Equations for Inventory Management

Chapter 1 Stocks and inventories

Empirical observation for the amount of stock held in a number of locations:

$$AS(N_2) = AS(N_1) \times \sqrt{\frac{N_2}{N_1}}$$

where:

N_2 = number of planned future facilities
N_1 = number of existing facilities
$AS(N_i)$ = aggregate stock with N_i facilities

Chapter 3 Economic order quantity

The variables used here, and throughout the book, are:

Q = order quantity
D = demand
UC = unit cost
RC = reorder cost
HC = holding cost
T = cycle length
VC = variable cost per unit time
TC = total cost per unit time
ROL = reorder level
LT = lead time

Qo = optimal order quantity
To = optimal cycle length
VCo = optimal variable cost per unit time
TCo = optimal total cost per unit time

- Economic order quantity:

$$Qo = \sqrt{\frac{2 \times RC \times D}{HC}}$$

- Optimal stock cycle length:

$$To = Qo/D = \sqrt{\frac{2 \times RC}{D \times HC}}$$

- Variable cost per unit time:

$$VC = \frac{RC \times D}{Q} + \frac{HC \times Q}{2}$$

- Optimal value of variable cost per unit time:

$$VCo = HC \times Qo = \frac{2 \times RC \times D}{Qo} = \sqrt{2 \times RC \times HC \times D}$$

- Total cost per unit time:

$$TC = UC \times D + VC$$

- Optimal cost per unit time:

$$TCo = UC \times D + VCo$$

- Change of variable cost moving away from the EOQ:

$$\frac{VC}{VCo} = \frac{1}{2} \times \left[\frac{Qo}{Q} + \frac{Q}{Qo}\right]$$

- Reorder level:

$$\text{Reorder level} = \text{lead time demand} - \text{stock on order}$$
$$ROL = LT \times D - n \times Qo$$

Chapter 4 Models for known demand

Model for finite replenishment rate, P

- Optimal order quantity:

$$Qo = \sqrt{\frac{2 \times RC \times D}{HC}} \times \sqrt{\frac{P}{P - D}}$$

- Optimal time cycle time:

$$To = \sqrt{\frac{2 \times RC}{HC \times D}} \times \sqrt{\frac{P}{P - D}}$$

- Optimal variable cost:

$$VCo = \sqrt{2 \times RC \times HC \times D} \times \sqrt{\frac{P - D}{P}}$$

- Optimal total cost:

$$TCo = UC \times D + VCo$$

Model for planned shortages and backorders

$$SC = \text{shortage cost per unit per unit time}$$

- Optimal order quantity:

$$Qo = \sqrt{\frac{2 \times RC \times D \times (HC + SC)}{HC \times SC}}$$

- Optimal amount to be backordered:

$$So = \sqrt{\frac{2 \times RC \times HC \times D}{SC \times (HC + SC)}}$$

- Time during which demand is met:

$$T_1 = (Qo - So)/D$$

- Time during which demand is backordered:

$$T_2 = So/D$$

- Cycle time;

$$T = T_1 + T_2$$

Model for shortages with lost orders

$$R = \text{revenue}$$

$$Z = \text{proportion of demand met}$$

- Cost of each unit of lost sales including loss of profits:

$$LC = DC + SP - UC$$

- Optimal revenue:

$$Ro = Z \times [D \times LC - \sqrt{2 \times RC \times HC \times D}]$$

Model for constraints on space

$$AC = \text{additional cost related to the storage area (or volume) used by each unit of the item.}$$

$$S_i = \text{amount of space occupied by one unit of item i.}$$

- The total holding cost per unit per unit time:

$$HC + AC \times S_i$$

- Optimal order quantities:

$$Q_i = \sqrt{\frac{2 \times RC_i \times D_i}{HC_i + AC \times S_i}}$$

Model for constraint on investment

$$UL = \text{upper limit on the total average investment}$$

- Optimal order quantities:

$$Q_i = Qo_i \times \frac{2 \times UL \times HC}{UC \times \sum_{i=1}^{N} VCo_i}$$

Model for discrete variable demand

- Test for the point where it is more expensive to order for N + 1 periods than to order for N periods:

$$N \times (N+1) \times D_{N+1} > \frac{2 \times RC}{HC}$$

- Confirming that it is more expensive to order for N + 2 periods than to order for N periods:

$$N \times (N+2) \times [D_{N+1} + D_{N+2}] > \frac{4 \times RC}{HC}$$

- Variable cost per period:

$$VC_N = \frac{RC}{N} + \frac{HC \times \sum_{i=1}^{N} D_i}{2}$$

Chapter 5 Models for uncertain demand

Model for the newsboy problem

$$SP = \text{selling price}$$

$$SV = \text{scrap value}$$

- Test for the optimal order size:

$$\text{Prob}(D \geq Qo) > \frac{UC - SV}{SP - SV} > \text{Prob}(D \geq Qo + 1)$$

- Expected profit with buying Q units:

$$EP(Q) = SP \times \left[\sum_{D=0}^{Q} D \times \text{Prob}(D) + Q \times \sum_{D=Q+1}^{\infty} \text{Prob}(D) \right] - Q \times UC$$

Model for discrete demand with shortages

$$A = \text{Actual stock level}$$

- Test for the optimal stock level:

$$\text{Prob}(D \leq Ao) \geq \frac{SC}{HC + SC} \geq \text{Prob}(D \leq Ao - 1)$$

Approach to intermittent demand

- Service level = 1 − Prob(shortage)
  = 1 − [Prob(there is a demand) × Prob(demand > A)]

Joint calculation of order quantity and reorder level with shortages

- Calculation for order quantity:

$$Q = \sqrt{\frac{2 \times D}{HC} \times \left[RC + SC \times \sum_{D=ROL}^{\infty} (D - ROL) \times \text{Prob}(D)\right]}$$

- Calculation for reorder level:

$$\frac{HC \times Q}{SC \times D} = \sum_{D=ROL}^{\infty} \text{Prob}(D)$$

Model for order quantity with shortages

- Order quantity:

$$Q = \sqrt{\frac{2 \times D}{HC} \times \left[RC + SC \times \sum_{D=ROL}^{\infty} (D - ROL) \times \text{Prob}(D)\right]}$$

Model for uncertain lead time demand

- Safety stock:

$$SS = Z \times \text{standard deviation of lead time} = Z \times \sigma \times \sqrt{LT}$$

- Reorder level:

$$ROL = \text{lead time demand} + \text{safety stock} = LT \times D + Z \times \sigma \times \sqrt{LT}$$

Model for service level with uncertain lead time

- Service level = Prob (LT × D < ROL) = Prob(LT < ROL/D)

Model for periodic review method

- Target stock level:

$$TSL = D \times (T + LT) + Z \times \sigma \times \sqrt{(T + LT)}$$

Chapter 6 Sources of information

Accounting information

- Cost of products sold = opening stock + net purchases − closing stock
- Value of stock = number of units in stock × unit value
- Average cost = $\dfrac{\text{Total cost of units}}{\text{Number of units bought}}$
- Closing stock = opening stock + purchases − sales
- Gross profit = sales revenue − cost of units sold

Chapter 7 Forecasting demand

Value of demand in a time series

$$\text{Actual demand} = \text{underlying pattern} + \text{random noise}$$

Linear relationship

$$\text{dependent variable} = a + b \times \text{independent variable}$$

$$y = a + bx$$

x = value of the independent variable

y = value of the dependent variable

a = intercept, where the line crosses the y axis

b = gradient of the line.

- Equations for linear regression:

$$b = \frac{n \times \sum (x \times y) - \sum x \times \sum y}{n \times \sum x^2 - \left(\sum x\right)^2}$$

$$a = \frac{\sum y}{n} - b \times \frac{\sum x}{n}$$

- coefficient of determination = (coefficient of correlation)^2

Multiple regression

$$y = a + b_1 \times \text{variable } 1 + b_2 \times \text{variable } 2 + b_3 \times \text{variable } 3 + b_4 \times \text{variable } 4 \ldots.$$

Exponential smoothing

- New Forecast = $\alpha \times$ latest demand + $(1 - \alpha) \times$ previous forecast
- α is the smoothing constant (usually between 0.1 and 0.2)
- Tracking signal = $\dfrac{\text{sum of forecast errors}}{\text{mean absolute deviation}}$
- Seasonal index = $\dfrac{\text{seasonal value}}{\text{deseasonalized value}}$
- Demand = (underlying value + trend) × seasonal index + noise

Chapter 8 Planning and stocks

Stock and planning

Stock at end Stock at Production Demand Backorders Backorders of this = end of last + during − met during − from earlier + met in later period period this period this period periods periods

Chapter 9 Material requirements planning

- Basic calculation

Gross requirements = number of units made × amount of material for each unit

Net requirements = gross requirements – current stock – stock on order

- Batching rule to find N

$$N \times (N + 1) \times D_{N+1} > \frac{2 \times RC}{HC}$$

Where:

N = the period number in a cycle
D_{N+1} = demand in period N + 1 of a cycle

Chapter 10 Just-in-time

- Number of *kanbans* to maintain smooth operations

$$\text{Number of } \textit{kanbans} = \frac{\text{demand in the cycle}}{\text{size of each container}}$$

$$K = \frac{D \times (TP + TD)}{C}$$

Where:

C = number of units held in each container
TP = time container spends in production part of a cycle (waiting, being filled and moving to the store of work in progress)
TD = time container spends in demand part of a cycle (waiting, being emptied and moving to the store of work in progress).
Totalcyclelength $= TP + TD$

- Number of *kanbans* with safety factor

$$SF = \text{safety factor (generally less than 0.1)}$$

$$K < \frac{D \times (TP + TD) \times (1 + SF)}{C}$$

- Maximum stock of work in progress

$$\text{Maximum stock level} = K \times C = D \times (TP + TD) \times (1 + SF)$$