Measurement is an operation of data acquisition and presentation, aimed at expressing in symbolic form the information empirically obtained on a system about a quantity, the measurand (we accept the common ambiguities of calling 'measurand' both the system under measurement and the measured quantity, and the latter in both its general and specific forms, for example, length and length of a given object in a given time).

Peculiar to measurement is the requirement of being objective and intersubjective, where objectivity implies that measurement results convey information only related to the system under measurement and not its environment, and intersubjectivity requires that measurement results convey the same information to different subjects. As such, these properties appear an ideal target, justifying the efforts to constantly enhance measurement devices and procedures.

To achieve an acceptable degree of objectivity and intersubjectivity, measuring systems are adopted, which include selective and repeatable sensors and traceable standards. Indeed:

- Although human beings are able to directly sense a fair amount of quantities and are well trained to express in linguistic form their perception (e.g., 'it is rather cold', 'this is heavier than that'), their statements are affected by subjectivity, that is, they report information on both the sensed system and the perceiver state; to avoid the influence of the latter, and thus to enhance the objectivity of the operation, the measurand is transduced by a sensing system whose output ideally depends only on the measurand and is unaffected by influence quantities and internal imperfections.

- While related to the measurand, the quantity provided by sensors still depends on their specific behavior; as a consequence, distinct sensors, even if perfectly repeatable, produce different outputs from the same input; furthermore, in many cases the sensor output quantity, appropriate for signal conditioning and for driving presentation devices, is not dimensionally homogeneous to the measurand. The sensor output must then be dealt with as an instrument reading, not as a measurand value. To make the information obtained by the measurement intersubjective, a common reference must be adopted so that measurand values are expressed in comparison to such a standard. Therefore, the possibility of tracing the readings to the agreed standard is critical, a condition operatively ensured by instrument calibration.

The requirement of empirical comparison to traceable standards is so fundamental that it can be assumed as distinctive of measurement; generic scale-preserving evaluations can be formalized as homomorphisms from empirical to symbolic relational systems, as shown in Figure 1 (see also Article 8, Formal Theory of Measurement, Volume 1). In the case of measurement, such mappings are

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2 Modelling Measuring Systems

Empirical RS → Homomorphic Evaluation → Symbolic RS

Figure 1. A generic scale-preserving evaluation.

RS of measurand states → RS of symbols used as measurand values

Empirical comparison → RS of standard states

Standard definition

Figure 2. Measurement as a scale-preserving evaluation obtained by the comparison to a standard.

RS of measurand states → RS of symbols used as measurand values

3. Empirical comparison → RS of derived standard states

2. Traceability chain → RS of primary standard states

Figure 3. Measurement as a scale-preserving evaluation obtained by the comparison to a standard derived by a primary standard.

not direct but mediated by the standard to a standard derived by a primary standard.

measurement involves (at least) two interactions: standard-instrument and measurand-instrument.

In its interaction with the measurand, the instrument generates an output; a general problem of measurement can then be stated as follows: from the output of the measuring instrument (‘the reading’) its input (the state of the system under measurement and its environment) must be reconstructed, and from this state a measurand value must be inferred.

To cope with this input–output inference problem, two basic strategies can, in principle, be followed:

• The analytical model of the measuring system behavior is identified and the obtained characteristic function is inverted, so that from the output readings the input signals are computed. Because of its complexity, this approach is seldom adopted.

• The system is regarded as a black box and only its input–output behavior is taken into account: the instrument operates by interacting with a set of (known) standard states and the corresponding output readings are recorded; by a suitable interpolation, this collection of couples becomes the so-called calibration curve, shown in Figure 5, which can be thought of as a mapping from measurand values to instrument readings. This function is then inverted so that each instrument reading can be associated with a measurand value.

2 THE OUTPUT/INPUT BLACK BOX MODEL

It is a well-known fact that different methods of measurement exist, each of them corresponding to a specific technique to perform the comparison between the measurand and the standard (see Figure 4). While some methods require the synchronous presence of the measurand and the standard (e.g. following the paradigm of the two-arm balance provided with a set of standard weights: a direct comparison), many others are based on the usage of devices acting as serializers of the comparison, so that a

Figure 4. The different usages of the measuring systems as comparators.

Figure 5. A diagram with the example of a curve generated by calibration.
The interactions standard-instrument and measurand-instrument have therefore a complementary function: while the former is aimed at creating a calibration diagram, the latter uses the inverted diagram, shown in Figure 6, to find the measurand value that corresponds to the obtained reading.

To enhance the user-friendliness of the measuring systems, it is customary to set up their presentation component so that the data they display are expressed directly in measurement units, that is, the calibration diagram is embedded into the systems. While measurement always requires calibration information, in these cases one can specifically speak of calibrated instruments.

3 SET-THEORETICAL MODEL

The sensor behavior, therefore critical for both calibration and measurement, is usually expressed as a characteristic function formalizing the input–output conversion performed by the sensor itself.

The sensor input, a couple \((x, \overrightarrow{w})\) where \(x = x(t) \in X\) is the measurand and \(\overrightarrow{w} = \langle w_1, \ldots, w_n \rangle = \langle w_1(t), \ldots, w_n(t) \rangle \in \overrightarrow{W}\) is a collection of further quantities influencing the sensor behavior, is transformed to its output \(y \in Y\). Therefore, the sensor characteristic function:

\[
f : X \times \overrightarrow{W} \times T \rightarrow Y
\]

(1)

takes the measurand \(x(t)\), the influence quantities \(\overrightarrow{w}(t)\) and the current time \(t\), included to take into account possible time-dependent effects, and associate them with the output signal \(y(t) = f(x(t), \overrightarrow{w}(t), t)\) to which both the measurand (‘the signal’) and the influence quantities (‘the noise’) contribute.

This simple formalization allows us to introduce some basic parameters describing the static behavior of a sensor:

- **Sensitivity**: ideally, \(x_1 \neq x_2\) implies \(f(x_1, \overrightarrow{w}, t) \neq f(x_2, \overrightarrow{w}, t)\), that is, distinct measurand values always produce distinct outputs; the ratio \(\Delta y/\Delta x\) expresses the aptitude of the sensor to reproduce measurand variations to output values.

- **Selectivity**: ideally \(f(x, \overrightarrow{w_1}, t) = f(x, \overrightarrow{w_2}, t)\) even if \(\overrightarrow{w_1} \neq \overrightarrow{w_2}\), that is, the sensor output is not affected by the variations of influence quantities; the less is the variability of \(y\) due to \(\overrightarrow{w}\), the better is the sensor (therefore, selectivity corresponds to nonsensitivity to influence quantities: the relative contribution of the measurand to the output can be formalized as a signal-to-noise ratio).

- **Repeatability and stability**: ideally \(f(x, \overrightarrow{w}, t_1) = f(x, \overrightarrow{w}, t_2)\) even if \(t_1 \neq t_2\), that is, the sensor output is not affected by short-term (fluctuations) and long-term (aging) time effects; the less is the variability of \(y\) due to \(t\) the better is the sensor (a stable sensor does not require frequent recalibrations).

- **Linearity**: ideally \(y = ax + b\) (where \(a\) and \(b\) are given coefficients, possibly with \(b = 0\)), that is, \(f\) is a straight line, the better the actual sensor behavior is approximated by this equation, the better the sensor is usually considered to be (a linear, zero-crossing sensor is calibrated in a single operation, aimed at determining the slope \(a\)).

In addition to these static parameters, the dynamic behavior of the sensor is synthesized by parameters such as its frequency response (see also Article 29, Relationship Between Signals in the Time and Frequency Domain, Volume 1; Article 27, Signals in the Frequency Domain, Volume 1; and Article 36, Systems in the Frequency Domain, Volume 1).

The technical specifications for sensors usually include some quantitative evaluation for these parameters in the nominal conditions of usage, expressed by the allowed ranges of measurand and influence quantities.

4 GENERALIZED MODEL

The inference process that leads to the evaluation and the expression of a measurand value is always only plausible in its results, and in general nothing can be inferred with certainty about the measurand value. The causes of this lack of certainty are various, and in particular the following:

- The model of the measurement system has not identified all of the relevant influence quantities, and any one of them may have a significant variability, such that the environmental conditions (including human operators) change after the calibration.
The measuring system is less stable than expected when the calibration procedure was defined, that is, the instrument would require a recalibration before its usage.

The interpolation shape of the calibration curve does not adequately map the actual instrument behavior (e.g., it is significantly nonlinear where a piecewise linear interpolation was chosen), so that for some instrument reading subsets the instrument is wrongly calibrated — see Article 58, Description of Accuracy, Linearity, and Drift, Volume 1.

All these cases can be formally characterized by recognizing that the certainty implied in the choice of a single-valued association between instrument readings and measurand values is not adequate. In the interaction with the measuring system during calibration, each measurand value generates an instrument reading that should be considered a sample drawn from a whole set of possible readings.

Such variability can be formalized according to a set-theoretical model, so that the information obtained in the calibration is expressed by a calibration strip, in which an interval of possible readings, whose center and width can be considered as the nominal reading and an uncertainty interval respectively, is associated with each measurand value (see Figure 7). (Note the changes of the calibration strip width along the measurand axis, taking into account nonuniformities in the uncertainty evaluation.)

As in the previous (certain, and therefore ideal) case, this diagram is used in its inverted form during measurement: for any given instrument reading, an uncertainty interval of possible measurand values is obtained together with a nominal value (see also Article 54, Explanation of Key Error and Uncertainty Concepts and Terms, Volume 1).

Figure 7. A diagram with the example of a strip generated by a calibration in which uncertainty has been taken into account.

An even more general approach could be adopted by expressing the uncertainty estimation as a standard deviation, and therefore in a probabilistic framework, as recommended by the ISO Guide to the Expression of Uncertainty in Measurement (1993) (GUM). The Guide, based on a recommendation by the International Committee for Weights and Measures (CIPM, 1981), states that measurement uncertainty can be estimated on the basis of both statistical and nonstatistical methods, and specifies a procedure to combine such components into a combined standard uncertainty. The set-theoretical formalization can then be regarded as a specialization of this framework: if the combined standard uncertainty is multiplied by a coverage factor, then an expanded uncertainty is obtained, which is thought of as the half-width of an uncertainty interval.

The inherent presence of uncertainty justifies the fundamental assumption that the result of a measurement must state not only a (nominal) measurand value but also its uncertainty estimation. Uncertainty in measurement is covered in Article 54, Explanation of Key Error and Uncertainty Concepts and Terms, Volume 1.

**RELATED ARTICLES**

Article 54, Explanation of Key Error and Uncertainty Concepts and Terms, Volume 1.

**REFERENCES**


**FURTHER READING**
