5.1

In Problem 5.1 an alternative way of evaluating the flow rates in the pipes is presented. This alternative method of evaluating pipe flow rates may also be used in Problems 5.2–5.6. See Figure 5.12 in text.

(a) Flow rates in pipes calculated from

\[
Q = -2Ah_{ij} \sqrt{\frac{2gD}{L}} \log \left( \frac{k}{3.7D} + \frac{2.51v_D}{D_{ij}/L} \right)
\]

Where minor losses are present \( h_{ij} \) is determined by initially equating \( h_{ij}, I, J \) to \( Z_I - Z_J \) and making successive corrections as in Chapter 4 (Example 4.2)

i.e. \( \Delta z = \frac{2(\sum Q - F)}{\sum Q/H} \)

Estimate \( Z_B = 80.0 \) m.

<table>
<thead>
<tr>
<th>Junction</th>
<th>Pipe (I, J)</th>
<th>( Z_I - Z_J ) =H_m</th>
<th>Q (L/s)</th>
<th>Q/H</th>
<th>( \Delta Z ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B first</td>
<td>AB</td>
<td>20.00</td>
<td>167.35</td>
<td>8.367</td>
<td></td>
</tr>
<tr>
<td>correction</td>
<td>BC_1</td>
<td>-20.00</td>
<td>-41.11</td>
<td>2.006</td>
<td>13.11</td>
</tr>
<tr>
<td></td>
<td>BC_2</td>
<td>-20.00</td>
<td>-43.61</td>
<td>2.180</td>
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</tr>
<tr>
<td></td>
<td>∑</td>
<td></td>
<td>82.63</td>
<td>12.603</td>
<td></td>
</tr>
<tr>
<td>B second</td>
<td>AB</td>
<td>6.89</td>
<td>96.27</td>
<td>13.97</td>
<td></td>
</tr>
<tr>
<td>correction</td>
<td>BC_1</td>
<td>-33.11</td>
<td>-53.43</td>
<td>1.61</td>
<td>-1.64</td>
</tr>
<tr>
<td></td>
<td>BC_2</td>
<td>-33.11</td>
<td>-57.01</td>
<td>1.72</td>
<td></td>
</tr>
<tr>
<td></td>
<td>∑</td>
<td></td>
<td>-14.17</td>
<td>17.30</td>
<td></td>
</tr>
</tbody>
</table>
After further similar corrections until $\sum Q - F < 0.1 \text{ L/s}$, final values:

$$Z_B = 91.48 \text{ m}$$

<table>
<thead>
<tr>
<th>Pipe</th>
<th>$Q$ (L/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>107.56</td>
</tr>
<tr>
<td>BC_1</td>
<td>52.05</td>
</tr>
<tr>
<td>BC_2</td>
<td>55.51</td>
</tr>
</tbody>
</table>

1(b) Solution by solving the equation

$$b_1 = \frac{\lambda LV^2}{2gD} + \frac{K_m V^2}{2g}$$

in the form

$$Q = AV = A \sqrt{2g(Z_I - Z_J) / \left( \frac{\lambda L}{D} + K_m \right)} = \sqrt{\frac{Z_I - Z_J}{K_{IJ}}}$$

in which $\lambda$ is determined from Barr’s equation, or from the Moody chart or the Colebrook–White equation.

In the following examples Barr’s equation is used but the final solution is similar to that of method (a). The difference is due to the different friction factor formulae used.

Estimate $Z_B = 80.0 \text{ m}$.

<table>
<thead>
<tr>
<th>Junction</th>
<th>$Z_I$ (m)</th>
<th>$Z_I - Z_J$ = (H (m)) Velocity (estimate)</th>
<th>$\lambda$ ($x 10^{-4}$)</th>
<th>$K$ (L/s)</th>
<th>$Q$ (L/s)</th>
<th>$Q/H$ (m/s)</th>
<th>$V$ (m/s)</th>
<th>$\Delta Z$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B first</td>
<td>AB 80.00</td>
<td>AB 20.00 2.0 0.01665 6.716 172.57 8.629 1.37</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>correction</td>
<td>BC_1 −20.00 1.0 0.01930 114.22 −41.82 2.091 0.85 13.37</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>BC_2 −20.00 1.0 0.01721 101.96 −44.29 2.214 0.90</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sum$ 86.46 12.934</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B second</td>
<td>AB 6.63 1.37 0.0170 6.856 98.34 14.833 0.78</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>correction</td>
<td>BC_1 −33.37 0.85 0.01956 115.83 −53.67 1.608 1.09 −1.35</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>BC_2 −33.37 0.90 0.0174 103.12 −56.88 1.704 1.16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sum$ −12.21 18.145</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B third</td>
<td>AB 7.98 0.80 0.0177 7.131 105.79 13.256 0.84</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>correction</td>
<td>BC_1 −32.02 1.10 0.0192 113.523 −53.11 1.659 1.08 −0.47</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>BC_2 −32.02 1.20 0.0169 99.882 −56.62 1.768 1.15</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sum$ −3.94 16.683</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Final values:

$$Z_B = 91.58 \text{ m}$$

$$Q_{AB} = 108.94 \text{ L/s}; \quad Q_{BC_1} = 52.78 \text{ L/s}; \quad Q_{BC_2} = 56.07 \text{ L/s}$$

5.2

See Figure 5.12 in text.

Equivalent system:
The effect of the pump in BC\(_1\) is equivalent to the gain in head across BC\(_1\) produced by lowering the level of the reservoir by the pump head, i.e. 5 m. The system shown in (b) is therefore analysed as in Problem 1 with final values

\[
Z_B = 90.98 \text{ m} \\
Q_{AB} = 110.82 \text{ L/s}; \quad Q_{BC_1} = 55.78 \text{ L/s}; \quad Q_{BC_2} = 55.03 \text{ L/s}
\]

### 5.3

See Figure 5.13 in text.

Method as in Problem 5.1(a).

Estimate \(Z_B = 90.0 \text{ m}\).

<table>
<thead>
<tr>
<th>Junction</th>
<th>(Z_j)</th>
<th>Pipe ((I, J))</th>
<th>(Z_j - Z_f = H (m))</th>
<th>(Q (L/s))</th>
<th>(Q/H)</th>
<th>(\Delta Z_B (m))</th>
</tr>
</thead>
<tbody>
<tr>
<td>B first</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>correction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AB</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BC</td>
<td>-20.00</td>
<td>-111.27</td>
<td>5.563</td>
<td>-15.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BD(_1)</td>
<td>-30.00</td>
<td>-83.54</td>
<td>2.785</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BD(_2)</td>
<td>-30.00</td>
<td>-58.60</td>
<td>1.953</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sum)</td>
<td></td>
<td></td>
<td></td>
<td>-155.35</td>
<td>20.107</td>
<td></td>
</tr>
<tr>
<td>B second</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>correction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AB</td>
<td>25.45</td>
<td>158.89</td>
<td>6.243</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BC</td>
<td>-4.55</td>
<td>-51.96</td>
<td>11.420</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>74.55</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>BD(_1)</td>
<td>-14.55</td>
<td>-57.67</td>
<td>3.963</td>
<td>0.77</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BD(_2)</td>
<td>-14.55</td>
<td>-39.88</td>
<td>2.741</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>(\sum)</td>
<td></td>
<td></td>
<td></td>
<td>9.38</td>
<td>24.367</td>
<td></td>
</tr>
</tbody>
</table>

Continue, and finally,

\[
Z_B = 75.31 \text{ m} \\
Q_{AB} = 156.43 \text{ L/s}; \quad Q_{BC} = 56.29 \text{ L/s} \\
Q_{BD_1} = 59.20 \text{ L/s}; \quad Q_{BD_2} = 40.98 \text{ L/s}
\]

### 5.4

See Figure 5.14 in text.

Note that the flow along BC (40 L/s) is simply treated as an external outflow at B.
Method as in Problem 5.1 (a)

Estimate $Z_B = 130.00$ m

$\Delta Z_B = 2 \left( \frac{\sum (Q - F) \cdot Q}{\sum Q/H} \right)$

<table>
<thead>
<tr>
<th>Junction</th>
<th>Pipe $(I, J)$</th>
<th>$Z_I - Z_J = H$ (m)</th>
<th>$Q$ (L/s)</th>
<th>$Q/H$</th>
<th>$\Delta Z$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B first</td>
<td>AB</td>
<td>20.00</td>
<td>126.32</td>
<td>6.316</td>
<td>2.49</td>
</tr>
<tr>
<td>correction DB</td>
<td></td>
<td>−40.00</td>
<td>−76.09</td>
<td>1.902</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sum$</td>
<td></td>
<td>50.23</td>
<td>8.218</td>
<td></td>
</tr>
<tr>
<td>$Z_J = 132.49$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B Second</td>
<td>AB</td>
<td>17.51</td>
<td>117.69</td>
<td>6.722</td>
<td>−0.2</td>
</tr>
<tr>
<td>correction DB</td>
<td></td>
<td>−42.49</td>
<td>−78.54</td>
<td>1.848</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sum$</td>
<td></td>
<td>39.15</td>
<td>8.570</td>
<td></td>
</tr>
</tbody>
</table>

Continue, and finally,

$Z_B = 132.3$ m

$Q_{AB} = 118$ L/s; $Q_{BD} = 78$ L/s

The pump in BC is required to overcome the difference in pressure head elevation between the ends of the pipe (i.e. $145.0 - 132.3 = 12.7$ m) in addition to the head loss along the pipe.

$V_{BC} = 0.8148$ m/s; $Re = 2.884 \times 10^5$

Using Barr’s equation (or the Moody diagram)

$\lambda = 0.01672$ and $h_f = \frac{0.01672 \times 4000 \times 0.8148^2}{19.62 \times 0.25}$

$= 9.05$ m

$\Rightarrow$ Pump total head ($H_m$) $= 12.7 \times 9.05 = 21.75$ m

Power consumption, $P = \frac{\rho g Q H_m}{\eta \times 1000}$ kW

where $\eta$ is the efficiency (decimal).

$\Rightarrow P = \frac{9.81 \times 0.04 \times 21.75}{0.6} = 14.22$ kW

5.5

See Figure 5.15 in text.

Estimate $Z_B = 120.0$ m; $Z_D = 90.0$ m.
\[ \text{Junction} \quad Z_{ij} \quad \text{Pipe (I, J)} \quad Z_i - Z_j = H \text{ (m)} \quad Q \text{ (L/s)} \quad Q/H \quad \Delta Z_j \text{ (m)} \\
\]

<table>
<thead>
<tr>
<th>Junction</th>
<th></th>
<th>AB</th>
<th>80.00</th>
<th>528.96</th>
<th>6.612</th>
</tr>
</thead>
<tbody>
<tr>
<td>B first correction</td>
<td></td>
<td>CB</td>
<td>-40.00</td>
<td>-122.83</td>
<td>3.071</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DB</td>
<td>-30.00</td>
<td>-494.52</td>
<td>16.484</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>\sum</strong></td>
</tr>
<tr>
<td>D first correction</td>
<td></td>
<td>BD</td>
<td>23.24</td>
<td>433.34</td>
<td>18.646</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ED</td>
<td>-30.00</td>
<td>-170.77</td>
<td>5.692</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FD</td>
<td>-40.00</td>
<td>-122.83</td>
<td>3.071</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td><strong>\sum</strong></td>
</tr>
<tr>
<td>B second correction</td>
<td></td>
<td>AB</td>
<td>86.76</td>
<td>551.85</td>
<td>6.361</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CB</td>
<td>-33.24</td>
<td>-111.56</td>
<td>3.356</td>
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<td></td>
<td>DB</td>
<td>-13.04</td>
<td>-320.89</td>
<td>24.608</td>
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<td></td>
<td><strong>\sum</strong></td>
</tr>
<tr>
<td>D second correction</td>
<td></td>
<td>BD</td>
<td>20.00</td>
<td>400.88</td>
<td>20.044</td>
</tr>
<tr>
<td></td>
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<td>ED</td>
<td>-40.00</td>
<td>-198.78</td>
<td>4.945</td>
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<tr>
<td></td>
<td></td>
<td>FD</td>
<td>-50.20</td>
<td>-138.19</td>
<td>2.753</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>\sum</strong></td>
</tr>
</tbody>
</table>

**Final values:**

\[ Z_B = 126.73 \text{ m}; \quad Z_D = 109.41 \text{ m} \]
\[ Q_{AB} = 505.3 \text{ L/s} \]
\[ Q_{BC} = 133.2 \text{ L/s}; \quad Q_{BD} = 372.1 \text{ L/s} \]
\[ Q_{DE} = 221.2 \text{ L/s}; \quad Q_{DF} = 150.8 \text{ L/s} \]

5.6

See Figure 5.15 in text.

Estimate \( Z_B = 120.0 \text{ m}; \quad Z_D = 90.0 \text{ m} \).
### Junction ZJ

<table>
<thead>
<tr>
<th>Pipe (I, J)</th>
<th>ZI − ZJ = H (m)</th>
<th>Q (L/s)</th>
<th>Q/H</th>
<th>ΔZJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>BD</td>
<td>16.20</td>
<td>112.92</td>
<td>6.97</td>
<td>−16.70</td>
</tr>
<tr>
<td>ED</td>
<td>−30.00</td>
<td>−85.55</td>
<td>2.85</td>
<td></td>
</tr>
<tr>
<td>FD</td>
<td>−40.00</td>
<td>−138.25</td>
<td>3.456</td>
<td></td>
</tr>
<tr>
<td>∑</td>
<td>−110.88</td>
<td>13.277</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AB</td>
<td>93.80</td>
<td>270.11</td>
<td>2.88</td>
<td></td>
</tr>
<tr>
<td>CB</td>
<td>−26.20</td>
<td>−144.99</td>
<td>5.534</td>
<td>−5.86</td>
</tr>
<tr>
<td>DB</td>
<td>−32.90</td>
<td>−164.43</td>
<td>4.998</td>
<td></td>
</tr>
<tr>
<td>∑</td>
<td>−39.31</td>
<td>13.412</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BD</td>
<td>27.04</td>
<td>148.24</td>
<td>5.482</td>
<td></td>
</tr>
<tr>
<td>ED</td>
<td>−13.30</td>
<td>−55.70</td>
<td>4.188</td>
<td>−1.64</td>
</tr>
<tr>
<td>FD</td>
<td>−23.30</td>
<td>−104.12</td>
<td>4.469</td>
<td></td>
</tr>
<tr>
<td>∑</td>
<td>−11.58</td>
<td>14.139</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Final values:**

\[ Z_B = 100.19 \text{ m}; \quad Z_D = 71.71 \text{ m} \]

\[ Q_{AB} = 279 \text{ L/s} \]

\[ Q_{BC} = 126.7 \text{ L/s}; \quad Q_{BD} = 152.4 \text{ L/s} \]

\[ Q_{DE} = 52.1 \text{ L/s}; \quad Q_{DF} = 100.3 \text{ L/s} \]

### 5.7

See Figure 5.16 in text. Analysis by head balance method.

Initially estimated pipe flow rates are shown in table below.

Friction head losses in pipes calculated from the Darcy–Weisbach equation with friction factor \( \lambda \) determined from the Colebrook–White equation. Alternatively, \( \lambda \) could be obtained from the Moody diagram or Barr’s equation or other explicit formula.

### Loop Pipe \( k/D \) (x10^{-4}) \( Q \) (L/s) \( Re \) (x10^4) \( \lambda \) \( h_l \) (m) \( h_l/Q \) \( \Delta Q \)

<table>
<thead>
<tr>
<th>Loop correction</th>
<th>Pipe</th>
<th>k/D</th>
<th>Q (L/s)</th>
<th>Re</th>
<th>( \lambda )</th>
<th>( h_l ) (m)</th>
<th>( h_l/Q )</th>
<th>( \Delta Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 first</td>
<td>AB</td>
<td>1.5</td>
<td>120.00</td>
<td>6.76</td>
<td>0.0146</td>
<td>27.14</td>
<td>0.2262</td>
<td></td>
</tr>
<tr>
<td>BE</td>
<td>3.0</td>
<td>5.00</td>
<td>0.56</td>
<td>0.0215</td>
<td>0.89</td>
<td>0.1777</td>
<td>−10.94</td>
<td></td>
</tr>
<tr>
<td>EA</td>
<td>1.5</td>
<td>−80.00</td>
<td>4.50</td>
<td>0.0152</td>
<td>−15.07</td>
<td>0.1884</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∑</td>
<td></td>
<td>12.96</td>
<td>0.5923</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(Continued)</td>
</tr>
</tbody>
</table>
After further correction ($\sum h_L < 0.01$ m in any loop)

<table>
<thead>
<tr>
<th>Pipe</th>
<th>$Q$ (L/s)</th>
<th>Junction</th>
<th>Head elevation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>104.8</td>
<td>B</td>
<td>39.03</td>
</tr>
<tr>
<td>EB</td>
<td>0.6</td>
<td>C</td>
<td>17.37</td>
</tr>
<tr>
<td>AE</td>
<td>95.2</td>
<td>D</td>
<td>18.14</td>
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<tr>
<td>BC</td>
<td>45.4</td>
<td>E</td>
<td>39.06</td>
</tr>
<tr>
<td>DC</td>
<td>4.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ED</td>
<td>44.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.8

See Figure 5.17 in text. Initial estimates of flow rates:

$Q_{BCE} = 50$ L/s; $Q_{BE} = 50$ L/s; $Q_{BDE} = 100$ L/s

1. Determine the flow rates in BCE, BE and BDE.

Procedure as in Problem 5.7.
Flow rate in AB = 200 L/s ($= Q_{EF}$)

$V = 1.258 \text{ m/s}; \quad Re = 5.01 \times 10^5$

$k/D = 0.00033; \quad \lambda = 0.0165; \quad h_l = 2.96 \text{ m}$

Total head loss = $2 \times 2.96 + 6.58 = 12.5 \text{ m}$

### 5.9

See Figure 5.16 in text.

The procedure is identical with that of Problem 5.7 except that the net head drop along BC is $h_l - H_p = (b_l - 15.0)$.

**Solution:**

<table>
<thead>
<tr>
<th>Pipe</th>
<th>$Q$ (L/s)</th>
<th>Junction</th>
<th>Head elevation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>106.4</td>
<td>B</td>
<td>38.50</td>
</tr>
<tr>
<td>EB</td>
<td>6.1</td>
<td>C</td>
<td>25.03</td>
</tr>
<tr>
<td>AE</td>
<td>93.6</td>
<td>D</td>
<td>24.77</td>
</tr>
<tr>
<td>BC</td>
<td>93.6</td>
<td>E</td>
<td>39.78</td>
</tr>
<tr>
<td>CD</td>
<td>2.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ED</td>
<td>37.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5.10
See Figure 5.18 in text.
Initial flow estimates shown in table.

<table>
<thead>
<tr>
<th>Loop</th>
<th>Pipe</th>
<th>(k/D) ((\times 10^{-4}))</th>
<th>(Q) (L/s)</th>
<th>(Re) ((\times 10^3))</th>
<th>(\lambda)</th>
<th>(b_L) (m)</th>
<th>(b_L/Q)</th>
<th>(\Delta Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>3.0</td>
<td>120.00</td>
<td>6.76</td>
<td>0.0160</td>
<td>23.80</td>
<td>0.1983</td>
<td></td>
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</tr>
<tr>
<td>BH</td>
<td>3.0</td>
<td>20.00</td>
<td>1.13</td>
<td>0.0192</td>
<td>0.30</td>
<td>0.0149</td>
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<td></td>
</tr>
<tr>
<td>1</td>
<td>HF</td>
<td>4.0</td>
<td>-20.00</td>
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<td>0.0189</td>
<td>-1.23</td>
<td>0.0617</td>
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<tr>
<td>FG</td>
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<td>-70.00</td>
<td>5.26</td>
<td>0.0170</td>
<td>-36.26</td>
<td>0.5179</td>
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<td></td>
</tr>
<tr>
<td>GA</td>
<td>3.0</td>
<td>-110.00</td>
<td>6.20</td>
<td>0.0161</td>
<td>-15.09</td>
<td>0.1372</td>
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</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>(\sum)</td>
<td>-24.48</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9300</td>
</tr>
<tr>
<td>2</td>
<td>BC</td>
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<td>0.0174</td>
<td>14.20</td>
<td>0.2840</td>
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</tr>
<tr>
<td>CD</td>
<td>4.0</td>
<td>30.00</td>
<td>2.25</td>
<td>0.0181</td>
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<td>0.0886</td>
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<tr>
<td>DH</td>
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<td>0.75</td>
<td>0.0208</td>
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<td>0.0679</td>
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<tr>
<td>HB</td>
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<td>0.0178</td>
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<td>(\sum)</td>
<td>15.32</td>
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<td>4.21</td>
<td>0.1590</td>
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<td>20.00</td>
<td>1.50</td>
<td>0.0189</td>
<td>1.23</td>
<td>0.0617</td>
<td>-4.23</td>
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<tr>
<td>EF</td>
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<td>1.50</td>
<td>0.0189</td>
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<td>FH</td>
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<td>0.35</td>
<td>0.0238</td>
<td>0.09</td>
<td>0.0182</td>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>(\sum)</td>
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<td></td>
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<td></td>
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</tr>
</tbody>
</table>

Final values:

<table>
<thead>
<tr>
<th>Pipe</th>
<th>Flow rate (L/s)</th>
<th>(b_L) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>136.41</td>
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<tr>
<td>BH</td>
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<tr>
<td>HF</td>
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<tr>
<td>GF</td>
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<td>21.62</td>
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<td>AG</td>
<td>93.59</td>
<td>11.06</td>
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<tr>
<td>BC</td>
<td>29.84</td>
<td>5.26</td>
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<tr>
<td>CD</td>
<td>9.84</td>
<td>0.33</td>
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<td>HD</td>
<td>24.06</td>
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<tr>
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<td>0.62</td>
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<tr>
<td>FE</td>
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<td>4.09</td>
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</table>

<table>
<thead>
<tr>
<th>Junction</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
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<tbody>
<tr>
<td>Head elevation (m)</td>
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</tbody>
</table>
5.11

See Figure 5.19 in text.

Create a ‘dummy’ pipe between A and F with zero flow and constant head loss of $-10$ m.

<table>
<thead>
<tr>
<th>Loop</th>
<th>Pipe</th>
<th>$k/D$ ($\times 10^{-4}$)</th>
<th>$Q$ (L/s)</th>
<th>$Re$ ($\times 10^4$)</th>
<th>$\lambda$</th>
<th>$h_L$ (m)</th>
<th>$h_L/Q$</th>
<th>$\Delta Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 first</td>
<td>AB</td>
<td>2.4</td>
<td>100.00</td>
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<td>0.1354</td>
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</tr>
<tr>
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<td>BE</td>
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</tr>
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<td>—</td>
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<td>-10.00</td>
<td>0.0000</td>
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</tr>
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<td>—</td>
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<tr>
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<td>0.0218</td>
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<td>SUM</td>
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<td>0.016</td>
<td>12.14</td>
<td>0.1282</td>
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</tr>
<tr>
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<td>0.0099</td>
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<td>0.0000</td>
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</tr>
<tr>
<td></td>
<td>SUM</td>
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<td>—</td>
<td>—</td>
<td>—</td>
<td>-0.23</td>
<td>0.1914</td>
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<tr>
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<td>EB</td>
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<td>0.0253</td>
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<td>0.0010</td>
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<td></td>
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<tr>
<td></td>
<td>SUM</td>
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<td>—</td>
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</table>

Final values:

<table>
<thead>
<tr>
<th>Pipe</th>
<th>$Q$ (L/s)</th>
<th>$h_L$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>95.31</td>
<td>12.30</td>
</tr>
<tr>
<td>BE</td>
<td>5.17</td>
<td>0.05</td>
</tr>
<tr>
<td>FE</td>
<td>44.69</td>
<td>2.35</td>
</tr>
<tr>
<td>BC</td>
<td>90.14</td>
<td>16.03</td>
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<td>CD</td>
<td>30.14</td>
<td>6.20</td>
</tr>
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<td>ED</td>
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</table>

<table>
<thead>
<tr>
<th>Head (m)</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>87.70</td>
<td>71.67</td>
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<td>87.66</td>
<td>90.00</td>
<td></td>
</tr>
</tbody>
</table>
5.12

All the matrices and vectors needed for the gradient method are (see Figure 5.18):

\( NT = 10 \)
\( NN = 7 \)
\( NS = 1 \)

\([A_{12}]\) = connectivity matrix; its dimension is \((10 \times 7)\)
\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & -1 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\([A_{21}]\) = transposed matrix of \([A_{12}]\)
\[
\begin{bmatrix}
1 & -1 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\([A_{10}]\) = topologic matrix node to node; dimension \((10 \times 1)\)
\([Q]\) = discharges vector; dimension \((10 \times 1)\)
\([H]\) = unknown piezometric head vector; dimension \((7 \times 1)\)
\([H_0]\) = fixed piezometric head vector; dimension \((1 \times 1)\)
\([q]\) = water demand vector; dimension \((7 \times 1)\)

\[
\begin{bmatrix}
-1 \\
0 \\
0 \\
1 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
-1 \\
0 \\
0 \\
0 \\
1 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
100 \\
0.05 \\
0.02 \\
0.02 \\
0.04 \\
0.03 \\
0.04 \\
0.03 \\
0.03 \\
0.03
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 \\
0.02 \\
0.02 \\
0.04 \\
0.03 \\
0.04 \\
0.03 \\
0.03 \\
0.03 \\
0.03
\end{bmatrix}
\]
\[ [N] = \text{diagonal matrix; dimension } (10 \times 10), \text{ having } 2 \text{ in the diagonal} \]

\[ \begin{pmatrix}
2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
\end{pmatrix} \]

\[ [I] = \text{identity matrix; dimension } (10 \times 10) \]

\[ \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{pmatrix} \]

**First iteration:**
The previous matrixes and vectors are valid for all the iterations. The following matrices change in each iteration:

\[ [A_{11}] = \text{diagonal matrix; dimension } (10 \times 10). \text{ It has the value } \alpha_i Q^{(n-1)} \text{ on the diagonal; coefficients } \beta \text{ and } \gamma \text{ are zero as no pumps exist in the network} \]

The following table shows the calculated values for \( \alpha \):

<table>
<thead>
<tr>
<th>Pipe</th>
<th>( Q (\text{m}^3/\text{s}) )</th>
<th>( \lambda )</th>
<th>( V (\text{m/s}) )</th>
<th>( h_t (\text{m}) )</th>
<th>( h_t + h_m (\text{m}) )</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>0.1</td>
<td>0.01615</td>
<td>3.1831</td>
<td>16.691</td>
<td>16.691</td>
<td>1669.1</td>
</tr>
<tr>
<td>BH</td>
<td>0.1</td>
<td>0.01615</td>
<td>3.1831</td>
<td>6.259</td>
<td>6.259</td>
<td>625.9</td>
</tr>
<tr>
<td>HF</td>
<td>0.1</td>
<td>0.01667</td>
<td>5.6588</td>
<td>27.223</td>
<td>27.223</td>
<td>2722.3</td>
</tr>
<tr>
<td>FG</td>
<td>0.1</td>
<td>0.01667</td>
<td>5.6588</td>
<td>72.593</td>
<td>72.593</td>
<td>7259.3</td>
</tr>
<tr>
<td>GA</td>
<td>0.1</td>
<td>0.01615</td>
<td>3.1831</td>
<td>12.518</td>
<td>12.518</td>
<td>1251.8</td>
</tr>
<tr>
<td>BC</td>
<td>0.1</td>
<td>0.01667</td>
<td>5.6588</td>
<td>54.445</td>
<td>54.445</td>
<td>5444.5</td>
</tr>
<tr>
<td>CD</td>
<td>0.1</td>
<td>0.01667</td>
<td>5.6588</td>
<td>27.223</td>
<td>27.223</td>
<td>2722.3</td>
</tr>
<tr>
<td>DH</td>
<td>0.1</td>
<td>0.01667</td>
<td>5.6588</td>
<td>54.445</td>
<td>54.445</td>
<td>5444.5</td>
</tr>
<tr>
<td>DE</td>
<td>0.1</td>
<td>0.01667</td>
<td>5.6588</td>
<td>27.223</td>
<td>27.223</td>
<td>2722.3</td>
</tr>
<tr>
<td>EF</td>
<td>0.1</td>
<td>0.01667</td>
<td>5.6588</td>
<td>54.445</td>
<td>54.445</td>
<td>5444.5</td>
</tr>
</tbody>
</table>
Matrix $[A_{11}]$:
\[
\begin{pmatrix}
166.91 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 62.59 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 272.23 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 725.93 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 125.18 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 544.45 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 272.23 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 544.45 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 272.23 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 544.45 \\
\end{pmatrix}
\]

$[A_{11}]'$ = diagonal matrix; dimension (10 $\times$ 10). It has the value $\alpha_i Q^{(n-1)}$ on the diagonal.

For this network $[A_{11}]' = [A_{11}]$.

To find $[H_{i+1}]$ by Equation 5.17 following a step-by-step analysis, the following matrices can be found:

$[N][A_{11}]'$
\[
\begin{pmatrix}
333.82 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 125.18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 544.46 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1451.86 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 250.36 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1088.9 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 544.46 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1088.9 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 544.46 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1088.9 \\
\end{pmatrix}
\]
\[
\begin{pmatrix}
0.0030 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.0080 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.0018 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.0007 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0040 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0009 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.0018 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0009 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0009
\end{pmatrix}^{-1}
\]

\[
\begin{pmatrix}
0.0030 & -0.0080 & 0.0000 & 0.0000 & 0.0000 & -0.0009 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
\end{pmatrix}
\]

\[
\left[\frac{[A21]}{[N][A11]}\right]^{-1}\frac{[A12]}{[N][A11]}^{-1}
\]

\[
\begin{pmatrix}
0.0119 & -0.0009 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.0080 \\
-0.0009 & 0.0028 & -0.0018 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & -0.0018 & 0.0046 & -0.0018 & 0.0000 & 0.0000 & 0.0000 & -0.0009 \\
0.0000 & 0.0000 & -0.0018 & 0.0028 & -0.0009 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.0009 & 0.0034 & -0.0007 & -0.0018 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.0007 & 0.0047 & 0.0000 & 0.0000 \\
-0.0080 & 0.0000 & 0.0000 & -0.0009 & 0.0000 & -0.0018 & 0.0000 & 0.0107 \\
\end{pmatrix}
\]

\[
-(\frac{[A21]}{[N][A11]}\right)^{-1}\frac{[A12]}{[N][A11]}^{-1}
\]

\[
\begin{pmatrix}
-223.8870 & -314.3725 & -359.6152 & -434.0857 & 583.0267 & 85.9591 & 297.2507 \\
-33.0090 & -46.3498 & -53.0202 & -63.9998 & 85.9591 & 226.3487 & 43.8254 \\
\end{pmatrix}
\]
\[
\begin{pmatrix}
\end{pmatrix}
\begin{pmatrix}
-100 & 0 & 0 & 100 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\end{pmatrix}
\]

\[
(A_{21}(N)A_{11}')^{-1} \times (A_{21}(N)A_{11}')^{-1} \times (A_{21}(N)A_{11}')^{-1} - (q)
\]

\[
\begin{pmatrix}
-0.29957 & 0.02000 & -0.03000 & 0.04000 & 0.08000 & -0.35942 & 0.08000
\end{pmatrix}
\begin{pmatrix}
-0.10 & 0.00 & -0.10 & 0.00 & 0.10 & 0.00 & 0.10
\end{pmatrix}
\begin{pmatrix}
-0.19957 & 0.02000 & 0.07000 & 0.04000 & -0.02000 & -0.35942 & -0.02000
\end{pmatrix}
\]

Thus,

\[
H_{i+1} = -((A_{21}(N)A_{11}')^{-1}A_{12})^{-1}
\times ((A_{21}(N)A_{11}')^{-1}A_{11}[Q] + [A_{10}][H_0]) - ([A_{21}][Q] - [q])
\]

<table>
<thead>
<tr>
<th>Node</th>
<th>(H) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>45.615</td>
</tr>
<tr>
<td>C</td>
<td>2.955</td>
</tr>
<tr>
<td>D</td>
<td>-7.486</td>
</tr>
<tr>
<td>E</td>
<td>-4.879</td>
</tr>
<tr>
<td>F</td>
<td>43.890</td>
</tr>
<tr>
<td>G</td>
<td>83.206</td>
</tr>
<tr>
<td>H</td>
<td>42.642</td>
</tr>
</tbody>
</table>
To find $[Q_{i+1}]$ by Equation 5. 18 following a step-by-step analysis, the following matrices can be found:

\[
\begin{align*}
[A_{21}] & [H_{i+1}] & [A_{12}] & [H_{i+1}] + [A_{10}][H_0] & \quad ([[N][A_{11}')]^{-1}) \times \\
(45.615 & -54.385) & (-2.9723 & -2.9723) & (-0.16292) \\
1.2478 & 1.2478 & 0.00229 \\
39.316 & 39.316 & 0.02708 \\
-83.206 & 16.794 & 0.06708 \\
-42.66 & -42.66 & -0.03918 \\
-10.441 & -10.441 & -0.01918 \\
50.128 & 50.128 & 0.04604 \\
2.6066 & 2.6066 & 0.00479 \\
48.769 & 48.769 & 0.04479
\end{align*}
\]

\[
(\langle[[N][A_{11}')]^{-1}\rangle\langle A_{11} \rangle)
\]

\[
\begin{array}{cccccccccccccccc}
0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5
\end{array}
\]

\[
([I] - \langle[[N][A_{11}')]^{-1}\rangle\langle A_{11} \rangle)
\]

\[
\begin{array}{cccccccccccccccc}
0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5
\end{array}
\]
\[ ([I] - (([N][A11']^{-1})[A11])[Q]) \]

\[
\begin{pmatrix}
0.05 \\
0.05 \\
0.05 \\
0.05 \\
0.05 \\
0.05 \\
0.05 \\
0.05 \\
0.05 \\
0.05
\end{pmatrix}
\]

Thus,

\[ [Q_{i+1}] = ([I] - ([N][A11']^{-1})[A11])[Q] - (([N][A11']^{-1}) \times ([A12][H_{i+1}] + [A10][H_0])) \]

<table>
<thead>
<tr>
<th>Pipe</th>
<th>( Q ) (( m^3/s ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>0.21292</td>
</tr>
<tr>
<td>BH</td>
<td>0.07374</td>
</tr>
<tr>
<td>HF</td>
<td>0.04771</td>
</tr>
<tr>
<td>FG</td>
<td>0.02292</td>
</tr>
<tr>
<td>GA</td>
<td>-0.01708</td>
</tr>
<tr>
<td>BC</td>
<td>0.08918</td>
</tr>
<tr>
<td>CD</td>
<td>0.06918</td>
</tr>
<tr>
<td>DH</td>
<td>0.00396</td>
</tr>
<tr>
<td>DE</td>
<td>0.04521</td>
</tr>
<tr>
<td>EF</td>
<td>0.00521</td>
</tr>
</tbody>
</table>

After only six iterations the following are the results:

<table>
<thead>
<tr>
<th>Iteration</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node B</td>
<td>0.0</td>
<td>45.615</td>
<td>72.944</td>
<td>69.630</td>
<td>69.503</td>
<td>69.511</td>
<td>69.511</td>
</tr>
<tr>
<td>Node C</td>
<td>0.0</td>
<td>2.955</td>
<td>69.080</td>
<td>65.287</td>
<td>64.255</td>
<td>64.252</td>
<td>64.253</td>
</tr>
<tr>
<td>Node D</td>
<td>0.0</td>
<td>-7.486</td>
<td>71.397</td>
<td>65.645</td>
<td>63.941</td>
<td>63.924</td>
<td>63.924</td>
</tr>
<tr>
<td>Node E</td>
<td>0.0</td>
<td>-4.879</td>
<td>73.308</td>
<td>64.774</td>
<td>63.345</td>
<td>63.301</td>
<td>63.301</td>
</tr>
<tr>
<td>Node F</td>
<td>0.0</td>
<td>43.890</td>
<td>75.484</td>
<td>68.069</td>
<td>67.408</td>
<td>67.390</td>
<td>67.390</td>
</tr>
<tr>
<td>Node G</td>
<td>0.0</td>
<td>83.206</td>
<td>96.082</td>
<td>89.075</td>
<td>88.985</td>
<td>88.981</td>
<td>88.981</td>
</tr>
<tr>
<td>Node H</td>
<td>0.0</td>
<td>42.642</td>
<td>71.946</td>
<td>67.685</td>
<td>67.414</td>
<td>67.418</td>
<td>67.418</td>
</tr>
</tbody>
</table>

*Note:* Values are given in metres.
Pipe discharges

<table>
<thead>
<tr>
<th>Pipe</th>
<th>Iter 0</th>
<th>Iter 1</th>
<th>Iter 2</th>
<th>Iter 3</th>
<th>Iter 4</th>
<th>Iter 5</th>
<th>Iter 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>0.10</td>
<td>0.21292</td>
<td>0.14599</td>
<td>0.13667</td>
<td>0.13641</td>
<td>0.13638</td>
<td>0.13638</td>
</tr>
<tr>
<td>BH</td>
<td>0.10</td>
<td>0.07374</td>
<td>0.04744</td>
<td>0.05451</td>
<td>0.05642</td>
<td>0.05655</td>
<td>0.05655</td>
</tr>
<tr>
<td>HF</td>
<td>0.10</td>
<td>0.04771</td>
<td>0.01080</td>
<td>0.00010</td>
<td>0.00243</td>
<td>0.00250</td>
<td>0.00251</td>
</tr>
<tr>
<td>FG</td>
<td>0.10</td>
<td>0.02292</td>
<td>−0.04401</td>
<td>−0.05333</td>
<td>−0.05359</td>
<td>−0.05362</td>
<td>−0.05362</td>
</tr>
<tr>
<td>GA</td>
<td>0.10</td>
<td>−0.01708</td>
<td>−0.08401</td>
<td>−0.09333</td>
<td>−0.09359</td>
<td>−0.09362</td>
<td>−0.09362</td>
</tr>
<tr>
<td>BC</td>
<td>0.10</td>
<td>0.08918</td>
<td>0.04855</td>
<td>0.03216</td>
<td>0.02998</td>
<td>0.02984</td>
<td>0.02983</td>
</tr>
<tr>
<td>CD</td>
<td>0.10</td>
<td>0.06918</td>
<td>0.02855</td>
<td>0.01216</td>
<td>0.00998</td>
<td>0.00984</td>
<td>0.00983</td>
</tr>
<tr>
<td>DH</td>
<td>0.10</td>
<td>0.00396</td>
<td>−0.00664</td>
<td>−0.02441</td>
<td>−0.02400</td>
<td>−0.02404</td>
<td>−0.02405</td>
</tr>
<tr>
<td>DE</td>
<td>0.10</td>
<td>0.04521</td>
<td>0.01519</td>
<td>0.01657</td>
<td>0.01398</td>
<td>0.01388</td>
<td>0.01387</td>
</tr>
<tr>
<td>EF</td>
<td>0.10</td>
<td>0.00521</td>
<td>−0.02481</td>
<td>−0.02343</td>
<td>−0.02602</td>
<td>−0.02612</td>
<td>−0.02613</td>
</tr>
</tbody>
</table>

Note: Values are given in cubic metres per second.

Finally,

<table>
<thead>
<tr>
<th>Node</th>
<th>H (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>69.511</td>
</tr>
<tr>
<td>C</td>
<td>64.253</td>
</tr>
<tr>
<td>D</td>
<td>63.924</td>
</tr>
<tr>
<td>E</td>
<td>63.301</td>
</tr>
<tr>
<td>F</td>
<td>67.390</td>
</tr>
<tr>
<td>G</td>
<td>88.981</td>
</tr>
<tr>
<td>H</td>
<td>67.418</td>
</tr>
</tbody>
</table>

| Pipe | Q (m³/s) | | Q| (L/s) |
|------|----------|---|------|
| AB   | 0.13638  | 136.38|
| BH   | 0.05655  | 56.55|
| HF   | 0.00251  | 2.51|
| FG   | −0.05362 | 53.62|
| GA   | −0.09362 | 93.62|
| BC   | 0.02983  | 29.83|
| CD   | 0.00983  | 9.83|
| DH   | −0.02405 | 24.05|
| DE   | 0.01387  | 13.87|
| EF   | −0.02613 | 26.13|

5.13

All the matrices and vectors needed for the gradient method are (see Figure 5.3)

NT = 7
NN = 5
NS = 1
$[A_{12}] = \text{connectivity matrix; dimension (10 \times 7)}$

$$
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 1 \\
-1 & 0 & 0 & 1 & 0
\end{pmatrix}
$$

$[A_{21}] = \text{transposed matrix of } [A_{12}]$

$$
\begin{pmatrix}
1 & -1 & 0 & 0 & 0 & 0 & -1 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -1 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0
\end{pmatrix}
$$

$[A_{10}] = \text{topologic matrix node to node; dimension (7 \times 1)}$

$[Q] = \text{discharges vector; dimension (7 \times 1)}$

$[H] = \text{unknown piezometric head vector; dimension (5 \times 1)}$

$[H_0] = \text{fixed piezometric head vector; dimension (1 \times 1)}$

$[q] = \text{water demand vector; dimension (5 \times 1)}$

$$
\begin{pmatrix}
-1 \\
0 \\
0 \\
0 \\
-1 \\
0
\end{pmatrix}
\begin{pmatrix}
0.1 \\
0.1 \\
0.1 \\
0.1 \\
0.1
\end{pmatrix}
\begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix}
\begin{pmatrix}
70
\end{pmatrix}
\begin{pmatrix}
0.06 \\
0.04 \\
0.03 \\
0.05 \\
0.04
\end{pmatrix}
$$

$[N] = \text{diagonal matrix; dimension (7 \times 7). It has 2 in the diagonal (from the Darcy–Weisbach head loss equation)}$

$$
\begin{pmatrix}
2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2
\end{pmatrix}
$$
First iteration: The previous matrices and vectors are valid for all the iterations. The following matrices change in each iteration:

\[ \[A_{11}\] = \text{diagonal matrix; dimension } (7 \times 7). \text{ It has the value } \alpha_i Q^{(n-1)}_i \text{ on the diagonal; coefficients } \beta \text{ and } \gamma \text{ are zero as no pumps exist in the network} \]

The following table shows the calculated values for \( \alpha \):

<table>
<thead>
<tr>
<th>Pipe</th>
<th>( Q ) (m(^3)/s)</th>
<th>( \lambda )</th>
<th>( V ) (m/s)</th>
<th>( h_1 ) (m)</th>
<th>( h_1 + h_m ) (m)</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>0.1</td>
<td>0.01593</td>
<td>2.0372</td>
<td>8.0916</td>
<td>8.0916</td>
<td>809.16</td>
</tr>
<tr>
<td>BC</td>
<td>0.1</td>
<td>0.01667</td>
<td>5.6588</td>
<td>108.89</td>
<td>108.89</td>
<td>10889</td>
</tr>
<tr>
<td>CD</td>
<td>0.1</td>
<td>0.01780</td>
<td>12.732</td>
<td>294.33</td>
<td>294.33</td>
<td>29433</td>
</tr>
<tr>
<td>DE</td>
<td>0.1</td>
<td>0.01667</td>
<td>5.6588</td>
<td>108.89</td>
<td>108.89</td>
<td>10889</td>
</tr>
<tr>
<td>EF</td>
<td>0.1</td>
<td>0.01667</td>
<td>5.6588</td>
<td>108.89</td>
<td>108.89</td>
<td>10889</td>
</tr>
<tr>
<td>AF</td>
<td>0.1</td>
<td>0.01615</td>
<td>3.1831</td>
<td>8.3454</td>
<td>8.3454</td>
<td>834.54</td>
</tr>
<tr>
<td>BE</td>
<td>0.1</td>
<td>0.01780</td>
<td>12.732</td>
<td>294.33</td>
<td>294.33</td>
<td>29433</td>
</tr>
</tbody>
</table>

Matrix \( [A_{11}] \):

\[
\begin{pmatrix}
80.916 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1088.9 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2943.3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1088.9 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1088.9 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 83.454 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2943.3
\end{pmatrix}
\]

[\( A_{11} \) = \text{diagonal matrix; dimension } (10 \times 10). \text{ It has the value } \alpha_i Q^{(n-1)}_i \text{ on the diagonal}]

For this network, \( [A_{11}]' = [A_{11}] \)

\[
\begin{pmatrix}
80.916 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1088.9 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2943.3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1088.9 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1088.9 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 83.454 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2943.3
\end{pmatrix}
\]
To find $[H_{t+1}]$ by Equation 5.17 following a step-by-step analysis, the following matrices can be found:

$$\mathbf{[N][A11']}$$

\[
\begin{pmatrix}
161.832 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2177.8 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 5886.6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2177.8 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2177.8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2177.8 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 5886.6
\end{pmatrix}
\]

$$\mathbf{([N][A11'])^{-1}}$$

\[
\begin{pmatrix}
0.00618 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.00046 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.00017 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.00046 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.00046 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.00599 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.00017
\end{pmatrix}
\]

$$\mathbf{([A21][N][A11'])^{-1}}$$

\[
\begin{pmatrix}
0.00618 & -0.00046 & 0 & 0 & 0 & 0 & -0.00017 \\
0 & 0.00046 & -0.00017 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.00017 & -0.00046 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.00046 & -0.00046 & 0 & 0.00017 \\
0 & 0 & 0 & 0 & 0.00046 & 0.00017 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.00046 & 0.00599 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.00017
\end{pmatrix}
\]

$$\mathbf{([A21][N][A11'])^{-1}[A12]}$$

\[
\begin{pmatrix}
0.00681 & -0.00046 & 0 & -0.00017 & 0 \\
-0.00046 & 0.00063 & -0.00017 & 0 & 0 \\
0 & -0.00017 & 0.00063 & -0.00046 & 0 \\
-0.00017 & 0 & -0.00046 & 0.00109 & -0.00046 \\
0 & 0 & 0 & -0.00046 & 0.00645
\end{pmatrix}
\]

$$\mathbf{-(A21)([N][A11'])^{-1}[A12]}^{-1}$$

\[
\begin{pmatrix}
-157.6146 & -137.0258 & -81.3150 & -60.7262 & -4.3309 \\
-81.3150 & -773.4514 & -2646.2911 & -1164.5144 & -83.0506 \\
-60.7262 & -358.5738 & -1164.5144 & -1462.3621 & -104.2925 \\
-4.3309 & -25.5727 & -83.0506 & -104.2925 & -162.4767
\end{pmatrix}
\]
\[
\begin{bmatrix}
8.092 \\ 108.890 \\ 294.330 \\ 108.890 \\ 108.890 \\ 8.345
\end{bmatrix}
\begin{bmatrix}
10 \end{bmatrix} =
\begin{bmatrix}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\begin{bmatrix}
-0.42255 \\ 0.04000 \\ 0.03000 \\ 0.10000 \\ -0.27939
\end{bmatrix} \\
\begin{bmatrix}
-0.1 \\ 0 \\ 0 \\ 0.2
\end{bmatrix} \\
\begin{bmatrix}
-0.32255 \\ 0.04000 \\ 0.03000 \\ 0.00000 \\ -0.47939
\end{bmatrix}
\end{bmatrix}
\]

Thus,

\[
[H_{i+1} = -([A21][N][A11])^{-1}[A12]^{-1}
\times([A21][Q] + [A10][H0]) - ([A21][Q] - [q])
\]

<table>
<thead>
<tr>
<th>Node</th>
<th>( H ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>44.997</td>
</tr>
<tr>
<td>C</td>
<td>-42.752</td>
</tr>
<tr>
<td>D</td>
<td>-44.472</td>
</tr>
<tr>
<td>E</td>
<td>20.225</td>
</tr>
<tr>
<td>F</td>
<td>75.758</td>
</tr>
</tbody>
</table>

To find \([Q_{i+1}\] by Equation 5.18 following a step-by-step analysis, the following matrices can be found:

\[
\begin{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
44.997 \\ -87.749 \\ -1.7206 \\ 64.698 \\ 55.533 \\ 75.758
\end{bmatrix} \\
\begin{bmatrix}
-25.003 \\ -87.749 \\ -1.7206 \\ 64.698 \\ 55.533 \\ -24.772
\end{bmatrix} \\
\begin{bmatrix}
-0.1545 \\ 0.04029 \\ 0.000029 \\ 0.02971 \\ 0.00255 \\ 0.0345
\end{bmatrix}
\end{bmatrix} \begin{bmatrix}
\begin{bmatrix}
-0.00421
\end{bmatrix}
\end{bmatrix}
\]
\[
\begin{pmatrix}
0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.5 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.5 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
0.05 \\
0.05 \\
0.05 \\
0.05 \\
0.05 \\
0.05 \\
0.05 \\
\end{pmatrix}
\]

Thus,
\[
[Q_{i+1}] = ([I] - (([N][A11]')^{-1})[A11])[[Q] - (([N][A11]')^{-1}) \times ([A12][H_{i+1}] + [A10][H_0])]
\]

<table>
<thead>
<tr>
<th>Pipe</th>
<th>Q (m³/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>0.20450</td>
</tr>
<tr>
<td>BC</td>
<td>0.09029</td>
</tr>
<tr>
<td>CD</td>
<td>0.05029</td>
</tr>
<tr>
<td>DE</td>
<td>0.02029</td>
</tr>
<tr>
<td>EF</td>
<td>0.02450</td>
</tr>
<tr>
<td>AF</td>
<td>0.01550</td>
</tr>
<tr>
<td>BE</td>
<td>0.05421</td>
</tr>
</tbody>
</table>
After only seven iterations the following are the results:

<table>
<thead>
<tr>
<th>Iteration</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node B</td>
<td>44.997</td>
<td>54.467</td>
<td>56.693</td>
<td>56.288</td>
<td>56.294</td>
<td>56.294</td>
<td>56.294</td>
<td>56.294</td>
</tr>
<tr>
<td>Node C</td>
<td>0</td>
<td>−42.752</td>
<td>22.703</td>
<td>32.899</td>
<td>31.637</td>
<td>31.649</td>
<td>31.650</td>
<td>31.650</td>
</tr>
<tr>
<td>Node D</td>
<td>0</td>
<td>−44.472</td>
<td>34.547</td>
<td>36.381</td>
<td>30.183</td>
<td>30.124</td>
<td>30.124</td>
<td>30.124</td>
</tr>
<tr>
<td>Node E</td>
<td>0</td>
<td>20.225</td>
<td>44.009</td>
<td>40.668</td>
<td>36.834</td>
<td>36.796</td>
<td>36.795</td>
<td>36.795</td>
</tr>
<tr>
<td>Node F</td>
<td>0</td>
<td>75.758</td>
<td>68.064</td>
<td>63.654</td>
<td>63.423</td>
<td>63.418</td>
<td>63.418</td>
<td>63.418</td>
</tr>
</tbody>
</table>

Note: Values are given in metres.

Pipe discharges:

<table>
<thead>
<tr>
<th>Pipe</th>
<th>Iter 0</th>
<th>Iter 1</th>
<th>Iter 2</th>
<th>Iter 3</th>
<th>Iter 4</th>
<th>Iter 5</th>
<th>Iter 6</th>
<th>Iter 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>0.10</td>
<td>0.20450</td>
<td>0.15155</td>
<td>0.13176</td>
<td>0.13156</td>
<td>0.13156</td>
<td>0.13156</td>
<td>0.13156</td>
</tr>
<tr>
<td>BC</td>
<td>0.10</td>
<td>0.09029</td>
<td>0.06123</td>
<td>0.04801</td>
<td>0.04654</td>
<td>0.04654</td>
<td>0.04654</td>
<td>0.04654</td>
</tr>
<tr>
<td>CD</td>
<td>0.10</td>
<td>0.05029</td>
<td>0.02123</td>
<td>0.00801</td>
<td>0.00654</td>
<td>0.00654</td>
<td>0.00654</td>
<td>0.00654</td>
</tr>
<tr>
<td>DE</td>
<td>0.10</td>
<td>0.02029</td>
<td>−0.00877</td>
<td>−0.02199</td>
<td>−0.02338</td>
<td>−0.02346</td>
<td>−0.02346</td>
<td>−0.02346</td>
</tr>
<tr>
<td>EF</td>
<td>0.10</td>
<td>0.02450</td>
<td>−0.02845</td>
<td>−0.04824</td>
<td>−0.04840</td>
<td>−0.04844</td>
<td>−0.04844</td>
<td>−0.04844</td>
</tr>
<tr>
<td>AF</td>
<td>0.10</td>
<td>0.01550</td>
<td>0.06845</td>
<td>0.08824</td>
<td>0.08840</td>
<td>0.08844</td>
<td>0.08844</td>
<td>0.08844</td>
</tr>
<tr>
<td>BE</td>
<td>0.10</td>
<td>0.05421</td>
<td>0.03032</td>
<td>0.02375</td>
<td>0.02498</td>
<td>0.02502</td>
<td>0.02502</td>
<td>0.02502</td>
</tr>
</tbody>
</table>

Note: Values are given in cubic metres per second.

Finally,

<table>
<thead>
<tr>
<th>Node</th>
<th>H (m)</th>
<th>Pressure head (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>56.294</td>
<td>31.294</td>
</tr>
<tr>
<td>C</td>
<td>31.650</td>
<td>11.650</td>
</tr>
<tr>
<td>D</td>
<td>30.124</td>
<td>10.124</td>
</tr>
<tr>
<td>E</td>
<td>36.795</td>
<td>14.795</td>
</tr>
<tr>
<td>F</td>
<td>63.418</td>
<td>38.418</td>
</tr>
</tbody>
</table>

| Pipe | Q (m³/s) | | Q (L/s) |
|------|----------|----------------|
| AB   | 0.13156  | 131.56         |
| BC   | 0.04654  | 46.54          |
| CD   | 0.00654  | 6.54           |
| DE   | −0.02346 | 23.46          |
| EF   | −0.04844 | 48.44          |
| AF   | 0.08844  | 88.44          |
| BE   | 0.02502  | 25.02          |