Toolbox 7
APPLIED HYDRAULICS

1. The volume flow rate of fluid \( V \) [m\(^3\)/s] is:

\[
V = w \cdot A \quad (7.1)
\]

The mass flow rate of fluid \( M \) [kg/s] is:

\[
M = w \cdot A \cdot \rho \quad (7.2)
\]

where:

- \( A \) = Cross-sectional area of the flow, [m\(^2\)]
- \( w \) = Mean fluid velocity at the same location, [m/s]
- \( \rho \) = Fluid density, [kg/m\(^3\)]

For circular pipes, Eqs (1) and (2) become:

\[
V = \frac{\pi \cdot d^2}{4} \cdot w \quad \text{and} \quad M = \frac{\pi \cdot d^2}{4} \cdot \rho \cdot w \quad (7.3)
\]

where \( d \) is internal diameter of pipe in [m].

2. The approximate values of the velocities used in calculations for industrial piping are shown in Table 7.1:

<table>
<thead>
<tr>
<th>Table 7.1: Velocities in Pipes</th>
<th>Velocity m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>WATER</td>
<td>--------------</td>
</tr>
<tr>
<td>1. Water flowing by gravity</td>
<td>0.1 – 0.5</td>
</tr>
<tr>
<td>2. Tap water (low noise)</td>
<td>0.5 – 0.7</td>
</tr>
<tr>
<td>3. Tap water (normal)</td>
<td>1.0 – 2.5</td>
</tr>
<tr>
<td>4. Cooling water</td>
<td>1.5 – 2.5</td>
</tr>
</tbody>
</table>

1 The thermo-physical properties of fluids and solids have to be taken from Toolbox 4, 5 and 6 or looked up in the publications listed in the chapter references.
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5.</td>
<td>Boiler feed water (suction)</td>
<td>0.5 – 1.0</td>
</tr>
<tr>
<td>6.</td>
<td>Boiler feed water (discharge)</td>
<td>1.5 – 2.5</td>
</tr>
<tr>
<td>7.</td>
<td>Condensate</td>
<td>1.0 – 2.0</td>
</tr>
<tr>
<td>8.</td>
<td>Circulation of hot water</td>
<td>1.0 – 3.0</td>
</tr>
<tr>
<td>9.</td>
<td>High pressure saturated steam</td>
<td>25 – 40</td>
</tr>
<tr>
<td>10.</td>
<td>Medium and low pressure saturated steam</td>
<td>30 – 40</td>
</tr>
<tr>
<td>11.</td>
<td>Saturated steam at peak load</td>
<td>up to 50</td>
</tr>
<tr>
<td>12.</td>
<td>Wet steam</td>
<td>up to 25</td>
</tr>
<tr>
<td>13.</td>
<td>Superheated steam</td>
<td>up to 50</td>
</tr>
<tr>
<td>14.</td>
<td>Suction line of pump</td>
<td>up to 0.5</td>
</tr>
<tr>
<td>15.</td>
<td>Suction line of pump (low pressure)</td>
<td>0.1 – 0.2</td>
</tr>
<tr>
<td>16.</td>
<td>Discharge line of booster pump</td>
<td>1.0 – 2.0</td>
</tr>
<tr>
<td>17.</td>
<td>Discharge line of burner pump</td>
<td>up to 1.0</td>
</tr>
<tr>
<td>18.</td>
<td>Combustion air duct</td>
<td>12 – 20</td>
</tr>
<tr>
<td>19.</td>
<td>Air inlet to boiler house</td>
<td>1.0 – 3.0</td>
</tr>
<tr>
<td>20.</td>
<td>Compressed air pipes</td>
<td>20 – 30</td>
</tr>
<tr>
<td>21.</td>
<td>Air with natural draft</td>
<td>2.0 – 4.0</td>
</tr>
<tr>
<td>22.</td>
<td>Ventilation ducts (hospitals, theaters, etc.)</td>
<td>1.8 – 4.0</td>
</tr>
<tr>
<td>23.</td>
<td>Ventilation ducts (office buildings, etc.)</td>
<td>2.0 – 4.5</td>
</tr>
<tr>
<td>24.</td>
<td>Air with natural draft</td>
<td>up to 4.0</td>
</tr>
<tr>
<td>25.</td>
<td>Stack at minimum load</td>
<td>up to 5.0</td>
</tr>
<tr>
<td>26.</td>
<td>Boiler with one-step burner (on-off)</td>
<td>5.0 – 8.0</td>
</tr>
<tr>
<td>27.</td>
<td>Boiler with two-step burner (high-low)</td>
<td>10 – 15</td>
</tr>
<tr>
<td>28.</td>
<td>Boiler with modulating burner (3:1)</td>
<td>15 – 25</td>
</tr>
</tbody>
</table>

To keep the surface free of soot, the velocities should always exceed 3.0 – 4.0 m/s

3. The form of the continuity equation for one-dimensional and incompressible steady fluid flow applied to any two sections gives:

\[ A_1 \cdot w_1 = A_2 \cdot w_2 = V \]  \hspace{1cm} (7.4)

and for variable density (compressible fluid), the continuity equation can be written as:

\[ \rho_1 \cdot A_1 \cdot w_1 = \rho_2 \cdot A_2 \cdot w_2 = M \]  \hspace{1cm} (7.5)

where subscripts 1 and 2 refer to the cross-sectional area of the pipe. The equations are valid for any rigid conduit as long as there is no addition or loss of liquid between the sections.

4. The general form of energy equation (Bernoulli equation) applicable to viscous (real) incompressible pipe or duct flow is:

\[ \frac{p_1}{g \cdot \rho} + z_1 + \frac{w_1^2}{2 \cdot g} = \frac{p_2}{g \cdot \rho} + z_2 + \frac{w_2^2}{2 \cdot g} + h_L \]  \hspace{1cm} (7.6)

where:

- \( \frac{p}{g \cdot \rho} \) = Pressure head, [m]; (static head)
- \( z \) = Elevation head above the same datum, [m]; (geometrical, position head)
- \( \frac{w}{2 \cdot g} \) = Velocity head, [m]; (dynamic head)
- \( h_L \) = Hydraulic loss head, [m]
The sum of the first three terms in Eq. (6) is defined as the total head, and the sum of pressure and elevation heads is referred to as the piezometric head.

The units are energy per unit weight of liquid [N·m/N] which reduce to [m].

The system of pipes and apparatuses throughout the fluid flows is called the network. The total hydraulic losses of the network are the sum of all of the particular pressure losses which occur in the network’s elements.

For example, in the network shown in Fig. 7.1, the following hydraulic losses can be recognized:
- Pressure loss for overcoming the friction resistance in the pipes;
- Pressure losses for overcoming the local resistance (four valves, four elbows, one sharp-edged inlet, and one sudden enlargement outlet of fluid);
- Expenditure of pressure for lifting the fluid from Tank 1 to Tank 2.

The sum of all of the above-mentioned losses has to be overcome by the pump.

5. Type of Flow

Laminar flow is also known as streamline, steady or viscous flow. The characteristics of laminar flow in a pipe are concentric layers of flow sheared slowly by the action of fluid viscosity. The fluid velocity is greatest at the center of the pipe and decreases sharply to zero at the pipe wall.

Turbulent flow is chaotic in nature and involves the cross mixing of molecules across the main stream without an observable pattern. The velocity distribution over the cross section is uniform, but if the wall is smooth there is always a layer near the pipe wall moving in the laminar flow.

Between laminar and turbulent flow lies a critical zone where flow losses cannot be predicted. The exact limits of laminar and turbulent flow can be only loosely defined. They depend somewhat on pipe roughness and the existence of bends and fittings.

Laminar flow in pipes is unusual. For example, for water flowing in a 0.3 m diameter pipe, the velocity will have to be below 0.02 m/s for laminar flow to exist. Comparing this value with typical velocity values in industrial pipelines (Table 7.1), one can conclude that the most practical pipe flow problems are in the turbulent region.

6. The calculation of friction loss in pipes and ducts depends on whether the flow is laminar or turbulent. The Reynolds number is a convenient parameter for predicting if the flow condition will be laminar or turbulent. It is defined as:

![Figure 7.1: Example of Hydraulic Network](image-url)
Re = \frac{\rho \cdot w \cdot d}{\mu} \quad (7.7)

Where:
- \( w \) = Mean fluid velocity, [m/s]
- \( d \) = Pipe diameter, [m]
- \( \rho \) = Fluid density, [kg/m³]
- \( \mu \) = Dynamic viscosity, [Pa s]

Flows through straight pipes are characterized by the following values of Reynolds number:
- Laminar flow: \( Re < 2,300 \)
- Transient region: \( 2,300 < Re < 10,000 \)
- Developed turbulent flow: \( Re > 10,000 \)

Flows through bent pipes (coils) have critical values of the Reynolds number that are higher than for straight pipes (Fig. 7.2).

![Figure 7.2: Critical Reynolds Number (Re_cr) in Bent Pipes (Coils) versus Ratio d/D](image)

7. For flows through pipes that are non-circular, the hydraulic or equivalent diameter is used in the expression for the Reynolds number. If there is a non-circular duct, the cross-sectional geometry can be converted to hydraulic (equivalent) diameters. The hydraulic (equivalent) diameter (\( d_h \)) is defined as:

\[
d_h = \frac{4 \cdot A}{P} \quad (7.8)
\]

Where:
- \( A \) = Cross-sectional area, [m²]
- \( P \) = Wetted perimeter, [m]

For example, the hydraulic diameters for a rectangular and annular cross-section (Fig. 7.3) are as follows:
- Rectangular:
  \[
d_h = \frac{4 \cdot A}{P} = \frac{2 \cdot W \cdot H}{W + H} \quad (7.9)
  \]
Annular:

\[ d_h = \frac{4 \cdot A}{P} = \frac{4 \cdot \pi \cdot \left( \frac{d_2^2 - d_1^2}{2} \right)}{\pi \cdot \left( \frac{d_2}{2} + d_1 \right)} = d_2 - d_1 \]  

(7.10)

Figure 7.3: Rectangular and Annular Cross-Section of the Pipe

If the flow rate is known, then the velocity can be computed using actual area and vice versa.

8. The relationship between the mean velocity \( w \) and the maximum axial velocity \( w_{\text{max}} \) in a pipe is:

- For laminar flow: \( w = 0.5 \cdot w_{\text{max}} \)
- For turbulent flow: the ratio \( w/w_{\text{max}} \) depends on the Reynolds number calculated for \( w_{\text{max}} \) (Fig. 7.4).

Figure 7.4: Ratio of \( w/w_{\text{max}} \) versus \( Re \)

It can be assumed for turbulent flow that \( w \) is approximately equal from \( 0.8 \cdot w_{\text{max}} \) to \( 0.9 \cdot w_{\text{max}} \), although the ratio \( w/w_{\text{max}} \) may exceed 0.9 at the high values of \( Re \).

The velocity profile for laminar and turbulent flows is presented in Fig. 7.5.
Figure 7.5: Velocity Profile in a Tube for Laminar (A) and Turbulent flow (B)

9. Friction loss ($h_f$) depends on:
   - pipe diameter; $d$ [m];
   - length, $L$ [m];
   - roughness of the pipe or duct, $e$ [m];
   - fluid density, $\rho$ [kg/m$^3$];
   - dynamic or kinetic viscosity, $\mu$ [Pa s] or $\nu$ [m$^2$/s];
   - fluid velocity, $w$ [m/s].

   Dimensional analysis is normally used to provide the functional relationship between the friction loss ($h_f$), pipe dimensions, fluid properties and flow parameters. The resulting equation (Darcy equation) is as follows:

$$h_f = \lambda \frac{L}{d} \left( \frac{w^2}{2g} \right)$$

(7.11)

The friction factor $\lambda$ is the measure of pipe roughness and it has to be evaluated experimentally for numerous pipes.

Pipe roughness can only be estimated and may increase over time due to corrosion and deposits.

I. ISOTHERMAL FLOW. Temperature of the fluid flowing through a pipe is constant.

Laminar flow ($Re < 2300$)

For $Re < 2300$, the flow in a pipe will be laminar and $\lambda$ is only the function of $Re$. It can be calculated by using the following equation:

$$\lambda = \frac{64}{Re}$$

(7.12)

For conduits that are not circular the friction factor can be calculated as:

$$\lambda = \frac{B}{Re}$$

(7.13)
The values of constant B for the various shapes of cross-section of conduits are as follows:

**Table 7.2: Values of Hydraulic Diameter \((d_h)\) and Coefficient \((B)\) for Friction Factor Calculation of Laminar Flow**

<table>
<thead>
<tr>
<th>Shape of cross-section</th>
<th>(d_h)</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle with diameter (d)</td>
<td>(d)</td>
<td>64</td>
</tr>
<tr>
<td>Square with side (a)</td>
<td>(a)</td>
<td>57</td>
</tr>
<tr>
<td>Equilateral triangle with side (a)</td>
<td>(0.58 \cdot a)</td>
<td>53</td>
</tr>
<tr>
<td>Ring of width (a)</td>
<td>(2 \cdot a)</td>
<td>96</td>
</tr>
<tr>
<td>Rectangle with side (a) and (b):</td>
<td>(</td>
<td>a/b</td>
</tr>
<tr>
<td></td>
<td>(a/b = 0.1)</td>
<td>(1.81 \cdot a)</td>
</tr>
<tr>
<td></td>
<td>(a/b = 0.25)</td>
<td>(1.6 \cdot a)</td>
</tr>
<tr>
<td></td>
<td>(a/b = 0.5)</td>
<td>(1.3 \cdot a)</td>
</tr>
<tr>
<td>Ellipse ((a - \text{minor semi-axis}; b - \text{major semi-axis}))</td>
<td>(</td>
<td>a/b</td>
</tr>
<tr>
<td></td>
<td>(</td>
<td>a/b</td>
</tr>
<tr>
<td></td>
<td>(</td>
<td>a/b</td>
</tr>
</tbody>
</table>

For the isothermal laminar flow of fluids through pipes, the loss of pressure due to friction can also be calculated according to the Hagen-Poiseuille law:

\[
\Delta p_f = g \cdot \rho \cdot h_f = 32 \cdot \frac{w \cdot \mu \cdot L}{d^2}
\]  

(7.14)

**Turbulent flow \((Re > 2300)\)**

- Hydraulically smooth pipes made of glass, copper and lead:

\[
\lambda = \frac{0.316}{Re^{0.25}}
\]  

(7.15)

The equation holds for \(Re < 100,000\).

- Hydraulically rough pipes made of steel and iron.

The dimensionless geometrical characteristic of hydraulically rough pipes is relative roughness, i.e. the ratio of the average height of roughness projections on the inner surface of pipe \((e)\) and its hydraulic (equivalent) diameter \((d_h)\):

\[
\varepsilon = \frac{e}{d_h}
\]  

(7.16)

The approximate average values of the roughness of pipes \(e [\text{mm}]\) are presented in **Table 7.3**.

**Table 7.3: Average Values of Pipe Wall Roughness**

<table>
<thead>
<tr>
<th>PIPE</th>
<th>(e [\text{mm}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Steel pipes: seamless and welded, with insignificant corrosion</td>
<td>0.2</td>
</tr>
<tr>
<td>2. Old and corroded steel pipes</td>
<td>0.67 and more</td>
</tr>
<tr>
<td>3. Pipes of roofing steel coated with varnish or drying oil</td>
<td>0.125</td>
</tr>
<tr>
<td>4. Iron water main pipes that have been in service</td>
<td>1.4</td>
</tr>
<tr>
<td>5. Aluminum technical smooth pipes and tubes</td>
<td>0.015 – 0.06</td>
</tr>
<tr>
<td>6. Finished seamless pipes of brass, copper, lead and glass</td>
<td>0.0015 – 0.01</td>
</tr>
</tbody>
</table>
7. Concrete pipes in medium condition and pipes for saturated steam 0.2
8. Steam pipes in operation periodically 0.5
9. Pipes for compressed air installation 0.8
10. Condensate pipes in operation periodically 1.0

For a turbulent flow \(000 \leq \text{Re} \leq 10^8\) and \(0^{-6} \leq \varepsilon \leq 10^{-2}\), \(\lambda\) can be calculated by using the following formula:

\[
\lambda = \frac{1.325}{\left[ \ln \left( \frac{\varepsilon}{3.7} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2}
\] (7.17)

**These values should be considered only as guides and not used if more exact values can be obtained from pipe manufacturers.**

**II. NON-ISOTHERMAL FLOW** occurs when the liquid flowing through a pipe becomes heated or cooled (the temperature of pipe wall differs from that of the fluid). In that case, the right-hand side of Eqs (7.12), (7.13) and (7.15) should be multiplied by dimensionless correction factor \(x\):

- For laminar flow:

\[
x = \left( \frac{\text{Pr}_w}{\text{Pr}_f} \right)^{1/3} \left[ 1 + 0.22 \times \left( \frac{\text{Gr}_f \times \text{Pr}_f}{\text{Re}_f} \right)^{0.15} \right]
\] (7.18)

- For turbulent flow in hydraulically smooth pipes:

\[
x = \left( \frac{\text{Pr}_w}{\text{Pr}_f} \right)^{1/3}
\] (7.19)

where:
- \(\text{Re}_f\) = Reynolds number for average fluid temperature
- \(\text{Pr}_f\) = Prandtl number for average fluid temperature (see Toolbox III-9)
- \(\text{Gr}_f\) = Grashof number for average fluid temperature (see Toolbox III-9)
- \(\text{Pr}_w\) = Prandtl number of fluid calculated for the wall temperature

By cooling the fluid \((t_w < t_f)\), the correction factor \(x\) is greater than unity and less than unity when the fluid is heated \((t_w > t_f)\).

As for gases, the value of the Prandtl number remains practically constant with the change of fluid temperature, the correction factor \(x\) for gases will be equal to unity.

10. The friction loss in bent pipes (coils) is greater than in straight pipes. The pressure loss in a coil can be calculated by calculating the pressure loss in a straight pipe of the same diameter and length as the coil and by introducing the correction factor, i.e.:

\[
\Delta p_{\text{coil}} = \psi \cdot \Delta p_{\text{straight}}
\] (7.20)

The dimensionless correction factor \((\psi > 1)\) can be calculated as follows:

\[
\psi = 1 + 3.54 \frac{d}{D}
\] (7.21)
Where \( d \) is internal diameter of the pipe and \( D \) is diameter of the turn of the coil (see Fig. 7.2).

### 11. Local Losses

Flow through valves, orifices, elbows, transitions, etc. causes flow separation which results in the generation and dissipation of turbulent eddies. For short systems containing many bends, valves, tees, etc. local losses can exceed friction losses.

The loss of pressure in each local obstacle is the sum of two losses: (1) the loss due to friction and (2) the additional loss due to the change in the direction or in the cross-sectional area of the flow. Since the entire length of a pipe is taken into account when calculating the pipe friction resistance (\( h_f \) or \( \Delta p_f \)) (including the local resistances), then (\( h_i \) or \( \Delta p_i \)) is the sum of these additional pressure losses.

The pressure loss due to the resistance of local obstacles is calculated by the formula:

\[
\Delta p_i = g \cdot \rho \cdot h_i = \sum \xi \cdot \frac{w^2 \cdot \rho}{2}
\]  

(7.22)

where:

\( \xi \) = dimensionless local resistance coefficient, [-]

The values of dimensionless local resistance coefficients are given in Tables 7.4 and 7.5.

The loss coefficient \( \xi \) is analogous to \( \lambda \cdot \frac{L}{d} \) in the Eq. (11).

When available, the manufacturers’ data should be used, particularly for valves, because of the wide variety of designs for the same type.

**Table 7.4: Dimensionless Local Resistance Coefficients for Bends and Valves**

<table>
<thead>
<tr>
<th>Item</th>
<th>Local resistance coefficient, ( \xi ) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bends</strong></td>
<td></td>
</tr>
<tr>
<td>Short radius, ( r/d = 1.0 )</td>
<td></td>
</tr>
<tr>
<td>90°</td>
<td>0.24</td>
</tr>
<tr>
<td>45°</td>
<td>0.10</td>
</tr>
<tr>
<td>35°</td>
<td>0.06</td>
</tr>
<tr>
<td>Long radius, ( r/d = 1.5 )</td>
<td></td>
</tr>
<tr>
<td>90°</td>
<td>0.19</td>
</tr>
<tr>
<td>45°</td>
<td>0.09</td>
</tr>
<tr>
<td>35°</td>
<td>0.06</td>
</tr>
<tr>
<td><strong>Valves</strong></td>
<td></td>
</tr>
<tr>
<td>Check valve (full-open)</td>
<td>0.8</td>
</tr>
<tr>
<td>Swing check (full-open)</td>
<td>1.0</td>
</tr>
<tr>
<td>Tilt disk (full-open)</td>
<td>4.6</td>
</tr>
<tr>
<td>Lift (full-open)</td>
<td>1.2</td>
</tr>
<tr>
<td>Double door (full-open)</td>
<td>1.32</td>
</tr>
<tr>
<td>Fully-open gate</td>
<td>0.15</td>
</tr>
<tr>
<td>Fully-open butterfly</td>
<td>0.2</td>
</tr>
<tr>
<td>Fully-open globe</td>
<td>4.0</td>
</tr>
</tbody>
</table>

**Table 7.5: Dimensionless Local Resistance Coefficients for Pipe Inlet, Reduction and Enlargement**

<table>
<thead>
<tr>
<th>Item</th>
<th>Local resistance coefficient, ( \xi ) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet to pipe</td>
<td>( \xi = 0.5 ) (sharp edges)</td>
</tr>
<tr>
<td>Outlet from pipe</td>
<td>( \xi = 0.2 ) (rounded edges)</td>
</tr>
<tr>
<td></td>
<td>( \xi = 1.0 )</td>
</tr>
</tbody>
</table>
Inward projecting pipe

\( \zeta = 1.0 \)

Rounded inlet

\( \zeta = 0.05 \)

Sudden contraction

<table>
<thead>
<tr>
<th>D/d</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta )</td>
<td>0.28</td>
<td>0.36</td>
<td>0.40</td>
<td>0.42</td>
<td>0.44</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Sudden enlargement

\[ \zeta = 1 - \left( \frac{d}{D} \right)^2 \]

Gradual reduction

\( \zeta = 0.5 \)

Gradual enlargement

\[ \zeta = \xi' \left[ 1 - \left( \frac{d}{D} \right)^2 \right] \]

\[ \frac{(D-d)/(2L)}{0.05} \]
\[ \begin{array}{cccccc}
0.10 & 0.20 & 0.30 & 0.40 & 0.50 & 0.80 \\
0.14 & 0.20 & 0.47 & 0.76 & 0.95 & 1.05 & 1.10 \\
\end{array} \]

Exit loss (sharp edged, projecting, rounded)

\( \zeta = 1.0 \)

12. The hydraulic resistance of the bank tubes with the lateral flow of the stream around them is as follows:

- Square pitch (Fig. 7.6A)

\[ \text{Eu} = b \cdot (4.5 + 4.5 \cdot m) \left( \frac{s_1}{d} \right)^{-3.23} \cdot \text{Re}^{-0.26} \]  \hspace{1cm} (7.23)

- Triangular pitch (Fig. 7.6B)

\[ \text{Eu} = b \cdot (0 + 3.3 \cdot m) \cdot \text{Re}^{-0.28} \hspace{1cm} \text{for} \hspace{0.5cm} \frac{s_1}{d} < \frac{s_2}{d} \]  \hspace{1cm} (7.24)

\[ \text{Eu} = b \cdot (7 + 1.7 \cdot m) \cdot \text{Re}^{-0.28} \hspace{1cm} \text{for} \hspace{0.5cm} \frac{s_1}{d} > \frac{s_2}{d} \]  \hspace{1cm} (7.25)

where:

- \( b \) = Correction factor depending on the angle of attack \( \phi \) \([-\] as is defined in Table 7.6,
- \( m \) = Number of tube rows in the bank in the direction of the flow, \([-\]
- \( d \) = External diameter of tube, \([\text{m}]\)
- \( s_1 \) = Lateral tube pitch, \([\text{m}]\)
- \( s_2 \) = Longitudinal tube pitch, \([\text{m}]\)
- \( \text{Eu} \) = Euler number (see Toolbox III-9)
- \( \text{Re} \) = Reynolds number
The fluid velocity is calculated for the narrowest section of the bank A-A (Fig. 7.3). The values of the fluid properties are taken for the mean temperature of the flow. The Reynolds number is calculated according to the external diameter of the pipe.

13. **Hydraulic resistance of shell-and-tube heat exchangers**

For the tube side of the heat exchanger and for the shell side (inter-tubular space) without lateral baffles (Fig. 7A), the pressure loss is as follows:

$$\Delta p = \lambda \cdot \frac{n \cdot L}{d_h} \cdot \frac{\rho \cdot w^2}{2} + \sum \xi \cdot \frac{\rho \cdot w^2}{2}$$

(7.26)

Where:
- $L$ = Length of one pass, [m]
- $N$ = Number of passes, [-]

The remaining symbols are the same as in the preceding equations.

**Tube side**
- Inlet or outlet header 1.5
- 180° bend between passes or sections 2.5
- Tube inlet or outlet 1.0

**Shell side**
- Shell side inlet and outlet 1.5
- 180° bend through baffle in shell side 1.5
- 90° bend shell side 1.0

---

**Table 7.6: Correction Factor \(b\) versus Angle of Attack**

<table>
<thead>
<tr>
<th>Angle [°]</th>
<th>90</th>
<th>80</th>
<th>70</th>
<th>60</th>
<th>50</th>
<th>40</th>
<th>30</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td>1.0</td>
<td>1.0</td>
<td>0.95</td>
<td>0.83</td>
<td>0.69</td>
<td>0.53</td>
<td>0.38</td>
<td>0.15</td>
</tr>
</tbody>
</table>

---

**Figure 7.6: Tube Bank Arrangements**
When the shell side has baffles (Fig. 7.7B), the hydraulic resistance in it is determined by using Eqs (7.20) – (7.22).

The velocities of fluid in pipe connections are usually close to those in the tubes or on the shell side. If the velocity is greater, then the pressure losses at the inlet and outlet of the heat exchanger are calculated according to the velocities in the connections.

14. **Hydraulic resistance of scrubber pickings**

The resistance of dry layer packing with height $H$ is:

$$\Delta p_{\text{dry}} = \frac{\lambda}{d_h} \cdot \frac{H \cdot \rho \cdot w_g^2}{2}$$  

(7.27)

Where:
- $\lambda$ = Dimensionless resistance coefficient when gas passes through the layer of packing, [-]
- $H$ = Height of packing layer, [m]
- $d_h$ = Hydraulic diameter, [m]
- $w_g$ = Actual velocity of the gas through the void of the packing, [m/s]
- $\rho$ = Gas density, [kg/m$^3$]

The hydraulic diameter can be expressed through the characteristics of the packing as follows:

$$d_h = \frac{4 \cdot \varepsilon}{\alpha}$$  

(7.28)

Where:
- $\varepsilon$ = Ratio of the volume of voids [m$^3$] and the total volume of the apparatus in which the packing is set down, [m$^3$]; [m$^3$/m$^3$]
- $\alpha$ = Ratio of the total wetted area of package [m$^2$] and packing volume [m$^3$]; [m$^2$/m$^3$]

The actual velocity of gas is $w_g = w / \varepsilon$.

For the randomly dumped packing of rings, the following relations can be used:

$$\lambda = \frac{140}{\text{Re}_g} \quad \text{at} \quad \text{Re}_g < 40$$  

(7.29)
\[ \lambda = \frac{16}{\text{Re}_g} \quad \text{at} \quad \text{Re}_g > 40 \]  \hspace{1cm} (7.30)

where \( \text{Re}_g = \frac{w_g \times d_h \times \rho}{\mu} = \frac{4 \times w \times \rho}{\alpha \times \mu} \).

The calculation of the hydraulic resistance of irrigated packing is more complicated.

15. **Hydraulic resistance of plate column apparatus**

The resistance of a bubble plate \( \Delta p \) can be calculated as follows (Fig. 7.8):

\[ \Delta p = \Delta p_{\text{dry}} + \Delta p_\sigma + \Delta p_{\text{fl}} \]  \hspace{1cm} (7.31)

Where:
- \( \Delta p_{\text{dry}} \) = Resistance of dry plate, [Pa]
- \( \Delta p_\sigma \) = Resistance due to the forces of surface tension, [Pa]
- \( \Delta p_{\text{fl}} \) = Resistance of the froth layer of plate, [Pa]

The resistance of a dry plate is as follows:

\[ \Delta p_{\text{dry}} = \xi \frac{w_h^2 \cdot \rho_g}{2} \]  \hspace{1cm} (7.32)

where:
- \( w_h \) = Velocity of gas in the slots of the cap or in the holes of plate, [m/s]
- \( \rho_g \) = Density of the gas, [kg/m³]
- \( \xi \) = Resistance coefficient
  - Bubble-cap plates: 4.5–5.0
  - Perforated (sieve) plates:
    - 7–10% free section of holes: 1.82
    - 11–25% free section of holes: 1.45
  - Grid trays: 1.4–1.5

Figure 7.8: Scheme of Bubble-Cap Plate (A) and Sieve Plate (B)

The surface tension resistance is as follows:

\[ \Delta p_\sigma = \frac{\sigma}{d_h} \]  \hspace{1cm} (7.33)
where:
\[
\begin{align*}
\sigma &= \text{Surface tension, [N/m]} \\
d_h &= \text{Hydraulic diameter of the hole, [m]}
\end{align*}
\]

- Bubble-cap plates, \(d_h = \frac{4 \cdot A}{P}\); \(A\) is area of the clear cross-section of the slot; \(P\) is perimeter of the slot
- Sieve and perforated plates, \(d_h\) is equal to hole diameter
- Grid trays, \(d_h\) is equal to double slot width

The froth layer resistance could be determined as:

**Bubble-cap plate (Fig. 7.8A)**

\[
\Delta p_n = 1.3 \cdot k \cdot \rho_l \cdot g \cdot \left( L + \frac{e}{2} + \Delta h \right) \quad (7.34)
\]

Where:
\[
\begin{align*}
k &= \text{Relative density of the froth layer (foam). If there is no more precise data, it can be assumed that } k \approx 0.5, [-] \\
\rho_l &= \text{Liquid density, [kg/m}^3]\] \\
L &= \text{Distance from the top edge of the slots to the weir crest, [m]} \\
e &= \text{Height of the slot, [m]} \\
\Delta h &= \text{Height of the liquid level above the weir crest, [m]} \\
g &= \text{Gravity acceleration, [m/s}^2]\]

**Sieve plate (Fig. 8B)**

\[
\Delta p_n = 1.3 \cdot k \cdot \rho_l \cdot g \cdot \left( h_w + \Delta h \right) \quad (7.35)
\]

where:
\[
\begin{align*}
h_w &= \text{Height of the weir crest, [m]}
\end{align*}
\]

The value \(\Delta h\) is determined by the formula for outflow over the weir:

\[
\Delta h = \left( \frac{V_l}{1.85 \cdot P \cdot k} \right)^{2/3} \quad (7.36)
\]

where:
\[
\begin{align*}
V_l &= \text{Volumetric flow rate of liquid, [m}^3]/[s]\] \\
P &= \text{Width of the weir, [m]} \\
k &= 0.5 \text{ Relative density of the froth layer (foam), [-]}
\]
16. Examples

Example 1: The water flows throughout the coil (Fig. 9) with a velocity of 1 m/s. The coil is made of used steel pipe with a diameter of 43 × 2.5 mm. The diameter of the coil turn is D = 1 m. The number of coil turns is 10. The average water temperature is 25 °C.

Pressure loss has to be calculated.

\[
\text{Solution:}
\]

The dynamic viscosity of water at 25 °C is \(0.8963 \times 10^{-3}\) [Pa s] and density is 997.0 [kg/m³]. The pressure of water is 1.013 bar. The viscosity of water (and other liquids) depends slightly on pressure (for < 40 bar).

The Reynolds number is:

\[
\text{Re} = \frac{\omega \cdot d \cdot D}{\mu} = \frac{1.0 \cdot (43 - 2 \cdot 2.5) \cdot 10^{-3} \cdot 997.0}{0.8963 \cdot 10^{-3}} = 42,270
\]

(7.37)

For seamless steel pipe with insignificant corrosion it is \(e = 0.2\) [mm] (Table 3). The relative roughness is:

\[
e = \frac{e}{d} = \frac{0.2}{38} = 0.00526
\]

(7.38)

According to Eq. (17) at \(\text{Re} = 42,270\) and \(e = 0.00526\), the dimensionless resistance coefficient is as follows:

\[
\lambda = \frac{1.325}{\left[\ln \left(\frac{\varepsilon + 5.74 \cdot \text{Re}^{0.9}}{3.7}\right)\right]^2} = \frac{1.325}{\left[\ln \left(\frac{0.00526 + 5.74}{42,270^{0.9}}\right)\right]^2} = 0.0333
\]

(7.39)

The length of the coil is approximately:

\[
L = \pi \cdot D \cdot n = \pi \cdot 1.0 \cdot 10 = 31.4 \text{ [m]}
\]

(7.40)

The friction pressure loss in a straight pipe is:

\[
\Delta p_{\text{straight}} = \lambda \cdot \frac{L}{d} \cdot \rho \cdot \frac{w^2}{2} = 0.0333 \cdot \frac{31.4}{0.038} \cdot \frac{997 \cdot 1.0^2}{2} = 13,718 \text{ [Pa]}
\]

(7.41)

The dimensionless correction factor is (Eq. 21):

\[
\psi = 1 + 3.54 \cdot \frac{d}{D} = 1 + 3.54 \cdot \frac{0.038}{1.0} = 1.1345
\]

(7.42)
Finally, the friction pressure loss in the coil is as follows:

\[ \Delta P_{\text{coil}} = \psi \cdot \Delta P_{\text{straight}} = 1.1345 \cdot 13,718 = 15,563 \text{ [Pa]} \]  

(7.43)

**Example 2:** Ethylene-glycol flows down from the tank into the apparatus along a pipe with a diameter of 29 × 2 mm (Fig. 10). The difference between the ethylene-glycol levels in the tank and in the apparatus is 10 m. The total length of the pipeline is 110 m. The flow rate of the solution has to be calculated. The level of the solution in the tank is constant. The temperature of the ethylene-glycol is 20 °C.

The mass flow rate of ethylene-glycol has to be determined.

**Solution:**

The Bernoulli equation (Eq. 7.6) for the steady liquid flow written for the cross-section at the liquid level in the tank (1-1) and for the cross-section at the liquid level in the apparatus (2–2), is as follows:

\[ \frac{p_1}{g \cdot \rho} + z_1 + \frac{w_1^2}{2 \cdot g} = \frac{p_2}{g \cdot \rho} + z_2 + \frac{w_2^2}{2 \cdot g} + h_L \]  

(7.44)

Since \( p_1 = p_2 \) and \( w_1 = w_2 \approx 0 \), then:

\[ H = z_1 - z_2 = h_L \]  

(7.45)

or the head \( H \) is used to overcome all hydraulic resistance of the pipeline.

As the velocity of ethylene-glycol is constant in the pipeline, the total hydraulic resistance is as follows:

\[ h_L = \frac{w^2}{2 \cdot g} \left( \frac{\lambda \cdot L}{d} + \xi_{\text{inlet}} + \xi_{\text{elbow}} + 2 \cdot \xi_{\text{valve}} + \xi_{\text{outlet}} \right) \]  

(7.46)

The local resistance coefficients are (Tables 4 and 5):

1. Liquid inlet to the pipeline: \( \xi = 0.5 \)
2. Check valve (full-open): \( \xi = 0.8 \cdot 2 = 1.6 \)
3. Elbow (90°): \( \xi = 0.24 \)
4. Pipe outlet: \( \xi = 1.0 \)

**Total:** \( \xi_{\text{total}} = 3.34 \)

For the given temperature of ethylene-glycol (20 °C), the density and dynamic viscosity are as follows:

\[ \rho = 1114.1 \text{ [kg/m}^3]\quad \mu = 0.02014 \text{ [Pa s]} \]  

(7.47)
Assuming that the flow is laminar and introducing Eq. (7.12) and numerical values into Eq. (7.46), it is:

\[
10 = \frac{w^2}{2 \cdot 9.81} \cdot \left( \frac{64}{w \cdot 0.025 \cdot 1114.1} + \frac{110}{0.025} + 3.34 \right) \tag{7.48}
\]

or

\[
w^2 + 60.9487 \cdot w - 58.7425 = 0 \tag{7.49}
\]

The real solution of this quadratic equation is \( w = 0.949 \) m/s. The Reynolds number is \( \text{Re} = 1313 \) and, as it is less than 2300, the flow is laminar. That means that the assumption introduced for the calculation of the friction factor is proven.

The mass flow rate of ethylene-glycol is as follows:

\[
M = \frac{d^2 \cdot \pi}{4} \cdot w \cdot \rho = 0.519 \text{ [kg/s]} \tag{7.50}
\]

**Example 3:** The pump feeds 25 tons per hour of water at a temperature of 25 °C from an open tank into a reactor where a gage pressure of 0.2 bar is maintained (Fig. 11). The pipeline is made of steel pipes with an outside diameter of 89 mm and tube wall thickness of 4 mm. The inside surface of the tube has insignificant corrosion. The length of the entire pipeline is 45 m. The pipeline is provided with three gate valves and two pipes bends at the angle of 90° with a bending radius of 80 mm. The water is lifted to the height of 15 m. The pump head has to be calculated.

**Figure 7.11**

**Solution:**

The Bernoulli equation (Eq. 6) for the steady flow of liquid written for the cross-section at the level of the liquid in tank 1-1 and for the cross-section at the liquid level in reactor 2-2 is as follows:

\[
\frac{P_1}{g \cdot \rho} + z_1 + \frac{w_1^2}{2 \cdot g} + H_{\text{pump}} = \frac{P_2}{g \cdot \rho} + z_2 + \frac{w_2^2}{2 \cdot g} + h_L \tag{7.51}
\]

Since \( w_1 = w_2 \approx 0 \), then:

\[
H_{\text{pump}} = \frac{P_2 - P_1}{g \cdot \rho} + z_2 - z_1 + h_L \tag{7.52}
\]

As the temperature of water is 20 °C and the average pressure is approximately 1.10 bar, the density is 998.2 [kg/m³] and the dynamic viscosity is 0.000985 [Pa s].
The volume flow rate in the pipe is as follows:

\[
V = \frac{M}{\rho} = \frac{25,000}{3,600} = 0.0069570 \text{ [m}^3/\text{s]} \tag{7.53}
\]

The velocity of water in the pipe is:

\[
w = \frac{V}{\pi \cdot d^2} = \frac{0.0069570}{\pi \cdot 0.081^2} = 1.35 \text{ [m/s]} \tag{7.54}
\]

The Reynolds number is:

\[
Re = \frac{w \cdot d \cdot \rho}{\mu} = \frac{1.35 \cdot 0.081 \cdot 998.2}{0.000985} = 110,815 \tag{7.55}
\]

The local resistance coefficients are (Table 7.4):
1. Liquid inlet to the pipeline: \( \xi = 0.5 \)
2. Check valve (full-open): \( \xi = 0.8 \cdot 3 = 2.4 \)
3. Elbow (90°): \( \xi = 0.24 \cdot 2 = 0.48 \)
4. Pipe outlet: \( \xi = 1.0 \)

**Total:** \( \xi_{\text{Total}} = 4.38 \)

According to Table 7.3, the roughness of steel pipes with insignificant corrosion is \( \varepsilon = 0.2 \text{ mm} \). The relative roughness is \( \varepsilon = 0.2/81 = 0.002469 \). The flow is turbulent and Eq. (7.17) can be used for the friction factor calculation as follows:

\[
\lambda = \frac{1.325}{\left[ \ln \left( \frac{\varepsilon + 5.74}{3.7} \right) + Re^{0.5} \right]^2} = \frac{1.325}{\left[ \ln \left( \frac{0.002469 + 5.74}{3.7} \right) + 110815^{0.5} \right]^2} = 0.02635 \tag{7.56}
\]

The total hydraulic resistance of the pipeline is:

\[
h_L = \frac{w^2}{2 \cdot g} \cdot \left( \lambda \cdot \frac{L}{d} + \xi_{\text{Total}} \right) = 1.35^2 \cdot \frac{1}{2 \cdot 9.81} \left( 0.02635 \cdot \frac{45}{0.081} + 4.38 \right) = 1.767 \text{ [m]} \tag{7.57}
\]

The pump head is:

\[
H_{\text{Pump}} = \frac{P_2 - P_1}{g \cdot \rho} + z_2 - z_1 + h_L = \frac{0.2 \cdot 10^5}{9.81 \cdot 998.2} + 15 + 1.767 = 18.8 \text{ [m]} \tag{7.58}
\]

**Example 4:** Aniline flows on the shell side of a shell-and-tube heat exchanger (Fig. 7.5A) parallel to the axes of the tubes. The velocity of the aniline is 0.5 m/s and it is cooled by water from 100 to 40 °C. The exchanger has 19 steel tubes with a diameter of 26 × 2.5 mm and a length of 2.7 m. The internal diameter of the shell is 200 mm. The roughness of the tubes is 0.2 mm.

The inlet water temperature is 20 °C.

The aniline pressure loss for overcoming the friction resistance has to be calculated.
Solution:

- **Shell side (isothermal flow):**

  The average temperature of the aniline is \((100 + 40)/2 = 70 \, ^\circ C\). The hydraulic diameter of the shell side of the heat exchanger is:

  \[
  d_h = \frac{4 \cdot A}{P} = \frac{D^2 - n \cdot d^2}{D + n \cdot d} = \frac{200^2 - 19.26^2}{200 + 19.26} = 39.12 \, \text{mm}
  \]  
  (7.59)

  The density and dynamic viscosity of aniline for a temperature of 70 \(^\circ\)C are:

  \[
  \rho = 980.2 \, [\text{kg/m}^3] \quad \mu = 1.4 \times 10^{-3} \, [\text{Pa s}]
  \]  
  (7.60)

  The Reynolds number is:

  \[
  \text{Re} = \frac{w \cdot d \cdot \rho}{\mu} = \frac{0.5 \cdot 0.03912 \cdot 980.2}{1.4 \cdot 10^{-3}} = 13,695
  \]  
  (7.61)

  The relative roughness is \(\varepsilon = e/d_h = 0.2/39.12 = 0.00511\), and, by using Eq. (17), the friction factor is as follows:

  \[
  \lambda = \frac{1.325}{\ln \left( \frac{0.00511 + 5.74}{3.7} \cdot \frac{13,695^{0.8}}{13,695^{0.5}} \right)^2} = 0.03675
  \]  
  (7.62)

  The pressure loss for overcoming the friction losses is:

  \[
  \Delta P_{\text{isothermal}} = \lambda \cdot \frac{L}{d_h} \cdot \frac{w^2 \cdot \rho}{2} = 0.03675 \cdot \frac{2.7 \cdot 980.2}{0.03912 \cdot 2} = 310.8 \, [\text{Pa}]
  \]  
  (7.63)

- **Shell side (non-isothermal flow):**

  If the temperature of the wall is different than the fluid temperatures, then the pressure loss has to be corrected because the flow becomes non-isothermal. In the analyzed case, there is no data for wall temperature. For the calculation of the wall temperature, the conditions of heat transfer for both fluids have to be known. The correction factor in the case of non-isothermal flow is defined by Eq. (7.19). As the Prandtl number of liquids depends strongly on temperature, the correction factor will also depend on temperature. The possible average wall temperature can in this case be from 25 \(^\circ\)C to approximately 45 \(^\circ\)C, depending on the water mass flow rate and the conditions of flow inside the tubes. The approximate temperature profile of aniline, water and the wall versus length of the heat exchanger is shown in Fig. 7.12.
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Figure 7.12: Temperature Profile of Fluids and the Wall versus Heat Exchanger Length

The results of the correction factor calculation for the non-isothermal flow of aniline are given in Table 7.7.

Table 7.7: Correction Factor for Different Average Wall Temperature

<table>
<thead>
<tr>
<th>Temperature [°C]</th>
<th>Density [kg/m³]</th>
<th>Dynamic Viscosity [Pa s]</th>
<th>Thermal Conductivity [W/(m K)]</th>
<th>Specific Heat [J/(kg K)]</th>
<th>Prandtl Number [-]</th>
<th>Correction Factor, [-]</th>
<th>Pressure Loss ∆p [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1022.0</td>
<td>0.0044</td>
<td>0.182</td>
<td>2070</td>
<td>50.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>1018.2</td>
<td>0.0040</td>
<td>0.181</td>
<td>2080</td>
<td>45.6</td>
<td>1.366</td>
<td>424.4</td>
</tr>
<tr>
<td>30</td>
<td>1014.3</td>
<td>0.0035</td>
<td>0.180</td>
<td>2090</td>
<td>41.0</td>
<td>1.318</td>
<td>409.7</td>
</tr>
<tr>
<td>35</td>
<td>1010.5</td>
<td>0.0031</td>
<td>0.180</td>
<td>2110</td>
<td>36.4</td>
<td>1.266</td>
<td>393.6</td>
</tr>
<tr>
<td>40</td>
<td>1006.7</td>
<td>0.0027</td>
<td>0.179</td>
<td>2120</td>
<td>31.6</td>
<td>1.208</td>
<td>375.6</td>
</tr>
<tr>
<td>45</td>
<td>1002.8</td>
<td>0.0022</td>
<td>0.178</td>
<td>2130</td>
<td>26.7</td>
<td>1.143</td>
<td>355.2</td>
</tr>
<tr>
<td>50</td>
<td>999.0</td>
<td>0.0018</td>
<td>0.177</td>
<td>2140</td>
<td>21.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>980.2</td>
<td>0.0014</td>
<td>0.173</td>
<td>2210</td>
<td>17.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>952.0</td>
<td>0.0008</td>
<td>0.167</td>
<td>2320</td>
<td>11.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is evident from Table 7.7 that the pressure loss in the case of non-isothermal flow increases with the decrease in the average wall temperature.

Example 5: The optimum diameter of a pipeline for the transportation of 6000 m³/h (at 0 °C and 1.013 bar) of methane over a distance of 4 km. The overall efficiency of the blower and its electrical motor is 0.5. The cost of electrical energy is 0.0625 $US per kWh. The cost of depreciation of the pipeline is 3.75 $US and pipeline maintenance is 2.9 $US per meter of length and per meter of diameter.

The pipe resistance coefficient is $\lambda = 0.03$ and the local resistance is estimated to be approximately 10 % of friction losses. The average temperature of gas is 25 °C.

The gas content (by volume) is: Methane (CH₄) 74.28 %; Ethane (C₂H₆) 5.71 %; Propane (C₃H₈) 1.45 %; Butanes Plus (C₄H₁₀) 0.9 %; Nitrogen (N₂) 2.27 %; Carbon Dioxide (CO₂) 15.38 %.

Solution:

Following the condition in this example that local losses are 10 % of the friction loss, the total pipeline's pressure loss is:
\[ \Delta p_{\text{Total}} = \Delta p_I + \Delta p_t = 1.1 \cdot \Delta p_I \]  

(7.64)

The density of gas at a given temperature and pressure is 0.9896 kg/m\(^3\). The mass flow rate of gas is now:

\[ M = \rho \cdot V = 0.9896 \cdot \frac{6000}{3600} = 1.65 \text{ [kg/s]} \]  

(7.65)

The density at a gas temperature of 25 °C is \( \rho = 0.9896 \cdot 273.15 / (273.15 + 25) = 0.9066 \text{ [kg/m}^3\text{]} \). Assuming that the gas pressure is not much higher than atmospheric pressure (1.013 bar), the gas velocity for a given temperature is:

\[ w = \frac{M}{d^2 \cdot \pi \cdot \rho} = \frac{2.317}{d^2} \]  

(7.66)

Now, the pressure loss can be expressed as the function of pipe diameter:

\[ \Delta p_{\text{Total}} [\text{Pa}] = 1.1 \cdot \frac{\rho}{d} \cdot \frac{L}{\lambda} \cdot \frac{w^2}{2} \cdot \frac{d^2 \cdot \rho}{\eta} = \frac{1.1 \cdot 0.03 \cdot 4000 \cdot 2.317^2 \cdot 0.9066}{d^2} = \frac{321.2}{d^2} \]  

(7.67)

where \( d \) is in [m].

The power consumed by the blower is:

\[ P = \frac{V \cdot \Delta p_{\text{Total}}}{\eta} = \frac{1.65 \cdot 321.2}{0.9066 \cdot d^3} = \frac{1169.16}{d^3} \text{ [W]} \]  

(7.68)

Assuming 330 working days and 24 working hours per day, the cost of running the system is as follows:

\[ C_R = 0.0625 \text{ [$US/kWh$] \cdot 330 \text{ [d/y]} \cdot 24 \text{ [h/d]} \cdot \frac{1169.16}{1000 \times d^2} \text{ [kW]} = \frac{578.7}{d^2} \text{ [$US/y$]} \]  

(7.69)

The annual cost of pipeline depreciation is:

\[ C_D = 3.75 \text{ [$US/(m \times m)$] \cdot 4000 \text{ [m]} \cdot d \text{ [m]} = 15,000 \cdot d \text{ [$US/y$]} \]  

(7.70)

and for pipeline maintenance is:

\[ C_M = 2.90 \text{ [$US/(m \times m)$] \cdot 4000 \text{ [m]} \cdot d \text{ [m]} = 11,600 \cdot d \text{ [$US/y$]} \]  

(7.71)

Finally, the total annual cost of pipeline as the function of the diameter is:

\[ C_{\text{Total}} = C_R + C_D + C_M = \frac{578.7}{d^2} + 26,600 \times d \]  

(7.72)
The solution of this equation gives the optimum diameter as 0.70 [m]. The total annual cost is 22,063 $US/year (Fig. 7.13). The pressure loss is 0.01284 bar or 1284 Pa.

![Figure 7.13: Total Annual Cost of Running the Natural Gas Pipeline versus Pipe Diameter](image_url)

**Example 6:** The sufficient initial pressure of gas for its transport over a pipeline 100 km long has to be calculated. The flow rate of gas is 5000 kg/h. The actual average temperature of gas is 18 °C. The pipeline diameter is 300 mm, and the pipe resistance is $\lambda = 0.0253$. The absolute outlet pressure of the gas should be $p_{out} = 1.5$ bar.

The gas content (by volume) is: Methane ($\text{CH}_4$) 92.60 %; Ethane ($\text{C}_2\text{H}_6$) 3.60 %; Propane ($\text{C}_3\text{H}_8$) 0.80 %; Butanes Plus ($\text{C}_4\text{H}_{10}$) 0.30 %; Ethene ($\text{C}_2\text{H}_4$); Nitrogen ($\text{N}_2$) 2.60 %; Carbon Dioxide ($\text{CO}_2$) 0.10 %.

**Solution:**

If it is assumed that pressure loss occurs only for overcoming friction resistance and knowing that the density and velocity of gas will be changed along the length of pipeline, the pressure loss equation has to be written in a differential form. It is as follows:

$$-d\rho = \frac{\lambda}{d} \cdot \frac{w^2 \cdot \rho}{2} \cdot dL$$

(7.73)

The variable density ($\rho$) and velocity ($w$) are functions of gas pressure, which drops as the gas flows through the pipe. The minus sign shows that pressure drops with increased pipe length.

The gas can be analyzed as perfect gas (relatively low pressure and high temperature), thus the gas density can be expressed as the function of temperature and pressure in the following way:

$$\rho = \frac{p}{R \cdot T}$$

(7.74)

where:

- $p$ = Absolute gas pressure, [Pa]
- $T$ = Absolute gas temperature, [K]
- $R$ = Gas constant, [J/(kg K)]

For the given gas content, the gas density for standard conditions (15 °C; 1.013 bar) is calculated as follows:

$$\rho_s = 0.7276 \text{ [kg/m}^3\text{]}$$

(7.75)
The gas density in the pipeline is the function of its pressure and temperature as:

$$\rho \left[ \text{kg/m}^3 \right] = \rho_s \cdot \frac{p}{p_s} \cdot \frac{T_s}{T} = 206.71 \cdot 10^{-5} \cdot \frac{p}{T} \left[ \text{K} \right]$$ (7.76)

From Eq. (5) (continuity equation), it can be concluded that in any pipe the cross-section must be 
$$\rho \times w = \text{const}$$ The mass gas flow rate is given (5000 kg/h) and the pipe cross-section is 
$$A = \frac{\pi \cdot d^2}{4} = \frac{\pi \cdot 0.3^2}{4} = 0.070686 \left[ \text{m}^2 \right]$$. For standard conditions, the ‘standard’ velocity will be:

$$w_s = \frac{5000}{3600 \cdot 0.726 \cdot 0.070686} = 27.00 \left[ \text{m/s} \right]$$ (7.77)

and, the velocity of gas in any cross-section of the pipeline is now:

$$w = w_s \cdot \frac{p_s}{p}$$ (7.78)

Now, by replacing the expressions for $\rho$ and $w$ in the differential equation this equation is transformed as follows:

$$-dp = \left( \frac{\lambda}{d} \cdot \frac{w_s^2 \cdot \rho_s}{2} \cdot \frac{T}{206.71 \cdot 10^{-5}} \right) \cdot \frac{dL}{p}$$ (7.79)

All of the variables in the brackets are known (the flow is isothermal and the gas temperature is constant) and the above mentioned equations become:

$$-p \cdot dp = 22.921 \cdot 10^5 \cdot dL$$ (7.80)

By integration of this equation it is possible to get:

$$p_{in}^2 - p_{out}^2 = 22.921 \cdot 10^5 \cdot L$$ (7.81)

Finally, the inlet pressure of gas has to be:

$$p_{in} = \sqrt{p_{out}^2 + 2 \cdot 22.921 \cdot 10^5 \cdot L} = \sqrt{150,000^2 + 2 \cdot 22.921 \cdot 10^5 \cdot 100,000}$$

$$= 693,484 \left[ \text{Pa} \right] \text{ or } 6.93 \left[ \text{bar} \right]$$ (7.82)

References


