Applied Gas Dynamics

Basic Facts

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Definition of Gas Dynamics

*Gas dynamics* is the science of fluid flow in which both density and temperature changes become significant. Taking 5 percent change in temperature as significant, it can be stated that, at standard sea level, Mach number 0.5 is the lower limit of gas dynamics. Thus, gas dynamics is the science of flow fields with speeds Mach 0.5 and above. Therefore, gas dynamic regimes consist of both subsonic and supersonic Mach numbers. Further, when the flow is supersonic, any change of flow property or direction is caused by waves.
These waves are isentropic and nonisentropic compression waves (shock waves), expansion waves and Mach waves. Among these, the compression and expansion waves can cause finite changes but the flow property changes caused by a Mach wave are insignificant. The essence of gas dynamics is that, when the flow speed is supersonic, the entire flow field is dominated by Mach waves, expansion waves and shock waves. It is through these waves, the change of flow properties from one state to another takes place.
**Introduction**

*Compressible flow* is the science of fluid flow where the density change associated with pressure change is significant. Fluid mechanics is the science of fluid flow in which the temperature changes associated with the flow are insignificant. Fluid mechanics is essentially the science of *isenthalpic* flows and thus the main equations governing a fluid dynamic stream are only the continuity and momentum equations and the second law of thermodynamics. The energy equation is passive as for as fluid dynamic streams are concerned.
At standard sea level conditions, considering less than five percent change in temperature as insignificant, flow with Mach number less than 0.5 can be termed as a fluid mechanic stream. A fluid mechanic stream may be compressible or incompressible. For an incompressible flow both temperature and density changes are insignificant. For a compressible flow, the temperature change may be insignificant but density change is finite.
But in many engineering applications, such as design of airplane, missiles and launch vehicles, the flow Mach numbers associated are more than 0.5. Hence both temperature and density changes associated with the flow become significant. Study of such flows where both density and temperature changes associated with pressure change become appreciable is called *gas dynamics*. 
In other words, gas dynamics is the science of fluid flow in which both density and temperature changes become significant. The essence of gas dynamics is that the entire flow field is dominated by Mach waves, expansion waves and shock waves, when the flow speed is supersonic. It is through these waves, the change of flow properties from one state to another takes place.
In the theory of gas dynamics, change of state in flow properties is achieved by three means; (a) with area change, treating the fluid to be inviscid and passage to be frictionless, (b) with friction, treating the heat transfer between the surroundings and system to be negligible and (c) with heat transfer, assuming the fluid to be inviscid. These three types of flows are called *isentropic flow, frictional* or *Fanno type flow* and *Rayleigh type flow*, respectively.
All problems in gas dynamics can be classified under the three flow processes described above, of course with the assumptions mentioned. Although it is impossible to have a flow process which is purely isentropic or Fanno type or Rayleigh type, in practice, it is justified in assuming so, since the results obtained with these treatments prove to be accurate enough for most practical problems in gas dynamics.
Compressibility

Fluids such as water are incompressible at normal conditions. But under conditions of high pressure (e.g. 1000 atmospheres) they are compressible. The change in volume is the characteristic feature of a compressible medium under static condition. Under dynamic conditions, i.e. when the medium is moving, the characteristic feature for incompressible and compressible flow situations are: the volume flow rate, \( \dot{Q} = AV = \text{constant at any cross-section of a streamtube for incompressible flow} \) and the mass flow rate, \( \dot{m} = \rho AV = \text{constant at any cross-section of a streamtube for compressible flow} \). In these relations, \( A \) is the cross-sectional area of the streamtube, \( V \) and \( \rho \) are respectively the velocity and density of the fluid at that cross-section (Figure 1.1).
In general, the flow of an incompressible medium is called *incompressible flow* and that of a compressible medium is called *compressible flow*. Though this statement is true for incompressible media at normal conditions of pressure and temperature, for compressible medium like gases it has to be modified.
As long as a gas flows at a sufficiently low speed from one cross-section to another of a passage the change in volume (or density) can be neglected and, therefore, the flow can be treated as incompressible. Although the fluid is compressible, this property may be neglected when the flow is taking place at low speeds. In other words, although there is some density change associated with every physical flow, it is often possible (for low speed flows) to neglect it and idealize the flow as incompressible. This approximation is applicable to many practical flow situations, such as low-speed flow around an airplane and flow through a vacuum cleaner.
From the above discussion it is clear that compressibility is the phenomenon by virtue of which the flow changes its density with change in speed. Now, we may question what are the precise conditions under which density changes must be considered? We will try to answer this question now.

A quantitative measure of compressibility is the volume modulus of elasticity $E$, defined as

$$E = -\frac{\Delta p}{\Delta V/V_i} \quad (1.1)$$

where $\Delta p$ is the change in static pressure, $\Delta V$ is the change in volume and $V_i$ is the initial volume. For ideal gases, the equation of state is

$$\rho V = RT$$
For isothermal flows this reduces to

\[ \rho V = p_i V_i = \text{constant} \]

where \( p_i \) is the initial pressure.

The above equation may be written as

\[ (p_i + \Delta p)(V_i + \dot{V}) = p_i V_i \]

Expanding this equation and neglecting the second order terms, we get

\[ \Delta \rho V_i + \dot{V} p_i = 0 \]
Therefore,

\[ \Delta p = -p_i \frac{\Delta V}{V_i} \]  

(1.2)

For gases, from Eqs. (1.1) and (1.2), we get

\[ E = p_i \]  

(1.3)

Hence, by Eq. (1.2), the compressibility may be defined as the volume modulus of the pressure.
Limiting Conditions for Compressibility

By mass conservation, we have \( \dot{m} = \rho V = \text{constant} \), where \( \dot{m} \) is mass flow rate per unit area, \( V \) is the flow velocity and \( \rho \) is the corresponding density. This can also be written as

\[
(V_i + \Delta V)(\rho_i + \Delta \rho) = \rho_i V_i
\]

Considering only first order terms, this simplifies to

\[
\frac{\Delta \rho}{\rho_i} = - \frac{\Delta V}{V_i}
\]
Substituting this in to Eq. (1.1) and noting that \( V = V \) for unit area per unit time in the present case, we get

\[
\Delta \rho = E \frac{\Delta \rho}{\rho_i}
\]  

(1.4)

From Eq. (1.4), it is seen that the compressibility may also be defined as the density modulus of the pressure.
For incompressible flows, by Bernoulli’s equation, we have

\[ p + \frac{1}{2} \rho V^2 = \text{constant} = p_{\text{stag}} \]

where the subscript “stag” refers to stagnation condition. The above equation may also be written as

\[ p_{\text{stag}} - p = \Delta p = \frac{1}{2} \rho V^2 \]
i.e., the change of pressure from stagnation to static states is equal to $\frac{1}{2} \rho V^2$. Using Eq. (1.4) in the above relation, we obtain

$$\frac{\Delta p}{E} = \frac{\Delta \rho}{\rho_i} = \frac{\rho_i V_i^2}{2E} = \frac{q_i}{E}$$

(1.5)

where $q_i = \frac{1}{2} \rho_i V_i^2$ is the dynamic pressure. Equation (1.5) relates the density change with flow speed.
The compressibility effects can be neglected if the density changes are very small, i.e. if

$$\frac{\Delta \rho}{\rho_i} \ll 1$$

From Eq. (1.5) it is seen that, for neglecting compressibility

$$\frac{q}{E} \ll 1$$

For gases, the speed of sound “a” may be expressed in terms of pressure and density changes as [see Eq. (1.11)]

$$a^2 = \frac{\Delta p}{\Delta \rho}$$
Using Eq. (1.4) in the above relation, we get

\[ a^2 = \frac{E}{\rho_i} \]

With this, Eq. (1.5) reduces to

\[ \frac{\Delta \rho}{\rho_i} = \frac{\rho_i}{2} \frac{V_i^2}{E} = \frac{1}{2} \left( \frac{V}{a} \right)^2 \]  

(1.6)

The ratio \( V/a \) is called the Mach number \( M \). Therefore, the condition of incompressibility for gases becomes

\[ M^2/2 \ll 1 \]
Thus, the criterion determining the effect of compressibility for gases is that the magnitude of the mach number $M$ should be negligibly small. Indeed, mathematics would stipulate this limit as $M \to 0$. But for Mach number zero corresponds to stagnation state. Therefore, in engineering sciences flows with very small Mach number are treated as incompressible. To have a quantification of this limiting value of Mach number to treat a flow as incompressible, Mach number corresponding to 5% change in flow density is usually taken as the limit.
It is widely accepted that compressibility can be neglected when

\[ \frac{\Delta \rho}{\rho_i} \leq 0.05 \text{ or } 5\% \]

i.e. when \( M \leq 0.3 \). In other words, the flow may be treated as incompressible when \( V \leq 100 \text{ m/s, i.e., when } V \leq 360 \text{ kmph under standard sea level conditions.} \) The above values of \( M \) and \( V \) are widely accepted values and they may be re-fixed at different levels, depending upon the flow situation and the degree of accuracy desired.
Supersonic Flow - What is It?

The Mach number $M$ is defined as the ratio of the local flow speed $V$ to the local speed of sound $a$

$$M = \frac{V}{a}$$  \hspace{1cm} (1.7)

Thus $M$ is a dimensionless quantity. In general both $V$ and $a$ are functions of position and time. Therefore, Mach number is not just the flow speed made nondimensional by dividing by a constant. In other words, the flow Mach number is the ratio of $V$ by $a$ and this relation should not be viewed as $M$ proportional to $V$ or inversely proportional $a$, in isolation. That is, we cannot write $M \propto V$ or $M \propto 1/a$, in isolation. However, it is almost always true that $M$ increases monotonically with $V$. 
A flow with Mach number is greater than unity is termed *supersonic flow*. In a supersonic flow \( V > a \) and the flow upstream of a given point remains unaffected by changes in conditions at that point.
Speed of Sound

Sound waves are infinitely small pressure disturbances. The speed with which sound propagates in a medium is called *speed of sound* and is denoted by \( a \). If an infinitesimal disturbance is created by the piston, as shown in Figure 1.2, the wave propagates through the gas at the velocity of sound relative to the gas into which the disturbance is moving. Let the stationary gas at pressure \( p_i \) and density \( \rho_i \) in the pipe be set in motion by moving the piston.
The infinitesimal pressure wave created by piston movement travels with speed $a$, leaving the medium behind it at pressure $p_1$ and density $\rho_1$ to move with velocity $V$.

**Figure 1.2**
Propagation of pressure disturbance.
As a result of compression created by the piston, the pressure and density next to the piston are infinitesimally greater than the pressure and density of the gas at rest ahead of the wave. Therefore,

\[ \Delta p = p_1 - p_i, \quad \Delta \rho = \rho_1 - \rho_i \]

are small.

Choose a control volume of length \( b \), as shown in Figure 1.2. Compression of volume \( Ab \) causes the density to rise from \( \rho_i \) to \( \rho_1 \) in time \( t = b/a \). The mass flow into volume \( Ab \) is

\[ \dot{m} = \rho_1 AV \quad (1.8) \]
For mass conservation, \( \dot{m} \) must also be equal to the mass flow rate \( Ab(\rho_1 - \rho_i)/t \) through the control volume. Thus,

\[
Ab(\rho_1 - \rho_i)/t = \rho_1 AV
\]

or

\[
a(\rho_1 - \rho_i) = \rho_1 V \quad \text{(1.9)}
\]

because \( b/t = a \).
The compression wave caused by the piston motion travels and accelerates the gas from zero velocity to $V$. The acceleration is given by

$$\frac{V}{t} = V \frac{a}{b}$$

The mass in the control volume $Ab$ is

$$m = Ab \bar{\rho}$$

where

$$\bar{\rho} = \frac{\rho_i + \rho_1}{2}$$
The force acting on the control volume is \( F = A(p_1 - p_i) \). Therefore, by Newton’s law,

\[
A(p_1 - p_i) = m \left( V \frac{a}{b} \right)
\]

\[
A(p_1 - p_i) = (A b \bar{\rho}) \left( V \frac{a}{b} \right)
\]

or

\[
\bar{\rho} V a = p_1 - p_i
\]

(1.10)
Because the disturbance is very weak, $\rho_1$ on the right-hand side of Eq. (1.9) may be replaced by $\bar{\rho}$ to result in

$$a(\rho_1 - \rho_i) = \bar{\rho}V$$

Using this relation, Eq. (1.10) can be written as

$$a^2 = \frac{p_1 - p_i}{\rho_1 - \rho_i} = \frac{\Delta p}{\Delta \rho}$$

In the limiting case of $\Delta p$ and $\Delta \rho$ approaching zero, the above equation leads to

$$a^2 = \frac{dp}{d\rho}$$

(1.11)

This is Laplace equation and is valid for any fluid.
The sound wave is an isentropic pressure wave, across which only infinitesimal change in fluid properties occur. Further, the wave itself is extremely thin and changes in properties occur very rapidly. The rapidity of the process rules out the possibility of any heat transfer between the system of fluid particles and its surrounding.

For very strong pressure waves, the traveling speed of disturbance may be greater than that of sound. The pressure can be expressed as

$$ p = p(\rho) \quad (1.12) $$
For isentropic process of a gas,

\[
\frac{p}{\rho^\gamma} = \text{constant}
\]

where the isentropic index \( \gamma \) is the ratio of specific heats and is a constant for a perfect gas. Using the above relation in Eq. (1.11), we get

\[
a^2 = \gamma \frac{p}{\rho} \tag{1.13}
\]

For a perfect gas, by the state equation

\[
p = \rho RT \tag{1.14}
\]

where \( R \) is the gas constant and \( T \) the static temperature of the gas in absolute units.
Equations (1.13) and (1.14) together lead to the following expression for the speed of sound. 

\[ a = \sqrt{\gamma RT} \]  

(1.15)

Perfect gas assumption is valid so long as the speed of gas stream is not too high. However, at hypersonic speeds the assumption of perfect gas is not valid and we must consider Eq. (1.13) to calculate the speed of sound.
Temperature Rise

For a perfect gas,

\[ p = \rho RT, \quad R = c_p - c_v \]

where \( c_p \) and \( c_v \) are specific heats at constant pressure and constant volume, respectively. Also, \( \gamma = c_p / c_v \), therefore,

\[ R = \frac{\gamma - 1}{\gamma} c_p \quad (1.16) \]

For an isentropic change of state, an equation not involving \( T \) can be written as

\[ p / \rho^\gamma = \text{constant} \]
Now, between state 1 and any other state the relation between the pressures and densities can be written as

\[
\left( \frac{p}{p_1} \right) = \left( \frac{\rho}{\rho_1} \right)^\gamma
\]

(1.17)

Combining Eqs. (1.17) and (1.14), we get

\[
\frac{T}{T_1} = \left( \frac{\rho}{\rho_1} \right)^{\gamma^{-1}} = \left( \frac{p}{p_1} \right)^{(\gamma-1)/\gamma}
\]

(1.18)

The above relations are very useful for gas dynamic studies. The temperature, density and pressure ratios in Eq. (1.18) can be expressed in terms of the flow Mach number.
Let us examine the flow around a symmetrical body, as shown in Figure 1.3.

In a compressible medium, there will be change in density and temperature at point 0. The temperature rise at the stagnation point can be obtained from the energy equation.
The energy equation for an isentropic flow is

\[ h + \frac{V^2}{2} = \text{constant} \quad (1.19) \]

where \( h \) is the enthalpy.

Equating the energy at far upstream \( \infty \) and the stagnation point \( 0 \), we get

\[ h_\infty + \frac{V_\infty^2}{2} = h_0 + \frac{V_0^2}{2} \]

But \( V_0 = 0 \), thus

\[ h_0 - h_\infty = \frac{V_\infty^2}{2} \]
For a perfect gas \( h = c_p T \), therefore, from the above relation we obtain

\[
c_p(T_0 - T_\infty) = \frac{V_\infty^2}{2}
\]

i.e.

\[
\Delta T = T_0 - T_\infty = \frac{V_\infty^2}{2c_p}
\] (1.20)
Combining Eqs. (1.15) and (1.16), we get

$$c_p = \frac{1}{\gamma - 1} \frac{a_\infty^2}{T_\infty}$$

Hence,

$$\Delta T = \frac{\gamma - 1}{2} T_\infty M_\infty^2$$

(1.21)

i.e.,

$$T_0 = T_\infty \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)$$

(1.22)
For air, $\gamma = 1.4$, hence

$$T_0 = T_\infty \left(1 + 0.2M_\infty^2\right)$$

(1.23)

where $T_0$ is the temperature at the stagnation point on the body. It is also referred to as total temperature. For example, the flow at the stagnation point 0 on the body shown in Figure 1.3, the flow will attain the stagnation temperature.
Mach Angle

The presence of a small disturbance is felt throughout the field by means of disturbance waves traveling at the local velocity of sound relative to the medium. Let us examine the propagation of pressure disturbance created by a moving object shown in Figure 1.4. The propagation of disturbance waves created by an object moving with velocity $V = 0$, $V = a/2$, $V = a$ and $V > a$ is shown in Figures 1.4(a), (b), (c), (d), respectively. In a subsonic flow the disturbance waves reach a stationary observer before the source of disturbance could reach him, as shown in Figures 1.4(a) and 1.4(b).
Figure 1.4

Propagation of disturbance waves.
But in supersonic flows it takes considerable amount of time for an observer to perceive the pressure disturbance, after the source has passed him. This is one of the fundamental differences between subsonic and supersonic flows. Therefore, in a subsonic flow the streamlines sense the presence of any obstacle in the flow field and adjust themselves well ahead of the obstacles and flow around it smoothly. But in a supersonic flow, the streamlines feel the obstacle only when they hit it. The obstacle acts as a source and the streamlines deviate at the Mach cone as shown in Figure 1.4(d). That is in a supersonic flow the disturbance due to an obstacle is sudden and the flow behind the obstacle has to change abruptly.
Flow around a wedge shown in Figures 1.5(a) and 1.5(b) illustrate the smooth and abrupt change in flow direction for subsonic and supersonic flow, respectively. For $M_\infty < 1$, the flow direction changes smoothly and the pressure decreases with acceleration. For $M_\infty > 1$, there is a sudden change in flow direction at the body and the pressure increases downstream of the shock.

(a) Subsonic flow  
(b) Supersonic flow

**Figure 1.5**  
Flow around a wedge.
In Figure 1.4(d), it is shown that for supersonic motion of an object there is a well-defined conical zone in the flow field with the object located at the nose of the cone and the disturbance created by the moving object is confined only to the field included inside the cone. The flow field zone outside the cone does not even feel the disturbance. For this reason, von Karman termed the region inside the cone as the *zone of action* and the region outside the cone as the *zone of silence*. 
The lines at which the pressure disturbance is concentrated and which generate the cone are called *Mach waves* or *Mach lines*. The angle between the Mach line and the direction of motion of the body is called the Mach angle $\mu$. From Figure 1.4(d), we have

$$\sin \mu = \frac{at}{Vt} = \frac{a}{V}$$

de i.e.

$$\sin \mu = \frac{1}{M}$$

(1.24)
From the disturbance waves propagation shown in Figure 1.4, we can infer the following features of the flow regimes.

- When the medium is *incompressible* ($M = 0$, Figure 1.4a) or when the speed of the moving disturbance is negligibly small compared to the local sound speed, the pressure pulse created by the disturbance spreads uniformly in all directions.

- When the disturbance source moves with a *subsonic speed* ($M < 1$, Figure 1.4b), the pressure disturbance is felt in all directions and at all points in space (neglecting viscous dissipation), but the pressure pattern is no longer symmetrical.
For *sonic velocity* \( M = 1 \), Figure 1.4c) the pressure pulse is at the boundary between subsonic and supersonic flow and the wave front is a plane.

For *supersonic speeds* \( M > 1 \), Figure 1.4d) the disturbance wave propagation phenomenon is totally different from those at subsonic speeds. All the pressure disturbances are included in a cone which has the disturbance source at its apex and the effect of the disturbance is not felt upstream of the disturbance source.
Small Disturbance

When the apex angle of wedge $\delta$ is vanishingly small the disturbances will be small and we can consider these disturbance waves to be identical to sound pulses. In such a case, the deviation of streamlines will be small and there will be infinitesimally small increase of pressure across the Mach cone shown in Figure 1.6.

$M_\infty > 1$

Figure 1.6
Mach cone.
Finite Disturbance

When the wedge angle $\delta$ is finite the disturbances introduced are finite, then the wave is not called Mach wave but a shock or shock wave (see Figure 1.7). The angle of shock $\beta$ is always smaller than the Mach angle. The deviation of the streamlines is finite and the pressure increase across a shock wave is finite.

![Figure 1.7
Shock wave.](image-url)
Entropy and temperature are the two fundamental concepts of thermodynamics. Unlike low-speed or incompressible flows, the energy change associated with a compressible flow is substantial enough to strongly interact with other properties of the flow. Hence the energy concept plays an important role in the study of compressible flows. In other words, the study of thermodynamics which deals with energy (and entropy) is an essential component in the study of compressible flow.
The following are the broad divisions of fluid flow based on thermodynamic considerations. (i) Fluid mechanics of perfect fluids, i.e., fluids without viscosity and heat (transfer) conductivity, is an extension of equilibrium thermodynamics to moving fluids. The kinetic energy of the fluid has to be considered in addition to the internal energy which the fluid possesses even when at rest. (ii) Fluid mechanics of real fluids (goes beyond the scope of classical thermodynamics). The transport processes of momentum and heat (energy) are of primary interest here. But, even though thermodynamics is not fully and directly applicable to all phases of real fluid flow, it is often extremely helpful in relating the initial and final conditions.
For low-speed flow problems, thermodynamic considerations are not needed because the heat content of the fluid flow is so large compared to the kinetic energy of the flow that the temperature remains nearly constant even if the whole kinetic energy is transformed into heat. In other words, the difference between the static and stagnation temperatures is not significant in low-speed flows. But in high-speed flows, the kinetic energy content of the fluid can be so large compared to its heat content that the difference between the static and stagnation temperature can become substantial. Hence, emphasis on the thermodynamic concepts assumes importance in high-speed flow analysis.
First Law of Thermodynamics (Energy Equation)

Consider a closed system, consisting of a certain amount of gas at rest, across whose boundaries no transfer of mass is possible. Let $\delta Q$ be an incremental amount of heat added to the system across the boundary (by thermal conduction or by direct radiation). Also, let $\delta W$ denote the work done on the system by the surroundings (or by the system on the surroundings). The sign convention is positive when the work is done by the system and negative when the work is done on the system. Due to the molecular motion of the gas, the system has an internal energy $U$. The first law of thermodynamics states that the heat added minus work done by the system is equal to the change in the internal energy of the system, i.e.

$$\delta Q - \delta W = dU$$  \hspace{1cm} (1.25)
This is an empirical result confirmed by laboratory experiments and practical experience. In Eq. (1.25), the internal energy $U$ is a state variable (thermodynamic property). Hence, the change in internal energy $dU$ is an exact differential and its value depends only on the initial and final states of the system. In contrast (the non-thermodynamic properties), $\delta Q$ and $\delta W$ depend on the process by which the system attained its final state from the initial state.
In general, for any given $dU$, there are infinite number of ways (processes) by which heat can be added and work can be done on the system. In the present course of study, we will be mainly concerned with the following three types of processes only.

1. **Adiabatic process** — a process in which no heat is added to or taken away from the system.

2. **Reversible process** — a process which can be reversed without leaving any trace on the surroundings, i.e., both the system and the surroundings are returned to their initial states at the end of the reverse process.

3. **Isentropic process** — a process which is adiabatic and reversible.
For an open system (e.g. pipe flow), there is always found a term \((U + pV)\) present instead of just \(U\). This term is referred to as enthalpy or heat function \(H\) given by

\[
H = U + pV
\]

\[
H_2 - H_1 = U_2 - U_1 + p_2 V_2 - p_1 V_1
\]

where \((p_2 V_2 - p_1 V_1)\) is termed *flow work* and subscripts 1 and 2 represent states 1 and 2, respectively.
In general, we can say that the following are the major differences between the open and closed systems.

1. The mass which enters or leaves an open system has kinetic energy, whereas there is no mass transfer possible across the boundaries of a closed system.

2. The mass can enter and leave an open system at different levels of potential energy.

3. Open systems are capable of delivering work continuously, because in the system the medium which transforms energy is continuously replaced. This useful work, which a machine continuously delivers, is called the *shaft work*. 
Energy Equation for an Open System

Consider the system shown in Figure 1.8.

![Figure 1.8](image)

**Figure 1.8**
Open system.
The total energy $E$ at the inlet station 1 and the outlet station 2 is given by

$$E_1 = U_1 + \frac{1}{2} m V_1^2 + m g z_1$$  \hspace{1cm} (1.28)

$$E_2 = U_2 + \frac{1}{2} m V_2^2 + m g z_2$$  \hspace{1cm} (1.29)
For an open system, the first-law expressions given by Eq. (1.25) has to be rewritten with the total energy \( E \) in place of the internal energy \( U \).

Thus, we have

\[
Q_{12} - W_{12} = E_2 - E_1
\]  

(1.30)

Substituting for \( E_1 \) and \( E_2 \) from Eqs. (1.28) and (1.29), respectively, we get

\[
Q_{12} - W_{12} = \left( U_2 + \frac{m}{2} V_2^2 + m g z_2 \right) - \left( U_1 + \frac{m}{2} V_1^2 + m g z_1 \right)
\]

(1.31)
For an open system, the shaft (useful) work is not just equal to $W_{12}$, but the work done to move the pistons at 1 and 2 must also be considered. Work done with respect to the system by the piston at state 1 is

\[
\begin{align*}
W'_1 & = -F_1 \Delta_1 \quad (F_1 = \text{force and } \Delta_1 = \text{displacement}) \\
W'_1 & = -p_1 A_1 \Delta_1 \quad (p_1 = \text{pressure at 1; } A_1 = \text{cross-sectional area of piston}) \\
W'_1 & = -p_1 V_1
\end{align*}
\]

Work delivered at 2 is $W'_2 = p_2 V_2$. Therefore,

\[
W_{12} = W_s + p_2 V_2 - p_1 V_1 \quad (1.32)
\]
In Eq. (1.32), $W_s$ is the shaft work, which can be extracted from the system and $(p_2 V_2 - p_1 V_1)$ is the flow work necessary to maintain the flow. Substituting Eq. (1.32) into Eq. (1.31), we get

$$Q_{12} - W_s = \left( U_2 + p_2 V_2 + \frac{m}{2} V_2^2 + m g z_2 \right) - \left( U_1 + p_1 V_1 + \frac{m}{2} V_1^2 + m g z_1 \right)$$

or

$$Q_{12} - W_s = \left( H_2 + \frac{m}{2} V_2^2 + m g z_2 \right) - \left( H_1 + \frac{m}{2} V_1^2 + m g z_1 \right)$$

where $H_1 = U_1 + p_1 V_1$ and $H_2 = U_2 + p_2 V_2$. This is the fundamental equation for an open system.
If there are any other forms of energy, such as, electrical energy or magnetic energy, their initial and final values should be added properly to this equation. The energy equation for an open system

\[
H_1 + \frac{m}{2} V_1^2 + m g z_1 = H_2 + \frac{m}{2} V_2^2 + m g z_2 + W_s - Q_{12}
\] (1.33)

is universally valid. This is the expression of the first law of thermodynamics for any open system. In most applications of gas dynamics, the gravitational energy is negligible compared to the kinetic energy. For working processes such as flow in turbines and compressors, the shaft work \(W_s\) in Eq. (1.33) is finite and, for flow processes like flow around an airplane, \(W_s = 0\).
Therefore, for a gas dynamic working process, Eq. (1.33) becomes

\[ H_1 + \frac{m}{2} V_1^2 = H_2 + \frac{m}{2} V_2^2 + W_s - Q_{12} \]  (1.34)

This is usually the case with turbo machines, internal combustion engines, etc., where the process is assumed to be adiabatic (i.e. \( Q_{12} = 0 \)). For a gas dynamic adiabatic flow process, the energy equation (1.33) becomes

\[ H_1 + \frac{m}{2} V_1^2 = H_2 + \frac{m}{2} V_2^2 \]  (1.35)

or

\[ H_1 + \frac{m}{2} V_1^2 = H_0 = \text{constant} \]  (1.36)

where \( H_0 \) is the stagnation enthalpy and \( H_1 \) is the static enthalpy. That is, the sum of static enthalpy and kinetic energy is constant in an adiabatic flow.
Adiabatic Flow Process

For an adiabatic process, the heat transfer associated with the process, \( Q = 0 \). Therefore, the energy equation is given by Eqs. (1.35) and (1.36). Dividing Eqs. (1.35) and (1.36) by \( m \), we can rewrite them as

\[
\frac{V_1^2}{2} = V_2^2
\]  
(1.37)

\[
\frac{V_2^2}{2} = h_0
\]  
(1.38)
or, in general,

\[ h + \frac{V^2}{2} = h_0 = \text{constant} \]  

(1.39)

where \( h = H/m \) is called specific static enthalpy and \( h_0 \) is the specific stagnation enthalpy. With \( h = p/\rho \), Eq. (1.39) represents Bernoulli’s equation for incompressible flow,

\[ \rho + \frac{1}{2} \rho V^2 = p_0 = \text{constant} \]

where \( p_0 \) is the stagnation pressure. That is, for incompressible flow of air the energy equation happens to be the Bernoulli equation, because we are not interested in the internal energy and the temperature for such flows.
In other words, Bernoulli’s equation is the limiting case of the energy equation for incompressible flows. Here it is important to realize that even though Bernoulli’s equation for incompressible flow of a gas is shown to be the limiting case of energy equation, it is essentially a momentum equation. For a closed system,

\[ Q_{12} - W_{12} = U_2 - U_1 \]

In terms of specific quantities this becomes

\[ q_{12} - w_{12} = u_2 - u_1 \]
For the processes of a closed system there is no shaft work, i.e., no useful work can be extracted from the working medium. There will only be compression or expansion work. Therefore, $w_{12}$ may be expressed as

$$w_{12} = \int_{1}^{2} p \, dV$$

Thus, the change in internal energy becomes

$$du = \delta q - p \, dv$$  \hspace{1cm} (1.40a)

Also, $h = u + pv$; $dh = du + p \, dv + v \, dp$. Using relation (1.40a), we can write the change in enthalpy as

$$dh = \delta q + v \, dp$$  \hspace{1cm} (1.40b)
For adiabatic change of state, Eqs. (1.40a) and (1.40b) reduce to

\[ du = -p \, dv, \quad dh = v \, dp \quad (1.40c) \]

where \( u, q \) and \( v \) in Eqs. (1.40) stand for specific quantities of internal energy, heat energy and volume, respectively.
The Second Law of Thermodynamics (Entropy Equation)

Consider a cold body coming in to contact with a hot body. From experience we can say that the cold body will get heated up and the hot body will cool down. However, Eq. (1.25) does not necessarily imply that this will happen. In fact, the first law allows the cold body to become cooler and the hot body to become hotter as long as energy is conserved during the process. However, in practice this does not happen; instead, the law of nature imposes another condition on the process, a condition that stipulates the direction in which a process should take place.
To ascertain the proper direction of a process, let us define a new state variable, the entropy, as follows.

\[ ds = \frac{\delta q_{\text{rev}}}{T} \]  

(1.41)

where \( s \) is the entropy (amount of disorder) of the system, \( \delta q_{\text{rev}} \) is an incremental amount of heat added reversibly to the system and \( T \) is the system temperature. The above definition gives the change in entropy in terms of a reversible addition of heat, \( \delta q_{\text{rev}} \).
Since entropy is a state variable, it can be used in conjunction with any type of process, reversible or irreversible. The quantity $\delta q_{\text{rev}}$ is just an artifice; an effective value of $\delta q_{\text{rev}}$ can always be assigned to relate the initial and final states of an irreversible process, where the actual amount of heat added is $\delta q$. In deed, an alternative and probably more lucid relation is

$$ds = \frac{\delta q}{T} + ds_{\text{irrev}}$$  \hspace{1cm} (1.42)$$

Equation (1.42) applies in general to all processes.
It states that, the change in entropy during any process is equal to the actual heat added $\delta q$, divided by the temperature, $\delta q/T$, plus a contribution from the irreversible dissipative phenomena of viscosity, thermal conductivity and mass diffusion occurring within the system, $ds_{\text{irrev}}$. These dissipative phenomena always cause increase of entropy, i.e.

$$ds_{\text{irrev}} \geq 0 \quad (1.43)$$

The equal sign in the inequality (1.43) denotes a reversible process where, by definition, the above dissipative phenomena are absent.
Hence, a combination of Eqs. (1.42) and (1.43) yields

\[ ds \geq \frac{\delta q}{T} \] (1.44)

Further, if the process is adiabatic, \( \delta q = 0 \) and Eq. (1.44) reduces to

\[ ds \geq 0 \] (1.45)

Equations (1.44) and (1.45) are two forms of the second law of thermodynamics.
The second law gives the direction in which a process will take place. Equations (1.44) and (1.45) imply that a process will always proceed in a direction such that the entropy of the system plus surroundings always increases, or at least remains unchanged. That is, in an adiabatic process the entropy can never decrease. This aspect of the second law of thermodynamics is important because it distinguishes between reversible and irreversible processes.
If $ds > 0$, the process is called an *irreversible process* and when $ds = 0$, the process is called a *reversible process*. A reversible and adiabatic process is called an *isentropic process*. However, in a non-adiabatic process, we can extract heat from the system and thus decrease the entropy of the system.
Thermal and Calorical Properties

The equation $pv = RT$ or $p/\rho = RT$ is called thermal equation of state, where $p$, $T$ and $v(= 1/\rho)$ are thermal properties and $R$ is the gas constant. A gas which obeys the thermal equation of state is called thermally perfect gas. Any relation between the calorical properties, $u$, $h$ and $s$ and any two thermal properties is called calorical equation of state. In general, the thermodynamic properties (the properties which do not depend on process) can be grouped into thermal properties ($p$, $T$, $v$) and calorical properties ($u$, $h$, $s$).
From Eqs. (1.40), we have

\[ u = u(T, v), \quad h = h(T, p) \]

In terms of exact differentials, the above relations become

\[
du = \left( \frac{\partial u}{\partial T} \right)_v \, dT + \left( \frac{\partial u}{\partial v} \right)_T \, dv \tag{1.46}
\]

\[
dh = \left( \frac{\partial h}{\partial T} \right)_p \, dT + \left( \frac{\partial h}{\partial p} \right)_T \, dp \tag{1.47}
\]
For a constant volume process, Eq. (1.46) reduces to

\[ du = \left( \frac{\partial u}{\partial T} \right)_v \, dT \]

where \( \left( \frac{\partial u}{\partial T} \right)_v \) is the specific heat at constant volume represented as \( c_v \), therefore,

\[ du = c_v \, dT \quad (1.48) \]

For an isobaric process, Eq. (1.47) reduces to

\[ dh = \left( \frac{\partial h}{\partial T} \right)_p \, dT \]

where \( \left( \frac{\partial h}{\partial T} \right)_p \) is the specific heat at constant pressure represented by \( c_p \), therefore,

\[ dh = c_p \, dT \quad (1.49) \]
From Eqs. (1.40a) for a constant volume (isochoric) process, we get

\[ \delta q = du = c_v \, dT \]  

and for a constant pressure (isobaric) process,

\[ \delta q = dh = c_p \, dT, \quad \delta q = dh = c_v \, dT + p \, dv \]  

For an adiabatic flow process \((q = 0)\), from Eq. (1.40c) we have

\[ dh = v \, dp \]
From Eqs. (1.50) it can be inferred that,

1. If heat is added at constant volume, it only raises the internal energy.
2. If heat is added at constant pressure, it not only increases the internal energy but also does some external work, i.e. it increases the enthalpy.
3. If the change is adiabatic, the change in enthalpy is equal to external work \( \nu dp \).
Thermally Perfect Gas

A gas is said to be thermally perfect when its internal energy and enthalpy are functions of temperature alone, i.e. for a thermally perfect gas,

\[ u = u(T), \quad h = h(T) \quad (1.51a) \]

Therefore, from Eqs. (1.48) and (1.49), we get

\[ c_v = c_v(T), \quad c_p = c_p(T) \quad (1.51b) \]

Further, from Eqs. (1.46), (1.47) and (1.51a), we obtain

\[ \left( \frac{\partial u}{\partial v} \right)_T = 0, \quad \left( \frac{\partial h}{\partial p} \right)_T = 0 \quad (1.51c) \]
The important relations of this section are

\[ du = c_v \, dT, \quad dh = c_p \, dT \]

These equations are universally valid so long as the gas is thermally perfect. Otherwise, in order to have equations of universal validity, we must add \( \left( \frac{\partial u}{\partial v} \right)_T \, dv \) to the first equation and \( \left( \frac{\partial h}{\partial p} \right)_T \, dp \) to the second equation.
The state equation for a thermally perfect gas is,

\[ pv = RT \]

In the differential form, this equation becomes

\[ p \, dv + v \, dp = R \, dT \]

Also,

\[ h = u + pv \]

\[ dh = du + p \, dv + v \, dp \]
Therefore,

\[ dh - du = p \, dv + v \, dp = R \, dT \]

i.e.

\[ R \, dT = c_p \, dT - c_v \, dT \]

Thus,

\[ R = c_p (T) - c_v (T) \quad (1.52) \]

For thermally perfect gases, Eq. (1.52) shows that, though \( c_p \) and \( c_v \) are functions of temperature, their difference is a constant with reference to temperature.
The Perfect Gas

This is a still more specialization than thermally perfect gas. For a perfect gas, both $c_p$ and $c_v$ are constants and are independent of temperature, i.e.

$$c_v = \text{constant} \neq c_v(T), \quad c_p = \text{constant} \neq c_p(T) \quad (1.53)$$

Such a gas with constant $c_p$ and $c_v$ is called a \textit{calorically perfect gas}. Therefore, a perfect gas should be thermally as well as calorically perfect.
From the above discussions it is evident that,

1. A perfect gas must be both thermally and calorically perfect.
2. A perfect gas must satisfy both *thermal equation of state*, \( p = \rho \, R \, T \) and *caloric equations of state*, \( c_p = (\partial h/\partial T)_p \) , \( c_v = (\partial u/\partial T)_v \).
3. A calorically perfect gas must be thermally perfect and a thermally perfect gas need not be calorically perfect. That is, thermal perfection is a prerequisite for caloric perfectness.
4. For a thermally perfect gas, \( c_p = c_p(T) \) and \( c_v = c_v(T) \); i.e. both \( c_p \) and \( c_v \) are functions of temperature. But even though the specific heats \( c_p \) and \( c_v \) vary with temperature, their ratio, \( \gamma \) becomes a constant and independent of temperature, i.e. \( \gamma = \text{constant} \neq \gamma(T) \).
5. For a calorically perfect gas, \( c_p \), \( c_v \) as well as \( \gamma \) are constants and independent of temperature.
Entropy Calculation

Entropy is defined by the relation (for a reversible process)

\[ \delta q = T \, ds \]

Using Eqs. (1.40), we can write

\[ T \, ds = du + p \, dv \quad (1.54) \]

\[ T \, ds = dh - v \, dp \quad (1.55) \]

Equations (1.54) and (1.55) are as important and useful as the original form of the first law of thermodynamics, viz. Eq. (1.25).
For a thermally perfect gas, from Eq. (1.49), we have \( dh = c_p \, dT \). Substituting this relation into Eq. (1.55), we obtain

\[
ds = c_p \frac{dT}{T} - \frac{v \, dp}{T} \tag{1.56}
\]

Substituting the perfect gas equation of state, \( p \, v = R \, T \), into Eq. (1.56), we get

\[
ds = c_p \frac{dT}{T} - R \frac{dp}{p} \tag{1.57}
\]

Integrating Eq. (1.57) between states 1 and 2, we obtain

\[
s_2 - s_1 = \int_{T_1}^{T_2} c_p \frac{dT}{T} - R \ln \left( \frac{p_2}{p_1} \right) \tag{1.58}
\]
Equation (1.58) holds for a thermally perfect gas. The integral can be evaluated if \( c_p \) is known as a function of \( T \). Further, assuming the gas to be calorically perfect, for which \( c_p \) is constant, Eq. (1.58) reduces to

\[
s_2 - s_1 = c_p \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{\rho_2}{\rho_1} \right)
\]

(1.59)

Using \( du = c_v \, dT \) in Eq. (1.54), the change in entropy can also be expressed as

\[
s_2 - s_1 = c_v \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{v_2}{v_1} \right)
\]

(1.60)
From the above discussion, we can summarize that a perfect gas is both thermally and calorically perfect. Further, a calorically perfect gas must also be thermally perfect, whereas a thermally perfect gas need not be calorically perfect.

For a thermally perfect gas, \( p = \rho RT \), \( c_v = c_v(T) \), \( c_p = c_p(T) \) and for a perfect gas, \( p = \rho RT \), \( c_v = \text{constant} \), \( c_p = \text{constant} \). Further, for a perfect gas all equations get simplified, resulting in the following simple relations for \( u \), \( h \) and \( s \).

\[
\begin{align*}
    u &= u_1 + c_v T \\
    h &= h_1 + c_p T \\
    s &= s_1 + c_v \ln \left( \frac{p}{p_1} \right) - c_p \ln \left( \frac{\rho}{\rho_1} \right)
\end{align*}
\]

where the subscript “1” refers to the initial state.
Equations (1.61a), (1.61b) and (1.52) combined with the thermal equation of state \((p = \rho RT)\) result in

\[
\begin{align*}
  u &= u_1 + \frac{1}{\gamma - 1} \frac{p}{\rho}, \\
  h &= h_1 + \gamma \frac{p}{\rho}
\end{align*}
\]

where \(\gamma\) is the ratio of specific heats, \(c_p/c_v\). For the most simple molecular model, the kinetic theory of gases gives the specific heats ratio, \(\gamma\) as

\[
\gamma = \frac{n + 2}{n}
\]

where \(n\) is the number of degrees of freedom of the gas molecules. Thus, for monatomic gases with \(n = 3\) (only 3 translational degrees of freedom), the specific heats ratio becomes

\[
\gamma = \frac{3 + 2}{3} = 1.67
\]
Diatomic gases like oxygen, nitrogen, etc. have \( n = 5 \) (3 translational degrees of freedom and 2 rotational degrees of freedom), thus,

\[ \gamma = \frac{5 + 2}{5} = 1.4 \]

Gases with extremely complex molecules, such as freon and gaseous compounds of uranium have large values of \( n \), resulting in values of \( \gamma \) only slightly greater than unity. Thus, the value of specific heats ratio \( \gamma \) varies from 1 to 1.67, depending on the molecular nature of the gas, i.e.

\[ 1 \leq \gamma \leq 1.67 \]
The above relations for \( u \) and \( h \) are important, because they connect the quantities used in thermodynamics with those used in gas dynamics. With the aid of these relations, the energy equation can be written in two different forms, as follows.

1. The energy equation for an adiabatic process, as given by Eq. (1.39), is

\[ h + \frac{V^2}{2} = h_0 = \text{constant} \]

when the gas is perfect, it becomes

\[ c_p T + \frac{V^2}{2} = c_p T_0 = \text{constant} \quad (1.62a) \]

2. Equation (1.62a), when combined with the state equation, yields

\[ \frac{\gamma}{\gamma - 1} \frac{p}{\rho} + \frac{V^2}{2} = \text{constant} \quad (1.62b) \]
Equation (1.62b) is the form of energy equation commonly used in gas dynamics. This is popularly known as *compressible Bernoulli’s equation* for isentropic flows.

From Eq. (1.62a), we infer that for an adiabatic process of a perfect gas,

\[ T_{01} = T_{02} = T_0 = \text{constant} \quad (1.63) \]

So far, we have not made any assumption about the reversibility or irreversibility of the process. Equation (1.63) implies that the stagnation temperature \( T_0 \) remains constant for an adiabatic process of a perfect gas, irrespective of the process being reversible or irreversible.
Consider the flow of gas in a tube with an orifice as shown in Figure 1.9. In such a flow process, there will be pressure loss. But if the stagnation temperature is measured before and after the orifice plate and if it remains constant, then the gas can be treated as perfect gas and all the simplified equations (Eq. 1.61) can be used. Otherwise, it cannot be treated as perfect gas and Eq. (1.61c) can be rewritten as

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^\gamma \exp\left[\frac{(s_2 - s_1)}{c_v}\right]$$  \hspace{1cm} (1.64)
Isentropic Relations

An adiabatic and reversible process is called isentropic process. For an adiabatic process, \( \delta q = 0 \) and for a reversible process, \( ds_{\text{irrev}} = 0 \). Hence, from Eq. (1.42), an isentropic process is one for which \( ds = 0 \), i.e., the entropy is constant. Important relations for an isentropic process can be directly obtained from Eqs. (1.59), (1.60) and (1.64) by setting \( s_2 = s_1 \). For example, from Eq. (1.59) we have

\[
0 = c_p \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{p_2}{p_1} \right)
\]

\[
\ln \left( \frac{p_2}{p_1} \right) = \frac{c_p}{R} \ln \left( \frac{T_2}{T_1} \right)
\]

\[
\frac{p_2}{p_1} = \left( \frac{T_2}{T_1} \right)^{c_p/R}
\]

(1.65)
From Eq. (1.52),

\[ c_p - c_v = R \]

\[ 1 - \frac{c_v}{c_p} = \frac{R}{c_p} \]

\[ \frac{\gamma - 1}{\gamma} = \frac{R}{c_p} \]

since \( c_p/c_v = \gamma \). Therefore,

\[ \frac{c_p}{R} = \frac{\gamma}{\gamma - 1} \]

Substituting this relation into Eq. (1.65), we obtain

\[ \frac{p_2}{p_1} = \left( \frac{T_2}{T_1} \right)^{\gamma/(\gamma - 1)} \]

(1.66)
Similarly, from Eq. (1.60),

\[ 0 = c_v \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{v_2}{v_1} \right) \]

\[ \ln \left( \frac{v_2}{v_1} \right) = -\frac{c_v}{R} \ln \left( \frac{T_2}{T_1} \right) \]

\[ \frac{v_2}{v_1} = \left( \frac{T_2}{T_1} \right)^{-c_v/R} \]  \hspace{1cm} (1.67)

But it can be shown that

\[ \frac{c_v}{R} = \frac{1}{\gamma - 1} \]
Substituting the above relation into Eq. (1.67), we get

\[
\frac{v_2}{v_1} = \left( \frac{T_2}{T_1} \right)^{-1/(\gamma-1)}
\]  \hspace{1cm} (1.68)

Since \( \rho_2/\rho_1 = v_1/v_2 \), Eq. (1.68) becomes

\[
\frac{\rho_2}{\rho_1} = \left( \frac{T_2}{T_1} \right)^{1/(\gamma-1)}
\]  \hspace{1cm} (1.69)
Substituting $s_1 = s_2$ into Eq. (1.64), we obtain

$$\frac{p_2}{p_1} = \left( \frac{\rho_2}{\rho_1} \right)^\gamma$$  \hspace{1cm} (1.70)

This relation is also called Poisson’s equation. Summarizing Eqs. (1.66), (1.69) and (1.70), we can write

$$\frac{p_2}{p_1} = \left( \frac{\rho_2}{\rho_1} \right)^\gamma = \left( \frac{T_2}{T_1} \right)^\gamma/(\gamma - 1)$$ \hspace{1cm} (1.71)

Equation (1.71) is an important equation and is used very frequently in the analysis of compressible flows.
Using the above discussed isentropic relations, several useful equations of total (stagnation) conditions can be obtained as follows. From Eqs. (1.15) and (1.62a),

$$\frac{T_0}{T} = 1 + \frac{V^2}{2c_p T} = 1 + \frac{V^2}{2\gamma R T/ (\gamma - 1)} = 1 + \frac{V^2}{2a^2/ (\gamma - 1)}$$

where $T$ is the static temperature, $T_0$ is the stagnation temperature and $V$ is the flow velocity. Hence,

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

(1.72)
Equation (1.72) gives the ratio of total to static temperature at a point in an isentropic flow field as a function of the flow Mach number $M$ at that point. Combining Eqs. (1.71) and (1.72), we get

$$\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}$$  \hspace{1cm} (1.73)

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/(\gamma-1)}$$  \hspace{1cm} (1.74)

Equations (1.73) and (1.74) give the ratio of total to static pressure and total and static density, respectively, at a point in an isentropic flow field as a function of the flow Mach number $M$ at that point.
Equations (1.72)-(1.74) form a set of most important equations for total properties, which are often used in gas dynamic studies. Their values as a function of \( M \) for \( \gamma = 1.4 \) (air at standard conditions) are tabulated in Table 1 of appendix.

At this stage we may ask how Eq. (1.71), which is derived on the basis of the concept of isentropic change of state (which is so restrictive – adiabatic as well as reversible – that it may find only limited applications) is so important and why it is frequently used.
In compressible flow processes such as flow through a rocket engine, flow over an airfoil, etc., large regions of the flow fields are isentropic. In the region adjacent to the rocket nozzle walls and the airfoil surface, a boundary layer is formed wherein the dissipative mechanisms of viscosity, thermal conduction and diffusion are strong. Hence, the entropy increases within these boundary layers. On the other hand, for fluid elements outside the boundary layer, the dissipative effects are negligible.
Further, no heat is being added to or removed from the fluid element at these points; hence the flow is adiabatic. Therefore, the fluid elements outside the boundary layer experience reversible adiabatic process, hence the flow is isentropic. Moreover, the boundary layers are usually thin; hence large regime of flow fields are isentropic. Therefore, a study of isentropic flow is directly applicable to many types of practical flow problems. Equation (1.71) is a powerful relation connecting pressure, density and temperature and is valid for calorically perfect gases.
Expressing all the quantities as stagnation quantities, Eq. (1.61c) can be written as

\[ s_{02} - s_{01} = c_v \ln \left( \frac{p_{02}}{p_{01}} \right) - c_p \ln \left( \frac{\rho_{02}}{\rho_{01}} \right) \]  

(1.75)

Also, by Eq. (1.52),

\[ R = c_p - c_v \]

and by the state equation

\[ \frac{\rho_{01}}{\rho_{02}} = \frac{\rho_{01}}{\rho_{02}} \frac{T_{01}}{T_{02}} \]
Substitution of the above relations into Eq. (1.75) yields

\[ s_{02} - s_{01} = R \ln \left( \frac{\rho_{01}}{\rho_{02}} \right) + c_p \ln \left( \frac{T_{02}}{T_{01}} \right) \]

For an adiabatic process of a perfect gas,

\[ T_{01} = T_{02} \]

Therefore,

\[ s_{02} - s_{01} = R \ln \left( \frac{\rho_{01}}{\rho_{02}} \right) \]

(1.76)
From Eq. (1.76) it is obvious that the entropy changes only when there are losses in pressure. It does not change with velocity and hence there is nothing like static and stagnation entropy. Also, by Eq. (1.63), the stagnation temperature does not change even when there are pressure losses. There is always an increase in entropy associated with pressure loss. In other words, when there are losses, there will be an increase in entropy, leading to a drop in stagnation pressure. These losses can be due to friction, separation, shocks, etc.
Example 1.1

Argon is compressed adiabatically in a steady-flow compressor from 101 kPa and 25°C to 505 kPa. If the compression work required is 475 kJ/kg, show that the compression process is irreversible. Assume argon to be an ideal gas.

Solution

As we know, the work required for a process is minimum when the process is isentropic, that is, when the process is adiabatic and reversible. Also, any process requiring more work than that required for an isentropic process is irreversible.
For an isentropic process, work transfer can be expressed as

\[
    w_{12} = \frac{\gamma}{\gamma - 1} R T_1 \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right]
\]

Given that,

\[
    T_1 = 25^\circ C = 298.15 \text{ K}, \quad p_1 = 101 \text{ kPa}, \quad p_2 = 505 \text{ kPa}
\]

For argon, \( \gamma = 1.67 \) and \( R = \frac{8314}{39.944} = 208.14 \text{ J/(kg K)} \), since the molecular weight of argon is 39.944.
Substituting these values into the work transfer equation, we get

\[
W_{12} = \frac{1.67 \times 208.14}{0.67} \times 298.15 \left[ 1 - \left( \frac{505}{101} \right)^{0.67} \right]
\]

\[
= -140.34 \text{ kJ/kg}
\]

The actual work required, 475 kJ/kg, is more than the isentropic work transfer. Hence, the process is irreversible.
Example 1.2

The Mach number of an aircraft is the same at all altitudes. If its speed is 90 kmph slower at 7000 m altitude than at sea level, what is its Mach number?

Solution

From standard atmospheric table, at 7000 m altitude, we get the local temperature $T_h$ as

$$T_h = 242.65 \text{ K}$$

Therefore, the speed of sound at 7000 m altitude is

$$a_h = \sqrt{\gamma R T_h} = \sqrt{1.4 \times 287 \times 242.65} = 312.24 \text{ m/s}$$

At sea level,

$$T_0 = 15^\circ \text{C} = 288.15 \text{ K}$$
The speed of sound at sea level is

\[ a_0 = \sqrt{\gamma R T_0} = \sqrt{1.4 \times 287 \times 288.15} = 340.26 \text{ m/s} \]

The Mach number is the same at these two altitudes. Thus,

\[ \frac{V_0}{a_0} = \frac{V_h}{a_h} = \left( \frac{V_0 - \frac{90}{3.6}}{a_h} \right) \]
\[ V_0 \frac{a_h}{a_0} = V_0 - \frac{90}{3.6} \]

\[ V_0 \left( \frac{a_h}{a_0} - 1 \right) = -\frac{90}{3.6} \]

\[ V_0 \left( \frac{312.24}{340.26} - 1 \right) = -25 \]

\[ V_0 = 303.59 \text{ m/s} \]

\[ M = \frac{V_0}{a_0} = \frac{303.59}{340.26} \]

\[ = 0.892 \]
Example 1.3

Air enters a compressor at a stagnation state of 100 kPa and 27°C. If it has to be compressed to a stagnation pressure of 900 kPa, calculate the power input to the compressor when the mass flow rate is 0.02 kg/s. Assume the compression process to be isentropic.

Solution Given that,

\[ p_{01} = 100 \text{ kPa}, \quad T_{01} = 27^\circ \text{C} = 300 \text{ K}, \quad p_{02} = 900 \text{ kPa} \]
The entropy change can be expressed as

\[ s_2 - s_1 = c_p \ln \left( \frac{T_{02}}{T_{01}} \right) - R \ln \left( \frac{p_{02}}{p_{01}} \right) \]

For an isentropic compression, \( s_2 - s_1 = 0 \). Therefore,

\[ c_p \ln \left( \frac{T_{02}}{T_{01}} \right) = R \ln \left( \frac{p_{02}}{p_{01}} \right) \]

\[ \ln \left( \frac{T_{02}}{T_{01}} \right) = \frac{R}{c_p} \ln \left( \frac{p_{02}}{p_{01}} \right) \]

\[ = \frac{287}{1004.5} \ln \left( \frac{900}{100} \right) = 0.628 \]

\[ T_{02} = \left( e^{0.628} \right) T_{01} = 562.16 \text{ K} \]
The power required is

\[
\text{Power} = \dot{m} \Delta h = \dot{m} c_p (T_{02} - T_{01})
\]

\[
= 0.02 \times 1004.5 \times (562.16 - 300)
\]

\[
= 5.27 \text{ kW}
\]
Example 1.4

Show that for air the difference between stagnation and static temperature in the kelvin scale is approximately \(5 \times (\text{speed in hundreds of meters per second})^2\).

Solution

By energy equation, we have

\[ h + \frac{V^2}{2} = h_0 \]

where \(h, h_0\) are the static and stagnation enthalpies and \(V\) is the flow speed. For a perfect gas, this equation can be written as

\[ c_p \, T + \frac{V^2}{2} = c_p \, T_0 \]
Therefore, the stagnation temperature-rise is

\[ T_0 - T = \frac{V^2}{2 c_p} = \frac{(\gamma - 1) V^2}{2 \gamma R} \]

since, \( c_p = \frac{\gamma}{\gamma - 1} R \).

For air, \( R = 287 \) J/(kg K) and \( \gamma = 1.4 \), under normal temperatures. Substituting for \( R \) and \( \gamma \) in the above equation, we get

\[ T_0 - T = \frac{0.4}{2 \times 1.4 \times 287} V^2 \]

\[ = 4.9776 \times 10^{-4} V^2 \text{ K} \]

\[ \approx 5 \left( V \times 10^{-2} \right)^2 \text{ K} \]

\[ \approx \text{5 (speed in hundreds of meters per second)}^2 \]
Limitations on Air as a Perfect Gas

1. When the temperature is less than 500 K, air can be treated as a perfect gas and the ratio of specific heats, $\gamma$, takes a constant value of 1.4.

2. When the temperature lies between 500 K and 2000 K, air is only thermally perfect (but calorically imperfect) and the state equation $p = \rho R T$ is valid, but $c_p$ and $c_v$ become functions of temperature, $c_p = c_p(T)$ and $c_v = c_v(T)$. Even though $c_p$ and $c_v$ are functions of temperature, their ratio $\gamma$ continues to be independent of temperature. That is, $c_p$ and $c_v$ vary with temperature in such a manner that their ratio continues to be the same constant as in temperatures below 500 K.

3. For temperatures more than 2000 K, air becomes both thermally and calorically imperfect. That is, $c_p$, $c_v$ as well as $\gamma$ become functions of temperature.
In supersonic flight with Mach number, say 2.0 at sea level, the temperature reached is already about 245°C (more than 500 K). But, for $500 \, \text{K} \leq T \leq 700 \, \text{K}$, we can still use perfect gas equations and the error involved in doing so will be negligible, i.e., for Mach number less than 2.68, perfect gas equations can be used with slight error. For temperatures more than 700 K, we must go for thermally perfect gas equations.
At this stage, we may have some doubt about the possible values of the isentropic index $\gamma$, when the flow medium is at a temperature which is quite high and the medium cannot be assumed as perfect. This doubt can be cleared if we consider the flow medium as an ideal gas, which satisfies perfect gas equations, has $\gamma = \text{constant}$, independent of temperature. For a monatomic gas (such as He, Ar, Ne, etc.), the simplest possible molecular structure gives $\gamma = 5/3$. This prediction is well confirmed by experiment.
At the other extreme of molecular complexity, very complicated molecules have large number of degrees of freedom and $\gamma$ may approach unity, which represents the minimum possible value, since $c_p \geq c_v$ by virtue of a general thermodynamic argument (Refer E. Rathakrishnan, Fundamentals of Engineering Thermodynamics, 2nd ed. 2005). Then $\gamma$ necessarily has a range of values

$$\frac{5}{3} \geq \gamma \geq 1$$

Experimental results show that most diatomic gases, nitrogen and oxygen in particular, have $\gamma = 7/5$ at room temperature, gradually tending to $\gamma = 9/7$ at a few thousand kelvin.
Wave Propagation

We studied that in incompressible flows the fluid particles could able to sense the presence of a body before actually reaching it. This fact suggests that a signaling mechanism exists, whereby the fluid particles can be informed, in advance, about the presence of a body ahead of it. The velocity of propagation of this signal must be apparently greater than the fluid velocity, since the flow is able to adjust to the presence of a body before reaching it. On the other hand, if the fluid particles were to move faster than the signal waves as in the case of supersonic flows, the fluid would not be able to sense the body before actually reaching it and abrupt changes in velocity and other properties would take place.
An understanding of the mechanism by which the signal waves are propagating through fluid medium along with an expression for the velocity of propagation of the waves will be extremely useful in deriving significant conclusions concerning the fundamental differences between incompressible and compressible flows.

When a fluid medium is allowed to vary its density, the consequence is that the fluid elements could able to occupy varying volumes in space. This possibility means that a set of fluid elements can spread into a larger region of space without requiring a simultaneous shift to be made to all fluid elements in the flow field, as would be required in the case of incompressible flow, in order to keep the density constant.
From studies on physics, we know that a small shift of fluid elements in compressible media will induce in due course similar small movements in adjacent elements and in this way a disturbance, referred to as an acoustic wave, propagates at a relatively high speed through the medium. Furthermore, in incompressible flows these waves propagate with infinitely large velocity; in other words, adjustments take place instantaneously throughout the flow and so in the conventional sense, there are no acoustic or elastic waves to be considered. With the introduction of compressibility, we thus permit the possibility of elastic waves having a finite velocity and the magnitude of this wave velocity is of great importance in compressible flow theory.
**Velocity of Sound**

Sound wave is a weak compression wave across which only infinitesimal changes in flow properties occur, i.e. across these waves there will be only infinitesimal pressure variations. In the ensuing chapters, we shall study waves where comparatively large pressure variation occurs over a very narrow front. Such waves are called shock waves, the flow process across them is nonisentropic and move relative to the fluid at speeds that exceed the speed of sound.
At this stage one may think of the sound waves as limiting cases of shock waves where the change in pressure across the wave becomes infinitesimal.

By Eq. (1.15), we have the speed of sound $a$ as $a = \sqrt{\gamma RT}$, where $T$ is the static temperature of the medium in absolute unit. The speed of sound in perfect gas may be computed by employing Eq. (1.15) and for the other fluids by employing Eq. (1.11).
Subsonic and Supersonic Flows

The velocity of sound is used as the limiting value for differentiating the subsonic flow from the supersonic flow. Flows with velocity more than the speed of sound are called *supersonic flows* and those with velocities less than the speed of sound are called *subsonic flows*. Flows with velocity close to the speed of sound are classified under a special category called *transonic flows*.
We saw the propagation of disturbance waves in flow fields with velocities from zero level to a level greater than the speed of sound, and that these disturbances will propagate along a “Mach cone”. For supersonic flow over two-dimensional objects, we will have a “Mach wedge” instead of Mach cone. The angle $\mu$ for such waves is measured in a counter-clockwise manner from an axis taken parallel to the direction of freestream as shown in Figure 1.10.
For an observer looking in the direction of flow towards the disturbance, the wave to his left is called *left-running wave* and the wave to his right is called *right-running wave* (Figure 1.10). Usually, the disturbance arises at a solid boundary where the fluid, having arrived supersonically without prior warning through pressure or sound signals, is made to undergo a change in direction, thus initiating a disturbance at the boundary which propagates along the Mach waves.
For historical interest, we should mention that Newton was the first to calculate the propagation speed of pressure waves. Based on the assumed isothermal process in a perfect gas, he found the speed of propagation of sound to be equal to the square root of the ratio of the pressure to the corresponding density involved in the process, i.e.

\[ a = \sqrt{\frac{p}{\rho}} \]

Because the science of thermodynamics was not known at Newton’s time, the 18% difference between his theory and experiment was never justified.
Nearly a century later, Marquis de Laplace rectified Newton’s calculation. The basic difference between Laplace’s theory and Newton’s theory is that the former considered an adiabatic process for propagation of pressure waves. This is fully justified since the compressions taking place in the propagation of pressure waves produce a very small temperature gradient and hence it is not possible for heat due to compression to be transferred to the surrounding region. The correction by Laplace from adiabatic process model multiplied Newton’s formula by $\sqrt{\gamma}$. The correct expression for speed of sound is $a = \sqrt{\gamma RT}$, which is the same as Eq. (1.15).
Similarity Parameters

In our discussions in the previous sections, we saw the Mach number $M$ as a primary parameter which dictates the flow pattern in the compressible regime of flow. In the chapters to follow, it will be seen that $M$ is also a parameter which appears almost in all equations of motion. Here the aim is to show that $M$ is an important parameter for experimental studies too.

Let us consider a prototype and a model which are geometrically similar. Now, it is our interest to find the condition which must be met in order to have the flow pattern around the model be similar to that around the prototype.
Examining the energy equation and taking into account the effect of viscosity and heat conductivity, it can be shown that the specific heats ratio, $\gamma$, must be the same for both the model and prototype. Thus, it can be concluded that the Mach number, $M$, must be the same for the model and prototype, if the flows are to be similar. When viscosity is present, an analysis applied to the inertia and viscous terms in the momentum equation leads to the criterion that the Reynolds number must be the same for ensuring similarity of flow pattern around the model and prototype.
Thus, by considering all the physical equations which govern the flow, namely, the continuity, momentum and energy equation and the equations of state, it is possible to arrive at four dimensionless parameters which must be the same for dynamic similarity of the model and prototype flow fields. They are
- Mach number, \( M = \frac{V}{a} \)

- Reynolds number, \( \text{Re}_L = \frac{\rho VL}{\mu} \)

- Ratio of specific heats, \( \gamma = \frac{c_p}{c_v} \)

- Prandtl number, \( \text{Pr} = \frac{\mu c_p}{k} \)

where \( V \) is the flow velocity, \( a \) is speed of sound, \( \rho \) is the flow density, \( L \) is a characteristic dimension of the body in the flow, \( \mu \) is the viscosity coefficient, \( c_p \) is specific heat at constant pressure, \( c_v \) is specific heat at constant volume and \( k \) is the thermal conductivity of the fluid.
In the potential flow region outside the boundary layer, where the viscous and heat conduction effects are relatively unimportant, it is usually necessary that only $M$ and $\gamma$ are to be the same between the model and prototype flow fields to establish similarity. Of the two, similarity in $M$ is more important than $\gamma$, since $\gamma$ has a relatively weak influence on the flow pattern.
Within the boundary layer, or in the interior of shock waves, viscous and heat conduction effects are significant. Hence, the Reynolds number and Prandtl number also must be included in the similarity conditions. But the Prandtl number is nearly the same for all gases and varies only slowly with temperature.
Continuum Hypothesis

From kinetic theory of gases, we know that matter is made up of a large number of molecules which are in constant motion and collision. But in the problems of engineering interest, we are concerned only with the gross behavior of the fluid, thought of as a continuous material and not in the motion of the individual molecules of the fluid. Even though the postulation of continuous fluid (continuum) is only a convenient assumption, it is a valid approach to many practical problems where only the macroscopic or phenomenological information is of interest.
The assumption of fluids as continua is valid only when the smallest volume of fluid of interest contains large number of molecules so that the statistical averages are meaningful. The advantage of continuum treatment is that, instead of dealing with the instantaneous states of large number of molecules, we have to deal with only certain properties describing the gross behavior of the substance. In compressible flows the relevant properties are the density, pressure, temperature, velocity, shear stress, coefficient of viscosity, internal energy, entropy and coefficient of thermal conductivity. The macroscopic approach with continuum hypothesis will fail whenever the mean free path of the molecules is of the same order of the smallest significant dimension of the problem under consideration.
The flow in which the mean free path of the molecules is of the same order or more than the characteristic dimension of the problem is termed as *rarefied flow*. To deal with highly rarefied gases, we should resort to microscopic approach of kinetic theory, since the continuum approach of classical fluid mechanics and thermodynamics is not valid there.

In order to determine whether the condition of continuum is valid, let us consider a steady flow and perform some approximate calculations of order-of-magnitude nature.
With kinetic theory, it can be shown using an order-of-magnitude approach that, the viscosity coefficient $\mu$ can be expressed as

$$\mu \approx \rho \overline{c} \lambda$$

and

$$\overline{c} \approx a$$

where $\overline{c}$ is the mean molecular velocity, $\lambda$ is the mean free path and $a$ is the speed of sound.
The Reynolds number of a flying vehicle can be expressed as

\[ \text{Re}_L = \frac{\rho VL}{\mu} = \frac{\rho c \lambda}{\mu} \frac{V}{a} \frac{a}{c} \frac{L}{\lambda} \approx \frac{V}{a} \frac{L}{\lambda} \]

where \( L \) is a characteristic length of the vehicle. But \( V/a = M \), therefore,

\[ \frac{L}{\lambda} \approx \frac{\text{Re}_L}{M} \quad (1.77) \]
Equation (1.77) shows that, the ratio of Reynolds number to Mach number is a dimensionless parameter indicative of whether a given problem is amenable to the continuum approach or not. From this ratio it is seen that, the continuum hypothesis is likely to fail when the Mach number is very large or the Reynolds number is extremely low. But we have to exercise caution while using Eq. (1.77), since $\text{Re}_L$ and $M$ depend on the nature of the problem considered. For example, when $\text{Re}_L$ is very low owing to low density, the continuum hypothesis is not valid, whereas when it is very low due to high viscosity, the continuum concept is perfectly valid and such a flow is termed *stratified flow*. 
However, the rules for determining the validity of the continuum concept in terms of $\text{Re}_L$ and $M$ can be illustrative by supposing that, in a given problem, the smallest *significant* body dimension is of the order of the boundary layer thickness, $\delta$. If $\text{Re}_L$ is large compared to unity and if the boundary layer flow is also laminar, then the boundary layer relations for a flat plate gives that,

$$\frac{\delta}{L} \approx \frac{1}{\sqrt{\text{Re}_L}}$$
Using Eq. (1.77), this can be expressed as

\[ \frac{\delta}{\lambda} \approx \frac{\sqrt{\text{Re}_L}}{M} \]

For this case Tsien (Tsien H.S., *Super-aerodynamics, Mechanics of Rarefied gases*, Jl. Aero. Sci., Vol. 13, No. 2 (1946), p. 653) suggests that the realm of continuum gas dynamics be limited to instances where the boundary layer thickness is at least 100 times the mean free path. That is

\[ \frac{\sqrt{\text{Re}_L}}{M} > 100 \]
Figure 1.11 shows the Reynolds number per unit length as a function of flight Mach number for various altitudes, based on the standard atmosphere.

![Figure 1.11](image)

**Figure 1.11**

Compressible Flow Regimes

The compressible flow regime can be subdivided into different zones based on the local flow velocity and the local speed of sound. To do this classification, we can make use of the energy equation, as follows.

Consider a streamtube in a steady compressible flow in which the flow does not exchange heat with the fluid in neighboring streamtubes, i.e. the flow process is adiabatic. The steady-flow energy equation for the adiabatic flow through such a streamtube is

\[ h + \frac{V^2}{2} = h_0 \]

where \( h \) and \( h_0 \) are the static and stagnation enthalpies and \( V \) is the flow velocity.
For a perfect gas, \( h = c_p T \), therefore,

\[
c_p T + \frac{V^2}{2} = c_p T_0
\]

But, \( c_p = \frac{\gamma}{\gamma - 1} R \) for perfect gases, thus,

\[
\frac{\gamma}{\gamma - 1} R T + \frac{V^2}{2} = \frac{\gamma}{\gamma - 1} R T_0
\]
This simplifies to

\[ V^2 + \frac{2}{\gamma - 1} a^2 = \frac{2}{\gamma - 1} a_0^2 = V_{\text{max}}^2 \]  

(1.78)

since \( a = \sqrt{\gamma RT} \), the local speed of sound and \( a_0 = \sqrt{\gamma R T_0} \), the speed of sound at the stagnation state (where \( V = 0 \)) and \( V_{\text{max}} \) is the maximum possible flow velocity in the fluid (where the absolute temperature is zero).
Equation (1.78) represents an ellipse and is called as *adiabatic steady-flow ellipse*. The adiabatic ellipse can be plotted as in Figure 1.12.

**Figure 1.12**
Steady-flow adiabatic ellipse.
Different realms of compressible flow having significantly different physical characteristics are represented schematically on the adiabatic ellipse. The zones highlighted on the adiabatic ellipse are the following.

- **Incompressible flow** is the flow in which the flow speed, $V$ is small compared to speed of sound, $a$ in the flow medium. The changes in $a$ are very small compared to changes in $V$.

- **Compressible subsonic flow** is that in which the flow speed and speed of sound of comparable magnitude, but $V < a$. Changes in flow Mach number $M$ is mainly due to changes is $V$. 

- **Transonic flow** is that in which the difference between the flow speed and speed of sound is small compared to either \( V \) or \( a \). Changes in \( V \) and \( a \) are of comparable magnitude.

- **Supersonic flow** is that with flow speed and speed of sound of comparable magnitude, but \( V > a \). Changes in Mach number \( M \) take place through substantial variation in both \( V \) and \( a \).

- **Hypersonic flow** is that where the flow speed is very large compared with speed of sound. Changes in flow velocity are very small and thus variations in Mach number \( M \) are almost exclusively due to changes in the speed of sound \( a \).
Summary

Compressible flow is defined as *variable density flow*; this is in contrast to incompressible flow, where the density is assumed to be invariant. Usually flows with Mach number less than 0.3 are treated as constant density (incompressible) flows.
Gas Dynamics is a science that primarily deals with the behavior of gas flows in which compressibility and temperature change become significant. Compressibility is a phenomenon by virtue of which the flow changes its density with change in speed. Compressibility may also be defined as the volume modulus of the pressure.
Mach number is the ratio of the local flow speed to the local speed of sound

\[ M = \frac{V}{a} \]

Flows with Mach number greater than unity are called supersonic flows.

Sound waves are infinitesimally small pressure disturbances. The speed with which sound waves propagate in a medium is called **speed of sound** \( a \). The speed of sound is given by

\[ a^2 = \frac{dp}{d\rho} \]
For a perfect gas, the speed of sound can be expressed as

\[ a = \sqrt{\gamma RT} \]

For a perfect gas, the state equation is

\[ p = \rho RT \]

For an isentropic flow of a perfect gas, the relation between the pressure, temperature and density between state 1 and any other state can be expressed as

\[ \frac{T}{T_1} = \left( \frac{\rho}{\rho_1} \right)^{\gamma-1} = \left( \frac{p}{p_1} \right)^{(\gamma-1)/\gamma} \]
For supersonic motion of an object, there is a well defined conical zone in the flow field with the object located at the nose of the cone. The region inside the cone is called the *zone of action* and the region outside the cone is termed the *zone of silence*. The lines at which the pressure difference is concentrated and which generate the cone are *Mach waves* or *Mach lines*. Therefore, Mach waves may be defined as weak pressure waves across which there is only insignificant change in flow properties.
The angle between the Mach line and the direction of motion of the body (flow direction) is called the *Mach angle* \( \mu \), given by

\[
\sin \mu = \frac{1}{M}
\]
Modern classification of the flow regimes is as follows:

1. Fluid flows with $0 < M < 0.8$ are called \textit{subsonic flow}.
2. The flow in the Mach number range $0.8 < M < 1.2$ is called \textit{transonic flow}.
3. The flow in the Mach number range $1.2 < M < 5$ is called \textit{supersonic flow}.
4. The flow with $M > 5$ is called \textit{hypersonic flow}.
Linearized theory can be used for studying subsonic and supersonic flows; the study of transonic and hypersonic flows is, however, complicated.

When a body is kept in transonic flow, it experiences subsonic flow over some portions of its surface and supersonic flow over other portions. There is also a possibility of shock formation on the body. It is this mixed nature of the flow field which makes the study of transonic flows complicated.
The temperature at stagnation point and over the surface of an object in the hypersonic flow becomes very high and, therefore, it requires special treatment. That is, we must consider the thermodynamic aspects of the flow along with gas dynamic aspects. That is why hypersonic flow theory is also called aerothermodynamic theory. Besides, because of high temperature, the specific heats become functions of temperature and hence the gas cannot be treated as perfect gas. If the temperature is quite high (of the order of more than 2000 K), even dissociation of gas can take place. The complexities due to high temperatures associated with hypersonic flow makes its study complicated.
Thermodynamics is the science that primarily deals with energy. The first law of thermodynamics is simply an expression of the conservation of energy principle.

The second law of thermodynamics asserts that actual processes occur in the direction of increasing entropy.

A system of fixed mass is called a closed system, or control mass and a system that involves mass transfer across its boundaries is called an open system, or control volume.
For an open system, by first law of thermodynamics, the energy equation can be expressed as

\[ H_1 + \frac{m}{2} V_1^2 + m g z_1 = H_2 + \frac{m}{2} V_2^2 + m g z_2 + W_s - Q_{12} \]

For a gas dynamic working process this equation becomes

\[ H_1 + \frac{m}{2} V_1^2 = H_2 + \frac{m}{2} V_2^2 + W_s - Q_{12} \]
A process during which there is no heat transfer is called an adiabatic process.

A reversible process is a process which can be reversed to its initial state without leaving any trace on the surroundings. That is, both the system and the surroundings are returned to their initial states at the end of the reverse process. Processes that are not reversible are called irreversible processes.
A process which is adiabatic and reversible is called an *isentropic process*. The factors that change the entropy of a system are heat transfer and irreversibility. Many engineering systems or devices such as nozzles, diffusers and turbines are essentially adiabatic in their operation and they perform best when the irreversibilities, such as the friction associated with the process, are minimized. Therefore, an isentropic process can serve as an appropriate model for actual processes.
A perfect gas is an *imaginary* substance that obeys the relation $p = \rho RT$.

The amount of energy needed to raise the temperature of a unit mass of a substance by one degree is called the specific heat at constant volume, $c_v$ for a constant volume process and the specific heat at constant pressure, $c_p$ for a constant-pressure process. They are defined as

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v, \quad c_p = \left(\frac{\partial h}{\partial T}\right)_p$$
The specific heats ratio $\gamma$ is defined as

$$\gamma = \frac{c_p}{c_v}$$

A gas is said to be perfect when it is \textit{thermally} as well as \textit{calorically} perfect. For a thermally perfect gas, $u$, $h$, $c_v$, and $c_p$ are functions of temperature alone.

For a calorically perfect gas, $c_p$ and $c_v$ are constants and are independent of temperature, i.e. a perfect gas has to be thermally as well as calorically perfect.
Entropy may be viewed as a measure of *disorder* or *randomness* in a system. The change in entropy can be expressed as

$$ds = \frac{\delta q}{T} + ds_{\text{irrev}}$$

The value of $ds$ can be used to determine whether a process is reversible, irreversible, or impossible:
For isentropic flow of a perfect gas,

\[
\frac{p_2}{p_1} = \left( \frac{\rho_2}{\rho_1} \right)^\gamma = \left( \frac{T_2}{T_1} \right)^{\gamma/(\gamma-1)}
\]

\[
\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2
\]

\[
\frac{p_0}{p} = \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{\gamma/(\gamma-1)}
\]

\[
\frac{\rho_0}{\rho} = \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{1/(\gamma-1)}
\]
For an adiabatic process of a perfect gas the entropy change in terms of stagnation pressure becomes

\[ s_{02} - s_{01} = R \ln \left( \frac{p_{01}}{p_{02}} \right) \]

Entropy changes only when there are losses in pressure. Entropy does not change with velocity and there is nothing like stagnation or static entropy.

The ratio of specific heats varies from 1 to 1.67. For monatomic gases like argon, \( \gamma = 1.67 \). Diatomic gases such as oxygen and nitrogen have \( \gamma = 1.4 \); for gases with extremely complex molecular structure \( \gamma \) is slightly more than unity.
When an object moves through a fluid medium, waves are emitted from each point on the object and travel outward at the speed of sound. In an incompressible fluid, the speed of sound is infinite; therefore, the entire flow field could able to feel the motion of the object instantaneously. In a compressible medium, the speed of sound has a finite value and hence, if a body travels at a velocity greater than that of sound, the fluid ahead of the body is not able to sense the motion of the object.
However, for subsonic motion of an object the fluid ahead of the object could sense the motion of the object. Therefore, for subsonic motion, the fluid elements adjust and move smoothly around the object, resulting in smooth, continuous streamline patterns. For the supersonic case, the fluid is forced to adjust rapidly to flow around a moving object, resulting in shock wave formation.
Exercise Problems

1.1 A gas has $p = 1$ atm, $T = 0^\circ$C and $\rho = 0.09$ kg/m$^3$. Determine, (a) the characteristic gas constant and molecular weight of the gas and (b) the specific volume of the gas at $70^\circ$C and 2.3 atm. Assume the gas to be thermally perfect.

[Ans. (a) 4.1217 kJ/(kg K), 2.017 kg, (b) 6.07 m$^3$/kg]

1.2 A rigid tank contains a certain amount of nitrogen gas at 120 kPa and $30^\circ$C, initially. Two kilograms of nitrogen gas is then added to the tank. If the final pressure and temperature of the gas in the tank are 240 kPa and $30^\circ$C, respectively, what is the volume of the tank?

[Ans. 1.5 m$^3$]
1.3 A barometer reads a local atmospheric pressure as 735 mm of mercury. How much is this pressure in pascals?

[Ans. 98.061 kPa]

1.4 A mercury manometer reads the pressure inside a vessel as 200 mm above atmosphere. The barometer reading is 750 mm Hg. Determine the absolute pressure in the vessel in kPa?

[Ans. 126.75 kPa]
1.5 A U-tube mercury manometer has its one limb connected to a pressure tank. The other end of the manometer is open to atmosphere. If the manometer measures 250 cm, determine the pressure in the tank. [Ans. 434.63 kPa]

1.6 A water manometer reads the pressure inside a vessel as 500 mm below atmospheric pressure. If the atmospheric pressure is 760 mm of Hg, determine the absolute pressure in the vessel in kPa. [Ans. 96.49 kPa]
1.7 A Bourdon pressure gauge records a pressure of 1.5 MPa gauge. If the atmospheric pressure is 740 mm Hg, determine the absolute value of the pressure read by the Bourdon gauge.

[Ans. 1.5987 MPa]

1.8 Determine the column height of a mercury manometer connected to a gas tank at a pressure of 50 kPa. Find the difference in the fluid level if water is used instead of mercury. Assume $\rho_{\text{Hg}} = 13600 \text{ kg/m}^3$ and $\rho_{\text{w}} = 1000 \text{ kg/m}^3$.

[Ans. $\triangle h = 4.7223 \text{ m}$]
1.9 A force of 8000 N is exerted uniformly on a piston of 100 mm diameter. Determine the pressure at the piston face.  
[Ans. 1018.6 kPa]

1.10 An oil of specific gravity 0.8 is contained in a vessel to a depth of 2.2 m. Determine the gauge pressure at the bottom of the vessel.  
[Ans. 17.27 kPa]
1.11 If the speed of sound at an altitude in earth's atmosphere is 299 m/s, determine the altitude.
[Ans. 10076.9 m]

1.12 The noise of an aircraft in level flight flown overhead is heard by an observer at sea level altitude, only when the aircraft has travelled 1 km from the location of the observer. Ignoring the temperature variation with altitude, estimate the flight altitude if the aircraft is flying at Mach 3.0.
[Ans. 333.45 m]
1.13 Two disturbance spheres created by an object moving at a supersonic speed are as shown in Figure 1.13. Assuming the medium to be standard sea level atmosphere, calculate the Mach number and velocity of the object and the Mach angle of the cone created by the disturbance.

Figure 1.13

[Ans. 3.286, 1117.24 m/s, 17.7°]
1.14 A jet plane flies at 1100 kmph. Determine the flight Mach number for (a) sea–level operation with an air temperature of 25°C and (b) high–altitude operation with an air temperature of −55°C. [Ans. (a) 0.883, (b) 1.03]

1.15 A stream of air drawn from a reservoir is flowing through an irreversible adiabatic process into a second reservoir in which the pressure is half of that in the first. Calculate the entropy difference between the two reservoirs, at the beginning of the process. [Ans. 198.933 N.m/(kg K)]

1.16 Air is expanded in an insulated cylinder equipped with a frictionless piston. The initial temperature of the air is 1400 K. The original volume is 1/10 of the final volume. Calculate (a) the change in temperature, (b) the work removed from the gas and (c) the pressure ratio. [Ans. (a) −842.65 K; (b) 6.04 × 10^5 N.m/kg; (c) 25.1189]
1.17 A tank of volume $15 \text{ m}^3$ contains air at $p_1 = 5.0 \times 10^5 \text{ Pa}$ and $T_1 = 500 \text{ K}$. The air is discharged into the atmosphere through a nozzle until the mass of air in the tank is reduced to one-half of its original value. Assuming the discharge be process adiabatic and frictionless, calculate the pressure and temperature of the air remaining in the tank. Assume the air as a perfect gas.

[Ans. $1.8946 \times 10^5 \text{ Pa}, 378.92 \text{ K}$]

1.18 Air is compressed isentropically in a centrifugal compressor from a pressure of $1.0 \times 10^5 \text{ Pa}$ to a pressure of $6.0 \times 10^5 \text{ Pa}$. The initial temperature is $290 \text{ K}$. Calculate (a) the change in temperature, (b) the change in internal energy, (c) the work imparted to the air, neglecting the velocity change.

[Ans. (a) $193.868 \text{ K}$; (b) $1.39 \times 10^5 \text{ N.m/kg}$; (c) $-1.39 \times 10^5 \text{ N.m/kg}$]
1.19 A perfect gas enclosed by an insulated (upright) cylinder and piston is in equilibrium at conditions \( p_1, v_1, T_1 \). A weight is placed on the piston. After a number of oscillations, the motion subsides and the gas reaches a new equilibrium at conditions \( p_2, v_2, T_2 \). Find the temperature ratio \( T_2/T_1 \) in terms of the pressure ratio \( \lambda = p_2/p_1 \). Show that, the change of entropy is given by

\[
s_2 - s_1 = R \ln \left( \frac{1 + (\gamma - 1)\lambda}{\gamma} \right)^{\gamma/(\gamma-1)} \frac{1}{\lambda}
\]

Also show that, if the initial disturbance is small i.e. \( \lambda = 1 + \varepsilon, \varepsilon \ll 1 \), then

\[
\frac{s_2 - s_1}{R} \approx \frac{\varepsilon^2}{2\gamma}
\]

\[
\left[ \text{Ans. } \frac{T_2}{T_1} = \frac{1 + (\gamma - 1)\lambda}{\gamma} \right]
\]
1.20 Unit weight of air is compressed adiabatically from an initial state with $p_1 = 10^5$ Pa and $T_1 = 303$ K to a final state of $p_2 = 2p_1$ and $T_2$. If the air enters and leaves the compressor with same velocity, calculate the shaft work necessary. Assume air as an ideal gas. 
[Ans. $w_s = -66.66$ kN-m/kg]

1.21 A fluid in a cylinder at a pressure of 6 atm and volume 0.3 m$^3$ is expanded at constant pressure to a volume of 2 m$^3$. Determine the work done by this expansion.
[Ans. 1.0335 MJ]
1.22 A gas at pressure 150 kPa and density 1.5 kg/m$^3$ is compressed to 690 kPa isentropically. Determine the final density. Assume the isentropic index to be 1.3. [Ans. 4.85 kg/m$^3$]

1.23 Air undergoes a change of state isentropically. The initial pressure and temperature are 101 kPa and 298 K, respectively. The final pressure is 7 times the initial pressure. Determine the final temperature. Assume air to be an ideal gas with specific heats ratio $\gamma = 1.4$. [Ans. 519.9 K]
1.24 Air at 30°C is compressed isentropically to occupy a volume which is 1/30 of its initial volume. Assuming air as an ideal gas, determine the final temperature.
[Ans. 908.55°C]

1.25 An ideal gas is cooled under constant pressure from 200°C to 50°C. Assuming constant specific heats with $c_p = 1000$ J/(kg K) and $\gamma = 1.4$, determine, (a) the molecular weight of the gas (b) the ratio of final to initial volume of the gas.
[Ans. (a) 29.1, (b) 0.683]
1.26 If the velocity of sound in an ideal gas of molecular weight 29 is measured to be 400 m/s at 100° C, determine the $c_p$ and $c_v$ of the gas at 100° C.

[Ans. $c_p = 860.1 \text{ J/(kg K)}$, $c_v = 573.4 \text{ J/(kg K)}$]

1.27 Air flows isentropically through a nozzle. If the speed and temperature at the nozzle exit are 390 m/s and 28° C, respectively, determine the Mach number and stagnation temperature there. What will be the Mach number just upstream of a station where the temperature is 92.5° C?

[Ans. 1.12, 103.29° C, 0.387]
1.28 Hydrogen gas at 7 atm and 300 K in a cylinder is expanded isentropically through a nozzle to a final pressure of 1 atm. Assuming hydrogen to be a perfect gas with $\gamma = 1.4$, determine the velocity and Mach number corresponding to the final pressure. Also, find the mass flow rate through the nozzle if the exit area is 10 cm$^2$. 
[Ans. 1923 m/s, 1.93, 0.275 kg/s]

1.29 Air in a cylinder changes state from 101 kPa and 310 K to a pressure of 1100 kPa according to the process

$$\rho v^{1.32} = \text{constant}$$

Determine the entropy change associated with this process. Assume air to be an ideal gas with $c_p = 1004 \text{ J/(kg K)}$ and $\gamma = 1.4$. 
[Ans. $-103.8 \text{ J/(kg K)}$]
1.30 Oxygen gas is heated from 25°C to 125°C. Determine the increase in its internal energy and enthalpy. Take $\gamma = 1.4$.
[Ans. 64950 J/(kg K), 90930 J/(kg K)]

1.31 Air enters a compressor at 100 kPa and 1.175 kg/m$^3$ and exits at 500 kPa and 5.875 kg/m$^3$. Determine the enthalpy difference between the outlet and inlet states.
[Ans. 0]
1.32 Show that, the entropy change for an ideal gas can be expressed as

$$ds = c_p \frac{dv}{v} + c_v \frac{dp}{p}$$

Using this result, show that for an ideal gas undergoing an isentropic change of state with constant specific heats, \(pv^\gamma = \text{constant}\).

1.33 Certain quantity of air at 0.7 MPa and 150°C occupies a volume of 0.014 m\(^3\). If the gas is expanded isothermally to a volume of 0.084 m\(^3\), calculate the change of entropy.

[Ans. 513.4 J/(kg K)]
1.34 0.3 kg of air at 350 kPa and 35°C receives heat energy at constant volume until its pressure becomes 700 kPa. It then receives heat energy at constant pressure until its volume becomes 0.2289 m³. Calculate the entropy change associated with each process. [Ans. 149.2 J/K, 333 J/K]

1.35 Air flowing through a frictionless diffuser. At a station in the diffuser the temperature, pressure and velocity are 0°C, 140 kPa and 900 m/s, respectively. At a downstream station the velocity decreases to 300 m/s. Assuming the flow to be adiabatic, calculate the increase in pressure and temperature of the flow between these stations. [Ans. 358.39 K, 2.491 MPa]
1.36 Nitrogen gas is compressed reversibly and isothermally from 100 kPa and 25°C to a final pressure of 300 kPa. Calculate the change in entropy.
[Ans. −0.3263 kJ/(kg K)]

1.37 Air undergoes a change of state isentropically from 300 K and 110 kPa to a final pressure of 550 kPa. Assuming ideal gas behavior, determine the change in enthalpy.
[Ans. 176.19 kJ/kg]
1.38 Air at low pressure inside a rigid tank is heated from 50°C to 125°C. What is the change in entropy associated with this heating process?
[Ans. 149.75 J/(kg K)]

1.39 Compute the temperature rise at the nose of an aircraft flying with Mach number 2 at an altitude of 10,000 m.
[Ans. 178.52]
1.40 A gas at an initial volume of 0.06 m³ and 15°C is expanded to a volume of 0.12 m³ while the pressure remains constant. Determine the final temperature of the gas.
[Ans. 303.15°C]

1.41 Certain quantity of a gas at 140 kPa and volume 0.15 m³ is compressed to 700 kPa at constant temperature. Determine the final volume of the gas.
[Ans. 0.03 m³]
1.42 Calculate the speed of sound in nitrogen gas at 100°C.  
[Ans. 393.85 m/s]

1.43 Find the value of the gas constant \( R \) for air in SI units.  
[Ans. 287.2 J/(kg K) or \( \text{m}^2/\text{(s}^2\text{K)} \)]
1.44 Methane gas enters a constant area duct at 4 atm, 200°C and 150 m/s. At a downstream station 2, \( T_2 = 400°C \) and \( V_2 = 250 \) m/s. Determine \( p_2 \), the mass flow rate and the entropy change between these stations.
[Ans. 3.414 atm, 247.9 kg/(m\(^2\) s), 835.86 J/(kg K)]

1.45 Gas flows through an insulated duct. At station 1 in the duct \( p_1 = 5 \) atm, \( T_1 = 350 \) K and \( V_1 = 55 \) m/s. At a downstream station 2, if \( p_2 = 1 \) atm and \( V_2 = 300 \) m/s, determine \( T_2 \) and the entropy difference \( s_2 - s_1 \) if the gas is (a) air and (b) argon.
[Ans. (a) 306.7 K, 329.25 J/(kg K), (b) 266.11 K, 192.71 J/(kg K)]
1.46 Helium gas at 200 kPa kept in a closed tank is heated from 50°C to 100°C. If the change in entropy is 1000 J/K, determine the new pressure and the mass of helium in the tank.
[Ans. 230.94 kPa, 2.21 kg]

1.47 An aircraft flying at an altitude of 10000 m experiences a temperature of 500°C at its nose. Determine the flight Mach number.
[Ans. 3.51]
1.48 Helium gas expands isentropically through a nozzle from state 1 to state 2. If the pressure, temperature at state 1 are 108 kPa and 92°C, respectively and the pressure and velocity at state 2 are 60 kPa and 903 m/s, respectively, determine $T_2$, $M_2$, $V_1$, $M_1$, $p_0$ and $T_0$.

[Ans. 289.05 K, 0.904, 139.02 m/s, 0.124, 109.38 kPa, 367 K]

1.49 Derive an expression for the entropy change in terms of pressures for a polytropic process of an ideal gas with constant specific heats.

\[
\text{Ans. } s_2 - s_1 = \frac{(n-\gamma) R}{n(\gamma-1)} \ln \left( \frac{p_2}{p_1} \right)
\]
1.50 The temperature and pressure of an unknown ideal gas in a cylinder of volume 1 m$^3$ are found to be 300 K and 6 atm, respectively. The mass of the gas in the cylinder is 975 g. Find the molecular weight of the gas.
[Ans. 4 kg/kmol]

1.51 Ten grams of oxygen gas undergoes a change of state at constant internal energy. Initial pressure and temperature are 2 atm and 330 K, respectively. The final volume occupied by the gas is twice the initial volume. Calculate the final temperature and pressure and the change in entropy due to the state change. Assume oxygen to be an ideal gas.
[Ans. 330 K, 1 atm, 1.8 J/K]
1.52 A cylinder of capacity 2 m³ contains oxygen gas at 600 kPa and 30°C. The gas is discharged from the cylinder till the pressure drops to 350 kPa, keeping the temperature at the same level. Assuming ideal gas behavior, determine the quantity of oxygen discharged.
[Ans. 6.35 kg]

1.53 Determine the specific volume of an ideal gas at 1 atm and 0°C.
[Ans. 22.4 m³/kgmol]
1.54 An ideal gas has a molecular weight of 39.948. If its specific heats ratio at a given temperature is 1.67, determine $c_p$ and $c_v$ of the gas at that temperature.

[Ans. $c_p = 518.4 \text{ J/(kg K)}$, $c_v = 310.4 \text{ J/(kg K)}$]

1.55 A rigid tank of volume 2 m$^3$ contains oxygen gas at 50 kPa and 50° C. Another rigid tank of same volume contains oxygen gas at 30 kPa and 25° C. The two tanks are then connected. If the final equilibrium temperature of the gas in both tanks is 25° C, what is the final pressure in both the tanks? Treat the process to be isentropic.

[Ans. 37.7 kPa]
1.56 If an airplane flies at 90 kmph at an altitude of 10000 m, find whether it flies at supersonic speed or subsonic speed?
[Ans. Flies at Mach 0.835]

1.57 Calculate the speed of sound in (a) air, (b) hydrogen, (c) methane, (d) oxygen, (e) carbon dioxide and (f) helium, at 100°C.
[Ans. (a) 387.21 m/s, (b) 1473.03 m/s, (c) 505.27 m/s, (d) 368.4 m/s, (e) 302.71 m/s, (f) 1134.25 m/s]
1.58 A missile in flight experiences a pressure and a temperature of 4 atm and 500 K, respectively, at its nose. Determine the density of air at its nose.  
[Ans. 2.824 kg/m³]

1.59 A pressure vessel of volume 15 m³ is filled with dry air at pressure 10 atm and temperature 300 K. Compute the mass of air in the tank.  
[Ans. 176.52 kg]
1.60 Calculate the enthalpy of the air stored in the pressure vessel in problem 59.
[Ans. 53.19 MJ]

1.61 For a calorically perfect gas show that,

\[ c_p - c_v = R \]
1.62 A gas initially at 140 kPa, 0.012 m$^3$ and 100$^\circ$C is compressed to a final state with a pressure and volume of 2.8 MPa and 0.0012 m$^3$, respectively, according to the law $pV^n = \text{constant}$. Determine (a) the value of $n$, (b) temperature of the final state, (c) the work transfer and (d) the change of internal energy of the gas if $R = 0.287$ kJ/(kg K) and $c_v = 0.717$ kJ/(kg K).
[Ans. (a) 1.3, (b) 472.54$^\circ$C, (c) $-5.6$ kJ, (d) 4.194 kJ]

1.63 One kilogram of gas at an initial pressure of 0.11 MPa and a temperature of 15$^\circ$C is compressed isothermally until the volume becomes 0.1 m$^3$. Determine, (a) the final pressure, (b) the final temperature and (c) the heat transfer associated with the process. Assume $c_p = 0.92$ kJ/(kg K) and $c_v = 0.66$ kJ/(kg K), for the gas.
[Ans. (a) 0.749 MPa, (b) 15$^\circ$C, (c) $-143.724$ kJ]
1.64 Certain quantity of gas has an initial volume and temperature of 1.2 liters and 150°C, respectively. If it is expanded to a volume of 3.6 liters according to the law $pV^{1.4} = \text{constant}$, determine the final temperature of the gas.
[Ans. $-0.486°C$]

1.65 Certain quantity of gas at an initial pressure of 0.1 MPa and volume 0.1 m$^3$ is compressed to a pressure of 1.4 MPa according to the law $pV^{1.26} = \text{constant}$. Determine the final volume of the gas.
[Ans. 12.31 liters]
1.66 Certain mass of air initially at 1.3 MPa, 0.014 m$^3$ and 135°C is expanded to a state where the pressure is 275 kPa and the volume is 0.056 m$^3$. Determine the mass and final temperature of the air.
[Ans. 0.1554 kg, 72.14°C]

1.67 Argon gas flows through a nozzle. At the nozzle inlet the pressure and density are 200 kPa and 1.2 kg/m$^3$, respectively. At the nozzle exit the pressure and temperature, respectively, are 100 kPa and 500°C. Calculate the initial temperature, the final density, the changes in entropy and enthalpy of the gas due to the flow process.
[Ans. 800.74 K, 0.621 kg/m$^3$, 126.08 J/(kg K), $-14.31$ kJ/kg]
1.68 An insulated tank is divided into two compartments of equal volume by a diaphragm. One of the compartments is kept empty at vacuum and the other contained a gas of unit mass. The diaphragm is bursted so that the gas fills both the compartments. Assuming the gas to be a perfect gas show that, the final temperature of the gas is the same as the initial temperature and that the entropy increases by an amount \( \frac{\gamma - 1}{\gamma} c_p \ln(2) \).

1.69 Show that, for an ideal gas the entropy change can be expressed as

\[
\Delta s = c_p \ln \left( \frac{v_2}{v_1} \right) + c_v \ln \left( \frac{p_2}{p_1} \right)
\]
1.70 Air flowing isentropically through a duct has a velocity and temperature of 100 m/s and 20°C, respectively, at a particular station. Find the velocity at a downstream station where the temperature is 5°C. [Ans. 200.34 m/s]

1.71 A pitot-static probe placed in an air stream measures 214 mm of mercury. If the stream pressure and temperature are 66 kPa and 9°C, calculate its velocity. [Ans. 247.5 m/s]