lithium photocell.
(c) Using only your graph, state your values for the following quantities. In each case, state what aspect of the graph you have used.
   (i) Planck’s constant, \( h \), in the units J s and eV s as determined from the graph
   (ii) The threshold frequency, \( f_0 \), for the metal surface in Hz as determined from the graph
   (iii) The work function, \( W \), for the metal surface as determined from the graph, in the units J s and eV s
(d) On the same axes, draw and label the graph you would expect to get when using a different photocell, given that it has a work function slightly larger than the one used to collect the data in the table above.
A new photocell is now investigated. When light of frequency $9.12 \times 10^{14}$ Hz is used, a stopping voltage of 1.70 V is required to stop the most energetic electrons.

(e) Calculate the work function of the new photocell, giving your answer in both joules and electron volts.

(f) When the battery voltage of the new photocell is set to 0 V, the photocurrent is measured to be 48 $\mu$A. The intensity of the light is now doubled. Describe what happens in the electric circuit with the power supply voltage set to 0 V when the light intensity is doubled.

(g) With the intensity still doubled, the voltage is now slowly increased from 0 and the photocurrent slowly reduces to 0 A. State the stopping voltage when the current first equals 0 A with the light intensity still doubled.

(a) Use $c = f \lambda$ to complete columns 1 and 2. Use $E = hf$ to complete column 3, and use the conversion factor for joules to eV to complete columns 4 and 5.

<table>
<thead>
<tr>
<th>Wavelength of light used (nm)</th>
<th>Frequency of light used $\times 10^{14}$ (Hz)</th>
<th>Photon energy of light used, $E_{\text{photon}}$ (eV)</th>
<th>Stopping voltage readings (V)</th>
<th>Maximum photo-electron energy $E_e$ (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>663</td>
<td>4.52</td>
<td>1.88</td>
<td>0.45</td>
<td>$7.20 \times 10^{-20}$</td>
</tr>
<tr>
<td>488</td>
<td>6.14</td>
<td>2.55</td>
<td>1.15</td>
<td>$1.84 \times 10^{-19}$</td>
</tr>
</tbody>
</table>

(b) The graph will contain two points representing the fact that light of frequency $4.52 \times 10^{14}$ Hz will produce electrons of energy 0.45 eV and light of frequency $6.52 \times 10^{14}$ Hz will produce electrons of energy 1.15 eV. A line drawn containing these two data points will give a work function of 1.5 eV and a threshold frequency of $3.5 \times 10^{14}$ Hz.

(c) (i) Planck’s constant = gradient of graph

$$= \frac{1.84 \times 10^{-19} - 7.20 \times 10^{-20}}{(6.14 - 4.52) \times 10^{14}} = 6.9 \times 10^{-34} \text{ J s},$$

which is close to the accepted value. It also has the value $4.3 \times 10^{-15}$ eV s.

(ii) From the line of best fit in graph (b), the threshold frequency $= x$-axis intercept $= 3.5 \times 10^{14}$ Hz.

(iii) From the line of best fit in the graph (b), the work function $= y$-axis intercept $= 2.4 \times 10^{-19} J = 1.5$ eV.
(e) Use $E_e = E_{\text{photon}} - W$ to calculate the work function, $W$.

\[
1.7 \times 1.6 \times 10^{-19} = 6.6 \times 10^{-34} \times 9.12 \times 10^{14} - W
\]

\[
W = 6.02 \times 10^{-19} - 2.72 \times 10^{-19}
\]

\[
= 3.3 \times 10^{-19} \text{ J}
\]

\[
= 2.1 \text{ eV}
\]

(f) With the light intensity doubled, the photocurrent would also double.

(g) The stopping voltage would remain the same, 1.7 V, as the colour and hence the frequency of the light source is unchanged.

### Revision question 11.9

The table below gives some data collected by students investigating the photoelectric effect using a photocell with a clean metallic cathode.

<table>
<thead>
<tr>
<th>Wavelength of light used (nm)</th>
<th>Frequency of light used $\times 10^{14}$ (Hz)</th>
<th>Photon energy of light used, $E_{\text{photon}}$ (eV)</th>
<th>Stopping voltage readings (V)</th>
<th>Maximum photo-electron energy $E_e$ (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>524</td>
<td>6.02 $\times 10^{-19}$</td>
<td>3.19</td>
<td>1.54</td>
<td>$3.78 \times 10^{-19}$</td>
</tr>
</tbody>
</table>

(a) Complete the table.

(b) Using only the two data points supplied in the table, plot a graph of maximum photo-electron energy in joules versus photon frequency in hertz for the photocell.

(c) Using only your graph, state your values for the following quantities. In each case, state what aspect of the graph you have used.

(i) Planck’s constant, $h$, in the units J s and eV s as determined from the graph

(ii) The threshold frequency, $f_0$, for the metal surface in Hz as determined from the graph

(iii) The work function, $W$, for the metal surface as determined from the graph, in the units J s and eV s

(d) On the same axes, draw and label the graph you would expect to get when using a different photocell, given that it has a work function slightly larger than the one used to collect the data in the table above.

A new photocell is now investigated. When light of frequency $8.25 \times 10^{14}$ Hz is used, a stopping voltage of 1.59 V is required to stop the most energetic electrons. In addition, when the battery voltage is set to 0 V, the photocurrent is measured to be 38 $\mu$A.

(e) Calculate the work function of the new photocell.

(f) Describe what happens in the electric circuit with the power supply voltage set to 0 V when the light intensity is halved.

(g) With the intensity still halved, the stopping voltage is now slowly increased from 0 V and the photocurrent slowly reduces to 0 A. State the stopping voltage when the current first equals 0 A with the light intensity still halved.

### What’s wrong with the wave model?

In the wave model picture of the photoelectric effect, the energy of light is shared between electrons and accumulated little by little with the arrival of each wavefront. If this were true, the photoelectric effect experiment results would be significantly different.

Higher intensity light, delivering energy at a greater rate, would produce electrons with higher kinetic energies, so the stopping potential difference would depend on intensity.
For example, the effect of waves on a beach is cumulative. As each wave breaks along the length of the beach, it adds to the effect of the previous waves until signs of erosion appear.

There would be a time delay while enough shared energy accumulated for electrons to escape, and this delay would be shorter for higher intensity light. There would be no lower limit on the frequency of light which could eject electrons. The waiting time for electrons to emerge would be longer using lower frequency light, since its wavefronts arrive less frequently; however, eventually a current would be detected.

**Great photoelectric effect results**

Einstein’s insights into using a particle model to explain the photoelectric effect led to his 1905 prediction. He predicted that a graph of stopping voltage versus frequency would be a straight line whose gradient was independent of the material emitting electrons.

\[
V_0 = \frac{1}{q_e} (hf - W)
\]

A ‘machine shop in a glass tube’ was needed to show that this prediction was correct. Robert Millikan, the same Millikan who had earlier measured the minimum value of electric charge, was the engineer of this machine shop, which is shown below. Strong monochromatic UV sources did not exist, so Millikan used the visible and near-UV lines of a mercury arc lamp. Since the visible and near-UV photons of the lamp have lower energy than UV photons, his studies were limited to materials with low work functions. He used the alkali metals: lithium, sodium and potassium.

![Millikan’s ‘machine shop in a glass tube’, and (b) his first published results](image)

Unfortunately, while a low work function makes their electrons accessible to visible light, it also made these materials vulnerable to reaction with the oxygen in air. The metals quickly become coated with a thin insulating layer of metal oxide. To overcome this problem, Millikan conducted his experiments in an evacuated glass container. Inside the container he placed an ingenious mechanism for rotating his electrodes past a sharp knife that scraped a clean metal surface for each experiment.

Part (a) of the above figure shows his experimental arrangement and part (b), his first published results. The gradient of the straight line is \( \frac{h}{q_e} \), where \( h \) is Planck’s constant and \( q_e \) is the magnitude of the electronic charge. Millikan determined \( \frac{h}{q_e} \) to be \( 4.1 \times 10^{-15} \text{ J s C}^{-1} \).
The graphs for different materials all have the same slope, $\frac{h}{q}\epsilon$, but are displaced to the right or left, depending on the work function. The cut-off frequency, $f_0$, is where the line meets the frequency axis. Its value is equal to $\frac{W}{h}$.

Einstein said:

*It seems to me that the observations associated with . . . the photoelectric effect, and other related phenomena . . . are more readily understood if one assumes that the energy of light is discontinuously distributed through space . . . the energy of a light ray spreading out from a point is not continuously spread out over an increasing space, but consists of a finite number of energy quanta which are localised at points in space, which move without dividing, and which can only be produced and absorbed as complete units.*

The word *quanta* is plural for *quantum*, a word meaning a small quantity of a fixed amount. These energy quanta of light are what we now call photons.

This need for a photon model to explain the workings of the photoelectric effect fitted very neatly with Planck’s black body radiation model, in which a particle model for light was required to make the theory fit with the experimental evidence of light radiated from hot objects. However, both these phenomena contradicted the enormously successful wave model for light summarised by Maxwell’s four equations for electromagnetic phenomena. The wave model for light in terms of perpendicular electric and magnetic fields is consistent with observed interference patterns and diffraction patterns, and with the propagation of light at a single speed universal speed, c. A wave model for light is also consistent with a large range of electrical and magnetic phenomena, for example electromagnetic induction.

Another chapter in physics was about to begin. The development of quantum mechanics would completely change the way in which scientists viewed the universe. The Newtonian mechanistic world was about to be overthrown. Confusion between particle and wave models for both light and matter would be resolved, but this would take another thirty years to achieve.
Solar cells

Telephone installations in remote locations extract energy from the Sun using technology based on the transfer of photon energy to electrons. They use solar cells to convert solar energy to electric energy. This is achieved by photons transferring their energy to electrons so that they are able to conduct electricity.

Solar cells are made using semiconductors like silicon. In semiconductors only about 1 in $10^6$ of the electrons have sufficient energy to be conduction electrons. In metals like silver this figure is about 1 in 30.

Conduction electrons are not bonded to any particular atom in the crystal. They can travel through the material when a potential difference is applied across it, producing an electric current.

The vital part of a solar cell is a sandwich of two different types of impure semiconductor material, called n-type and p-type. The sandwich slivers are only tens of microns thick. Electrons drift from the n-type material, containing electrons that are not attached to any particular atom, to the p-type material, where there are spaces for electrons in the bonding structure. This creates an electric field in the layer of material very close to the boundary between the two types, with the electrons in stable positions in the bonding structure of the semiconductor material.

When the electric circuit containing this cell is in the dark, the electric field has no effect; but in the sunshine photons stream into the cell.

If a photon has sufficient energy, then it can knock an electron out of its niche in the material, enabling it to become a conduction electron and leaving a hole behind in the bonding structure. If this occurs within the region where there is an electric field, the electric force sweeps the electron through the cell, and through the circuit, contributing to the electric current.

The efficiency of a solar cell is limited by many factors. If its surface is too shiny, photons are reflected, so the surfaces are usually roughened. The sun’s spectrum itself limits how well the cell can make use of the photons. In silicon, a transfer of 1.1 eV is needed to transform a bound electron into a conduction electron. This corresponds to a wavelength of $1.1 \times 10^{-6}$ m, just into the infra-red part of the spectrum. Photons having energy less than 1.1 eV pass straight through a simple silicon cell because their energy is too small to convert bound electrons into conduction electrons.
A photon model for the photoelectric effect

Almost thirty years after the first observation of the photoelectric effect, experimental measurements confirmed the need for a photon model for light. The wave model for light was incapable of explaining the observations of the photoelectric effect.

**TABLE 11.1** Timeline of key discoveries about the photoelectric effect

<table>
<thead>
<tr>
<th>Date</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1887</td>
<td>It all started with Hertz carefully noting the unusual behaviour of sparks across the gaps in his radio wave detector circuit. This was the first observation of the photoelectric effect.</td>
</tr>
<tr>
<td>1901</td>
<td>Max Planck solved the black-body radiation problem theoretically, paving the way for light to be modelled not only as a wave but also as a localised particle with energy proportional to the frequency of the light, $E = hf$.</td>
</tr>
<tr>
<td>1902</td>
<td>Philipp Lenard carried out experiments to accumulate knowledge about the behaviour of electrons emitted by light. There were several puzzling aspects to his results — electron energies did not depend on the light intensity and there was a unique cut-off frequency for each material.</td>
</tr>
<tr>
<td>1905</td>
<td>The flash of insight was Albert Einstein’s, when he realised that all of Lenard’s observations could be explained if he changed the way he thought about light — if light energy travelled as particles not waves. He used the particle model to predict that the graph of stopping voltage versus frequency would be straight, with a slope that was the same for all electron emitters.</td>
</tr>
<tr>
<td>1915</td>
<td>Robert Millikan sealed the success of Einstein’s theory with plots of $V_0$ versus $f$ for the alkali metals that were straight and parallel to one another. He used the plots to measure Planck’s constant. The photon energy was $hf$.</td>
</tr>
</tbody>
</table>

**TABLE 11.2** Observations made from the photoelectric effect and model predictions

<table>
<thead>
<tr>
<th>Observation</th>
<th>Wave model prediction</th>
<th>Photon model prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>For a given frequency of light, the photocurrent is dependent in a linear fashion on the brightness or intensity of light.</td>
<td>The wave model makes no significant predication other than that brighter light should produce electrons with greater energy, which is not the case.</td>
<td>Intensity of light relates to the number of photons per second striking the photocell. We would expect the photocurrent to be dependent on the intensity of light.</td>
</tr>
<tr>
<td>The energy of photoelectrons is independent of intensity of light and only linearly dependent on frequency.</td>
<td>The energy of electrons is dependent on the intensity of light: the bigger the amplitude of the wave, the larger the energy transferred to electrons.</td>
<td>The energy of photoelectrons is linearly dependent on the frequency of light, provided we interpret the energy of a single photon of light as equal to $hf$.</td>
</tr>
<tr>
<td>There is no significant time delay between incident light striking a photocell and subsequent emission of electrons, and this observation is independent of intensity.</td>
<td>Time delay to be shorter with increasing intensity</td>
<td>No time delay expected as individual photons of light strike photocell and transfer energy to individual electrons</td>
</tr>
<tr>
<td>There exists a threshold frequency below which the photoelectric effect does not occur, and this threshold is independent of intensity.</td>
<td>No threshold effect should exist, as energy transfer to electrons from light source is accumulative and eventually emission will occur.</td>
<td>A threshold frequency is predicted, as photons with energy less than the work function are incapable of freeing electrons from the photocell.</td>
</tr>
</tbody>
</table>
Summary

- The equation \( c = f \lambda \) describes the speed of a wave in terms of its frequency, \( f \), and wavelength, \( \lambda \).
- The photoelectric effect is the emission of electrons from materials, usually metals, by the action of light.
- The photoelectric effect is best explained by considering light as consisting of a stream of particles called photons. Each photon has an energy, \( E \), that is dependent on only the frequency of the light, \( f \), according to the equation \( E = hf \). This is the Einstein interpretation of the photoelectric effect.
- The electron volt is a unit of energy.
  \[ 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} \]
- When a photon hits an electron in a metal, it will transfer either all or none of its energy to an electron. This occurs within a time interval of typically \( 10^{-9} \text{ s} \) of a beam of light striking a surface.
- Below a threshold frequency \( f_0 \), the emission of electrons does not occur regardless of the intensity of light.
- The maximum kinetic energy of emitted electrons, \( E_{k_{\text{max}}} \), is given by the equation \( E_{k_{\text{max}}} = hf - W \), where \( f \) is the frequency of the light and \( W \) is the work function of the material.
- The maximum kinetic energy of the electrons emitted because of the photoelectric effect can be determined by measuring the stopping voltage, \( V_0 \).
  \[ E_{k_{\text{max}}} = qV_0 \]
- The intensity of light has no effect on the stopping voltage but only effects in direct proportion the size of the photocurrent. A wave model for light cannot account for this, but a particle model of light can.
- A graph of the maximum kinetic energy of emitted electrons plotted against frequency gives a straight line. The gradient of the graph is Planck’s constant, \( h \), the y-intercept is the work function, \( W \), and the x-intercept is the threshold frequency, \( f_0 \).
- The photoelectric effect is strong evidence for light consisting of a stream of particles.

Questions

Electromagnetic radiation

1. The light from a red light-emitting diode (LED) has a frequency of \( 4.59 \times 10^{14} \text{ Hz} \).
   (a) What is the wavelength of this light?
   (b) What is the period of this light?

2. We can detect light when our eye receives as little as \( 2 \times 10^{-17} \text{ J} \). How many photons of green light is this?

3. Fill in the gaps in table 11.3 with the missing wavelength, frequency, photon energy and photon momentum values for the five different sources of electromagnetic radiation.

4. A red laser emitting 600 nm light and a blue laser emitting 450 nm light emit the same power. Compare their rate of emitting photons.

The photoelectric effect

5. The diagram below shows a cathode, several electrons that have been ejected from the cathode by light, and an anode. The electrons leaving the cathode surface have been labelled with their kinetic energy and their initial velocity vector. The anode is 5 mm from the cathode.

![Diagram]

(a) What is the speed of the electrons which have a kinetic energy of 0.8 eV?

Copy the diagram and sketch the path you would expect each electron to take for each of the potential differences, \( V \), in parts (b) to (d) on the next page.

<table>
<thead>
<tr>
<th>Source</th>
<th>Wavelength</th>
<th>Frequency</th>
<th>Energy</th>
<th>Momentum</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Infra-red from CO(_2) laser</td>
<td>10.6 ( \mu \text{m} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b) Red helium–neon laser</td>
<td></td>
<td></td>
<td>1.96 eV</td>
<td></td>
</tr>
<tr>
<td>(c) Yellow sodium lamp</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(d) UV from eximer laser</td>
<td></td>
<td>1.55 ( \times 10^{15} \text{ Hz} )</td>
<td></td>
<td>1.125 ( \times 10^{-27} \text{ kg m s}^{-1} )</td>
</tr>
<tr>
<td>(e) X-rays from aluminium</td>
<td></td>
<td></td>
<td>2.01 ( \times 10^{-16} \text{ J} )</td>
<td></td>
</tr>
</tbody>
</table>
6. What is the stopping voltage when UV radiation having a wavelength of 200 nm is shone onto a clean gold surface? The work function of gold is 5.1 eV.

7. In the following diagram, the curve shows how the current measured in a photoelectric effect experiment depends on the potential difference between the anode and cathode.

(a) Explain the curve. Why does it reach a constant maximum value at a certain positive voltage, and why does it drop to zero at a certain negative voltage?
(b) If the intensity of the light was increased without changing its frequency, sketch the curve that would be obtained. Explain your reasoning.
(c) If the frequency of the light was increased without changing its intensity, sketch the curve that would be obtained. Explain your reasoning.
(d) If the material of the cathode was changed, but the light was not changed in any way, sketch the curve that would be obtained. Explain your reasoning.

8. The curve below shows the current in a photoelectric cell versus the potential difference between the anode and the cathode when blue light is shone onto the anode.

(a) State the current when the voltage is 0 V.
(b) State the current when the voltage is +1.0 V.
(c) State the current when the voltage is increased to +2.0 V.
(d) Why does increasing the voltage have no effect on the current in the circuit?
(e) The polarity is now reversed and the voltage increased until the current drops to 0 A. State the stopping voltage and hence the maximum energy of electrons emitted from the anode.
(f) The light source is now made brighter without changing the frequency. Copy the figure and sketch a second curve that illustrates the effect of increasing the intensity of the blue light.
(g) The light source is now returned to its original brightness and green light is used. A current is still detected. Sketch a third curve to illustrate the effect of using light of a lower frequency.
(h) The apparatus is altered so that the anode consists of a metal with a smaller work function. Again blue light is used. Sketch a fourth curve to illustrate the effect of changing the anode without changing either the brightness or colour of the light.

9. The work function for a particular metal is 3.8 eV. When monochromatic light is shone onto the photocell, electrons with energy 0.67 eV are emitted.

(a) What is the stopping voltage required to stop these electrons?
(b) What is the frequency of the monochromatic light used?
(c) What is the threshold frequency of the metallic surface?

10. In a photoelectric effect experiment, the threshold frequency is measured to be $6.2 \times 10^{14}$ Hz.

(a) Calculate the work function of the metal surface used.
(b) If electrons of maximum kinetic energy $3.4 \times 10^{-19}$ J are detected when light of a particular frequency is shone onto the apparatus, what is the stopping voltage?
(c) With the same source of light, what is the wavelength and hence the momentum of the photons?
11. When light of frequency $5.3 \times 10^{14}$ Hz is shone onto a metal surface, electrons with a maximum kinetic energy of 1.7 eV are emitted. A second photocell is now positioned and this time, using the same light, electrons of energy 1.3 eV are emitted. Calculate the difference between the work functions of the two photocells. Which cell has the greater work function: the first or the second?

12. One electron ejected from a clean zinc plate by ultraviolet light has a kinetic energy of $4.0 \times 10^{-19}$ J.
(a) What would be the kinetic energy of this electron when it reached the anode, if a retarding voltage of 1.0 V was applied between the anode and cathode?
(b) What is the minimum retarding voltage that would prevent this electron reaching the anode?
(c) All electrons ejected from the zinc plate are prevented from reaching the anode by a retarding voltage of 4.3 V. What is the maximum kinetic energy of electrons ejected from the zinc?
(d) Sketch a graph of photocurrent versus voltage for this metal surface. Use an arbitrary photocurrent scale.

13. The diagram below shows the energies of electrons in a block of copper. Zero energy is defined to be that for a stationary, free electron.

![Diagram of energy levels in copper]

(a) What is the work function of copper?
(b) A stream of light whose photons have an energy of 5.9 eV shines on the copper surface. Describe the possible outcome for electrons in the copper having energies of:
(i) −4.7 eV
(ii) −5.3 eV
(iii) −5.9 eV
(iv) −6.3 eV.

14. Robert Millikan performed his photoelectric experiment using a clean potassium surface, with a work function of 2.30 eV. He used a mercury discharge lamp. One wavelength of radiation emitted by the lamp was 254 nm, in the ultraviolet.
(a) What is the maximum kinetic energy of electrons ejected from the potassium surface by this UV radiation?
(b) What voltage would be required to reduce the photocurrent in the cell to zero?
(c) Sketch a graph of maximum electron kinetic energy versus frequency for potassium. Show the point on the graph obtained from the 254 nm UV radiation.
(d) Repeat this sketch for sodium, which has a work function of 2.75 eV.

15. When the surface of a material in a photoelectric effect experiment is illuminated with light from a mercury discharge lamp, the stopping voltages given in the table are measured.

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>Stopping voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>366</td>
<td>1.48</td>
</tr>
<tr>
<td>405</td>
<td>1.15</td>
</tr>
<tr>
<td>436</td>
<td>0.93</td>
</tr>
<tr>
<td>492</td>
<td>0.62</td>
</tr>
<tr>
<td>546</td>
<td>0.36</td>
</tr>
<tr>
<td>579</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Plot the stopping voltage versus the frequency of the light and use the graph to determine:
(a) the threshold frequency
(b) the threshold wavelength
(c) the work function of the material, in eV
(d) the value of Planck’s constant.

16. Give four reasons why a particle model for light better explains the observations made for the photoelectric effect. In particular, explain why a wave model is inadequate for each reason.