9.1. Using a reference such as [8], identify one aircraft with fixed tricycle landing gear, one aircraft with retractable tricycle landing gear, one aircraft with tail gear, one aircraft with quadricycle landing gear, and one aircraft with partially retractable landing gear (either main or nose gear is retracted). For each aircraft, provide name of the aircraft, type of the aircraft and its picture or three-view.

- **Fixed tricycle landing gear**
  
  ![Streak Shadow, Homebuilt (Courtesy of Jenny Coffey)](image)

  Streak Shadow, Homebuilt (Courtesy of Jenny Coffey)
De Havilland Canada DHC-6-300 Twin Otter, GA (Courtesy of A J Best)

➢ Retractable tricycle landing gear

Airbus A319-111, Transport (Courtesy of Anne Deus)
Lockheed MC-130H Hercules, Transport (Courtesy of Antony Osborne)

Space Shuttle, Spacecraft (Courtesy of Antony Osborne)
➢ Tail gear

Piper PA-17 Vagabond, GA (Courtesy of Jenny Coffey)

➢ Quadricycle landing gear

Sikorsky S-55, Boeing B-52H Stratofortress

Boeing B-52H Stratofortress, Bomber (Courtesy of Antony Osborne)
- Partially retractable landing gear (either main or nose gear is retracted)

Rutan Varieze, Blanik L-23, Hawker Sea Hurricane Mk1B-2

Hawker Sea Hurricane Mk1B-2, WWII British fighter (Courtesy of Jenny Coffey)
9.2. Using a reference such as [8], determine the followings:

a. The ratio between wheel track to fuselage length; and the ratio between wheel base to wing span for twin turboprop regional transport ATR 42 (Figure 3.8).

From page 177 of Jane’s 1996-1997 [8]:

- Fuselage length: \( L_f = 22.67 \) m
- Wheel base: \( B = 8.78 \) m
- Wheel track: \( l = 4.1 \) m
- Wing span: \( b = 24.57 \) m

- The ratio between wheel track to fuselage length = \( 4.1 / 22.67 = 0.181 \)
- The ratio between wheel base to wing span = \( 8.78 / 24.57 = 0.357 \)
- The ratio between wheel base to fuselage length = \( 8.78 / 22.67 = 0.387 \)
- The ratio between wheel track to wing span = \( 4.1 / 24.57 = 0.167 \)

b. The lateral angle between the main wheels off the cg (front-view) for fighter F-16 Falcon (Figure 3.12).

From side view (page 644 of Jane’s 1996-1997):

- The distance between cg and ground: 9 mm
- The distance between main wheel to cg: 6 mm
- The lateral angle between the main wheels off the cg = \( \tan^{-1}(6 / 9) = 33.7 \) deg

c. What percentage of aircraft weight is carried by the nose gear of jet transport Airbus A310? Assume that the aircraft cg is located at 20% of MAC.

From page 156 of Jane’s 1996-1997 (Top-view and side view):

- The distance between nose wheel to cg = 21 mm
- The distance between main wheel to cg = 6 mm
- The distance between main wheel to nose wheel (wheel base) = 27 mm

Using statics equations for equilibrium yields: \( \text{Load}_{\text{nose}} = 6 / (21 + 6) = 0.222 = 22.2 \% \)
9.3. Using a reference such as [8], describe the features of the landing gear of aircraft Harrier II AV-8B (Figure 4.19) in brief.


9.4. Using a reference such as [8], describe the features of the landing gear of aircraft Scaled Composites White Knight in brief.
9.5. A pilot of a prop-driven aircraft shown in figure 9.38 is going to take-off with 14 degrees of fuselage angle of attack.

Determine if the aircraft rear fuselage will hit the ground during take-off rotation. If yes, what must be the main gear height to achieve the clearance of 20 cm?

First, we need to determine the clearance angle:
\[
\alpha_c = \tan^{-1}\left(\frac{H_f}{AB}\right)
\]  
(9.3)

\[
H_f := 1.3 \text{m} \quad AB := 12 \text{m} \quad \alpha_c := \tan^{-1}\left(\frac{H_f}{AB}\right) = 0.108\text{rad} \quad \alpha_C = 6.183\text{deg}
\]

Since the clearance angle (6.183 deg) is less than fuselage rotation angle (14 deg), the fuselage will hit the ground during take-off rotation.

Next, a new value for the main gear height must be determined to prevent the occurrence of the fuselage hit.
\[
\alpha_c = \tan^{-1}\left(\frac{H_f}{AB}\right) \Rightarrow H_f = AB \times \tan(\alpha_c)
\]  
(9.3)

\[
\alpha_{TO} := 14\text{deg} \quad H := AB \times \tan(\alpha_{TO}) = 2.992\text{m}
\]

When the landing gear height is 2.99 m, the fuselage is about to have the contact with the ground.

\[
H_C := 20\text{cm} \quad H_L := H_C + H = 3.192\text{m}
\]

A landing gear height of 3.192 provides a 20 cm clearance during a 14 deg take-off rotation.
9.6. A fighter aircraft is taking off with 16 degrees of fuselage angle of attack. The height of the lowest point of the rear fuselage is 1.4 m and the distance between main gear and the fuselage tail point is 6.8 m. The landing gear is attached to the fuselage. Does the rear fuselage hit the ground during take-off rotation? If yes, determine the main gear height to achieve the clearance of 40 cm.

First, we need to determine the clearance angle:

\[
\alpha_c = \tan^{-1} \left( \frac{H_f}{AB} \right)
\]

(Equ 9.3)

\[
H_f = 1.4 \text{m} \quad AB = 6.8 \text{m} \quad \alpha_c = \tan^{-1} \left( \frac{H_f}{AB} \right) = 0.203 \text{rad} \quad \alpha_C = 11.63 \text{deg}
\]

Since the clearance angle (11.6 deg) is less than fuselage rotation angle (16 deg), the fuselage will hit the ground during take-off rotation.

Next, a new value for the main gear height must be determined to prevent the occurrence of the fuselage hit.

\[
\alpha_c = \tan^{-1} \left( \frac{H_f}{AB} \right) \Rightarrow H_f = AB \times \tan(\alpha_c)
\]

(Equ 9.3)

\[
\alpha_{TO} = 16 \text{deg} \quad H := AB \times \tan(\alpha_{TO}) = 1.95 \text{m}
\]

When the landing gear height is 1.95 m, the fuselage is about to have the contact with the ground.

\[
H_C := 40 \text{cm} \quad H_L := H_C + H = 2.35 \text{m}
\]

A landing gear height of 2.35 provides a 40 cm clearance during a 16 deg take-off rotation.
9.7. A utility aircraft with a mass of 7,000 kg has a tricycle landing gear configuration. The wheel base and wheel track is 11.6 m and 1.9 m respectively and the distance between main gear to aircraft cg is 0.65 m. Determine the static load on each gear. What percentage of the aircraft weight is carried by main gear?

\[ F_n = \frac{B_m}{B} W \]  \hspace{1cm} (9.6)

\[ F_m = \frac{B_n}{B} W \]  \hspace{1cm} (9.7)

\[
\begin{align*}
\text{m}_{TO} & := 7000 \text{kg} & B & := 11.6 \text{m} & B_m & := 0.65 \text{m} & g & = 9.807 \frac{\text{m}}{\text{s}^2} & W_{TO} & := \text{m}_{TO} g = 68646.6 \text{N} \\
F_n & := \frac{B_m}{B} W_{TO} = 3846.6 \text{N} & \hspace{1cm} \text{(Equ 9.6)} \\
B_n & := B - B_m = 10.95 \text{m} \\
F_m & := \frac{B_n}{B} W_{TO} = 64800 \text{N} & \hspace{1cm} \text{(Equ 9.7)} \\
\end{align*}
\]

\[
\begin{align*}
\frac{F_n}{W_{TO}} &= 5.6034\% \\
T1 & := 1.9 \text{m} & F_{m1} &= \frac{F_m}{2} = 32400 \text{N} & \hspace{1cm} \text{Each wheel of main gear}
\end{align*}
\]

Thus, 5.6% of the aircraft weight is carried by the nose gear.
9.8. A large transport aircraft with a mass of 70,000 kg has a tricycle landing gear configuration. The wheel base and wheel track is 25 m and 4.2 m respectively and the distance between main gear to aircraft cg is 1.2 m. Determine the static load on each gear. What percentage of the aircraft weight is carried by nose gear?

\[
m_{TO} := 70000 \text{ kg} \quad B := 25 \text{ m} \quad B_m := 1.2 \text{ m} \quad g = 9.807 \frac{\text{m}}{\text{s}^2} \quad W_{TO} := m_{TO} g = 686465.5 \text{ N}
\]

\[
F_m := \frac{B_m}{B} W_{TO} = 32950.3 \text{ N} \quad \text{(Equ 9.6)}
\]

\[
B_n := B - B_m = 23.8 \text{ m} \quad \text{(Figure 9.17)}
\]

\[
F_n := \frac{B_n}{B} W_{TO} = 653515.2 \text{ N} \quad \text{(Equ 9.7)}
\]

\[
\frac{F_n}{W_{TO}} = 4.8\% 
\]

\[
T1 := 4.2 \text{ m} \quad F_m := \frac{F_m}{2} = 326757.6 \text{ N} \quad \text{Each wheel of main gear}
\]

Thus, 4.8% of the aircraft weight is carried by the nose gear.
A twin turboprop aircraft with a take-off mass of 20,000 kg has a tricycle landing gear configuration. The aircraft cg is allowed to move between 0.8 m to 1.2 m from the main gear.

a. The nose gear is desired to carry a maximum of 10% of the aircraft weight in static equilibrium. Determine the wheel base.

b. The deceleration during landing brake is \(-5 \text{ m/s}^2\) and the acceleration during take-off is \(7 \text{ m/s}^2\). The distance between aircraft cg to the ground is 2.4 m. Determine the maximum dynamic load on each wheel.

### a. wheel base

\[
\begin{align*}
\text{m}_1 &= 20000\text{kg} & \text{Bm}_\text{min} &= 0.8\text{m} & \text{Bm}_\text{max} &= 1.2\text{m} & g &= 9.807\text{m/s}^2 & \frac{F_{nmax}}{W_{TO}} &= 10% \\
W_{TO} &= \text{m}_1 g = 196133\text{N} \\
F_{nmax} &= \frac{\text{Bm}_\text{max}}{B} W_{TO} \quad \Rightarrow \quad B = \text{Bm}_\text{max} \frac{W_{TO}}{F_{nmax}} \quad (\text{Equ 9.9}) \\
F_{nmax} &= 0.10 W_{TO} \quad \Rightarrow \quad B = \text{Bm}_\text{max} \frac{1}{0.10} = 12\text{m}
\end{align*}
\]

### b. Maximum load on the nose gear will be during landing braking:

\[
\begin{align*}
a_L &= -5 \frac{\text{m}}{\text{s}^2} & a_T &= 7 \frac{\text{m}}{\text{s}^2} & H_{cg} &= 2.4\text{m} \\
F_n &= W_{TO} \frac{\text{Bm}_\text{max}}{B} + W_{TO} \left| a_L \right| H_{cg} g \cdot B = 39613.\text{N} \quad (\text{Equ 9.13}) \\
\frac{F_n}{W_{TO}} &= 20.197% \\
\text{It is interesting to note that this load is 20.2 percent of the aircraft weight.}
\end{align*}
\]

- Maximum load on the main gear will be during take-off acceleration.

\[
\begin{align*}
\text{Bn}_\text{max} &= B - \text{Bm}_\text{min} = 11.2\text{m} \\
F_n &= W_{TO} \frac{\text{Bn}_\text{max}}{B} + W_{TO} \frac{a_T H_{cg}}{g \cdot B} = 211057.\text{N} \quad (\text{Equ 9.17}) \\
\frac{F_n}{W_{TO}} &= 107.609% \\
\end{align*}
\]
It is interesting to note that this load is 107.6 percent of the aircraft weight. This implies that the main gear during take-off has to carry a total load which is 7.6 percent greater than the aircraft weight.
9.10. A large transport aircraft with a take-off mass of 300,000 kg has a tricycle landing gear configuration. The aircraft cg is allowed to move between 1.2 m to 1.8 m from the main gear.

a. The nose gear is desired to carry a maximum of 18% of the aircraft weight in static equilibrium, determine wheel base.

b. The deceleration during landing brake is $-7 \text{ m/s}^2$ and the acceleration during take-off is $10 \text{ m/s}^2$. The distance between aircraft cg to the ground is $4 \text{ m}$. Determine the maximum load on each gear.

a. wheel base

\[ m_1 := 300000 \text{ kg} \quad B_{m_{\min}} := 1.2 \text{ m} \quad B_{m_{\max}} := 1.8 \text{ m} \quad g = 9.807 \text{ m/s}^2 \quad \frac{F_{n_{\max}}}{W_{TO}} = 18\% \]

\[ W_{TO} := m_1 g = 2941995 \text{ N} \]

\[ F_{n_{\max}} = \frac{B_{m_{\max}}}{B} W_{TO} \quad \Rightarrow \quad B = B_{m_{\max}} \frac{W_{TO}}{F_{n_{\max}}} \quad (\text{Equ 9.9}) \]

\[ F_{n_{\max}} = 0.18 W_{TO} \quad \Rightarrow \quad B := B_{m_{\max}} \frac{1}{0.18} = 10 \text{ m} \]

b. Maximum load on the nose gear will be during landing braking:

\[ a_L := -7 \frac{\text{m}}{\text{s}^2} \quad a_T := 10 \frac{\text{m}}{\text{s}^2} \quad H_{cg} := 4 \text{ m} \]

\[ F_n := W_{TO} \frac{B_{m_{\max}}}{B} + W_{TO} \frac{a_T H_{cg}}{g \cdot B} = 1369559.1 \text{ N} \quad (\text{Equ 9.13}) \]

\[ \frac{F_n}{W_{TO}} = 46.552\% \]

It is interesting to note that this load is 46.5 percent of the aircraft weight.

- Maximum load on the main gear will be during take-off acceleration.

\[ B_{n_{\max}} := B - B_{m_{\min}} = 8.8 \text{ m} \quad (\text{Figure 9.18}) \]

\[ F_n := W_{TO} \frac{B_{n_{\max}}}{B} + W_{TO} \frac{a_T H_{cg}}{g \cdot B} = 3788955.6 \text{ N} \quad (\text{Equ 9.17}) \]
$$\frac{F_n}{W_{TO}} = 128.789\%$$

It is interesting to note that this load is 128.8 percent of the aircraft weight. This implies that the main gear during take-off has to carry a total load which is 28.8 percent greater than the aircraft weight.
9.11. A jet transport aircraft with a mass of 40,000 kg and a wing area of 85 m² is turning on a runway. The ground speed is 15 knot and the turn radius is 25 m. The height of the aircraft center of gravity from the ground is 2.7 m.

a. Determine minimum overturn angle to prevent an overturn in this taxi maneuver.

\[ m_1 := 40000 \text{ kg} \quad S_w := 85 \text{ m}^2 \quad V_1 := 15 \text{ knot} \quad R_1 := 25 \text{ m} \quad H_{cg} := 2.7 \text{ m} \]

\[ F_C := m_1 \frac{V_1^2}{R_1} = 95275.1 \text{ N} \quad \text{(Equ 9.19)} \]

\[ \Phi_{ot} := \arctan \left( \frac{F_C}{m_1 g} \right) = 0.238 \quad \Phi_{ot} = 13.7 \text{ deg} \quad \text{(Equ 9.24)} \]

b. Determine the wheel track corresponding to this overturn angle.

\[ T_w := 2 \frac{F_C H_{cg}}{m_1 g} = 1.312 \text{ m} \quad \text{(Equ 9.22)} \]
9.12. A single engine prop-driven aircraft with a mass of 4,000 kg and a wing area of 14 m$^2$ is turning on a runway. The ground speed is 18 knot and the turn radius is 15 m. The height of the aircraft center of gravity from the ground is 0.8 m.

a. Determine minimum overturn angle to prevent an overturn in this taxi maneuver.

\[ m_1 := 4000 \text{ kg} \quad S_w := 14 \text{ m}^2 \quad V_g := 18 \text{ knot} \quad R_1 := 15 \text{ m} \quad H_{cg} := 0.8 \text{ m} \]

\[ F_C := m_1 \cdot \frac{V_g^2}{R_1} = 22866 \text{ N} \quad \text{(Equ 9.19)} \]

\[ \Phi_{ot} = \arctan \left( \frac{F_C}{m_1 g} \right) = 0.528 \quad \Phi_{ot} = 30.2 \text{ deg} \quad \text{(Equ 9.24)} \]

b. Determine the wheel track corresponding to this overturn angle.

\[ T_w := 2 \cdot \frac{F_C H_{cg}}{m_1 g} = 0.933 \text{ m} \quad \text{(Equ 9.22)} \]
9.13. Consider the aircraft in problem 11 is on a runway at 5,000 ft altitude. The aircraft side area is 120 m$^2$, and the height of the aircraft centroid of side area from the ground is 2.6 m. A cross wind with a speed of 35 knot is blowing. Assume the aircraft side drag coefficient is 1.1. Determine the minimum wheel track to prevent an overturn due to this cross wind. The lowest possible mass is 25,000 kg when there is no passenger on-board and zero fuel.

\[
\begin{align*}
\text{mass} &= 25000 \text{ kg} \\
\rho_5 &:= 0.002048 \frac{\text{slug}}{\text{ft}^3} \\
\rho_{5k} &= 1.055 \frac{\text{kg}}{\text{m}^3} \\
A_s &= 120 \text{ m}^2 \\
H_C &= 2.6 \text{ m} \\
V_W &= 35 \text{ knot} \\
C_{D_s} &= 1.1
\end{align*}
\]

\[
F_W := \frac{1}{2} \rho_{5k} V_W^2 A_s C_{D_s} = 22584.7 \text{ N} \quad \text{(Equ 9.25)}
\]

\[
Y_w := \frac{F_W H_C}{m g} = 0.24 \text{ m} \quad \text{(Equ 9.27)}
\]

\[
T_{w_m} := 2 Y_w = 0.479 \text{ m} \quad \text{(Equ 9.28)}
\]

Therefore, the minimum wheel track for this aircraft to avoid a rollover due to this cross-wind is 0.5 m.
9.14. Consider the aircraft in Problem 12 is on a runway at 3,000 ft altitude. The aircraft side area is 16 m\(^2\), and the height of the aircraft centroid of side area from the ground is 1.2 m. A cross wind with a speed of 30 knot is blowing. Assume the aircraft side drag coefficient is 0.7. Determine the minimum wheel track to prevent an overturn due to this cross wind. The lowest possible mass is 2,000 kg when there is no passenger on-board and zero fuel.

\[
\begin{align*}
  m_1 &:= 2000 \text{ kg} \\
  \rho_3 &:= 0.002175 \text{ slug/ft}^3 \\
  \rho_3 &:= 1.121 \text{ kg/m}^3 \\
  A_s &:= 16 \text{ m}^2 \\
  H_C &:= 1.2 \text{ m} \\
  V_W &:= 30 \text{ knot} \\
  C_d &:= 0.7
\end{align*}
\]

\[
F_W := \frac{1}{2} \rho_3 V_W^2 A_s C_d = 1495.2 \text{ N}
\]  \quad \text{(Equ 9.25)}

\[
Y_w := \frac{F_W H_C}{m_1 g} = 0.09 \text{ lm}
\]  \quad \text{(Equ 9.27)}

\[
T_w := 2 Y_w = 0.183 \text{ m}
\]  \quad \text{(Equ 9.28)}

Therefore, the minimum wheel track for this aircraft to avoid a rollover due to this cross-wind is 0.19 m.
9.15. An aircraft with a mass of 20,000 kg and wing span of 28 m has a tricycle landing gear configuration. The wheel base is 12 m, and the maximum distance between the aircraft cg and the nose gear is 11 m. The wing is made of aluminum with a modulus elasticity of 74 GPa. Assume that the wing can be modeled with a beam of I-section with a second moment of area of 0.0025 m\(^4\). If the maximum allowable wing deflection is 2 cm, determine the maximum allowable wheel track.

\[
m_{\text{max}} := 20000 \text{kg} \quad b := 28 \text{m} \quad B := 12 \text{m} \quad B_{\text{max}} := 11 \text{m} \quad E := 74 \text{GPa} \quad g = 9.807 \frac{\text{m}}{\text{s}^2}
\]

\[I := 0.0025 \text{m}^4 \quad y_{\text{max}} := 2 \text{cm}
\]

\[T_{w,\text{max}} := \left( \frac{48 E I B y_{\text{max}}}{m g B_{\text{max}}} \right)^{\frac{1}{3}} = 9.959 \text{m} \quad \text{(Equ 9.32)}
\]
9.16. An aircraft with a mass of 100,000 kg and wing span of 38 m has a tricycle landing gear configuration. The wheel base is 20 m, and the minimum distance between the aircraft cg and the main gear is 1.3 m. The wing is made of aluminum with a modulus elasticity of 70 MPa. Assume that the wing can be modeled with a beam of I-section with a second moment of area of 0.008 m$^4$. If the maximum allowable wing deflection is 3 cm, determine the maximum allowable wheel track.

\[ m_1 := 100000 \text{ kg} \quad b := 38 \pi \quad B := 20 \pi \quad B_{m, \text{min}} := 1.3 \pi \quad E := 70 \text{ GPa} \quad g = 9.807 \frac{\text{m}}{\text{s}^2} \]

\[ I := 0.008 \text{ m}^4 \quad y_{\text{max}} := 3 \text{ cm} \]

\[ B_{n, \text{max}} := B - B_{m, \text{min}} = 18.7 \text{ m} \quad \text{(Figure 9.18)} \]

\[ \frac{1}{T_{\text{w, max}}} = \left( \frac{4EIBy_{\text{max}}}{m1gB_{n, \text{max}}} \right)^{\frac{1}{3}} = 9.581 \text{ m} \quad \text{(Eqn 9.32)} \]
9.17. A business aircraft (Fig. 9.39) with a take-off mass of 20,000 kg and a wing area of 60 m$^2$ has two turbofan engines, each generating 25,000 N of thrust. The overall length of the aircraft is 25 m, it has a tricycle landing gear, and the runway is concrete. Assume that the forward cg is at 15% MAC, and wing-fuselage ac is at 22% MAC. The aircraft is equipped with a double slotted flap which is set to generate extra lift coefficient of 0.9 during take-off. The elevator deflection during take-off rotation is generating tail lift coefficient of -1.3.

Some dimensions of the aircraft are shown in Fig. 9.39, and other characteristics of the aircraft are as follows:

- $V_c = 350$ KTAS (at 25,000 ft), $V_s = 82$ KEAS, $C_{D_0} = 0.022$, $C_{D_{0,TO}} = 0.031$, $I_{yy,mg} = 30,000$ kg.m$^2$, $AR = 10$, $C_{m_o} = -0.05$, $e = 0.87$, $S_h = 13$ m$^2$

![Figure 9.39. Aircraft in problem 17](image)

The aircraft is required to rotate about the main gear with an angular acceleration of 6 deg/sec$^2$ during the take-off operation at sea level altitude; determine the distance between main wheel to the aircraft forward cg.

**Solution:**

From Fig. 9.39, we can extract the following dimensions:

- Vertical distance between cg and ground $h_{cg} := 2 \cdot m$
- Distance between Drag and ground $h_D := 3 \cdot m$
- Distance between Thrust line and ground $h_T := 2.4 \cdot m$
- Distance between tail ac to cg $l_h := 12 \cdot m$
According to the given locations of aircraft cg (i.e., 15% MAC), and wing-fuselage ac (i.e., 22% MAC):

\[ x_{Lg} = x_{mg} - (0.22 - 0.15)C \]

Given:

\[ m_{TO} := 20000 \text{kg} \quad S_w := 60 \text{m}^2 \quad T1 := 2.25000 \text{N} \quad L_f := 25 \text{m} \quad \Delta C_{L\text{flap}} := 0.5 \quad C_{Lh} := -1.3 \]

Concrete runway (Table 9.7): \( \mu := 0.03 \):

\[ x_{cg\text{bar}} := 0.1 \quad x_{ac\text{bar}} := 0.2 \]

\[ V_c := 350 \text{knot} \quad h_C := 25000 \text{ft} \quad \rho_{25} := 0.001066 \frac{\text{slug}}{\text{ft}^3} \quad \rho_25 := 0.549 \frac{\text{kg}}{\text{m}^3} \]

\[ V_s := 82 \text{knot} \quad \rho_o := 0.002378 \frac{\text{slug}}{\text{ft}^3} \quad C_{Do} := 0.02 \quad C_{Do\text{TO}} := 0.03 \quad I_{yy} := 30000 \text{kg} \cdot \text{m}^2 \]

\[ AR := 10 \quad C_{mo} := -0.0 \quad e_1 := 0.8 \quad S_h := 13 \text{m}^2 \quad \theta_{d\text{dot}} = 6 \frac{\text{deg}}{\text{sec}^2} \]

The air density at sea level is 1.225 kg/m³, and at 25,000 ft is 0.549 kg/m³. To obtain the wing mean aerodynamic chord:

\[ AR = \frac{b^2}{S} := \sqrt{AR \cdot S_w} = 24.495 \text{m} \quad (\text{Equ 5.19}) \]

\[ C_{\text{bar}} := \frac{S_w}{b} = 2.449 \text{m} \quad (\text{Equ 5.18}) \]

To find aircraft drag:

\[ K_1 := \frac{1}{\pi \cdot e_1 \cdot AR} = 0.037 \quad (\text{Equ 5.22}) \]

\[ W_{TO} := m_{TO}g = 196133 \text{N} \]

\[ C_{Lc} := \frac{2 \cdot W_{TO}}{\rho_{25} S_w V_{c}^2} = 0.367 \quad (\text{Equ 5.1}) \]
Other aerodynamic forces and moments:

\[ C_{LTO} := C_{Lc} + \Delta C_{L\text{flap}} = 1.267 \quad \text{(Equ 4.69c)} \]

\[ C_{DTO} := C_{D_0TO} + K_1 \cdot C_{LTO}^2 = 0.09 \quad \text{(Equ 4.68)} \]

\[ V_R := 1.1 \cdot V_s = 90.2 \text{ knot} \quad \text{(Equ 9.35)} \]

\[ D_{TO} := \frac{1}{2} \rho_o \left(V_R\right)^2 S_w C_{DTO} = 7104.41 \text{N} \quad \text{(Equ 9.44)} \]

Other aerodynamic forces and moments:

\[ L_{TO} := \frac{1}{2} \rho_o \left(V_R\right)^2 S_w C_{LTO} = 100310.31 \text{N} \quad \text{aircraft take off lift (Equ 9.41)} \]

\[ L_h := \frac{1}{2} \rho_o \cdot V_R^2 \cdot S_h \cdot C_{Lh} = -22298.98 \text{N} \quad \text{tail lift (Equ 9.43a)} \]

\[ M_o := \frac{1}{2} \rho_o \cdot V_R^2 \cdot S_w \cdot C_{mo} \cdot C_{bar} = -9696.061 \text{N-m} \quad \text{Aerodynamic Pitching moment (Equ 9.45)} \]

\[ L_{wf} := L_{TO} - L_h = 122609.30 \text{N} \quad \text{Wing-fuselage lift (Equ 9.42)} \]

Friction force:

\[ F_R := (W_{TO} - L_{TO}) \mu = 3353.79 \text{N} \quad \text{Friction (Equ 9.40)} \]

Aircraft linear acceleration at the time of take-off rotation:

\[ a := \frac{T_1 - D_{TO} - F_R}{m_{TO}} = 1.977 \text{ m/s}^2 \quad \text{(Equ 9.36)} \]

The distance between main gear and a reference line:

\[ x_{mg} = \frac{I_{yyac} \dot{\theta} - D(z_{D} - z_{mg}) + T(z_f - z_{mg}) - M_{acf} - ma(z_{cg} - z_{mg}) - Wx_{cg} + L_{wf} x_{acf} + L_h x_{acr}}{L_{wf} + L_h - W} \quad \text{(9.54)} \]
Contributing moments are:

Moment of the Weight: \[ M_W = -W_{TO} \left( x_{mg} - x_{cg} \right) \] (Equ 9.46)

assumption: \[ x_{cg} := 0 \]

Moment of aircraft drag \[ M_D := D_{TO} h_D = 21313.248 \text{N} \cdot \text{m} \] (Equ 9.47)

Moment of engine thrust \[ M_T := T_1 h_T = 120000 \text{J} \] (Equ 9.48)

Distance between wing-fuselage lift and main gear \[ x_{Lwf} = x_{mg} - (0.22 - 0.15) \cdot C_{bar} \]

\[ x_{ac cg} := x_{ac bar} - x_{cgb} = 0.07 \]

Moment of wing-fuselage lift \[ M_{Lwf} = L_{wf} \left( x_{mg} - 0.07 C_{bar} \right) \] (Equ 9.49)

Tail lift moment \[ M_{Lh} = -L_h \left( L_{h} - x_{mg} \right) \] (Equ 9.50)

Moment of reaction to acceleration: \[ M_{accel} := m_{TO} a \cdot h_{mg} = 79083.58 \text{N} \cdot \text{m} \] (Equ 9.51)

inertial moment: \[ M_{cg} := I_{yy} \cdot \ddot{\theta} = 3141.6 \text{N} \cdot \text{m} \] (Equ 9.38)

Please note that in this example, the x reference line is assumed to be the aircraft cg; thus \[ x_{cg} = 0 \]. Furthermore, for all moment arms, the absolute value is utilized. Now, all moments are substituted into the equation 9.54:

\[
\begin{align*}
    x_{mg} &= \frac{I_{yy} \ddot{\theta} - D \left( z_D - z_{mg} \right) + T \left( z_T - z_{mg} \right) - M_{ac} - m \left( z_{cg} - z_{mg} \right) - W \cdot x_{cg} + L_{wf} \cdot x_{ac} + L_{h} \cdot h_{a}}{L_{wf} + L_{h} - W} \\
    &\quad \cdot \left( x_{mg} - x_{cg} \right) \end{align*}
\] (9.54)

Rotation about main gear governing equation:

\[
M_W + M_D + M_T + M_{Lwf} + M_{Lh} + M_o + M_{accel} = I_{yy} \cdot \ddot{\theta}
\] (Equ 9.51)

\[
\begin{align*}
    x_{mg} &= \frac{M_{cg} - M_D - M_T - M_{accel} - W_{TO} \cdot x_{cg} + L_{wf} \cdot x_{ac cg} \cdot C_{bar} + \cdot L_{h} \cdot h_{h}}{L_{wf} + L_{h} - W_{TO}} \\
    &\quad \cdot \left( x_{mg} - x_{cg} \right) \end{align*}
\] (Equ 9.54)

\[ x_{mg} = 2.235 \text{m} \] distance between main gear and forward cg
This distance indicates (According to Figure 9.15) that the aircraft has the following tipback angle:

\[ \alpha_{tb} = \arctan \left( \frac{\frac{x_{mg}}{h_{cg}}}{x_{mg}} \right) = 0.84 \text{ rad} \]

\[ \alpha_{tb} = 48.17 \text{ deg} \]

(Equ 9.34)
9.18. A transport aircraft with a take-off mass of 15,000 kg and a wing area of 52 m$^2$ has two turbofan engines, each generating 24,000 N of thrust. The overall length of the aircraft is 17 m, it has a tricycle landing gear, and the runway is concrete. Assume that the forward cg is at 18% MAC, and wing-fuselage ac is at 26% MAC. The aircraft is equipped with a single slotted flap which is set to generate extra lift coefficient of 0.8 during take-off. The elevator deflection during take-off rotation is generating tail lift coefficient of -1.3. Other characteristics of the aircraft are as follows:

$V_c = 440$ KTAS (at 27,000 ft), $V_s = 85$ KEAS, $C_{D_0} = 0.023$, $C_{D_{0,TO}} = 0.032$, $I_{yy,mg} = 22,800$ kg.m$^2$, $C_{mo} = -0.06$, $AR = 12$, $e = 0.87$, $S_h = 12$ m$^2$, $h_{cg} = 2.2$ m, $h_D = 3.1$ m, $h_T = 1.7$ m, $l_h = 11$ m

The aircraft is required to rotate about the main gear with an angular acceleration of 9 deg/sec$^2$ during the take-off operation at 5000 ft altitude; determine the distance between main wheel to the aircraft forward cg.

Solution:

The following dimensions are given:

Vertical distance between cg and ground $h_{cg} = 2.2$ m

Distance between Drag and ground $h_D = 3.1$ m

Distance between Thrust line and ground $h_T = 1.7$ m

Distance between tail ac to cg $l_h = 11$ m

According to the given locations of aircraft cg (i.e., 18% MAC), and wing-fuselage ac (i.e., 26% MAC):

$x_{L_{cg}} = x_{mg} - (0.26 - 0.18)C$

$n_{TO} := 15000$ kg $S_w := 52$ m$^2$ $T1 := 2:24000$N $I_T := 17$ m $\Delta C_{L_{flap}} := 0.1$ $CL_h := -1.3$

Concrete runway (Table 9.7): $\mu := 0.03$

$x_{cgbar} := 0.18$ $x_{abar} := 0.2$

$V_c := 440$ knot $h_C := 27000$ft $\rho_{27} := 0.00993$ slug/ft$^3$ $\rho_{27} = 0.512$ kg/m$^3$

$V_s := 85$ knot $\rho_{5k} := 0.002048$ slug/ft$^3$ $C_{D_0} := 0.02$ $C_{D_{0,TO}} := 0.03$ $I_{yy} := 22800$ kg.m$^2$

$AR := 12$ $C_{mo} := -0.06$ $e_1 := 0.87$ $S_h := 12$ m$^2$ $\theta_{ddot} := 9 \frac{deg}{sec^2}$
The air density at 500 ft is 1.055 kg/m$^3$, and at 27,000 ft is 0.512 kg/m$^3$. To obtain the wing mean aerodynamic chord:

\[ \text{AR} = \frac{b^2}{S} \quad \text{and} \quad b := \sqrt{\text{AR} \cdot S_w} = 24.98 \text{m} \quad \text{(Equ 5.19)} \]

\[ \text{Cbar} := \frac{S_w}{b} = 2.082 \text{m} \quad \text{(Equ 5.18)} \]

To find aircraft drag:

\[ K_1 := \frac{1}{\pi \cdot e_l \cdot \text{AR}} = 0.03 \quad \text{(Equ 5.22)} \]

\[ W_{TO} := m_{TO} g = 147099.7 \text{N} \]

\[ C_{Lc} := \frac{2 \cdot W_{TO}}{\rho \cdot 27 \cdot S_w \cdot V_c^2} = 0.216 \quad \text{(Equ 5.1)} \]

\[ C_{LTO} := C_{Lc} + \Delta C_{Lflap} = 1.016 \quad \text{(Equ 4.69c)} \]

\[ C_{D_{TO}} := C_{D_{0_{TO}}} + K_1 \cdot C_{LTO}^2 = 0.063 \quad \text{(Equ 4.68)} \]

\[ V_R := 1.1 \cdot V_s = 93.5 \text{knot} \quad \text{(Equ 9.35)} \]

\[ D_{TO} := \frac{1}{2} \cdot \rho \cdot S_k \cdot (V_R)^2 \cdot S_w \cdot C_{D_{TO}} = 4029.2 \text{N} \quad \text{(Equ 9.44)} \]

Other aerodynamic forces and moments:

\[ L_{TO} := \frac{1}{2} \cdot \rho \cdot S_k \cdot (V_R)^2 \cdot S_w \cdot C_{L_{TO}} = 64494.57 \text{N} \quad \text{aircraft take off lift} \quad \text{(Equ 9.41)} \]

\[ L_h := \frac{1}{2} \cdot \rho \cdot S_k \cdot V_R^2 \cdot S_h \cdot C_{Lh} = -19048.08 \text{N} \quad \text{tail lift} \quad \text{(Equ 9.43a)} \]

\[ \text{Mac}_{wf} := \frac{1}{2} \cdot \rho \cdot S_k \cdot V_R^2 \cdot S_w \cdot C_{mo} \cdot \text{Cbar} = -7930.35 \text{N} \cdot \text{m} \quad \text{Aerodynamic Pitching moment} \quad \text{(Equ 9.45)} \]
Friction force:
\[ F_R := (W - L) \cdot \mu = 2891.18 \text{ N} \]  
(Equ 9.40)

Aircraft linear acceleration at the time of take-off rotation:
\[ a := \frac{T1 - D_{TO} - F_R}{m_{TO}} = 2.739 \text{ m/s}^2 \]  
(Equ 9.36)

The distance between main gear and a reference line:
\[
x_{mg} = \frac{I_{yy_m} D (z_D - z_{mg}) + T (z_T - z_{mg}) - M_{ac_{ff}} - ma (z_{cg} - z_{mg}) - W x_{cg} + L_{wf} x_{ac_{ff}} + L_h x_{ac_{fa}}}{L_{wf} + L_h - W}
\]  
(9.54)

Contributing moments are:
\[ x_{cg_{for}} := x_{cgb} \cdot C_{bar} = 0.375 \text{ m} \]  
\[ x_{ac} := x_{acb} \cdot C_{bar} = 0.54 \text{ m} \]

Moment of the Weight:
\[ M_W = -W_{TO} (x_{mg} - x_{cg}) \]  
(Equ 9.46)

assumption:
\[ x_{cg} := 0 \]

Moment of aircraft drag
\[ M_D := D_{TO} \cdot h_D = 12490.519 \text{ N m} \]  
(Equ 9.47)

Moment of engine thrust
\[ M_T := T1 \cdot b_T = 81600 \text{ N m} \]  
(Equ 9.48)

Distance between wing-fuselage lift and main gear
\[ x_{Lwf} = x_{mg} - (0.26 - 0.18) \cdot C_{bar} \]

\[ x_{ac_{cg}} := x_{acb} - x_{cgb} = 0.08 \]

Moment of wing-fuselage lift
\[ M_{Lwf} = L_{wf} (x_{mg} - 0.07 \cdot C_{bar}) \]  
(Equ 9.49)
Tail lift moment

\[ M_{th} = -I_h (l_h - x_{mg}) \]  
(Equ 9.50)

Moment of reaction to acceleration:

\[ M_{accel} := m_{TO} a \cdot h_{cg} = 90375.162 \text{N} \cdot \text{m} \]  
(Equ 9.51)

inertial moment:

\[ M_{cg} := I_{yy} \ddot{\theta} = 3581.4 \text{N} \cdot \text{m} \]  
(Equ 9.38)

Please note that in this example, the x reference line is assumed to be the aircraft cg; thus \( x_{cg} = 0 \). Furthermore, for all moment arms, the absolute value is utilized. Now, all moments are substituted into the equation 9.54:

\[
x_{mg} = \frac{I_{yy} \ddot{\theta} - D (z_d - z_{mg}) + T (z_f - z_{mg}) - M_{r_{ac}} - ma (z_{cg} - z_{mg}) - W x_{cg} + L_{wf} x_{ac} + L_h x_{ac}}{L_{wf} + L_h - W} 
\]  
(9.54)

Rotation about main gear governing equation:

\[
M_W + M_D + M_T + M_{L_{wf}} + M_{L_h} + M_B + M_{accel} = I_{yy} \ddot{\theta}''
\]  
(Equ 9.51)

\[
x_{mg} := \frac{M_{cg} - M_D + M_T - M_{L_{wf}} - M_{accel} - W_{TO} x_{cg} + L_{wf} x_{ac} + C_{bar} + L_h \cdot \dot{l}_h}{L_{wf} + L_h - W_{TO}}
\]  
(Equ 9.54)

\[ x_{mg} = 2.486 \text{m} \] distance between main gear and forward cg

This distance indicates (According to Figure 9.15) that the aircraft has the following tipback angle:

\[ \alpha_{tb} := \tan \left( \frac{x_{mg}}{h_{cg}} \right) = 0.846 \text{rad} \]

\[ \alpha_{tb} = 48.494 \text{deg} \]  
(Equ 9.34)
9.19. Design a landing gear for the following transport aircraft to carry 25 passengers. The aircraft has two turboprop engines, and is equipped with a single slotted flap which is deflected 20 degrees during the take-off operation on a concrete runway. Assume that the aircraft forward cg is at 14% MAC, aft cg is at 34% of MAC, and wing-fuselage aerodynamic center is located at 23% MAC. The distance between horizontal tail aerodynamic center to the wing-fuselage aerodynamic center is 18 m.

\[ m_{TO} = 40,000 \text{ kg}, \quad D_{fmax} = 2.8 \text{ m}, \quad V_{max} = 420 \text{ KTAS (at 30,000 ft)}, \quad V_s = 75 \text{ KEAS}, \quad D_{prop} = 3.4 \text{ m}, \]
\[ C_{Do\_clean} = 0.018, \quad C_{Do\_TO} = 0.032, \quad I_{yy} = 30,000 \text{ kg.m}^2, \quad P_{max} = 12,000 \text{ hp}, \quad C_{mo} = -0.02, \quad \eta_{P\_TO} = 0.5, \quad \alpha_{TO} = 15 \text{ deg} \]

Wing: airfoil: \( S = 100 \text{ m}^2, \) NACA 64-215, \( AR = 14, \quad e = 0.93, \quad \Delta C_{L\text{flap}} = 0.9, \quad \lambda = 1 \)

Horizontal tail: \( S_h = 25 \text{ m}^2, \) NACA 0009, \( AR_t = 6, \quad C_{Lh\_TO} = -0.9 \)

The following parameters must be determined: landing gear configuration; fixed or retractable; height; wheel track; wheel base; the distance between main wheel to aircraft cg; and applied load on each wheel.

This is an open-ended design problem, which has no single distinct solutions, and may have several acceptable designs. See the solution for Example 9.8 for an example of the design process.
9.20. Design a landing gear for the following early warning jet aircraft. The aircraft has two jet engines, and is equipped with a single slotted flap which is deflected 25 degrees during the takeoff operation on a concrete runway. Assume that the aircraft forward cg is at 15% MAC, aft cg is at 30% of MAC, and wing-fuselage aerodynamic center is located at 24% MAC. The distance between horizontal tail aerodynamic center to the wing-fuselage aerodynamic center is 26 m.

\[ m_{TO} = 180,000 \text{ kg}, \quad D_{\text{max}} = 3.5 \text{ m}, \quad V_{\text{max}} = 460 \text{ KTAS (at 35,000 ft)}, \quad V_s = 110 \text{ KEAS}, \quad C_{\text{Do, clean}} = 0.019, \quad C_{\text{Do, TO}} = 0.028, \quad I_{yy} = 3 \times 10^7 \text{ kg.m}^2, \quad T_{\text{max}} = 2 \times 270 \text{ kN}, \quad C_{\text{mo}} = -0.06, \quad \alpha_{\text{TO}} = 13 \text{ deg} \]

Wing: airfoil: \( S = 320 \text{ m}^2 \), NACA 652-415, AR = 10, \( e = 0.85 \), \( \Delta C_{L_{\text{flap}}} = 1.4 \), \( \lambda = 1 \)
Horizontal tail: \( S_h = 75 \text{ m}^2 \), NACA 0012, AR = 4, \( C_{L_{h, \text{TO}}} = -1.3 \)

The aircraft configuration and other geometry variables are illustrated in Figure 9.41. The following parameters must be determined: landing gear configuration; fixed or retractable; height; wheel track; wheel base; the distance between main wheel to aircraft cg; and applied load on each wheel.

![Figure 9.2. Aircraft in Problem 20](image_url)

This is an open-ended design problem, which has no single distinct solutions, and may have several acceptable designs. See the solution for Example 9.8 for an example of the design process.