Stop Loss Reinsurance

Stop loss is a **nonproportional** type of **reinsurance** and works similarly to **excess-of-loss** reinsurance. While excess-of-loss is related to single loss amounts, either per risk or per event, stop-loss covers are related to the total amount of claims X in a year – net of underlying excess-of-loss contracts and/or **proportional reinsurance**. The reinsurer pays the part of X that exceeds a certain amount, say R. The reinsurer's liability is often limited to an amount L so that the payment is no more than if the total claim exceeds $L + R$. Furthermore, it is common for the reinsurer's liability to be limited to a certain share $(1 - c)$ of the excess $X - R$, the remaining share c being met by the ceding company (*see* **Reinsurance**). The reinsurer's share Z of the claims may by summarized as follows

$$Z = \begin{cases} 0 & X \le R \\ (1-c)(X-R) & R < X < R+L \\ (1-c)L & X \ge R+L, \end{cases} \quad (1)$$

where X is the total claim amount in the contract period, typically a year

$$X = \sum_{i=1}^{N} Y_i \quad (2)$$

where the stochastic variables, $Y_i, i = 1, \ldots, N$, are the individual loss amounts, and the stochastic variable N is the number of losses.

Like for excess-of-loss covers, the stop-loss cover is denoted by L *xs* R. Both retention (*see* **Retention and Reinsurance Programmes**) and limit can be expressed as amounts, as percentages of the premiums (net of underlying excess-of-loss contracts and/or proportional reinsurance), or as percentages of the total sums insured, but mostly as percentages of the premiums.

While normal excess-of-loss reinsurance provides a ceding company with protection mainly against severity of loss and with no protection against an increase in the frequency of losses below the priority, stop-loss reinsurance offers protection against an increase in either or both elements of a company's loss experience.

One of the purposes of reinsurance is to reduce the variance of the underwriting results. Stop-loss reinsurance is the optimal solution among all types of reinsurance in the sense that it gives, for a fixed reinsurance risk premium, the smallest variance for the company's net retention [3, 10, 26, 29, 30] (*see* **Retention and Reinsurance Programmes**). It is cheaper in respect of risk premium given a fixed level of variance, and it gives a far greater reduction of variance than other forms of reinsurance given a fixed level of risk premium. With a sufficiently large limit, the stop-loss contract gives an almost total guarantee against ruin – if the ceding company is able to pay all losses up to the stop-loss priority.

Stop loss appears to be the ideal form of reinsurance. However, stop-loss reinsurance *maximizes* the dispersion for the reinsurer. Therefore, the reinsurer will prefer a proportional contract if he wants to reduce his costs, [6, 26], and will ask for a high loading of the stop-loss contract to compensate for the high variance accepted. The reinsurer will usually require the following:

1. The underwriting results should fluctuate considerably from year to year. Fluctuation should come from the claims side (not from the premium side) and be of a random nature.
2. The business should be short tailed.
3. The reinsurer will design the cover in such a way that the ceding company does not have a guaranteed profit, which means that the retention should be fixed at a level clearly above premiums minus administrative expenses plus financial revenue. In other words, the ceding company shall make a loss *before* the stop-loss cover comes into operation. A further method that stimulates the ceding company to maintain the quality of its business is that the stop-loss reinsurer does not pay 100% but 90 or 80% of the stop-loss claims, that is, requiring $c > 0$ in (1). The remaining 10 or 20% is a 'control quote' to be covered by the ceding company.
4. An upper limit is placed on the cover provided. For various reasons (such as **inflation**, growth of portfolio) the premium income can grow rapidly and the reinsurer may like to restrict the range of cover not only to a percentage of the premiums,

but also to a fixed amount per year to discourage the ceding company from writing more business than expected.

Stop-loss covers are mostly seen either for some property classes (*see* **Property Insurance – Commercial Only**, **Property Insurance – Personal**) or for life portfolios (*see* **Life Insurance**, **Life Reinsurance**). Stop-loss covers are particularly suitable for risks such as **hail**, and **crop**, which suffers from an occasional bad season and for which a single loss event would be difficult to determine, since changes in claims ratios are mostly due to strong fluctuations in the frequency of small- and medium-sized claims. Furthermore, these classes are characterized by a correlation between the risks. An assumption that the individual claims Y_i's are independent will therefore not necessarily be satisfied.

As mentioned above, reinsurers will not accept stop-loss contracts that will guarantee profits to the ceding companies by applying a low retention, such that the stop-loss contract covers losses even before the ceding company suffers from a loss. It should at least be required that the retention exceeds the expected loss ratio (or the risk premium of the ceding company), or $R \geq \mathrm{E}$ where E denotes the expected loss cost for the portfolio. Another possibility is to require $c > 0$, that is, forcing the ceding company to pay part of the losses that exceed the retention. However, this is not a very interesting possibility from a theoretical point of view and in the remaining part of this article it is assumed that $c = 0$.

Benktander [4] has studied a special (unlimited) stop-loss cover having the retention $R = \mathrm{E}$, and has found a simple approximation formula for the risk premium for such a stop-loss cover too

$$\pi(\mathrm{E}) \cong \mathrm{E}P_\lambda([\lambda]) \qquad (3)$$

with $\lambda = \mathrm{E}^2/V$ where P_λ is the Poisson distribution $P_\lambda(k) = \lambda^k e^{-\lambda}/\lambda!$ (*see* **Discrete Parametric Distributions**), $[\lambda]$ is the integer part of λ, E is the expected value of the total claims distribution, and $V = \sigma^2$ is the variance. It is shown that $\sigma/\sqrt{2\pi}$ is a good approximation for the risk premium of this special loss cover.

Stop-loss covers can be used internally in a company, as a tool for spreading exceptional losses among subportfolios such as tariff groups [2]. The idea is to construct an internal stop-loss cover excess of, for example, $k\%$ of the premiums. On the other hand, the tariff group has to pay a certain 'reinsurance premium'. The company will earmark for this purpose, a certain fund, which is debited with all excess claims falling due on the occurrence of a claim, and credited with all stop-loss premiums from the different tariff groups. The same idea can be used for establishing a catastrophe fund to spread extreme losses over a number of accounting years.

Since stop-loss reinsurance is optimal, in the sense that if the level V of the variance of the retained business is fixed, the stop-loss reinsurance leaves the maximum retained risk premium income to the ceding company; therefore, stop-loss treaties are the most effective tools to decrease the necessary **solvency margin**.

Variations of Stop-Loss Covers

In the reinsurance market, some variations from the typical stop-loss contract as described by (1) and (2) are seen from time to time. However, they are not frequently used. One of these variations is the form of aggregate excess-of-loss contracts covering the total loss cost for, for example, losses larger than a certain amount D, that is, having (2) replaced by

$$X = \sum_{i=1}^{N} I\{Y_i > D\}Y_i \qquad (4)$$

with $I\{Y_i > D\} = \begin{cases} 1 & Y_i > D \\ 0 & Y_i \leq D. \end{cases}$

Another variation covers the total loss cost for, for example, the k largest claims in a year, that is, having (2) replaced by

$$X = \sum_{i=1}^{k} Y_{[i]} \qquad (5)$$

where $Y_{[1]} \geq Y_{[2]} \geq \cdots \geq Y_{[N]}$ are the ordered claim amounts (*see* **Largest Claims and ECOMOR Reinsurance**).

More different from the standard stop-loss covers are the frequency excess-of-loss covers. Such covers will typically apply if the loss frequency rate f (number of claims in percent of the number of policies) exceeds a certain frequency a up to a further b

$$K = \begin{cases} 0 & f \leq a \\ (f - a) & a < f < b \\ (b - a) & f \geq b, \end{cases} \qquad (6)$$

The claim to the frequency excess-of-loss contract is then defined as K multiplied with the average claim amount for the whole portfolio. Such covers protect the ceding company only against a large frequency in losses – not against severity, and are typically seen for motor hull business (*see* **Automobile Insurance, Private**, **Automobile Insurance, Commercial**).

None of these variations seem to be described in the literature. However, they can be priced using the same techniques as the standard stop-loss cover (see below) by modifying the models slightly.

Stop-Loss Premiums and Conditions

As in the case of excess-of-loss contracts, the premium paid for stop-loss covers consists of the risk premium plus loadings for security, administration and profits, and so on (*see* **Reinsurance Pricing**). The premium is usually expressed as a percentage of the gross premiums (or a percentage of the sums insured if the retention and limit of the cover are expressed as percentages of the sums insured). **Burning cost** rates subject to minimum–maximum rates are known but not frequently seen. In the section 'Pricing stop loss' below, we shall therefore concentrate on estimating the risk premium for a stop-loss cover.

Stop-loss covers can also be subject to additional premiums paid in case the observed loss ratio exceeds a certain level and/or no claims bonus conditions (*see* **Reinsurance Pricing**).

Pricing Stop Loss

Let us denote the aggregate annual claims amount for a certain portfolio X and its distribution function $F(x)$ and let $\pi(R, L)$ be the risk premium of the layer L excess R, that is,

$$\pi(R, L) = \int_R^{R+L} (x - R)\,\mathrm{d}F(x) + L\int_{R+L}^{\infty} \mathrm{d}F(x). \tag{7}$$

By integrating, the above expression can be written as

$$LF(R + L) - \int_R^{R+L} F(x)\,\mathrm{d}x + L - LF(R + L),$$

so that the net premium becomes

$$\pi(R, L) = \int_R^{R+L} (1 - F(x))\,\mathrm{d}x = \pi(R) - \pi(L) \tag{8}$$

where $\pi(R)$ is the risk premium for the unlimited stop-loss cover in excess of R (*see* **Stop Loss Premium**).

The estimation of stop-loss premiums can therefore be based on some knowledge about the distribution function $F(x)$ of the sum of all claims in a year (assuming that the stop-loss cover relates to a period of one year). Generally speaking, there are two classes of methods to estimate $F(x)$:

1. Derivation of the distribution function using the underlying individual risk, using data concerning number of claims per year, and severity of individual claims amounts. This is by far the most interesting from a theoretical point of view, and is extensively covered in the literature.
2. Derivation of the distribution function from the total claims data. This is – at least for nonlife classes – the most interesting method from a practical point of view. Only a limited number of papers cover this topic.

Derivation of the Distribution Function Using the Individual Data

This class of methods can be divided into two subclasses:

1. In the collective risk model, the aggregate claims $X = Y_1 + \cdots + Y_N$ is often assumed to have a **compound distribution**, where the individual claim amounts Y_i are independent, identically distributed following a distribution $H(y)$, and independent of the number of claims N, [7, 17, 20, 24]. Often, N is assumed to be Poisson distributed, say with Poisson parameter λ (the expected number of claims). Thus

$$F(x) = \mathrm{e}^{-\lambda} \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} H^{*k}(x) \tag{9}$$

where $H^{*k}(x)$ is the kth ***convolution*** of $H(x)$. There is a large number of papers describing how to approximate the compound distribution function $F(x)$, and to compute the stop-loss premium. The aggregate claims distribution function can in some cases be calculated recursively, using, for example, the Panjer recursion formula [7, 19, 27] (*see* **Sundt and Jewell Class of Distributions**). But for large portfolios, recursive methods may be too time consuming. By introducing the concept of stop-loss ordering [8] (*see* **Ordering of Risks**), it is possible to derive lower and upper bound for stop-loss premiums. Stop-loss upper and lower bounds have been studied by a large number of authors, see, for example, [8, 15, 16, 18, 21, 24, 31, 32, 34].

2. The **individual risk model**. Consider a portfolio of n independent policies, labeled from 1 to n. Let $0 < p_i < 1$ be the probability that policy i produces no claims in the reference period and $q_i = 1 - p_i$ be the probability that this policy leads to at least one positive claim. Further, denote by

$$G_i(u) = \sum_{x=1}^{\infty} g_i(x)u^x, \quad i = 1, 2, \ldots, n, \quad (10)$$

the probability generating function of the total claim amount of policy i, given that this policy has at least one claim. The probability generating function of the aggregate claim X of the portfolio is then given by

$$P(x) = \prod_{i=1}^{n} [p_i + q_i G_i(x)]. \quad (11)$$

Many authors have studied these approximations, for example, [12–14, 17, 23, 25, 33] (*see* **De Pril Recursions and Approximations**).

Derivation of the Distribution Function from the Total Claims Data

When rating stop loss for nonlife classes of business, the information available in most practical situations is the claims ratios – claims divided by premiums (or claims divided by sums insured – similar to how the cover is defined). The second method will have to be used then, but is not necessarily straightforward. Do we have of enough information of the past to have a reasonable chance to calculate the risk of the future? Here also comes in the question of the number of years necessary or desirable as a basis for the rating. In excess-of-loss reinsurance, rating is often based on five years of statistical information. In stop loss, we would need a much longer period, ideally 20 to 25 years. If data are available for a large number of years, these data often turn out to be heterogeneous or to be correlated with time because conditions, premium rates, portfolio composition, size of portfolio, new crops, and so on, have changed during the period in question. If, in that case, only the data of the most recent years are used, the number of these data is often too small a base for construction of a distribution function.

Instead of cutting off the earlier years, we could take a critical look at the data. First, we will have to recalculate the claims ratios according to today's level of premium rates. On the other hand, it is not necessary to take inflation into account unless there is a different inflation working on the premiums and on the claims amounts. Is there still any visible trend in the claims ratio? If the claims ratio seems to vary around a constant or around a slightly decreasing level we can ignore the trend. But if the trend points upwards we will have to adjust expected claims ratio and retention accordingly. Do we fear that the change in portfolio composition has increased the variability in the underwriting results, or do we feel that a measure of variation calculated from past statistics is representative for the future?

Distributions in nonlife insurance are not symmetrical but show a positive skewness. This means that the variations around the average usually give stronger positive deviations (larger claims, negative underwriting results) than negative ones. This is very obvious not only for the claims-size distributions of importance in excess-of-loss reinsurance but also, however less markedly, in the distribution of the total claims amount protected by stop-loss reinsurance.

A number of distributions are suggested in the literature:

1. The exponential distribution [4] (*see* **Continuous Parametric Distributions**) giving the following risk premium for the unlimited stop-loss cover excess R

$$\pi(R) = E\mathrm{e}^{-R/E} \quad (12)$$

2. The lognormal distribution [4] (*see* **Continuous Parametric Distributions**) with mean μ and

variance σ^2, giving the risk premium

$$\pi(R) = E\left\{1 - \Phi\left(\frac{\ln R - \mu - \sigma^2}{\sigma}\right)\right\} - R\left\{1 - \Phi\left(\frac{\ln R - \mu}{\sigma}\right)\right\} \quad (13)$$

where Φ denotes the cumulative standard normal cumulative distribution function (*see* **Continuous Parametric Distributions**).

3. Bowers [5] has shown that the stop-loss risk premium for an unlimited layer excess R

$$\pi(R) \leq 0.5\sigma\left(\sqrt{1 + \left(\frac{R - E}{\sigma}\right)^2} - \frac{R - E}{\sigma}\right)$$

when $R \geq E$ (14)

4. A simple approximation is the normal approximation, where $F(x)$ is approximated by the normal distribution $\Phi((x - E)/\sigma)$ [9]. Since the normal distribution is symmetrical, a better approximation is the Normal Power approximation [3, 9, 10] (*see* **Approximating the Aggregate Claims Distribution**), where the stop-loss premium can be expressed in terms of the normal distribution function Φ. When $R > E + \sigma$, the risk premium $\pi(R)$ for the unlimited stop-loss cover in excess of R can be approximated by the following expression

$$\pi(R) = \sigma\left(1 + \frac{\gamma}{6}y_R\right)\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-(y_R^2/2)} - (R - E)(1 - \Phi(y_R)) \quad (15)$$

where E is the total risk premium (the mean value of the aggregate claim amount), σ and γ are the standard deviation and skewness of the aggregate claim amount. y_R is related to R via the Normal Power transformation, that is,

$$y_R = v_\gamma^{-1}\left(\frac{R - E}{\sigma}\right) \quad (16)$$

where

$$v_\gamma^{-1}(x) = \sqrt{1 + \frac{9}{\gamma^2} + \frac{6x}{\gamma}} - \frac{3}{\gamma}$$

$$F(X) \approx \Phi[v_\gamma^{-1}(x)]$$

Other distribution functions suggested in the literature are the Gamma distribution and the inverse Gaussian distribution, [4, 9, 28]. Other improvements of the normal distributions are the Edgeworth or the Esscher approximations [9] (*see* **Approximating the Aggregate Claims Distribution**).

New Portfolios

There is a problem when rating new portfolios where the claims ratios are only available for a few years (if any at all). In such cases, one must combine with market data. For instance, hail market data are available for a large number of countries, and the hail portfolios in the same market can always be assumed to be correlated to a certain extent, such that years that are bad for one company in the market will be bad for all companies in the market. However, one should always have in mind when using market data that the individual variance in the claims ratio for one company will always be larger than the variance in the claims ratio for the whole market, and the premium for a stop-loss cover should reflect the variance in the claims ratios for the protected portfolio.

Which Type of Model Is To Be used?

The individual risk model is based on the assumption that the individual risks are mutually independent. This is usually not the case for classes such as hail and crop portfolios. As shown in [1, 11, 18, 22] the stop-loss premium may increase considerably if the usual assumption of independence of the risks in the reinsured portfolio is dropped. In such cases, modeling the total claims amount or claims ratios should be preferred.

The independence assumption is more likely to be satisfied when looking at the largest claims only in a portfolio. Therefore, the individual models are much more suitable for pricing aggregate excess-of-loss covers for the k largest losses or aggregate excess-of-loss covering claims exceeding a certain amount.

The individual risk models are often used in stop loss covering life insurance, since the assumption of independency is more likely to be met and the modeling of the individual risks are easier: there are no partial losses and ‘known’ probability of death for each risk.

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(*See also* **Reinsurance Pricing**; **Retention and Reinsurance Programmes**; **Life Reinsurance**; **Stop Loss Premium**)

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